RF Systems

Alessandro Gallo, INFN - LNF

Lecture II

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RF Systems
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RF Power Sources
RF Power Sources:
Solid State Amplifiers

Various technologies available:
- Silicon bipolar transistors
- silicon LDMOS
- GaAsFET,
- Static Induction Transistors (SITs)

The power required is obtained by operating numerous transistors in parallel.

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<table>
<thead>
<tr>
<th>RF Source</th>
<th>Frequency / Bandwidth</th>
<th>Max Power</th>
<th>Class of operation / Efficiency</th>
<th>Features</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid State</td>
<td>dc ÷ 10 GHz / Large (up to multi-octave)</td>
<td>0.5 kW/pallet 200 kW/plant</td>
<td>A, AB ( \eta \leq 40 % )</td>
<td>Simplicity, Modularity, No HV</td>
<td>Large dc currents, Combiner Losses</td>
</tr>
</tbody>
</table>
RF Power Sources:
Solid State Amplifiers

352 MHz, 190 kW solid state amplifier for the SOLEIL RF System

*Schematics of the whole power plant*
RF Power Sources: Tetrodes (Grid Tubes)

Tetrodes are long-time, well established grid tubes RF sources.
- Evolution of Triodes;
- Widely used in industry, TV and communications;
- Electrons in the tube are produced by thermo-ionic effect at the cathode;
- The intensity of the captured current at the Anode electrode is modulated by the Control grid potential;
- Screen grid increases RF isolation between electrodes.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Tetrodes</td>
<td>50 ÷ 1000 MHz / few %</td>
<td>200 kW/tube</td>
<td>A, AB, B, C η ≤ 70 %</td>
<td>Simplicity, Cheap</td>
<td>HV, Transit-time limited</td>
</tr>
</tbody>
</table>
RF Power Sources: Tetrodes (*Grid Tubes*)

**Tests of the 73 MHz, 50 kW Tetrode Amplifier for the DAFNE damping ring**

<table>
<thead>
<tr>
<th>RF performance</th>
<th>CW</th>
<th>Long pulses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>345 MHz</td>
<td>40 MHz</td>
</tr>
<tr>
<td>Output power</td>
<td>60 kW</td>
<td>80 kW</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>cw 2 sec</td>
<td>2 sec</td>
</tr>
<tr>
<td>Anode voltage</td>
<td>10.5 kV</td>
<td>12 kV</td>
</tr>
<tr>
<td>Anode current</td>
<td>7.7 A</td>
<td>7.7 A</td>
</tr>
<tr>
<td>Screen-grid voltage</td>
<td>630 V</td>
<td>800 V</td>
</tr>
<tr>
<td>Control-grid voltage</td>
<td>-270 V</td>
<td>-120 V</td>
</tr>
<tr>
<td>Duty factor</td>
<td>- 10 %</td>
<td>10 %</td>
</tr>
</tbody>
</table>

**Maximum ratings**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>400 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anode voltage</td>
<td>15 kV</td>
</tr>
<tr>
<td>Anode direct current</td>
<td>10 A</td>
</tr>
<tr>
<td>Peak cathode current</td>
<td>56 A</td>
</tr>
<tr>
<td>Screen grid voltage</td>
<td>800 V</td>
</tr>
<tr>
<td>Control grid voltage</td>
<td>- 300 V</td>
</tr>
<tr>
<td>Anode dissipation</td>
<td>50 kW</td>
</tr>
<tr>
<td>Screen-grid dissipation</td>
<td>400 W</td>
</tr>
<tr>
<td>Control-grid dissipation</td>
<td>150 W</td>
</tr>
</tbody>
</table>
**RF Power Sources:**

**Inductive Output Tubes (Grid Tubes)**

Inductive Output Tubes (IOTs) are grid tubes (also known as “klystrodes”) commercially available since ’80s. They combine some design aspects of tetrodes and klystrons:

- Anode grounded and separated from collector;
- Tube beam current intensity modulated by the RF on the grid. RF input circuit is a resonant line;
- Short accelerating gap to reduce transit-time, beam emerging from a hole in the anode;
- RF output extracted from a tuned cavity between anode and collector decelerating the bunched beam;
- High efficiency, widely used in TV and communications.

<table>
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<th>Max Power</th>
<th>Class of operation / Efficiency</th>
<th>Features</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive Output Tubes (IOTs)</td>
<td>100÷2000 MHz / few %</td>
<td>500 kW/tube</td>
<td>B, C η ≤ 80 %</td>
<td>Efficient, Reliable, Cheap</td>
<td>HV (@ high freq.)</td>
</tr>
</tbody>
</table>
RF Power Sources: Inductive Output Tubes (Grid Tubes)

New 150 kW CW transmitter for the ELETTRA Synchrotron Light Source based on 2×80 kW IOTs
The klystron has been invented in late ’30s by Hansen and Varian Bros.

- Based on the velocity modulation concept;
- After being accelerated to a non-relativistic energy in an electrostatic field, a thermo-ionic beam is velocity-modulated crossing the gap of an RF cavity (buncher) excited by the RF input signal.
- Velocity modulation turns into density modulation after the beam has travelled a drift space. RF power output is extracted from a tuned output cavity (catcher) excited by the bunched beam.
- Other passive cavities are generally placed between buncher and catcher to enhance the bunching process.
- Beam particle transverse motion is focused all over the fly by solenoidal magnetic fields.
- Operation as oscillator is possible by feeding back an RF signal from catcher to buncher.
RF Power Sources: Klystrons (*Velocity Modulation Tubes*)

The klystron is the most widely diffused RF source in particle accelerators. A large variety of tubes exists for different applications, spanning wide ranges of frequency and output power, as well as different duty cycles. Principal tube categories are:

- **High power CW tubes** (up to 2 MW), for synchrotron, storage rings and CW linacs;
- **Very high peak power** (up to 100 MW) tubes for low duty-cycle machines (1÷4 μs, 100 Hz rep rate), such as S-band, C-band (and proposed X-band) normal-conducting linacs;
- **High peak power** (up to 10 MW) tubes for high duty cycle machines (1 ms, 10 Hz rep rate), such as SC linacs for FEL radiation production or for future linear colliders. Multi-beam klystrons have been developed for this task.

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<th>Max Power</th>
<th>Efficiency</th>
<th>Features</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Klystrons</td>
<td>0.3÷ 30 GHz / ≈1 %</td>
<td>≈2 MW rms</td>
<td>η =40÷60 %</td>
<td>High power, High gain</td>
<td>HV, Efficiency</td>
</tr>
</tbody>
</table>
RF Power Sources: Examples of klystrons

E3712 Toshiba S-band Klystron
\( f = 2856 \text{ MHz} \)
\( P = 80 \text{ MW pk} \)
\( \eta = 44 \% \)
\( \text{Gain} = 53 \text{ dB} \)

TH 2132 S-band Klystron
\( f = 2998.5 \text{ MHz} \)
\( P = 45 \text{ MW pk} / 20 \text{ kW rms} \)
\( \eta = 43 \% \)
\( \text{Gain} = 54 \text{ dB typical} \)

NLC Klystron
\( f = 11.4 \text{ GHz} \)
\( V_0 = 490 \text{ kV} \)
\( I_0 = 260 \text{ A} \)
\( P = 75 \text{ MW peak} \)
\( \eta = 55 \% \)

TH 2089 Klystron for LEP
\( f = 352 \text{ MHz} \)
\( P = 1.3 \text{ MW CW} \)
\( \eta = 65 \% \)
\( \text{Gain} = 40 \text{ dB min.} \)

TH 2089 Klystron for TESLA
\( f = 1300 \text{ MHz} \)
\( V_0 = 115 \text{ kV} \)
\( I_0 = 133 \text{ A} \)
\( P = 9.8 \text{ MW peak} \)
\( \eta = 64 \% \)
Beams = 6

MBK Klystron
\( f = 352 \text{ MHz} \)
\( P = 1.3 \text{ MW CW} \)
\( \eta = 65 \% \)

TH 2132 S-band Klystron
\( f = 2998.5 \text{ MHz} \)
\( P = 45 \text{ MW pk} / 20 \text{ kW rms} \)
\( \eta = 43 \% \)
\( \text{Gain} = 54 \text{ dB typical} \)
RF Power Sources: Klystron Pulsed Modulators

Pulsed modulators provide HV pulses to operate the klystrons in pulsed regime. To reach the highest klystron peak power the required HV can get close to 0.5 MV, with pulse duration in the 1-5 μs range, and acceptable pulse shape. HV pulsed modulators are then a technical and technological issue, and might in the end limit the maximum available RF power from a power plant.
RF Power Sources (Others): Magnetrons

In a Magnetron the cathode and anode have a coaxial structure, and a longitudinal static magnetic field (perpendicular to the radial DC electric field) is applied. The cylindrical anode structure contains a number of equally spaced cavity resonators and electrons are constrained by the combined effect of a radial electrostatic field and an axial magnetic field. The output power is coupled out from one of the cavities connected to a load through a waveguide.

The cavity oscillations produce electric fields that spread outward into the interaction space and energy is transferred from the radial DC field to the RF field by electrons whose trajectory is bent by the magnetic field. Magnetrons have a wide range of output powers, from 1 kW to 1 MW. Typical DC-to-RF power-conversion efficiency ranges from 50 to 85%.
RF Power Sources (Others): Travelling Wave Tubes (TWTs)

Helix Travelling Wave Tubes are widely used μ-wave amplifiers mainly consisting in:

- an electron gun;
- a focusing magnetic structure controlling the beam transverse size along the tube;
- an helix waveguide excited by the input RF signal whose fields interact with the electron beam, leading to the amplification process. The pitch to circumference of the helix is such that the longitudinal phase velocity of the wave equals that of the electrons;
- a collector to collect the electrons.

The axial phase velocity is relatively constant over a wide range of frequencies, and this accounts for the large TWT bandwidths.

Amplification is due to a continuous bunching of the beam allowing the energy transfer from the beam to the helix. Typical gains are 40 to 60 dB while DC-to-RF conversion efficiency is in the range of 50 ÷75 %.
RF Power Sources: Pulse Compression (SLED)

The Stanford Linac Energy Doubling (SLED) is a system developed to compress RF pulses in order to increase the peak power (and the available accelerating gradients) for a given total pulse energy. This is obtained by capturing the pulsed power reflected by a high-Q cavity properly excited by the RF generator (typically a klystron). In fact the wave reflected by an overcoupled cavity ($\beta \approx 5$) peaks at the end of the RF pulse to about twice the incident wave level (with opposite polarity).

By playing with this, and properly tailoring the incident wave, accelerating gradients integrated over a small portion of the original pulse duration almost doubled.
Beam Loading
We know from the longitudinal dynamics theory of storage rings that the motion of a single particle is stably focused around an equilibrium position known as “synchronous phase”.

The synchronous phase corresponds to an accelerating voltage on the particle equal to its average single-turn loss, which accounts for synchrotron radiation emission and interaction with the vacuum chamber wakefields. In synchrotrons this also accounts for the required energy gain/turn during ramping.

For storage rings with positive dilation factor the synchronous phase is placed on the accelerating voltage positive slope; the contrary for negative dilation factor rings.

The particle absorbs energy from the accelerating field but also contributes to the total voltage. The particle induced voltage can be easily calculated treating the cavity accelerating mode as a lumped resonant RLC resonator interacting with an impulsive current.
The beam-cavity interaction can be conveniently described by introducing the resonator RLC model. According to this model, the cavity fundamental mode interacts with the beam current just like a parallel RLC lumped resonator. The relations between the RLC model parameters and the mode field integrals $\omega_r$ (mode angular resonant frequency), $V_c$ (maximum voltage gain for a particle travelling along the cavity gap for a given field level), $U$ (energy stored in the mode), $P_d$ (average power dissipated on the cavity walls) and $Q$ (mode quality factor), are given by:

<table>
<thead>
<tr>
<th>Mode field integrals</th>
<th>Cavity RLC model</th>
<th>RLC parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_c = \int_{gap} E_z(z) e^{j\omega_r \frac{z}{\beta_c}} dz$</td>
<td>$R_s$</td>
<td>$R_s = \frac{V_c^2}{2P_d}$</td>
</tr>
<tr>
<td>$P_d = \frac{1}{2} \int_{\sigma} R_{surf} H_i^2 d\sigma$</td>
<td>$C$</td>
<td>$C = \frac{1}{\omega_r R/Q}$</td>
</tr>
<tr>
<td>$U = \frac{1}{4} \int_V (\varepsilon E^2 + \mu H^2) d\tau$</td>
<td>$L$</td>
<td>$L = \frac{1}{\omega_r^2 C} = \frac{R/Q}{\omega_r}$</td>
</tr>
<tr>
<td>$Q = \frac{\omega_r U}{P_d}$; $R/Q = \frac{V_c^2}{2\omega_r U}$</td>
<td>Beam</td>
<td>Note: we assume that positive voltages accelerate the beam, which is therefore modelled as a current generator discharging the cavity</td>
</tr>
</tbody>
</table>
The particle in a storage ring are gathered in bunches that typically show a gaussian longitudinal profile. The bunches are normally much shorter than the RF wavelength and interact with the accelerating field similarly to macroparticles. The voltage induced in the cavity by a short gaussian bunch is still an exponentially decaying cosine wave with a finite rise time (related to the bunch duration).

\[ \Delta V_q = \frac{q}{C} = q \omega_r \frac{R}{Q} \]

The particle in a storage ring are gathered in bunches that typically show a gaussian longitudinal profile. The bunches are normally much shorter than the RF wavelength and interact with the accelerating field similarly to macroparticles. The voltage induced in the cavity by a short gaussian bunch is still an exponentially decaying cosine wave with a finite rise time (related to the bunch duration).
It can be easily verified that both the amplitude and the slope of the voltage induced in a cavity by a single passage of a short bunch are almost negligible with respect to the cavity accelerating voltage. What can not in general be neglected is the voltage generated by the cumulative effect of many bunch passages.

In fact, the time distance between adjacent bunches $T_b$ is normally much smaller than the cavity filling time $\tau_f$. This is true even in single bunch operation whenever the ring revolution period is smaller than the cavity filling time. In this case the voltage kicks associated to the passage of the bunches add coherently, and the overall beam induced voltage can grow substantially and even exceed the generator induced voltage.

\[ N = \frac{\tau_f}{T_b} \]
The cumulative contribution of many bunch passages depends essentially on the RF harmonic of the beam spectrum. Also neighbour lines play a role in case of uneven bunch filling pattern (gap transient effect).

The static beam loading problem consists in computing:
- the overall contribution of the beam to the accelerating voltage;
- the power needed from the RF generator to refurnish both cavity and beam;
- the optimal values of cavity detuning and cavity-to-generator coupling to minimize the power request to the generator.
To solve the static beam loading problem we refer to the following circuital model:

\[
Y_L = \frac{I_T}{V_c} = \frac{1 + \beta}{R_L} + j\left(\frac{\omega C}{\omega L} - 1\right)
\]

\[
I_g = \frac{2V_{FWD}}{nZ_0} = \sqrt{\frac{8\beta P_{FWD}}{R_s}}
\]

\[
I_b \approx \frac{2q}{T_b} ; \quad \phi_b = \phi_s
\]

The beam current is represented by the phasor \(I_b\) at the frequency of the RF source, while the external RF source is represented by a current generator \(I_g\) whose amplitude is related to the power of the forward wave launched on the transmission line.

If \(\eta > 0\) the beam current phasor anticipates the total cavity voltage (i.e. \(\phi_s > 0\)), while it is retarded in the opposite case (\(\eta < 0 \Rightarrow \phi_s < 0\)).

The RF sources are normally specified by the maximum amount of power deliverable on a matched load. This is also the power actually dissipated in the system whenever a circulator is interposed to protect and match the source. The cavity is usually tuned near the optimal value that minimizes the request of forward RF power to the generator.
The analysis of the beam-cavity circuit model leads to the reported phasor diagram. The total current exciting the cavity is:

\[ I_T = I_g - I_b = V_c \cdot Y = V_c \cdot \left[ \frac{\beta + 1}{R_s} + j \left( \omega C - \frac{1}{\omega L} \right) \right] \]

Real and imaginary parts of the generator current are then given by:

\[
\text{Re}[I_g] = \text{Re}[I_T + I_b] = V_c \frac{\beta + 1}{R_s} + I_b \cos(\phi_s)
\]

\[
\text{Im}[I_g] = \text{Im}[I_T + I_b] = V_c \frac{\omega/\omega_r - \omega_r/\omega}{R/Q} + I_b \sin(\phi_s)
\]

Being the off-resonance parameter \( \delta \) defined as:

\[
\delta = \frac{\omega}{\omega_r} - \frac{\omega_r}{\omega} \approx 2 \frac{\omega - \omega_r}{\omega}
\]

the minimum generator current amplitude, corresponding to the optimal cavity tuning, is obtained when its imaginary part is zero:

\[
\text{Im}[I_g] = 0 \Rightarrow V_c \frac{\delta}{R/Q} = -I_b \sin(\phi_s) \Rightarrow \delta = -\frac{I_b R/Q}{V_c} \sin(\phi_s) = -\text{sgn}(\eta) \frac{2 q_b}{T_b} \frac{R/Q}{V_c} \sqrt{1 - \left( \frac{V_{\text{loss}}}{V_c} \right)^2}
\]
The optimal cavity tuning condition in a storage ring with negative dilation factor $\eta$ asks for positive values of $\delta$. This means that the cavity has to be tuned below the frequency of the RF generator, and the amount of detuning is proportional to the intensity of the stored current. The sign of the detuning is just opposite for positive values of the dilation factor $\eta$.

Under the optimal cavity tuning condition we also have:

$$ I_g = \text{Re}[\tilde{I}_g] = V_c \frac{\beta + 1}{R_s} + I_b \cos(\phi_s) = \sqrt{\frac{8\beta P_{\text{FWD}}}{R_s}} \Rightarrow P_{\text{FWD}} = \frac{R_s}{8\beta} \left[ V_c \frac{\beta + 1}{R_s} + I_b \cos(\phi_s) \right]^2 $$

leading to the optimal coupling condition:

$$ \beta_{\text{opt}} = 1 + \frac{I_b}{V_c} \frac{R_s}{V_c} \cos(\phi_s) = 1 + \frac{I_b}{V_c} \frac{R_s}{V_c^2} V_{\text{loss}} = 1 + \frac{P_{\text{beam}}}{P_{\text{cav}}} $$

$$ P_{\text{FWD}} = \frac{1}{2} \frac{V_c^2}{R_s} + \frac{1}{2} I_b V_c \cos(\phi_s) = P_{\text{cav}} + P_{\text{beam}} $$

It follows immediately that the forward power request to the external generator is:

The system is perfectly matched and no RF power is wasted because of reflections.
The beam can be also modelled as a complex admittance equal to the ratio between the current and voltage phasors. This model is less physical (the resistive part of the beam admittance does not enlarge the bandwidth of the cavity!) but allows more direct computation of the reflection coefficient at the coupling port.

\[ Y_{cav} = \frac{1}{R_s} + j \frac{\delta}{R/Q} \]

\[ Y_{beam} = \frac{I_b}{V_c} e^{j\phi_s} = \frac{I_b}{V_c} \cos \phi_s + j \frac{I_b}{V_c} \sin \phi_s \]

\[ \beta - 1 - \frac{R_s I_b \cos \phi_s}{V_c} - j R_s \left( \frac{\delta}{R/Q} + \frac{I_b}{V_c} \sin \phi_s \right) \]

\[ \beta + 1 + \frac{R_s I_b \cos \phi_s}{V_c} + j R_s \left( \frac{\delta}{R/Q} + \frac{I_b}{V_c} \sin \phi_s \right) \]
For a given required cavity voltage $V_c$, the optimal values of the cavity detuning $\delta$ and coupling coefficient $\beta$ are both dependent on the instantaneous stored current $I_b$. The accelerating cavities are equipped with tuners, that are devices allowing small and continuous cavity profile deformation to real time control the resonant frequency position.

Sometimes cavities are also equipped with variable input couplers but they are never operated in regime of continuous $\beta$ adjustment. In general the coupling coefficient is adjusted just once to match the maximum expected beam current. At lower currents the system is partially mismatched, but the available RF power is nevertheless sufficient to feed the cavity and the beam.

It turns out that the cavity tuning $\delta$ and the generator power $P_{FWD}$ have to follow the actual current value in order to keep the accelerating voltage constant preserving good matching conditions.

These tasks are normally accomplished by dedicated slow feedback systems, namely the “tuning loop” and the “amplitude loop”.
Low-Level RF control
The tuning loop restores automatically and continuously the cavity resonant frequency to compensate the beam reactive admittance. The loop controls the RF phase between the cavity voltage and the forward wave from the generator. Phase drifts are corrected by producing mechanical deformations of the cavity profile by means of dedicated devices (plungers, squeezers, ...). In fact the loop controls the phase of the transfer function:

\[
\frac{\dot{V}_c}{\dot{V}_{FWD}} = 2n \frac{1}{Z_0 + \frac{1}{n^2(Y_{cav} + Y_{beam})}} = \frac{2}{1 + \beta \sqrt{\frac{\beta R_s}{Z_0}}} \\
\angle \frac{\dot{V}_c}{\dot{V}_{FWD}} = -\arctan \frac{Q_L \delta + \frac{R_L I_b}{V_c} \sin \phi_s}{1 + \frac{R_L I_b}{V_c} \cos \phi_s}
\]

with

\[
Q_L = \frac{Q_0}{1 + \beta} \\
R_L = \frac{R_s}{1 + \beta}
\]

By controlling the set point of the variable phase shifter the phase of the transfer function can be locked to 0 or to any other value \( \phi_0 \).
The best cavity tuning is obtained by locking the loop to $\phi_0 = 0$. In this case we have:

$$\tan \left( \frac{Q_L \delta + \frac{R_L I_b}{V_c} \sin \phi_s}{1 + \frac{R_L I_b}{V_c} \cos \phi_s} \right) = 0 \Rightarrow Q_L \delta + \frac{R_L I_b}{V_c} \sin \phi_s = 0 \Rightarrow \delta = -\frac{I_b R/Q}{V_c} \sin \phi_s = 0$$

For dynamics considerations, sometimes it is useful to set a phase $\phi_0$ slightly different from zero. In this case we have:

$$\tan \left( \frac{Q_L \delta + \frac{R_L I_b}{V_c} \sin \phi_s}{1 + \frac{R_L I_b}{V_c} \cos \phi_s} \right) = \phi_0 \Rightarrow Q_L \delta = -\frac{R_L I_b}{V_c} \sin \phi_s - \tan(\phi_0) \left(1 + \frac{R_L I_b}{V_c} \cos \phi_s\right)$$

$$\rho = \frac{\rho_0 + j \tan(\phi_0)}{1 - j \tan(\phi_0)} \quad \text{with} \quad \rho_0 = \frac{\beta - 1 - \frac{R_L I_b}{V_c} \cos \phi_s}{\beta + 1 - \frac{R_L I_b}{V_c} \cos \phi_s}$$

The reflection at the input coupler port is not minimized in this case, and the overall efficiency of the system is reduced.

Tuning loops are generally very slow since they involve mechanical movement through the action of motors. Typical bandwidth of such systems are of the order of 1 Hz. Tuning systems are also necessary to stabilize the cavity resonance against thermal drifts.
The automatic regulation of the generator output level can be obtained by implementing amplitude loops. These are feedback systems which detect and correct variations of the level of the cavity voltage.

If the power amplifier is not fully saturated, the regulation can be obtained by controlling the RF level of the amplifier driving signal.

If the amplifier is saturated, the feedback has to act directly on the high voltage that sets the level of the saturated output power.

Referring to the reported model, the loop transfer function can be written in the form:

$$\frac{A_c}{A_{ref}} = \frac{H(s)}{1 + H(s)} \quad \text{with} \quad H(s) = \frac{A_{in}}{V_{mod}} K C(s) G(s)$$

For little cavity detuning ($Q_L \delta \ll 1$) the cavity response to an amplitude modulated signal is a single pole low-pass:

$$C(s) \approx \frac{C_0}{1 + s/\sigma} \quad \text{with} \quad \sigma = \frac{\omega_c}{2Q_L}$$
In heavy beam loading regime the cavity detuning is likely to be as large as the cavity bandwidth or even more, so that the single pole model for the cavity amplitude response is inappropriate. A more complex model applies in this case, involving cross-modulation blocks.

Amplitude loops may have bandwidths ranging from few Hz to 1 MHz. Sometimes it is useful to have large residual loop gain at line frequencies to correct spurious modulation introduced by the power stages. The gain and bandwidth can be tailored by properly designing the error amplifier transfer function $G(s)$. For instance, if the cavity single-pole model is adequate, high loop gains in the low frequency region are obtainable by implementing an integrator providing a zero for compensating the cavity pole.

The low frequency gain can be boosted by properly treating the error signal. However, the coupling between the various RF servo loops induced by the large cavity detuning may result in a global instability of the beam-cavity system. To avoid it, gain and bandwidth of individual loops have to be reduced.
The cavity RF phase (or the power station RF phase) can be locked to the reference RF clock by another dedicated servo loop. The need for a phase loop is not strictly related to beam loading effects but more to ensure synchronization between different RF cavities or between RF voltage and other sub-systems of the accelerator (such as injection system, beam feedback systems, ...).

The phase is locked to the reference by measuring the relative phase deviation by means of a phase detector and applying a continuous correction through a phase shifter. For loop gain and bandwidth the same considerations expressed in the amplitude loop case hold.

\[ \phi_c = \phi_{in} \frac{C(s)}{1 + H(s)} + \phi_{ref} \frac{H(s)}{1 + H(s)} \]

with \( H(s) = k_{mod} k_{det} C(s) G(s) \)

\[ \phi_c \approx \phi_{in} \text{ if } |H(s)| >> |C(s)|; |H(s)| >> 1 \]
Beam Phase Loop

The beam phase loops are feedback systems aimed at adding a damping (frictional) term in the synchrotron equation for the beam center-of-mass coherent motion.

In the basic scheme the phase of the beam is detected and, after a manipulation to introduce a 90° phase shift at the synchrotron frequency, is applied back to the cavity.

Ideally, if the cavity modulation were exactly proportional to the time derivative of the beam phase we would get:

\[ \phi_c = k \dot{\phi}_b; \quad \ddot{\phi}_b + \omega_s^2 \phi_b = \omega_s^2 \phi_c \quad \Rightarrow \quad \ddot{\phi}_b - \omega_s^2 k \dot{\phi}_b + \omega_s^2 \phi_b = 0 \]

making a frictional term appearing in the synchrotron equation.
An Example: PEP-II
Low Level RF Control Loops

Tuner loops - standard tuning for minimum reflected power

Klystron operating point support
- Ripple loop adjusts a complex modulator to maintain constant gain and phase shift through the klystron/modulator system.
- Klystron saturation loops maintain constant saturation headroom

Direct feedback loop (analog)
- Causes the station to follow the RF reference adding regulation of the cavity voltage
- Extends the beam-loading Robinson stability limit
- Lowers the effective fundamental impedance seen by the beam

Comb filter (digital)
- Adds narrow gain peaks at synchrotron sidebands to further reduce the residual impedance

Gap feedback loop (digital)
- Removes revolution harmonics from the feedback error signal to avoid saturating the klystron on gap synchronous phase transients

Longitudinal feedback uses RF as low-frequency “woofer” kicker