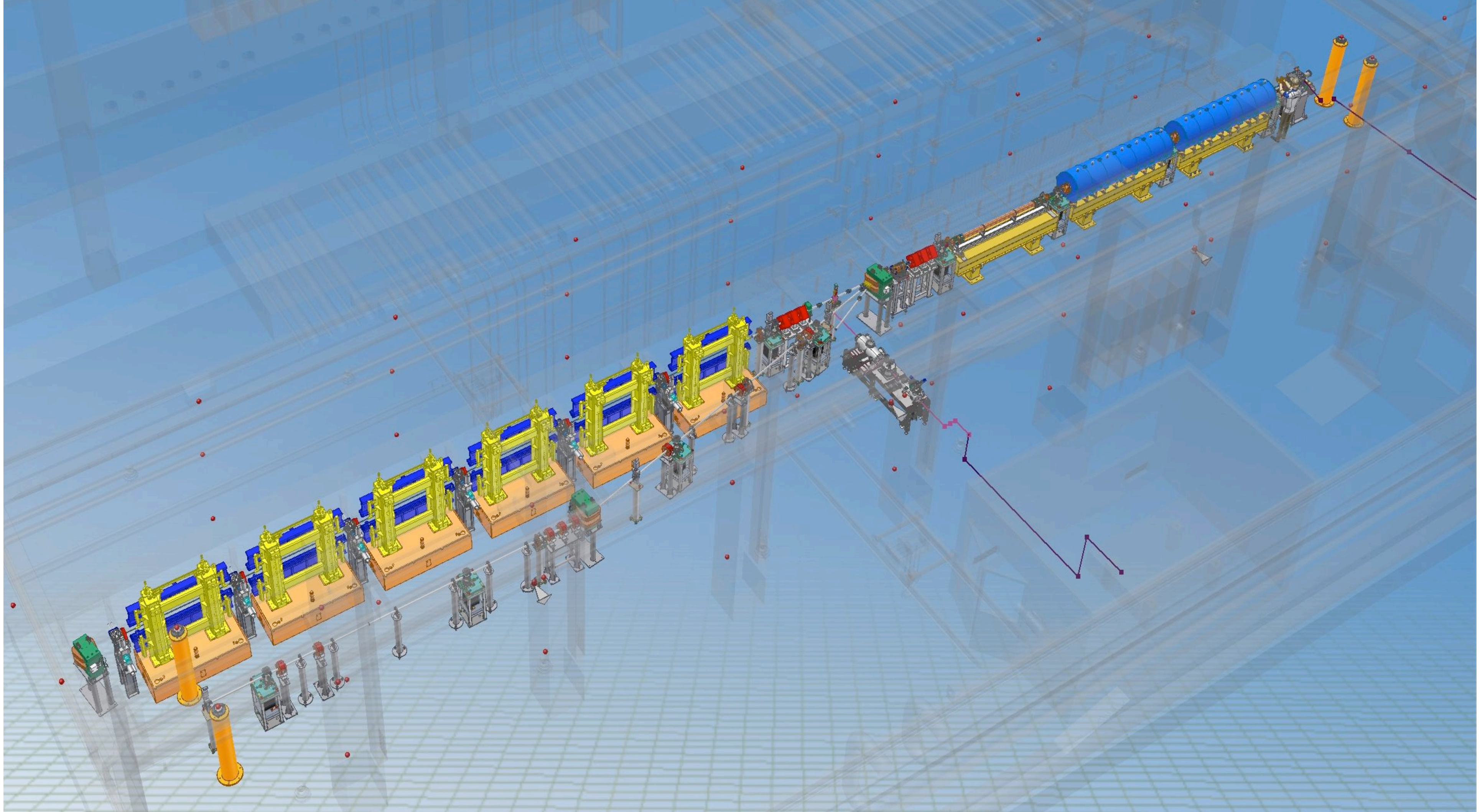
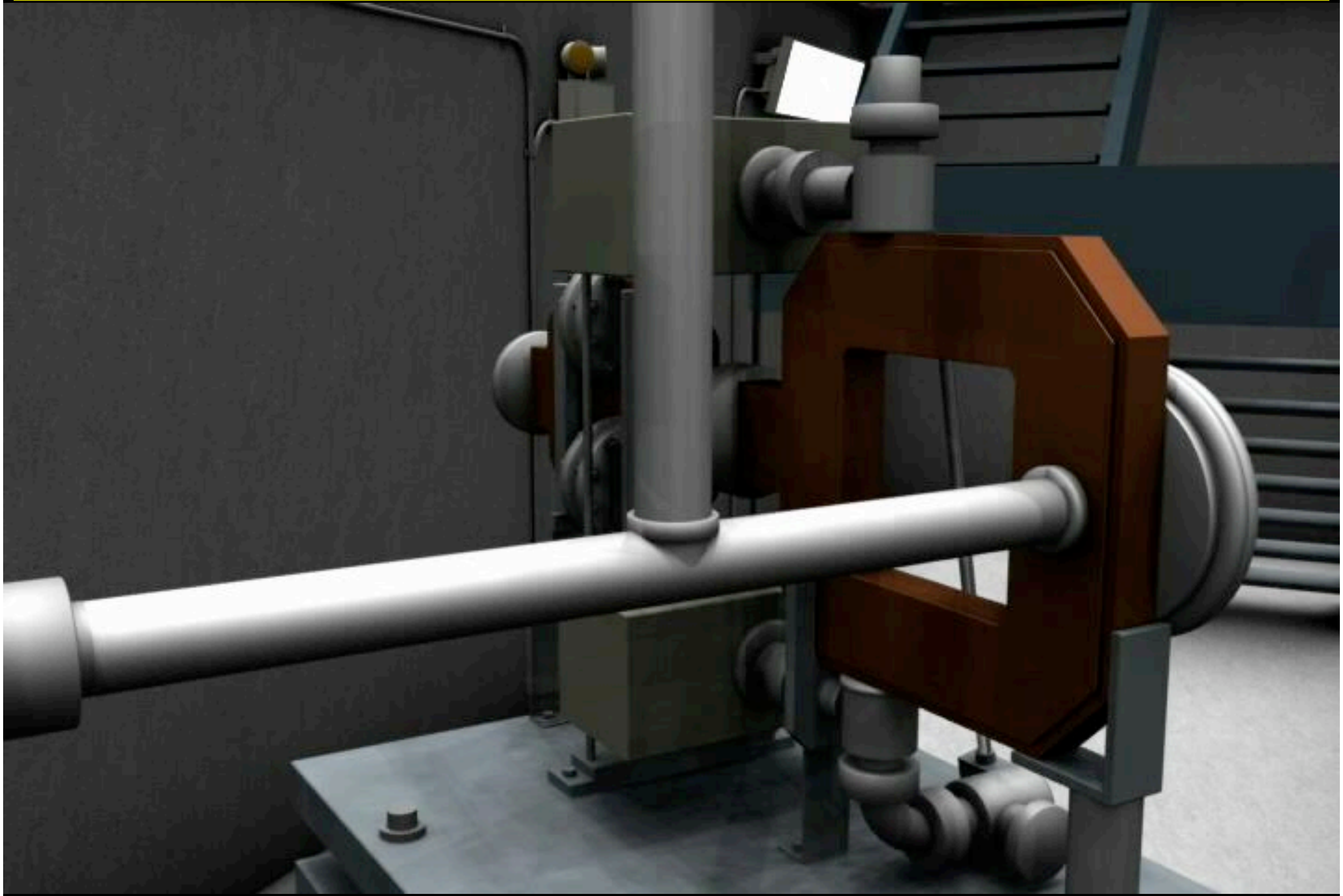
Linac Driven Free Electron Lasers (I)

Massimo.Ferrario@lnf.infn.it



CAS THE CERN ACCELERATOR SCHOOL

Injector

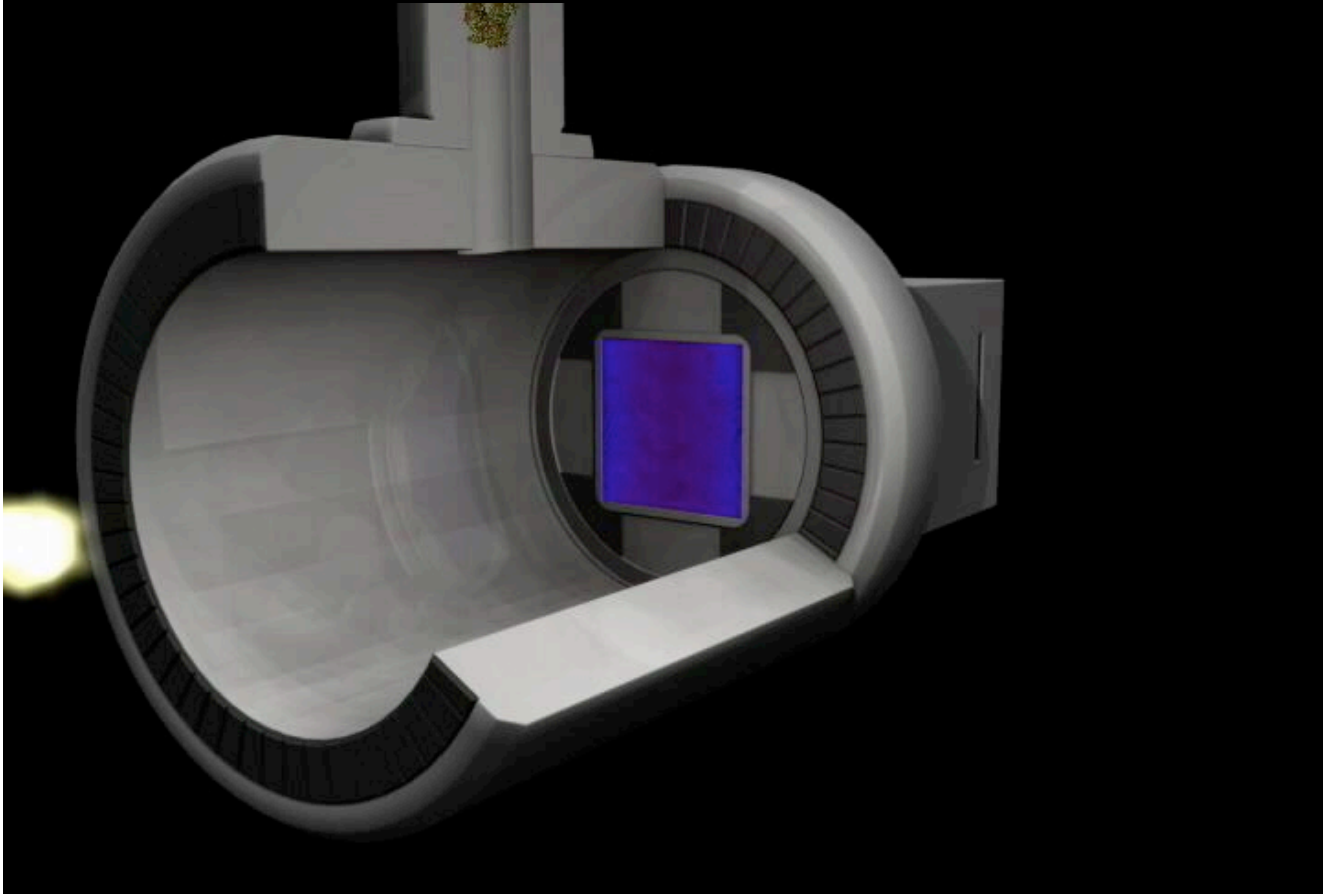


Magnetic bunch compressor (Chicane)

Long undulators chain

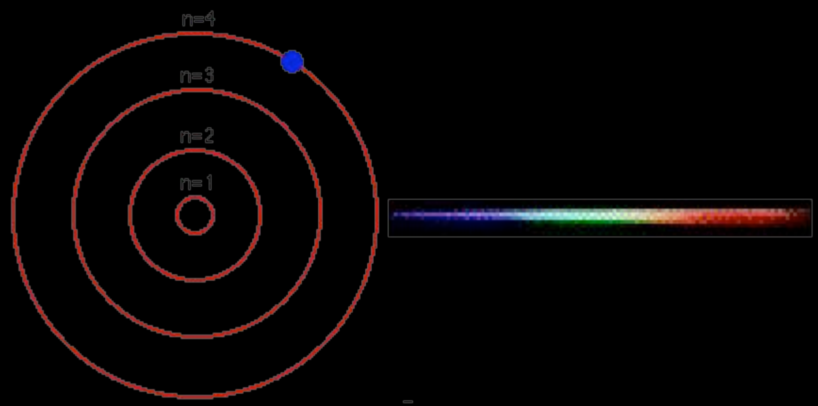
Beam separation

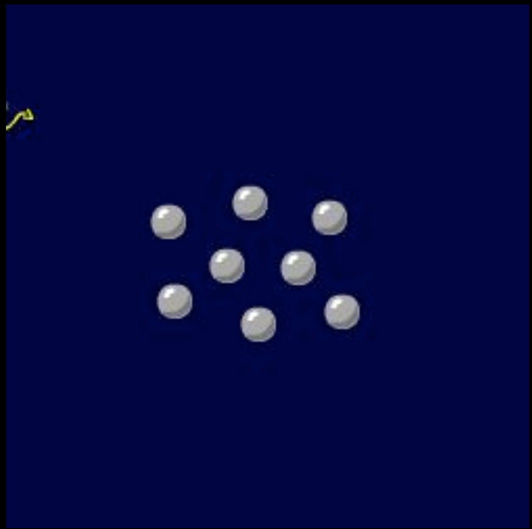
Experimental hall (Single Protein Imaging)



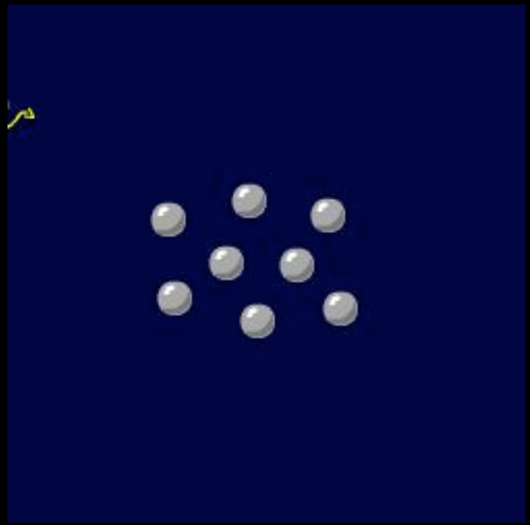
Atomic Laser ==> Bounded Electrons

Light Amplification by Stimulated Emission of Radiation

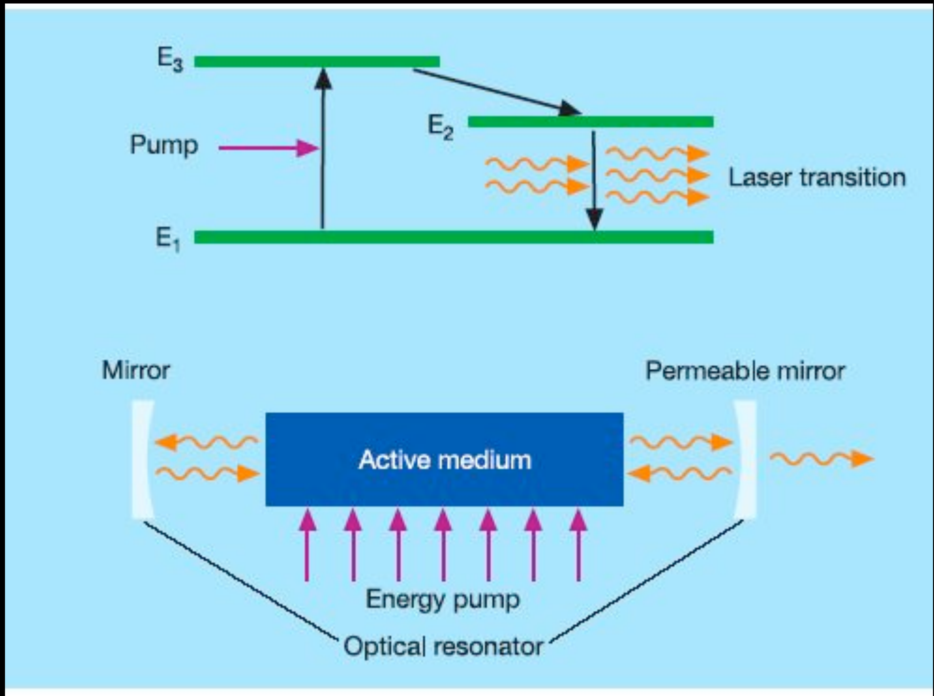


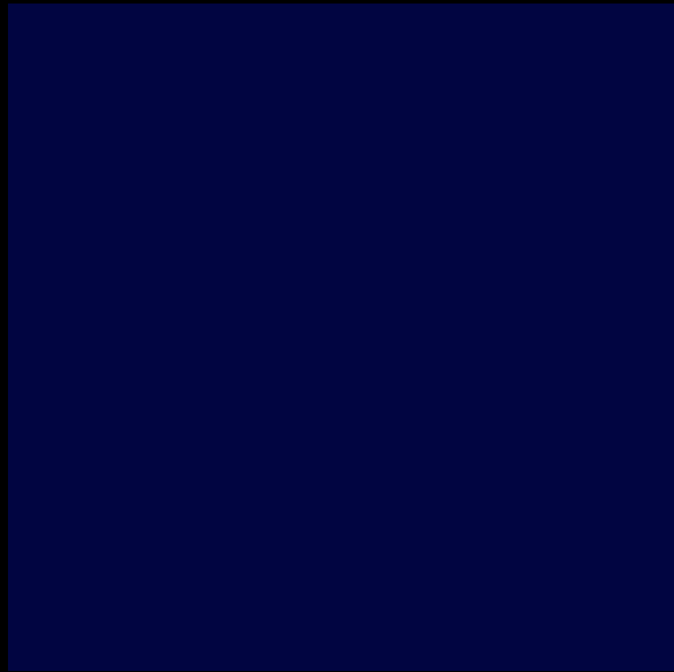


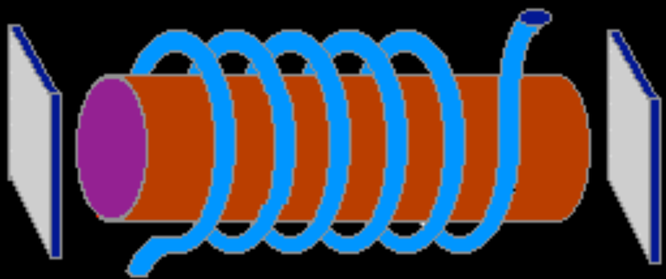
Spontaneous Emission

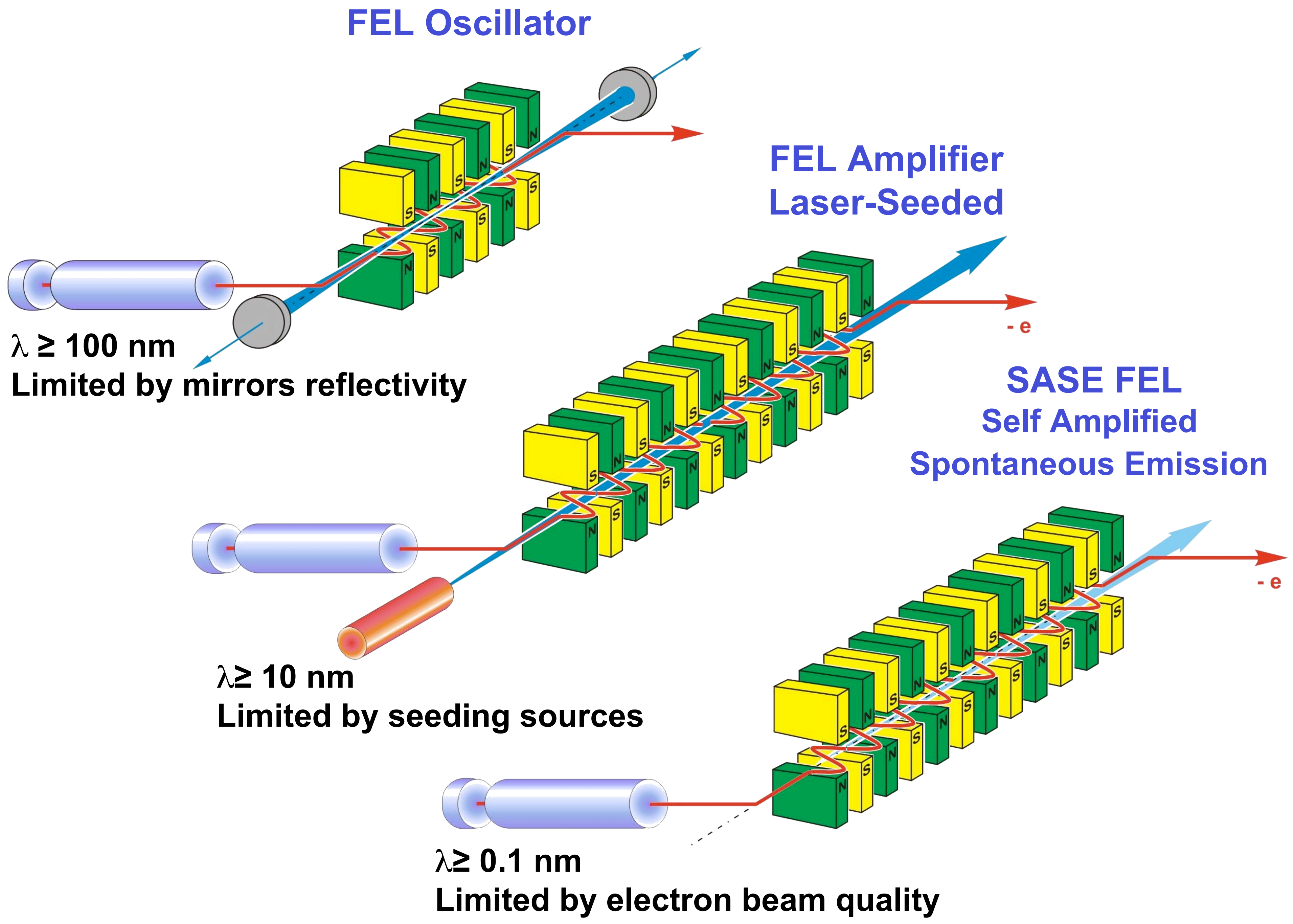


Stimulated Emission


E_3
E_2
Pump
Laser transition
E_1
Mirror
Permeable mirror
Active medium
Energy pump
Optical resonator

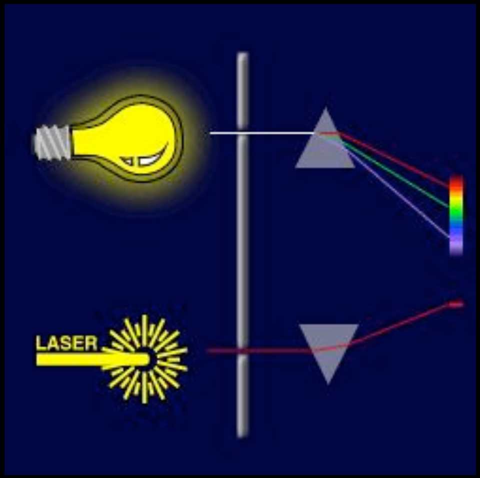




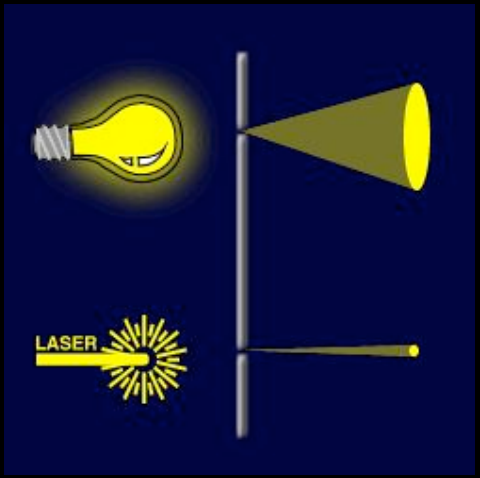


FEL Oscillator
λ ≥ 100 nm
Limited by mirrors reflectivity
FEL Amplifier
Laser-Seeded
- e
λ≥ 10 nm
Limited by seeding sources
SASE FEL
Self Amplified
Spontaneous Emission
- e
λ≥ 0.1 nm
Limited by electron beam quality

Properties of Stimulated Emission

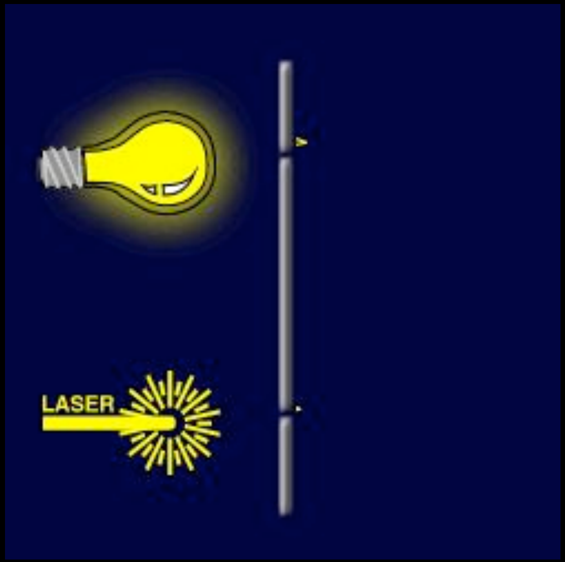


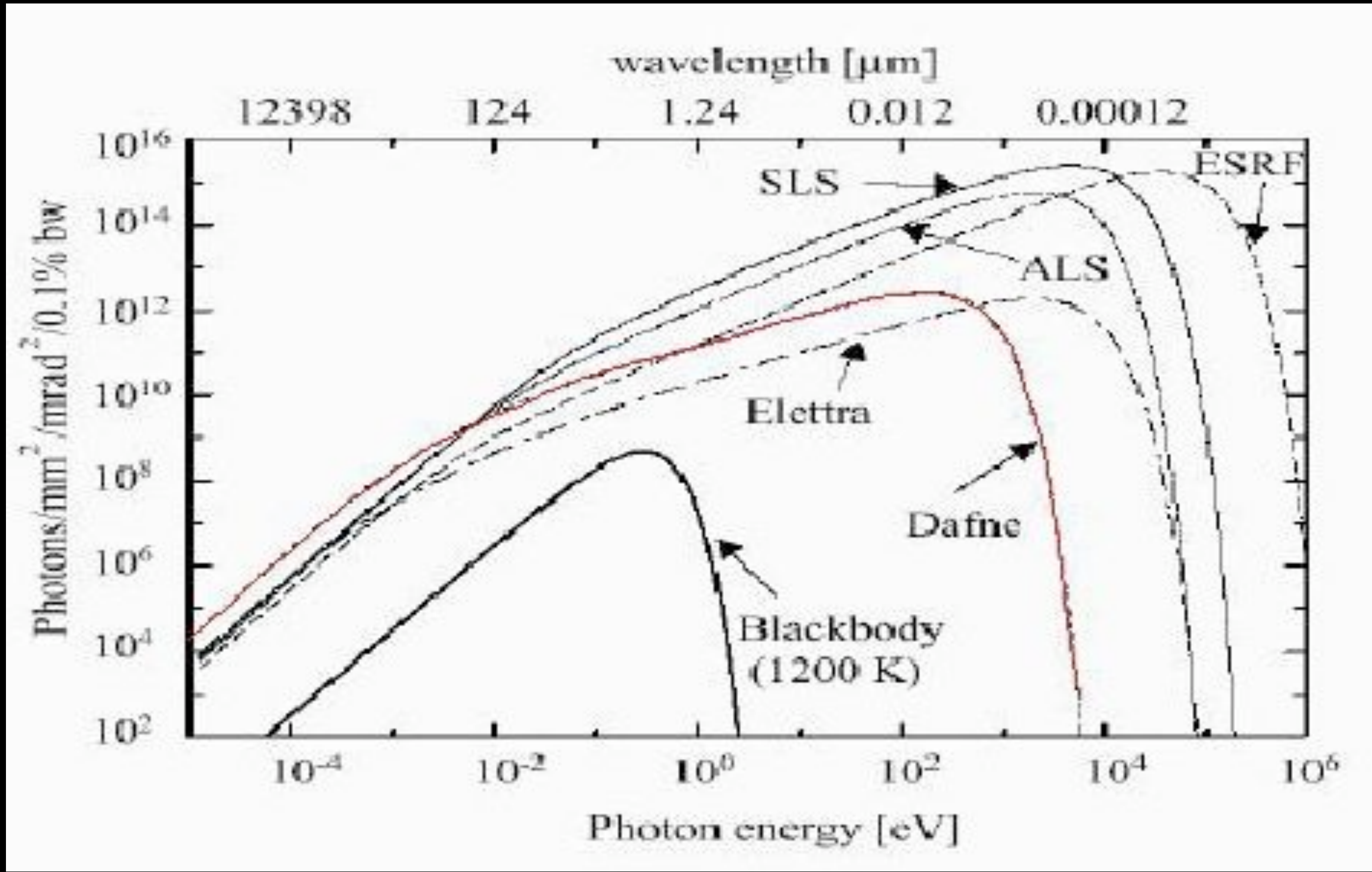
1. Monochromaticity.



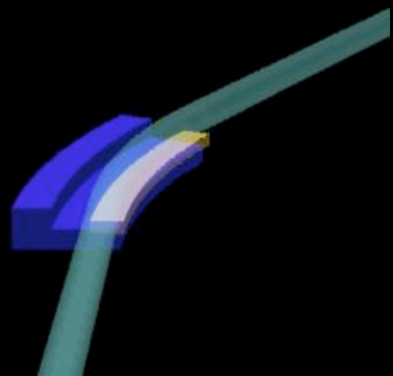
2. Directionality.

3. Coherence.

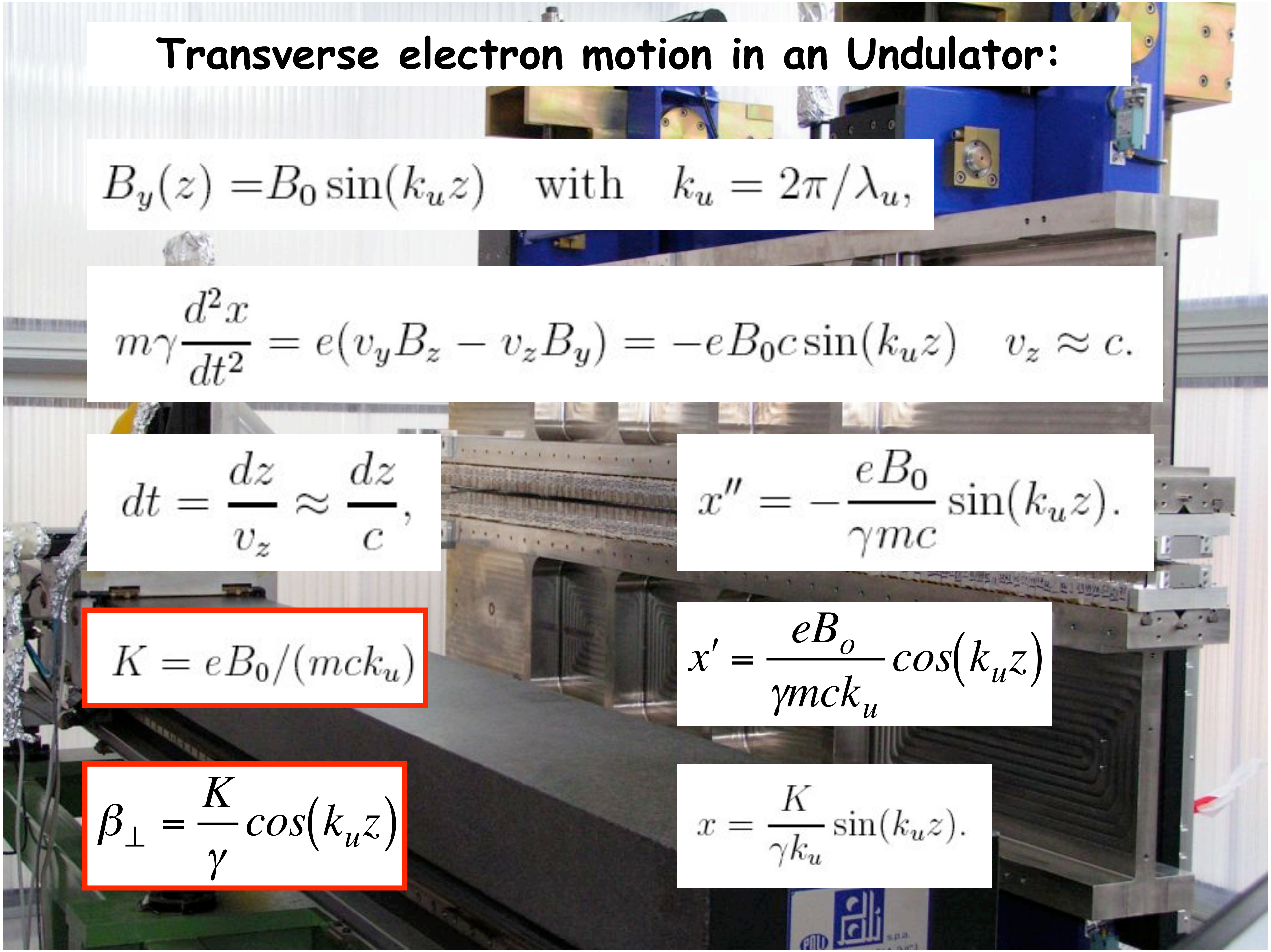




wavelength [µm]
12398
124
1.24
0.012
0.00012
Photons/mm²/mrad²/0.1% bw
SLS
ESRF
ALS
Elettra
Dafne
Blackbody
(1200 K)
Photon energy [eV]



Transverse electron motion in an Undulator:



$$B_y(z) = B_0 \sin(k_u z) \quad \text{with} \quad k_u = 2\pi/\lambda_u,$$

$$m\gamma \frac{d^2 x}{dt^2} = e(v_y B_z - v_z B_y) = -eB_0 c \sin(k_u z) \quad v_z \approx c.$$

$$dt = \frac{dz}{v_z} \approx \frac{dz}{c},$$

$$x'' = -\frac{eB_0}{\gamma mc} \sin(k_u z).$$

$$K = eB_0/(mck_u)$$

$$x' = \frac{eB_o}{\gamma mck_u} cos(k_u z)$$

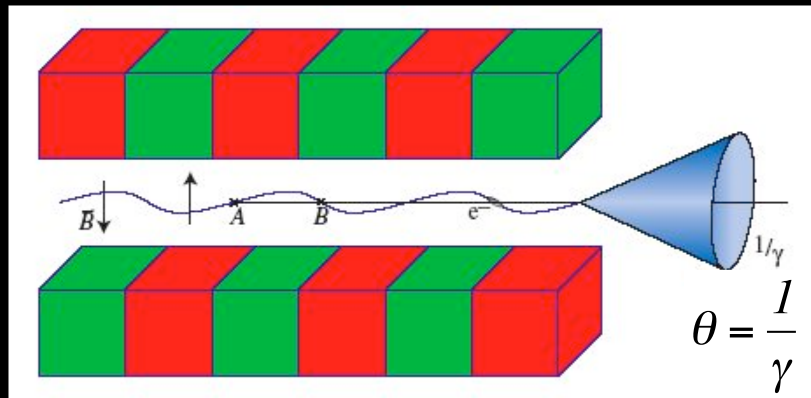
$$\beta_\perp = \frac{K}{\gamma} cos(k_u z)$$

$$x = \frac{K}{\gamma k_u} \sin(k_u z).$$

Undulator Radiation

The electron trajectory is determined by the undulator field and the electron energy

$$x = \frac{K}{\gamma k_u} \sin(k_u z).$$



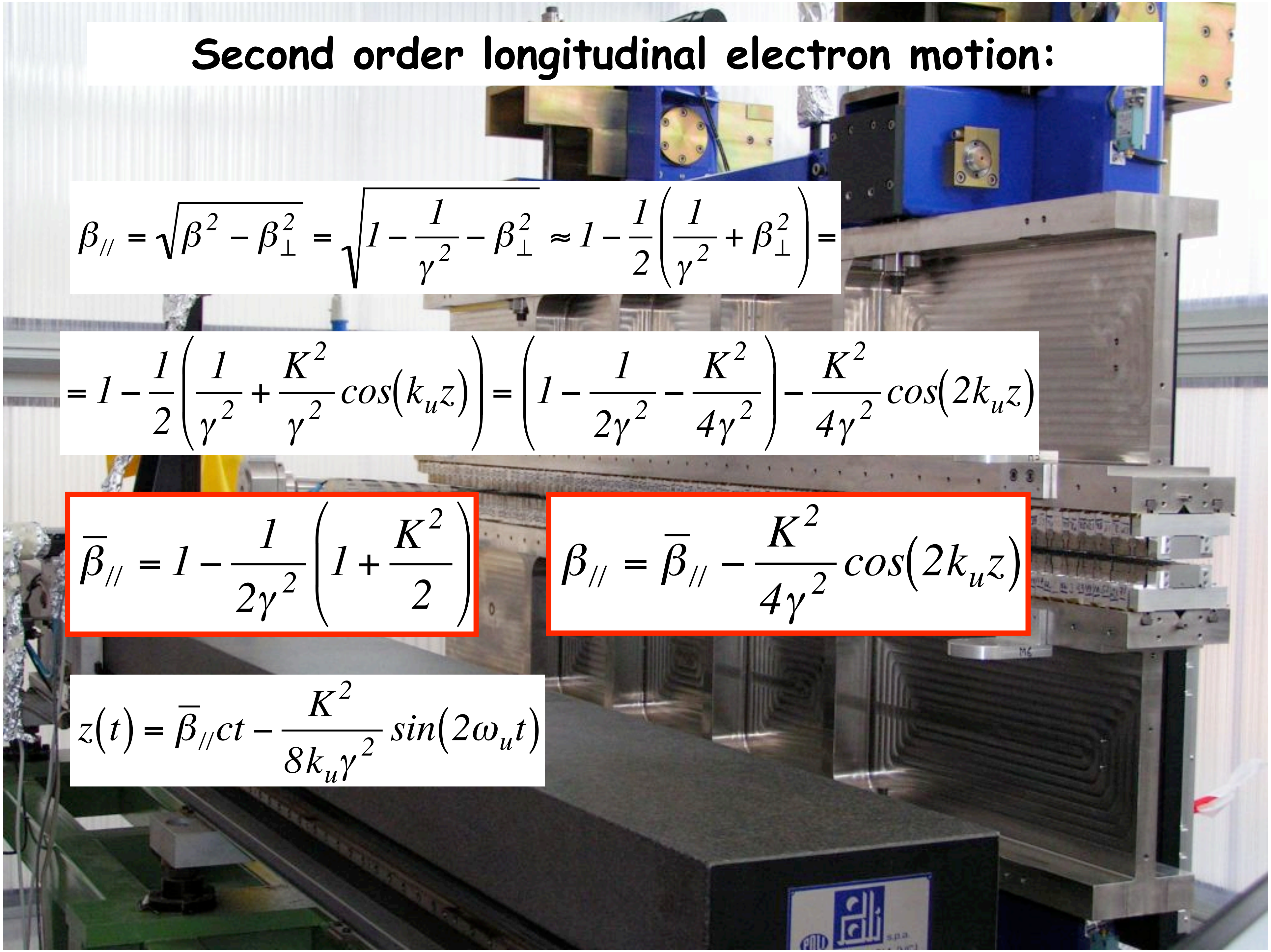
$$\theta = \frac{1}{\gamma}$$

$$\beta_{x\,Max} = \frac{K}{\gamma}$$

$$K = \frac{e\tilde{B}_o \lambda_u}{2\pi mc}$$

The electron trajectory is inside the radiation cone if $K \leq 1$

Second order longitudinal electron motion:



$$\beta_{//} = \sqrt{\beta^2 - \beta_\perp^2} = \sqrt{1 - \frac{1}{\gamma^2} - \beta_\perp^2} \approx 1 - \frac{1}{2}\left(\frac{1}{\gamma^2} + \beta_\perp^2\right) =$$

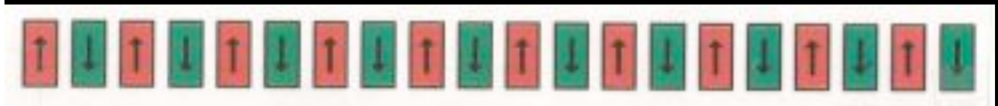
$$= 1 - \frac{1}{2}\left(\frac{1}{\gamma^2} + \frac{K^2}{\gamma^2}\cos(k_u z)\right) = \left(1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}\right) - \frac{K^2}{4\gamma^2}\cos(2k_u z)$$

$$\bar{\beta}_{//} = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

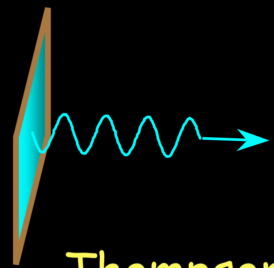
$$\beta_{//} = \bar{\beta}_{//} - \frac{K^2}{4\gamma^2}\cos(2k_u z)$$

$$z(t) = \bar{\beta}_{//} ct - \frac{K^2}{8k_u\gamma^2}\sin(2\omega_u t)$$

Relativistic Mirrors



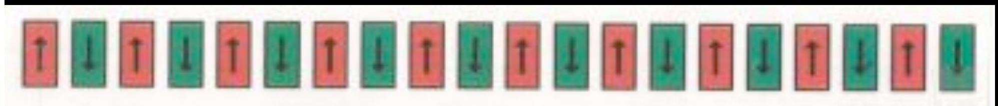
$$\lambda'_u = \frac{\lambda_u}{\gamma_{//}}$$



Counter propagating pseudo-radiation

$$\lambda'_{rad} = \lambda'_u$$

Thompson back-scattered radiation in the mirror moving frame

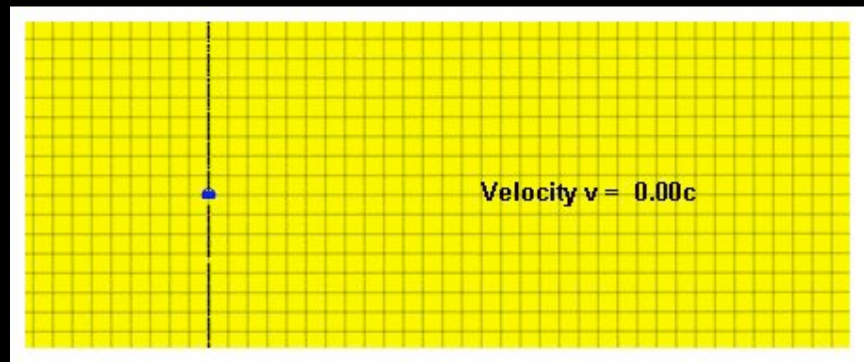


$$\lambda_{rad} = \gamma\lambda'_{rad}\left(1-\beta\cos\vartheta\right) \approx \lambda_u\left(1-\overline{\beta}_{//}\cos\vartheta\right)$$

Doppler effect in the laboratory frame

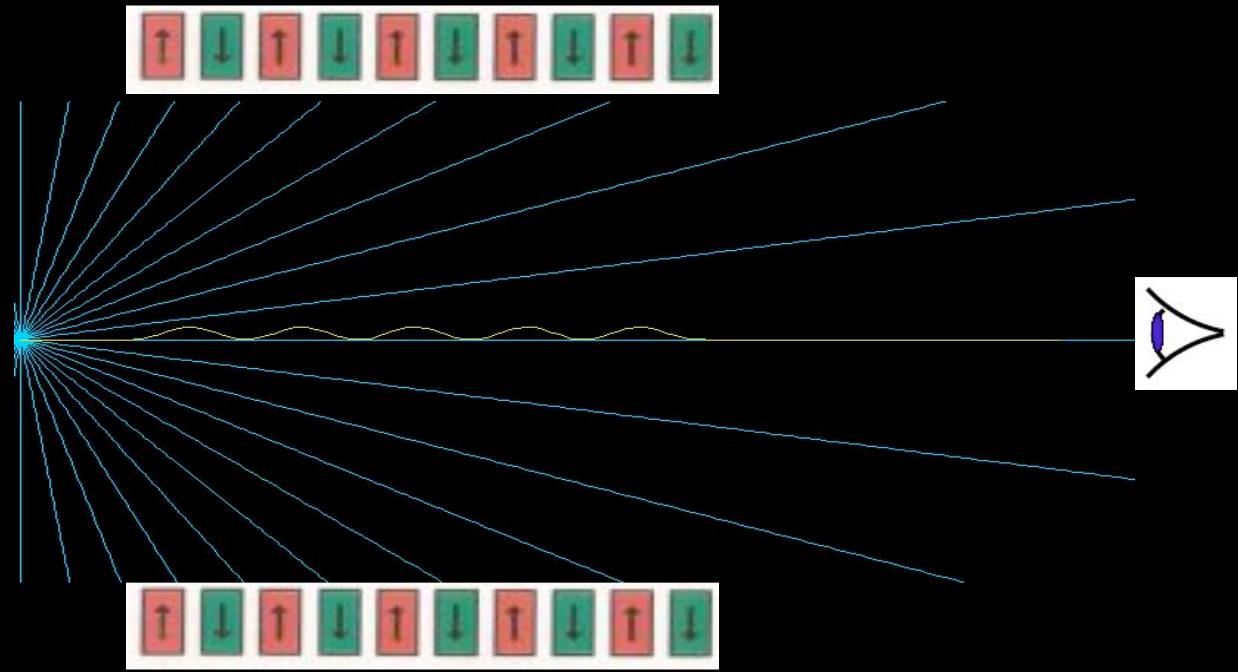
$$\overline{\beta}_{//} = 1 - \frac{1}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$\cos\vartheta \approx 1 - \frac{\vartheta^2}{2}$$

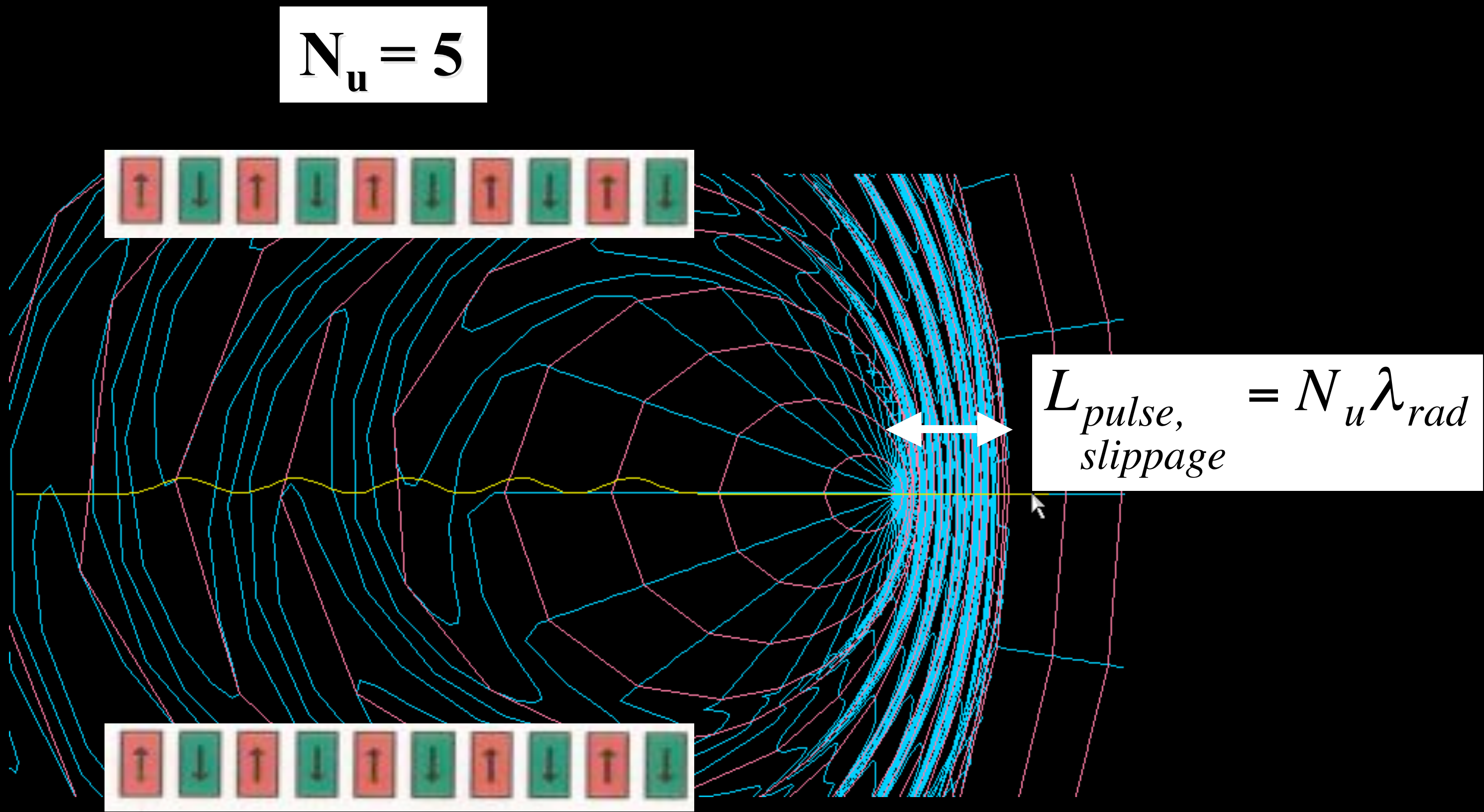


$$\lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\vartheta^2\right)$$

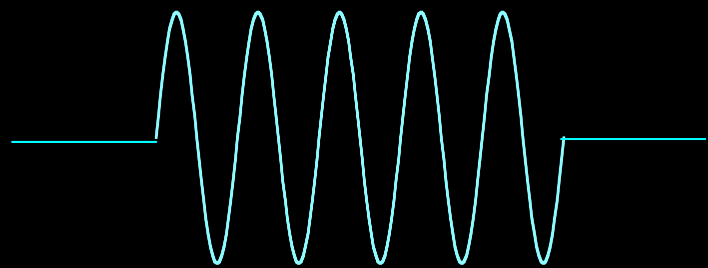
TUNABILITY & RED SHIFT



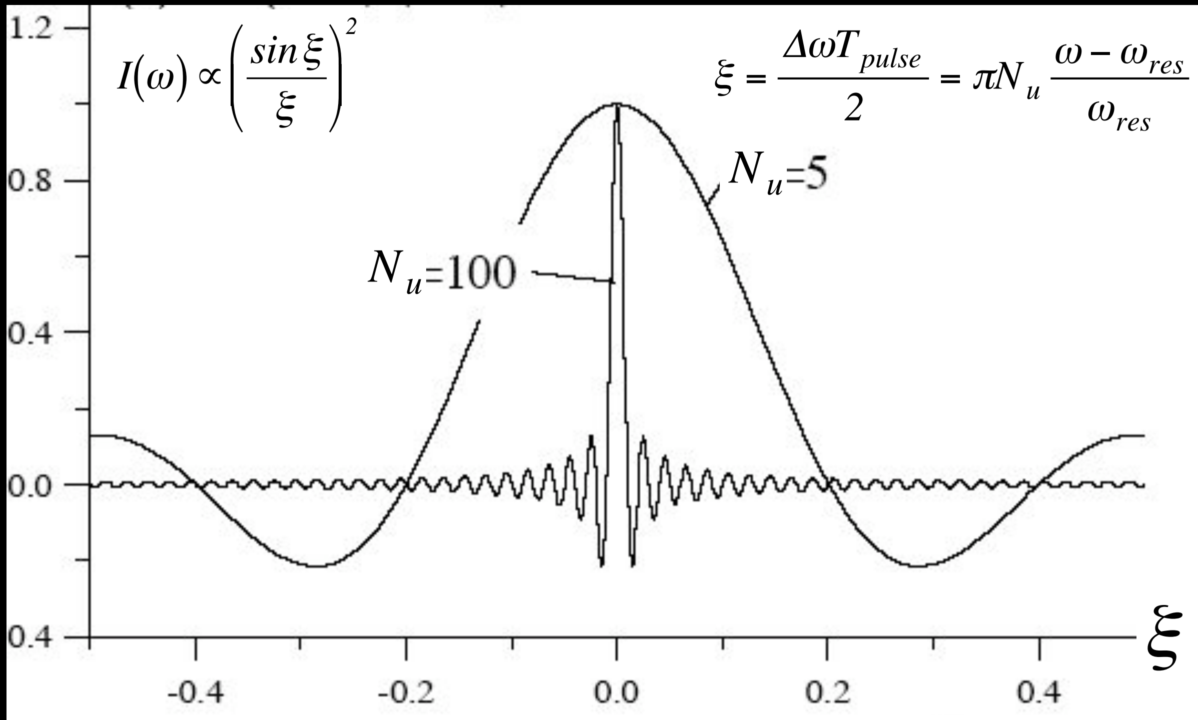
Radiation Simulator – T. Shintake, @ http://www-xfel.spring8.or.jp/Index.htm



Due to the finite duration the radiation is not monochromatic but contains a frequency spectrum which is obtained by Fourier transformation of a truncated plane wave



Spectral Intensity



$$\frac{\Delta\omega}{\omega} \approx \frac{1}{N_u}$$

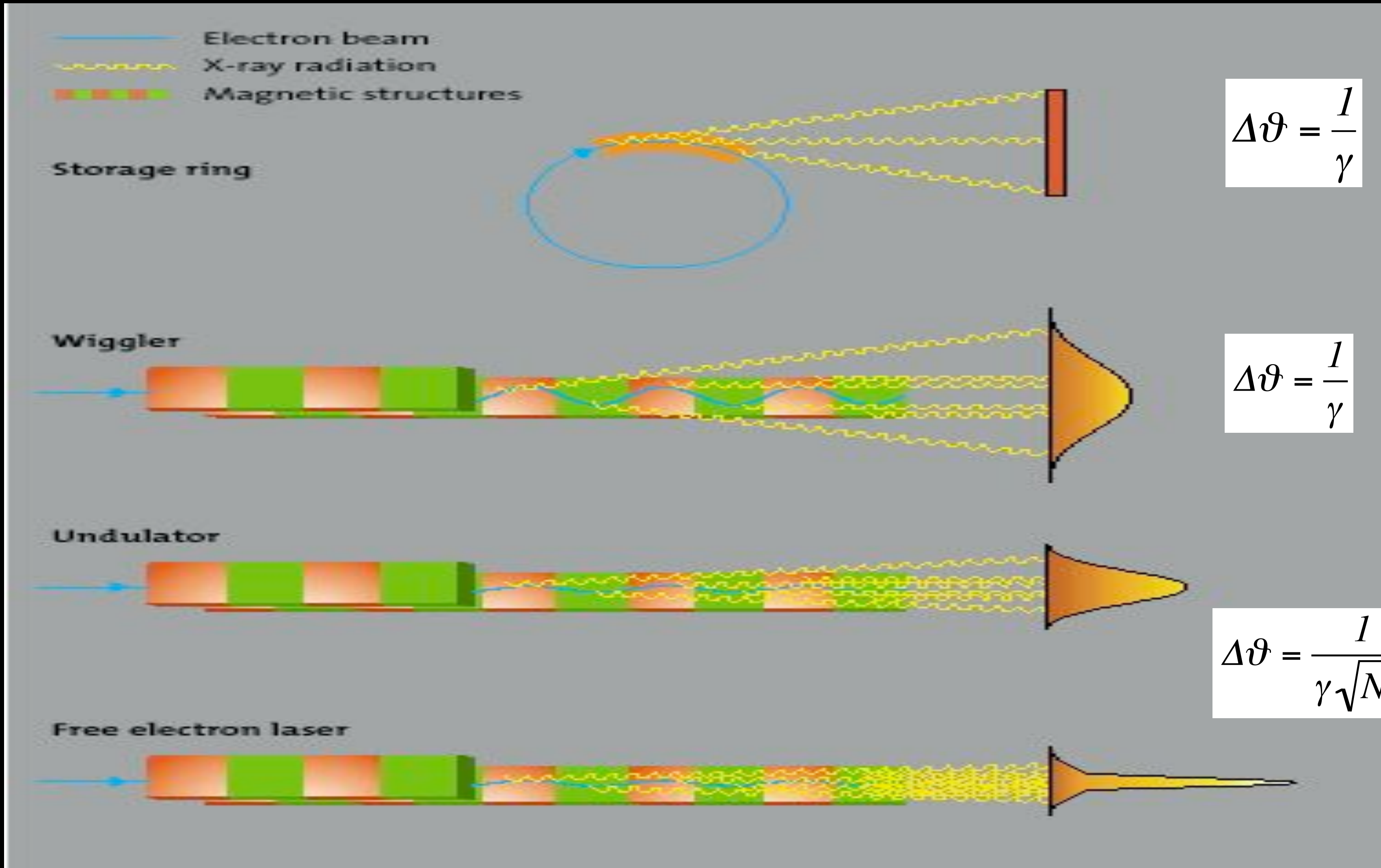
Line width

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda(\vartheta) - \lambda(0)}{\lambda(0)} = \frac{\gamma^2\vartheta^2}{1 + \frac{K^2}{2}} \approx \frac{1}{N_u}$$

==>

$$\vartheta \approx \sqrt{\frac{1}{\gamma^2 N_u}\left(1 + \frac{K^2}{2}\right)} \approx \frac{1}{\gamma\sqrt{N_u}}$$

Angular width



$$\Delta\vartheta = \frac{1}{\gamma}$$

$$\Delta\vartheta = \frac{1}{\gamma}$$

$$\Delta\vartheta = \frac{1}{\gamma\sqrt{N_u}}$$

Peak power of one accelerated charge:

$$P_1 = \frac{e^2}{6\pi\varepsilon_o c^3}\gamma^4\left\langle \dot{v}_\perp^2 \right\rangle$$

$$\left\langle \dot{v}_\perp^2 \right\rangle = \frac{1}{T}\int_{-T/2}^{T/2}\left(-\frac{K}{\gamma k_u}\omega^2 \sin(\omega t)\right)^2 = \frac{1}{T}\int_{-T/2}^{T/2}\left(-\frac{K\gamma c^2 k_u}{1+K^2/2}\sin(\omega t)\right)^2 = \frac{1}{2}\frac{K^2\gamma^2 c^4 k_u^2}{\left(1+K^2/2\right)^2}$$

$$P_1 = \frac{1}{12}\frac{e^2 K^2 \gamma^2 c \; k_u^2}{\pi\varepsilon_o\left(1+K^2/2\right)^2}$$

Different electrons radiate indepedently hence the total power depends linearly on the number N_e of electrons per bunch:

Incoherent Spontaneous Radiation Power:

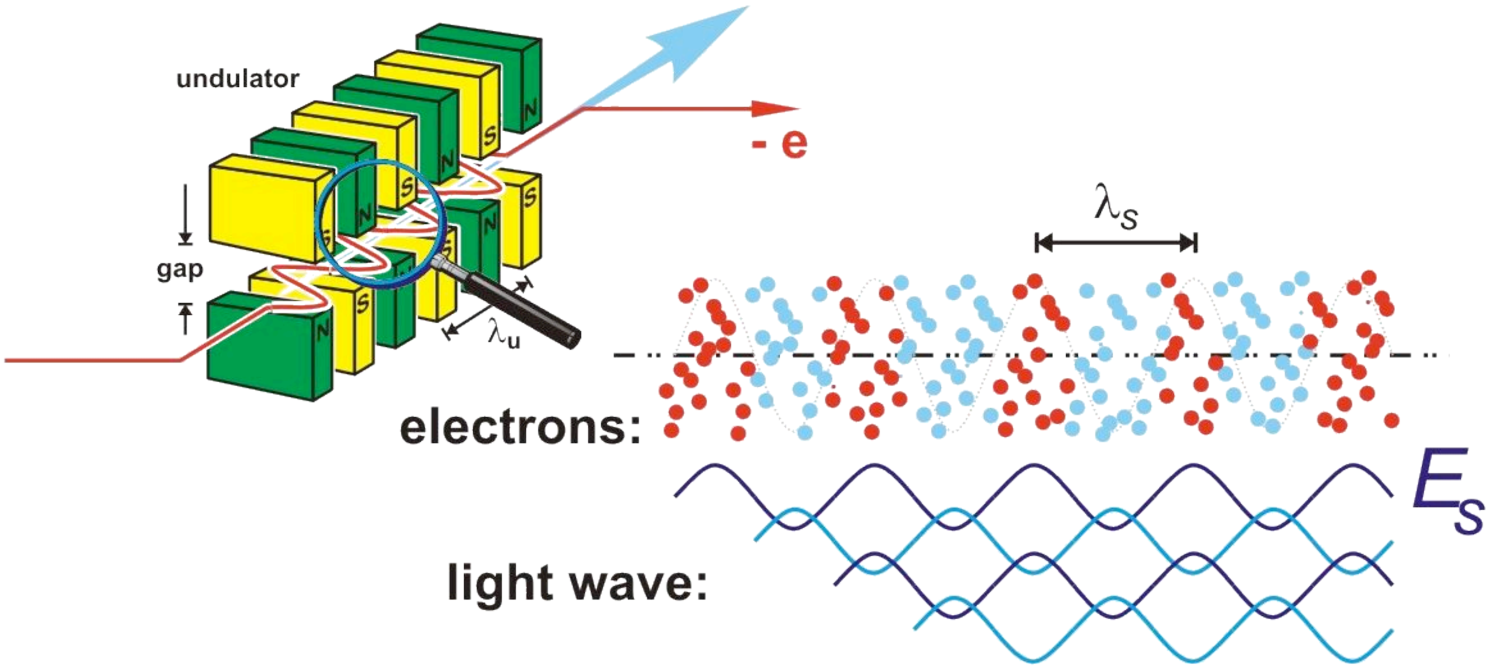
$$P_T = N_e\frac{e^2}{6\pi\varepsilon_o c^3}\gamma^4\left\langle \dot{v}_\perp^2 \right\rangle$$

Coherent Stimulated Radiation Power:

$$P_T = \frac{N_e^2 e^2}{6\pi\varepsilon_o c^3}\gamma^4\left\langle \dot{v}_\perp^2 \right\rangle$$

WE NEED micro-BUNCHING !

Spontaneous Emission ==> Random phases



Radiated Power:

$P \propto N$

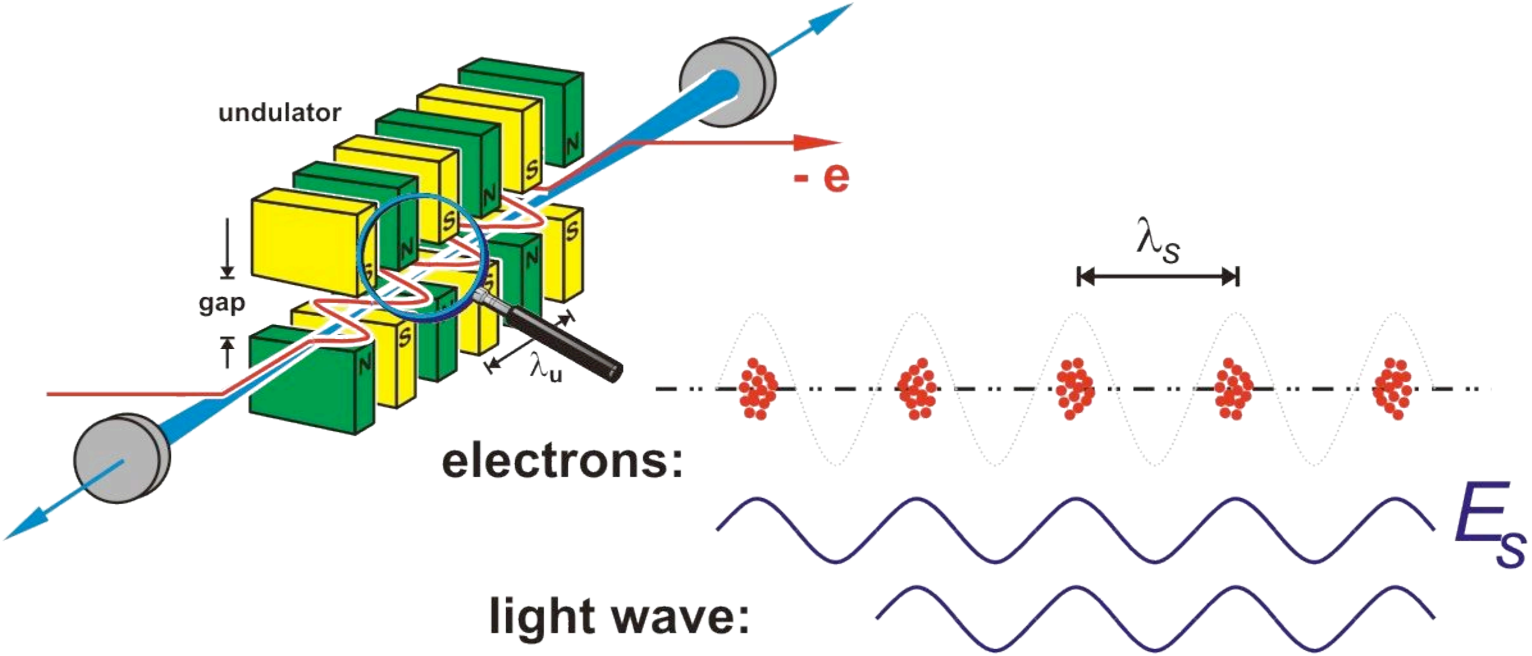
destructive interference

⟶ **shotnoise radiation**

Spontaneous Emission ==> Random Claps

Coherent Claps ==> Stimulated Emission

Coherent Light ==> Stimulated Emission

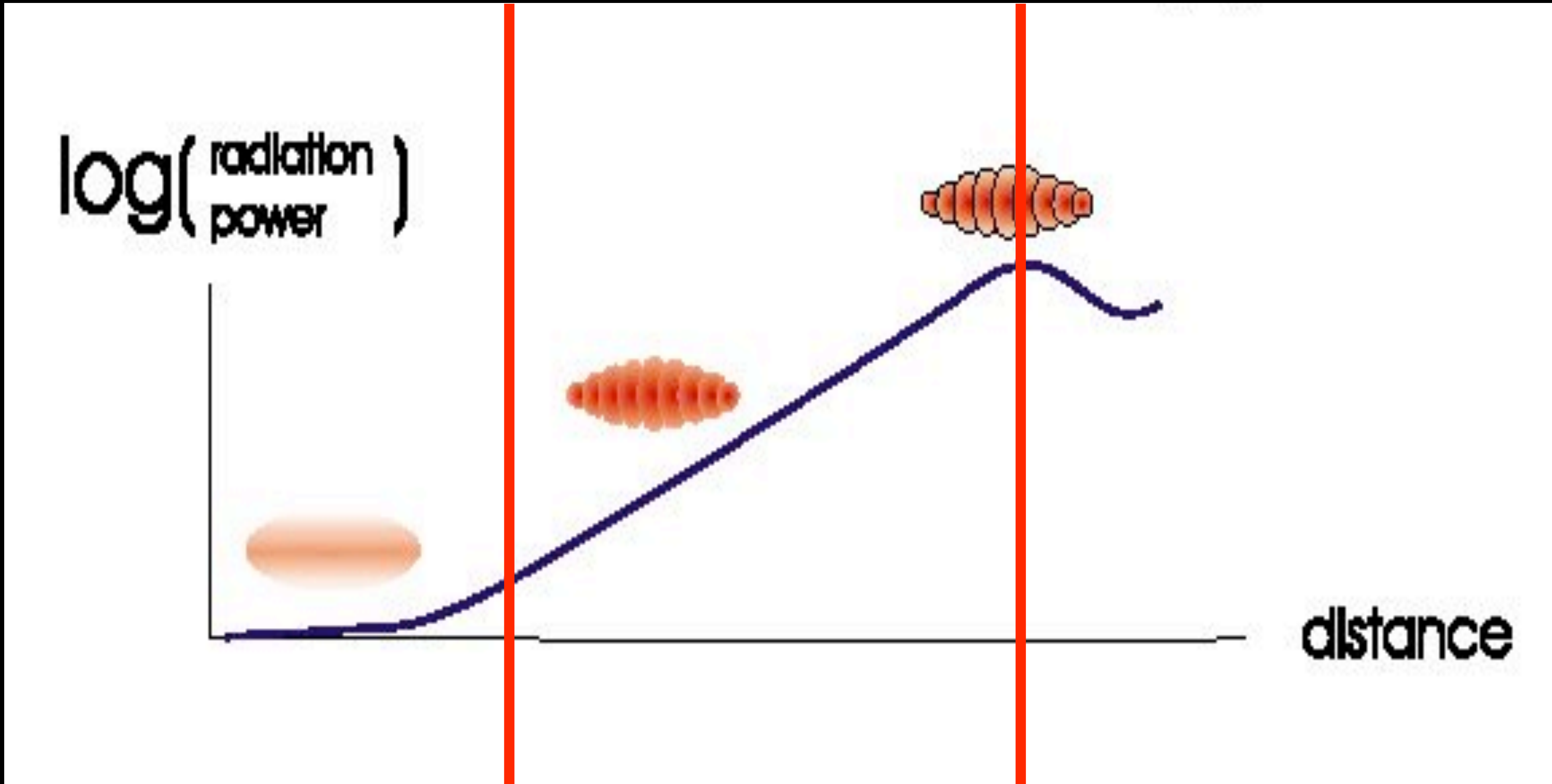


Radiated Power :

$P \propto N^2$

constructive interference

→ **enhanced emission**



Letargy

Spontaneous Emission

Low Gain

Slow Bunching

Exponential Growth

Stimulated emission

High Gain

Enhanced Bunching

Saturation

Absorption

No Gain

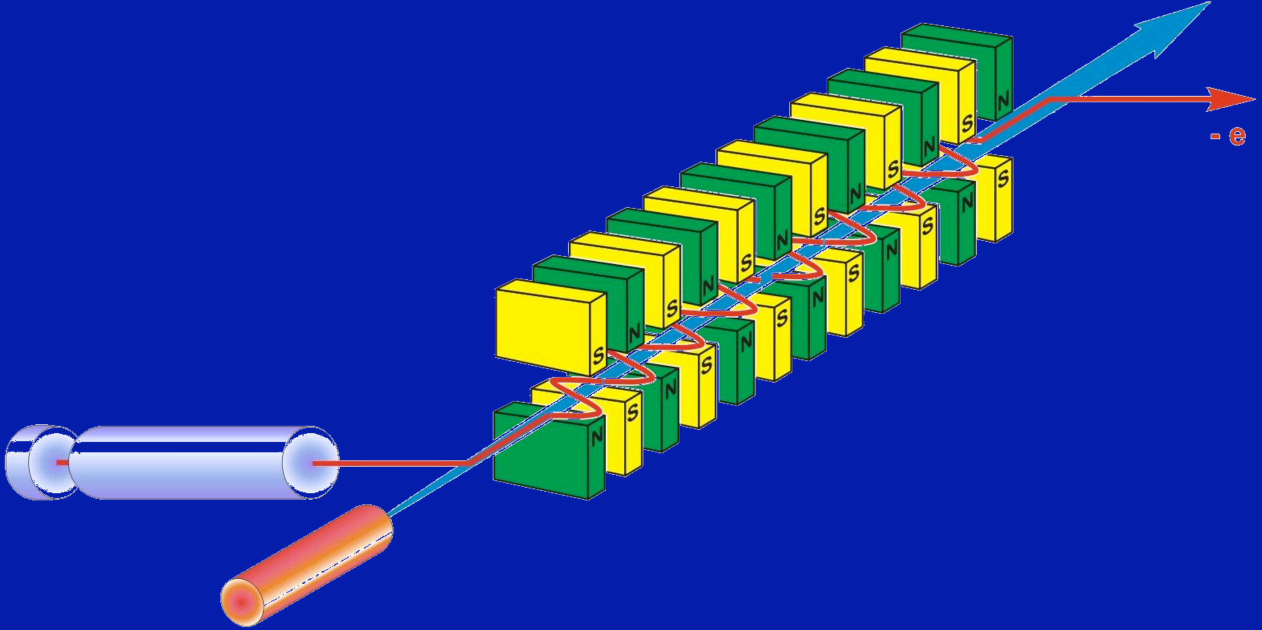
Debunching

Free Electron Laser 1D Self Consistent Model

Consider"seeding"by an external light source with wavelength λ_r
The light wave is co-propagating with the relativistic electron beam

$$\frac{d\gamma}{dt} = -\frac{e}{mc}\vec{E}\cdot\vec{\beta} = -\frac{e}{mc}E_{\perp}\beta_{\perp}$$

Energy exchange occurs only if there is transverse motion



Newton Lorentz Equations

E, B

Problem: electrons are slower than light

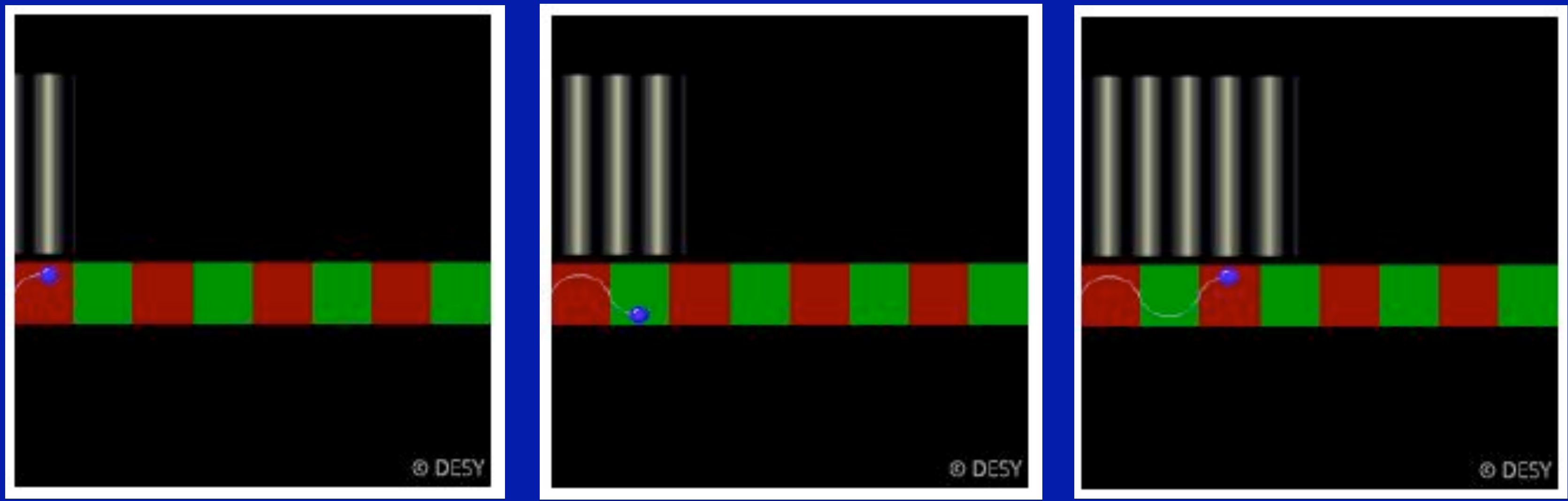
Question: can there be a continuous energy transfer from electron beam to light wave?

Answer: We need a Self Consistent Model

$J_\perp$

Maxwell Equations

(R. Bonifacio, C.Pellegrini, L.Narducci, Opt. Comm., 50, 373 (1984))



After one wiggler period the electron sees the radiation with the same phase if the flight time delay is exactly one radiation period: $\Delta t = t_e - t_{ph} = T_{rad}$

$$\Delta t = \frac{\lambda_u}{c\beta_{//}} - \frac{\lambda_u}{c} = \frac{\lambda_{rad}}{c} \longrightarrow \lambda_{rad} = \frac{1-\bar{\beta}_{//}}{\bar{\beta}_{//}}\lambda_u \xrightarrow{\bar{\beta}_{//}\approx 1} \lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$\gamma_{res} \approx \sqrt{\frac{\lambda_u}{2\lambda_{rad}}\left(1+\frac{K^2}{2}\right)}$$

The relative slippage of the radiation envelope through the electron beam can be neglected, provided that $l_b \gg N_u\lambda_r$ (Steady State Regime)

Plane wave with constant amplitude, co-propagating with the electron beam:

$$E_x(z,t) = E_o \cos(k_l z - \omega_l t + \psi_o)$$
$$k_l = \frac{\omega_l}{c} = \frac{2\pi}{\lambda_l}$$

$$\frac{d\gamma}{dt} = -\frac{e}{m_e c} E_x \beta_x = -\frac{e}{m_e c}\frac{K}{\gamma}\cos(k_u z) E_o \cos(k_l z - \omega_l t + \psi_o)$$
$$= -\frac{eE_o K}{2\gamma m_e c}\left[\cos((k_l + k_u)z - \omega_l t + \psi_o) + \cos((k_l - k_u)z - \omega_l t + \psi_o)\right]$$
$$= -\frac{eE_o K}{2\gamma m_e c}\left[\cos\psi - \cos\overline{\psi}\right]$$

Ponderomotive phase:

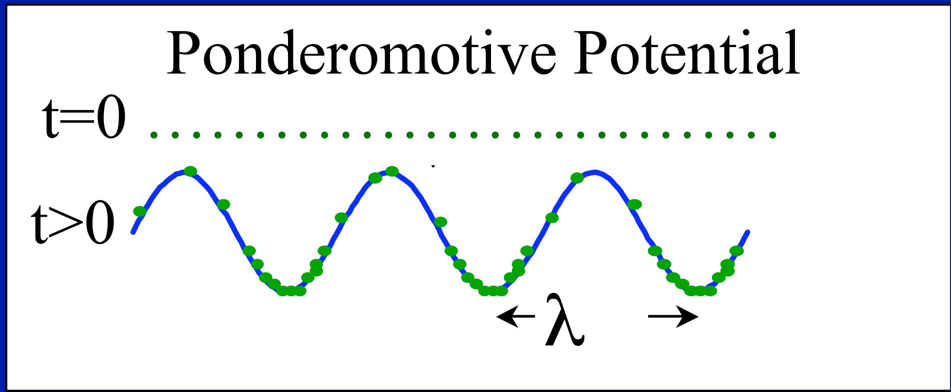
$$\psi(t) = (k_l + k_u)z - \omega_l t + \psi_o$$

Fast oscillating phase (we can neglect it):

$$\overline{\psi} = \psi - 2k_u z$$

In a resonant and randomly phased electron beam, nearly one half of the electrons absorbs energy and one half loses energy, with no net energy exchange.

If the undulator is sufficiently long the energy modulation becomes a phase modulation: the electrons self-bunch on the scale of a radiation wavelength.



The phase of the combined "ponderomotive" (radiation + undulator) field, propagates in forward direction with a phase velocity that corresponds to the velocity of the resonant particle:

$$\frac{d\psi}{dt} = (k_l + k_u)\bar{v}_z - k_l c = 0 \longrightarrow \bar{v}_z = \frac{k_l c}{k_l + k_u} = c\left(1 - \frac{1}{2\gamma_r^2}\left(1 + \frac{K^2}{2}\right)\right)$$

The particles bunch around a phase ψ_r for which there is weak coupling with the radiation:

Bunching Parameter:

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i\psi_j} = \left\langle e^{-i\psi_j} \right\rangle$$

$|b| \approx 0$ Spontaneous emission

$|b| \to 1$ Stimulated emission

Motion in the potential well: the electron pendulum equations

For particles with off resonance energy $\gamma \neq \gamma_r$, the ponderomotive phase is no longer constant

$$\frac{d\psi}{dt} = (k_l + k_u)\bar{v}_z - k_l c \overset{k_u << k_l}{\approx} k_l c\left(\frac{k_u}{k_l} - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right) = \frac{k_l c}{2}\left(\frac{1}{\gamma_r^2} - \frac{1}{\gamma^2}\right)\left(1 + \frac{K^2}{2}\right)$$

$$\frac{d\psi}{dt} \approx k_u c\,\frac{\gamma^2 - \gamma_r^2}{\gamma_r^2} \approx 2k_u c\,\frac{\gamma - \gamma_r}{\gamma_r} = 2k_u c\eta$$

$$\eta = \frac{\gamma - \gamma_r}{\gamma_r} << 1$$

$$\frac{d\eta}{dt} = \frac{1}{\gamma_r}\frac{d\gamma}{dt} = -\frac{eE_o K}{2\gamma_r^2 m_e c}\cos\psi$$

Two coupled first order differential equations

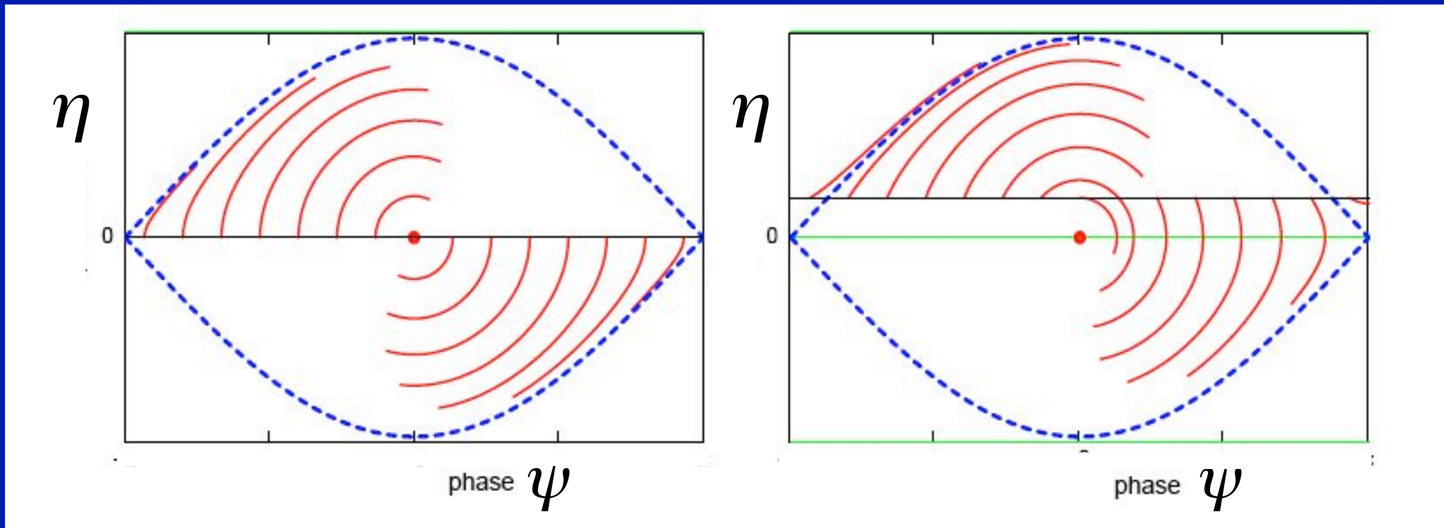
Combining the two coupled first order differential equations:

$$\begin{cases} \dfrac{d\psi}{dt} = 2k_u c\eta \\ \dfrac{d\eta}{dt} = -\dfrac{eE_o K}{2\gamma_r^2 m_e c}\cos\psi \end{cases}$$

$$\frac{d^2\psi}{dt^2} = -\frac{eE_o K k_u}{\gamma_r^2 m_e}\cos\psi$$

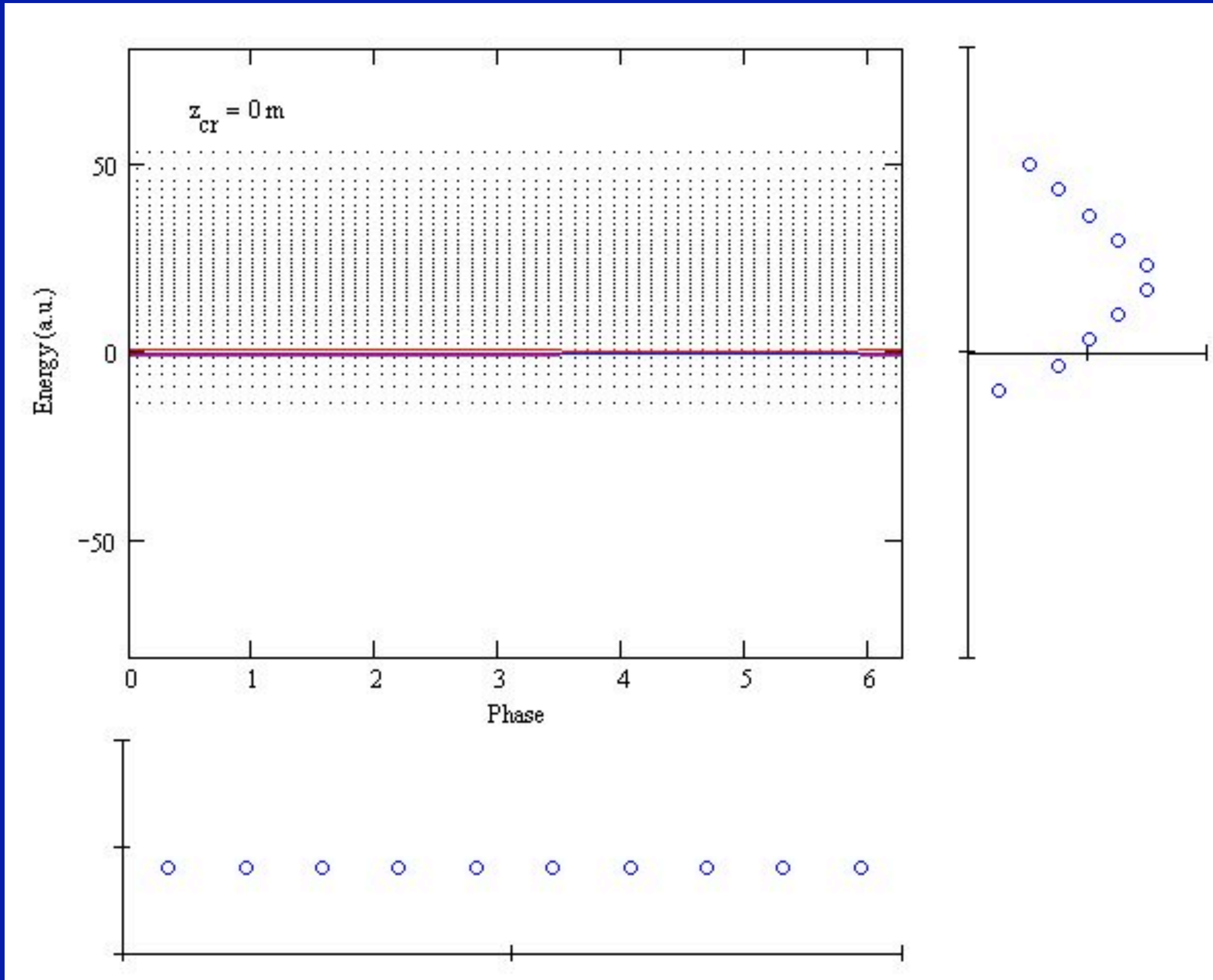
$$\frac{d^2\psi}{dt^2} + \Omega^2 \cos\psi = 0$$

$$\Omega^2 = \frac{eE_o K k_u}{\gamma_r^2 m_e}$$

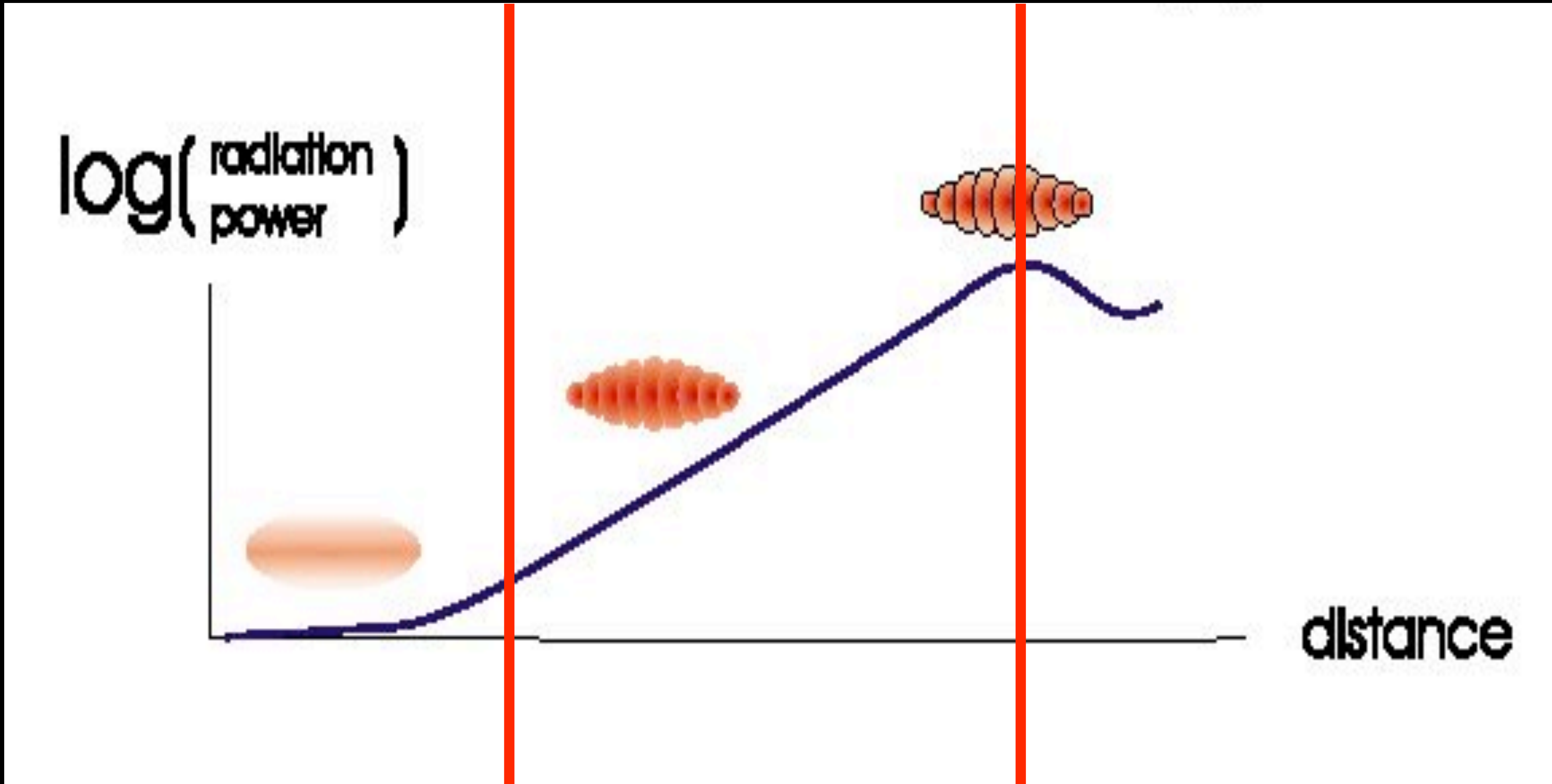


Separatrix

$$\eta_{sep} = \pm\sqrt{\frac{eEK}{k_u m_e c^2 \gamma_r^2}}\cos\left(\frac{\psi-\psi_r}{2}\right)$$



Courtesy L. Giannessi (Perseo in 1D mode http://www.perseo.enea.it)



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No Gain

Debunching

High gain FEL regime

$$\left[\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\tilde{E}_x(z,t) = \mu_o \frac{\partial j_x}{\partial t}$$

Test solution

$$\tilde{E}_x(z,t) = \tilde{E}_x(z)e^{i(k_l z - \omega_l t)} = \frac{E_o(z)e^{i\varphi}}{2}e^{i(k_l z - \omega_l t)}$$

$$\left[2ik_l\tilde{E}'_x(z) + \tilde{E}''_x(z)\right]e^{i(k_l z - \omega_l t)} = \mu_o \frac{\partial j_x}{\partial t}$$

Slowly Varying Envelope Approximation (SVEA):

the amplitude variation within one undulator period is very small

$$\tilde{E}'_x(z) << \frac{\tilde{E}_x(z)}{\lambda_u} \quad \Rightarrow \quad \tilde{E}''_x(z) << \frac{\tilde{E}'_x(z)}{\lambda_u}$$

$$\frac{d\tilde{E}_x(z)}{dz} = -\frac{i\mu_o}{2k_l}\frac{\partial j_x}{\partial t}e^{-i(k_l z - \omega_l t)}$$

To be consistent with SVEA we should average also the source term over a time $T \approx n\lambda_l/c$ in which $\tilde{E}_x(z)$ could be considered constant

$$2ik_l\tilde{E}'_x = \mu_o \frac{1}{T}\int_t^{t+T}\frac{\partial \tilde{j}_x}{\partial t}e^{-i(k_l z - \omega_l t)}dt$$

$$\frac{1}{T}\int_t^{t+T}\frac{\partial\tilde{j}_x}{\partial t}e^{-i(k_l z-\omega_l t)}dt = \frac{-i\omega_l}{T}\int_t^{t+T}\tilde{j}_x e^{-i(k_l z-\omega_l t)}dt$$

Integration by parts

$$\tilde{j}_x = \frac{e}{S}\sum_{j=1}^{N} v_{xj}\delta\left(z - z_j(t)\right) = \frac{e}{Sv_z}\sum_{j=1}^{N} v_{xj}\delta\left(t - t_j(z)\right)$$

Beam model

S: transverse beam area

Exercise: verify there are not misprints (~mistakes):

$$\frac{1}{T}\int_t^{t+T}\tilde{j}_x e^{-i(k_l z-\omega_l t)}dt = \frac{e}{Sv_zT}\int_t^{t+T}\sum_{j=1}^{N} v_{xj}\delta\left(t - t_j(z)\right)e^{-i(k_l z-\omega_l t)}dt$$

$$= \frac{e}{V}\sum_{j=1}^{N} v_{xj}e^{-i(k_l z-\omega_l t_j)} \qquad \textbf{where: } V = Sv_zT$$

$$= \frac{e}{V}\sum_{j=1}^{N}\frac{Kc}{\gamma_j}cos(k_u z)e^{-i(k_l z-\omega_l t_j)} \qquad \textbf{using } v_{xj} = ...$$

$$= \frac{eKc}{V\gamma_r}\sum_{j=1}^{N} e^{-i((k_l+k_u)z-\omega_l t_j)} = \frac{eKc}{V\gamma_r}\sum_{j=1}^{N} e^{-i\psi_j} \qquad \textbf{using } \gamma_j \approx \gamma_r$$

$$= \frac{eKc}{V\gamma_r}N\left\langle e^{-i\psi_j}\right\rangle = \frac{eKc}{\gamma_r}n_e\left\langle e^{-i\psi_j}\right\rangle \qquad \textbf{where } n_e = \frac{N}{V}$$

Three coupled first order differential equations.

They describe a collective instability of the system which leads to electron self-bunching and to exponential growth of the radiation until saturation effects set a limit on the conversion of electron kinetic energy into radiation energy.

$$j = 1, N_e$$

$$\begin{cases} \dfrac{d\tilde{E}_x}{dz} = \dfrac{\omega_l \mu_o}{2k_l} \dfrac{eKc}{\gamma_r} n_e \left\langle e^{-i\psi_j} \right\rangle \\ \dfrac{d\psi_j}{dz} = 2k_u \eta_j \\ \dfrac{d\eta_j}{dz} = -\dfrac{eK}{2m_e c^2 \gamma_r^2} \Re e\left(\tilde{E}_x e^{i\psi_j} \right) \end{cases}$$

$$b = \frac{1}{N} \sum_{j=1}^{N} e^{-i\psi_j} = \left\langle e^{-i\psi_j} \right\rangle$$

Bunching parameter

Saturation effects prevent the beam to radiate as N^2, limting the radiated power scaling to $N^{4/3}$, due to a competition between neighbours slices .

When propagation effects and slippage are relevant, i.e. when the elctron beam is as short as a slippage length, the emitted radiation leaves the bunch before saturation occurs and the power scaling becomes N^2 (Super-radiant or Single Spike regime)

Can there be a continuous energy transfer from electron beam to light wave?

$$\frac{d\eta_j}{dz} \propto \Re e\left(\tilde{E}_x e^{i\psi_j}\right) \propto \Re e\left(E_o(z) e^{i(\psi_j + \varphi)}\right) = E_o(z)\cos\left(\psi_j + \varphi\right) = E_o(z)\cos(\Psi)$$

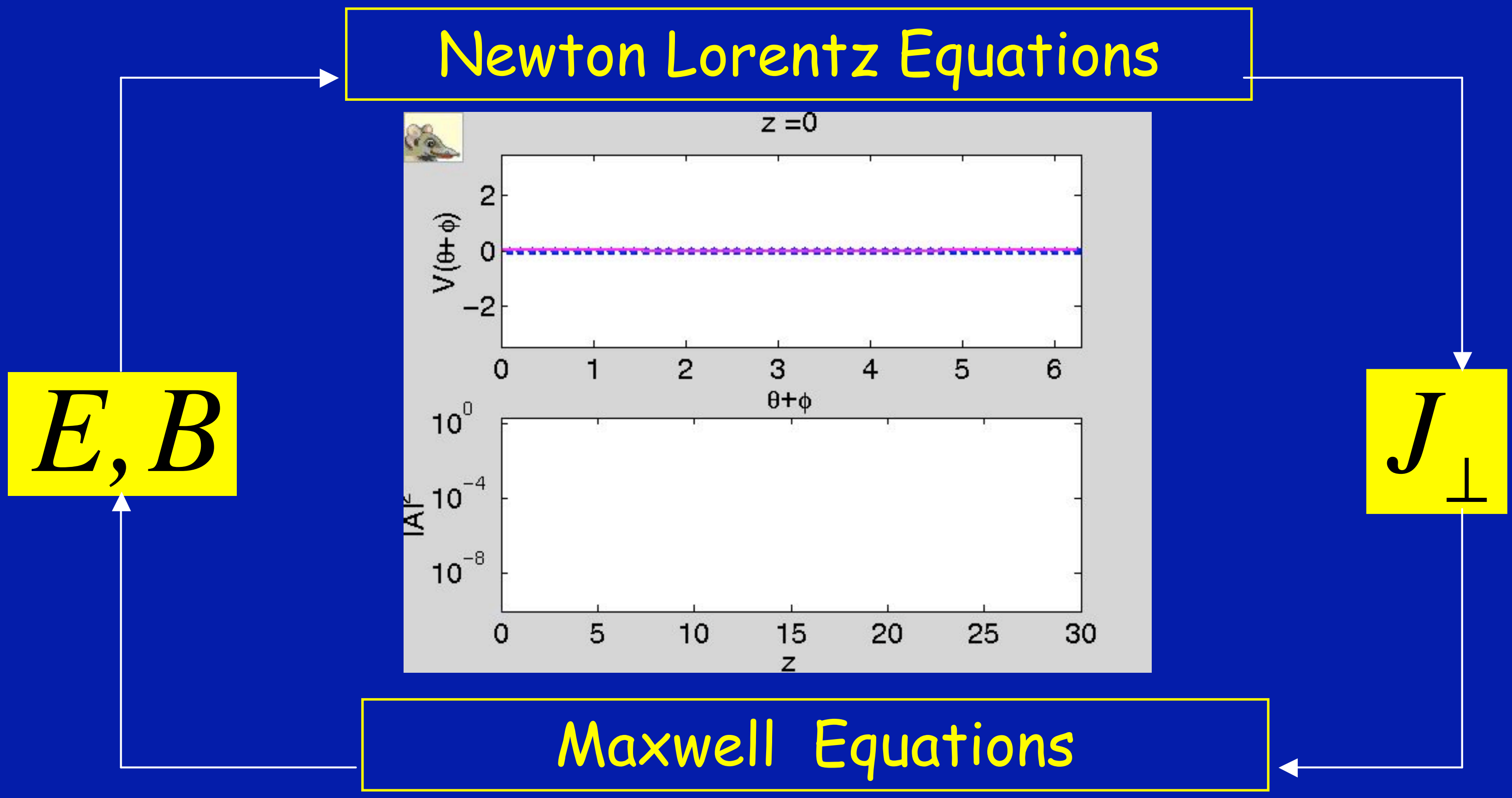
$$\Psi = \left(k_l + k_u\right)z - k_l ct - \frac{d\varphi}{dt}t$$

$$v_{phase} = \frac{k_l c - \dot{\varphi}}{k_l + k_u} < \bar{v}_z$$

The electron beam acts as a dielectric medium which slows down the phase velocity of the ponderomotive field compared to the average electron longitudinal velocity.

Hence the bunching turns out to occur around a phase corresponding to radiation energy gain.

The particles within a micro-bunch radiate coherently. The resulting strong radiation field enhances the micro-bunching even further. Result: collective instability, exponential growth of radiation power.

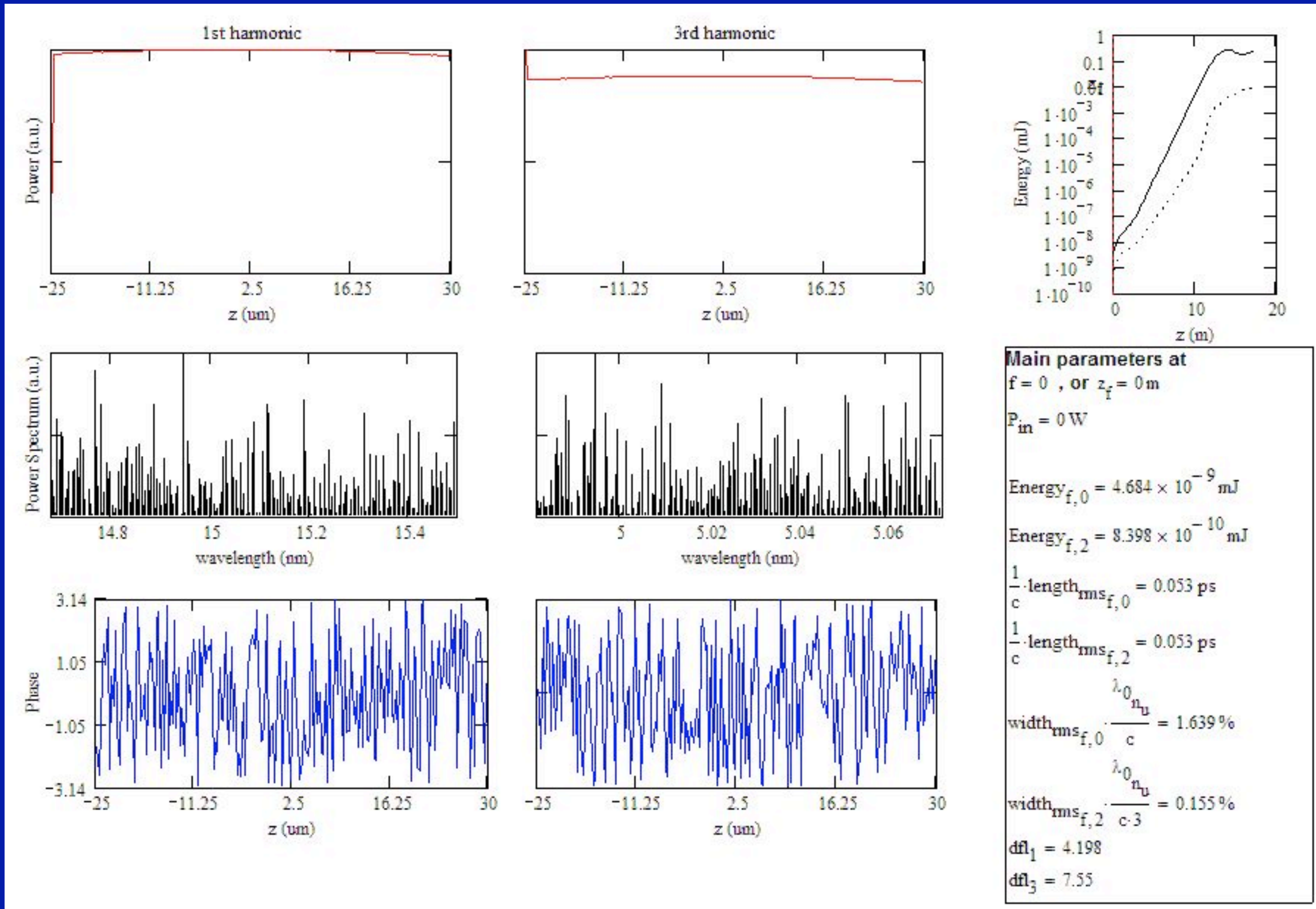


SASE FEL at short wavelengths require a very intense, high quality e-beam

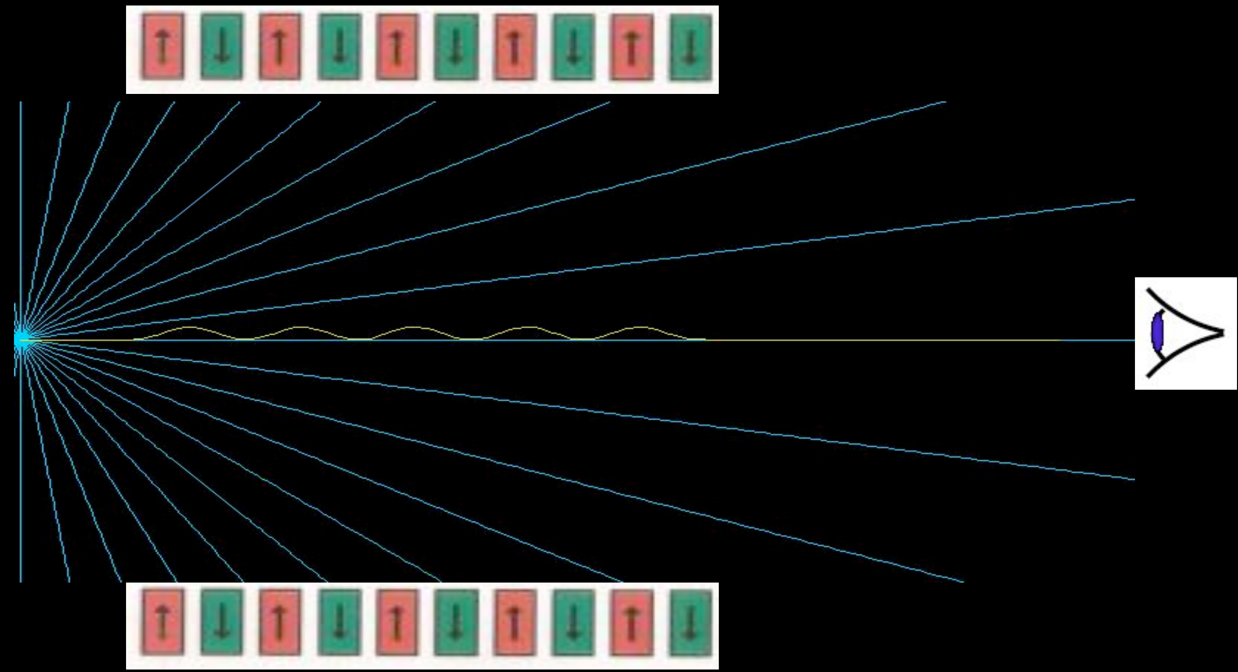
- FEL Parameter $\rho = 0.136 \frac{1}{\gamma_r} J^{1/3} B_u^{2/3} \lambda_u^{4/3}$
- Exponential growth $P(z) = \frac{P_0}{9} \exp\left(\frac{z}{L_G}\right)$
- Gain Length $L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$
- Saturation power $P_{sat} = \rho P_{beam} \propto N_e^{4/3}$
- Constraint on emittance $\varepsilon = \frac{\varepsilon_n}{\gamma} < \frac{\lambda_0}{4\pi}$
- Constraint on energy spread $\Delta\gamma/\gamma < \rho$
- Relative bandwidth $\frac{\Delta\omega}{\omega} = \sqrt{\frac{\rho}{N_u}}$

http://xfel.desy.de/

SASE

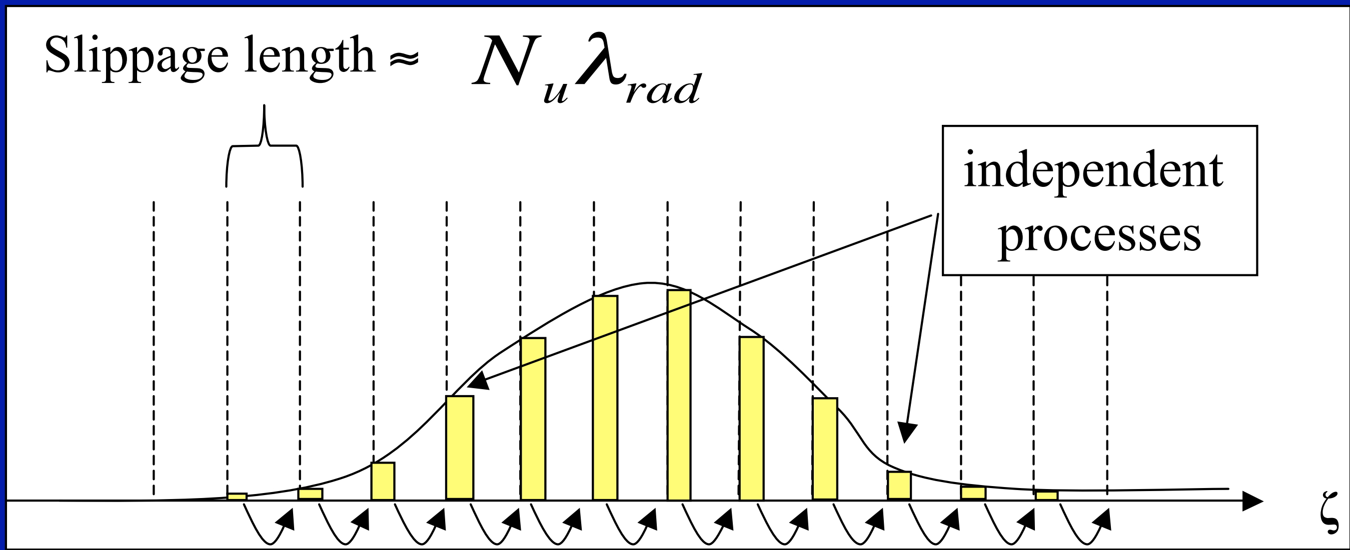


Courtesy L. Giannessi (Perseo in 1D mode http://www.perseo.enea.it)



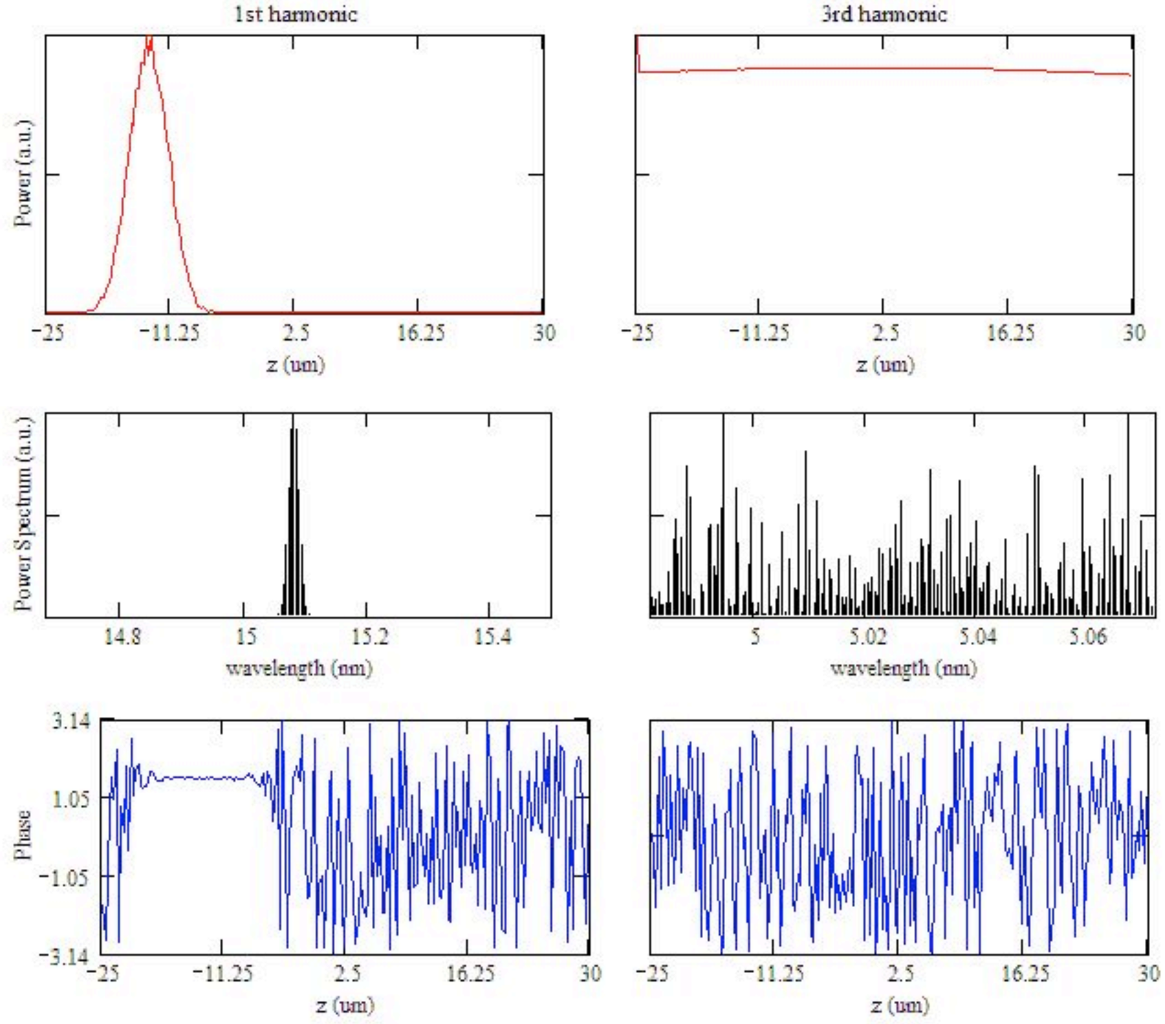
Radiation Simulator – T. Shintake, @ http://www-xfel.spring8.or.jp/Index.htm

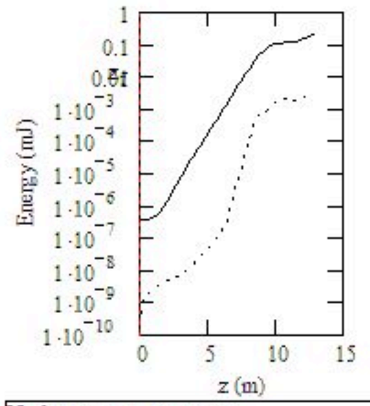
SASE Longitudinal coherence



The radiation "slips" over the electrons for a distance $N_u\lambda_{rad}$

SEEDING





Main parameters at

f = 59 , or z_f = 12.936 m

$P_{in} = 2 \times 10^4$ W

$Energy_{f,0} = 0.212$ mJ

$Energy_{f,2} = 3.751 \times 10^{-3}$ mJ

$\frac{1}{c} \cdot length_{rms_{f,0}} = 0.035$ ps

$\frac{1}{c} \cdot length_{rms_{f,2}} = 0.038$ ps

$width_{rms_{f,0}} \cdot \frac{\lambda 0_{n_u}}{c} = 0.117\,\%$

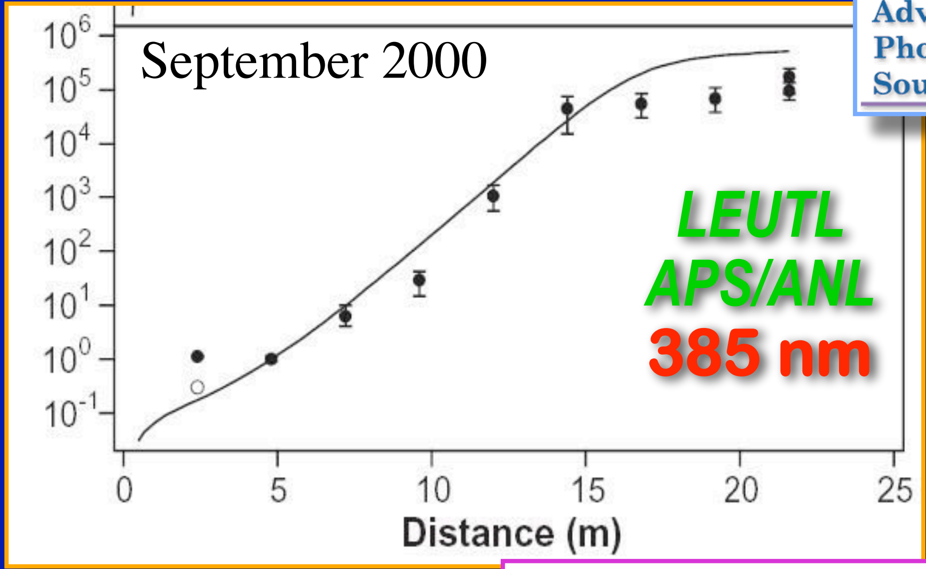
$width_{rms_{f,2}} \cdot \frac{\lambda 0_{n_u}}{c \cdot 3} = 0.05\,\%$

$dfl_1 = 1.639$

$dfl_3 = 2.282$

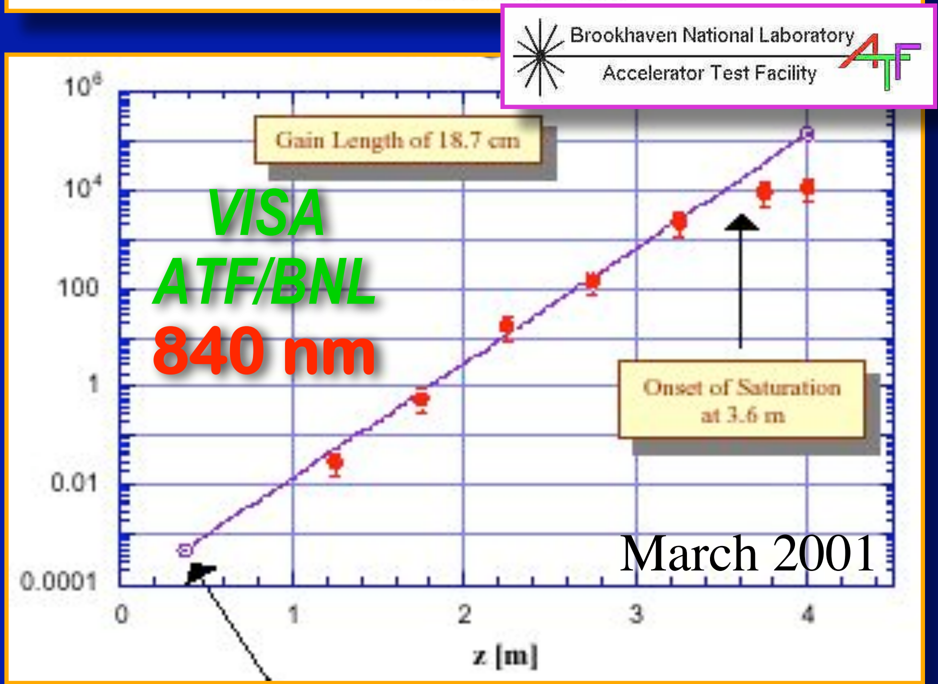
Courtesy L. Giannessi (Perseo in 1D mode http://www.perseo.enea.it)

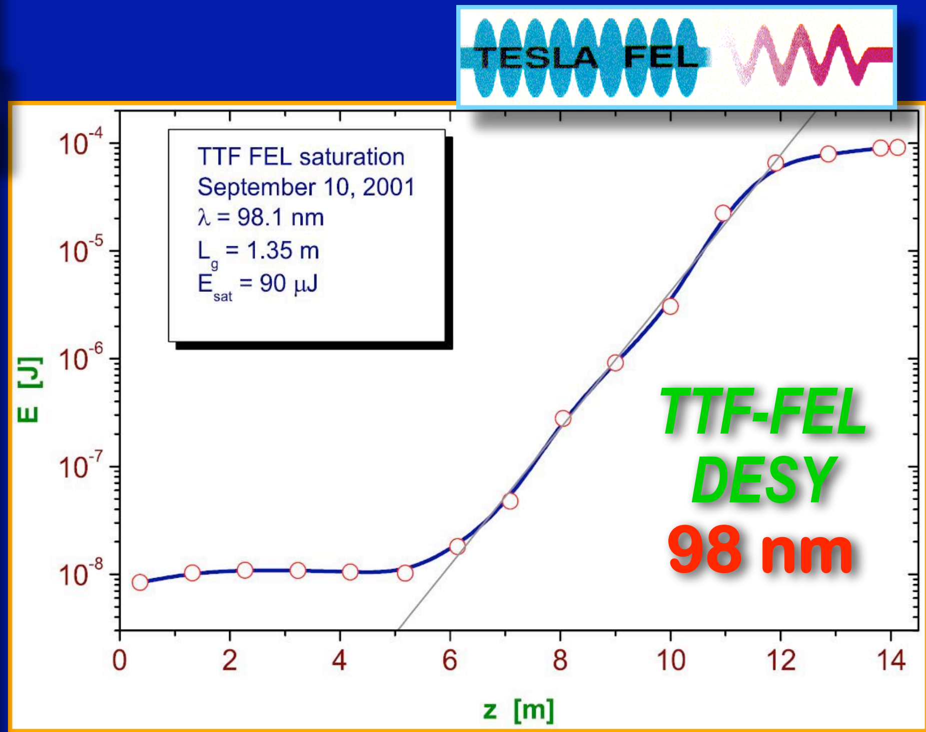
SASE Saturation Results



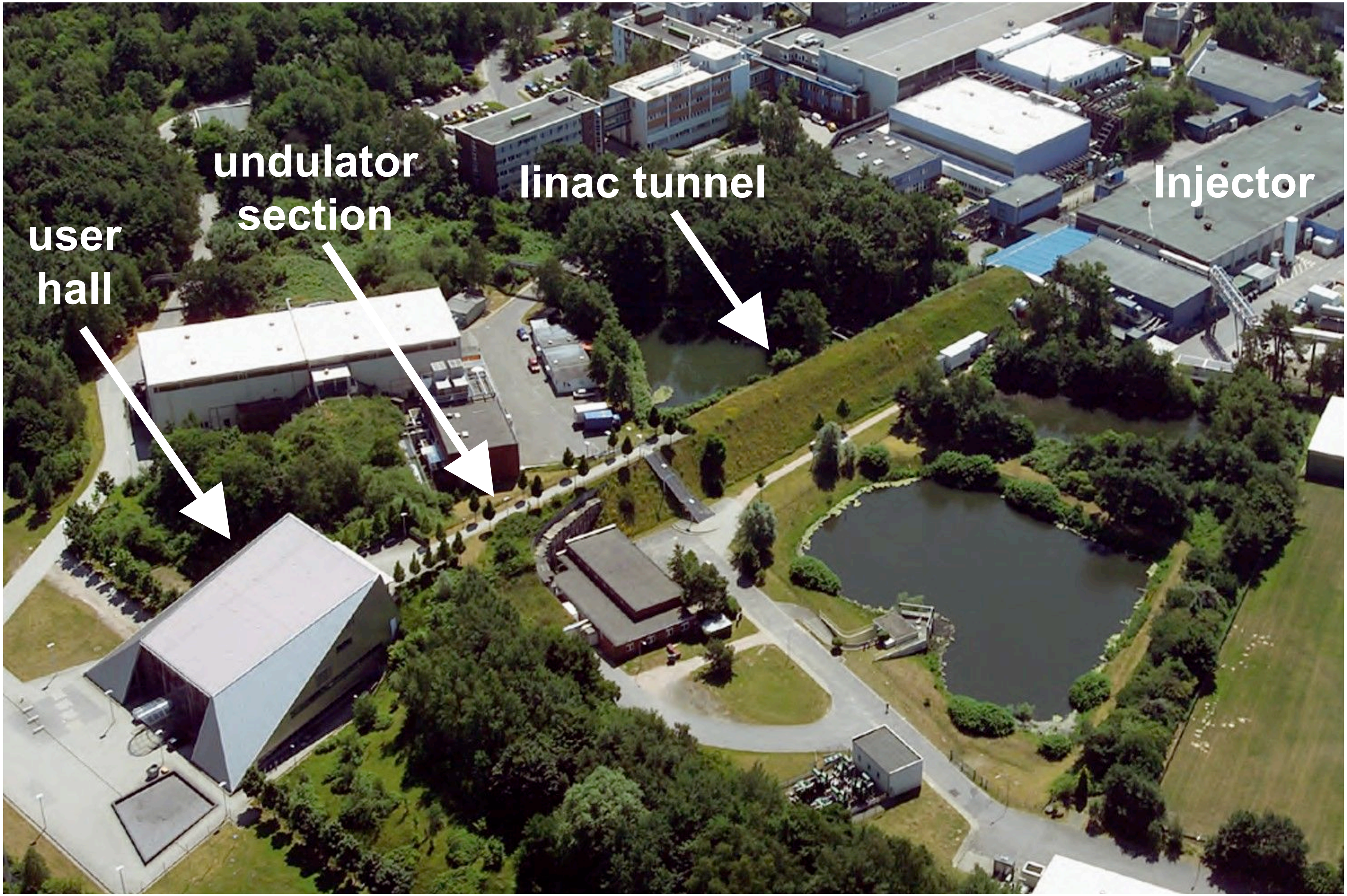
Since September 2000:

3 SASE FEL's demonstrate saturation



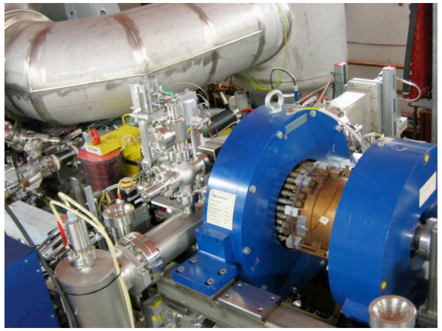


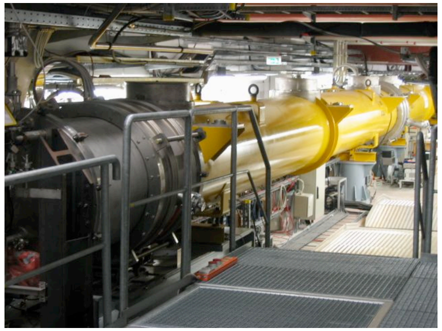
FLASH – VUV Single-Pass FEL, Hamburg

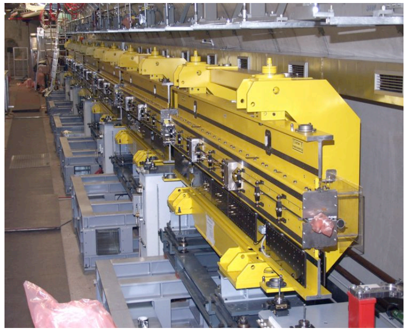


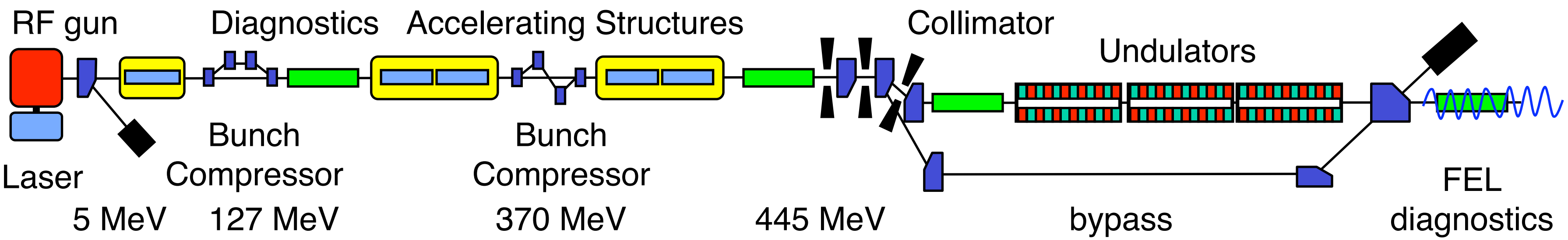
Courtesy Bart Faatz (DESY)

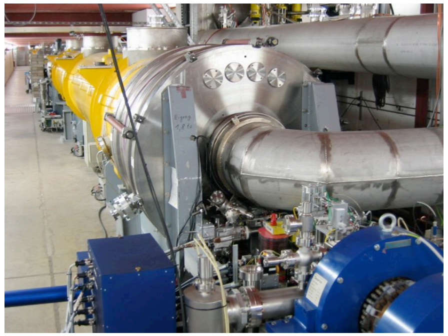
Layout of the VUV-FEL

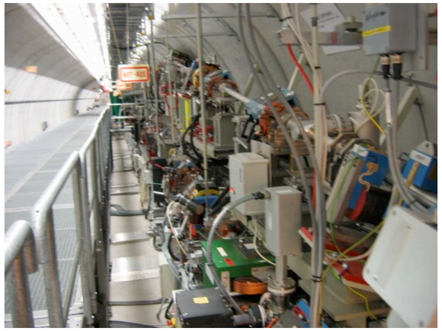


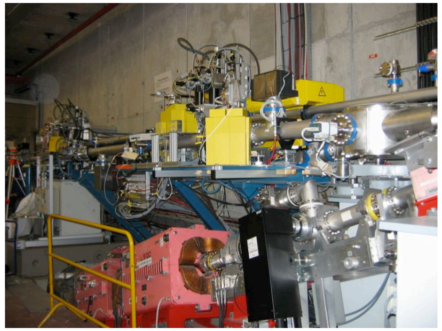








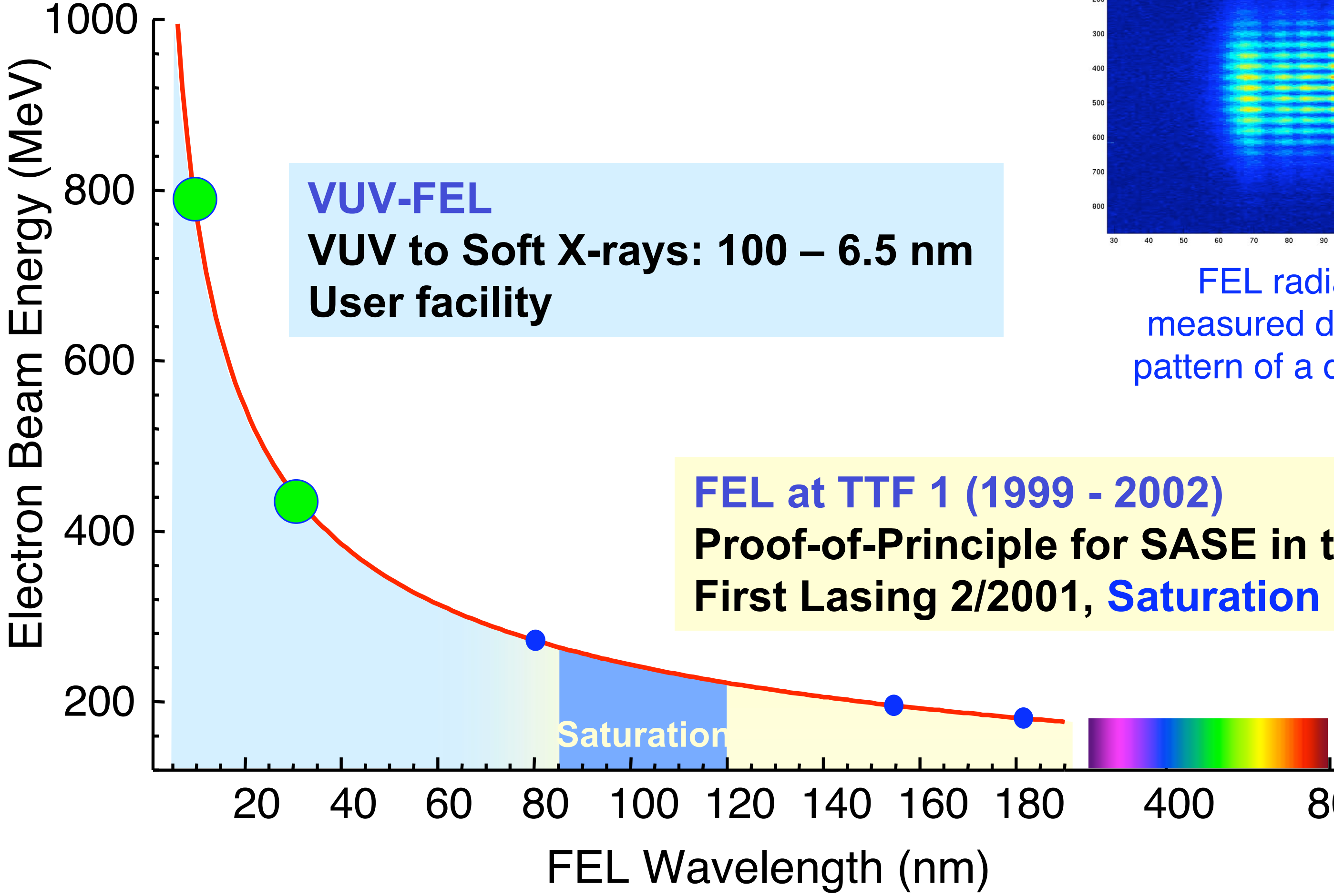


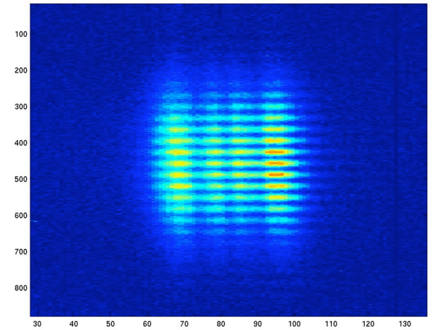


250 m

Courtesy Bart Faatz (DESY)

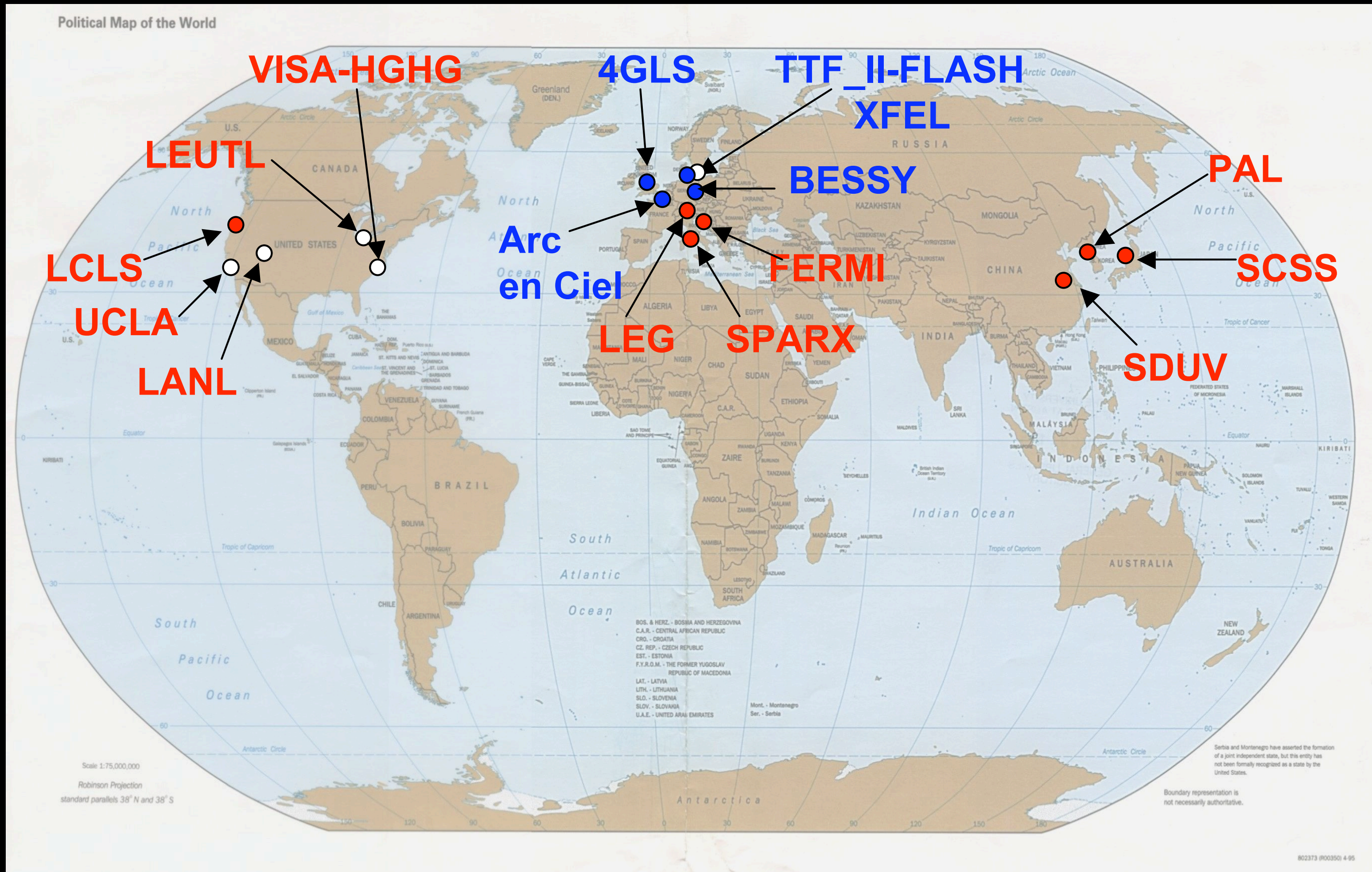
Beam Energy and Wavelength



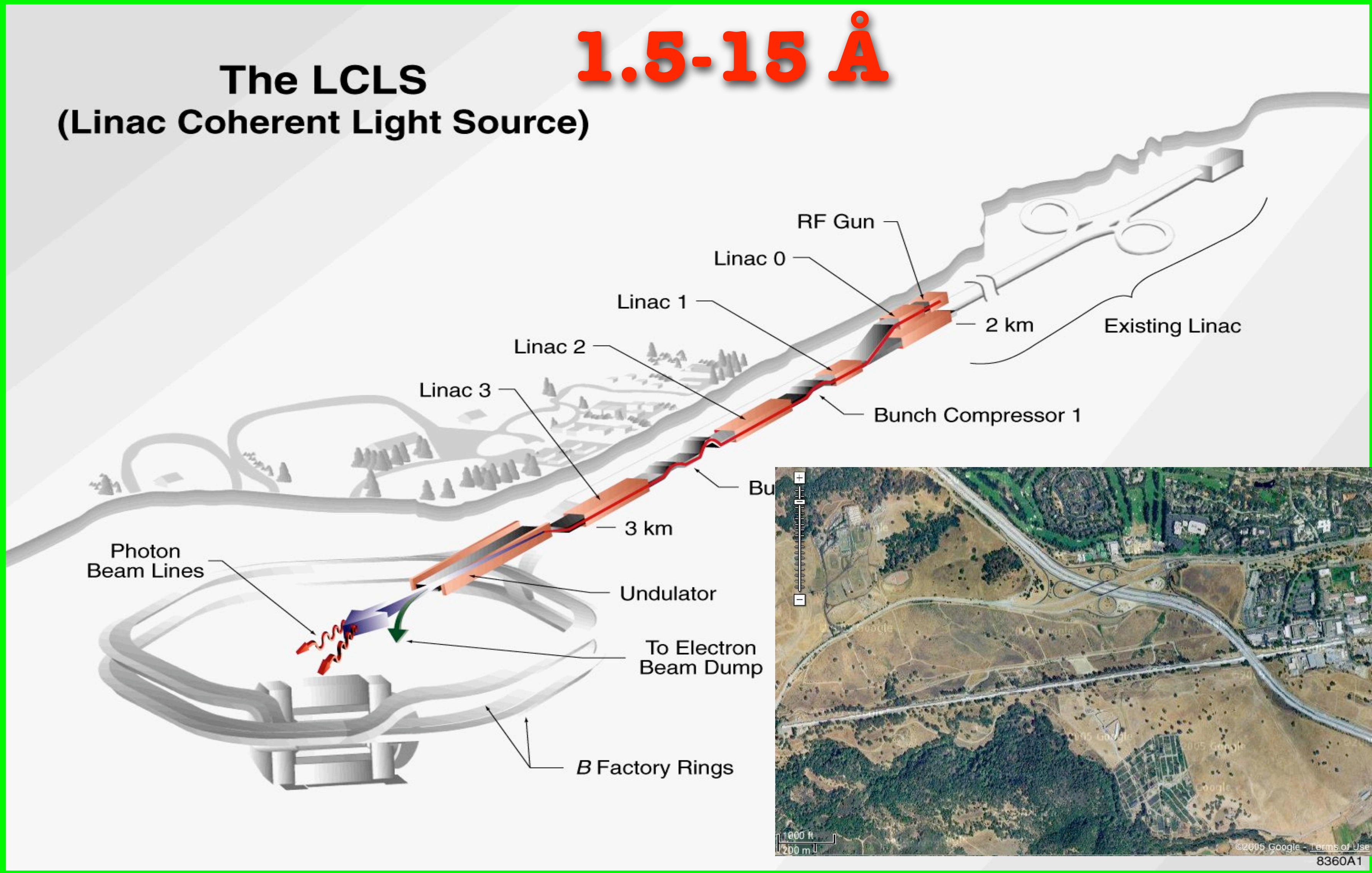


FEL radiation:
measured diffraction
pattern of a double slit

Short Wavelength SASE FEL

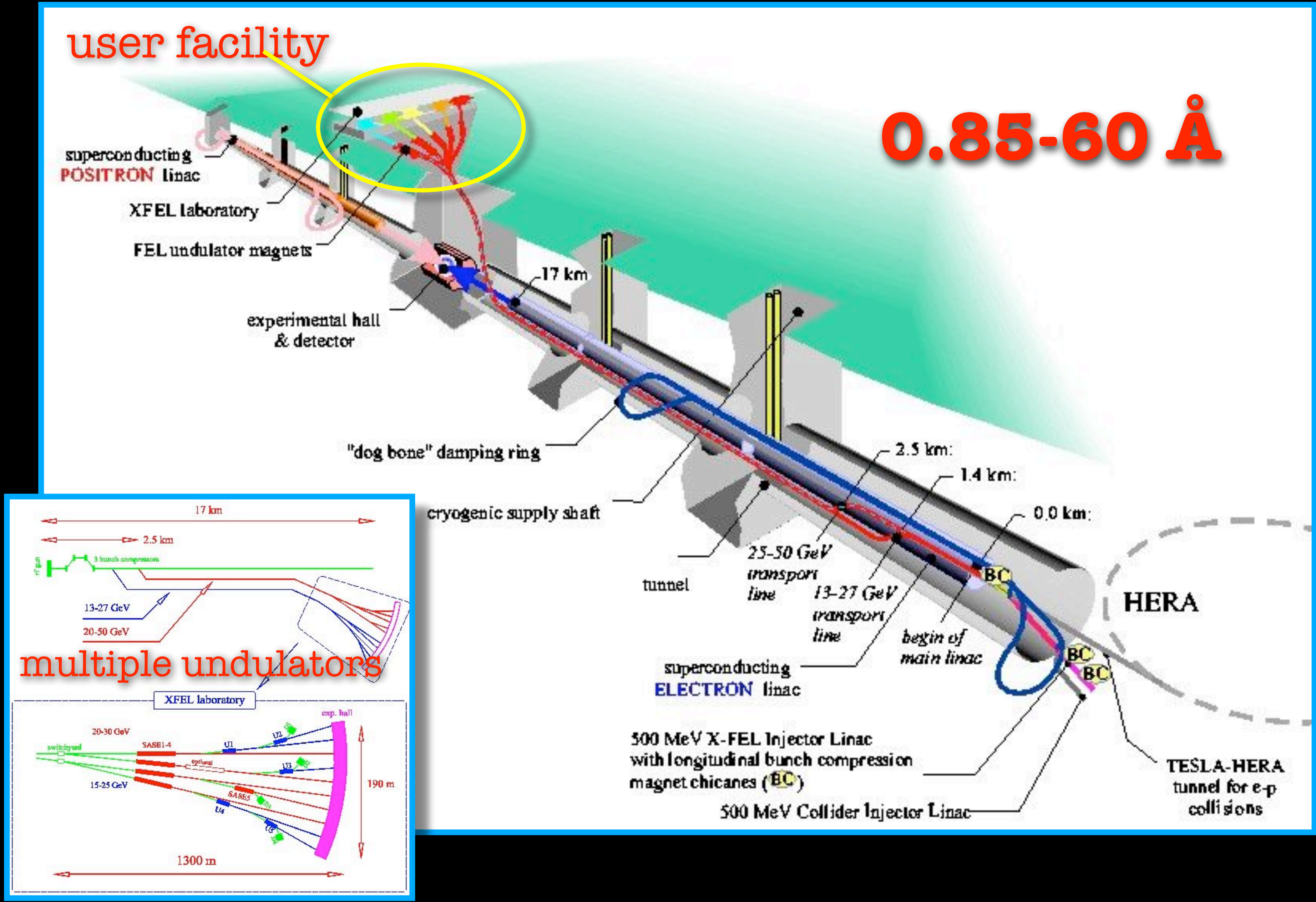


LCLS at *SLAC*



X-FEL based on last 1-km of existing *SLAC* linac

XFEL at *DESY*


user facility
0.85-60 Å
superconducting POSITRON linac
XFEL laboratory
FEL undulator magnets
experimental hall & detector
17 km
"dog bone" damping ring
cryogenic supply shaft
tunnel
25-50 GeV transport line
13-27 GeV transport line
2.5 km:
1.4 km:
0.0 km:
begin of main linac
superconducting ELECTRON linac
BC
HERA
500 MeV X-FEL Injector Linac with longitudinal bunch compression magnet chicanes (BC)
500 MeV Collider Injector Linac
TESLA-HERA tunnel for e-p collisions
17 km
2.5 km
3 bunch compressors
13-27 GeV
20-50 GeV
multiple undulators
XFEL laboratory
exp. hall
20-30 GeV
switchyard
SASE1-4
U1
U2
U3
U4
U5
SASE5
15-25 GeV
190 m
1300 m

THE END

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