

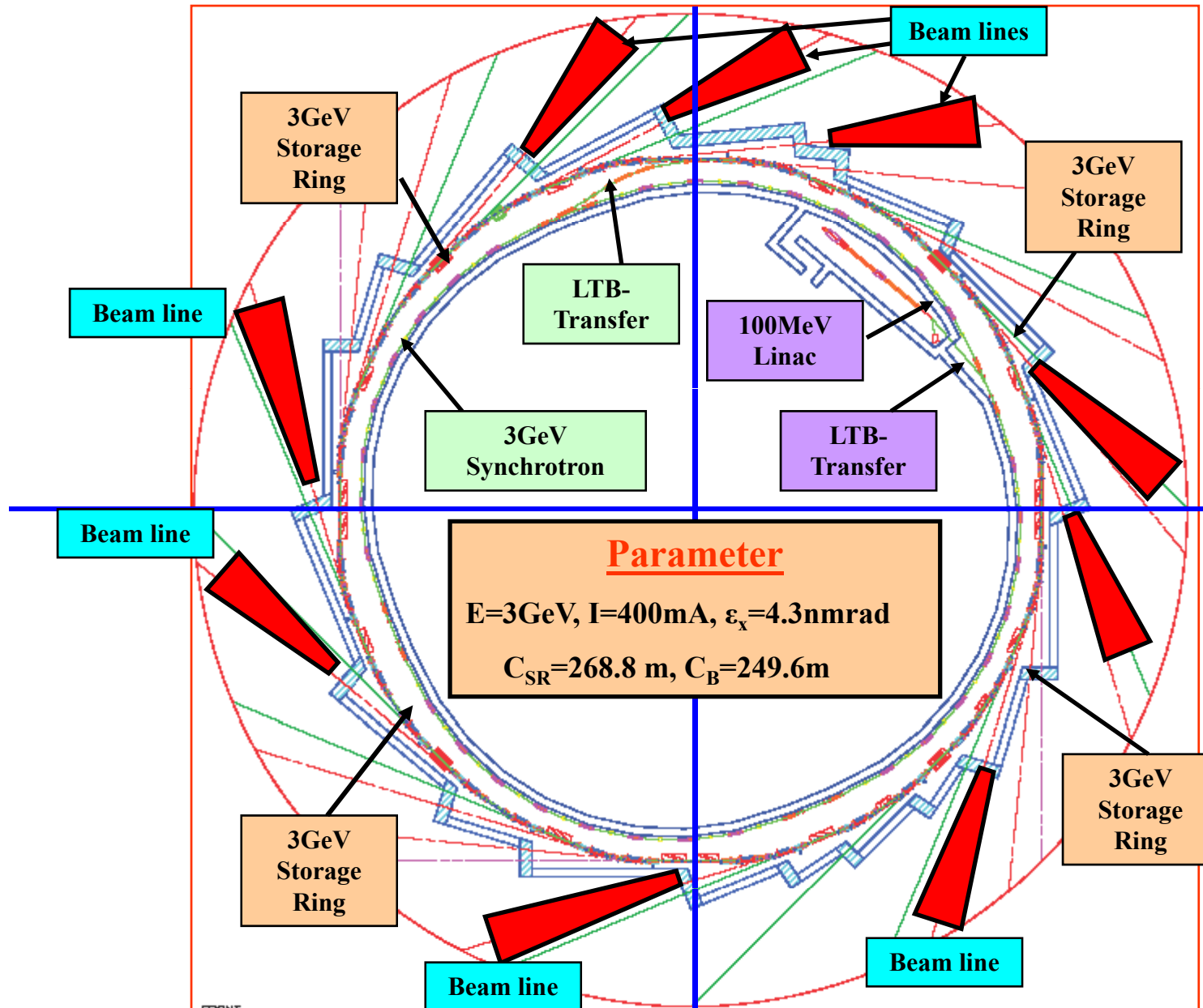


# Magnets (Warm)

Dieter Einfeld

CELLS / ALBA  
Spain / Barcelona  
[einfeld@cells.es](mailto:einfeld@cells.es)

# The Accelerator Complex of ALBA





## **Contents:**

**Introduction**

**Dipole Magnets**

**Quadrupoles**

**Combined Function Dipols**

**Sextupoles**

**Higher Harmonics**

**Correctors**

**Girders**

**Cooling**

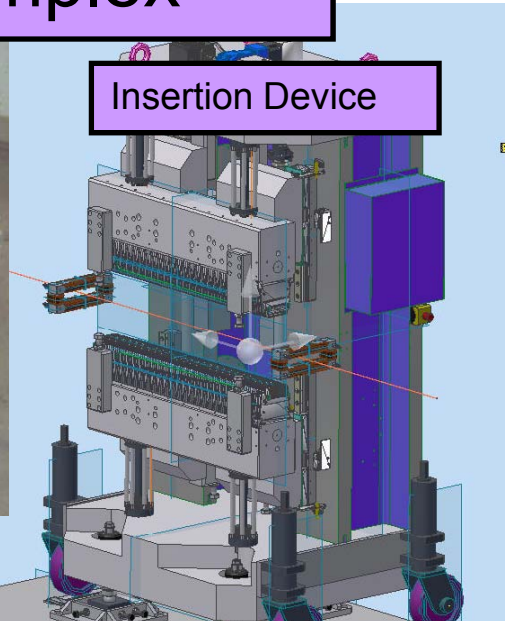
# Magnets within an Accelerator Complex



Quadrupole



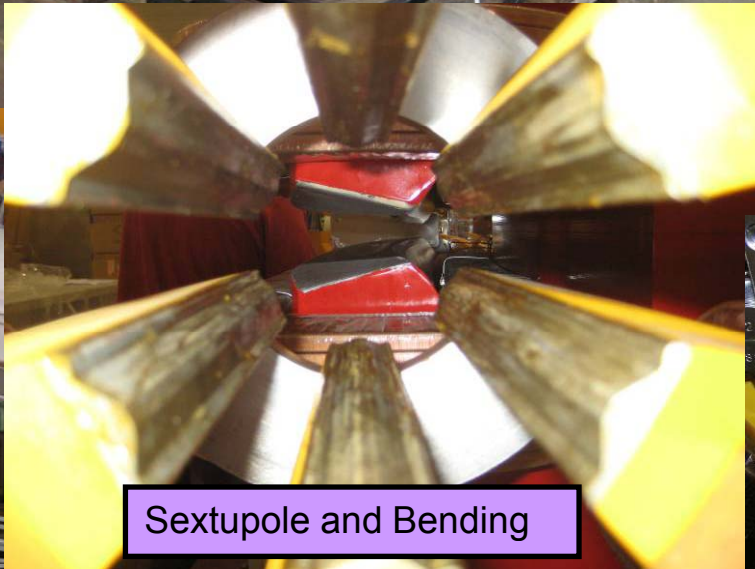
Bending



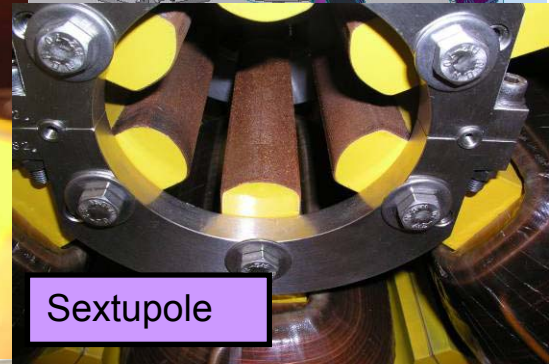
Insertion Device



Septum



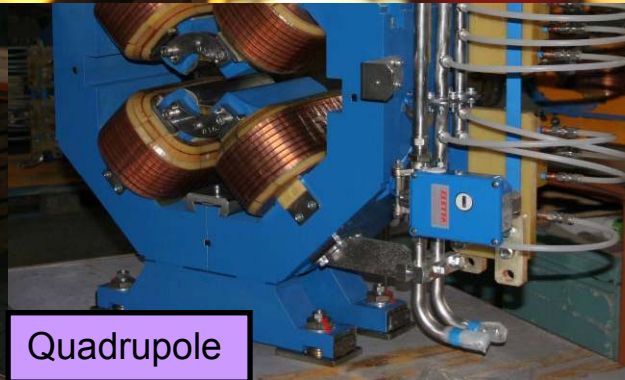
Sextupole and Bending



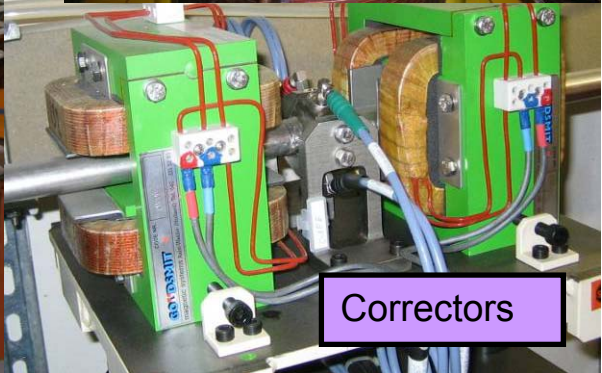
Sextupole



Sextupoles



Quadrupole



Correctors

# Magnets within an Accelerator Complex

Bendings, Quadrupoles, Sextupoles and Correctors

Bendings, Quadrupoles and Correctors

## Storage Ring

- Beam Optics
- Magnets
- Vacuum Syst.
- Girders
- Alignment
- Power Supplies
- Diagnostics
- Rad-Freq-Sytem
- Pulsed Magnets
- Puls-Power-Suppl.
- Timing
- Control System
- Insertion Devices
- Shielding

## Boost. Synchr.

- Beam Optics
- Magnets
- Vacuum Syst.
- Girders
- Alignment
- Power Supplies
- Diagnostics
- Rad-Freq-System
- Pulsed Magnets
- Puls-Power-Suppl.
- Timing
- Control Syst.
- Shielding

## Transfer Lines

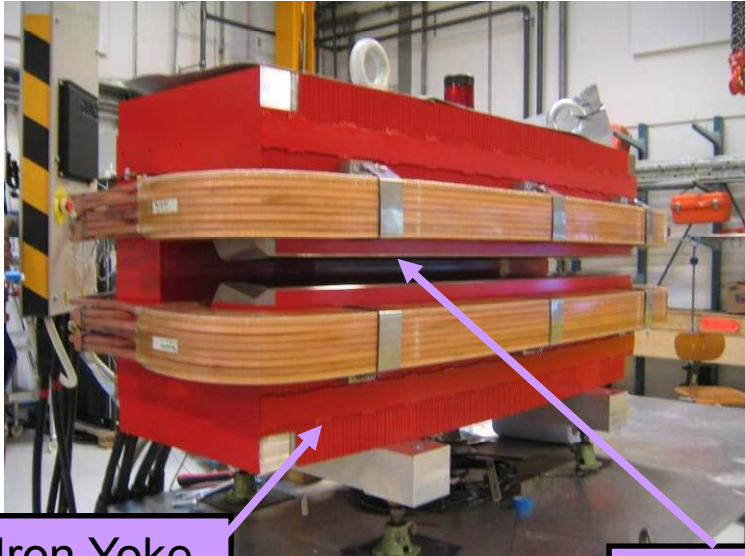
- Beam Optics
- Magnets
- Vacuum Syst.
- Girders
- Alignment
- Power Supplies
- Dagnostic
- Control Syst.
- Shielding

## Linac

- Beam Optics
- Gun
- Bunching Section
- Acceler.-Struct.
- Rad.-Freq.-Syst.
- Magnets
- Girders
- Alignment
- Power Suppl.
- Dagnostic
- Timing
- Control Syst.
- Shielding

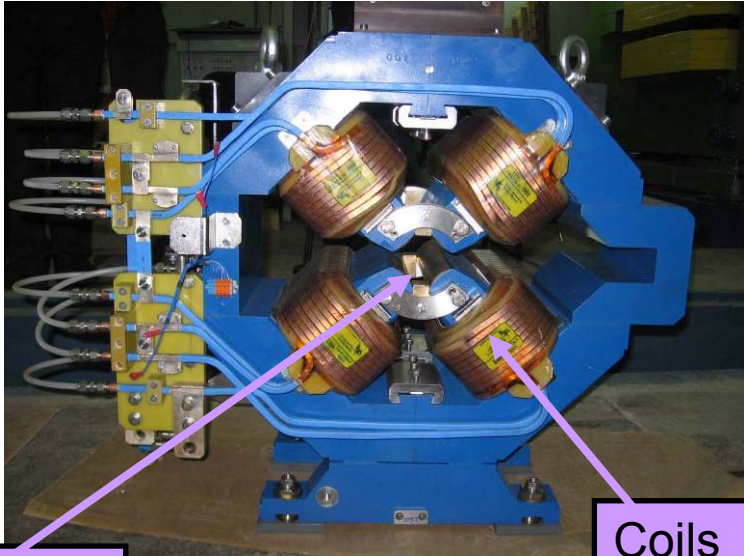


# Main Components of a Magnet

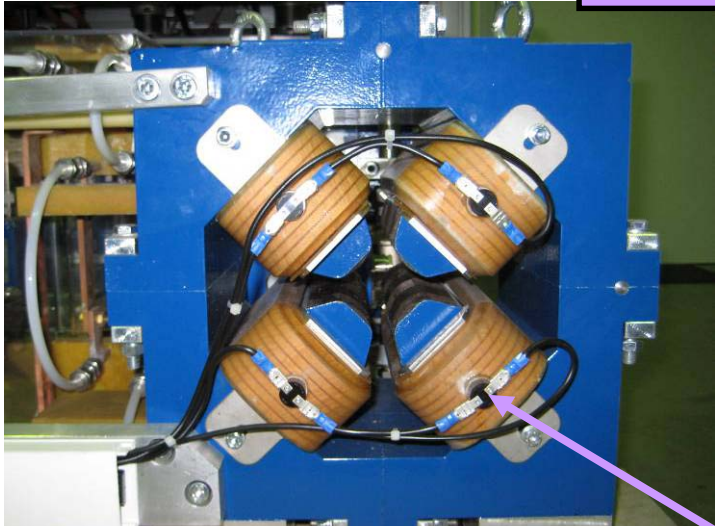


Iron Yoke

Pole profile

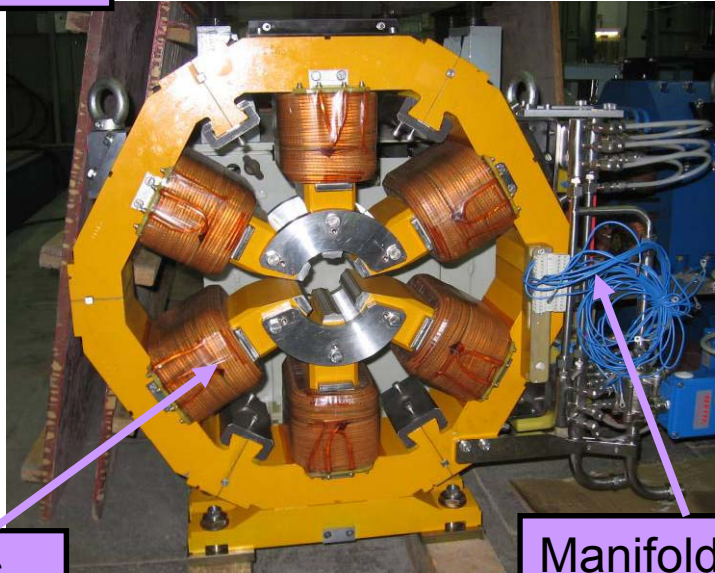


Coils



Supports

Sensors

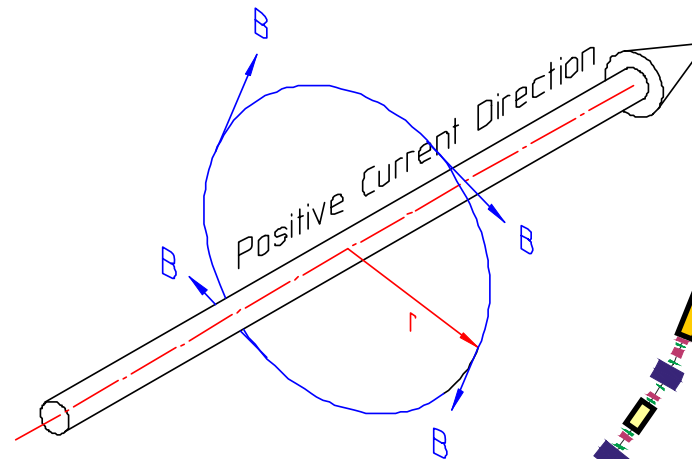


Manifolds

# Principles of the Magnet Design

Bending, quadrupoles, sextupoles  
correctors and combined magnets

$$I = \oint \vec{H} \cdot d\vec{l}$$
$$= \frac{B}{\mu_0} 2\pi r$$



Integral form of the  
Magnetic Field Equation

## But first – nomenclature!

**Magnetic Field:** (the magneto-motive force produced by electric currents)

symbol is  $\underline{\mathbf{H}}$  (as a vector);

units are Amps/metre in S.I units;

**Magnetic Induction or Flux Density:** (the density of magnetic flux driven through a medium by the magnetic field)

symbol is  $\underline{\mathbf{B}}$  (as a vector);

units are Tesla (Webers/m<sup>2</sup>)

**Note:** induction is frequently referred to as "Magnetic Field".

**Permeability of free space:**

symbol is  $\mu_0$  ;  $\mu_0 = 4\pi \cdot 10^{-7}$

units are Henries/metre;

**Permeability** (abbreviation of **relative permeability**):

symbol is  $\mu_r$ ;

the quantity is dimensionless;



# Calculation of the Magnetic Fields

Everything starts with the Maxwell Equations:

$$\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_L \vec{E} ds = -\frac{d\Phi}{dt} \dots \text{with} \dots \Phi = \int_A \vec{B} dA$$

$$\text{rot } \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\oint_L \vec{H} ds = I + \frac{d\Psi}{dt} \dots \text{with} \dots \Psi = \int_A \vec{D} dA$$

$$\text{div } \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\int_s \vec{D} dA = Q$$

$$\text{div } \vec{B} = 0$$

$$\int_s \vec{B} dA = 0$$

With the following relations:

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad \vec{B} = \mu_0 \mu_r \vec{H}$$

# Calculation of the Magnetic Fields

In a material free region the magnetic fields can be calculated by a potential function  $V(x,y,z)$ , which is determined by the Laplace equation:

$$\Delta V = 0 = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

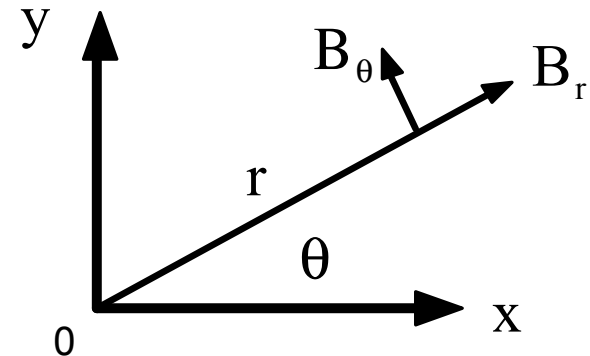
With some mathematical manipulations one gets the following field components:

$$B_r = -\sum_n (A_n r^{n-1} \cos n\Theta + B_n r^{n-1} \sin n\Theta)$$

$$B_\varphi = -\sum_n (-A_n r^{n-1} \sin n\Theta + B_n r^{n-1} \cos n\Theta)$$

$B_n$ : are the normal components,

$A_n$ : are the skew components,



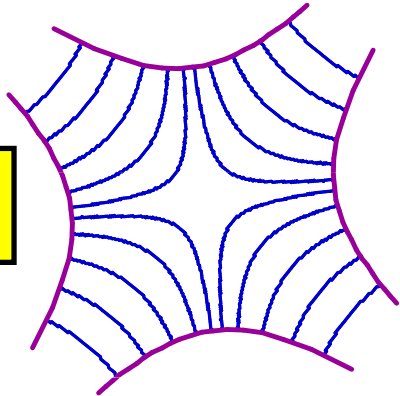
$$B_x = B_r \cos \theta - B_\theta \sin \theta,$$
$$B_y = B_r \sin \theta + B_\theta \cos \theta,$$

# Normal and Skew Magnets

Normal quadrupole



Rotation by 45 degrees

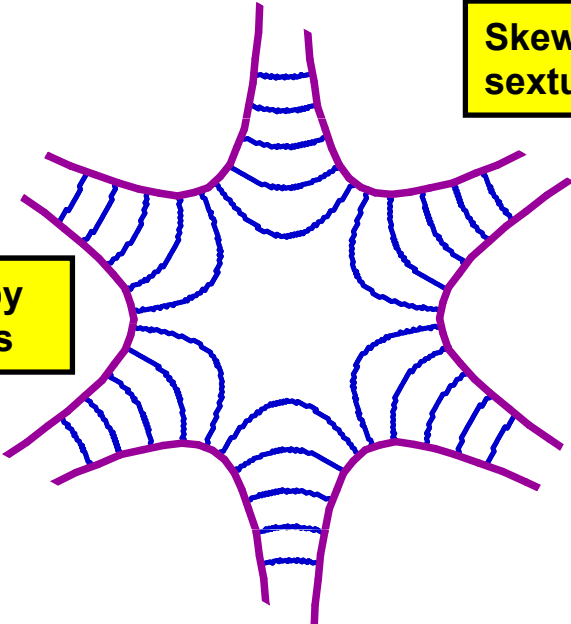


Skew quadrupole

Normal sextupole



Rotation by 90 degrees



Skew sextupole

# Description of a Magnetic Field

In general for the magnetic field one has the following description:

$$B = B_1 + B_2 * x + B_3 * x^2 + B_4 * x^3 + B_5 * x^4 + B_6 * x^5 + B_7 * x^6 + B_8 * x^7 + B_9 * x^8 + B_{10} * x^9$$

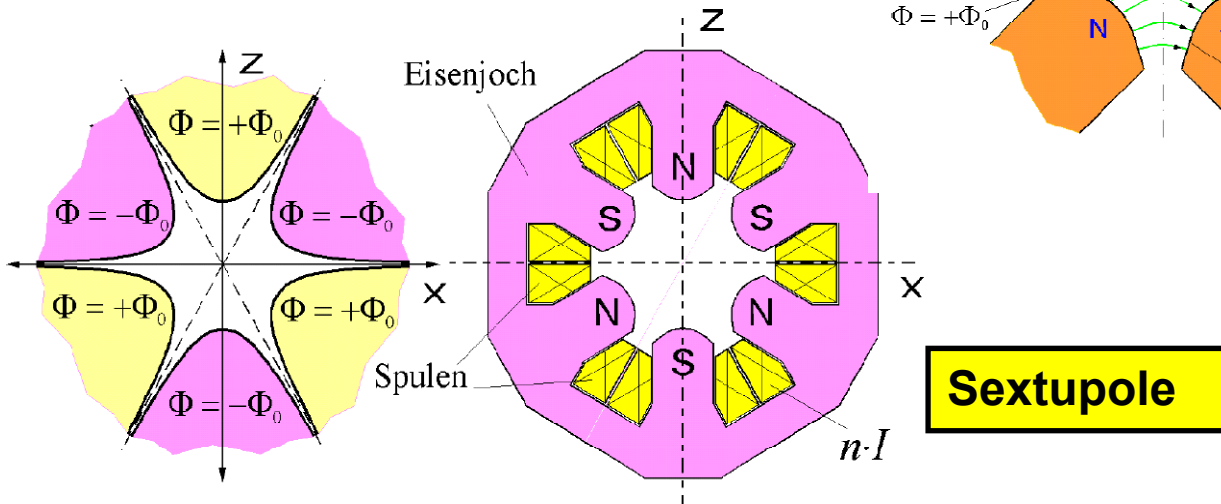
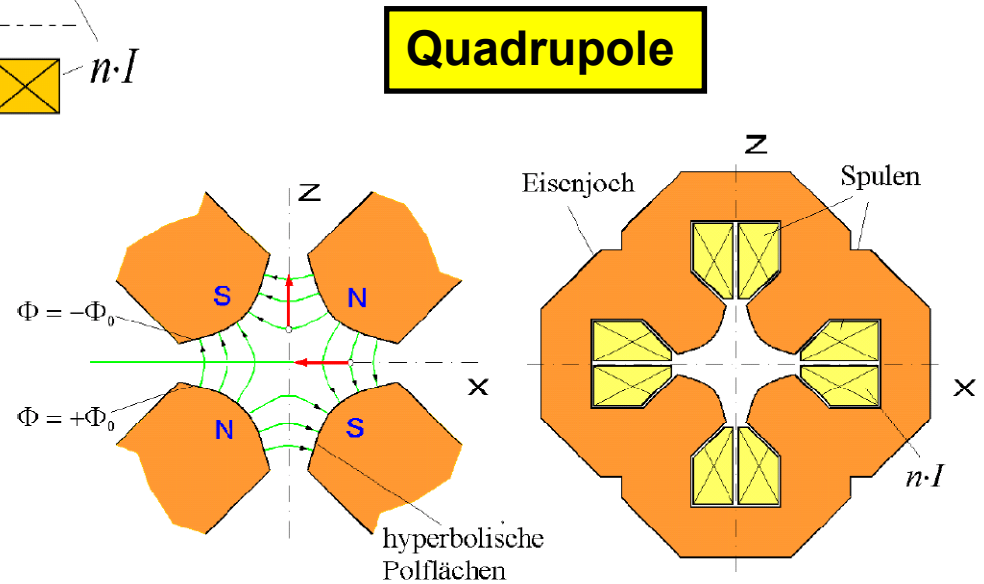
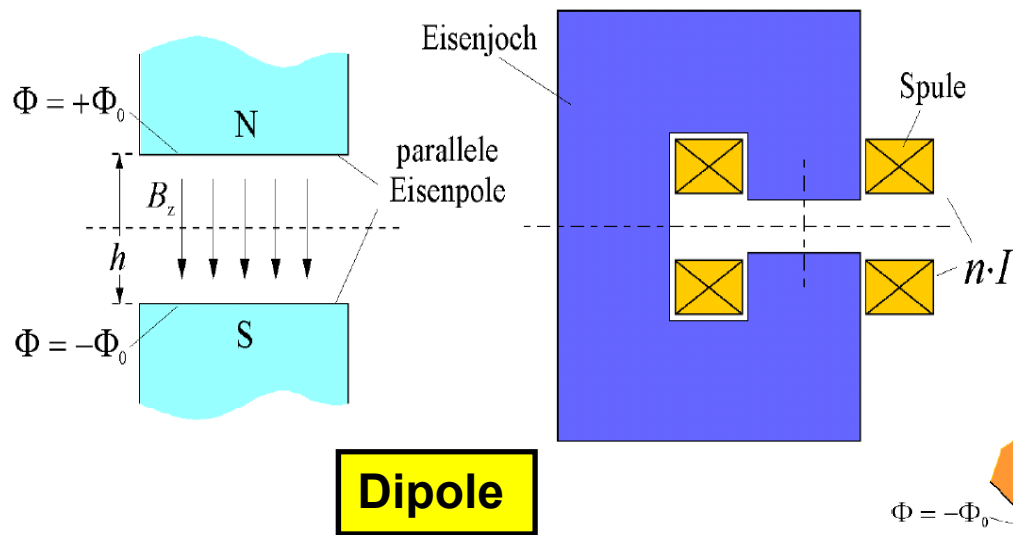
$$B = B_0 + \frac{1}{1!} \cdot \frac{dB}{dx} x + \frac{1}{2!} \cdot \frac{d^2B}{dx^2} x^2 + \frac{1}{3!} \cdot \frac{d^3B}{dx^3} x^3 + \frac{1}{4!} \cdot \frac{d^4B}{dx^4} x^4 + \frac{1}{5!} \cdot \frac{d^5B}{dx^5} x^5 + \frac{1}{6!} \cdot \frac{d^6B}{dx^6} x^6 + etc$$

$$B = B_1 + B_2 \cdot x + \frac{1}{2} \cdot B^2 \cdot x^2 + \frac{1}{6} \cdot B^3 \cdot x^3 + \frac{1}{24} \cdot B^4 \cdot x^4 + \frac{1}{120} \cdot B^5 \cdot x^5 + \frac{1}{720} \cdot B^6 \cdot x^6 + etc$$

$$B = B_1(2\text{pole}) + B_2(4\text{pole}) + B_3(6\text{pole}) + B_4(8\text{pole}) + B_5(10\text{pole}) + B_6(12\text{pole}) + etc$$



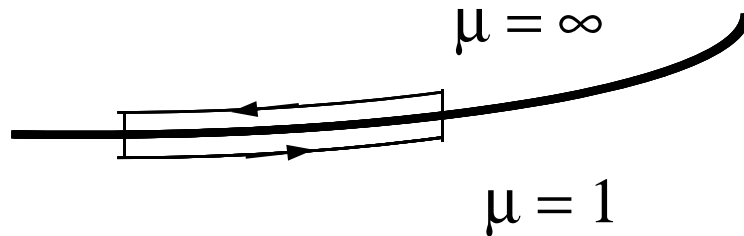
# Dipole, Quadrupole and Sextupole



## Introducing Iron Yokes

What is the ideal pole shape?

Flux is normal to a ferromagnetic surface with infinite  $\mu$ :



$$\text{curl } \mathbf{H} = 0$$

$$\text{therefore } \int \mathbf{H} \cdot d\mathbf{s} = 0;$$

$$\text{in steel } \mathbf{H} = 0;$$

$$\text{therefore parallel } \mathbf{H} \text{ air} = 0$$

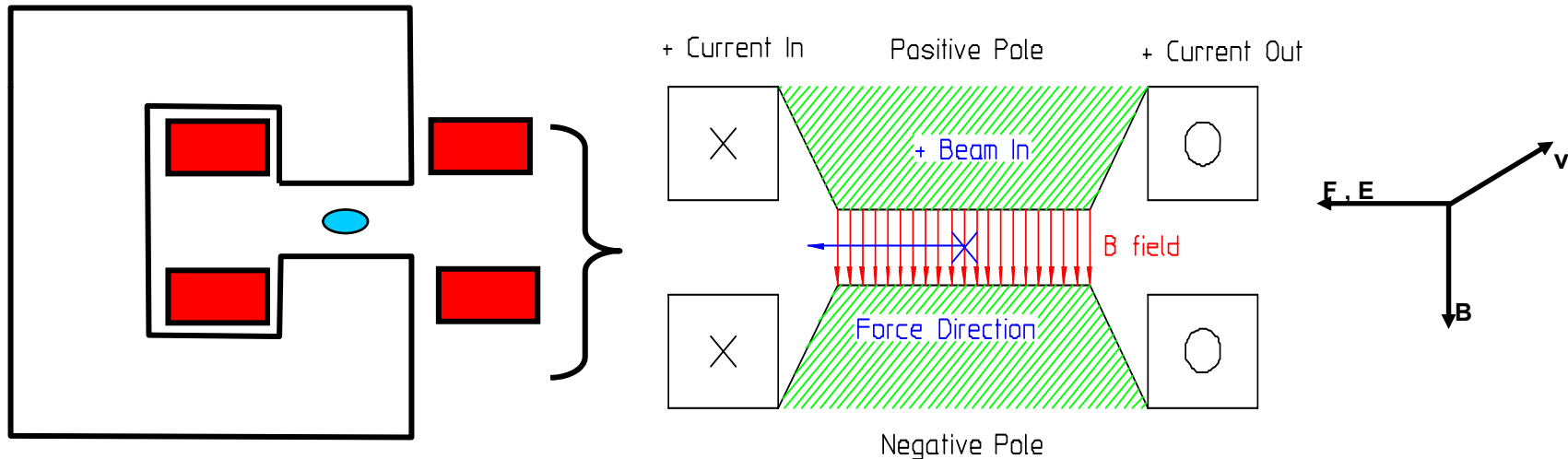
**therefore  $\mathbf{B}$  is normal to surface.**

Flux is normal to lines of scalar potential, ( $\mathbf{B} = -\nabla\phi$ );

So the lines of scalar potential are the ideal pole shapes!

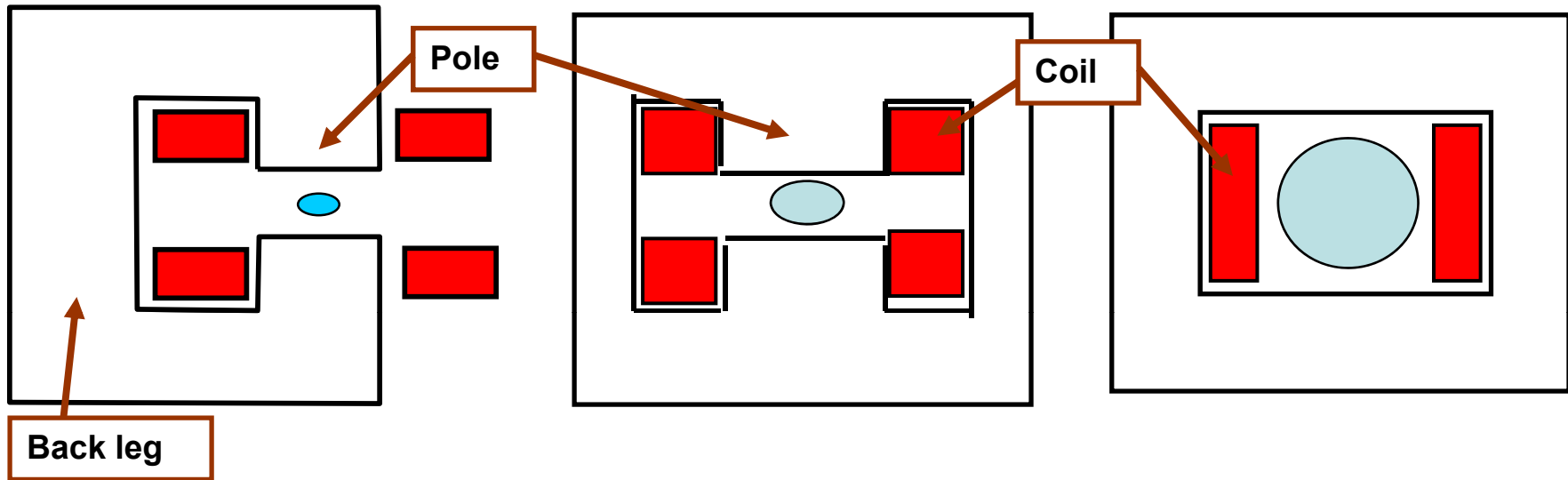
# The Dipole Magnet

The dipole magnet has two poles, a constant field and steers a particle beam. The purpose of all bending magnets in a ring is to bend the beam by exactly 360 degrees. Using the right hand rule, the positive dipole steer the rotating beam toward the left.



The 'shim' is a small, additional piece of ferro-magnetic material added on each side of the two poles – it compensates for the finite cut-off of the pole, and is optimised to reduce the 6, 10, 14..... pole error harmonics.

# Types of magnets



## "C core":

### Advantages:

- Easy access;
- Classic design;
- Low A-Turns;

### Disadvantages:

- Less rigid;
- Still needs shims;
- Asymmetric.

## "H core":

### Advantages:

- Symmetric;
- More rigid;
- Low A-Turns

### Disadvantages:

- Still needs shims;
- Access problems.

## "Window Frame"

### Advantages:

- High quality field;
- No pole shim;
- Symmetric & rigid;

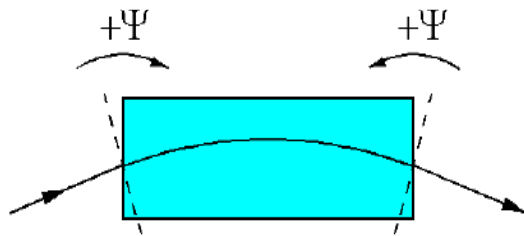
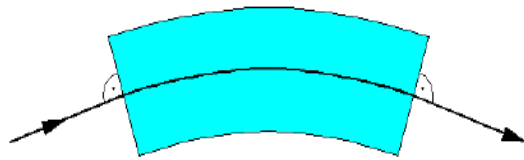
### Disadvantages:

- High A-Turns
- Major access problems;
- Insulation thickness

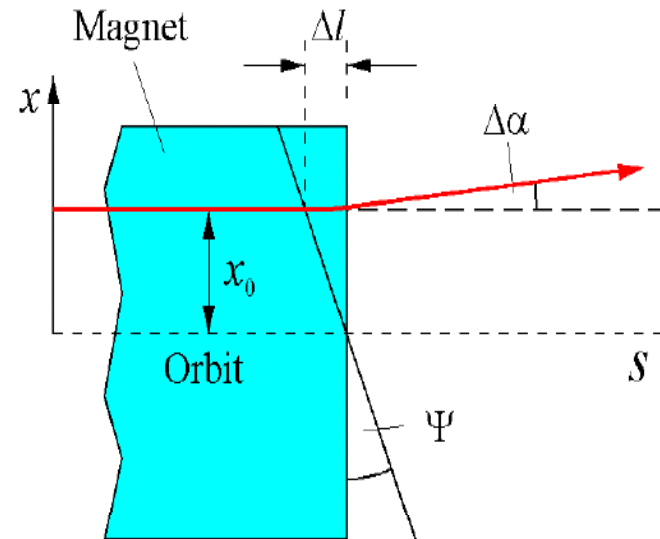


## More Types of Bending Magnets

**Sector magnet**



**Rectangular magnet**



**Defocusing in a rectangular magnet**

Within the sector bending magnet the trajectory length for positive  $x$  will be larger and therefore the sector magnet is a focussing one.

Within the rectangular bending magnet it is vice versa and it is a defocusing one.

# Dipole Excitation

According to Maxwell it is:

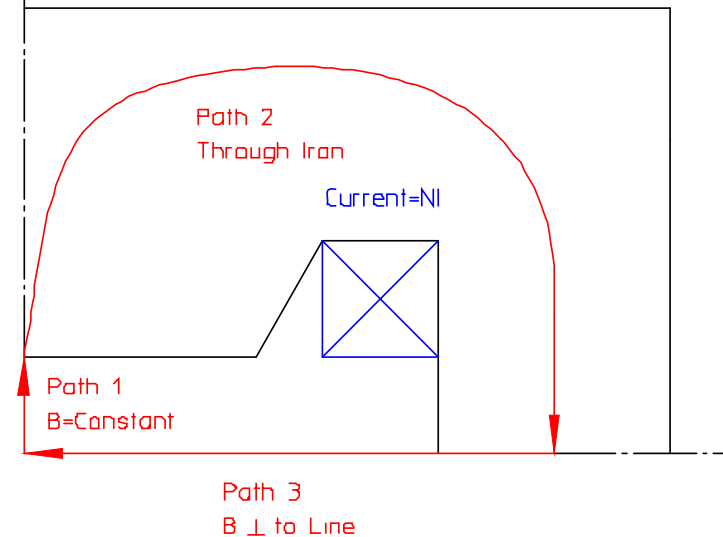
$$\oint \vec{H} \cdot \vec{dl} = \oint_{Path1} \vec{H} \cdot \vec{dl} + \oint_{Path2} \vec{H} \cdot \vec{dl} + \oint_{Path3} \vec{H} \cdot \vec{dl} = NI$$

Along Path 1  $|H| = \frac{B}{\mu_0}$  and  $H \parallel l$

Therefore:  $\oint_{Path1} \vec{H} \cdot \vec{dl} = \frac{Bh}{\mu_0}$

Along path 2,  $|H| = \frac{B}{\mu \mu_0}$  For iron;  $\mu \approx 1000$

Therefore:  $\oint_{Path2} \vec{H} \cdot \vec{dl} = |H|_{iron} l_{iron} \ll \frac{Bh}{\mu_0} \approx 0$



# Dipole Excitation

Along path 3:  $\overline{H} \perp \overline{dl}$  and  $\oint_{Path3} \overline{H} \cdot \overline{dl} = 0$

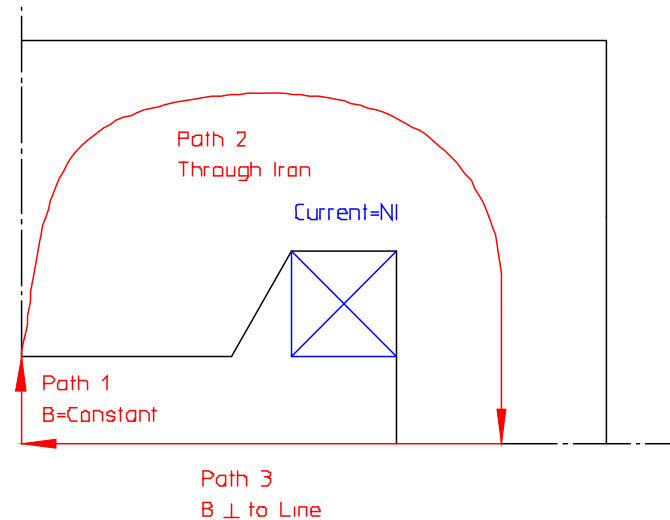
Therefore:  $\overline{H} \cdot \overline{dl} = 0$

Finally:  $\oint \overline{H} \cdot \overline{dl} = NI \approx \frac{Bh}{\mu_0}$

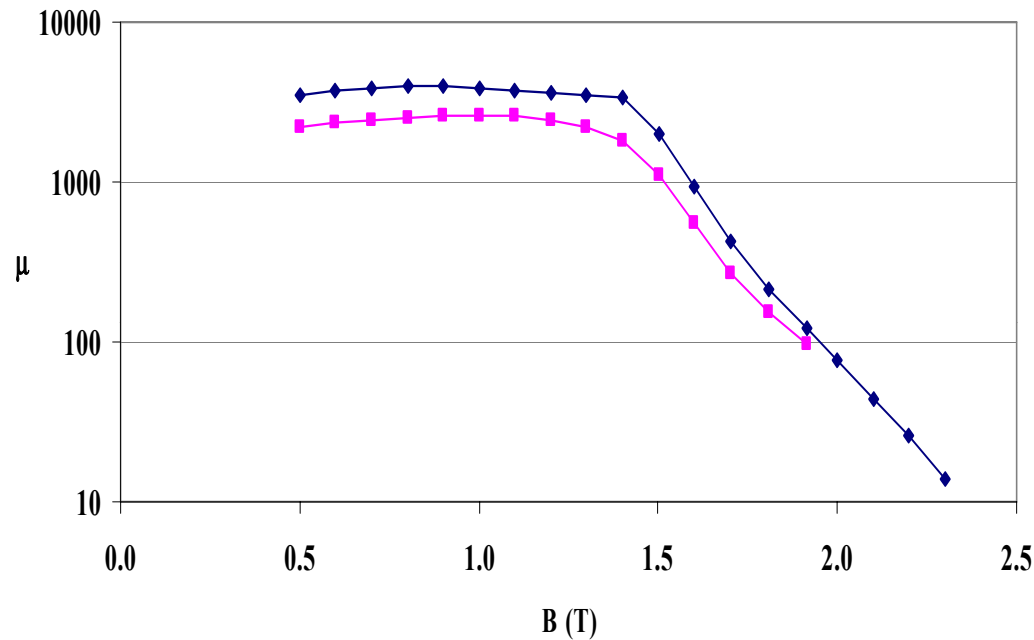
$$B/l \approx \mu_0 N/h = \text{constant}$$

$B_{\text{air}} = \mu_0 NI / (h + l_{\text{iron}}/\mu)$ ; is the exact solutions

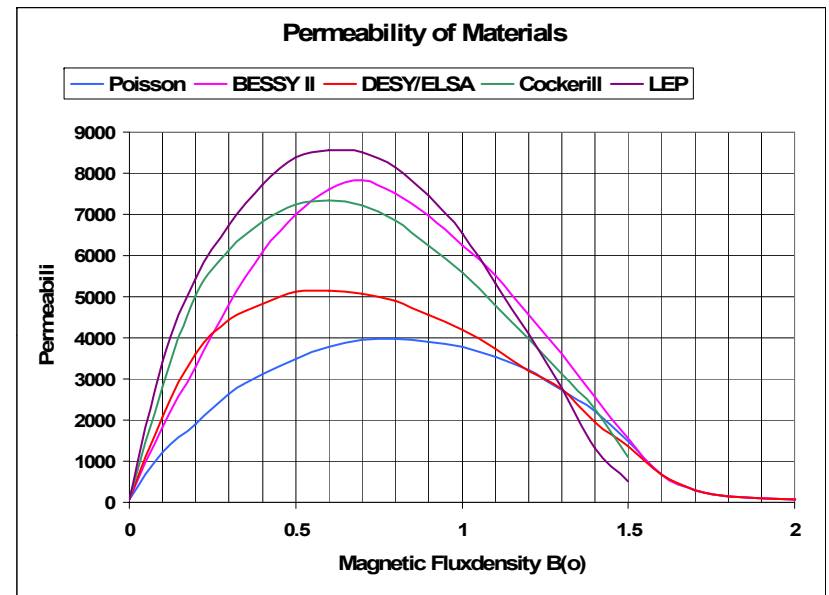
The same procedure can be used for the calculation of the excitation for the quadrupoles, sextupoles and correctors



# Yoke - Permeability of low silicon steel



◆ Parallel to rolling direction    ■ Normal to rolling direction.

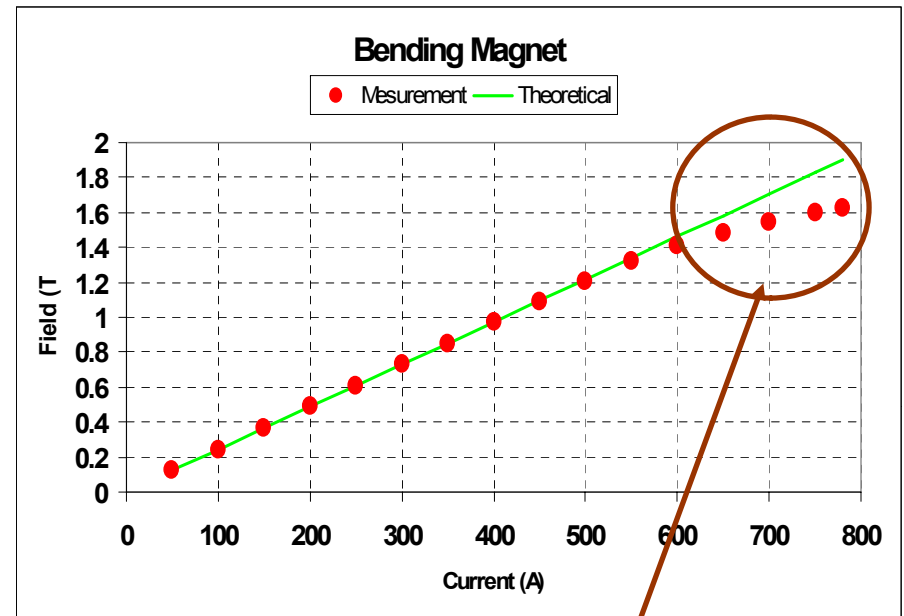


Without saturation the factor  $l_{iron}/\mu$  should be much smaller as the half gap height  $h$ . For a synchrotron light bending magnet the length of the field line within half of the magnet is roughly 750 mm. At 1.5 T  $\mu_r$  is roughly 1000, hence the factor  $l_{iron}/\mu_r$  is roughly 1 mm, which makes roughly an saturation of 5 % to  $h=20$  mm.

# Excitation curve of the ANKA Bending magnet

Flux density	1.40 T
Radius	5.956 m
Deflection Angle	22.5 degree
Magnetic length	2.340 m
Iron length	2.274 m
Total length	2.47 m

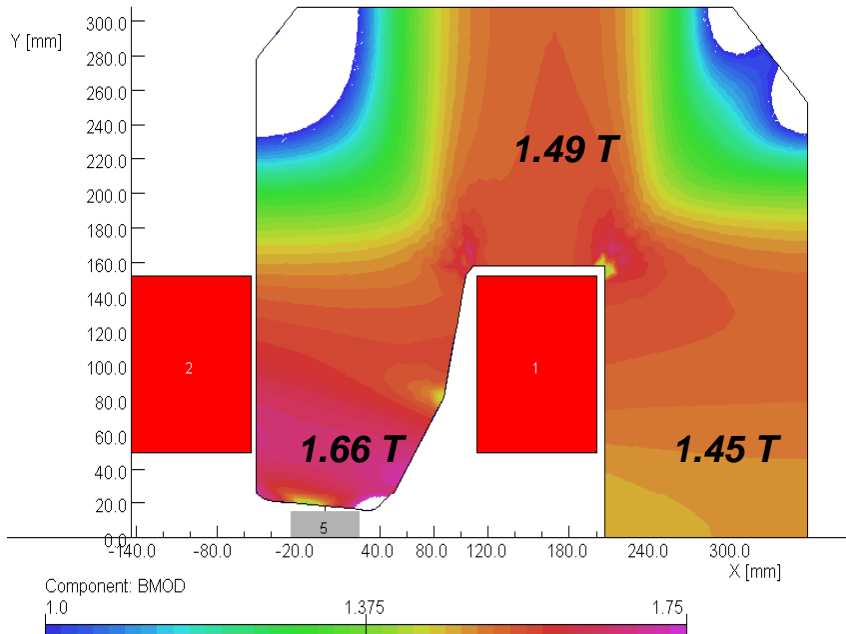
Strength	0.3411 m <sup>-2</sup>
Gradient	2.84 T/m
Gap height	42 mm
Current	643 A
Turns	80
Conductor	13 * 13 mm <sup>2</sup>



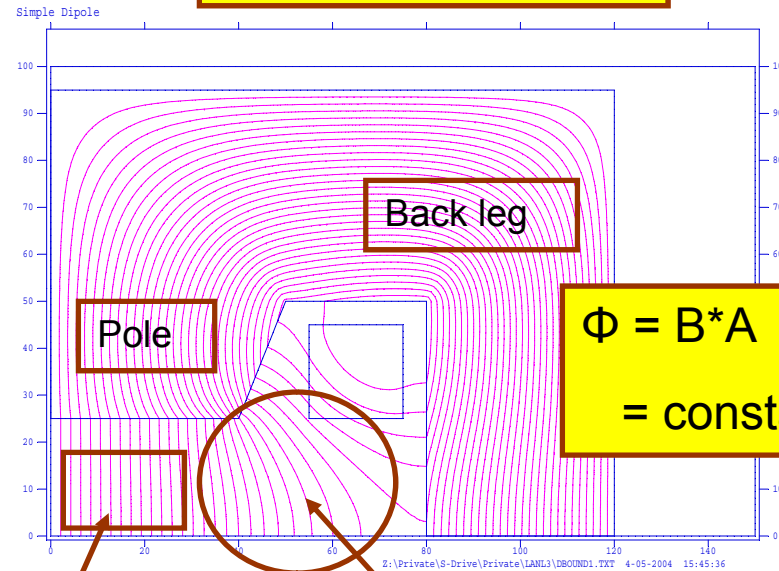
This is the region with some saturation, which means  $h$  (the gap height) is not any more much larger as the factor  $I_{\text{iron}}/\mu$ .

# 2 D Flux density distribution in the ALBA Dipole.

Flux densities



Magnetic field lines



$$\Phi = B \cdot A$$

$$= \text{constant}$$

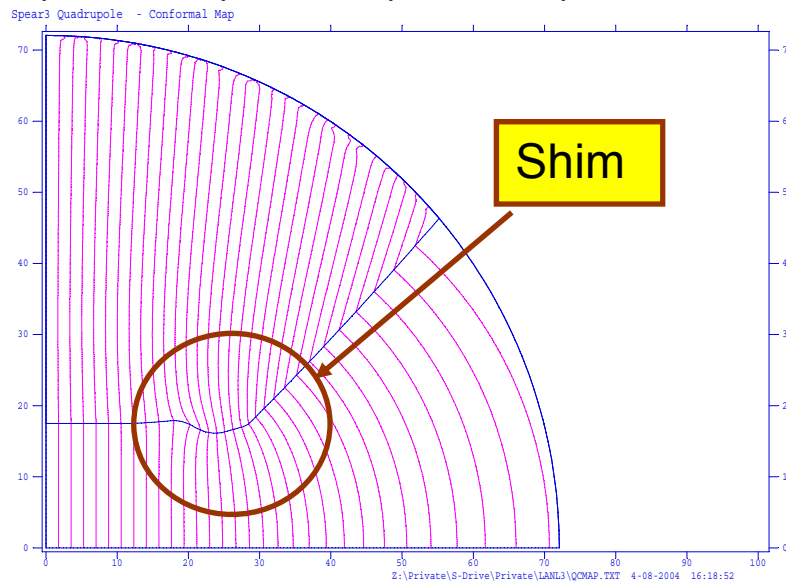
Homogenous field

Stray field

The flux of the stray field is roughly 30 % of the homogenous part. In order to have a constant flux density within the pole, the thickness of the pole has to be increased to the upper part. The thickness of the back leg must be the same or larger as the maximum width of the pole. If  $B < 1$  T an increase of the pole width is not needed.

## Size of a Dipole.

The size of a dipole is given by the required so called “good field area” of the magnet. The good field area is given by the accelerator physicist. For example in a bending magnet for a synchrotron light source a good field area of roughly +/- 15 to 20 mm is required. In heavy ion - or other machines it can be completely different (much larger). To optimize the pole profile one uses to the end of the poles so called shims (as given below). The contour of the shims have to be determined with a 2D (Poisson) or 3D (TOSCA) code

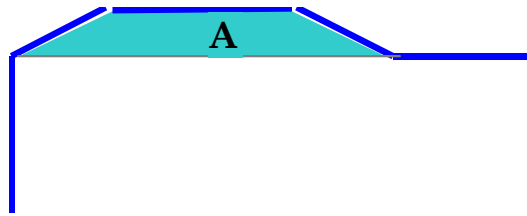


For accelerators operating with a fixed energy the flux density  $B$  can be up to 1.5 T or larger. For ramping machines the flux density should not be larger as roughly 1 T in order to avoid saturation effects.

## Shimming of Pole Profiles

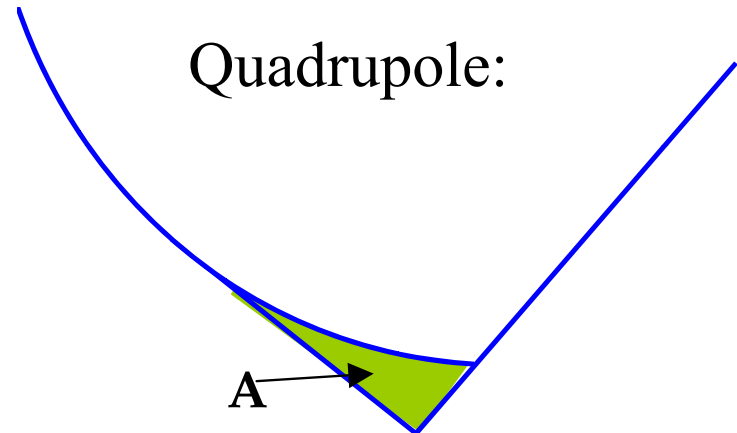
To compensate for the non-infinite pole, shims are added at the pole edges. The area and shape of the shims determine the amplitude of error harmonics which will be present.

Dipole:



The designer optimises the pole by ‘predicting’ the field resulting from a given pole geometry and then adjusting it to give the required quality.

Quadrupole:

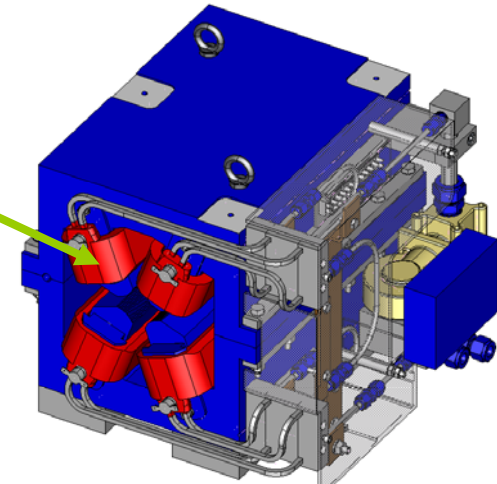
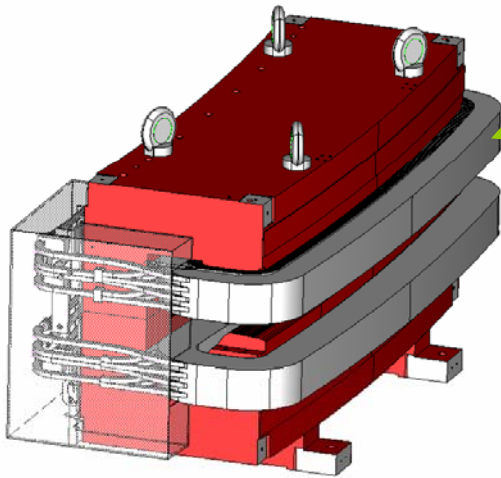


When high fields are present, chamfer angles must be small, and tapering of poles may be necessary



# Size of Coils.

Coils



The currents and the windings of the coil have to make the excitation

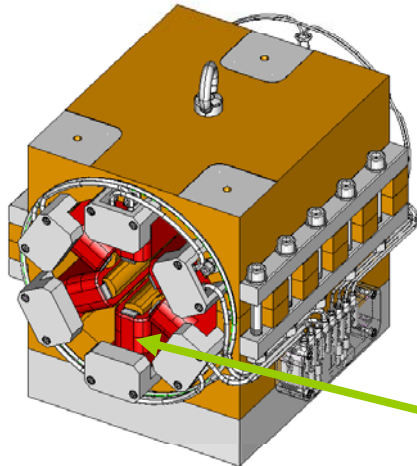
$$N \cdot I = B_0(g + l_{Fe}/\mu_r)/\mu_0$$

$$N \cdot I \approx B_0 g / \mu_0$$

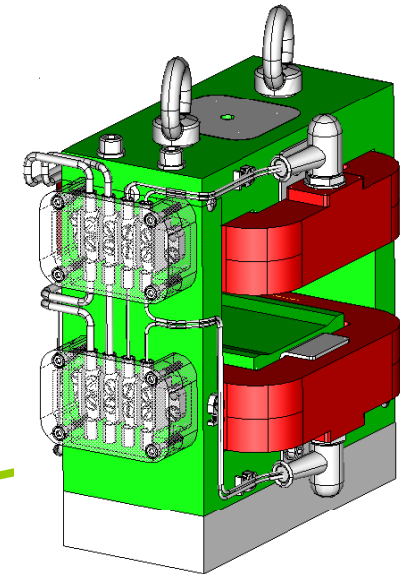
For the ALBA bending with a gap of 36 mm and a flux density of 1.42 T the excitation is:

$$N \cdot I = 40680 \text{ A} \cdot \text{Wdgs}$$

This can be done by a larger number of windings or a high current.

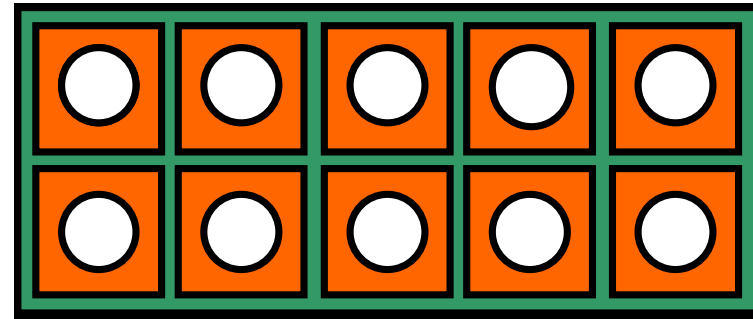


Coils



## Coil Geometry

Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.



Amp-turns ( $NI$ ) are determined, but total copper area ( $A_{\text{copper}}$ ) and number of turns ( $N$ ) are two degrees of freedom and need to be decided.

Current density:  
 $j = NI/A_{\text{copper}}$   
Optimum  $j$   
determined from  
**economic** criteria.

## Number of turns, N

The value of number of turns (N) is chosen to match power supply and interconnection impedances.

Factors determining choice of N:

Large N (low current)

Small, neat terminals.

Thin interconnections-hence low cost and flexible.

More insulation layers in coil, hence larger coil volume and increased assembly costs.

High voltage power supply  
-safety problems.

Small N (high current)

Large, bulky terminals

Thick, expensive connections.

High percentage of copper in coil volume. More efficient use of space available

High current power supply.  
-greater losses.

## Current density - optimisation

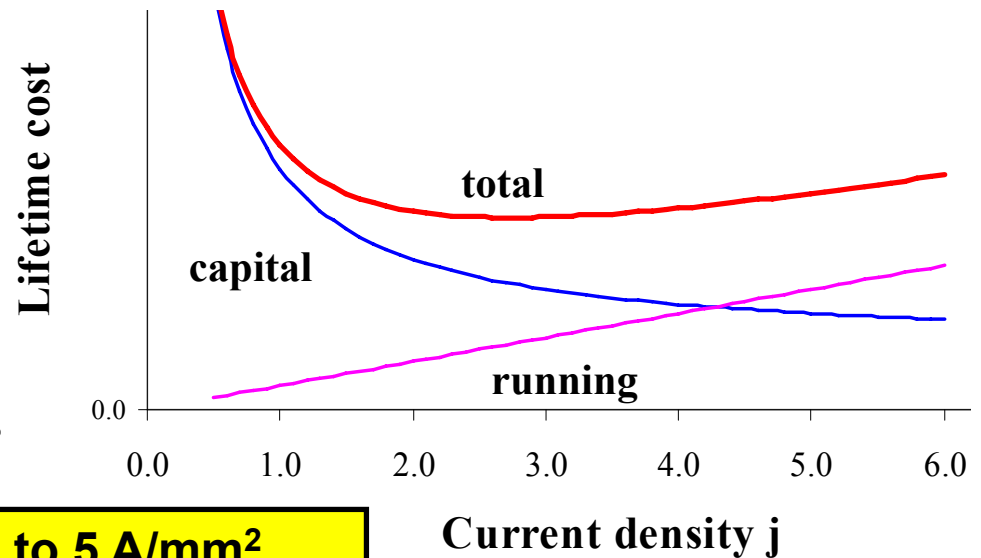
Advantages of low  $j$ :

- **lower power loss** – power bill is decreased;
- **lower power loss** – power converter size is decreased;
- **less heat** dissipated into magnet tunnel.

Advantages of high  $j$ :

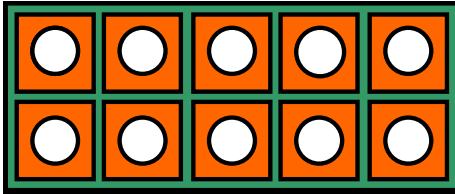
- **smaller coils**;
- **lower capital cost**;
- **smaller magnets**.

Chosen value of  $j$  is an optimisation of magnet capital against power costs



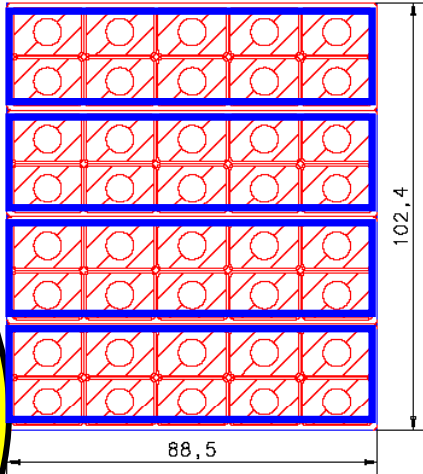
At ALBA we current densities of 3.5 to 5 A/mm<sup>2</sup>

# Size of the Coil

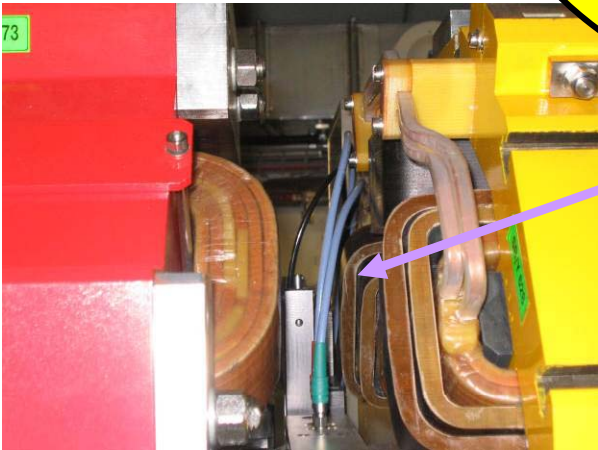


2 layers

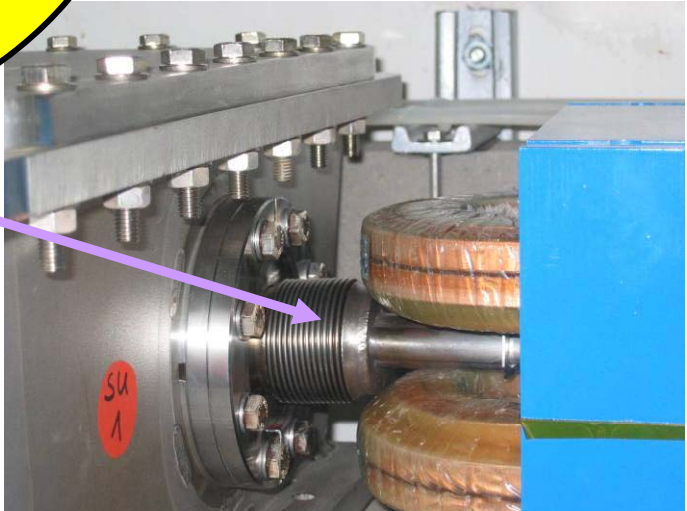
A small number of layers needs more space in the longitudinal directions. A high number of layers needs more space for the magnets in the vertical direction. For the number of layers one has to make a compromise and it depend upon the machine



4 layers/pancakes

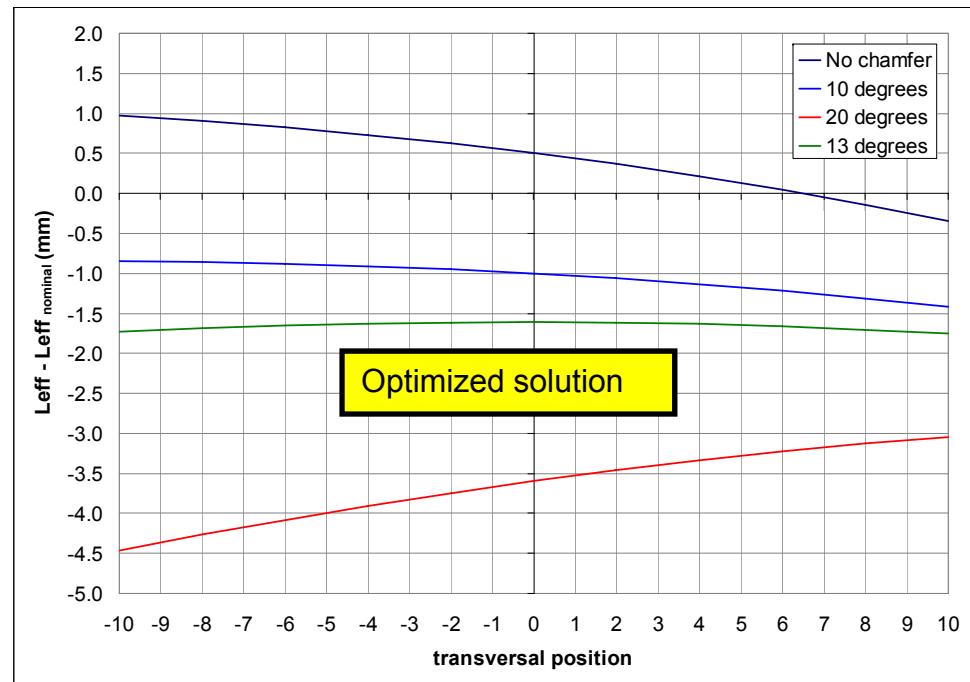
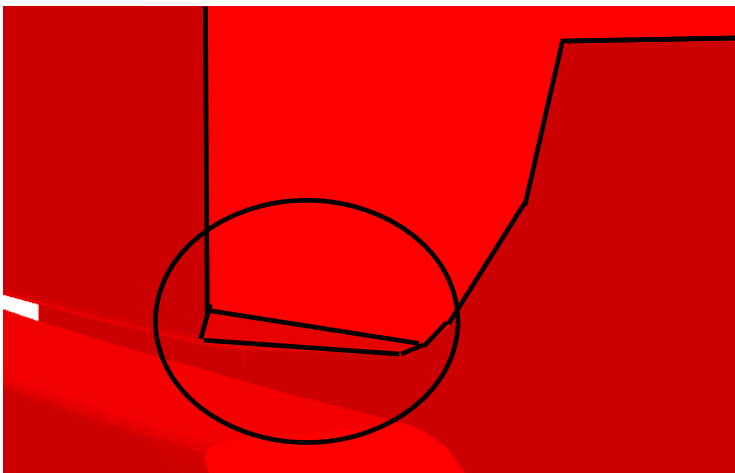


Space within the machine for the components should be as small as possible.

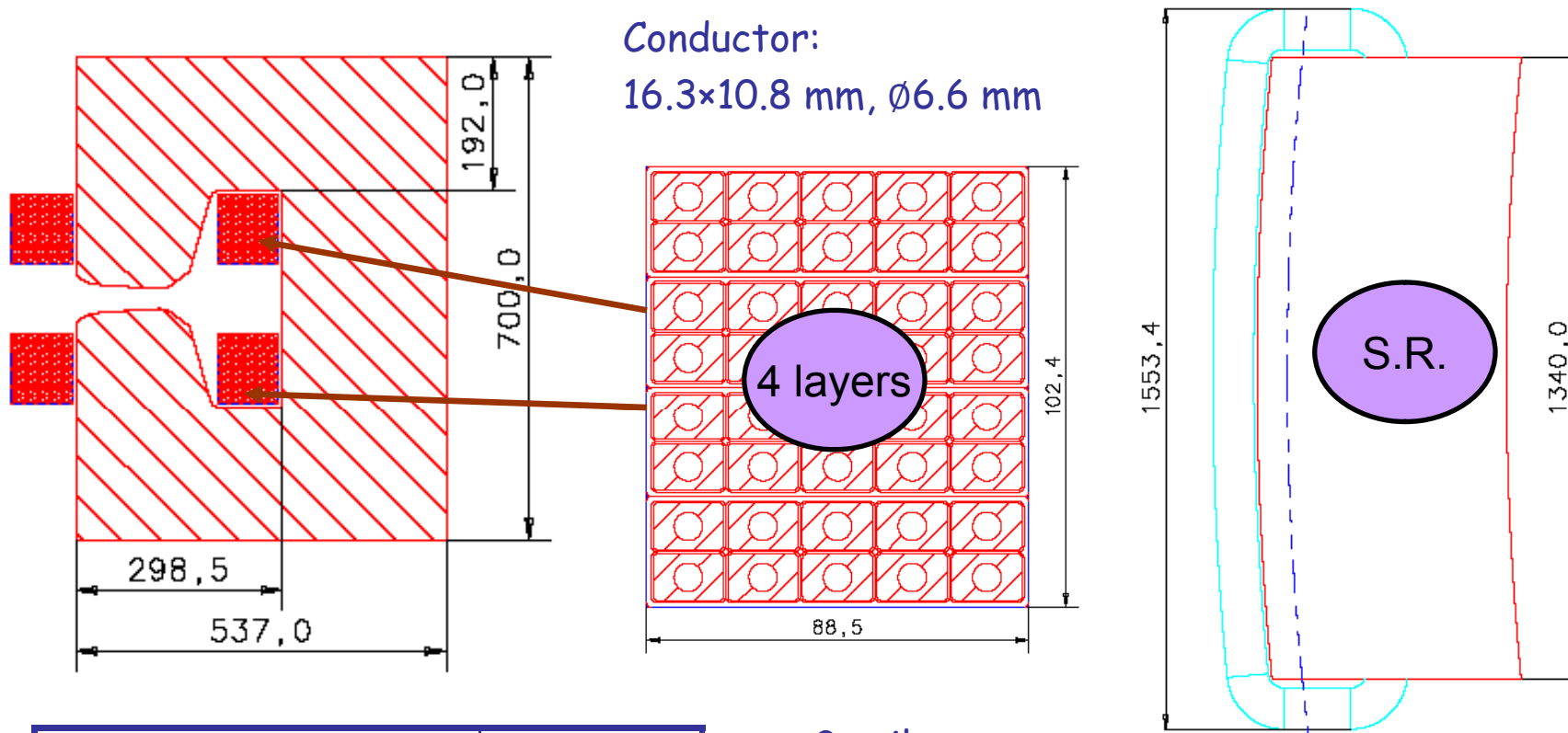


# SR Dipole End Chamfer

- Cut at 45° and then rotate the cut up to 20°
- Use OPERA-3D to model the magnet
- Calculate  $B_y(s)$  at different transversal trajectories
- Beam Dynamics uses this field to do the machine simulation with a sliced magnet
- The best chamfer is that which makes the effective length constant along the transversal direction of the magnet.



# Layout of the ALBA combined Bending Magnet



Bending angle	11.25°
Curvature radius	7.047
Central gap	36 mm
Central magnetic field	1.42 T
Gradient	5.65 T/m

2 coils

40 turns/coil (5 turns wide x 8 turns height)

4 pancakes/coil

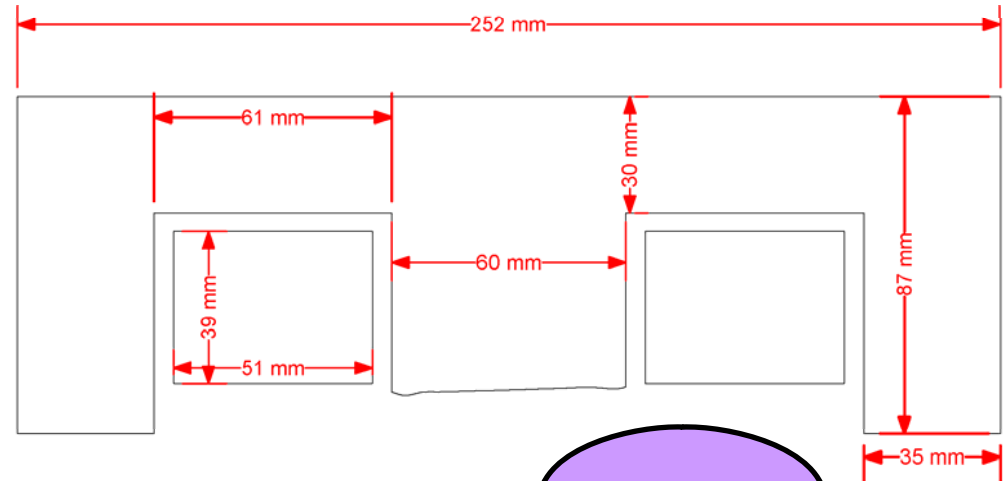
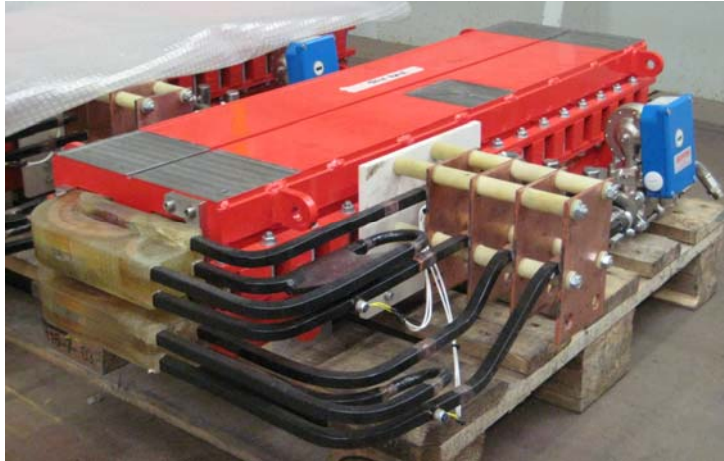
2 hydraulic circuits/coil,  $\Delta T=8.6$  deg for

$\Delta P=7$  bars

$P=9.6$  kW

$I=527$  A

# Dimensions of the ALBA combined Bending Magnet

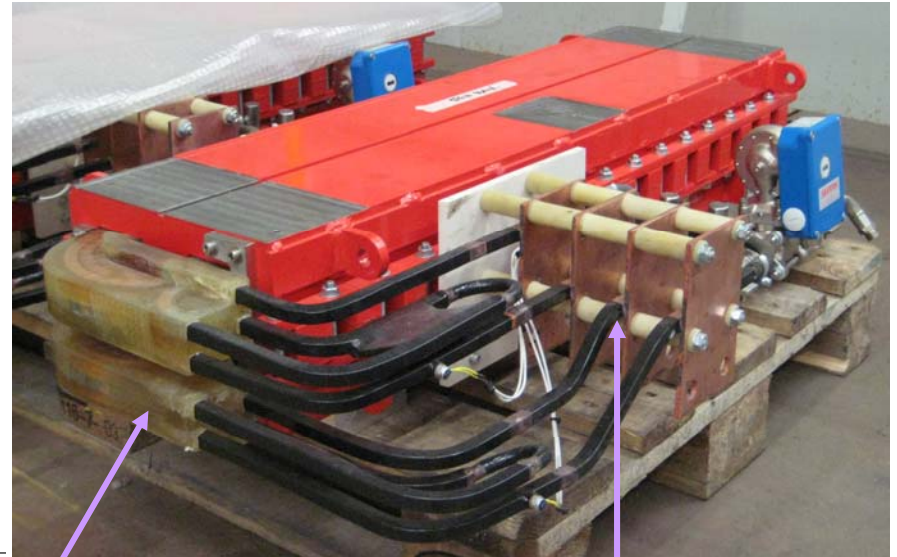
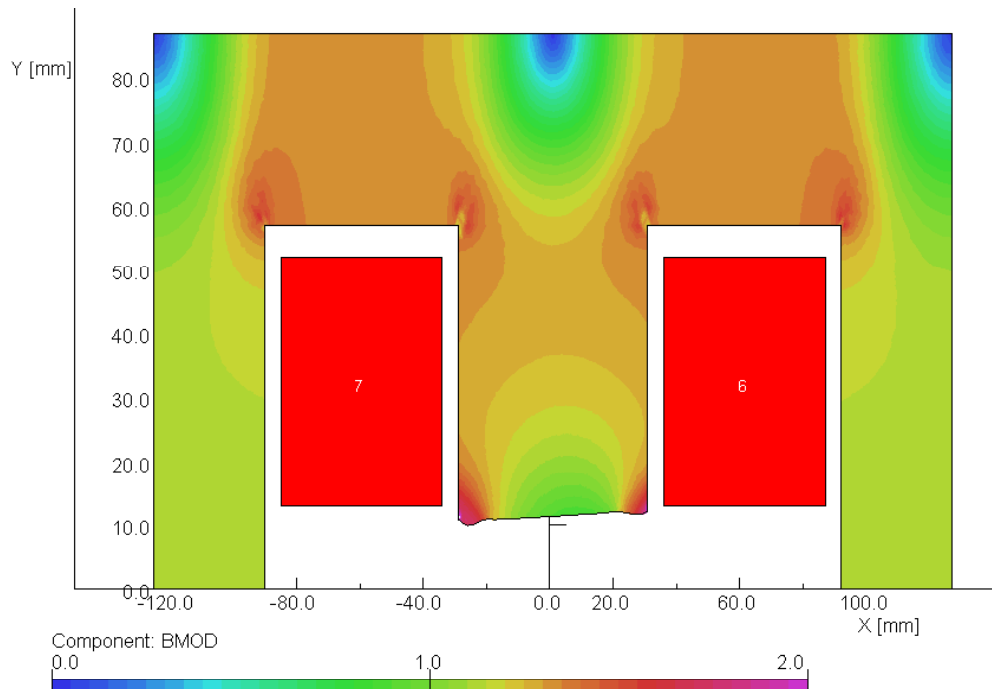


Number of magnets	8 / 32
Effective length (m)	1 / 2
Bending angles (°)	5 / 10
Curvature radius (m)	11.4592
Central gap (mm)	22.6
Central dipolar field (T)	0.873
Central Quadrupolar field (T/m)	-2.29
Central Sextupolar field (T/m <sup>2</sup> )	-9.0

- 2 coils  
12 turns/coil  
(4 turns wide x 3 turns height)
- 1 / 2 hydraulic circuits/coil  
 $\Delta T = 8^\circ\text{C}$  ,  $\Delta P = 6$  bars
- $P_{\text{average}} = 1.9 / 3.7$  kW
- $I_{\text{peak}} = 660$  A



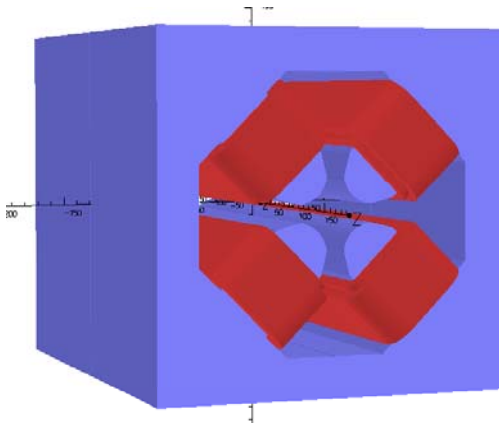
# Booster Bending Dipoles (Opera – 2D models)



Coils with only two layers.

Space for the many fold and connectors

# Quadrupole Design.

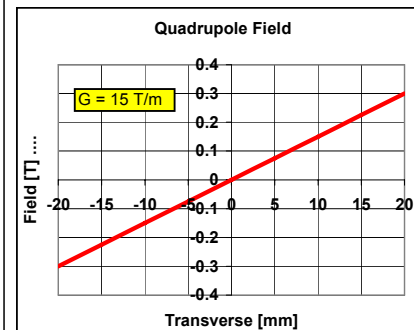
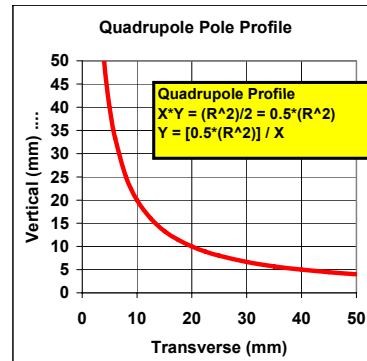
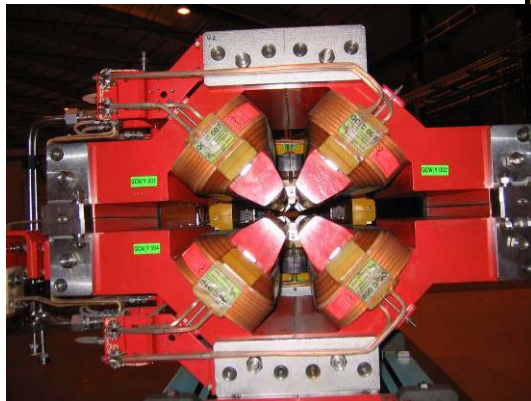
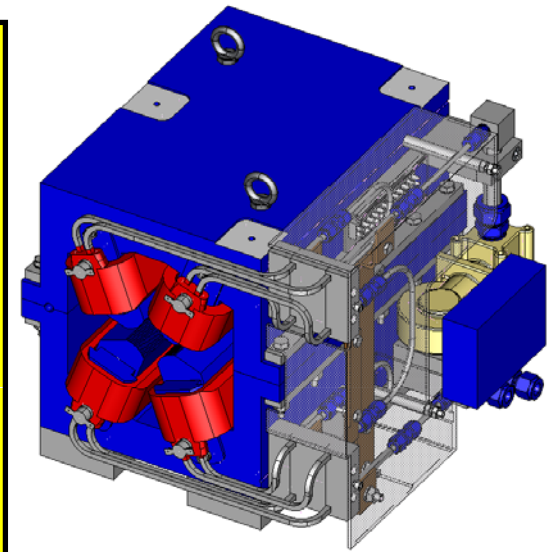


The Quadrupole Magnet has four poles. The field varies *linearly* with the distance from the magnet center. It focuses the beam along one plane while defocusing the beam along the orthogonal plane. An *F* or focusing quadrupole focuses the particle beam along the *horizontal* plane.

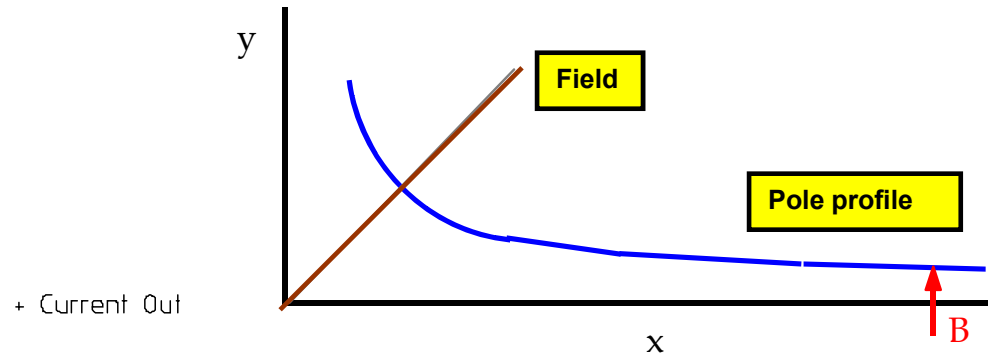
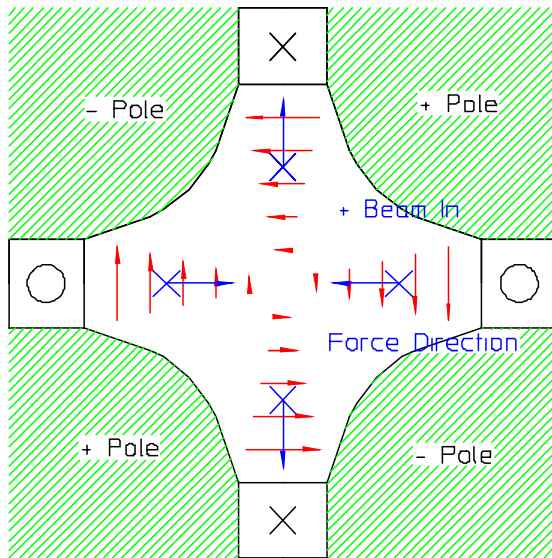
The field of the quadrupole has to be proportional to the distance from the centre (x or y). The excitation in general is given by:

$$B_0 = \mu_0 N \cdot I / g \text{ or } B(x) = \mu_0 N \cdot I / x$$

This means the pole profile of a quadrupole is a hyperbolic one.



# Excitation current in Quadrupole



For quadrupoles the required excitation can be calculated by considering fields and gap at large  $x$ . For example:

Pole equation:  $x \cdot y = R^2 / 2$ ; on  $x$  axes:  $B_y = g \cdot x$ ; where  $g$  is gradient (T/m).

At large  $x$  (to give vertical lines at  $B$ ):  $N I = (g \cdot x) ( R^2 / 2x) / \mu_0$

or  $N \cdot I = g \cdot R^2 / 2 \mu_0$  (per pole).

The magnetic flux at the pole tip is given by:  $B_{\text{pole}} = 2 \cdot \mu_0 \cdot N \cdot I / R$ .

The gradient is given by:  $g = 2 \cdot \mu_0 \cdot N \cdot I / R^2 = B_{\text{pole}} / R$  and  $m = g / (\rho \cdot B)$

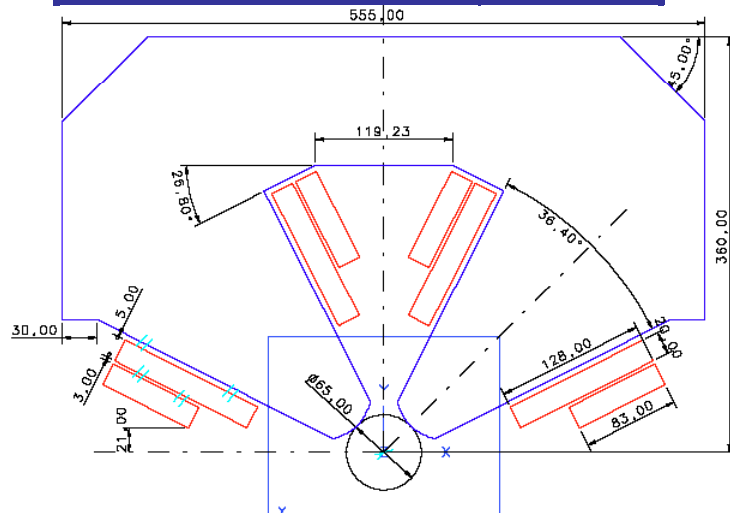
With  $R = 35$  mm and  $B_{\text{pole}} = 0.6$  T the gradient is roughly 17 T/m.

Today one can go up with the gradient to 22 T/m

# Dimensions of the ALBA Quadrupoles

## Former Quadrupole Magnets

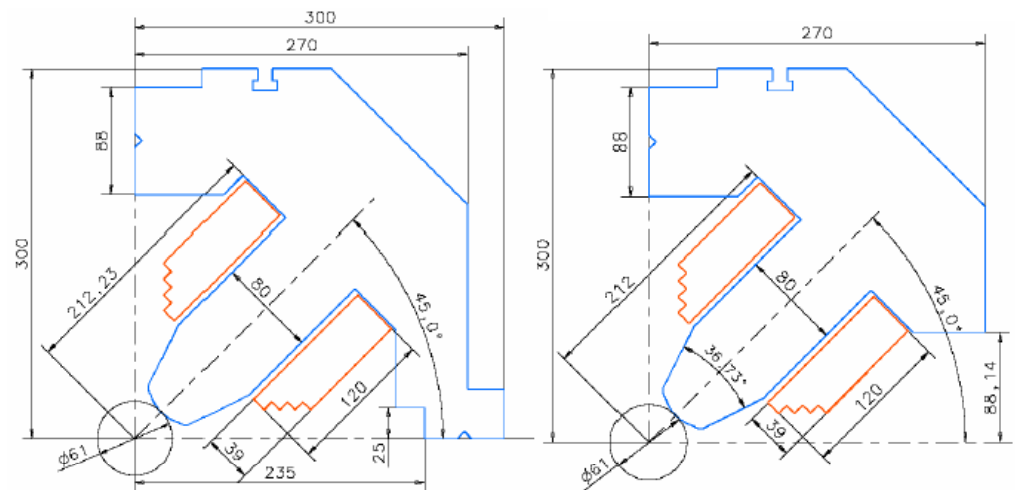
Number of magnets	112
Aperture	65 mm
Max. gradient	22 T/m
Max. current	200 A



- 1 type of lamination, 2 coil pancakes (56 turns, 8x8 mm,  $\varnothing$ 4.5 mm)
- Mechanically made of 2 pieces
- All opened magnets with spacers
- Iron Overall dimensions are 550x720 mm

## Present Quadrupole Magnets

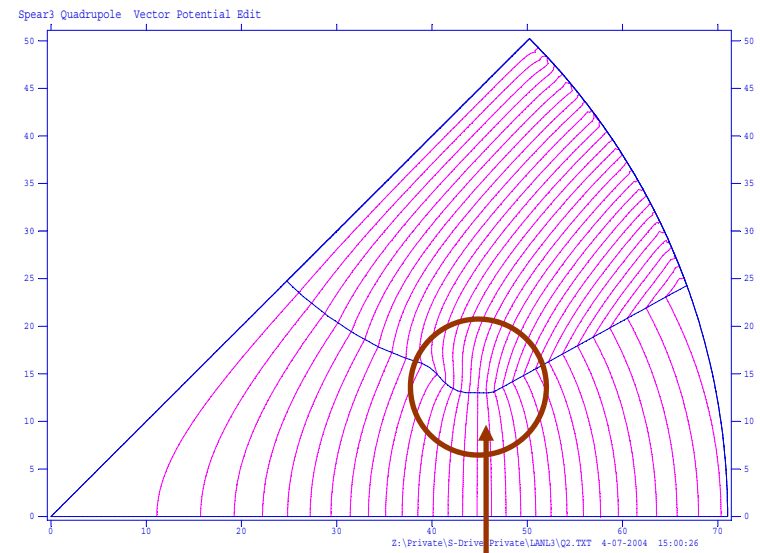
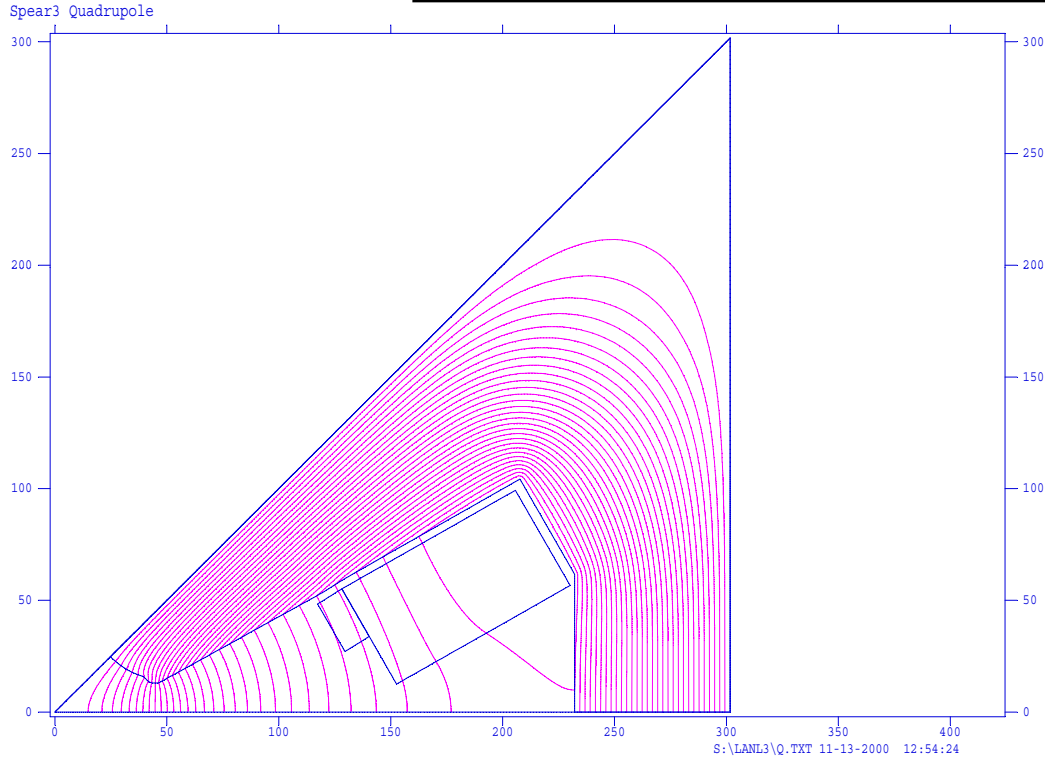
Number of magnets	112
Aperture	61 mm
Max. gradient	22.4 T/m
Max. current	185 A



- 2 types of laminations, 1 coil (46 turns, 8x8 mm,  $\varnothing$ 5 mm)
- Mechanically made of 4 pieces
- 100 closed magnets, 12 opened magnets
- Iron Overall dimensions are 600x600 mm

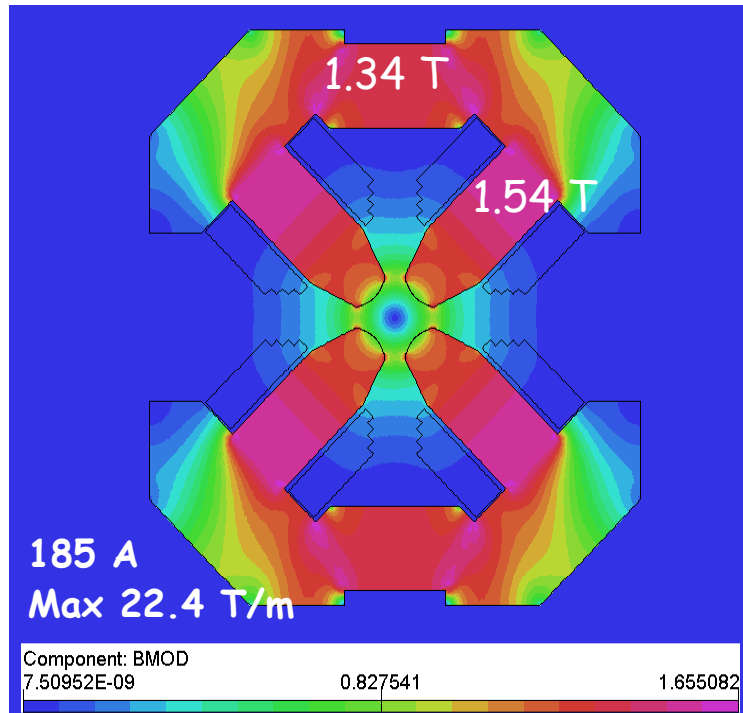
# Symmetric Quadrupole

## Magnetic field lines in a symmetric quadrupole

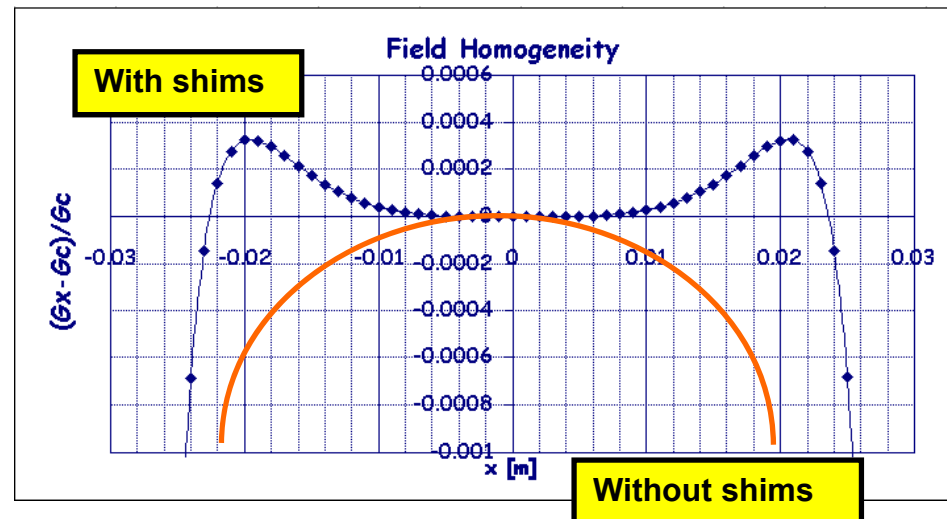
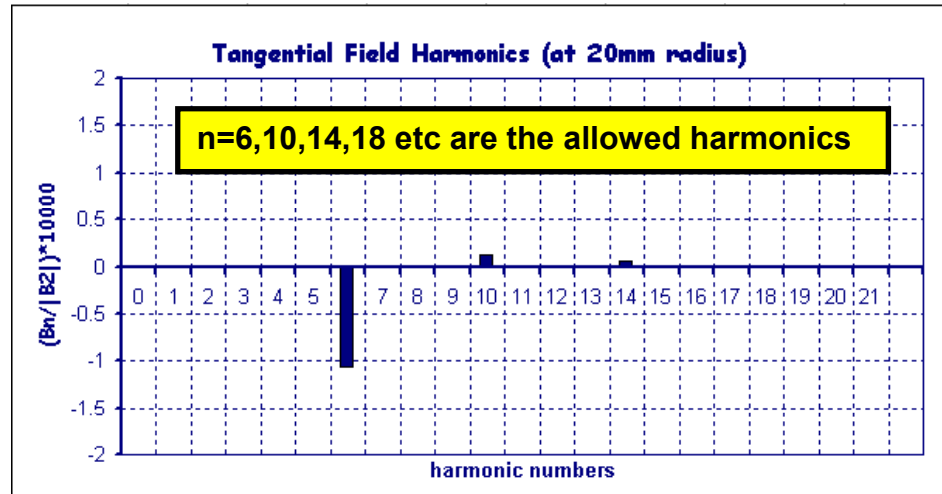


**Introducing of a shim to increase the good field region**

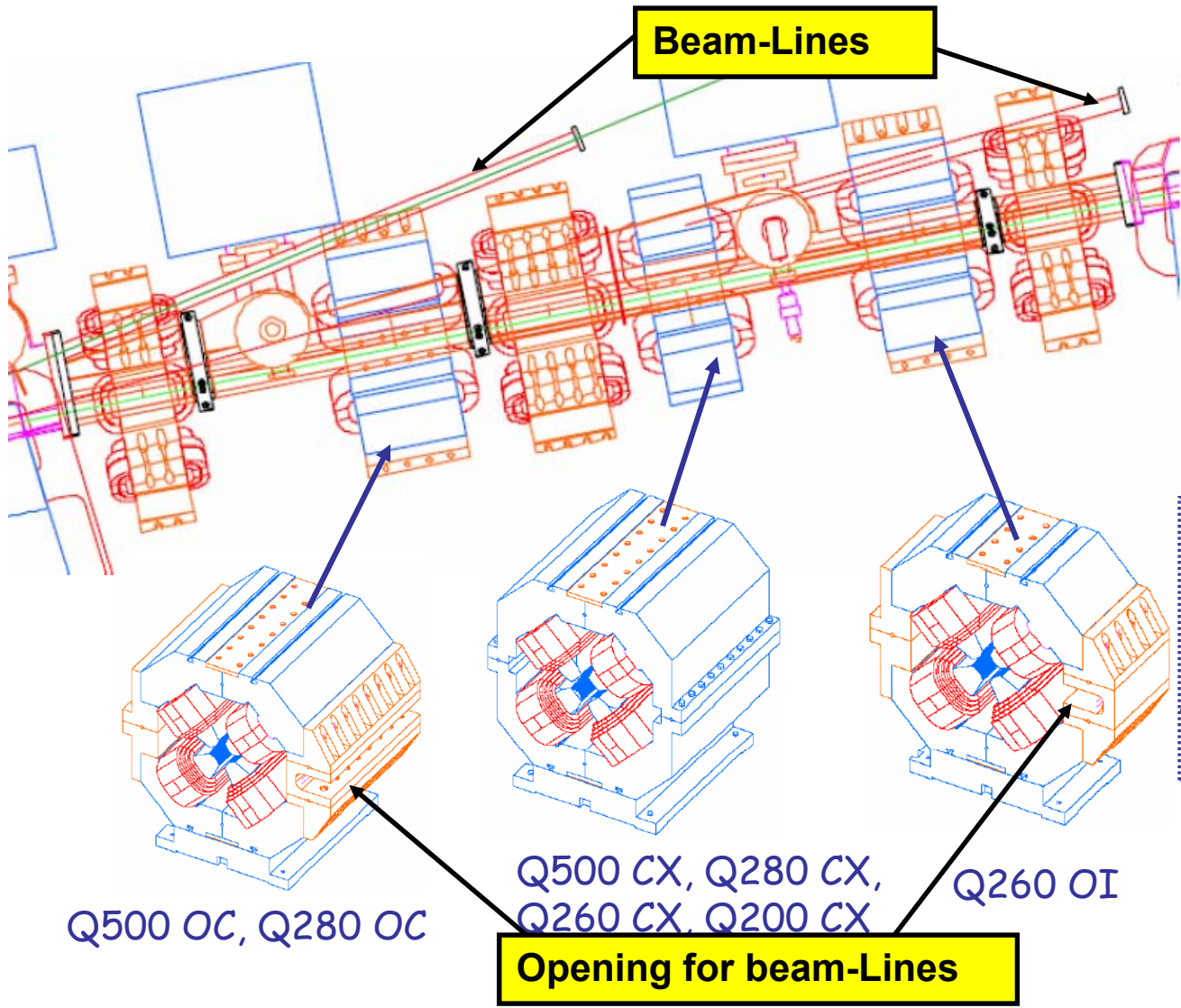
# Quadrupole Magnets (2D-OPERA models)



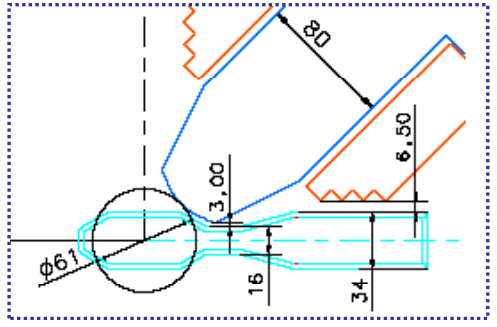
$$\frac{\Delta g}{g} \leq \pm 4 \cdot 10^{-4} \quad |x| \leq 24.5 \text{ mm}$$



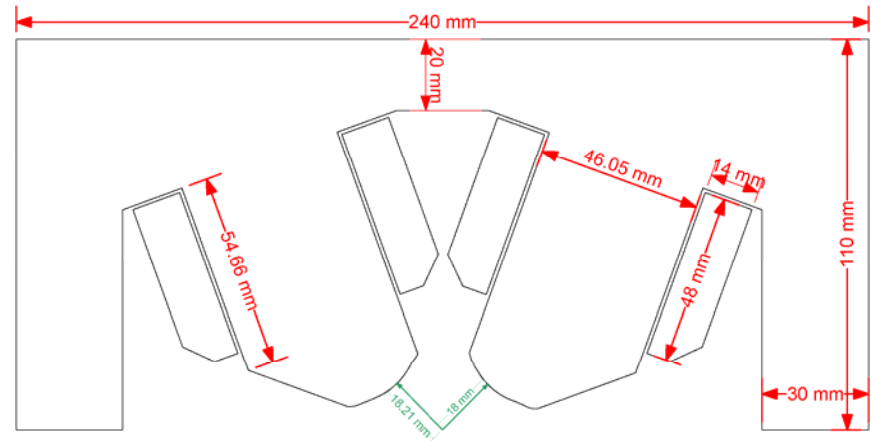
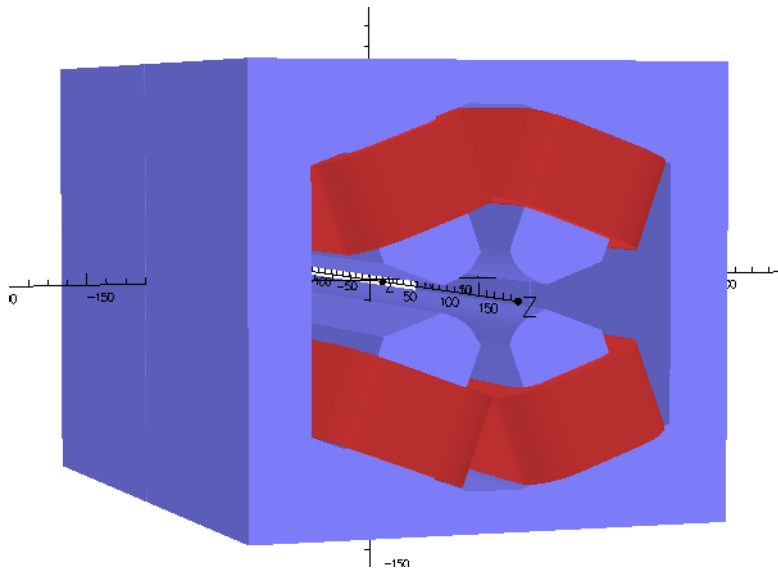
# Quadrupole and Beam-Lines: Spacers



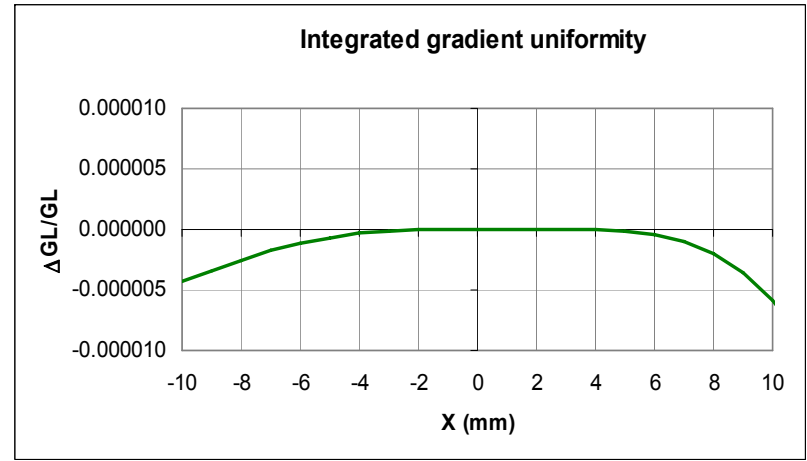
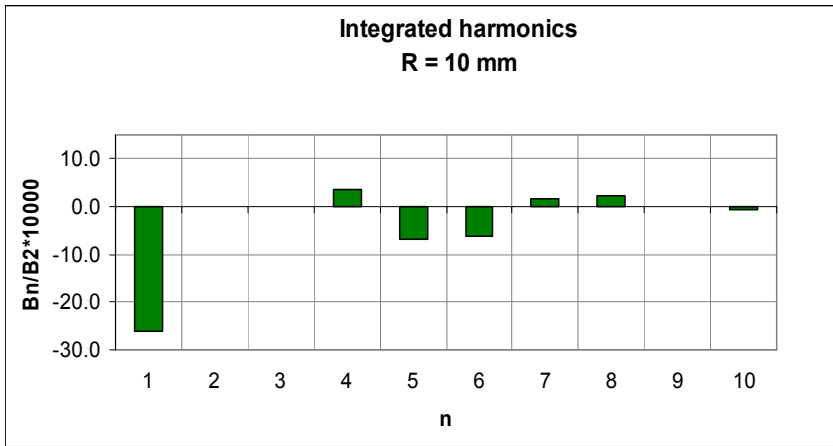
Type	Number
Q200CX	16+1
Q260CX	42+2
Q260OI	6+1
Q280CX	20+1
Q280OC	4+1
Q500CX	22+1
Q500OC	2+1



# Combined Quadrupoles for the ALBA Booster



## Introduction of a sextupole field

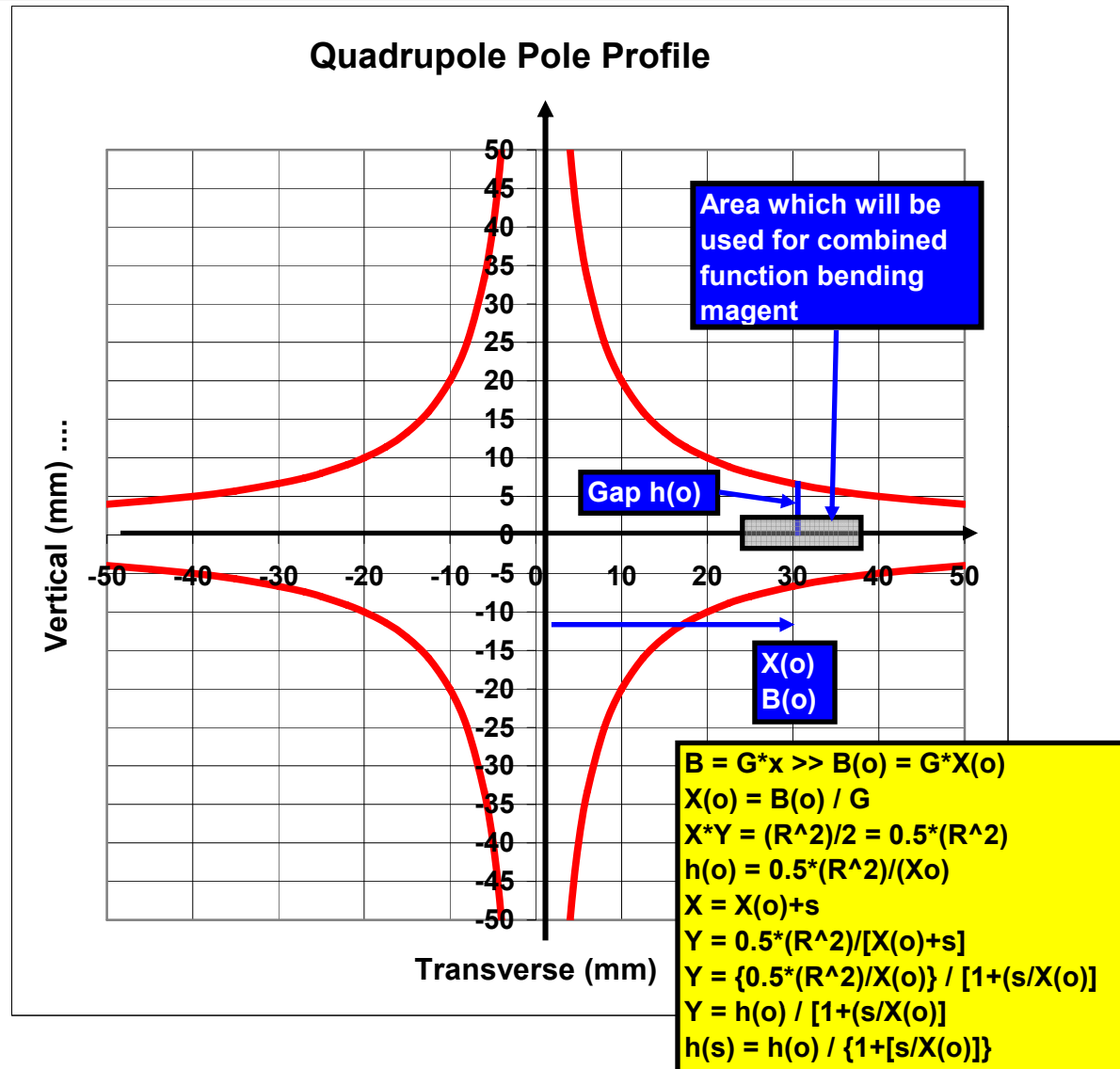




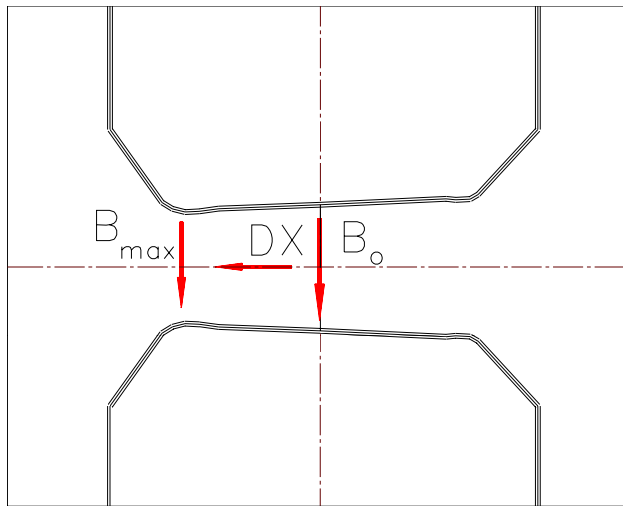
# Combined function Bending Magnet.

The combined function bending magnet combines the function of a bending magnet (bending a beam) and a quadrupole (focussing a beam) in one unit, the so called combined bending magnet. It is combined a "bending" in the horizontal direction with a "focussing" in the vertical direction. With combined bending magnet it is possible to build a compact machine, because some of the defocussing quadrupoles are not needed.

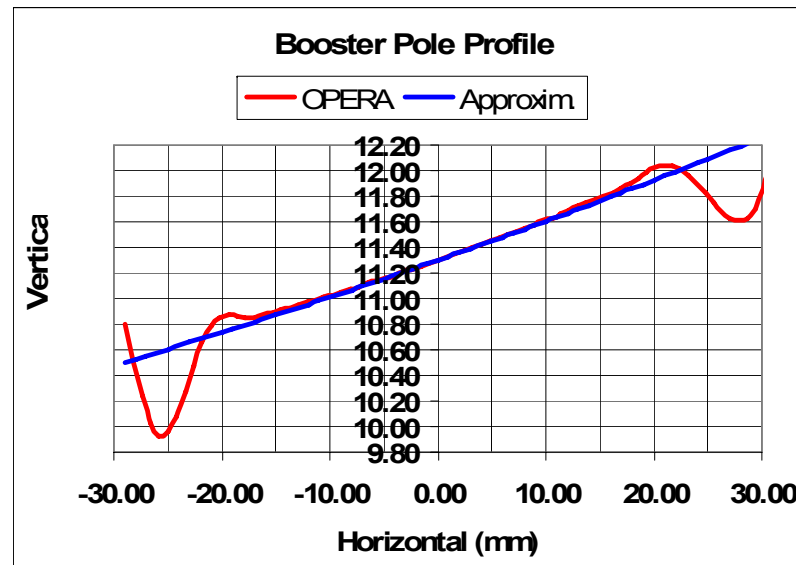
At Alba we used the combined bending magnet in the storage ring as well in the booster synchrotron.



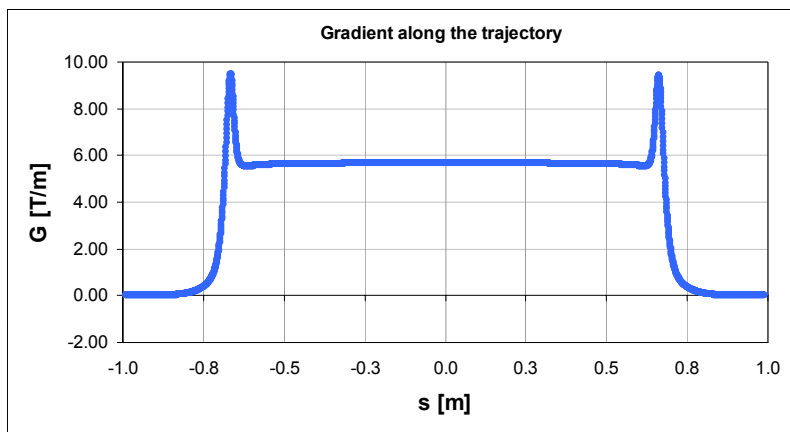
# Combined function Bending Magnet.



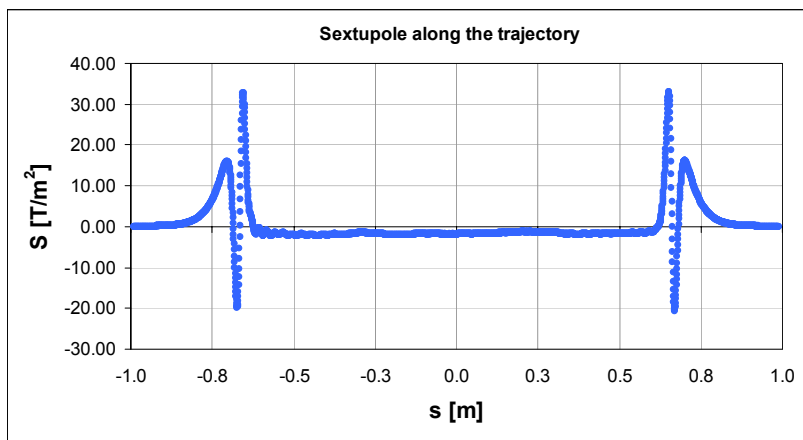
Source	Energy (GeV)	$B_0$ (T)	G (T/m)	$B_{max}$ (T)
ALS	1.9	1.279	5.133	1.58
Elettra	2.3	1.38	3.303	1.58
Boomerang	3.0	1.30	3.335	1.50
CLS	2.9	1.354	3.867	1.586
SPEAR III	3.3	1.4	3.60	1.62
ALBA	3.0	1.42	5.66	1.66



# Longitudinal Field Distribution



Gradient: Focusing strength



Sextupole:  
Change in chromaticity

# Parameters of the ALBA Bending Magnets

Storage Ring Bending Magnet		
<b>Magnetic properties</b>		
Beam Energy (E)	GeV	3
Central Field (Bo)	T	1.42
Field gradient (Go)	T/m	5.656
Sextupole component (B <sup>''</sup> )	T/(m <sup>2</sup> )	0
Effective length (Lo)	m	1.384
<b>Mechanical properties</b>		
Bending radius (Ro)	m	7.047
Bending angle (phi)	degrees	11.25
Central Gap (h)	mm	36
Length of Fe-yoke L(Fe)	mm	1340
<b>Coil and conductor</b>		
Number of coils		2
Number of pancakes per coil		4
Number of turns per pancake		10
Conductor size	mm <sup>2</sup>	16.3*10.8
Cooling channel diameter (D)	mm	6.6
Number of ampere turns per coil	A-turns	21000
Current (I)	A	527
Current density (j)	A/mm <sup>2</sup>	3.72
Resistance at 23 degrees	Ohm	
Inductivity	mH	
Voltage drop	V	
Power	kW	
<b>Cooling</b>		
Maximim DT	Celsius	8.6
Nominal input temperature	Celsius	23
Number of cooling circuits per coil		2
Maximum pressure drop per magnet	bar	7

Booster Bending Magnets			
		5 Degr.	10 Degr.
<b>Magnetic properties</b>			
Beam Energy (E)	GeV	3	3
Central Field (Bo)	T	0.8733	0.8733
Field gradient (Go)	T/m	2.292	2.292
Sextupole component (B <sup>''</sup> )	T/(m <sup>2</sup> )	18	18
Effective length (Lo)	m	1	2
<b>Mechanical properties</b>			
Bending radius (Ro)	m	11.4592	11.4592
Bending angle (phi)	degrees	5	10
Central Gap (h)	mm	22.6	22.6
Length of Fe-yoke L(Fe)	mm	0.972	1.972
<b>Coil and conductor</b>			
Number of coils		2	2
Number of pancakes per coil		1	1
Number of turns per pancake		12	12
Conductor size	mm <sup>2</sup>	12*12	12*12
Cooling channel diameter (D)	mm	5	5
Number of ampere turns per coil	A-turns	7906	7906
Current (I)	A	659	659
Current density (j)	A/mm <sup>2</sup>	6.08	6.08
Resistance at 23 degrees	mOhm	9.2	18.2
Inductance	mH	1.3	2.6
Voltage drop	V	6.1	31.8
Power	kW	2	3.94
<b>Cooling</b>			
Maximim DT	Celsius	11	11
Nominal input temperature	Celsius		23
Number of cooling circuits per coil		1	2
Maximum pressure drop per magnet	bar	7	7

# Parameters of the ALBA Quadrupoles

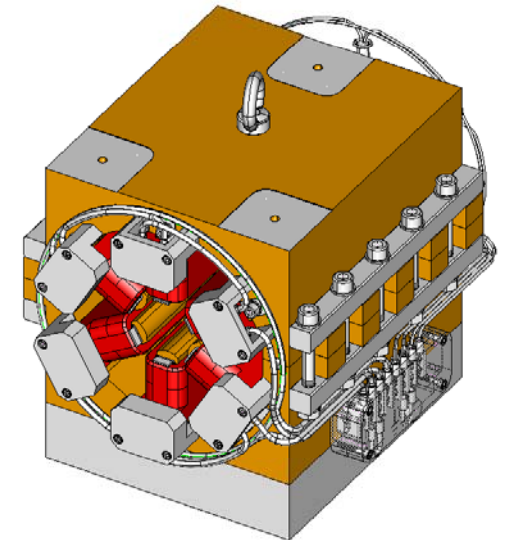
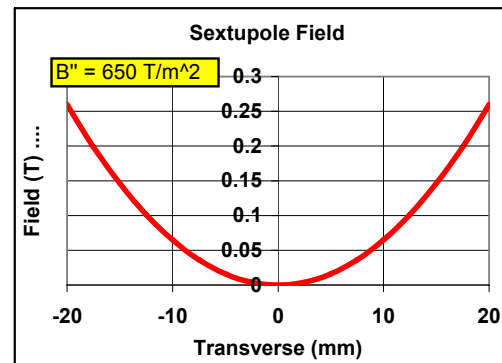
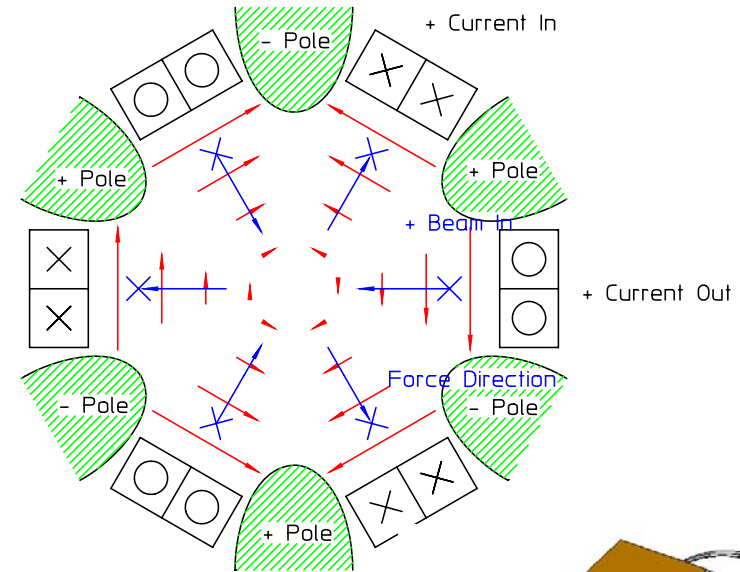
Storage Ring Quadrupole Magnets					
		Q200	Q260	Q280	Q500
<b>Magnetic properties</b>					
Beam Energy (E)	GeV	3	3	3	3
Field gradient (Go)	T/m	19.8	21	21.4	21.9
Sextupole component (B <sup>''</sup> )	T/(m <sup>2</sup> )	0	0	0	0
Effective length (Lo)	m	0.23	0.29	0.31	0.53
<b>Mechanical properties</b>					
Aperture radius	mm	30.5	30.5	30.5	30.5
Length of Fe-yoke L(Fe)	m	0.2	0.26	0.28	0.5
Maximum length of magnet	m	0.298	0.358	0.378	0.598
<b>Coil and conductor</b>					
Number of coils		4	4	4	4
Number of turns per coil		46	46	46	46
Conductor size	mm <sup>2</sup>	8*8	8*8	8*8	8*8
Cooling channel diameter (D)	mm	5	5	5	5
Number of ampere turns per coil	A-turns	7801	8274	8432	8629
Current (I)	A	167.8	178.7	199.6	187.4
Current density (j)	A/mm <sup>2</sup>	3.78	4.02	4.5	4.22
Resistance at 23 degrees	mΩ				
Inductivity	mH				
Voltage drop	V				
Power	kW				
<b>Cooling</b>					
Maximim DT	Celsius	8	8	8	8
Nominal input temperature	Celsius	23	23	23	23
Number of cooling circuits per coil		4	4	4	4
Maximum pressure drop per magnet	bar	7	7	7	7

Booster Quadrupole Magnets				
		QS180	QS340	QC340
<b>Magnetic properties</b>				
Beam Energy (E)	GeV	3	3	3
Field gradient (Go)	T/m	17.45	17.45	17.45
Sextupole component (B <sup>''</sup> )	T/(m <sup>2</sup> )	0	0	5
Effective length (Lo)	m	0.2	0.36	0.36
<b>Mechanical properties</b>				
Aperture radius	mm	18	18	18
Length of Fe-yoke L(Fe)	m	0.18	0.34	0.34
Maximum length of magnet	m	0.28	0.44	0.44
<b>Coil and conductor</b>				
Number of coils		4	4	4
Number of turns per coil		17	17	17
Conductor size	mm <sup>2</sup>	5*5	5*5	5*5
Cooling channel diameter (D)	mm	3	3	3
Number of ampere turns per coil	A-turns	2250	2250	2250
Current (I)	A	132.4	132.4	132.4
Current density (j)	A/mm <sup>2</sup>	3.78	4.02	4.22
Resistance at 23 degrees	mΩ	34.6	59	59
Inductivity	mH	3	6	6
Voltage drop (resistive)	V	4.6	7.8	7.8
Power	W	606	1034	1034
<b>Cooling</b>				
Maximim DT	Celsius	8	8	8
Nominal input temperature	Celsius	23	23	23
Number of cooling circuits per coil		1	1	1
Maximum pressure drop per magnet	bar	7	7	7

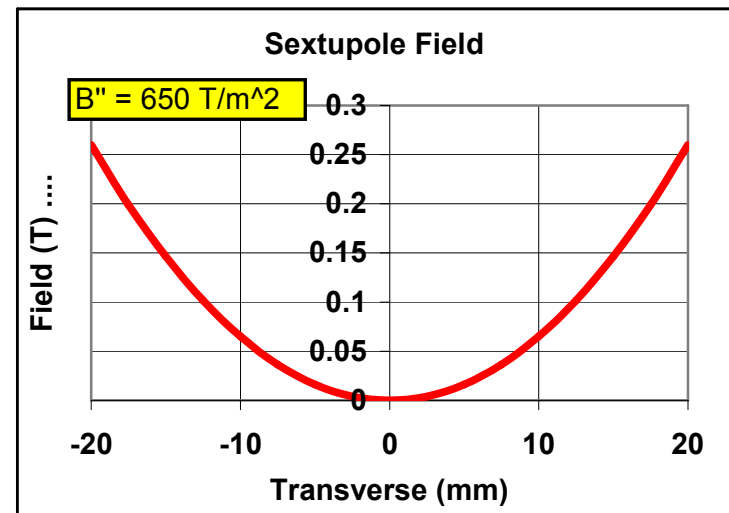
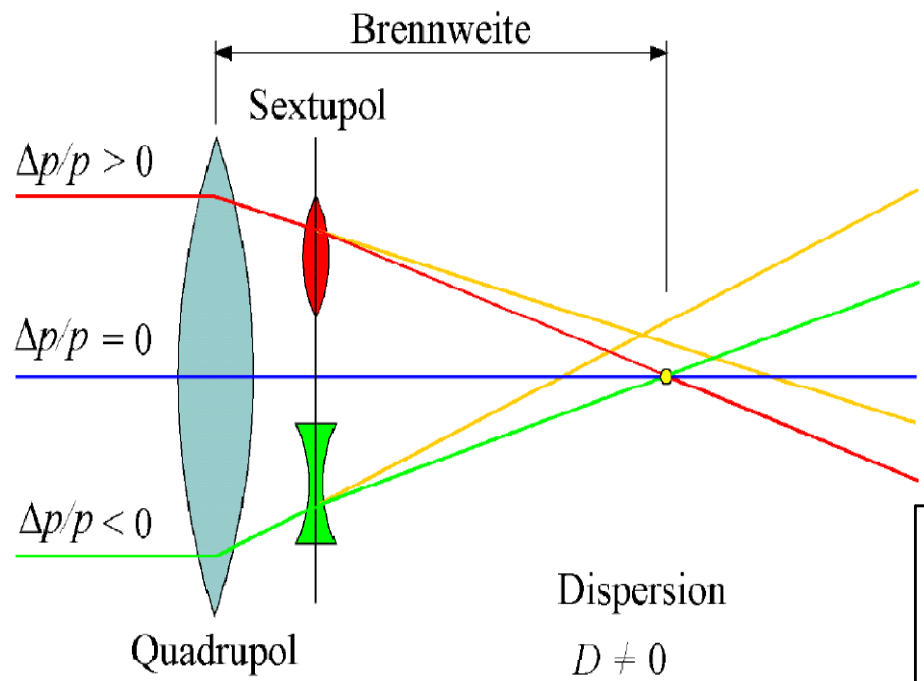
# Sextupole Design.

The Sextupole Magnet has six poles. The field varies *quadratically* with the distance from the magnet center. It's purpose is to affect the beam at the edges, much like an optical lens which corrects chromatic aberration. Sextupole are needed for the compensation of the chromaticity to make in a small range the focusing of the machine energy independent. An *F* sextupole will steer the particle beam toward the center of the ring.

Note that the sextupole also steers along the 60 and 120 degree lines.



# Sextupole Design.



# Sextupole Excitation

Using arguments similar to those used for the dipole:

$$\oint_{Path2} \overline{H} \cdot \overline{dl} + \oint_{Path3} \overline{H} \cdot \overline{dl} \approx 0$$

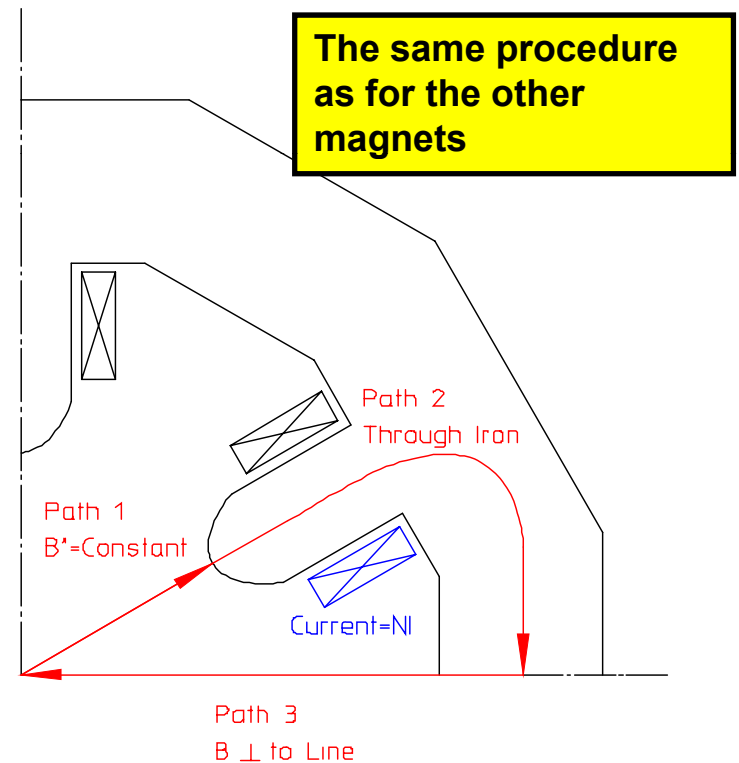
Along Path 1:  $B'(r) = \int B'' dr = B'' r$

$$B(r) = \int B'' r dr = \frac{B'' r^2}{2}$$

$$H \parallel r \quad \text{and} \quad |H(r)| = \frac{B'' r^2}{2\mu_0}$$

$$\oint_{Path1} \overline{H} \cdot \overline{dl} = \int_0^R \frac{B'' r^2 dr}{2\mu_0} = \frac{B'' R^3}{6\mu_0}$$

$$\text{Finally; } \oint \overline{H} \cdot \overline{dl} = NI \approx \frac{B'' R^3}{6\mu_0}$$





## Excitation current in a Sextupole

$$\oint \vec{H} \cdot d\vec{l} = NI \approx \frac{B'' R^3}{6\mu_0}$$

$$N \cdot I = B'' \cdot R^3 / (6 \cdot \mu_0)$$

$$B_{\text{pole}} = B'' \cdot R^2 / 2 \quad \text{and} \quad B'' = 2 \cdot B_{\text{pole}} / R^2$$

The magnetic flux at the pole tip is given by:  $B_{\text{pole}} = 3 \cdot \mu_0 \cdot N \cdot I / R$ .

The differential gradient is given by:  $B'' = 6 \cdot \mu_0 \cdot N \cdot I / R^3 = 2 \cdot B_{\text{pole}} / R^2$

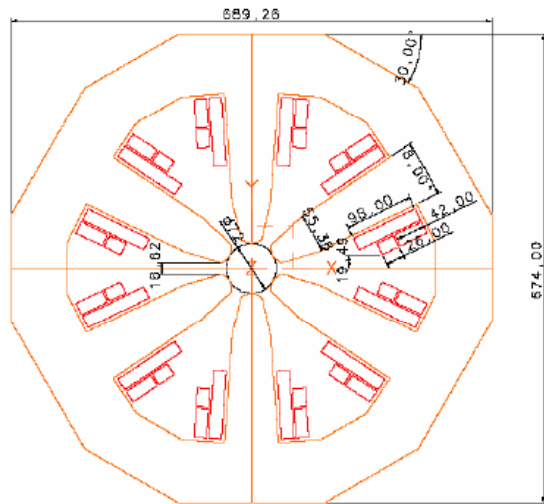
With  $R = 35 \text{ mm}$  and  $B_{\text{pole}} = 0.4 \text{ T}$  the differential gradient is roughly  $653 \text{ T/m}^2$ .

Today one can go up with the gradient to  $750 \text{ T/m}^2$

# Design of ALBA Sextupoles

## Former Sextupole Magnets

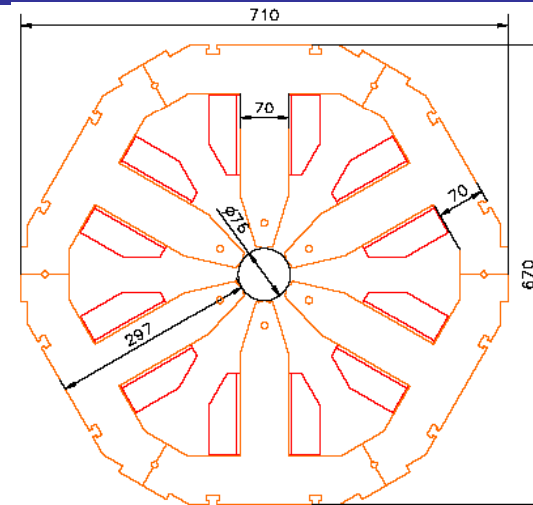
Number of magnets	120
Aperture	72 mm
Max. differential gradient	600 T/m <sup>2</sup>
Max. current	200 A



- 2 sextupole cross section needed.
- Sextupolar field: 1 coil per pole (28 turns, 7×7 mm, Ø3.5 mm).
- Correctors: 2 coils per pole (10 & 6 turns, 7×7 mm, Ø3.5 mm).

## Present Sextupole Magnets

Number of magnets	120
Aperture	76 mm
Max. differential gradient	700 T/m <sup>2</sup>
Max. current	200 A

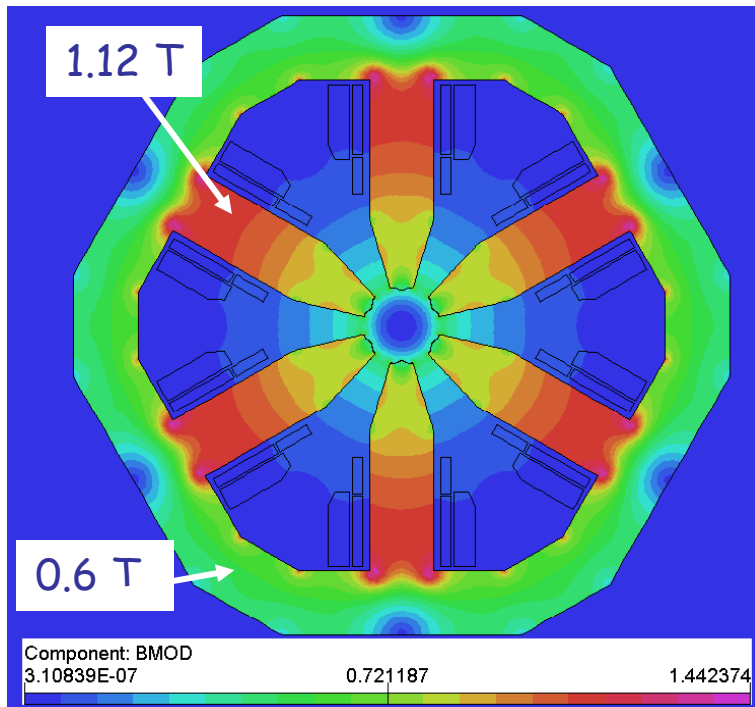


- 1 sextupole cross section.
- Sextupolar field: 1 coil per pole (28 turns, 7×7 mm, Ø3.5 mm).
- Correctors: 2 coils per pole (224 & 112 turns, 0.8×4.5mm solid conductor).

# ALBA Sextupole Magnets (2D-OPERA models)

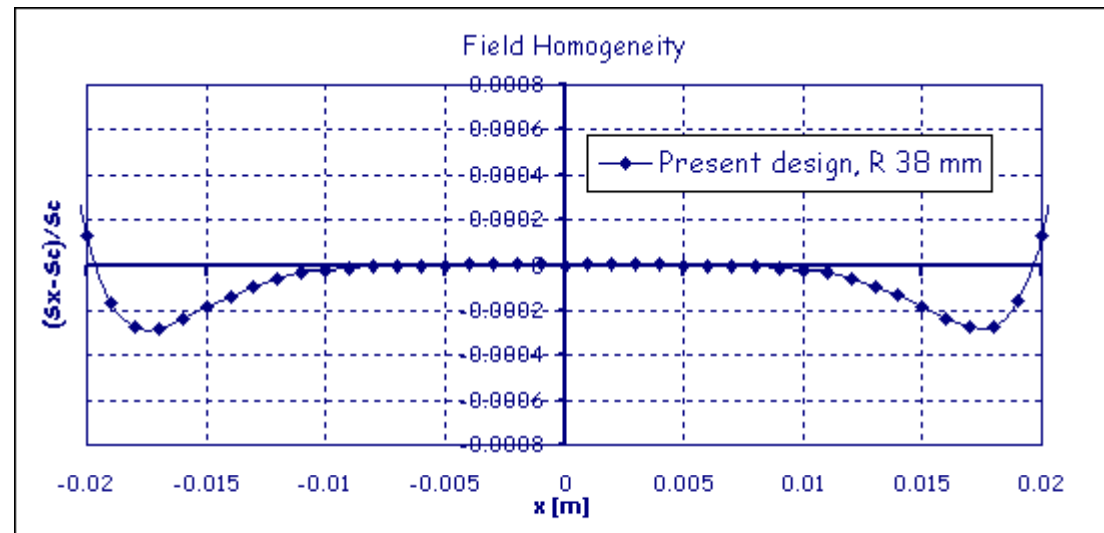
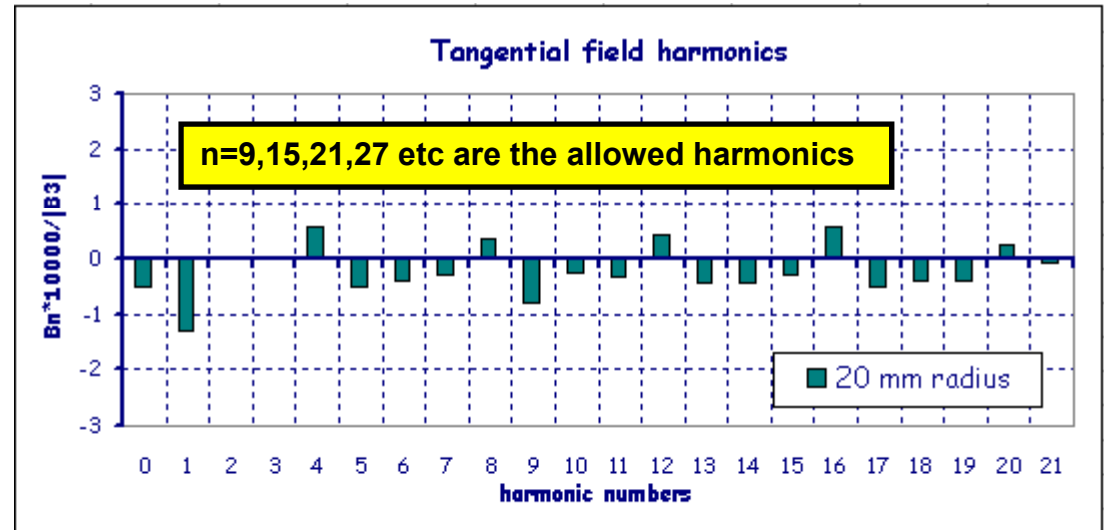
## Sextupolar field:

Max 4020 A-turns/coil  
 $S = 532 \text{ T/m}^2$  (+5% included)



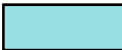
Required good field region


$$-20 \leq x \leq 20 \text{ mm} \quad \frac{\Delta S}{S} \leq \pm 1 \cdot 10^{-3}$$



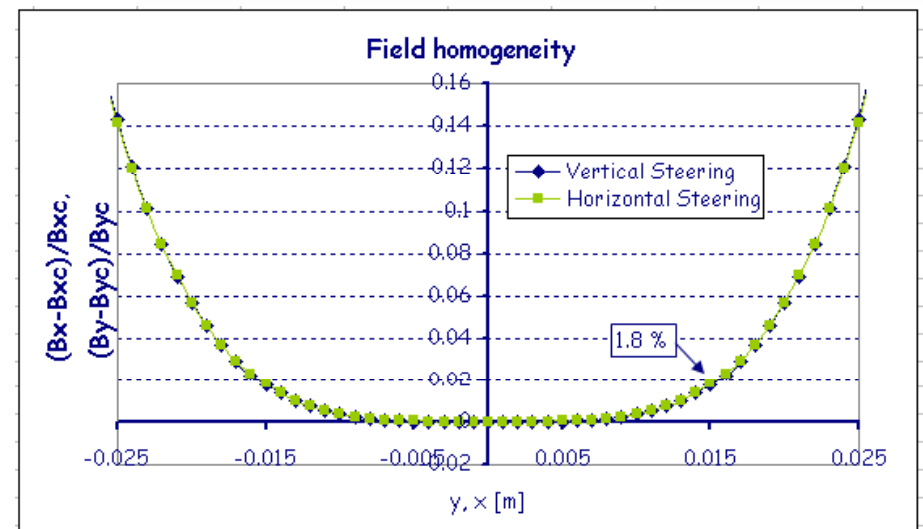
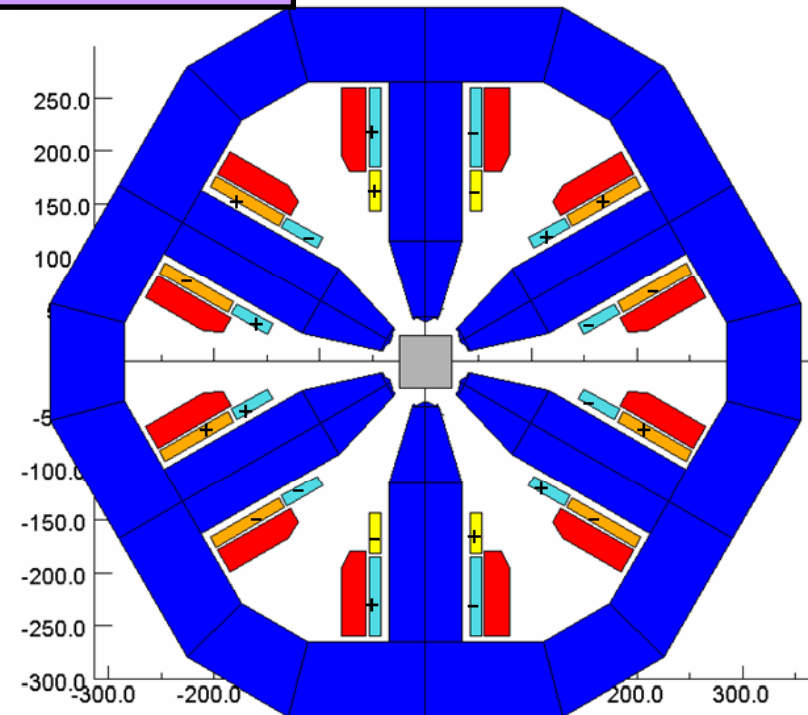
## Steering (2D-models)

All sextupoles will be equipped with steering coils for:

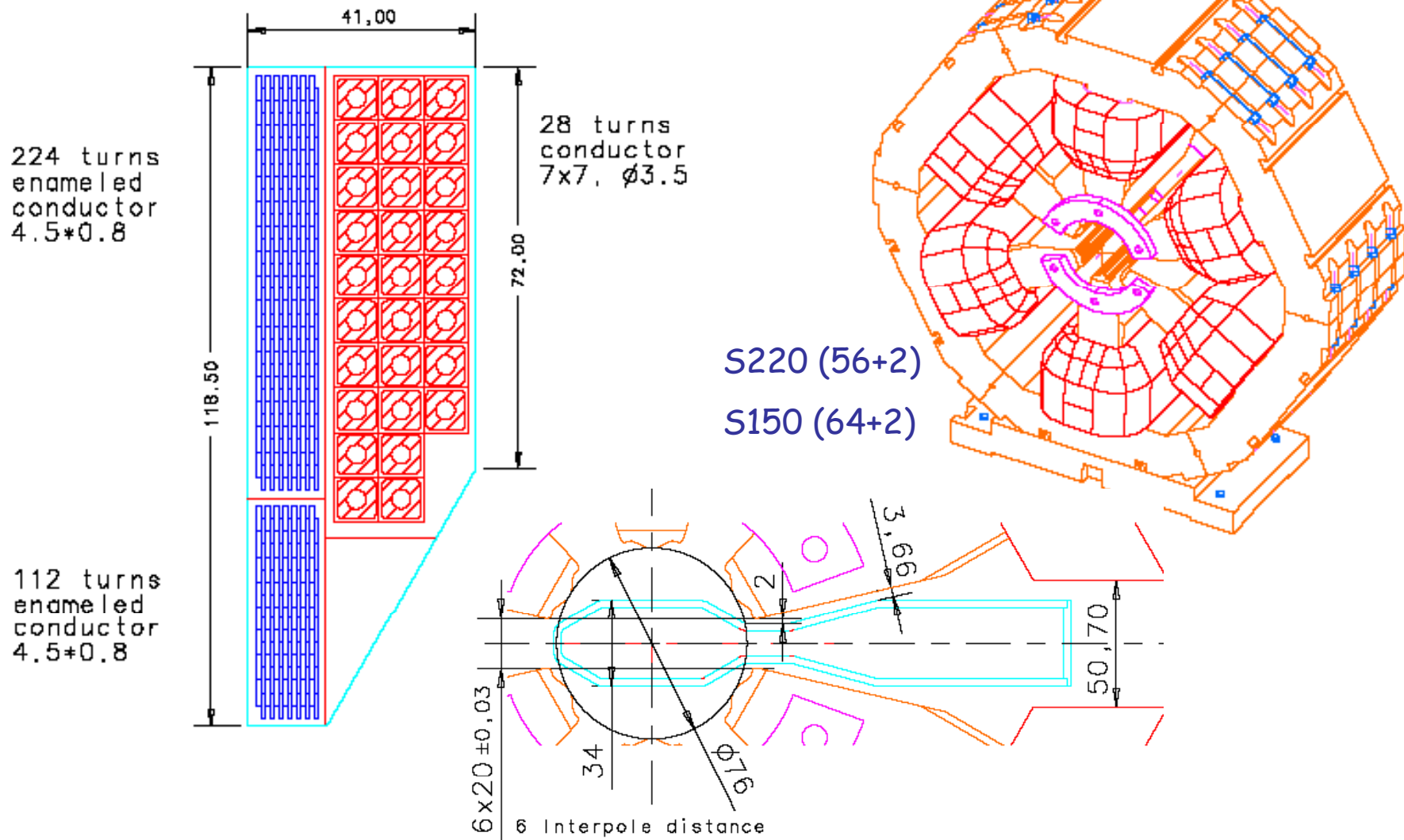
 Horizontal Steering 0.8 mrad  
2 coil types (1806 A-turn, 903 A-turn)  
 $B_y(x=0) = 0.0514 \text{ T}$

 Vertical Steering 0.8 mrad  
1 coil type (1520 A-turn)  
 $B_x(y=0) = 0.0499 \text{ T}$

 Skew Quadrupole  $g_x=0.2 \text{ T/m}$   
1 coil type (225 A-turn)



# ALBA-Sextupoles: Mechanical design



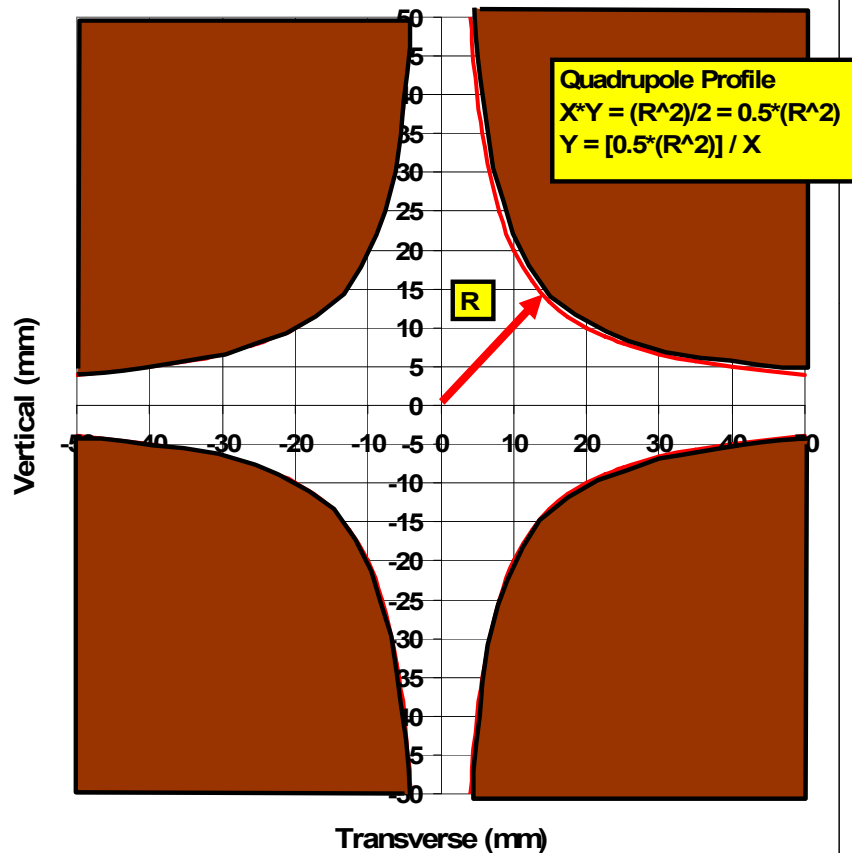
# Parameters of the ALBA Sextupoles

Storage Ring Sextupole Magnets			
		S-150	S-220
<b>Magnetic properties</b>			
Beam Energy (E)	GeV	3	3
Sextupole component (B'')	T/(m <sup>2</sup> )	700	700
Magnetic field at pole tip	T	0.51	0.51
Effective length (L <sub>o</sub> )	m	0.175	0.245
<b>Mechanical properties</b>			
Aperture radius	mm	38	38
Length of Fe-yoke L(Fe)	m	0.15	0.22
Maximum length of magnet	m	0.252	0.322
<b>Coil and conductor</b>			
Number of coils		6	6
Number of turns per coil		28	28
Conductor size	mm <sup>2</sup>	7*7	7*7
Cooling channel diameter (D)	mm	3.5	3.5
Number of ampere turns per coil	A-turns	5400	5400
Current (I)	A	192.9	192.9
Current density (j)	A/mm <sup>2</sup>	4.9	4.9
Resistance at 23 degrees	mΩ		
Inductivity	mH		
Voltage drop	V		
Power	kW		
<b>Cooling</b>			
Maximim DT	Celsius	9	12
Nominal input temperature	Celsius	23	23
Number of cooling circuits per coil		3	3
Maximum pressure drop per magnet	bar	7	7

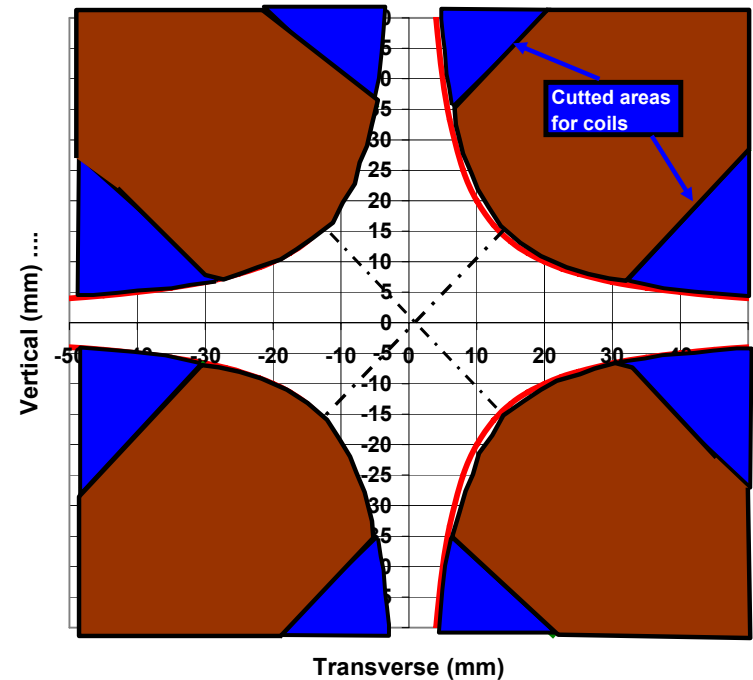
Booster Sextupole Magnets		
		S-200
<b>Magnetic properties</b>		
Beam Energy (E)	GeV	3
Sextupole component (B'')	T/(m <sup>2</sup> )	400
Magnetic field at pole tip	T	0.065
Effective length (L <sub>o</sub> )	m	0.2
<b>Mechanical properties</b>		
Aperture radius	mm	18
Length of Fe-yoke L(Fe)	m	0.2
Maximum length of magnet	m	0.3
<b>Coil and conductor</b>		
Number of coils		6
Number of turns per coil		50
Conductor size	mm <sup>2</sup>	2.8*1
Cooling channel diameter (D)	mm	
Number of ampere turns per coil	A-turns	310
Current (I)	A	6.2
Current density (j)	A/mm <sup>2</sup>	2.21
Resistance at 23 degrees	mΩ	886
Inductivity	mH	34
Voltage drop (resistive)	V	5.5
Power	W	33
<b>Cooling</b>		
Maximim DT	Celsius	
Nominal input temperature	Celsius	
Number of cooling circuits per coil		
Maximum pressure drop per magnet	bar	

# Explanation for higher Harmonics

Quadrupole Pole Profile



Quadrupole Pole Profile



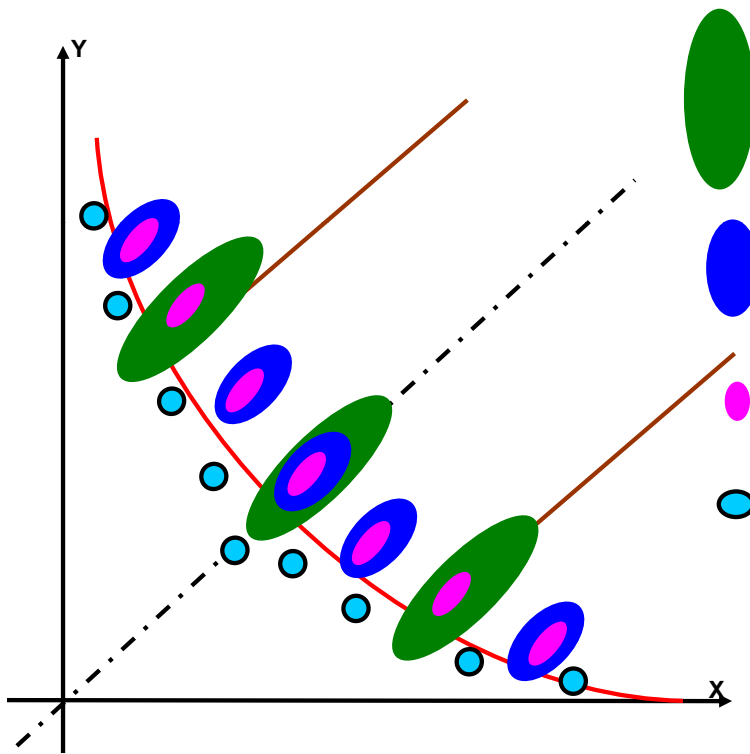
By cutting the pole profile in order to have space for the introduction of the coils the field distribution will be disturbed and higher multipoles will be introduced.

# Higher Harmonics in Magnets

A dipole has overall 2 poles, which is  $n = 1$

A quadrupole has overall 4 poles, which is  $n = 2$

A sextupole has overall 6 poles, which is  $n = 3$



Dipole: Introduction of 3 poles, that means overall 6 poles, which is  $n = 3$

Quadrupole: Introduction of 3 poles, that means overall 12 poles, which is  $n = 6$

Sextupole: Introduction of 3 poles, that means overall 18 poles, which is  $n = 9$

Dipole: Introduction of 5 poles, that means overall 10 poles, which is  $n = 5$

Quadrupole: Introduction of 5 poles, that means overall 20 poles, which is  $n = 10$

Sextupole: Introduction of 5 poles, that means overall 30 poles, which is  $n = 15$

Dipole: Introduction of 7 poles, that means overall 14 poles, which is  $n = 7$

Quadrupole: Introduction of 7 poles, that means overall 28 poles, which is  $n = 14$

Sextupole: Introduction of 7 poles, that means overall 42 poles, which is  $n = 21$

Dipole: Introduction of 9 poles, that means overall 18 poles, which is  $n = 9$

Quadrupole: Introduction of 9 poles, that means overall 36 poles, which is  $n = 18$

Sextupole: Introduction of 9 poles, that means overall 54 poles, which is  $n = 27$



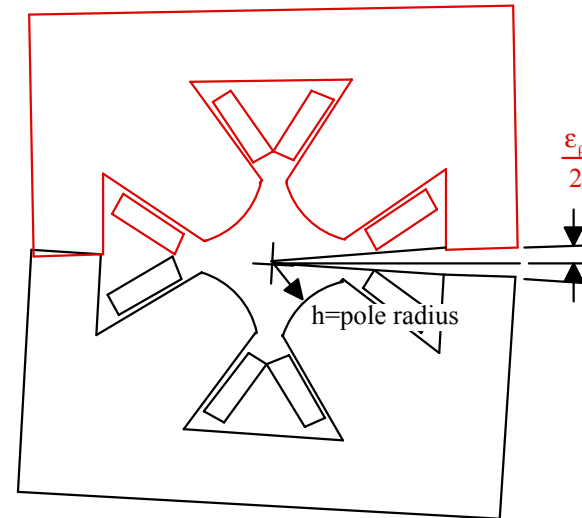
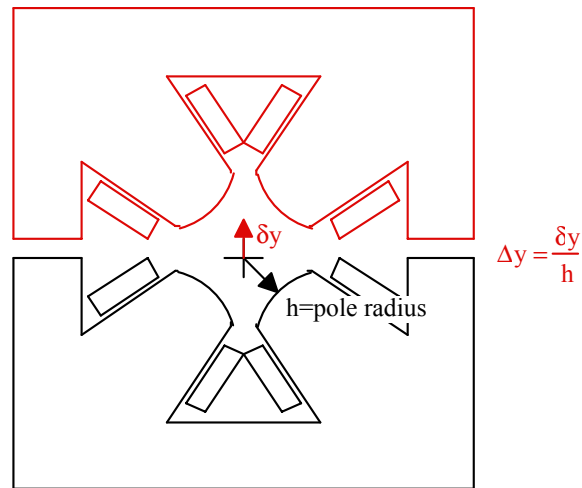
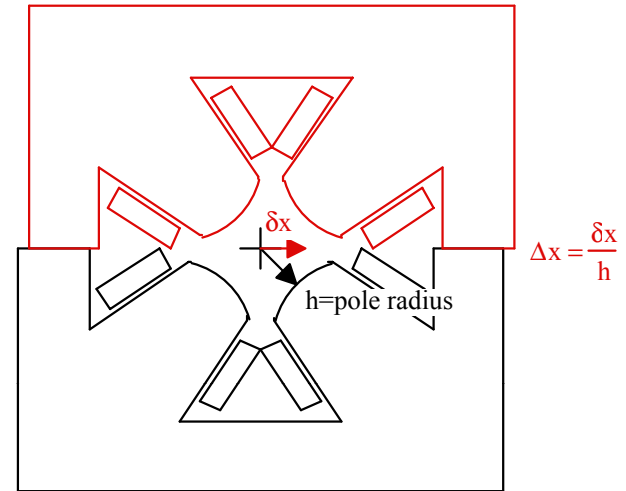
## Summary - 'Allowed' Harmonics

Summary of 'allowed harmonics' in fully symmetric magnets:

Fundamental geometry	'Allowed' harmonics
Dipole, $n = 1$	$n = 3, 5, 7, \dots$ ( 6 pole, 10 pole, etc.)
Quadrupole, $n = 2$	$n = 6, 10, 14, \dots$ (12 pole, 20 pole, etc.)
Sextupole, $n = 3$	$n = 9, 15, 21, \dots$ (18 pole, 30 pole, etc.)
Octupole, $n = 4$	$n = 12, 20, 28, \dots$ (24 pole, 40 pole, etc.)

# Higher Harmonics in Magnets

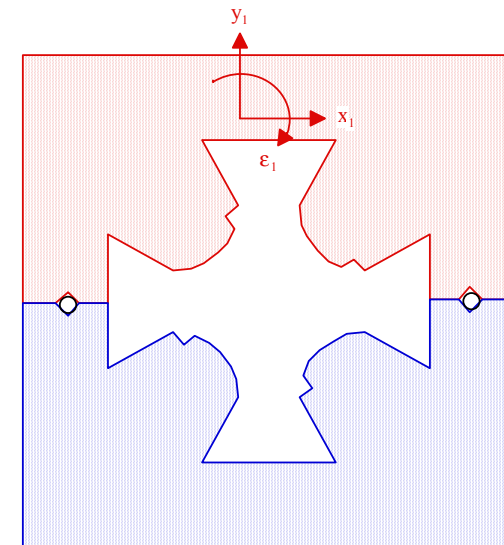
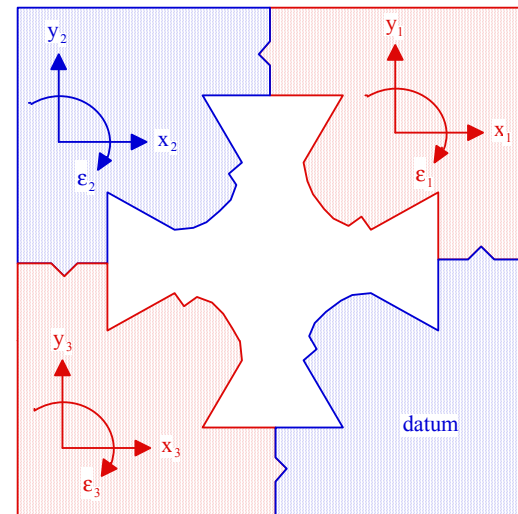
**Assembling errors introduce higher harmonics**



# Assembling of Quadrupoles

Each segment can be assembled with errors with three kinematic motions,  $x$ ,  $y$  and  $e$  (rotation). Thus, combining the possible errors of the three segments with respect to the datum segment, the core assembly can be assembled with errors with  $3 \times 3 \times 3 = 27$  degrees of freedom.

This assembly has the advantage that the two core halves can be assembled kinematically with *only* three degrees of freedom for assembly errors. Thus, assembly errors are more easily measured and controlled.



# Higher Harmonics in Magnets

## Typical results from a quadrupole

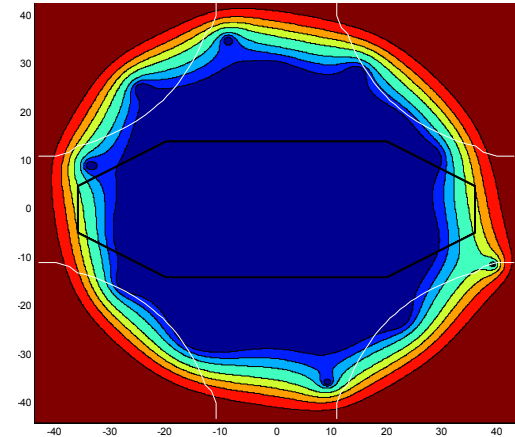
### QUADROPOLE REGISTRATION CERTIFICATE OF Q500CX\_021\_180A\_02

Date: 10:08:00 12.05.2008  
 Quadrupole effective length (L): 50.00 cm. Coil radius (R): 2.65 cm. Horizontal shift: 0.0094 mm.  
 Main current (I<sub>M</sub>): 0.0000 A. Vertical shift: 0.0131 mm.  
 Correction current: I = 0.0000 A. Angle of slope: 0.00001 rad.  
 D:\Measurements\====ALDA====\04\June\quadrupoles\Fourth\Q500CX\_021\_180A\_02\_180A\_02.imf

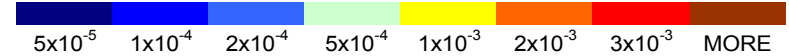
Harmonic (n)	Magnetic field amplitude at radius 1.00 cm.		Magnetic field amplitude at reference radius 2.500 cm.				Phase (rad.)
	$\sqrt{A_n^2 + B_n^2}$ (Gs)	relative	$B_n$ (Gs)	$A_n$ (Gs)	$\sqrt{A_n^2 + B_n^2}$ (Gs)	relative	
1	3.686146	0.001616	-2.15540	-2.99030	3.686146	0.000646	0.94624
2	2281.200	1.000000	-5703.00	-0.15592	5703.001	1.000000	0.00002
3	0.536546	0.000235	1.404338	3.045198	3.353416	0.000588	1.13869
4	0.084272	0.000037	1.312289	0.108339	1.318754	0.000231	0.08237
5	0.015485	0.000007	0.552235	0.248880	0.804900	0.000108	0.42037
6	0.009183	0.000004	0.890467	0.087789	0.894784	0.000157	0.09827
7	0.001125	0.000000	0.235104	-0.14181	0.274563	0.000048	-0.5427
8	0.001022	0.000000	-0.53130	0.326749	0.623742	0.000109	-0.5513
9	0.000080	0.000000	-0.07418	0.054830	0.082131	0.000016	-0.8347
10	0.000326	0.000000	1.230373	0.189723	1.244915	0.000218	0.15299
11	0.000038	0.000000	0.080772	0.351094	0.360266	0.000063	1.34467
12	0.000010	0.000000	0.068473	0.228560	0.238030	0.000042	1.28777
13	0.000006	0.000000	0.021216	0.380901	0.381491	0.000067	1.51515
14	0.000003	0.000000	0.390377	0.071631	0.396894	0.000070	0.18147
15	0.000001	0.000000	0.201315	0.075037	0.214845	0.000038	0.35678
16	0.000000	0.000000	-0.03192	0.144512	0.147997	0.000026	-1.3533
17	0.000000	0.000000	0.066092	0.200132	0.210763	0.000037	1.25183
18	0.000000	0.000000	0.188873	0.078697	0.202000	0.000035	0.38945
19	0.000000	0.000000	0.091177	0.083107	0.123369	0.000022	0.73913
20	0.000000	0.000000	0.063095	0.019988	0.066185	0.000012	0.30679

Operator: Blinov Semenov

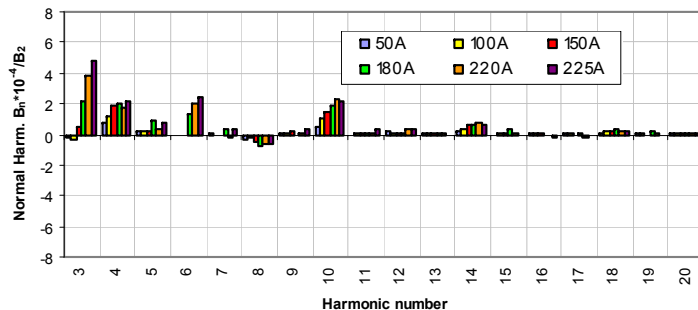
$$\int_{-L}^{+L} H_z dz = \int_{-L}^{+L} H_z dz + \int_{-L}^{+L} H_z dz \quad \text{where} \quad \int_{-L}^{+L} H_z dz = \sum_{n=1}^{\infty} \left( \frac{L}{n} \right)^{n-1} (A_n \cos(n\phi) - B_n \sin(n\phi)) \quad \int_{-L}^{+L} H_z dz = \sum_{n=1}^{\infty} \left( \frac{L}{n} \right)^{n-1} (A_n \sin(n\phi) + B_n \cos(n\phi))$$



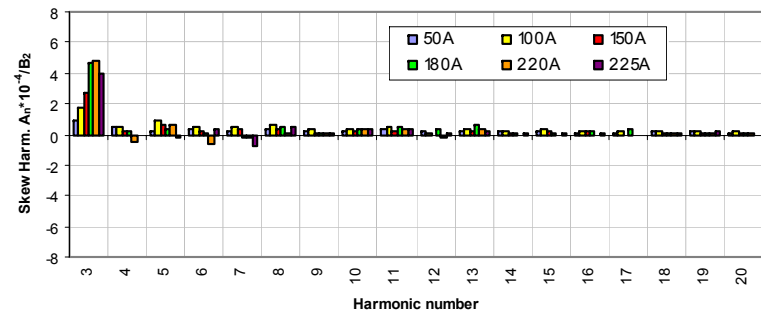
$\Delta B/B$ , for n>2



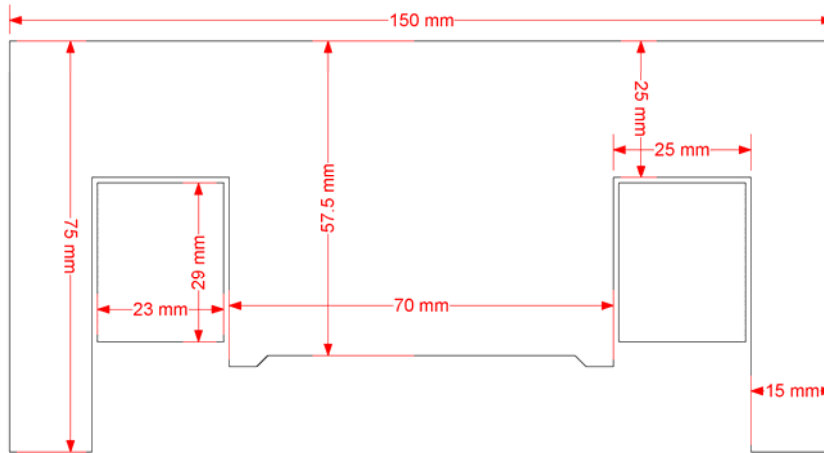
Q500CX-021-NORMAL COMPONENTS



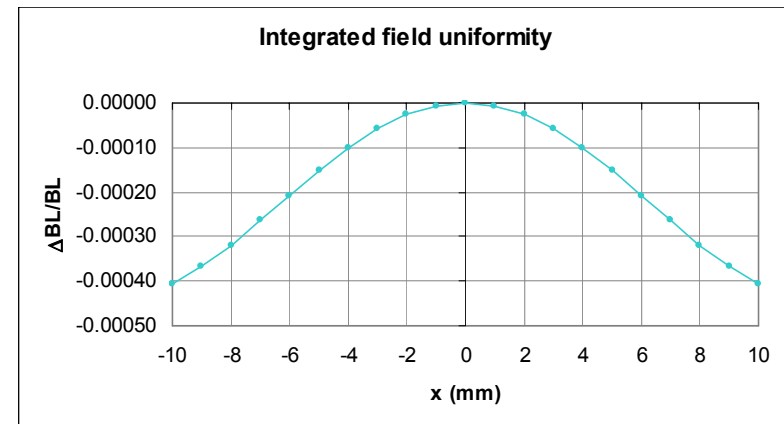
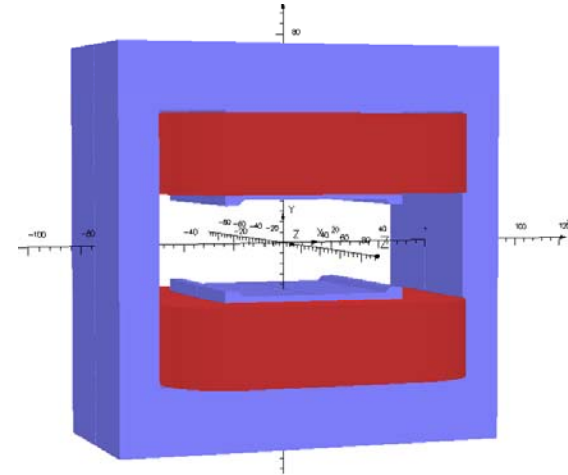
Q500CX-021-SKEW COMPONENTS



# Correctors



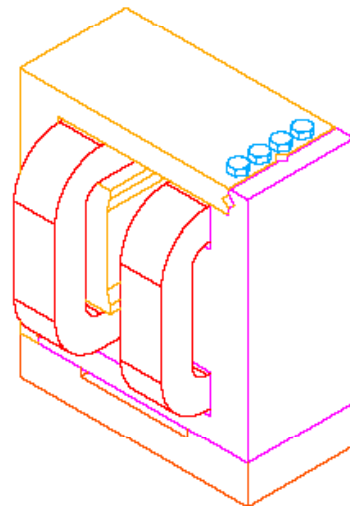
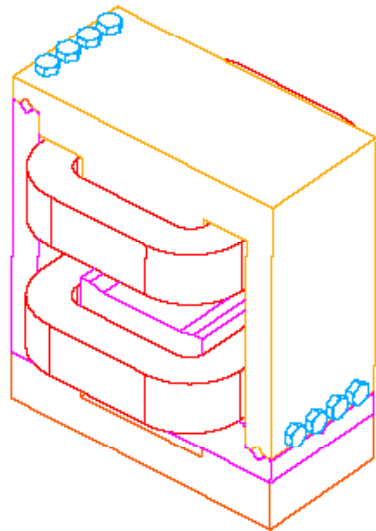
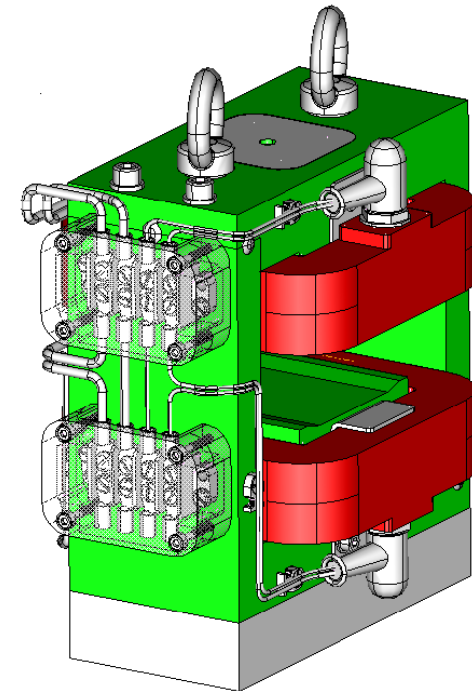
Number of magnets	104
Physical length (mm)	70
Maximum field (T)	0.05
Max deflection @ 3 GeV (mrad)	0.5



- 2 coils  
174 turns/coil (solid conductor)
- $P_{\text{average}} = 8 \text{ W}$
- $I_{\text{max}} = 4 \text{ A}$

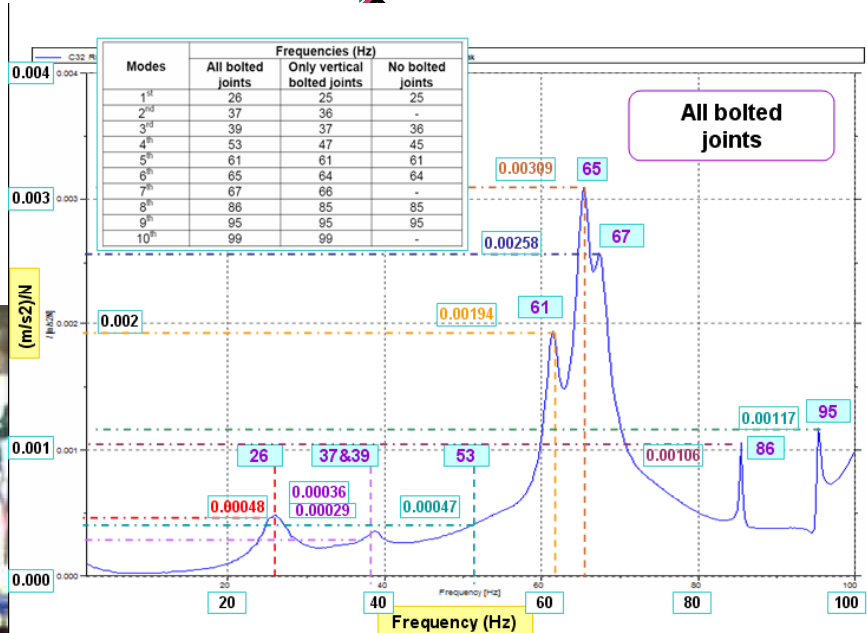
# ALBA-Booster-Correctors

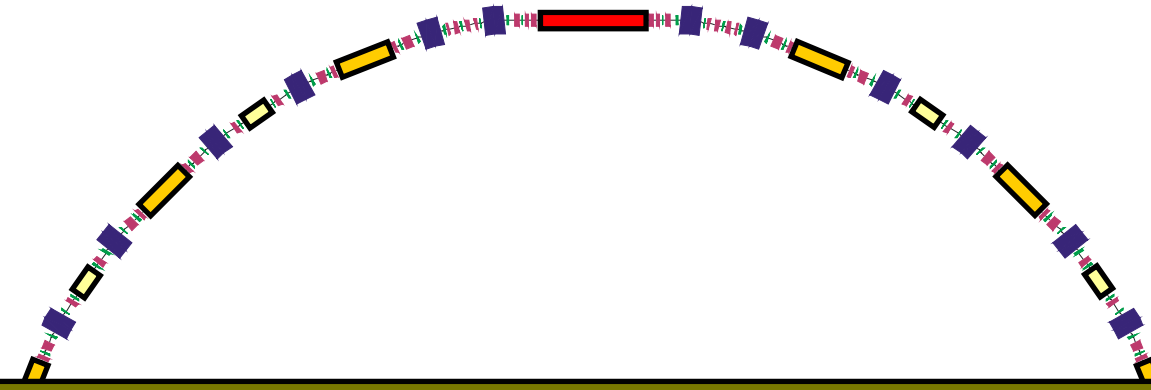
Bending angle	mrad	1
Bending field	T	0.0503
Length of Fe yoke	mm	70
Gap	mm	35
Solid conductor size	mm <sup>2</sup>	2
Number of ampere turns	amp-turn/coil	700
Number of turns per coil		174
Current	A	4
Dissipated power	W	20



# Girder System

More or less it is the SLS design, but the number of feet's (pedestals) has been increased to six in order to move up the eigen-frequency



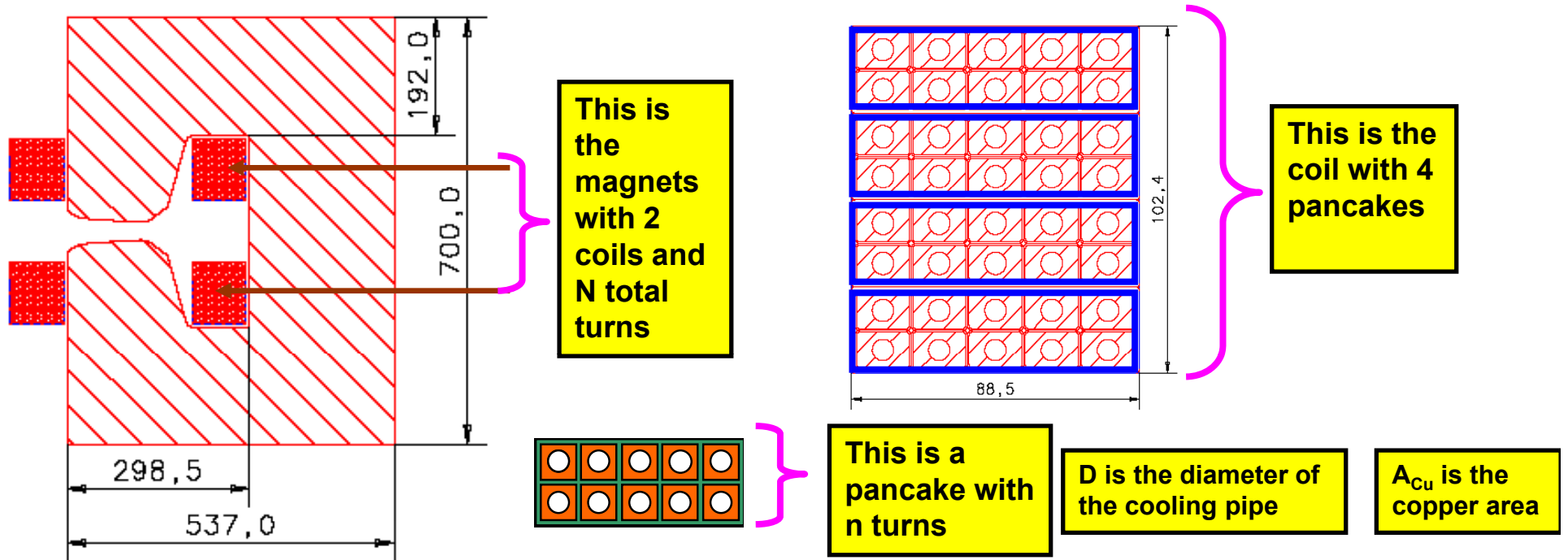


# **Additional Material for the Power and Cooling of Magnets**





# Calculation of the Power Consumption



$L_{Cu}$  = Average length of the conductor around the poles (m).

N = Total number of turns around the poles.

$A_{Cu}$  = Copper area (mm)

D = Diameter of the cooling pipe within the conductor (mm)

$\rho$  = Specific resistance of the conductor ( $\Omega \cdot \text{mm}^2/\text{m}$ )

# Calculation of the Power Consumption

$\Theta$  = Excitation of the magnet ( $\Theta = N \cdot I = B \cdot \text{gap} / \mu_0$ ).

$I$  = Excitation-current ( $I = \Theta / N$ ).

$J$  = Current density in the conductor ( $j = I / A_{Cu}$ )

$\Phi$  = Magnetic flux of the magnet ( $\Phi = B \cdot A$ ).

$L$  = Inductivity of the magnet ( $L = N \cdot \Phi / I$ ).

$R_{Cu}$  = Resistance of the conductor ones around the poles ( $R_{Cu} = \rho \cdot L_{Cu} / A_{Cu}$ )

$R_M$  = Resistance of the coils around the poles ( $R_M = N \cdot R_{Cu}$ )

$V_{Cu}$  = Voltage drop for one turn around the poles ( $V_{Cu} = I \cdot R_{Cu}$ )

$V_M$  = Voltage drop for the magnet ( $V_M = I \cdot R_M = V_{Cu} \cdot N = N \cdot I \cdot R_{Cu}$ )

$P_M$  = Power consumption of the magnet ( $P_M = I \cdot V_M = I^2 \cdot R_M = N \cdot I^2 \cdot R_{Cu}$ )

$P_M = N \cdot I^2 \cdot R_{Cu} = N \cdot (j \cdot A_{Cu})^2 \cdot R_{Cu} = N \cdot j^2 \cdot A_{Cu} \cdot \rho \cdot L_{Cu} \approx N \cdot j^2 \cdot A_{Cu}$

# Calculation of the Power Consumption

$P_M = \text{Power consumption of the magnet } (P_M = I \cdot V_M = I^2 \cdot R_M = N \cdot I^2 \cdot R_{Cu})$

$$P_M = N \cdot I^2 \cdot R_{Cu} = N \cdot (j \cdot A_{Cu})^2 \cdot R_{Cu} = N \cdot j^2 \cdot A_{Cu} \cdot \rho \cdot L_{Cu} \approx N \cdot j^2 \cdot A_{Cu}$$

We can write the power consumption also in an other way

$$P_M = N \cdot j^2 \cdot A_{Cu} \cdot \rho \cdot L_{Cu} = (N \cdot I / A_{Cu}) \cdot j \cdot A_{Cu} \cdot L_{Cu} = (N \cdot I) \cdot j \cdot L_{Cu}$$

$(N \cdot I)$  is the required excitation of the magnet, hence the power consumption is proportional to the current density and the length of the magnet

The excitation currents are:

$$(N \cdot I)_{\text{dipole}} = B \cdot g / \mu_0$$

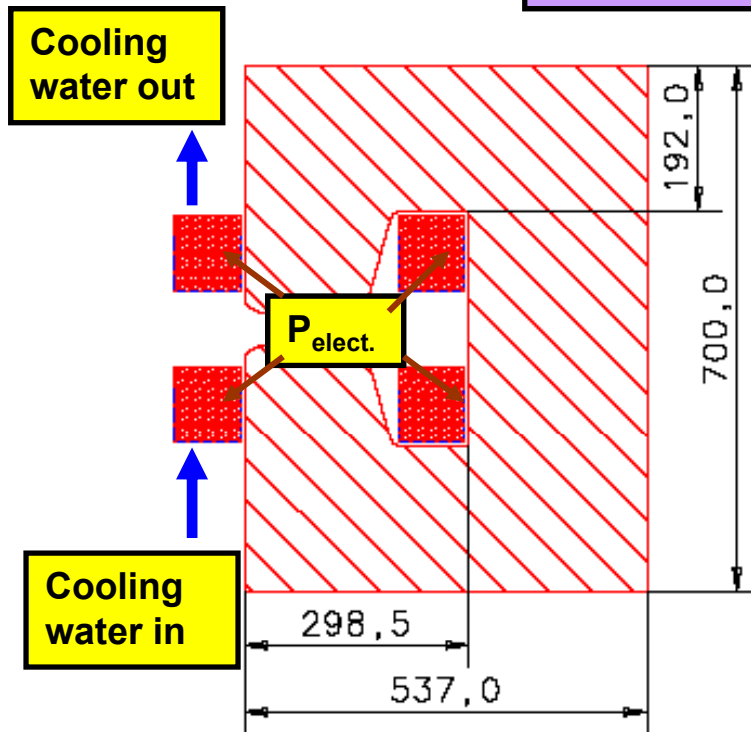
$$(N \cdot I)_{\text{quadrup.}} = B' \cdot R^2 / (2\mu_0)$$

$$(N \cdot I)_{\text{sextup.}} = B'' \cdot R^3 / (6\mu_0)$$

Design criteria's: The inductivity should be low ( $N$  : small,  $I$  : high)

The power consumption should be low ( $g, R$ : small,  $R$ : small, and length: small)

# Coil Cooling



Concerning the cooling we have the following definitions:

$Q$  = Heat quantity

$M$  = Mass [gr]

$c$  = Specific heat capacity [cal / (gr\*Kelvin)]

$\Delta\delta$  = Temperature change [Kelvin]

$\rho$  = Specific gravity [gr / cm<sup>3</sup>]

$V$  = Volume [cm<sup>3</sup>]

Caloric equivalent : 1cal = 4.186 VAs

$Q = M*c*\Delta\delta$  [ cal ] =  $\rho*V*C*\Delta\delta$  [cal]

According to the conversation of energy it must be:

$$(dQ/dt) = P_{electr} = P_M / \text{number of cooling circuits}$$

$$P_{electr.} = \rho*c*\Delta\delta*(\Delta V/dt)$$

## Calculation of the Cooling

$$P_{\text{electr.}} = \rho * c * \Delta\delta * (\Delta V / \Delta t)$$

Or the required flow rate:

$$\Delta V / \Delta t = \{ [P_{\text{electr.}} / W] / [\Delta\delta / K] \} * \{ 10^{-6} / 4.186 \} \text{ [m}^3/\text{s]}$$

$$\Delta V / \Delta t = \{ [P_{\text{electr.}} / W] / [\Delta\delta / K] \} * 0.2389 * 10^{-6} \text{ [m}^3/\text{s]}$$

$$\Delta V / \Delta t = \{ [P_{\text{electr.}} / W] / [\Delta\delta / K] \} * 1.433 * 10^{-2} \text{ [l/min]}$$

$$\Delta V / \Delta t = \{ [P_{\text{electr.}} / W] / [\Delta\delta / K] \} * 0.86 \text{ [l/h]}$$

The corresponding flow velocity is:

$$v = (\Delta V / \Delta t) / A_{\text{cool}} ,$$

$A_{\text{cool}}$  = cross section of the cooling pipe in the conductor.

$$v = \{ (\Delta V / \Delta t) / 10^{-6} \text{ m}^3/\text{s} \} / \{ A_{\text{cool}} / \text{mm}^2 \} \text{ [m/s]} ,$$

## Calculation of the Cooling

For an optimized cooling we need turbulent flow within the cooling pipes of the copper conductor. Turbulent flow does exist if the Reynolds's number (Re) is larger as 1160:

$$Re > 1160$$

Calculation of the Reynolds number:

$r$  = Radius in the cooling pipe of the conductor

$\rho$  = Specific gravity ( $\rho$  from water = 1 gr/cm<sup>3</sup>)

$v$  = velocity (cm/s)

$\eta$  = viscosity ( $\eta$  from water = 0.75 ... 1 mPas)

Introducing all the numbers one get:

$$Re = [r/mm] * [v/(m/s)] * 10^3$$

The corresponding critical velocity is:

$$v_c > 5.0 / (r/mm) [m/s]$$

## Calculation of the Cooling

For an optimized cooling we need turbulent flow within the cooling pipes of the copper conductor. Turbulent flow does exist if the Reynolds's number (Re) is larger as 1160:

$$\text{Re} > 1160$$

Calculation of the Reynolds number:

$r$  = Radius in the cooling pipe of the conductor

$\rho$  = Specific gravity ( $\rho$  from water = 1 gr/cm<sup>3</sup>)

$v$  = velocity (cm/s)

$\eta$  = viscosity ( $\eta$  from water = 0.75 ... 1 mPas)

Introducing all the numbers one get:

$$\text{Re} = [r/\text{mm}] * [v/(\text{m/s})] * 10^3$$

# Calculation of the Reynolds number

For an optimized cooling we need turbulent flow within the cooling pipes of the copper conductor. Turbulent flow does exist if the Reynolds number (Re) is larger as 1160:

$$Re > 1160$$

Calculation of the Reynolds number:

$r$  = Radius in the cooling pipe of the conductor

$\rho$  = Specific gravity ( $\rho$  from water = 1 gr/cm<sup>3</sup>)

$v$  = velocity (cm/s)

$\eta$  = viscosity ( $\eta$  from water = 0.75 ... 1 mPas)

Introducing all the numbers one get:

$$Re = [r/mm] * [v/(m/s)] * 10^3$$

The corresponding critical velocity is:

$$v_c > 5.0 / (r/mm) [m/s]$$



# Calculation of the Pressure Drop

For a turbulent flow the pressure drop within a pipe is given by Blasius:

$r$  = Radius in the cooling pipe of the conductor

$\rho$  = Specific gravity ( $\rho$  from water = 1 gr/cm<sup>3</sup>)

$v$  = velocity (cm/s)

$\eta$  = viscosity ( $\eta$  from water = 0.75 ... 1 mPas)

Introducing all the numbers one get:

$$Re = [r/mm] * [v/(m/s)] * 10^3$$

The corresponding critical velocity is:

$$v_c > 5.0 / (r/mm) [m/s]$$

## Calculation of the Pressure Drop

According to the law of Blasius, the pressure drop in a pipe is the following:

$$\Delta P = \frac{0.1582}{2^4 \sqrt{2}} \cdot L \cdot \eta^{1/4} \cdot \rho^{3/4} \cdot \frac{v^{7/4}}{r^{5/4}}$$

$$\Delta P = 0.1582 \cdot L \cdot \eta^{1/4} \cdot \rho^{3/4} \cdot \frac{v^{7/4}}{(2r)^{5/4}}$$

$$\Delta P = 5.0 \cdot 10^{-5} \text{ bar} \cdot (L / m) \cdot [v / m / s]^{1.75} \frac{1}{[2r / m]^{1.25}}$$



# Acknowledgment

Many thanks to

**Montse Pont**

**Niels Mark**

**Jack Tanabe**



**Thank you very much**