Timing, Synchronization & Longitudinal Aspects

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CERN

CAS Course on
Beam Injection, Extraction and Transfer

13 March 2017
Outline

• Introduction

• General concepts
  • Signals with noise, transmission of RF signals
  • Phase detectors and dividers

• Beam transfer
  • Fundamental periodicity
  • Transfer between circular lepton accelerators

• Transfer between hadron accelerators
  • Beam phase loop, bucket numbering
  • Transfer process: Synchronization, transfer triggers
  • Longitudinal matching

• Summary
Introduction
Introduction

• Two or more people must be synchronized to meet
  → Calendar item: date, time and location
  → Typical uncertainty: some minutes

• Slightly more precision required to have a meeting with a particle beam
  → Typical uncertainty: some nanoseconds down to femtoseconds

→ To be at the right time in the right place

→ Set conditions and generate timings and RF signals with a given time relation with respect to the beam
→ Make beam feel comfortable in its new accelerator
Timescales

Proton synchrotrons at medium energy

10 ns
1 ns
100 ps
10 ps
1 ps
100 fs
10 fs

Proton bunches in low energy synchrotrons

Hadron colliders

Electron storage rings

SASE FELs

Plasma wakefield experiments

Pump-probe FELs

Electronics

Low-level RF systems

→ Geometrical size: few meters to some km
Synchronization for beam transfer

- How to get the beam through the accelerator?

  Source  ➔ Exit

- How to transfer beam from accelerator A to B?

  Accelerator A  ➔ Accelerator B

- Beam passes many elements on its way:
  ➔ RF structures ➔ Must be in phase
  ➔ Septa, bumper, and kicker magnet ➔ Trigger
  ➔ Fast beam instrumentation ➔ Trigger
  ➔ RF systems in source and target accelerator ➔ Correct phase with respect to beam
Particle velocity

- Particle velocity depends on its type: 
  \[ \beta = \frac{v}{c} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2} \]

- Old television set (30 kV):
  - Electrons at 30% of \( c_0 \)
  - Protons just at 0.7%

- Small synchrotron (500 MeV):
  - Electrons at \( 99.999995\% \)
  - Protons at 75.8%

→ Many electron accelerators at ‘fixed’ frequency
## Synchronization needs for particle types

<table>
<thead>
<tr>
<th>Lepton accelerators</th>
<th>Hadron accelerators</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Velocity $v \approx c$ in high energy accelerators</td>
<td>• Slow, even velocity change relevant to the multi-GeV range</td>
</tr>
<tr>
<td>• Synchrotron radiation <strong>damping</strong> (mainly circular accelerators)</td>
<td>• Negligible or small damping from synchrotron radiation</td>
</tr>
<tr>
<td>• Short bunches</td>
<td>• Long bunches</td>
</tr>
<tr>
<td>• Storage rings: $\sim 10...100$ ps</td>
<td>• Synchrotrons: $1...1000$ ns (depends on RF frequency)</td>
</tr>
<tr>
<td>• Linear free electron lasers: $50...200$ fs</td>
<td>• Linear accelerators: typically <strong>few ns</strong></td>
</tr>
</tbody>
</table>

→ Fixed frequencies
→ High precision

→ Variable (sweeping) frequencies
→ Moderate precision
Bunch-to-bucket transfer

• Bunch from sending accelerator into the bucket of receiving

Advantages:
→ Particles always subject to longitudinal focusing
→ No need for RF capture of de-bunched beam in receiving accelerator
→ No particles at unstable fixed point
→ Time structure of beam preserved during transfer to the next
Noise on signals
Noisy signals

- Degradation of signal quality due to noise
  - Amplitude and/or phase jitter
- What is the difference between a coherent signal and noise?

→ Amplitude of coherent, quasi monochromatic signal (at 200 MHz) is independent of observation bandwidth
→ Incoherent noise power (dominated by spectrum analyzer front-end amplifier/mixer) is proportional to bandwidth
→ Thermal noise power \( \frac{P}{\Delta f} = k_B T = 1.38 \cdot 10^{-23} \text{ J/K} \cdot 296 \text{ K} \approx -174 \text{ dBm/Hz} \)
Analysis of phase noise

- Compare noise power with carrier power as reference

![Graph](image)

- Noise power density
  \[ \mathcal{L}(f) = \frac{\text{Power density}}{\text{Carrier power}} \left[ \frac{\text{dBc}}{\text{Hz}} \right] = \frac{1}{2} S_{\phi}(f) \]

  \( \Delta t = \frac{\Delta \phi}{2\pi f_c} \)

  \( \Delta t_{\text{rms}} = \frac{1}{2\pi f_c} \sqrt{\int_{f_1}^{f_2} S_{\phi}(f) \, df} \)
Typical phase noise plots

- Measure phase noise of a synthesized lab generator

\[ \Delta t_{\text{rms}} = \sqrt{\Delta t_{\text{rms},1}^2 + \Delta t_{\text{rms},2}^2 + \cdots} \]

- Note: jitter values can be added as square root of quadratic sum

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>( \Delta t_{\text{rms}} ) [fs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10...100 Hz</td>
<td>12.4</td>
</tr>
<tr>
<td>100 Hz ... 1 kHz</td>
<td>5.4</td>
</tr>
<tr>
<td>1...10 kHz</td>
<td>5.4</td>
</tr>
<tr>
<td>10...100 kHz</td>
<td>11.1</td>
</tr>
<tr>
<td>100 kHz...1 MHz</td>
<td>13.0</td>
</tr>
<tr>
<td>Total</td>
<td>31.0</td>
</tr>
</tbody>
</table>

Convenient split to relevant ranges
Signal transmission
Transmission of reference signals

- Thermal drift of long coaxial cables or optical fibres

- Thermal coefficient of delay:
  \[ TCD = \frac{\Delta \tau}{\tau} \cdot \frac{1}{\Delta T} = \frac{\Delta \phi}{\phi} \cdot \frac{1}{\Delta T} \]

- Example: 2 km long RG223 cable with \( \sim 10 \mu s \) delay
  \[ \Delta T \text{ of only } 1^\circ C \text{ (room temperature) changes delay by } \sim 0.5 \text{ ns} \]
  \[ 1.8^\circ \text{ at } 10 \text{ MHz (CERN PS), but } 73^\circ \text{ at } 400 \text{ MHz (LHC)} \]

- Optical fibres are typically 10...100 times more stable
- What to do if this is still not sufficient?
Transmission of reference signals

• Measured drift of optical fibres over long distance standard optical fibre

- Measured temperature and delay drift of ~6.3 km fiber

- Drift by about 1 ns insufficient for requirements of setup

  → Active compensation of delay
**Example: Active drift compensation**

- Precise synchronization of proton beam from CERN SPS with plasma wake-field experiment AWAKE

Prototype hardware

→ Expect picosecond precision over several kilometres

D. Barrientos, J. Molendijk
Transmission of reference signals

- Total delay composed of coarse (steps of 10 ps) and fine ~30 ps range: \( \tau = \tau_{\text{coarse}} + \tau_{\text{fine}} \)

\[ \begin{align*}
\tau_{\text{coarse}} &\quad \text{[ns]} \\
4.95 &\quad 5.00 &\quad 5.05 \\
\end{align*} \]

\[ \begin{align*}
\tau_{\text{fine}} &\quad \text{[ps]} \\
0 &\quad 5 &\quad 10 &\quad 15 &\quad 20 &\quad 25 \\
00:00 &\quad 06:00 &\quad 12:00 &\quad 18:00 \\
\end{align*} \]

Precision difficult to evaluate without 2\textsuperscript{nd} ‘reference’ link

Arrival of two beams in AWAKE experiment stable to better ~100 ps over months

D. Barrientos, J. Molendijk
Overview of transmission methods

Various approaches:

1) RF distribution
   - \( f \sim 100\text{MHz} \ldots \text{GHz} \)
   - \( \Delta t \sim \Delta f = f \)

2) Carrier is optically
   - \( f \sim \text{GHz} \)

3) Carrier is optically + detection
   - \( f \sim 200\text{THz} \)

4) Pulsed optical source
   - \( \Delta f \sim 5\text{THz} \)
   - SLAC, FLASH, E-XFEL, SwissFEL

H. Schlarb
Phase detection
Two signals at different frequencies $\omega_1$ and $\omega_2$ change linearly in phase difference, $\Delta \phi$, between both signals. Ambiguity to distinguish between $\Delta \phi = -\pi, \pi, -3\pi, 3\pi, \ldots$ Saw-tooth in phase means constant frequency difference.

Equivalence of frequency and phase:

$$\omega = \frac{d\phi}{dt} \quad \Leftrightarrow \quad \phi = \int \omega \, dt$$
How to detect phase differences?

• Example: analogue 4 quadrant multiplier and low pass filter

\[
\sin(\omega_1 t + \phi_1) \rightarrow \frac{1}{2} \{ \cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] - \cos[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)] \}
\]

\sin(\omega_2 t + \phi_2)

• Signals:
How to detect phase differences?

- Example: analogue 4 quadrant multiplier and low pass filter

\[ \sin(\omega_1 t + \phi_1) \quad \xrightarrow{\text{Example}} \quad \frac{1}{2} \{ \cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] \} \]

\[ \sin(\omega_2 t + \phi_2) \quad \xrightarrow{\text{Remove ripple \rightarrow Low-pass filter}} \quad \cos[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)] \]

- Signals:
**How to detect phase differences?**

- **Example:** analogue 4 quadrant multiplier and low pass filter

\[
\sin(\omega_1 t + \phi_1) \rightarrow \frac{1}{2} \{\cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] - \cos[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)]\}
\]

- **Signals:**

- **Phase discriminator in approximately \(+/-90^\circ\) range**

- **Relative:** arbitrary shift by \(90^\circ\)

- **Phase discriminator in approximately \(+/-90^\circ\) range**
### Further phase detection techniques

#### Multitude of different phase discriminators

<table>
<thead>
<tr>
<th>Type</th>
<th>Range</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analogue 4 quadrant multiplier</td>
<td>$\pi$</td>
<td><strong>Sinusoidal</strong>: $s_{out} \sim \cos \phi$</td>
</tr>
<tr>
<td>Exclusive OR gate</td>
<td>$\pi$</td>
<td><strong>Linear</strong>: $s_{out} \sim \phi - 3\pi/2$, or $s_{out} \sim -\phi + \pi/2$</td>
</tr>
<tr>
<td>Sample and hold</td>
<td>$\pi$</td>
<td><strong>Sinusoidal</strong>: $s_{out} \sim \sin \phi$</td>
</tr>
<tr>
<td>Flip-flop phase detector</td>
<td>$\pi$</td>
<td><strong>Linear</strong>: $s_{out} \sim \phi - \pi$</td>
</tr>
<tr>
<td>Tri-state double flip-flop</td>
<td>$2\pi$</td>
<td><strong>Linear</strong>: $s_{out} \sim \phi$</td>
</tr>
<tr>
<td>Balanced optical microwave phase detector (Sagnac loop)</td>
<td>$&lt;\pi$</td>
<td><strong>Sinusoidal</strong>: $s_{out} \sim \sin \phi$ (clipped)</td>
</tr>
</tbody>
</table>

- **Full phase coverage of $2\pi$ range excludes ambiguity of $\pm \pi$**

$\rightarrow$ **Avoids locking of phase loop with unwanted offset**

- **Measure phase at high frequencies for precision**
Dividers
Frequency dividers

- Generate signals using frequency division from $f_{RF}$

- Works (well, on paper), so what is the problem?

  $\rightarrow$ Dividers are nothing but counters! Initial value?
Synchronizing multiple dividers

- Generate signals using frequency division from $f_{RF}$

- How to fix?
  - Reset from master to slave divider(s) to force initial condition
  - $\rightarrow$ Never more than one divider without reset!
Multiple divider with counting offset

- Counter with programmable offset value

\[ f_{RF} \]

\[ \text{Counter to } n \]

\[ \text{Offset} \]

\[ x = 0? \]

\[ f_{RF}/n \]

\[ x = 0? \]

\[ f_{RF}/n \]

\[ \text{Adder with } \text{Mod}(x,n) \]

- Single counter/divider split in two output branches
- Impossible to lose relative phase of outputs
- More complicated set-up allows also \( f_{RF}/m \) and \( f_{RF}/n \), etc.
Fundamental periodicity
Example: BESSY II booster and storage ring

- Storage ring circumference 240 m, $f_{RF} = 499.6$ MHz
- Circumference ratio of Booster and storage ring: $2/5$

$\rightarrow$ Everything repeats with periodicity of

- 5 turns in booster
- 2 turns in storage ring
Example: SLS booster and storage ring

- Storage ring circumference 288 m, $f_{RF} = 499.6$ MHz
- Circumference ratio of Booster and storage ring: $15/16$

→ Fundamental periodicity (super-period)
  16 turns of booster corresponding to 15 turns in storage ring
Fundamental periodicity for transfer

- Two accelerators with revolution periods $T_{\text{rev},1}$ and $T_{\text{rev},2}$

$$T_{\text{rev},2} = \frac{m}{n} T_{\text{rev},1} \quad \rightarrow \quad T_{\text{super}} = T_{\text{common}} = T_{\text{fiducial}} = mT_{\text{rev},1} = nT_{\text{rev},2}$$

→ Beam transfer may take place at every period $mT_{\text{rev},1}$ or $nT_{\text{rev},2}$

→ This periodicity is, depending on the accelerator and laboratory, called \textit{super-period}, \textit{common} or \textit{fiducial period}

→ In case of \textbf{integer ratio} of revolution frequencies, beam can be transferred once every turn of the larger accelerator

<table>
<thead>
<tr>
<th>Sending</th>
<th>Receiving</th>
<th>Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>BESSY booster</td>
<td>BESSY SR</td>
<td>2/5</td>
<td>Fixed frequency</td>
</tr>
<tr>
<td>SLS booster</td>
<td>SLS SR</td>
<td>15/16</td>
<td>Fixed frequency</td>
</tr>
<tr>
<td>J-PARC RCS</td>
<td>J-PARC MR</td>
<td>2/9</td>
<td>Profit from ratio for bucket selection</td>
</tr>
<tr>
<td>PS booster</td>
<td>PS</td>
<td>1/4</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>SPS</td>
<td>1/11</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>AD</td>
<td>3/1</td>
<td>Particle type and energy change at transfer</td>
</tr>
<tr>
<td>SPS</td>
<td>LHC</td>
<td>7/27</td>
<td>$f_c$ as low 1.6 kHz</td>
</tr>
</tbody>
</table>
Synchronous triggers

How to generate beam synchronous triggers?
→ Chains of counters to re-synchronize timings

Each step re-synchronizes with respect counter clock
• ‘Start engine button’ synchronous to nothing
• Complete system of two accelerators periodic with timing #1
• Timing #2 marks, e.g., a delay in number of turns
• Timing #3 counts $f_{RF}$ clocks to fine adjust, e.g., bucket number
Timming counters may use different clocks, as long as the clocks are derived from the same source.

- Reproducible delay between clock #2 and #3
- Tree structures of timings
Circular electron/lepton accelerators

• Simplification for most electron accelerators:
  ➔ Leptons are *practically* at speed of light
  ➔ Synchrotron radiation *damping forces bunches into buckets*
  ➔ Beam synchronous *timing triggers* can be derived by counting RF master clock (or its sub-multiples)
  ➔ Everything is predictable from the beginning

→ *Let’s get frequencies moving*
Transfer between hadron accelerators
Synchronous triggers and bucket counting

• Circular hadron accelerators: master clock sweeps
• Need again synchronous timings with respect to beam
  → Kicker magnets
  → Beam instrumentation
• RF manipulations require bunches in certain buckets
  → Beating pattern due to multiple RF harmonics
    → Splits behaviour for different buckets
  → Bucket numbering
• Need to know longitudinal beam position for transfer
  → Where (in phase/in time) is the beam?
Phase-locked loop

- Frequency re-generation and multiplication
- Voltage controlled oscillator (VCO) locked in phase to input

\[ \omega_{VCO} = 2\pi f_{VCO} = \frac{d\phi}{dt} = K_{VCO} V_{in} \]

\[ f_{\text{out}} = n \cdot f_{\text{in}} \]

→ Fixed phase relationship: \( \phi_{\text{out}}/n - \phi_{\text{in}} = \text{const.} \)
→ Optional divider:
Beam phase loop

Phase pick-up

Beam phase loop

RF cavity

Cavity phase

Power amplifier

Digital synthesizer

Loop corr.

DDS

Precision VFO

Synchronous phase, $\phi_s$

$\phi_{err} \sim \Delta f$

$f_{RF}$

$h \cdot f_{rev}$ from $B$

$f_{out} = f_{in} \pm \Delta f$

$\Delta f$

$\Delta \phi$

→ Phase-locked loop with beam phase as reference for RF system

$\rightarrow$ RF

$\rightarrow$ Slow signal

$\rightarrow$ $h \cdot f_{rev}$
Benefits of beam phase loop at transfer

- Adapt RF phase to bunch phase before beam blows-up
- Fast compared to timescale of synchrotron frequency, $f_s$

Rigid RF, no phase loop

- Even large transients (injection, transition) can be controlled
- Small longitudinal emittance blow-up
Start counting with injection

\[ f_{\text{RF}} \]

• Start of divider/counter?

→ Get it right from injection
→ Use output from divider as reference for incoming beam

Beam synchronous \( f_{\text{rev}} \)
Start counting with injection

- **Start of divider/counter?**
  - Get it right from injection
  - Use output from divider as reference for incoming beam

- **Before injection:**
  - Distribute delayed revolution frequency to sending accelerator
  - Bunches are injected synchronously with \( f_{\text{rev, delayed}} \)
  - Shifted with respect to \( f_{\text{RF}} \) and \( f_{\text{rev}} \)

\[ f_{\text{RF}} \]

\[
\begin{array}{c}
\text{Delay} \\
\Delta \tau
\end{array}
\]

\[
\begin{array}{c}
f_{\text{rev, delayed}} \\
f_{\text{rev}}
\end{array}
\]

To sending accelerator
Start counting with injection

1. Start of divider/counter?
   - Get it right from injection
   - Use output from divider as reference for incoming beam

2. Before injection:
   - Distribute delayed revolution frequency to sending accelerator
   - Bunches are injected synchronously with $f_{\text{rev, delayed}}$
   - Shifted with respect to $f_{\text{RF}}$ and $f_{\text{rev}}$
Beam phase loop without beam?

→ Just replace beam by a simple RF generator!

Phase pick-up

RF cavity

Power amplifier

DDS VCO

Precision VFO

Beam synchronous \( f_{rev} \)

\[ h \cdot f_{rev}, \text{ from } B \]

\[ f_{out} = f_{in} \pm \Delta f \]
Synchronization chain for bucket counting

- Incoming beam has reproducible phase with respect to RF bucket, synchronous $f_{rev}$ and beam phase emulating generator
  - Straightforward switch to beam signals, already locked in phase
**Synchronization chain for bucket counting**

- **Incoming beam** has reproducible phase with respect to RF bucket, synchronous \( f_{\text{rev}} \) and beam phase emulating generator
  
  → **Straightforward switch to beam signals, already locked in phase**
  
  → **Beam phase with respect to \( f_{\text{rev}} \) always known**
Bucket numbering
### Bucket numbering for RF manipulations

<table>
<thead>
<tr>
<th></th>
<th>Triple splitting</th>
<th>Batch compression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Injection harmonic</strong></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td><strong>Periodicity of RF manipulation</strong></td>
<td>Every bucket</td>
<td>Only one beating along circumference</td>
</tr>
<tr>
<td><strong>Injection bucket selection</strong></td>
<td>4 buckets difference between both injections</td>
<td>Both injections into independently defined buckets</td>
</tr>
</tbody>
</table>

> Must inject into the correct bucket numbers
Example: PS injection bucket selection

- Bunches must be placed into the correct buckets numbers
- Harmonic number change only for even number of bunches

→ Bucket number control during both transfers PSB → PS

→ How to handle changing number of bunches?
Intermediate summary

• Basic techniques of signal synchronizations
  → Beware of dividers

• Beam transfer between circular lepton accelerators
  → Constant frequency
  → Predictable, independently from beam
  → Fundamental periodicity

• Beam transfer between circular hadron accelerators
  → Beam is reference, keep track
Timing, Synchronization & Longitudinal Aspects

II

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CAS Course on Beam Injection, Extraction and Transfer

13 March 2017
• Introduction

• General concepts
  - Signals with noise, transmission of RF signals
  - Phase detectors and dividers

• Beam transfer
  - Fundamental periodicity
  - Transfer between circular lepton accelerators

• Transfer between hadron accelerators
  - Beam phase loop, bucket numbering
  - Transfer process: Synchronization, transfer triggers
  - Longitudinal matching

• Summary
Synchronization and transfer
Steps of beam transfer synchronization

1. • Set bending fields in both accelerators to the same magnetic rigidity

2. • Synchronize sending or receiving accelerator

→ Ready for transfer

3. • Start counting clock of fundamental periodicity
   • Trigger bump and septum elements

4. • Start counting $f_{\text{rev}}$ clock (sending/receiving accelerator)
   • Start counting bucket clock

5. • Fine delay
   • Ejection and injection kickers triggers

→ Transfer
Match bending field of both accelerators

- Same magnetic rigidity $\rho B$ of sending (1) and receiving (2) accelerators

$$F_Z = F_L \rightarrow \frac{p}{q} = \rho B$$

- $\rho_1 B_1 = \rho_2 B_2$

→ No rule without exception: Particle type change at transfer
  - Proton to anti-proton conversion, e.g.,
    $120 \text{ GeV/c} \neq 8 \text{ GeV/c}$ (Fermilab), $26 \text{ GeV/c} \neq 3.6 \text{ GeV/c}$ (CERN),
  - Charge state change at transfer, e.g. LHC ion injector chain
    Pb$^{54+}$ in LEIR/PS $\rightarrow$ Pb$^{82+}$ (in SPS)
Match RF frequencies

- RF frequencies of both accelerators must have appropriate ratio assuming that the beam velocity is unchanged

\[ f_{\text{rev}} = \frac{f_{RF}}{h} = \frac{\beta c}{2\pi R} \]

\[ \beta c = 2\pi R_1 f_{\text{rev},1} = 2\pi R_2 f_{\text{rev},2} \]

\[ R_1 \frac{f_{RF,1}}{h_1} = R_2 \frac{f_{RF,2}}{h_2} \]

→ Common choice of most circular electron accelerators \( f_{RF,1} = f_{RF,2} \)
→ Harmonic number, \( h \), proportional to circumference, \( 2\pi R \)
→ Again no rule without exception: Production of antiprotons in target in transfer line
Distance between bunches

- Distance of bunches (bunch spacing, $\tau_{\text{bunch}}$) from source accelerator must match distance of buckets

- Example: $\tau_{\text{bunch}} = 2/f_{\text{RF}}$

- Example: $\tau_{\text{bunch}} = 5/f_{\text{RF}}$

- Common case: $f_{\text{RF,2}} = n \cdot f_{\text{RF,1}}$
  \[ \Rightarrow \quad f_{\text{RF,LHC}} = 2 \cdot f_{\text{RF,SPS}} \text{ and } f_{\text{RF,SPS}} = 5 \cdot f_{\text{RF,PS}} \]

- Several exceptional cases:
  - No bunch distance with single bunch $\Rightarrow$ more flexibility
  - Adjust bunch spacing using multiple RF systems
Exception: double-harmonic RF at transfer

- Was used at CERN PSB-to-PS to transfer 2 bunches at once
- Circumference ratio $C_{PS}/C_{PSB} = 4$
  → Ratio virtually moved to $2/7$: use $h_{RF} = 2 + 1$

1. Add $h_1$ component such that bunches approach to 245 ns (small spacing) → big spacing becomes 327 ns
2. Synchronize on $h_1$ to the PS
3. Trigger extraction kicker in-between the small spacing
4. Eject two bunches per ring at a distance of 327 ns

Spacing larger than $C_{PSB}/2$ → $h_{PS} = 7$, $C_{PS}/7$

Christian Carli
## Steps of beam transfer synchronization

1. Set bending fields in both accelerators to the **same** magnetic rigidity

2. **Synchronize** sending or receiving accelerator

→ Ready for transfer

3. Start counting clock of **fundamental periodicity**
   - Trigger bump and septum elements

4. Start counting $f_{rev}$ **clock** (sending/receiving accelerator)
   - Start counting **bucket clock**

5. **Fine delay**
   - Ejection and injection kickers **triggers**

→ Transfer
Before synchronization

- Even with magnetic rigidity matched: revolution frequencies not at theoretical ratio due to imperfections

→ Bunches and buckets slip in phase

But: important question left unanswered!

Slippage of bunches and bucket

Rotation in source accelerator subtracted
Who is the boss?

- Transfer beam to a downstream machine: **Bunch-to-bucket**
  
1. Protons between synchrotrons → **Synchronize accelerators**

2. Move relative phase of RF together with beam between both machines to hit the empty buckets

---

Sending accelerator is, the boss?  
Receiving accelerator is the boss?
Choice of master for transfer synchronization

- **Sending accelerator is master of transfer**
  - Receiving accelerator adapts to incoming beam
  - Common choice when receiving accelerator has no beam before transfer
  - Interesting for only single beam transfer, e.g., protons from PS → AD for antiproton production

- **Receiving accelerator is master of transfer**
  - Sending accelerator adapts to incoming beam
  - Common choice when receiving accelerator has already beam before transfer (multiple injections)
  - Most common at CERN, e.g., proton injector chain PSB → PS → SPS → LHC
Before synchronization

- Simple test case of circumference ratio 2: $C_2 = 2C_1$
Before synchronization

- Simple test case of circumference ratio $2$: $C_2 = 2C_1$

$\rightarrow$ Synchronize both accelerators to force: $f_{\text{rev,1}} = 2f_{\text{rev,2}}$
Simple synchronization process

1. Move beam to off-momentum ($B$ const.): 
   \[
   \frac{df}{f} = \frac{\gamma^2_{tr} - \gamma^2}{\gamma^2_{tr}} \frac{dp}{p}
   \]
   → Well defined frequency difference between accelerators

2. Measure azimuth error, when beam at correct azimuth
   → Close synchronization loop
   → Moves beam to ref. momentum

\[
\Delta \phi
\]

Beam azimuth (from phase loop)

Ref. azimuth (from master divider)

Act on $f_{RF}$ of slave

Locked! 200 ms

Bunch should be here

$\Delta \phi$

$f_{rev} (h = 1)$
Example: Synchronization of SPS to LHC

→ **LHC is master** for beam transfer from SPS

Arrival at flat-top +30 ms  
Measure $\Delta \tau$ of azimuth SPS-LHC  
Re-measure $\Delta \tau$ of azimuth

Close fine re-phasing loop at $f_{RF,LHC}$  
Ready for transfer

Set $f_{rev,SPS} = \frac{27}{7} f_{rev,LHC}$  
Apply frequency bump  
Apply frequency bump

→ **Coarse and fine re-phasing** to perfectly align bunches with respect to target buckets (400 MHz, 2.5 ns) in LHC
→ **Complete synchronization process** takes about 500 ms
Example: Fast cogging of booster at FNAL

- **Rapid cycling synchrotron from 400 MeV to 8 GeV**
- **Total cycle length is only 25 ms** → **How to synchronize fast?**

1. Measure beam phase early in the cycle and **predict azimuth at flat-top**
2. Apply **radial/frequency bumps already during acceleration**
After synchronization

- Simple test case of circumference ratio 2: $C_2 = 2C_1$

  Source or target accelerator is master at transfer

\[ \rightarrow \text{Revolution frequencies coupled: } f_{\text{rev},1} = 2f_{\text{rev},2} \]

\[ \rightarrow \text{Transfer can be triggered every turn of the target accelerator} \]
Example: Ejection bucket numbering in PS

- Azimuthal position of 1st bunch ambiguous after RF manipulations
  → Number of buckets and bunches changes during acceleration

- But: Synchronous $f_{\text{rev,PS}}$ signal with reproducible phase to beam
  → ‘Re-numbering’ of buckets by shifting reference from SPS

→ Shift of external reference $f_{\text{rev,PS}}$ adjustable in SPS bucket units
→ Synchronize external and beam synchronous $f_{\text{rev,PS}}$
Example: Ejection synchronization chain

→ Multiple ‘batches’ are transferred from PS to 11 times larger SPS

Outgoing beam $\phi_{\text{const.}}$ $f_{\text{rev,PS}}$ internal $\phi_{\text{const.}}$ $f_{\text{rev,PS}}$ reference $\phi_{\text{const.}}$ $f_{\text{rev,SPS}}; f_{\text{RF,SPS}}$ for synchro.

Beam phase loop

Synchronization loop

Ejection divider

Last turn before PS $\rightarrow$ SPS transfer

→ Beam phase with respect to $f_{\text{rev}}$ always known

$\phi_{\text{const.}}$ $f_{\text{rev,SPS}}$ marker

Outgoing beam $\phi_{\text{const.}}$
Steps of beam transfer synchronization

1. • Set bending fields in both accelerators to the same magnetic rigidity

2. • Synchronize sending or receiving accelerator

→ Ready for transfer

3. • Start counting clock of fundamental periodicity
   • Trigger bump and septum elements

4. • Start counting $f_{\text{rev}}$ clock (sending/receiving accelerator)
   • Start counting bucket clock

5. • Fine delay
   • Ejection and injection kickers triggers

→ Transfer
Synchronous triggers

→ Cascade of trigger counters for fast transfer elements
  - Very similar to transfer with lepton synchrotrons
Steps of beam transfer synchronization

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   • Ejection and injection kickers triggers

→ Transfer
Example: Turn count control at extraction

- J-PARC rapid cycling synchrotron and main ring ratio: 4.5
  - Transfer possible once every two turns of main ring
  - Transfer of 4 times two bunches

Counter on $f_{\text{rev}}$ of source (RCS) set normally

Counter on $f_{\text{rev}}$ of source (RCS) delayed by one turn

→ Beam synchronous timing can also be used to control target azimuth (bucket number) of transferred beam
Energy matching
Energy matching of incoming beam

- **Ideal beam** circulates with the expected revolution frequency ($\Delta f = 0$) on the central orbit ($\Delta R = 0$) $\Rightarrow \Delta p = 0$

- **Real beam** behaviour is calculated using

<table>
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**Choice of two parameters from $B, p, R, f$ directly constrains all others**

$\Rightarrow$ Example: at **fixed** magnetic field ($\Delta B = 0$), revolution frequency and radial position are directly linked
Energy matching without RF

- Observe de-bunching (no RF) with periodic trigger at $n \cdot f_{\text{rev}}$

  → Does the beam circulate with the expected $f_{\text{rev}}$? at the central orbit?

De-bunching with $f_{\text{rev}}$ error

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<td>200 ns drift in 1.8 ms</td>
<td>$\Delta f_{\text{rev}} \approx 50$ Hz</td>
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<td>Badly matched, $\Delta R \approx 5.3$ mm</td>
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De-bunching expected $f_{\text{rev}}$

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- Changing $B$ alone insufficient, since $f_{\text{rev}}$ and $R$ linked (const. $p$)

  → Change two parameters to fix the others, e.g., $B$ and $p$ or $B$ and $f$

  → All parameters are constrained
Longitudinal matching equations
Recap of longitudinal beam dynamics (1)

For a single harmonic RF system

\[ H \left( \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) = -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos \phi - \cos \phi_0 + (\phi - \phi_0) \sin \phi_0 \right] \]

with \( \phi = \phi_0 + \Delta \phi \) it becomes

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) = -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos(\phi_0 + \Delta \phi) - \cos \phi_0 + \Delta \phi \sin \phi_0 \right] \]

using \( \cos(\phi_0 + \Delta \phi) = \cos \phi_0 \cos \Delta \phi - \sin \phi_0 \sin \Delta \phi \)

\[ \simeq \cos \phi_0 \left( 1 - \frac{1}{2} \Delta \phi^2 \right) - \sin \phi_0 \Delta \phi \]

The Hamiltonian simplifies to

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) \simeq -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 - \frac{1}{2} \frac{qV}{2\pi} \Delta \phi^2 \cos \phi_0 \]
Recap of longitudinal beam dynamics (2)

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\text{rev}}}\right) \approx -\frac{1}{2} \frac{h\eta \omega_{\text{rev}}}{pR} \left(\frac{\Delta E}{\omega_{\text{rev}}}\right)^2 - \frac{1}{2} \frac{qV}{2\pi} \Delta\phi^2 \cos \phi_0$$

• In the centre of the bucket, particles move on elliptical trajectories in $\Delta\phi$-$\Delta E$ phase space
• Hamiltonian is constant on these trajectories

→ Aspect ratio of the elliptical trajectories must be identical in sending and receiving accelerator
• Compare two particles on the same trajectory
  1. No phase deviation
  2. No energy deviation

\[ H \left( \Delta \phi = 0, \frac{\Delta E}{\omega_{\text{rev}}} \right) = -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 \]

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} = 0 \right) = -\frac{1}{2} \frac{qV}{2\pi} \Delta \phi^2 \cos \phi_0 \]

→ \( \Delta \phi \) depends on frequency → use physical duration \( \Delta \tau \) instead

\[ \Delta \phi = 2\pi f_{\text{RF}} \Delta \tau = h \omega_{\text{rev}} \Delta \tau \]

→ Also replacing \( pR = \frac{E \beta^2}{\omega_{\text{rev}}} \)
Physical aspect ratio of bucket trajectories (2)

→ Hamiltonian equal for both extreme particles, hence

\[-\frac{1}{2} \frac{h \eta \omega_{\text{rev}}^2}{E \beta^2} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 = -\frac{1}{2} \frac{qV}{2\pi} \hbar^2 \omega_{\text{rev}}^2 \Delta \tau^2 \cos \phi_0\]

which can be simplified to

\[\left( \frac{\Delta E}{\Delta \tau} \right)^2 = \frac{qV}{2\pi} E \beta^2 \hbar \omega_{\text{rev}}^2 \frac{\cos \phi_0}{\eta}\]

→ This aspect ratio $\Delta E/\Delta \tau$ must remain unchanged at transfer
Matched bunch-to-bucket transfer

→ Equating \( \left( \frac{\Delta E}{\Delta \tau} \right)^2 = \frac{qV}{2\pi} E \beta^2 h\omega_{\text{rev}}^2 \frac{\cos \phi_0}{\eta} \) for sending (1) and receiving (2) accelerator gives a general matching condition

\[
q_1 V_1 E_1 \beta_1^2 h_1 \omega_{\text{rev},1}^2 \frac{\cos \phi_{0,1}}{\eta_1} = q_2 V_2 E_2 \beta_2^2 h_2 \omega_{\text{rev},2}^2 \frac{\cos \phi_{0,2}}{\eta_2}
\]

→ For most cases (fixed energy and no particle type change)

\[q_1 = q_2 \quad \beta_1 = \beta_2 \quad E_1 = E_2 \quad \cos \phi_{0,1} = \cos \phi_{0,2} = 1\]

It simplifies to the voltage ratio between RF systems:

\[
\frac{V_1}{V_2} = \left( \frac{R_1}{R_2} \right)^2 \left| \frac{\eta_1}{\eta_2} \right| \frac{h_2}{h_1}
\]
Simple matched transfer example

• Transfer between to accelerators with $f_{RF,2} = f_{RF,1}/2$

→ Phase space aspect ratio:

$$\Delta E = \beta \omega_{rev} \left( \sqrt{\frac{qV}{2\pi}} E \hbar \frac{\cos \phi_0}{\eta} \right) \cdot \Delta \tau$$

Source (1) bucket

Target (2) bucket
Simple matched transfer example

- Transfer between to accelerators with $f_{RF,2} = f_{RF,1}/2$

→ Phase space aspect ratio:

$$\Delta E = \beta \omega_{rev} \sqrt{\frac{qV}{2\pi} Eh \left| \frac{\cos \phi_0}{\eta} \right|} \cdot \Delta \tau$$

→ Obvious case of matched bunch-to-bucket transfer
Longitudinal matching
Longitudinal matching at injection

- Long. emittance is only preserved for **correct RF voltage**

**Matched case**

- Bunch is fine, longitudinal emittance remains constant

**Longitudinal mismatch**

- Dilution of bunch results in increase of long. emittance
Longitudinal matching

Matched case

\[ \Delta \phi = 0, \frac{V_{\text{inj}}}{V_{\text{RF}}} = 1 \]

→ Bunch is fine, longitudinal emittance remains constant

Longitudinal mismatch

\[ \Delta \phi = 0, \frac{V_{\text{inj}}}{V_{\text{RF}}} = 2 \]

→ Dilution of bunch results in increase of long. emittance
Matching of phase and energy

- What is the difference?

- $-45^\circ$ phase error at injection
- Can be easily corrected by bucket phase
- Equivalent energy error
- Phase does not help: requires beam energy change
Example: mismatch at injection to PS

- Deliberate longitudinal mismatch at injection for blow-up

→ Intentional mismatch contributes to controlled longitudinal blow-up
No problem with electron accelerators

- Synchrotron radiation damping matches bunches by itself
- Phase and energy oscillations decay

LEP synchrotron injection

Proposed FCC-ee injection

→ Mismatched injection can be a useful tool
Summary

• Basic techniques of signal synchronizations
  → Beware of dividers

• Beam transfer between circular lepton accelerators
  → Constant frequency

• Beam transfer between circular hadron accelerators
  → Variable frequency
  → Moving target

• Follow the beam
  → No need to measure → keep track
  → Matching between accelerators
A big Thank You

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\[ \approx \cos \phi_0 \left( 1 - \frac{1}{2} \Delta \phi^2 \right) - \sin \phi_0 \Delta \phi \]

this simplifies to

\[ H(\Delta \phi, \dot{\phi}) \approx \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \omega_s^2 \Delta \phi^2 \]