Injection and Extraction in Cyclotrons

CERN Accelerator School – Specialised Course
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Outline

• Cyclotron Basics
  scaling and isochronicity, focusing, turn separation, classical cyclotrons and derived types

• Injection for Cyclotrons
  internal source, electrostatic inflectors, horizontal injection, optics matching, bunching

• Extraction for Cyclotrons
  electrostatic septum, stepwidth calculation, charge exchange extraction
The Classical Cyclotron

- invented 1930, Lawrence, Nobel Prize
- powerful concept:
  - simplicity, compactness
  - continuous injection/extraction
  - multiple usage of accelerating voltage

- two capacitive electrodes "Dees", two gaps per turn
- internal ion source
- homogenous B field
- constant revolution time
  (for low energy, \( \gamma \approx 1 \))

\[ \omega_c = \frac{eBz}{\gamma m} \]
wide spectrum of cyclotrons ...

compact and cost optimized for series production
e.g. medical nuclide production
→ Internal source, extraction or internal target

huge and complex for variable research purposes, e.g. R.I.B.
production or high intensity
→ External source, injection
cyclotron basics: isochronicity and scalings

continuous acceleration \( \rightarrow \) revolution time should stay constant, though \( E_k, R \) vary

magnetic rigidity:

\[
BR = \frac{1}{e} p = \beta \gamma \frac{m_0 c}{e}
\]

orbit radius from isochronicity:

\[
R = \frac{c}{\omega_c} \beta = R_\infty \beta
\]

deduced scaling of \( B \):

\[
R \propto \beta; \quad BR \propto \beta \gamma \quad \rightarrow \quad B(R) \propto \gamma(R)
\]

thus, to keep the isochronous condition, \( B \) must be raised in proportion to \( \gamma(R) \); this contradicts the focusing requirements!

field index \( k \):

\[
k = \frac{R}{B} \frac{dB}{dR} = \frac{\beta}{\gamma} \frac{d\gamma}{d\beta} = \gamma^2 - 1
\]
cyclotron basics: stepwidth (nonrelativistic, B const)

relation between energy and radius

\[ qRB_z = \sqrt{2mE_k} \]
\[ \frac{dR}{R} = \frac{1}{2} \frac{dE_k}{E_k} \]

use:
\[ \Delta E_k = \text{const}; B_z = \text{const}; E_k \propto R^2 \]

thus:
\[ \Delta R \propto \frac{R}{E_k} \propto \frac{1}{R} \]

radius increment per turn decreases with increasing radius → extraction becomes more and more difficult at higher energies

“cyclotron language”

\[ R_{\infty} = \frac{R}{\beta} \]
focusing in a cyclotron

centrifugal force \(mv^2/r\)

Lorentz force \(qv\times B\)

\[ m\ddot{r} = m r\ddot{\theta}^2 - q r \dot{\theta} B_z \]

focusing: consider small deviations \(x\) from beam orbit \(R\) \((r = R + x)\):

\[
\begin{align*}
\ddot{x} + \frac{q}{m} v B_z (R + x) - \frac{v^2}{R + x} &= 0, \\
\ddot{x} + \omega_c^2 (1 + k) x &= 0.
\end{align*}
\]

using: \(\omega_c = \frac{q B_z}{m} = \frac{v}{R}\)

\(r\dot{\theta} \approx v\)

\(k = \frac{R}{B} \frac{dB}{dR}\)

thus in radial plane:

\(\omega_r = \omega_c \sqrt{1 + k} = \omega_c \nu_r\)

\(\nu_r = \sqrt{1 + k}\)

\(\approx \gamma\) using isochronicity condition

in vertical plane:

\(\nu_z = \sqrt{-k}\)

\(k < 0\) to obtain vertical focus.
Classical vs Isochronous Cyclotron

- insufficient vertical focusing
- limited energy reach

\[ \nu_z^2 = -\frac{R}{B_z} \frac{dB_z}{dR} + F^2(1 + 2\tan^2\delta) \]

\[ F^2 = \frac{B_z^2 - B_z^2}{B_z^2} \]
Azimuthally Varying Field vs. Separated Sector Cyclotrons

- AVF = single pole with shaping
- often spiral poles used
- internal source possible
- D-type RF electrodes, rel. low energy gain
- compact, cost effective
- depicted Varian cyclotron: 80% extraction efficiency; not suited for high power

- modular layout, larger cyclotrons possible, sector magnets, box resonators, stronger focusing, injection/extraction in straight sections
- external injection required, i.e. pre-accelerator
- box-resonators (high voltage gain)
- high extraction efficiency possible:
  e.g. PSI: 99.98% = (1 - 2·10⁻⁴)
classification of cyclotron like accelerators

- **classical cyclotron**
  \[ B(\theta) = \text{const} \]

- **Thomas cyclotron**
  [Azimuthally Varying Field, e.g. \( B(\theta) \propto b + \cos(3\theta) \), one pol]

- **separated sector cyclotron**
  [separated magnets, resonators]

- **synchro-cyclotron**
  [varying RF frequency]

- **Fixed Focus Alternating Gradient Accelerator (FFAG)**
  [varying RF, strong focusing]

- **AVF concept** – harmonic pole shaping, electron model, Richardson et al (1950), courtesy of Lawrence Berkeley National Laboratory

- high intensity
- high energy
- compact machine
next: injection for cyclotrons

- internal source, axial injection, horizontal injection
- electrostatic inflector, electrostatic deflectors
- transverse matching, bunching
- space charge
Injection – Overview

Injection Techniques
- **internal source**
- axial injection
  - mirror inflector
  - **spiral inflector**
  - hyperbolic inflector
- radial injection
  - **electrostatic septum**
  - stripping injection

Aspects to be considered
- overall central region design
- radial centering
- matching of beam optics
- vertical centering
- bunching / long. capture
- minimize overall losses for high intensity application
Internal Ion Source

Example: Cold Cathode, Penning Ionisation Gauge (PIG)

cylindrical „chimney“ with slit as extraction aperture for protons

**advantage:**
- simple concept
- no heating required

**critical:**
- reproducibility of captured current (geometry related sensitivity)
- current stability on short (ms) timescale
internal ion source

→ example COMET (Accel/Varian)

- Hydrogen is injected and ionized through chimney.
- First acceleration by puller, connected to one Dee (80kV).

**Diagram Notes:**
- Chimney = ion source.
- Deflector electrode for intensity regulation.
external source: axial vs. horizontal injection

axial: suited for compact cyclotron with field covering entire plane

horizontal: suited for sector cyclotron with gaps between magnets

B field results in desired radial deflection

Ideally field free region
Beam Deflection by Electric Field

momentum change: \[ \Delta p_{\perp} = \int F_{\perp} dt = \int \frac{E_{\perp}}{\beta c} ds, \quad F = qE \]

resulting angle: \[ \theta = \frac{\Delta p_{\perp}}{p} = \frac{q l E}{\gamma \beta^2 E_0} \]

bending radius: \[ \rho = \frac{l}{\theta} \]

electric rigidity: \[ E\rho = \frac{\gamma^2 - 1}{\gamma} \frac{E_0}{q} = \frac{\gamma + 1}{\gamma} \frac{E_k}{q} \]

low energy at source: \[ E\rho \approx 2U_{\text{acc}} \]

comparison electric and magnetic force on protons
\[ \vec{F}_E = e \cdot \vec{E}, \quad \vec{F}_B = e \cdot \vec{v} \times \vec{B} \]

table: bending radius, varying \( E_k \)

<table>
<thead>
<tr>
<th>( E_k )</th>
<th>( B = 1T )</th>
<th>( E = 10\text{MV/m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 keV</td>
<td>35 mm</td>
<td>12 mm</td>
</tr>
<tr>
<td>1 MeV</td>
<td>140 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>1 GeV</td>
<td>5.6 m</td>
<td>150 m</td>
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</table>
electrostatic inflectors

Mirror inflector: particle energy is variable, simple design

Spiral inflector: force always perpendicular to velocity vector, no energy change

Velocity vector rotates around vertical axis due to action of magnetic field; other solutions exist, e.g. hyperbolic inflector or even magnetostatic inflector.
injection schemes – spiral inflector

• an electrostatic component, basically a capacitor
• E-field arranged perpendicular to orbit, particles move on equipotential surfaces

simulation of orbits injected through a spiral inflector

[inflector IBA Cyclone 30 cyclotron]

[courtesy: W.Kleeven (IBA)]
Horizontal Injection – Example PSI Ring Cyclotron

Injection element

Injection path (72MeV) in region of low field, passing along 3rd-harmonic (150MHz) resonator

extraction
Bunching for Cyclotrons

Ion sources deliver DC beam; for acceleration in an RF field the beam must be bunched; unbunched beam should be removed at low energy (≤5MeV) to avoid uncontrolled losses and activation

Schemes applied in practice:

<table>
<thead>
<tr>
<th></th>
<th>bunching in cyclotron</th>
<th>external buncher cavities</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>internal source</td>
<td>x</td>
<td></td>
<td>lowest cost and complication</td>
</tr>
<tr>
<td>external source</td>
<td>x</td>
<td>x</td>
<td>higher intensity, variety of ions</td>
</tr>
<tr>
<td>DC pre-accelerator</td>
<td></td>
<td>x</td>
<td>low ΔE, costly</td>
</tr>
<tr>
<td>Cockcroft-Walton</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radio Freq. Quadrupole (RFQ)</td>
<td></td>
<td>x</td>
<td>compact, costly</td>
</tr>
</tbody>
</table>
Sketch of 870 keV Injektion Beam Line

Ion Source

Cockcroft – Walton
High voltage generator

50 MHz Buncher CWB

150 MHz Buncher CW3B

Injektion Point

Beamline

Accelerating System 50 MHz

Center region Injector 2

Phase 180°

Coll.
50 MHz and 50/150 MHz Harmonic Oscillation

→ by utilizing a harmonic buncher (3ω), a larger fraction of a DC beam can be captured in the cyclotron

only 50MHz buncher

additional 150MHz buncher

[M.Humbel, PSI]
Center Region of PSI Injector 2

collimation of low energy protons and intensity control

0.86 → 72MeV
max 2.5mA, 180kW
PSI Injector 2 and Injection Beamline
Transverse Matching

- Similar to a synchrotron the envelope function $\beta$ varies around the circumference; the beam at injection must be matched to avoid blow up and sub-optimal beam distributions.

Nonetheless of the short «storage» time of a beam in a cyclotron, the distribution starts to filament, if not properly matched.

Example: beam sizes around the circumference for Inj II cyclotron, PSI [Ch.Baumgarten, [7]]
transverse space charge

especially at low energy space charge effects are critical for the injection of high intensity beams

vertical force from space charge: \[ F_y = \frac{n_v e^2}{\varepsilon_0 \gamma^2} \cdot y, \quad n_v = \frac{N}{(2\pi)^{\frac{3}{2}} \sigma_y D_f R \Delta R} \]

[constant charge density, \( D_f = \frac{I_{avg}}{I_{peak}} \)]

thus, eqn. of motion: \[ \ddot{y} + \left( \omega_c^2 \nu_{y0}^2 - \frac{n_v e^2}{\varepsilon_0 m_0 \gamma^3} \right) y = 0 \]

\[ \rightarrow \text{tune shift results in intensity limit (see [6])!} \]

tune shift from forces: \[ \Delta \nu_y \approx -n_v \frac{2\pi r_p R^2}{\beta^2 \gamma^3 \nu_{y0}} \]
next: extraction for cyclotrons

• review of schemes: internal targets, electrostatic deflectors, stripping
• maximizing extraction efficiency: stepwidth, coherent oscillations, avoid tails
electrostatic septum and charge exchange extraction

- simplest solution: use beam without extraction $\rightarrow \text{internal target}$; use some mechanism to exchange target
- **electrostatic deflectors** with thin electrodes, deflecting element should affect just one turn, not neighboured turn $\rightarrow$ critical, cause of losses
- alternative: charge exchange by stripping foil; accelerate $\text{H}^-$ or $\text{H}_2^+$ to extract protons (problem: significant probability for unwanted loss of electron; Lorentz dissociation: B-field low, scattering: vacuum $10^{-8}$mbar)

<table>
<thead>
<tr>
<th>binding energies</th>
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<tbody>
<tr>
<td>$\text{H}^-$</td>
</tr>
<tr>
<td>0.75eV</td>
</tr>
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</table>

\[ \text{eg.: } \text{H}^- \rightarrow \text{H}^+ \]
\[ \text{H}_2^+ \rightarrow 2\text{H}^+ \]
derivation of relativistic turn separation in a cyclotron

starting point: bending strength
→ compute total log.differential
→ use field index $k = R/B \cdot dB/dR$

\[
BR = \sqrt{\gamma^2 - 1} \frac{m_0c}{e}
\]

\[
\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma d\gamma}{\gamma^2 - 1}
\]

\[
\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1} \frac{1}{1 + k}
\]

radius change per turn

\[
\frac{dR}{dn_t} = \frac{dR}{d\gamma} \frac{d\gamma}{dn_t}
\]

\[
= \frac{U_t}{m_0c^2} \frac{\gamma R}{(\gamma^2 - 1)(1 + k)}
\]

\[
= \frac{U_t}{m_0c^2} \frac{R}{(\gamma^2 - 1)\gamma}
\]

\[U_t = \text{energy gain per turn}\]

\{ \text{isochronicity not conserved (last turns)} \}

\{ \text{isochronicity conserved (general scaling)} \}
discussion: scaling of turn separation

for clean extraction a large stepwidth (turn separation) is of utmost importance; in the PSI Ring most efforts were directed towards maximizing the turn separation.

general scaling at extraction:

$$\Delta R(R_{\text{extr}}) = \frac{U_t}{m_0 c^2} \frac{R_{\text{extr}}}{(\gamma^2 - 1)\gamma}$$

scaling during acceleration:

$$\frac{dR}{dn_t} \approx \frac{U_t}{m_0 c^2 \beta^2} R \rightarrow \Delta R(R) \propto \frac{1}{R}$$

desirable:
- limited energy (< 1GeV)
- large radius $R_{\text{extr}}$
- high energy gain $U_t$

illustration:
**stepwidth vs. radius** in cyclotrons of different sizes; 100MeV inj $\rightarrow$ 800MeV extr
methods to enhance turn separation

several techniques were invented to „artificially“ increase turn separation beyond the magnitude achieved by simple acceleration

<table>
<thead>
<tr>
<th>„brute force“</th>
<th>resonant orbit distortion is excited by harmonic coils beyond a certain radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>precessional extraction</td>
<td>resonant excitation at ( v_r = 1 ) plus steep ( v_r ) slope in fringe field</td>
</tr>
<tr>
<td>regenerative extraction</td>
<td>using coherent excitation at half integer resonance by gradient bump</td>
</tr>
</tbody>
</table>

taken from Kleeven [1]
Resonant Extraction (Varian/Accel cyclotron)

Extraction efficiency: up to 80%

use $V_r = 1$

Electrostatic extraction elements

Field bumps

[M.Schippers, PSI]
extraction with coherent oscillations (PSI)

betatron oscillations around the “closed orbit” can be used to increase the radial stepwidth by a factor 3!

without orbit oscillations: stepwidth from $E_k$-gain

with orbit oscillations: extraction gap; up to 3 x stepwidth possible for $\nu_r = 1.5\pi$ (phase advance)

$\nu_r$ decreases from 1.75 to 1.5

phase vector of orbit oscillations $(r,r')$
extraction profile measured at PSI Ring Cyclotron

red: tracking simulation [OPAL]
black: measurement

turn numbers from simulation

dynamic range: factor 2.000 in particle density

position of extraction septum d=50µm

[Y.Bi et al]
vertical tune in Ring cyclotron supports extraction

radial tune vs. energy (PSI Ring)
typically $v_r \approx \gamma$ during acceleration; but decrease in outer fringe field

field map showing increase and steep decline of field with radius
coupling resonance – pass quickly!

$Q_r$ decreases towards extraction – enhance turn separation

**comments:**
- running on the coupling resonance would transfer the large radial betatron amplitude into vertical oscillations, which must be avoided
- special care has to be taken with fine-tuning the bending field in the extraction region
injection/extraction with electrostatic elements

**principle of extraction channel**

- **parameters**
  - Extraction channel:
    - $E_k = 590\text{MeV}$
    - $E = 8.8 \text{ MV/m}$
    - $\theta = 8.2 \text{ mrad}$
    - $\rho = 115 \text{ m}$
    - $U = 144 \text{ kV}$

- Major loss mechanism is scattering in 50$\mu\text{m}$ electrode!

**electrostatic rigidity:**

\[
E \rho = \gamma + \frac{1}{\gamma} \frac{E_k}{q}
\]
Electrostatic Elements for High Energy/High Intensity

- HV feedthrough 140-150 kV
- Actuator
- Cathode
- Isolator
- Loss electrodes
- Tungsten stripes 3 mm X 0.05 mm
- GND

[D.Götz, PSI]
longitudinal space charge (tails at extraction)

sector model:
→ accumulated energy spread transforms into transverse tails  
  • consider rotating uniform sectors of charge (overlapping turns)  
  • test particle “sees” only fraction of sector due to shielding of vacuum chamber with gap height $2w$

two factors are proportional to the number of turns:
  1) the charge density in the sector  
  2) the time span the force acts

$$\Delta U_{sc} \approx 2.800 \Omega \cdot e I_p \cdot \frac{n_{\text{max}}^2}{\beta_{\text{max}}}$$

 derivation see [4]: Joho 1981

in addition:
  3) the inverse of turn separation at extraction:

$$\frac{1}{\Delta R_{\text{extr}}} \propto n_{\text{max}}$$

→ the attainable current at constant losses scales as $n_{\text{max}}^{-3}$
extraction foil

- thin foil, for example carbon, removes the electron(s) with high probability
- new charge state of ion brings it on a new trajectory → separation from circulating beam
- lifetime of foil is critical due to heating, fatigue effects, radiation damage
- conversion efficiencies, e.g. generation of neutrals, must be considered carefully

Electrons removed from the ions spiral in the magnetic field and may deposit energy in the foil.

**How much power is carried by the electrons?**

→ velocity and thus $\gamma$ are equal for $p$ and $e$

$$E_k = (\gamma - 1)E_0$$

$$\rightarrow E_{k_e} = \frac{E_0^e}{E_0^p}E_{k_p} = 5.4 \cdot 10^{-4} E_{k_p}$$

**Bending radius of electrons?**

$$\rho^e = \frac{E_0^e}{E_0^p}\rho^p$$

→ typically mm
example: multiple H⁻ stripping extraction at TRIUMF

[R.Baartman]
example: $\text{H}_2^+$ stripping extraction in proposed Daedalus cyclotron [neutrino source]

purpose: pulsed high power beam for neutrino production, goals:
• 800MeV kin. energy
• 5MW avg. beam power

[L.Calabretta, A.Calanna et al]
Summary: Injection & Extraction for Cyclotrons

**Injection**
- Internal source
  - Axial injection: electrost. inflector
  - Horizontal injection: electrost. septum
  - Stripping injection

**Extraction**
- Internal target
  - Electrostatic element
  - Coherent oscillations
  - Resonant extraction
  - Stripping extraction: $H^-$, $H_2^+$, various ions

**Beam Physics Aspects:**
- Central region design, beam centering, transverse matching, bunching, beam blowup/tails & loss minimization & activation, space charge
**literature w.r.t. cyclotron injection/extraction**

| [1] | comprehensive review of inj./extr. concepts | W.Kleeven (IBA), Injection and Extraction for Cyclotrons  
[https://cds.cern.ch/record/1005057/files/p271.pdf](https://cds.cern.ch/record/1005057/files/p271.pdf) |
| [2] | many examples and calculations for compact machines | P.Heikkinen (Jyväsym), Injection and Extraction for Cyclotrons  
[http://accelconf.web.cern.ch/AccelConf/c81/papers/ei-03.pdf](http://accelconf.web.cern.ch/AccelConf/c81/papers/ei-03.pdf) |
[http://accelconf.web.cern.ch/AccelConf/CYCLOTRONS2013/papers/we2pb01.pdf](http://accelconf.web.cern.ch/AccelConf/CYCLOTRONS2013/papers/we2pb01.pdf) |
| [7] | formation of round bunches and matching approach | Ch.Baumgarten, transverse-longitudinal coupling by space charge in cyclotrons  
Thank you for your attention!