Presentation overview

- Why do we need another magnet type?

**Septum**
- Basic concept and terminology
- Key objectives
- Types of septa
- Electrostatic deflection
- Magnetic deflection
- Electrostatic vs Magnetic deflection

**Electrostatic septa**
- Types
- Examples, design specifics

What to remember

Literature

Additional material
- Electrostatic deflection derivation
- Magnetic deflection derivation
- Equivalence of electrostatic and magnetic deflection for relativistic particles
Why do we need another magnet type?

Synchrotron single bunch extraction (simplified)

The circulating beam is captured in the dipole’s aperture and bent, altering its original trajectory.

Having two distinctive field regions with and without deflecting field meets the desired requirements.
A septum (plural septa) is a partition, a wall, a barrier that separates two cavities or two chambers (biology, mechanics, part. physics, etc.).

- Latin origin - saepio (sēpiō) - surround, enclose, fence in.
- In particle accelerators, a septum separates two distinctive field regions in order to selectively deflect particle beams.

- Used for injection and extraction of the beam

- Often the device that embodies the septum is called septum as well (electrostatic septum, septum magnet, etc.)

**Septum**

The septum separates the left and the right side of the heart.
Basic concept and terminology

- A septum shares a lot with dipole (bending) magnets
- Has an abrupt field change between field and no-field region

Schematic representation of a septum

θ – Bending angle
R – Bending radius
s – Sagitta
w – Deflecting gap width
t – Septum thickness
l – Septum length

If deflecting gap does not follow the trajectory of the deflected beam (as shown here) the deflecting gap should be wide enough to accommodate the deflected beam trajectory.

Key objectives

- Field region (type, magnitude, direction, flatness)
- Field-free region (field leakage)
- Beam trajectory (sagitta), beam optics
- Beam impedance, wake fields
- Vacuum (UHV materials, conductance, bake-out)
- Positioning and mechanical stability
- Synchrotron radiation
- Radiation effects (beam loss region)
- Thermal management
- Machine / personnel protection
- Reliability / Serviceability / Reparability
- Economics (building costs vs exploitation costs)
Types of septa

How do we deflect charged particles beams?

Electric, magnetic and "exotic" (e.g. crystals - plane channeling*) Lorentz force** – the force exerted on a point charge by electromagnetic fields.

\[
\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}
\]

- Force exerted on the point charge [N]
- \(q\) – Electric charge [C]
- \(\vec{E}\) – Electric field [V/m]
- \(\vec{v}\) – Velocity of the point charge [m/s]
- \(\vec{B}\) – Magnetic flux density [T]

Often charged particles in accelerators move with relativistic speeds – relativistic dynamics should be applied.

Electric force

Deflecting force is collinear with the electric field – positive charges are deflected in the direction of the electric field lines, negative charges are deflected on the opposite direction.

\[
\vec{F} = q\vec{E}
\]

Conventions:
- Force on a positive point charge.
- Electric field lines go from positive electrode to the negative one.
- Opposite electric charges attract each other and like electric charges repel.
Electrostatic deflection

Electrostatic bending angle \( \theta_E \) of particles with single elementary charge \(^{[3, 20]}\)

\[
\theta_E \approx \frac{E \cdot l_{\text{eff}}}{10^9 \cdot \beta \cdot p} = \frac{U \cdot l_{\text{eff}}}{10^9 \cdot \beta \cdot p \cdot d}
\]

Where:

\( \theta_E \) – electrostatic bending angle* [rad]
\( E \) – deflecting electric field [V/m]
\( l_{\text{eff}} \) – effective length of the septum [m], usually different from the mechanical length due to fringe fields
\( \beta \) – relativistic coefficient that gives the fraction of the speed of light at which the particles travel [-]
\( p \) – particles momentum [GeV/c]
\( U \) – deflecting voltage [V]
\( d \) – distance between the deflecting electrodes [m]

*Small angle approximation
\[ \tan(\theta) \approx \theta \]
up to \( \approx 0.17 \text{ rad} \ (\approx 10^\circ) \) error is <1%

Don’t get caught by the units!!!

\[
p \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}} \right] = \frac{q_e \cdot eV}{c} \left[ \frac{\text{eV}}{c} \right]
\]

(Derivation in the additional material slides at the end of presentation)

Types of septa

How do we deflect charged particles beams?

Electric, magnetic and “exotic” (e.g. crystals - plane channeling*)

Lorentz force** – the force exerted on a point charge by electromagnetic fields.

\[
\vec{F} = q \vec{E} + q \vec{v} \times \vec{B}
\]

\( \vec{F} \) – Force exerted on the point charge [N]
\( q \) – Electric charge [C]
\( \vec{E} \) – Electric field [V/m]
\( \vec{v} \) – Velocity of the point charge [m/s]
\( \vec{B} \) – Magnetic flux density [T]

*Plane channeling of protons (> 10 GeV) in Si mono-crystals\(^{[21, 22]}\)

**First derivation is often attributed to Oliver Heaviside or James Maxwell

Often charged particles in accelerators move with relativistic speeds – relativistic dynamics should be applied.
**Magnetic force**

Deflecting force $F$ is cross product of $\mathbf{v}$ and $B$

$$F = q \mathbf{v} \times \mathbf{B}$$

Conventions:

- Force on a positive point charge.
- Magnetic field lines go from North pole to the South pole of the magnet.
- Right hand rule: If the thumb points in the direction of motion and the index finger is in the direction of the magnetic field, the force goes in the direction of the middle finger.

---

**Magnetic deflection**

Magnetic bending angle $\theta_M$ of particles with elementary charge $[\text{1, 3, 20, 23}]$

$$\theta_M \approx \frac{0.3 \cdot B \cdot l_{eff}}{p} \approx \frac{3.76 \cdot n \cdot l \cdot l_{eff}}{10^7 \cdot p \cdot d}$$

Where:

- $\theta_M$ – magnetic bending angle* [rad]
- $B$ – deflecting magnetic flux density [T]
- $l_{eff}$ – effective length of the septum [m], usually different from the mechanical length due to fringe fields
- $p$ – particles momentum [GeV/c]
- $n$ – number of turns [-]
- $l$ – current [V]  
- $\mu_0$ – permeability of free space [H/m]
- $d$ – distance between magnetic poles [m]

*Small angle approximation

$$\tan(\theta) \approx \theta$$

up to $0.17 \text{ rad (} \approx 10^\circ \text{)}$ error is $<1\%$

$$B \approx \frac{\mu_0 n I}{d}$$

Don’t get caught by the units!!!

$$p \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}} \right] = \frac{q \mu_0 n I}{\epsilon_0} \left[ \frac{\text{eV}}{\text{m}} \right]$$

*(Derivation in the additional material slides at the end of presentation)*
Electrostatic vs Magnetic deflection

Which one to use? How to compare?\textsuperscript{[24]} - Duality of electromagnetism

<table>
<thead>
<tr>
<th>Deflecting field</th>
<th>Stored energy per unit volume (free space)</th>
<th>Scalar form of Lorentz force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric</td>
<td>$W_E = \frac{\varepsilon_0 B^2}{2}$</td>
<td>$F_E = qE$</td>
</tr>
<tr>
<td>Magnetic</td>
<td>$W_M = \frac{B^2}{2\mu_0}$</td>
<td>$F_M = qvB$</td>
</tr>
</tbody>
</table>

For $W_E = W_M (E = cB)$ and relativistic particles ($v = c$)

$$F_E = qcB = F_M$$

(\textit{Derivation in the additional material slides at the end of presentation})

Erice, Italy, 16 March, 2017

M. Paraliev

Electrostatic vs Magnetic deflection

But something is different...

- It is more practical to use \textit{magnetic field}!
- Too high electric field in vacuum could provoke electric breakdown. It is widely accepted that 10 MV/m is a practical limit\textsuperscript{[24]}.
- Electric deflection could be beneficial for \textit{non-relativistic} particles (e.g. low energy beams, heavy ions etc.)

$$B[T] = \frac{3.3}{\beta} E[\text{GV/m}]$$

\textit{Old Greek dictum of wisdom:}
\textit{If you are clever, you use magnetic fields in an accelerator whenever it is possible.}

Dr. Bernhard Holzer\textsuperscript{[1]}

Comparison between electric and magnetic deflection for beams with different momentum $p$

<table>
<thead>
<tr>
<th>$\beta$, $\gamma$</th>
<th>$\gamma_r$, $\gamma_r$</th>
<th>$\text{Preliminary, MeV/c}$</th>
<th>$\text{Preliminary, GeV/c}$</th>
<th>Electric field, MV/m</th>
<th>Equivalent magnetic field, T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1.000</td>
<td>0.0005</td>
<td>0.0009</td>
<td>10.00</td>
<td>33.356</td>
</tr>
<tr>
<td>0.01</td>
<td>1.000</td>
<td>0.0051</td>
<td>0.0094</td>
<td>10.00</td>
<td>3.336</td>
</tr>
<tr>
<td>0.1</td>
<td>1.005</td>
<td>0.0514</td>
<td>0.0944</td>
<td>10.00</td>
<td>0.334</td>
</tr>
<tr>
<td>0.3</td>
<td>1.048</td>
<td>0.1607</td>
<td>0.2955</td>
<td>10.00</td>
<td>0.111</td>
</tr>
<tr>
<td>0.5</td>
<td>1.155</td>
<td>0.2950</td>
<td>0.5425</td>
<td>10.00</td>
<td>0.067</td>
</tr>
<tr>
<td>0.9</td>
<td>2.794</td>
<td>2.0592</td>
<td>3.9401</td>
<td>10.00</td>
<td>0.099</td>
</tr>
<tr>
<td>0.99</td>
<td>7.089</td>
<td>3.5664</td>
<td>6.9944</td>
<td>10.00</td>
<td>0.034</td>
</tr>
<tr>
<td>0.999</td>
<td>22.366</td>
<td>11.4185</td>
<td>20.9955</td>
<td>10.00</td>
<td>0.033</td>
</tr>
<tr>
<td>0.9999</td>
<td>70.712</td>
<td>36.1328</td>
<td>66.4386</td>
<td>10.00</td>
<td>0.033</td>
</tr>
<tr>
<td>0.99999</td>
<td>223.607</td>
<td>114.2698</td>
<td>210.1114</td>
<td>10.00</td>
<td>0.033</td>
</tr>
<tr>
<td>0.999999</td>
<td>707.107</td>
<td>361.3552</td>
<td>664.4349</td>
<td>10.00</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Erice, Italy, 16 March, 2017

M. Paraliev
**Electrostatic vs Magnetic deflection**

... and it goes further... There is no “universal” solution!

Advantages and disadvantages of the two schemes

<table>
<thead>
<tr>
<th>Electric septum</th>
<th>Magnetic septum</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Near perfect no-field region</td>
<td>+ Strong deflection</td>
</tr>
<tr>
<td>+ Thin septum</td>
<td>+ More effective for relativistic beams</td>
</tr>
<tr>
<td>+ Low mass density (low beam interaction)</td>
<td>+ In-vacuum and in-air design is possible</td>
</tr>
<tr>
<td>+ Better for non-relativistic beams</td>
<td>- Thick septum</td>
</tr>
<tr>
<td>- Difficult to have high fields</td>
<td>- Field leakage</td>
</tr>
<tr>
<td>- Less effective for relativistic beams</td>
<td>- Non-uniform field region</td>
</tr>
<tr>
<td>- High voltages handling</td>
<td>- Interaction with other magnets</td>
</tr>
<tr>
<td>- Strictly in-vacuum design</td>
<td>- High currents handling</td>
</tr>
</tbody>
</table>

Erice, Italy, 16 March, 2017

M. Paraliev
Basic scheme

Electric field is established between a HV electrode and a septum foil.

The extracted beam passes through the electric field region and it is deflected.

Using Faraday cage effect the foil and the foil support create a zero-field region for the circulating beam that goes straight.

- Thin foil is used to minimize the interaction with beam (reduce beam losses and radiation levels)
- To utilize precise alignment with respect to the circulating beam often the septum is mounted on precision mover system.
- Care should be taken to ensure good vacuum conduction in order to maintain low background pressure

Example I – Foil septum

Construction and technical data of “Septum 23” [PS, CERN][13, 15]

- Electrode length: 778 mm
- Gap width: 17 mm
- Beam momentum: 24 GeV/c
- Deflection angle: 0.28 mrad
- Septum thickness: 100 μm
- Vacuum: 10^-9 mbar
- Voltage: 260 kV
- Electric field: up to 15 MV/m
- Septum foil material: Molybdenum
- Electrode material: Anodised Peraluman 300 aluminum alloy
- In-situ bake-able
Example I – Foil septum

Electric field considerations and HV aspects

- Good field region 40 mm
- Polished molybdenum foil
- Polished st. steel deflectors to control field distribution
- Anodized HV electrode
- Good vacuum to avoid residual gas ionization triggered HV breakdown
- Every 3 months the insulating oil of the HV feedthroughs is changed (radiation deterioration) and septum is HV conditioned
- Annually the close HV cables are replaced
- Maintenance - radioactive activation

Example I – Foil septum

Positioning and mechanical considerations

- Septum positioning in radial and angular direction
- HV electrode positioning in radial direction
- Angular resolution: 0.01 mrad
- Translation resolution: 100 μm
- Initial problems with bake-out above 90°C (different thermal expansion coefficients)\(^\text{[15]}\)
- Later fixed by rebuilding the foil tensioner\(^\text{[13]}\)
Example I – Foil septum

General consideration

- Septum is activated – difficult to service
- In-situ bake-out system – mechanical stress due to different thermal expansion coefficients
- Thermal loading of septum (slow extraction systems)
- Beam impedance – reduce discontinuities using proper screening
- Machine protection system (vacuum interlock)

Example II – Foil septum with inverted design

Construction and technical data of injection septum for CNAO (PIMMS)\[12, 17\]

- Electrode length: 800 mm
- Gap width: 25 mm
- Beam momentum: 20 MeV/c
- Deflection angle: 60 mrad
- Septum thickness: 100 μm
- Vacuum: $10^{-9}$ mbar
- Voltage: 69 kV
- Electric field: up to 2.8 MV/m
- Septum: Molybdenum foil
- Electrode material: St. steel
Example II – Foil septum with inverted design

Design specifics\[^{[17]}\]

- HV electrode inside the septum support
- Mechanical support and remote positioning systems are from the same side, freeing up space for other equipment on the other side of the beam
- Electric field is fully enclosed – field distribution control, well defined exposed HV surfaces
- Two sections to follow the beam trajectory
- Electrodes positioning range ±5 mm

Example III – Diagonal foil septum

Construction and technical data of ER.SEH10 septum (LEIR, CERN)\[^{[11]}\]

- Electrode length: 720 mm
- Gap width: 40 mm
- Beam energy: 4.2 MeV/nucleon
- Deflection angle: 28.9 mrad
- Septum thickness: 100 μm
- Vacuum: $10^{-12}$ mbar
- Voltage: 51 kV
- Electric field: up to 1.12 MV/m
- Septum: Molybdenum foil
- Electrode material: Titanium
- Deflectors material: St. steel

\[^{[17]}\]:
- HV electrode inside the septum support
- Mechanical support and remote positioning systems are from the same side, freeing up space for other equipment on the other side of the beam
- Electric field is fully enclosed – field distribution control, well defined exposed HV surfaces
- Two sections to follow the beam trajectory
- Electrodes positioning range ±5 mm

\[^{[11]}\]:
- Electrode length: 720 mm
- Gap width: 40 mm
- Beam energy: 4.2 MeV/nucleon
- Deflection angle: 28.9 mrad
- Septum thickness: 100 μm
- Vacuum: $10^{-12}$ mbar
- Voltage: 51 kV
- Electric field: up to 1.12 MV/m
- Septum: Molybdenum foil
- Electrode material: Titanium
- Deflectors material: St. steel
Example III – Diagonal foil septum

Design specifics

- Polished molybdenum foil, HV electrode and deflectors
- Retractable septum and HV electrode
- Diagonal field
- Remote displacement at 30° from horizontal plane
- Good field region 40 mm (±0.5 % field uniformity)
- After bake out at 300°C the alignment of the foil remains within 100 μm, so that the effective septum foil thickness as seen by the beam is less than 200 μm.

Wire septum

The thin foil septum is substituted with array of thin wires

- An array of wires reduces the effective density of the septum, decreasing beam loss and radiation
- High electric field possible
- Field leakage in no-field region
- Wires array increase the vacuum conductivity to the screened volume.
- Individual tensioner on each wire
- Ionization of residual gas in the field-free region can provoke a HV breakdown. Care should be taken to remove ions from the volume.
Static field simulation - modeling

How dense should the wire array be to have sufficient field screening?

Wire density coefficient \( k \)

\[
k = \frac{d}{D}
\]

- \( d \) – wire diameter
- \( D \) – distance between wires (unit length)

Parametric electrostatic simulation – CST model

- Choose proper boundary conditions
- Use planes of symmetry
- Define areas of interest
- Use parameters when building the model in order to run parametric simulations easily

Static field simulation – potentials and meshing

Setting the field at nominal 1 V/m will make resulting field strength and field enhancement factor the same

- Keep aspect ratios as low as possible (but be aware of proximity effects!)
- Use adequate meshing (adaptive mesh, vacuum bodies with different mesh parameters etc.)
Static field simulation – field results

Field is uniform above the wire array and vanishes below it.

- Explore carefully the field enhancement around the elements
- Use post-processing to have more detailed (tabulated) results

Static field simulation – parametric study

Field profiles along the lines A and B for different wire density coefficient $k$

- Away from wire proximity the field profile is identical on both lines
- Even $k = 0.1$ gives significant screening (field reduction of ~53 at 2 units distance)
- At trajectories closer than 1 unit the field is not homogeneous
Static field simulation – parametric study

Field on the wires’ surface and field screening in function of $k$

- The field on the wires’ surface could be several times larger than the main field and it could provoke a HV breakdown
- Maximum wires’ surface field follows nicely $1/k$ dependence

Example IV – Wire septum

Construction and technical data of wire septum ZS (5 units, SPS, CERN)\(^{[10]}\)

- Electrode length: 2997 mm
- Wire material: W74Re26
- Wire diameter: 50 – 100 μm
- Wire spacing: 1.5 mm
- Gap width: 20 mm
- Beam momentum: 450 GeV/c
- Deflection angle: ~70 μrad
- Vacuum: $10^{-12}$ mbar
- Voltage: 220 kV
- Ion traps voltage: 3 to 6.5 kV
- Electric field: up to 11 MV/m
- Electrode material: Anodized aluminum

Erice, Italy, 16 March, 2017 33 M. Paraliev

Erice, Italy, 16 March, 2017 34 M. Paraliev
Example IV – Wire septum

Design specifics and general considerations

- Septa are activated – difficult to service
- Retractable design
- Powered by -300 kV (Cockcroft-Walton) HV generator
- The device is prone to breakdowns due to background gas ionization – ion traps (electrodes) clear the zero-field volume and partially neutralize the leakage field
- Spark protection – deactivates the electrodes in case of too frequent sparks

Thermal loading - often the accelerators are used in different operation modes (different beam momentum, beam intensity and so on). In case of system failure or operator’s error, high intensity beam could land on the septum and damage it.

Short pulses – instantaneous heating – no heat conduction

\[
\frac{dE}{dV} = \rho \int_{T_0}^{T_0+\Delta} c_p(T) dT
\]

Where:

- \( E \) - deposited energy
- \( V \) - volume of the material where the energy is deposited
- \( \rho \) - material density
- \( c_p(T) \) - specific heat of the material
- \( T_0 \) - initial temperature
- \( \Delta T \) - temperature change
**Example V – curved wire septum**

**Construction and technical data of ES Mini-Wire-Septum collimator**

- Electrode length: 500 mm
- Wire material: Beryllium ($Z = 4$)
- Wire diameter: 50 μm
- Wire spacing: 5 mm
- Gap width: 10 mm
- Beam momentum: 0.13 to 1.6 GeV/c
- Deflection angle: 10.2 to 1.14 mrad
- Voltage: 50 kV
- Electric field: 5 MV/m
- Wire polarization: 13.7 kV

---

**Example V – curved wire septum**

**Design specifics of ES Mini-Wire-Septum collimator**

- Ultra thin wire and low atomic number ($N_{Be} = 4$) for low beam interaction
- Wire curved due to electrostatic force of 28 mN/m
- Perfect alignment of the wires is difficult but even might not be necessary, reduces further beam interaction
- Wire septum is polarized (13.7 kV) – compensates the leakage field (no ion traps necessary, field on both sides of the septum)
**Practical considerations**

- Very thin septa – mechanical, thermal stress
- Low atomic number material – reduce beam interaction
- High electric field – surface finish and conditioning, spark protection
- Good field region – geometry of gap, deflectors
- Field leakage compensation (for wire septa only)
- Beam impedance – proper screening
- HV handling – creepage distances (feedthroughs, supports)
- Insulators degradation (insulation oil, cables)
- Alignment – remote positioning systems
- Good vacuum – bake-out capabilities, vacuum conductivity
- Background gas ionization – ion traps
- Machine protection (vacuum, system failure, operator mistakes)
- Activation – maintenance limitations

---

**What to remember**

*Septum is a wall!*

\[ \vec{F} = \text{const} \]

\[ \vec{F} = 0 \]

*When possible choose...*  
... *magnetic!*

---

Erice, Italy, 16 March, 2017 39 M. Paraliev

Erice, Italy, 16 March, 2017 40 M. Paraliev
Acknowledgments

Special thanks to
Dr. Mike Barnes (CERN)
for providing very useful material

Special thanks to
Dr. Sladana Dordevic (PSI)
for helping with the field simulations

Thank you for your attention

Erice, Italy, 16 March, 2017
41
M. Paraliev

Literature

[9] M. Plum, "Injection and extraction single, turn injection", USPAS Class on Injection and Extraction, USA, 2009

Erice, Italy, 16 March, 2017
42
M. Paraliev
Additional material – Electrostatic deflection derivation

Derivation of electrostatic deflection of moving charged particle with velocity \( v \), rest mass \( m_0 \) and charge \( q_0 \), in electric field \( E \) perpendicular to particle’s direction of travel. The electric force \( F_e \) balances the centrifugal force \( F_c \) and the particle travels on an arc trajectory with radius \( R \).

Bending angle \( \theta \) is found using bending radius \( R \) and the length of the field \( l \) in the limits of small angle approximation \( \tan(\theta) \approx \theta \)

\[
F_c = q_0E, \quad F_e = \frac{q_0v^2}{R}, \quad \rho = \gamma m_0 c, \quad v = \beta c
\]

Where:
- \( F_c \) - centrifugal force [N]
- \( F_e \) - electrostatic force [N]
- \( E \) - electric field [V/m]
- \( R \) - bending radius [m]
- \( q_0 \) - particle charge (elementary charge) [C]
- \( m_0 \) - particle's rest mass [kg]
- \( \rho \) - beam momentum [kg.m/s] or [GeV/c]
- \( \gamma \) - relativistic gamma [-]
- \( \beta \) - relativistic beta [-]
- \( c \) - speed of light [m/s]
- \( l \) - length of the field [m]

\[
\theta \approx \frac{l}{R} = \frac{0.3 B l}{p \left[ \frac{\text{GeV}}{c} \right]}
\]

Erice, Italy, 16 March, 2017

M. Paraliev

Additional material – Magnetic deflection derivation

Derivation of magnetic deflection of moving charged particle with velocity \( v \), rest mass \( m_0 \) and charge \( q_0 \), in magnetic field \( B \) going out of plane of paper. The magnetic force \( F_m \) balances the centrifugal force \( F_c \) and the particle travels on an arc trajectory with radius \( R \).

Bending angle \( \theta \) is found using bending radius \( R \) and the length of the field \( l \) in the limits of small angle approximation \( \tan(\theta) \approx \theta \)

\[
F_m = q_0vB, \quad F_c = \frac{q_0v^2}{R}, \quad \rho = \gamma m_0 c, \quad v = \beta c
\]

Where:
- \( F_c \) - centrifugal force [N]
- \( F_m \) - magnetic force [N]
- \( B \) - magnetic flux density [T]
- \( R \) - bending radius [m]
- \( q_0 \) - particle charge (elementary charge) [C]
- \( m_0 \) - particle's rest mass [kg]
- \( \rho \) - beam momentum [kg.m/s] or [GeV/c]
- \( \gamma \) - relativistic gamma [-]
- \( \beta \) - relativistic beta [-]
- \( c \) - speed of light [m/s]
- \( l \) - length of the field [m]

\[
\theta \approx \frac{l}{R} = \frac{0.3 B l}{p \left[ \frac{\text{GeV}}{c} \right]}
\]
**Additional material – Equivalence of electrostatic and magnetic deflection**

Comparing capabilities of electric and magnetic field with same volumetric energy density (in vacuum) to deflect relativistic charged particles

Where:
- $U_e$ - electric field energy density [$J/m^3$]
- $U_m$ - magnetic field energy density [$J/m^3$]
- $\varepsilon_0$ - vacuum permittivity [$F/m$]
- $\mu_0$ - vacuum permeability [$H/m$]
- $q_e$ - particle charge (elementary charge) [$C$]
- $E$ - electric field [$V/m$]
- $B$ - magnetic flux density [$T$]
- $c$ - speed of light [$m/s$]
- $F_e$ - electric force [$N$]
- $F_m$ - magnetic force [$N$]

Electric field energy density $U_e = \frac{\varepsilon_0 E^2}{2}$

Magnetic field energy density $U_m = \frac{\mu_0 B^2}{2}$

For $U_e = U_m$ and $c^2 = \frac{1}{\varepsilon_0 \mu_0}$, $\varepsilon_0 \frac{E^2}{2} = \frac{B^2}{2\mu_0}$ or $E = cB$

Magnetic (Lorentz) deflection force $F_m = q_e v B$

Relativistic regime $v \equiv c$

$F_m = q_e v B \equiv q_e c B = q_e E$

Electrostatic (Lorentz) deflection force $F_e = q_e E$

or $F_e \equiv F_m$

Electric and magnetic field, with same energy density, have same effectiveness in deflecting relativistic charged particles.