

Injection: Hadron Beams

C. Bracco

Acknowledgements: T. Argyroupoulos, W. Bartmann,
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A. Huschauer, V. Kain and F. Maria Velotti

Outline

- Introduction
- Single-turn injection
 - Principles and HW
 - Injection errors: mismatch, injection oscillations and emittance blowup
 - Aperture: protection devices and injection losses
 - Slip-stacking
- Multi-turn injection:
 - Phase space painting
 - Charge exchange H⁻ injection
 - Combined longitudinal and transverse injection
- Lessons learnt

Introduction

What do we mean by injection?

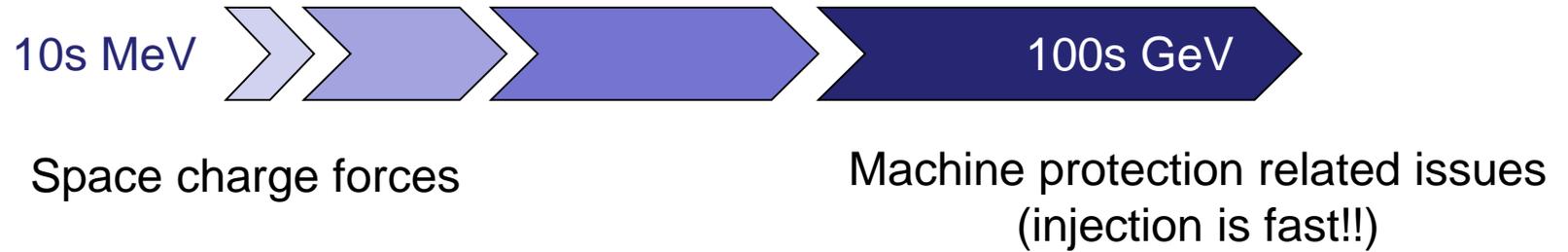
Place a particle beam into a circular accelerator or accumulator at the right time while:

1. Placing the newly injected particles onto the correct trajectory with the correct phase-space parameters
2. Minimizing the beam losses

Injection is one of the most complex parts of a ring and if not properly designed can bring to:

- Machine damage
- Compromised beam quality → reduced performance

Challenges

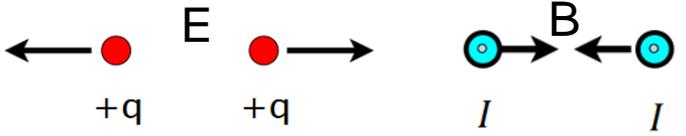


Challenges



Space charge forces

“Self-forces” produced by the beam itself



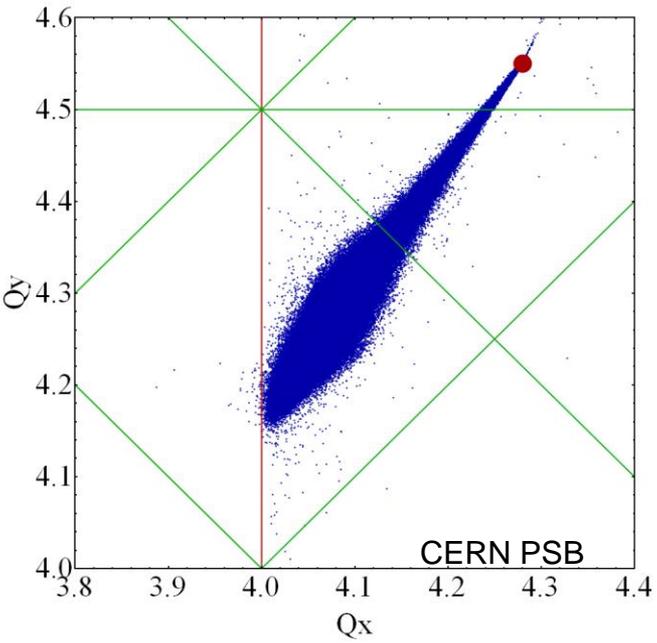
Particles in an unbunched beam, with uniform density and circular cross-section will experience an incoherent tune shift:

$$\Delta Q_{x,y} \propto \frac{N}{\gamma^2 \beta \epsilon_N} \quad [1]$$

This can lead to emittance blow up and losses. Effect is stronger at low energy and for high charge densities → limit achievable brightness!

N= number of particles per unit length
 γ and β = relativistic factors
 ϵ_N = normalised emittance

Machine protection related issues (injection is fast!!)

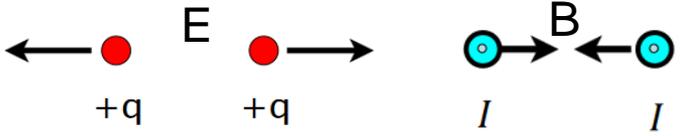


Challenges



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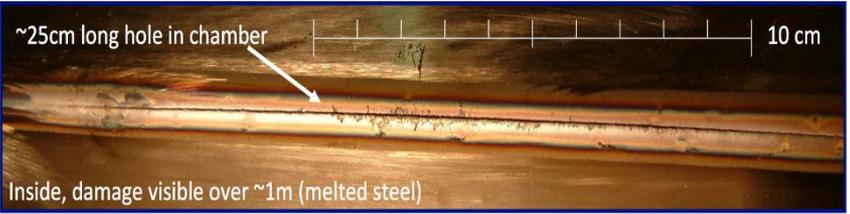
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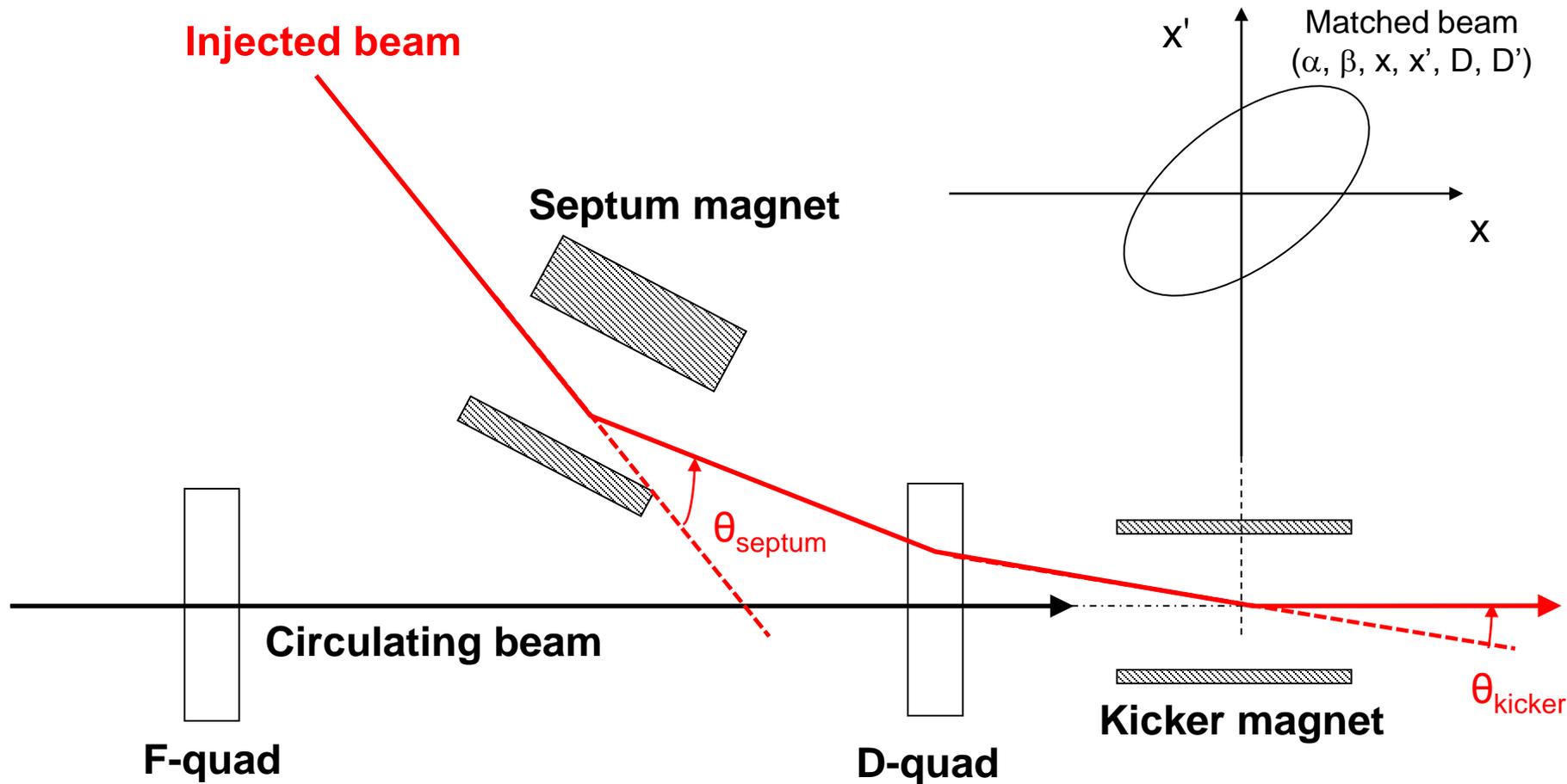
Machine protection related issues (injection is fast!!)

High stored energy: up to 2.4 MJ* for each LHC injection (x2 for HL-LHC)
 → ~ 0.5 kg of TNT!



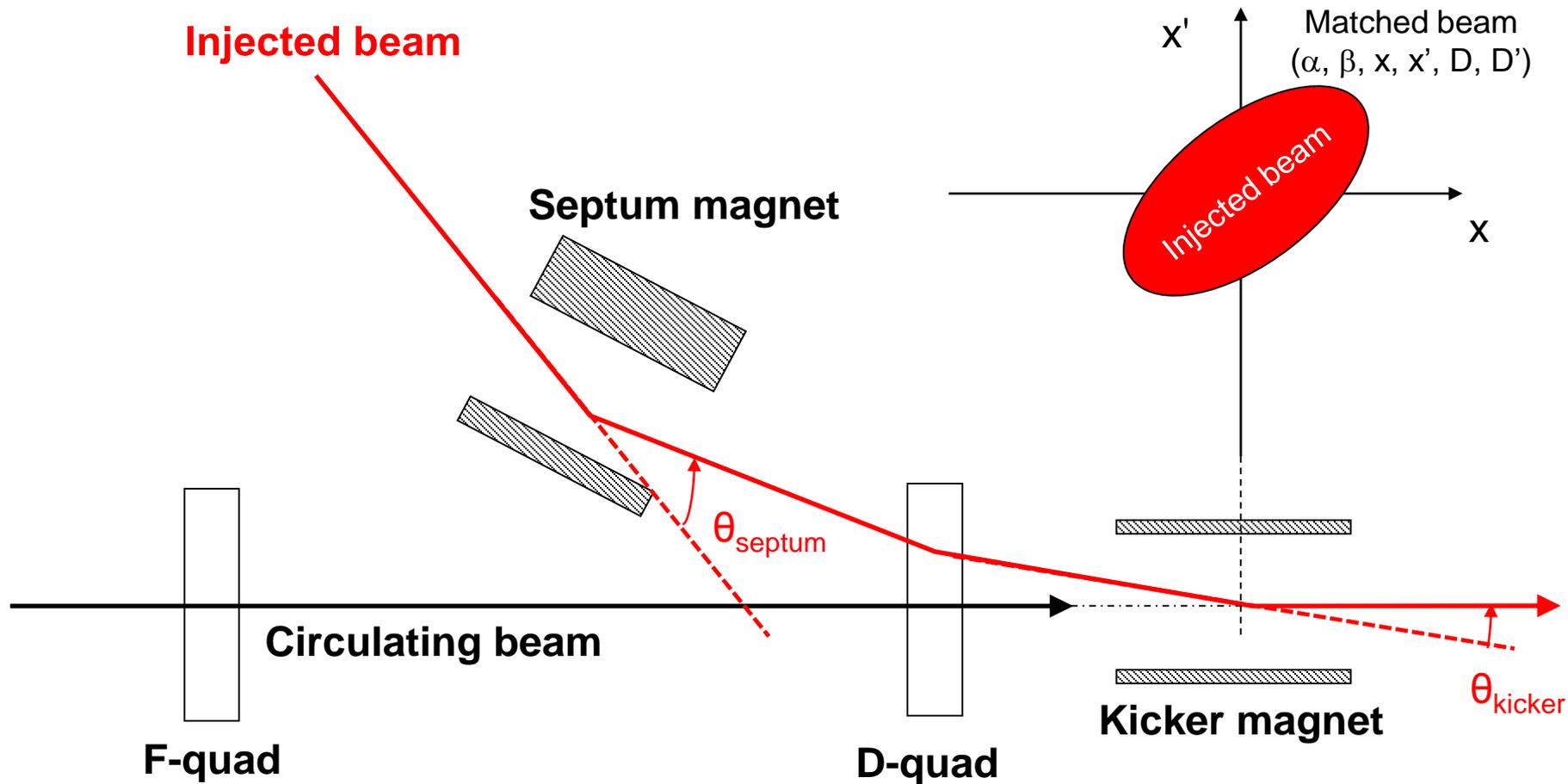
*(288 bunches) x (1.15x10¹¹ ppb) x (450 GeV)

Single-turn injection – same plane



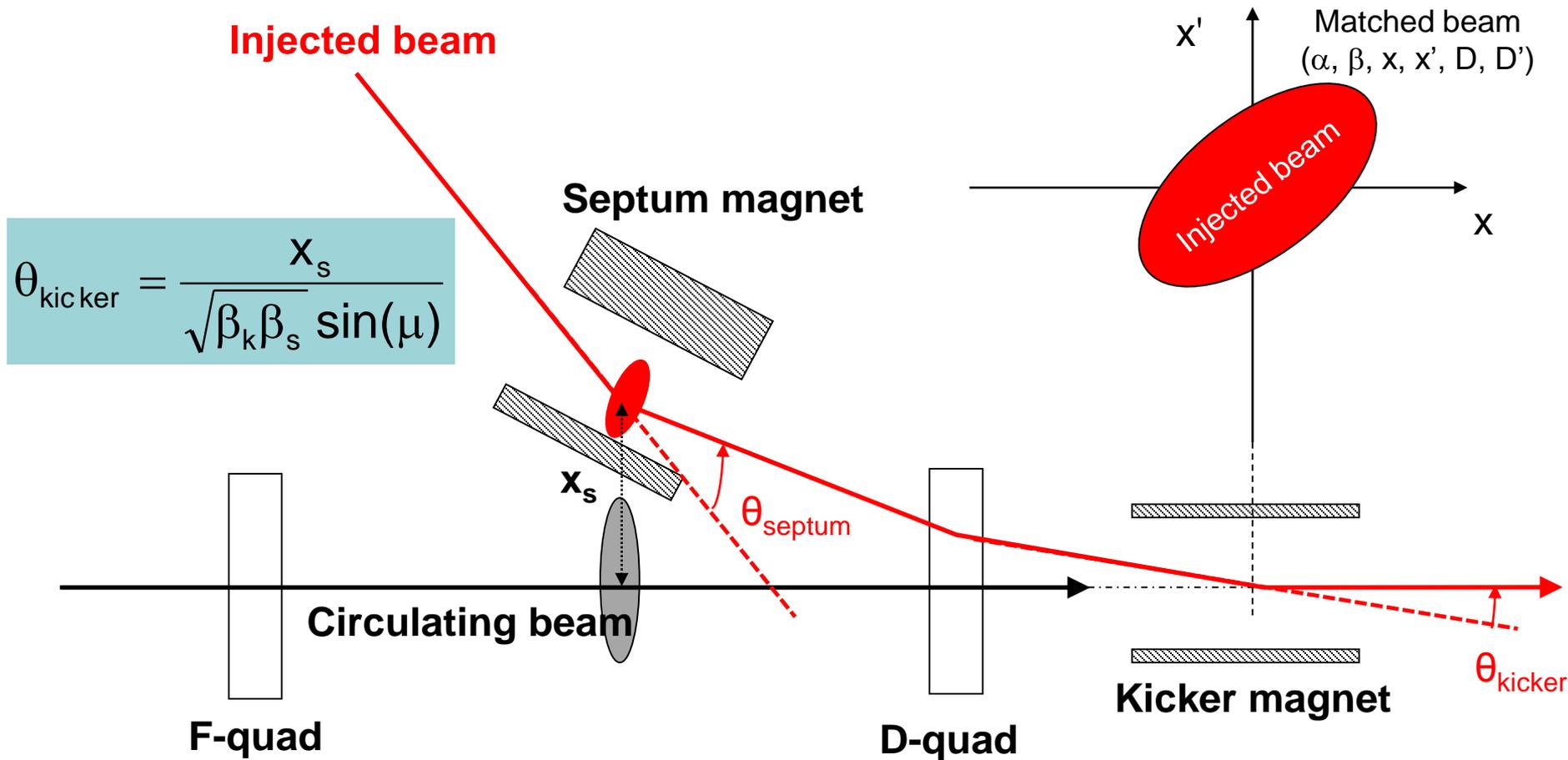
- Septum deflects the beam onto the closed orbit at the centre of the kicker
- Kicker compensates for the remaining angle
- Septum and kicker either side of D quad to minimise kicker strength

Single-turn injection – same plane



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Single-turn injection – same plane



$$\theta_{\text{kicker}} = \frac{x_s}{\sqrt{\beta_k \beta_s} \sin(\mu)}$$

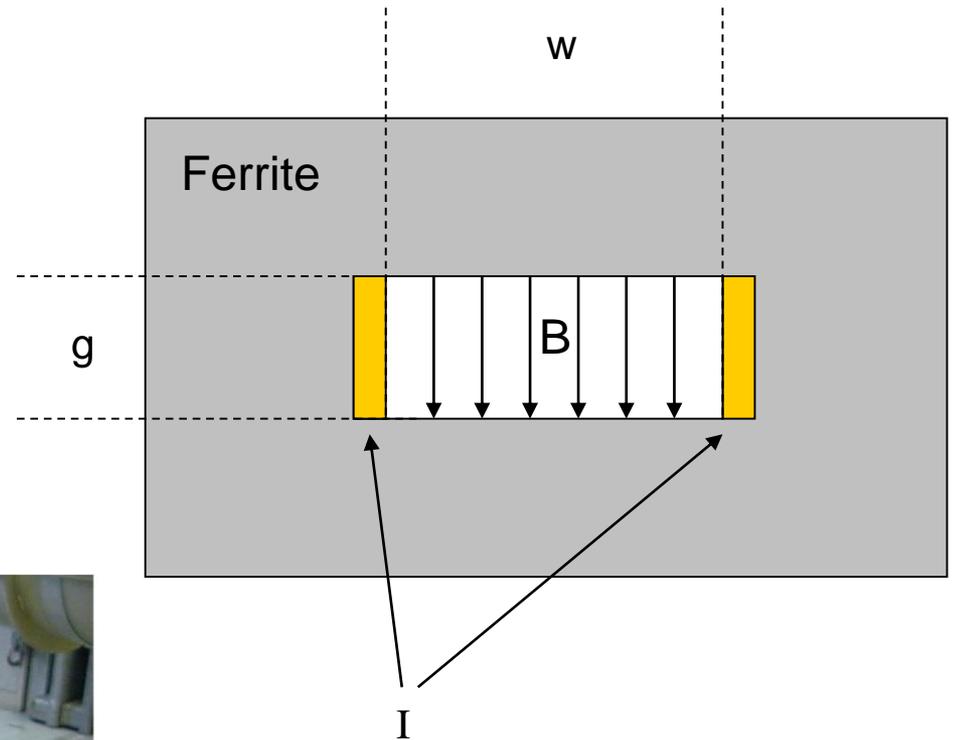
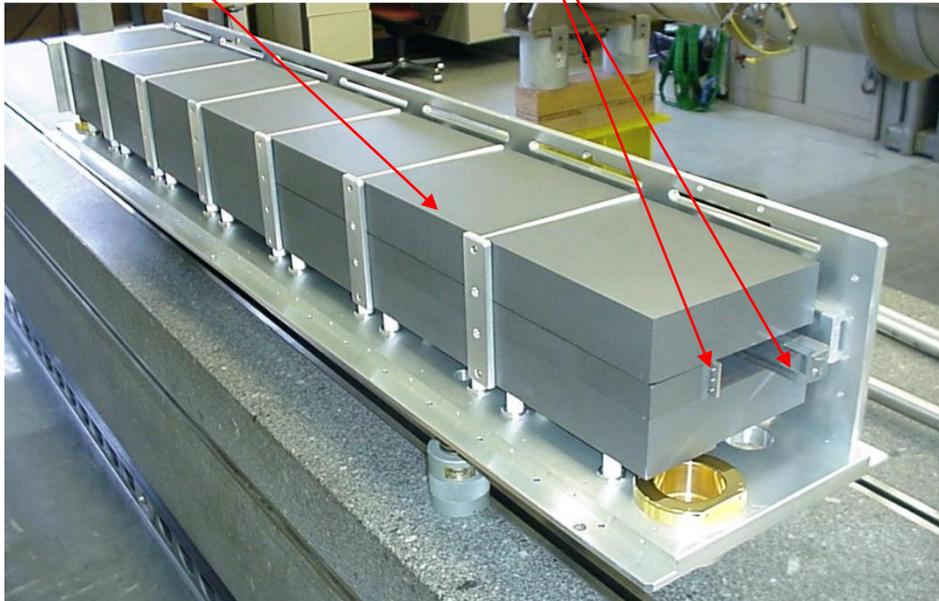
The position of the injected beam at the septum x_s has to be larger than:
injected beam envelope + circulating beam envelope + thickness of the septum blade + beam size increase due to energy spread + closed orbit distortion + alignment errors

Kicker magnet

Pulsed magnet with **very fast rise time**
(100 ns – few μ s)

Ferrite

Conductors



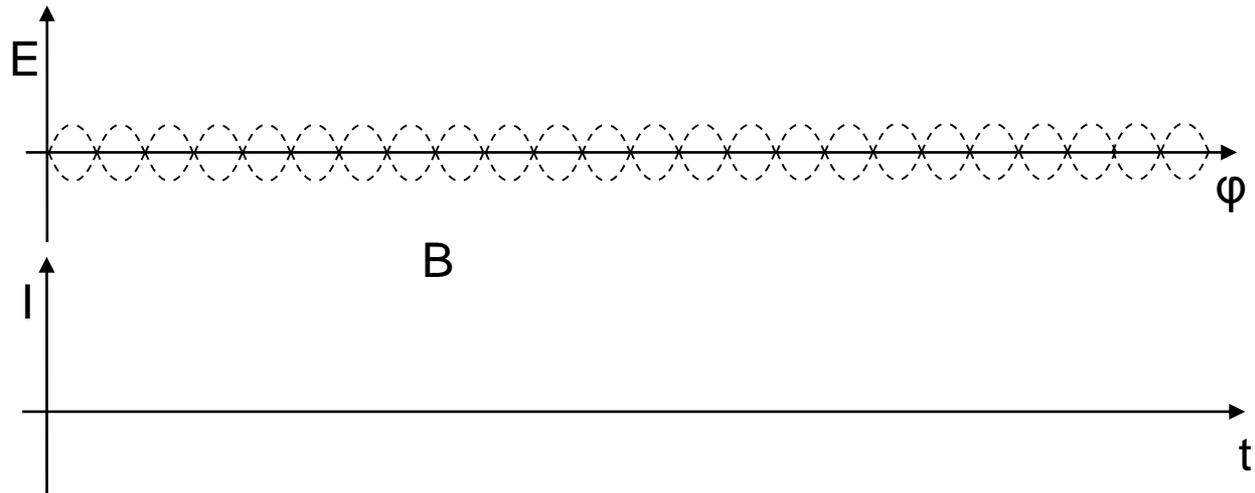
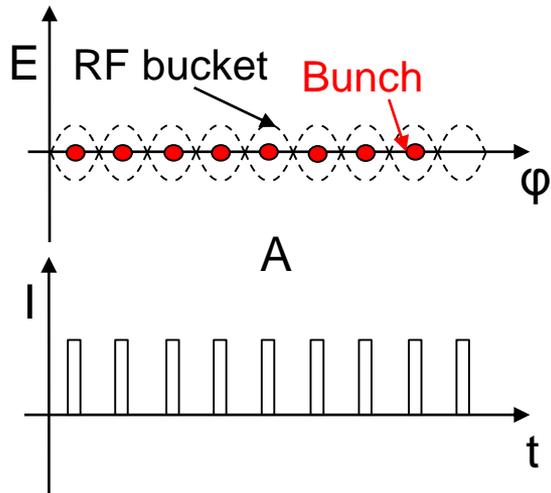
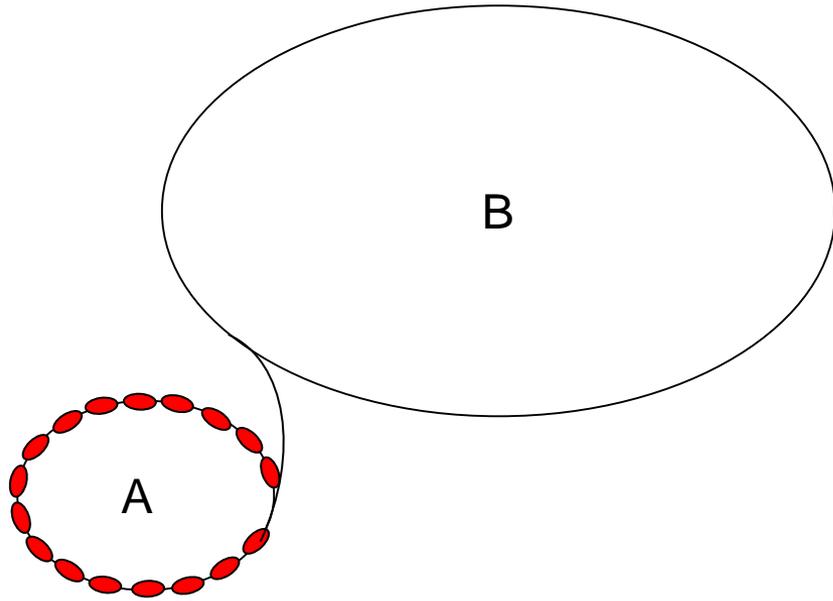
$$B = \mu_0 I / g$$

$$L \text{ [per unit length]} = \mu_0 w / g$$

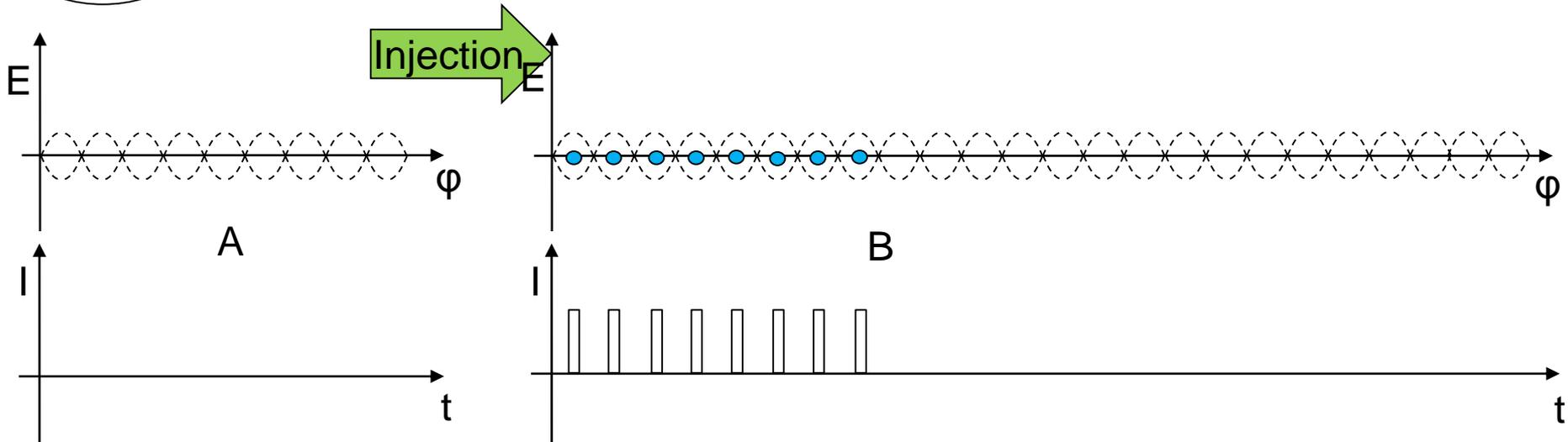
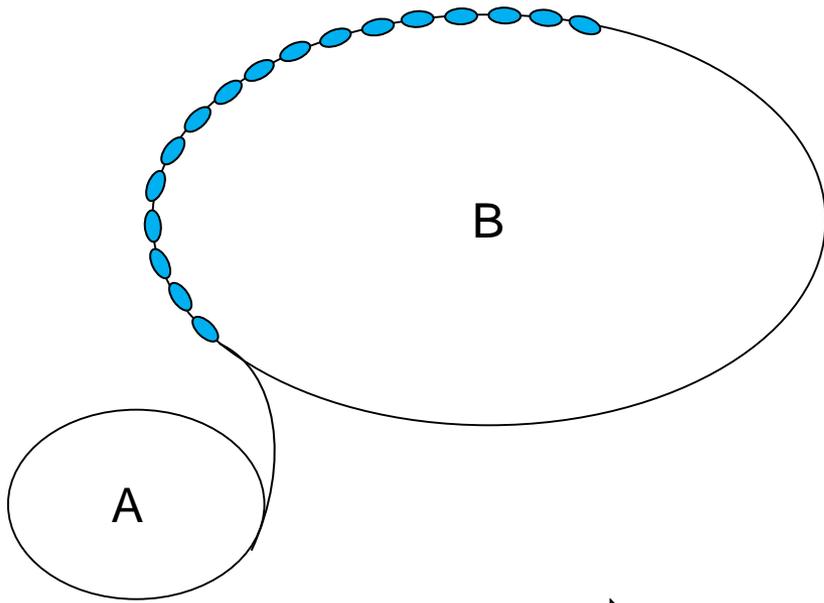
$$dI/dt = V / L$$

Typically 3 kA in 1 μ s rise time

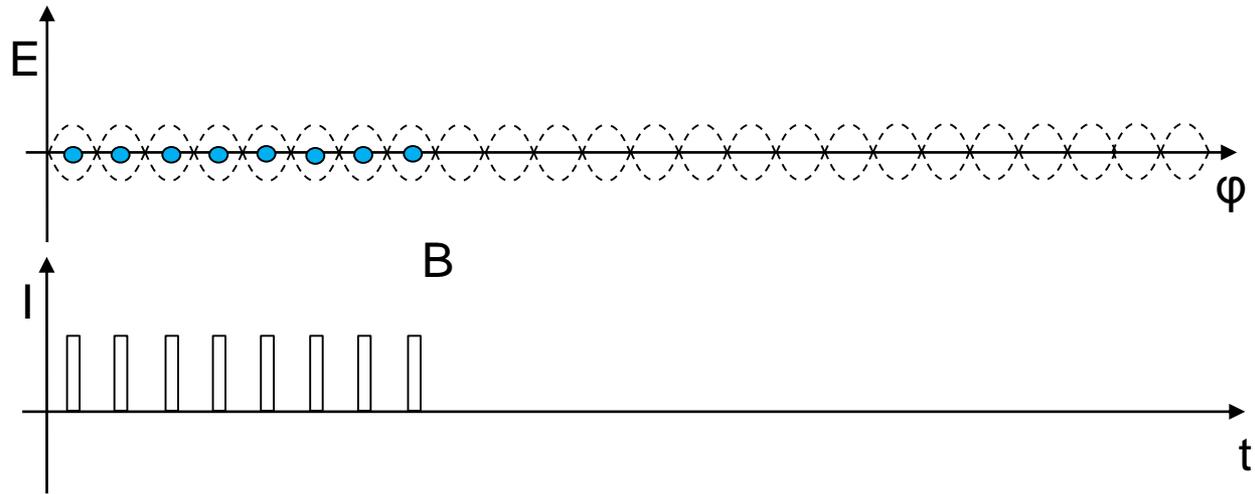
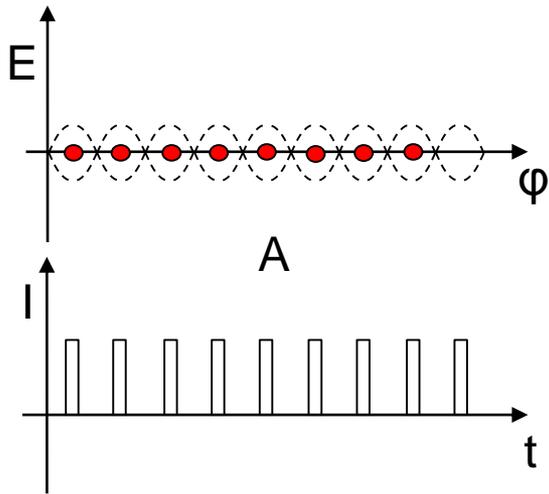
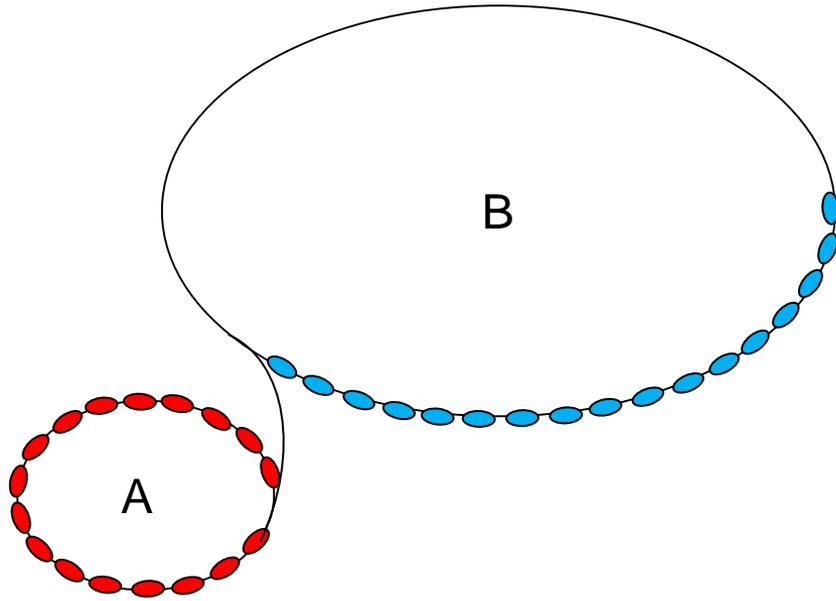
Filling factor



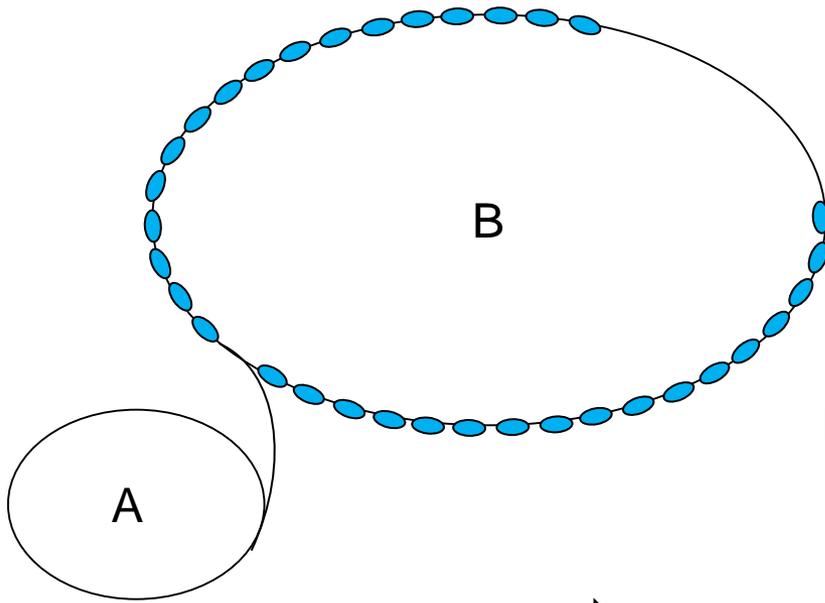
Filling factor



Filling factor



Filling factor

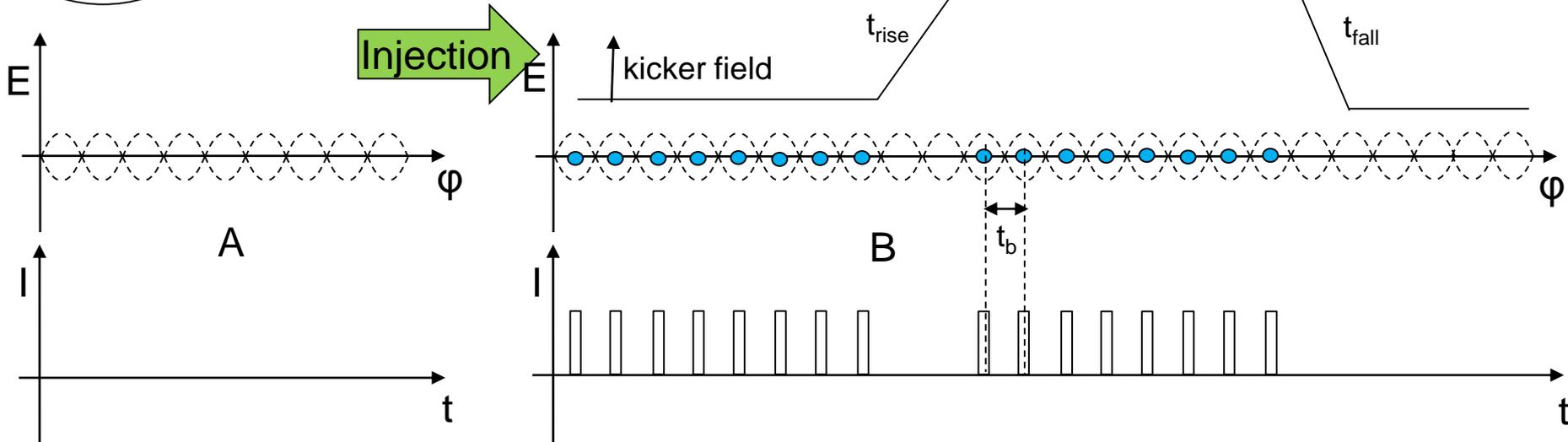


Ideal number of storable bunches: $n_b = \frac{L_B}{t_b}$

due to t_{rise} and t_{fall} the number of bunches which can be stored is:

$$\bar{n}_b = \frac{nt_{flattop} - [(n-1)t_{rise} + t_{fall}^*]}{t_b} \quad \text{where} \quad n \approx \frac{L_B}{L_A}$$

Filling factor [%]: $f = \frac{\bar{n}_b}{n_b}$ The lower t_{rise} the higher f

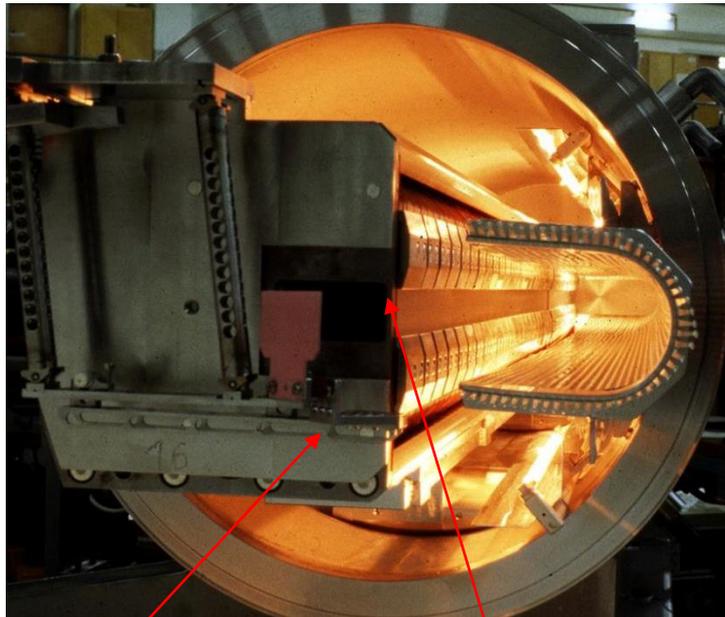
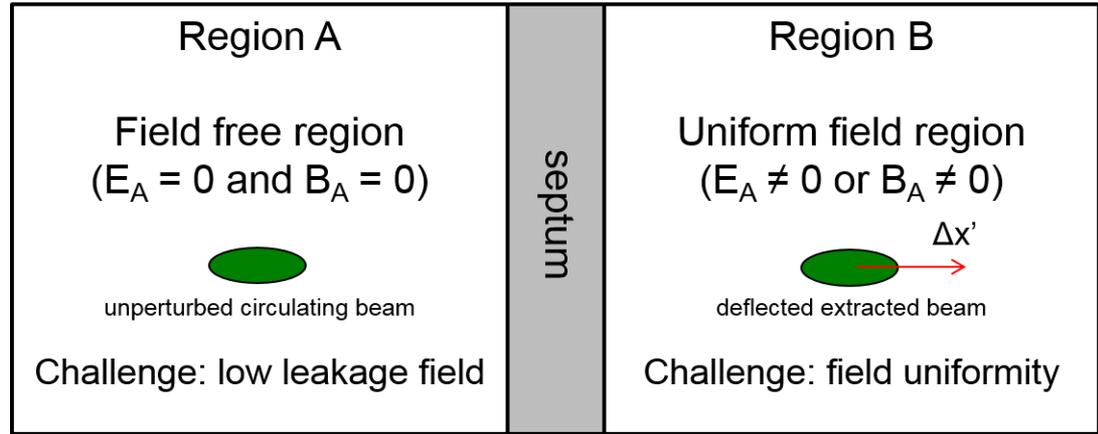


**In reality t_{fall} has to be replaced with $t_{abort} > t_{fall}$ which is the pulse time of the extraction kickers

Magnetic septum

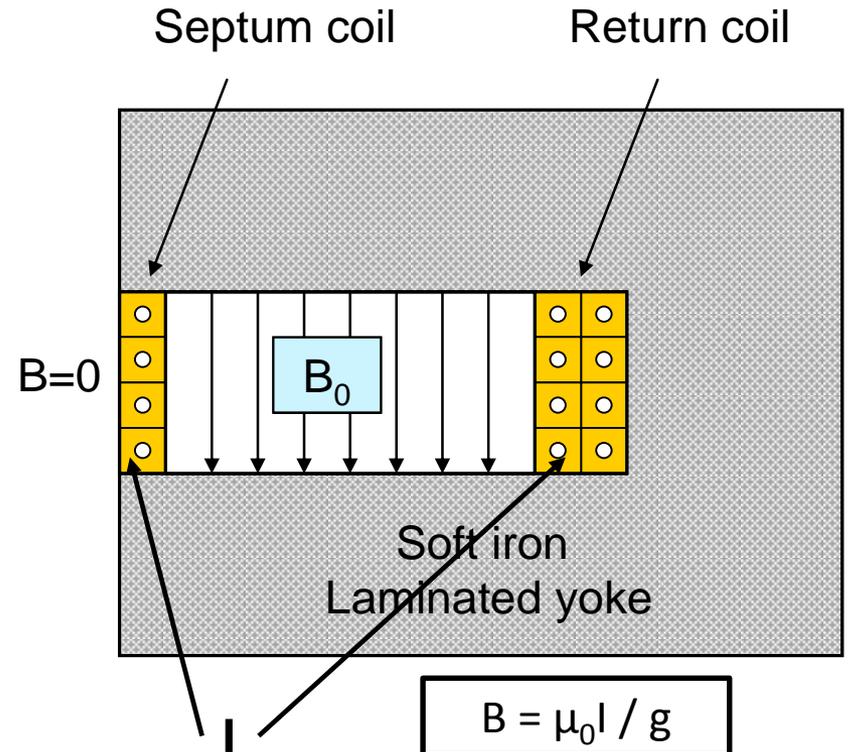
Pulsed or DC magnets with thin (2 -20 mm) septum between zero and high field region.
Typically ~x10 higher deflection compared to kickers. $I \sim 5-25$ kA

M. Paraliev's Lecture



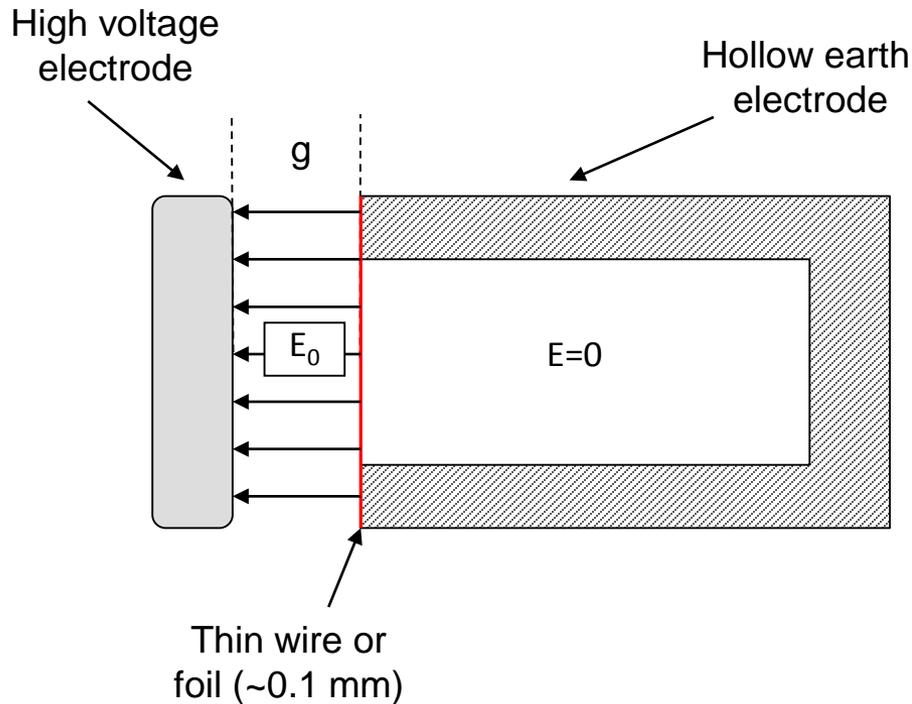
Yoke

Septum



Electrostatic septum

DC electrostatic device with very thin septum between zero field and high field region



$$E = V / g$$

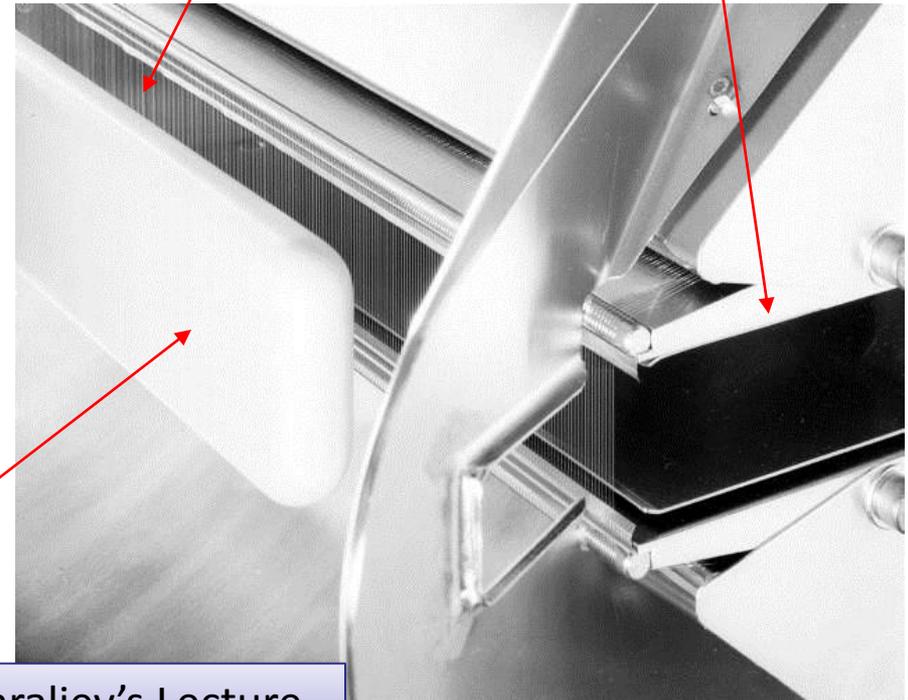
Typically $V = 200$ kV

$$E = 100$$
 kV/cm

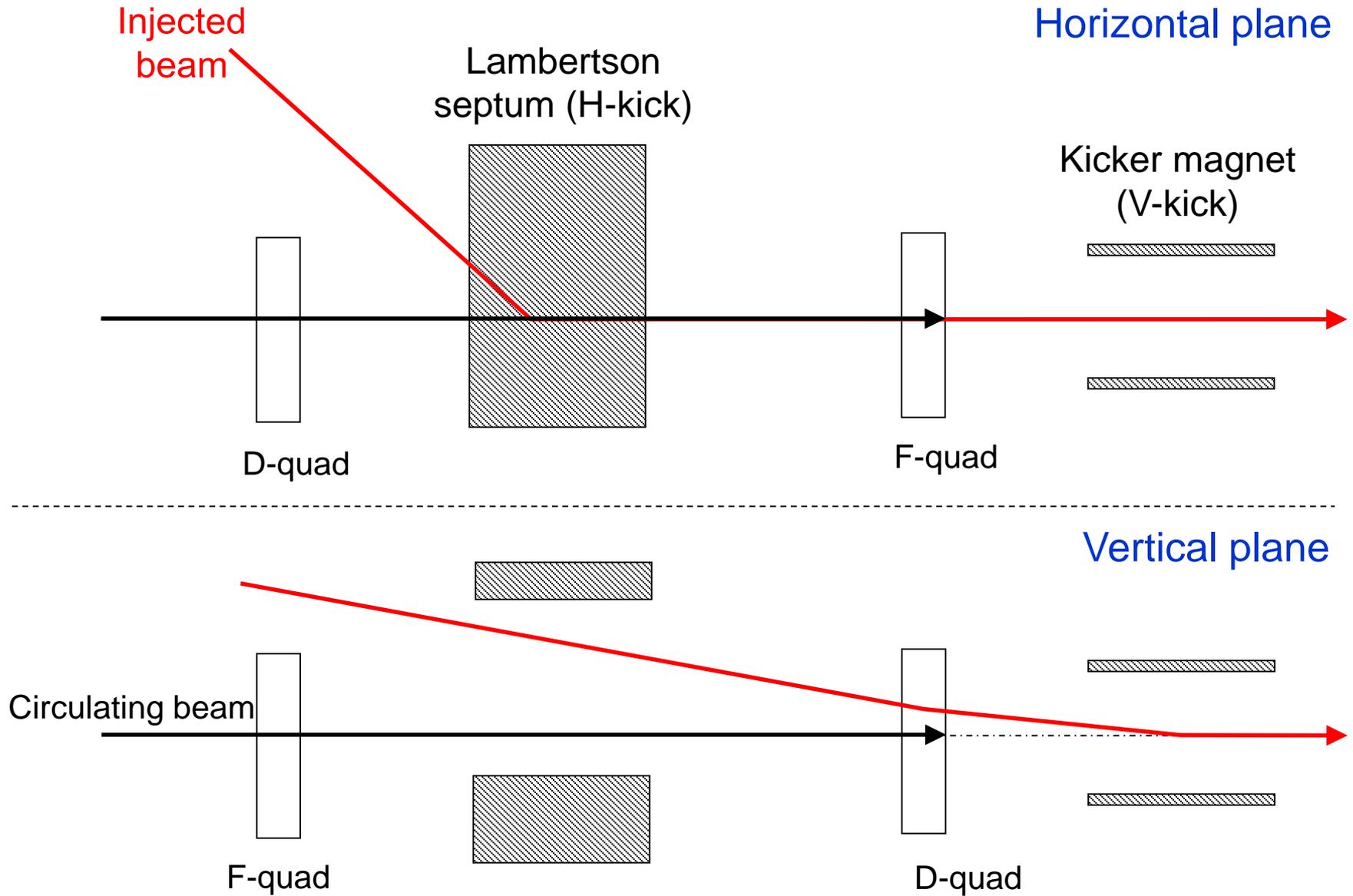
Septum wires (in SPS alignment of the 60 - 100 μ m over 15 m is challenging!

Hollow earth electrode

High voltage electrode

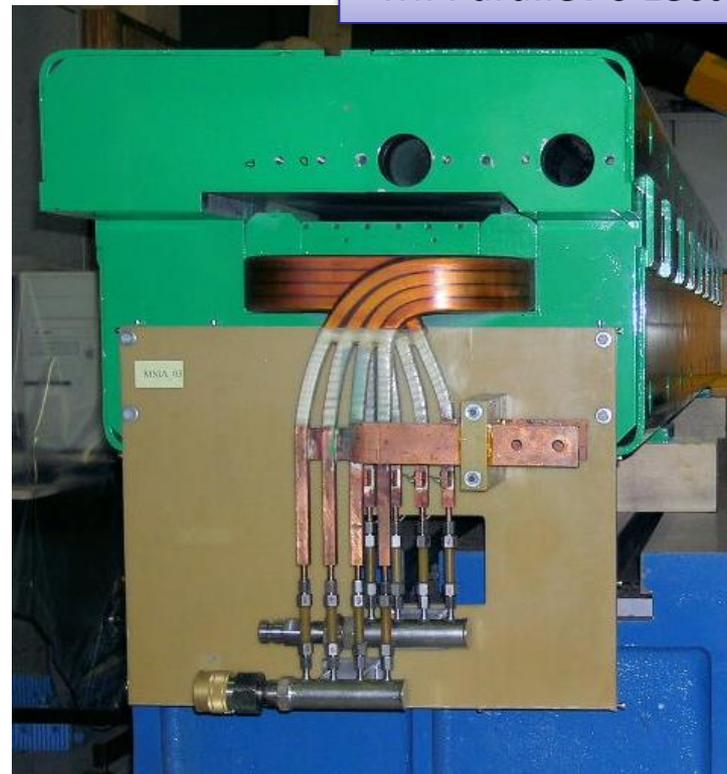
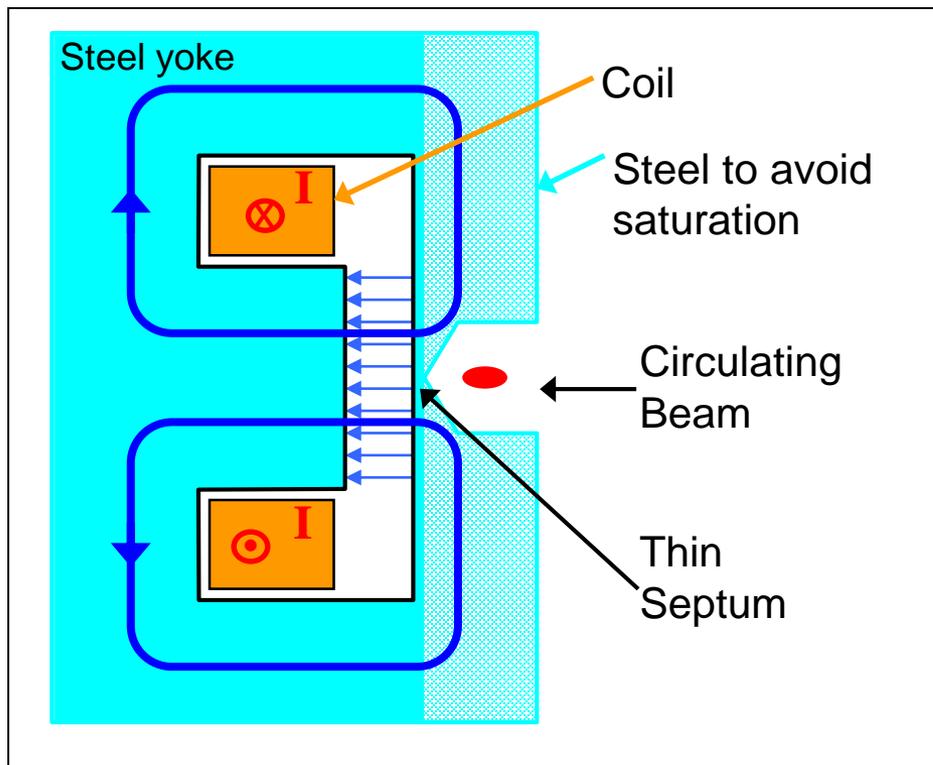


Two planes injection



Lambertson septum

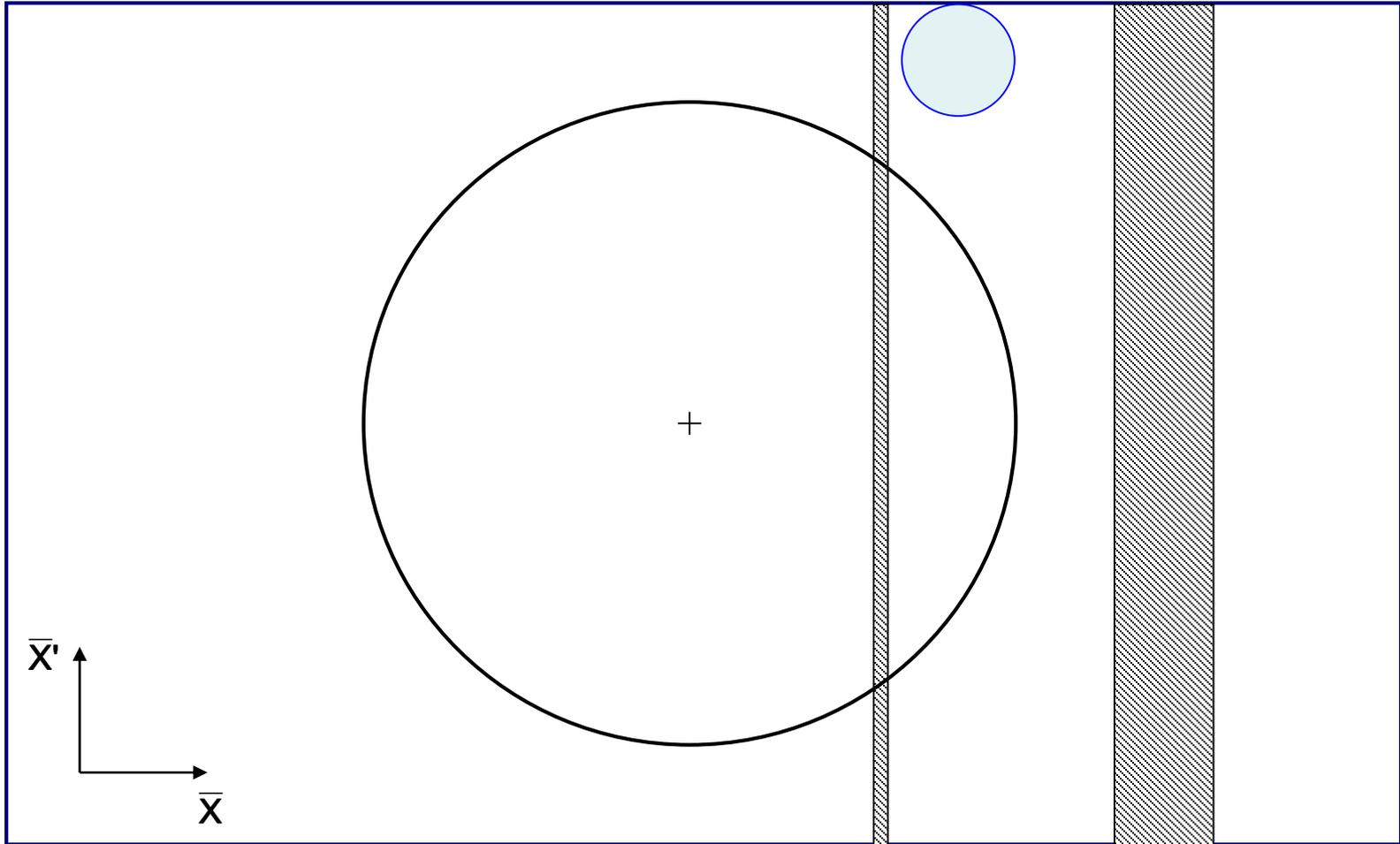
M. Paraliiev's Lecture



- Magnetic field in gap orthogonal to previous example of septa:
 - Lambertson deflects beam orthogonal to kicker: dual plane injection/extraction
- Rugged design: conductors safely hidden away from the beam
- Thin steel yoke between aperture and circulating beam – however extra steel required to avoid saturation, magnetic shielding often added

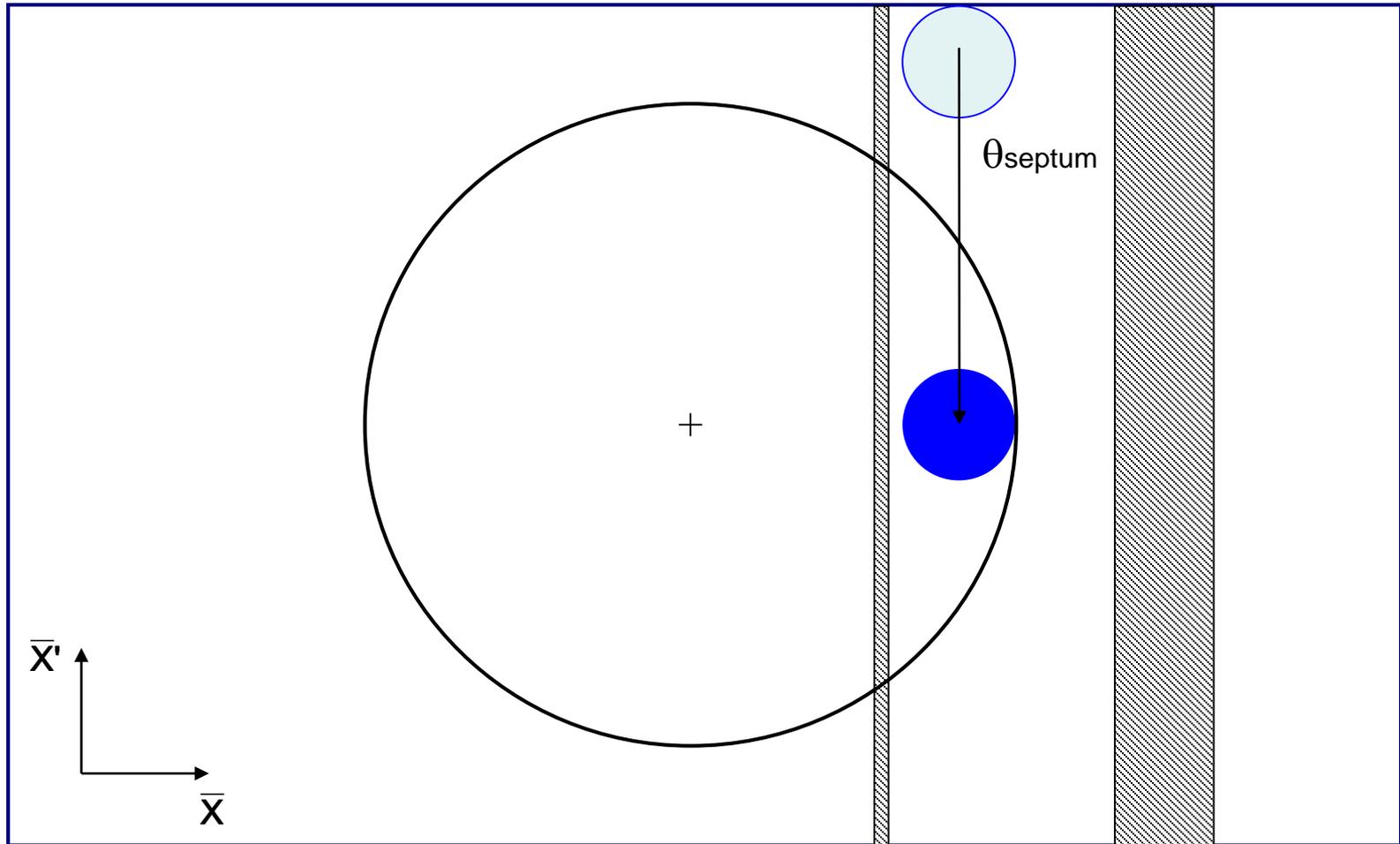
Single-turn injection

Normalised phase space at centre of idealised septum



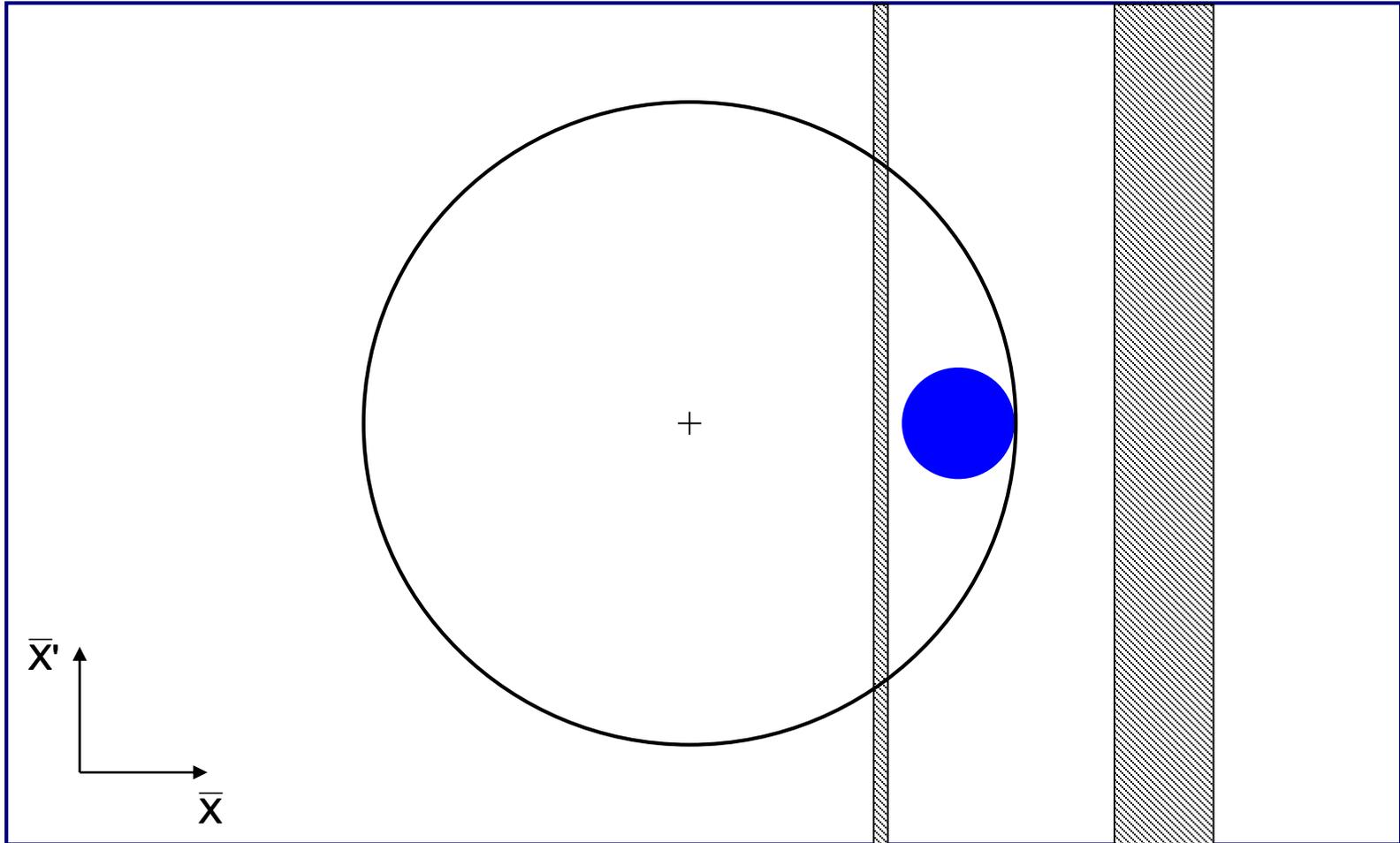
Single-turn injection

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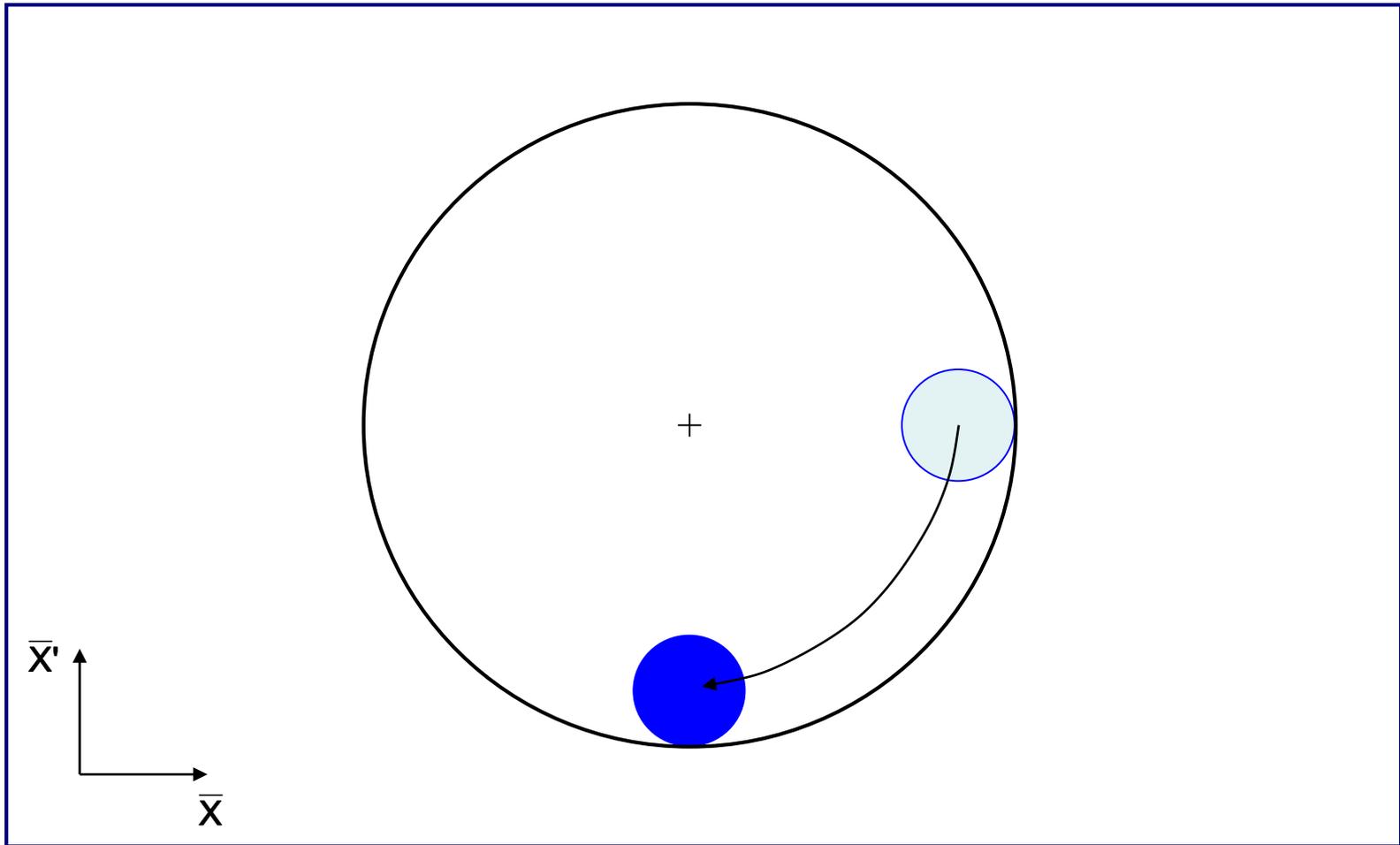
Single-turn injection

Normalised phase space at centre of idealised septum



Single-turn injection

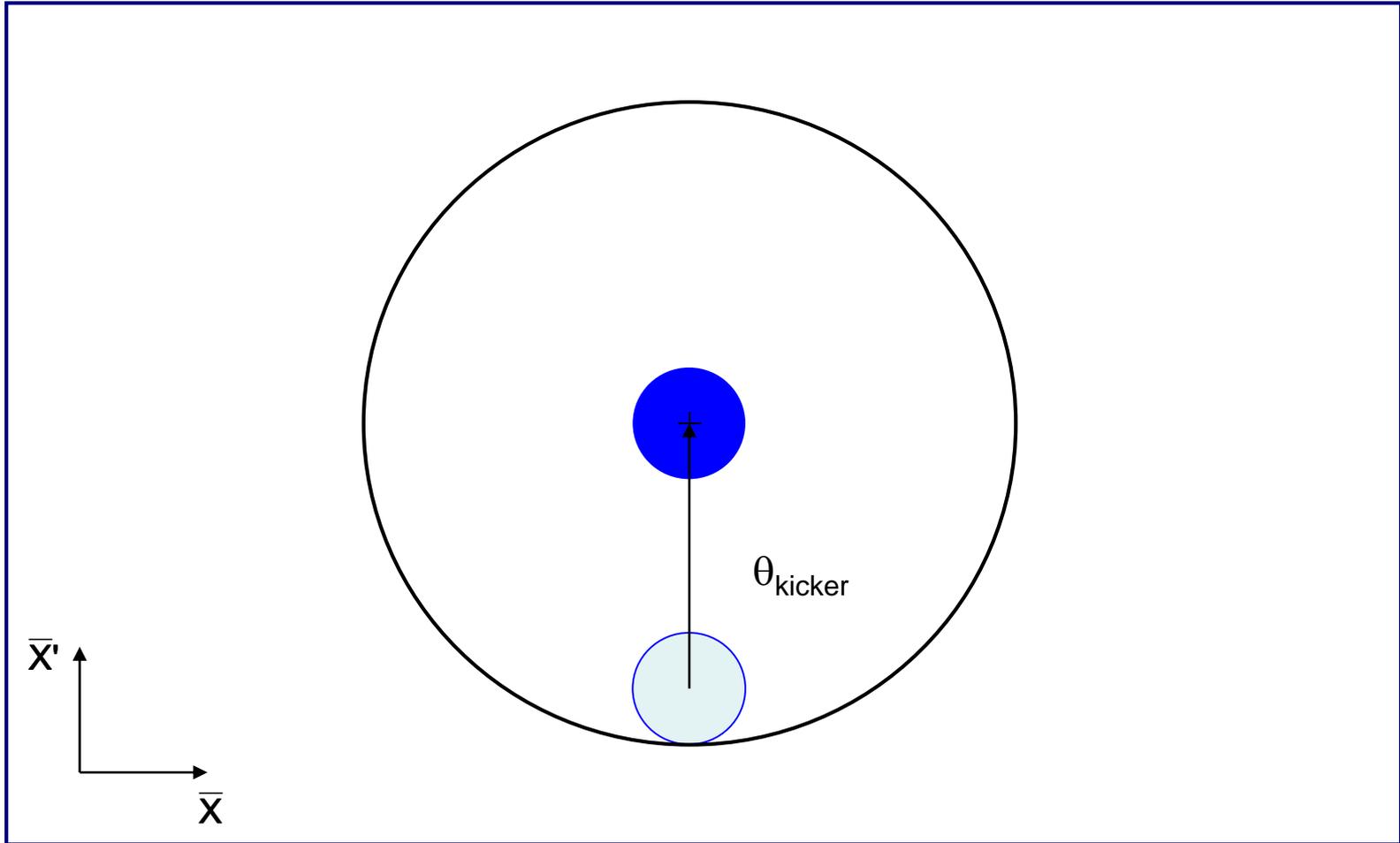
$\mu/2$ phase advance to kicker location



Single-turn injection

Normalised phase space at centre of idealised kicker

Kicker deflection places beam on central orbit:



Injection errors

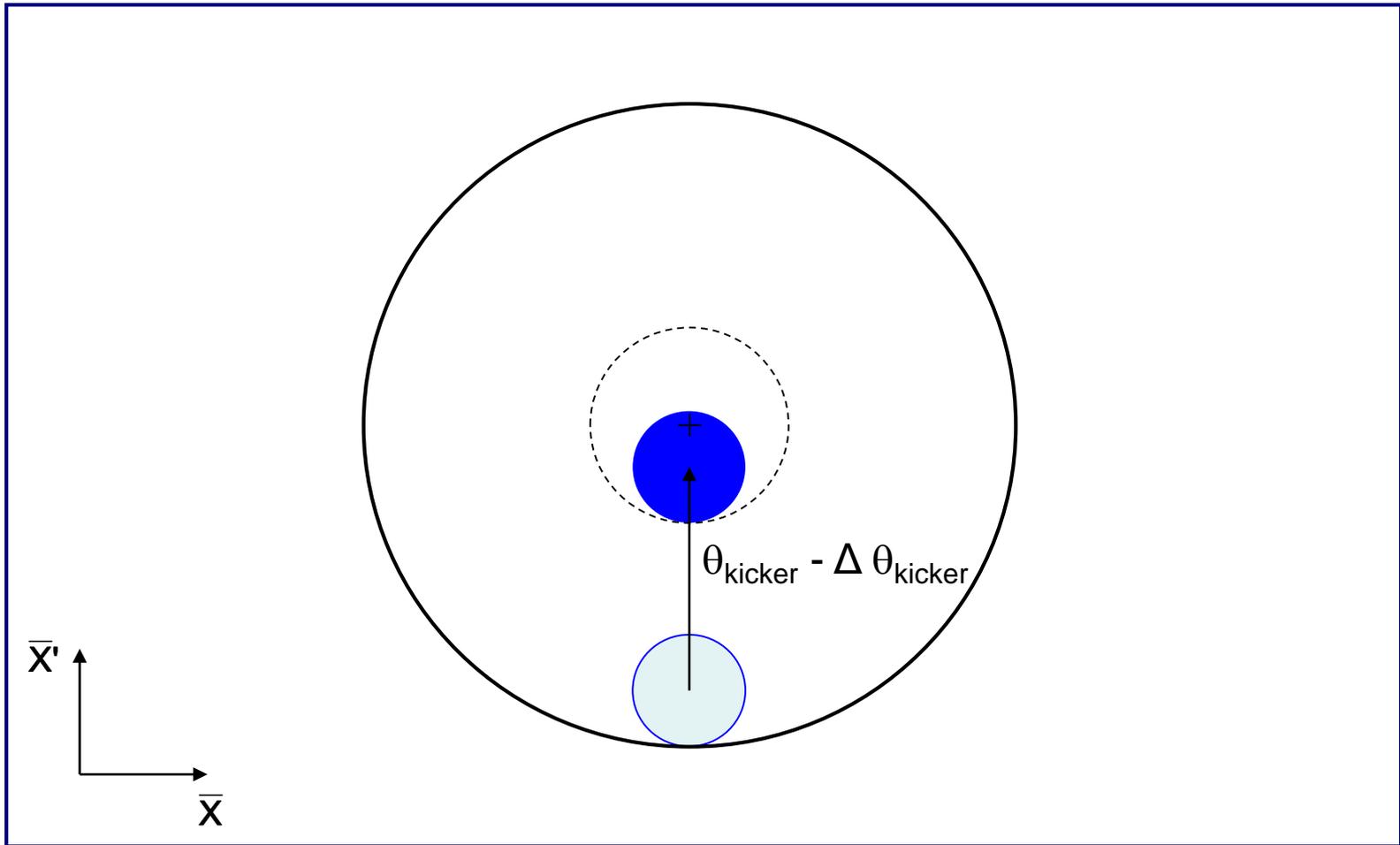
Transverse errors such as:

- Error in septum angle
- Error in ring kicker angle
- Steering error (including extraction kicker angle from previous accelerator)

Will lead to an emittance blow-up through filamentation due to the non-linear effects (e.g. higher-order field components) which introduce amplitude-dependent effects into particle motion.

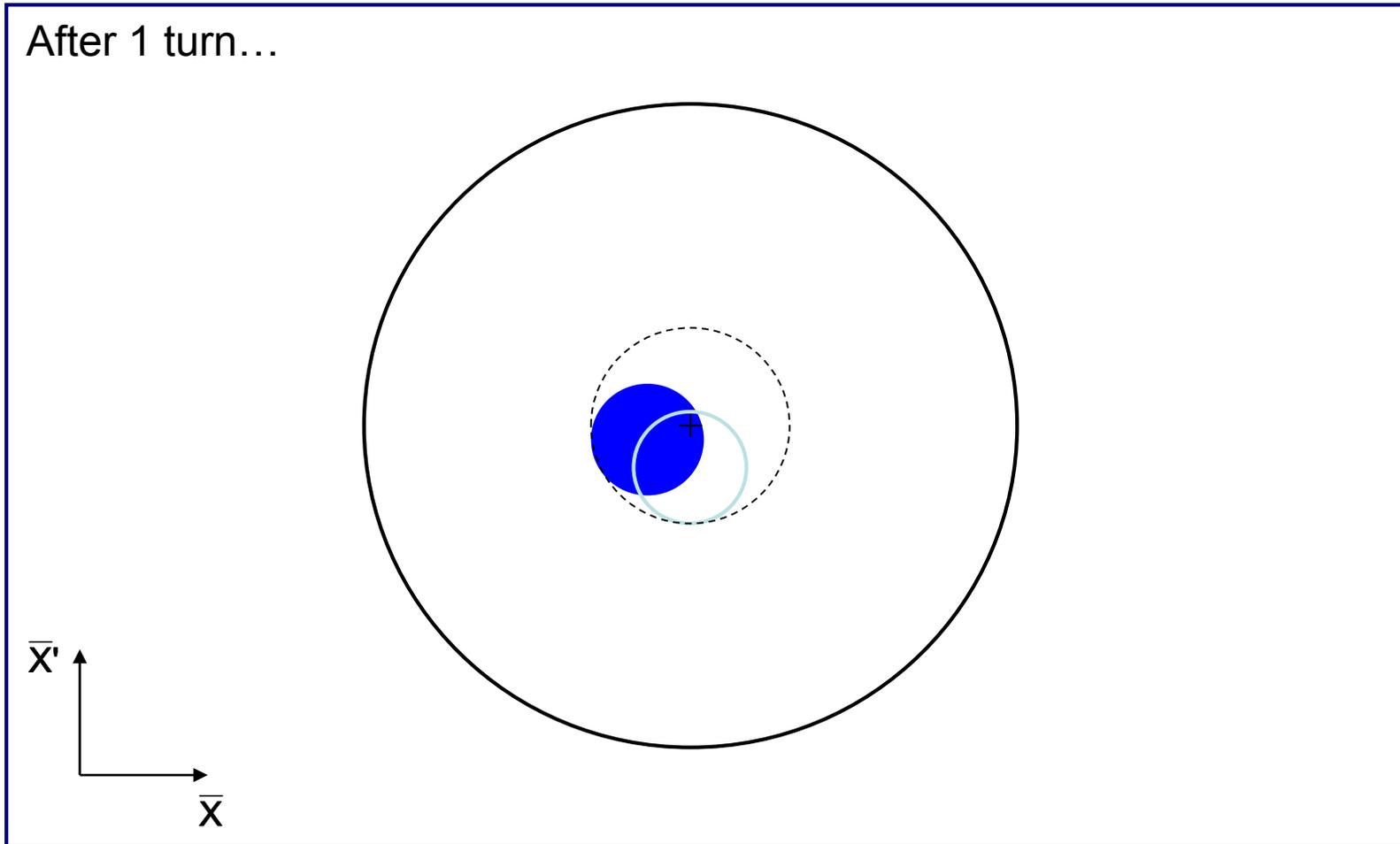
Injection oscillations

For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ :



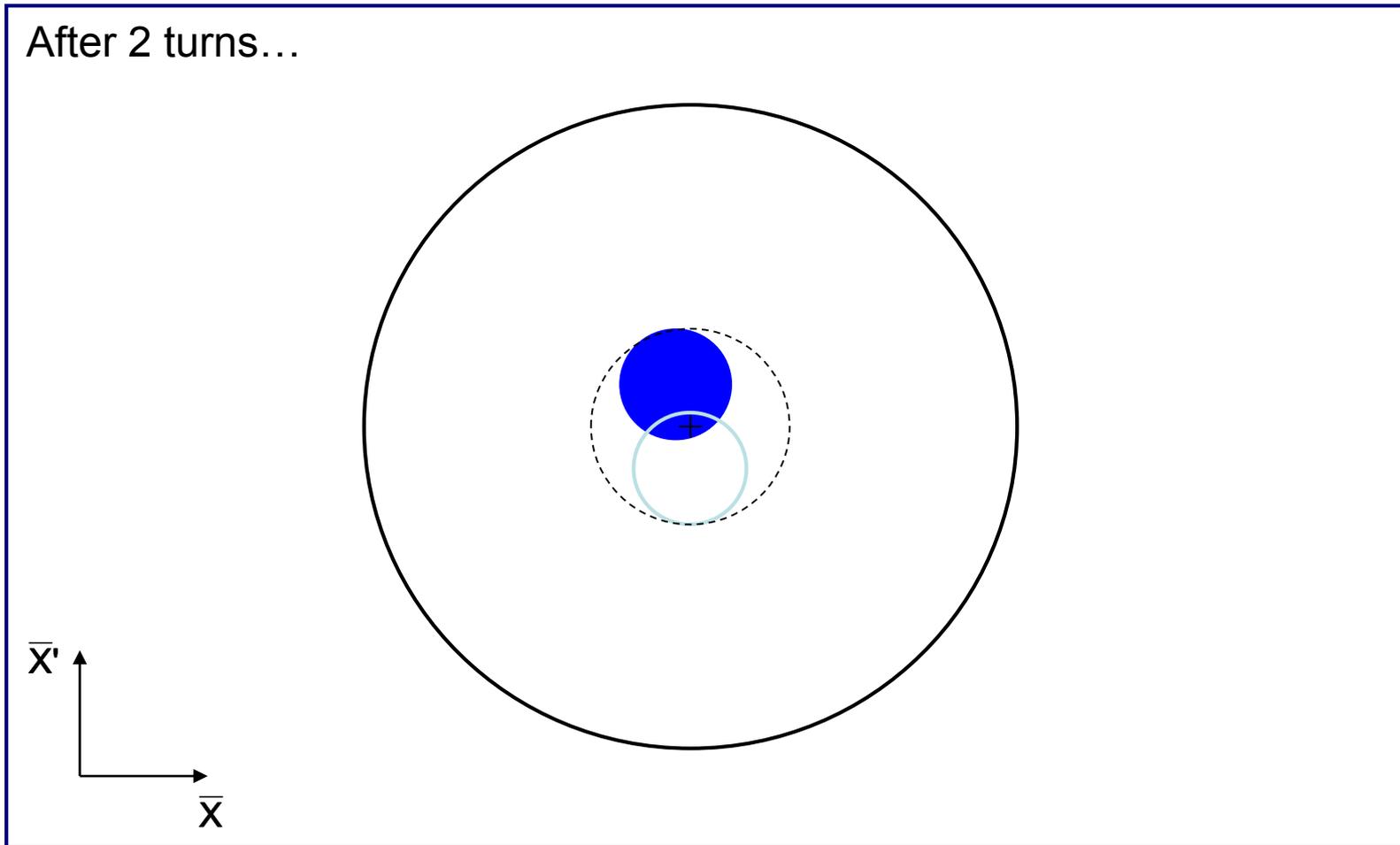
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Injection oscillations

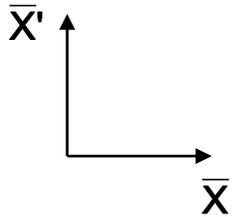
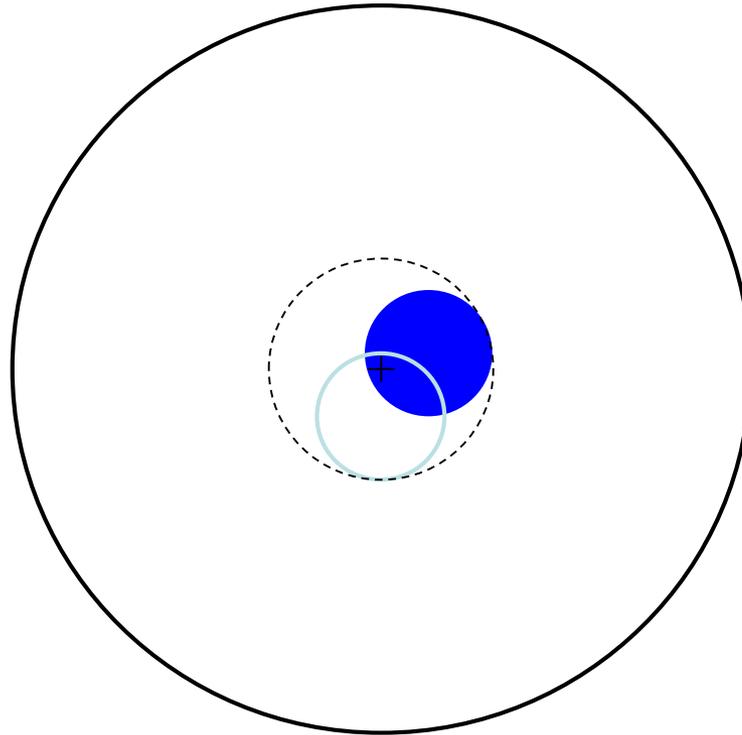
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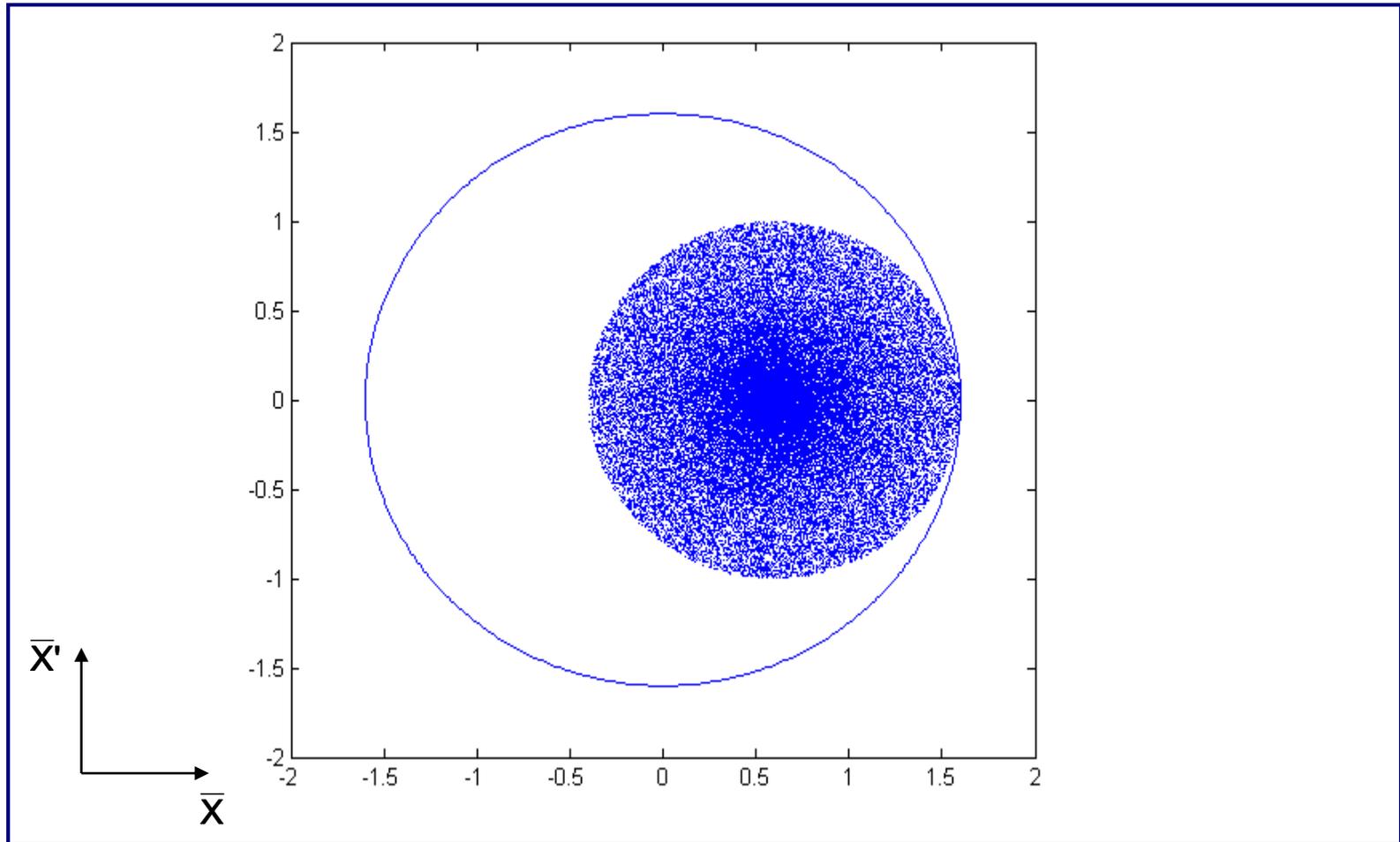
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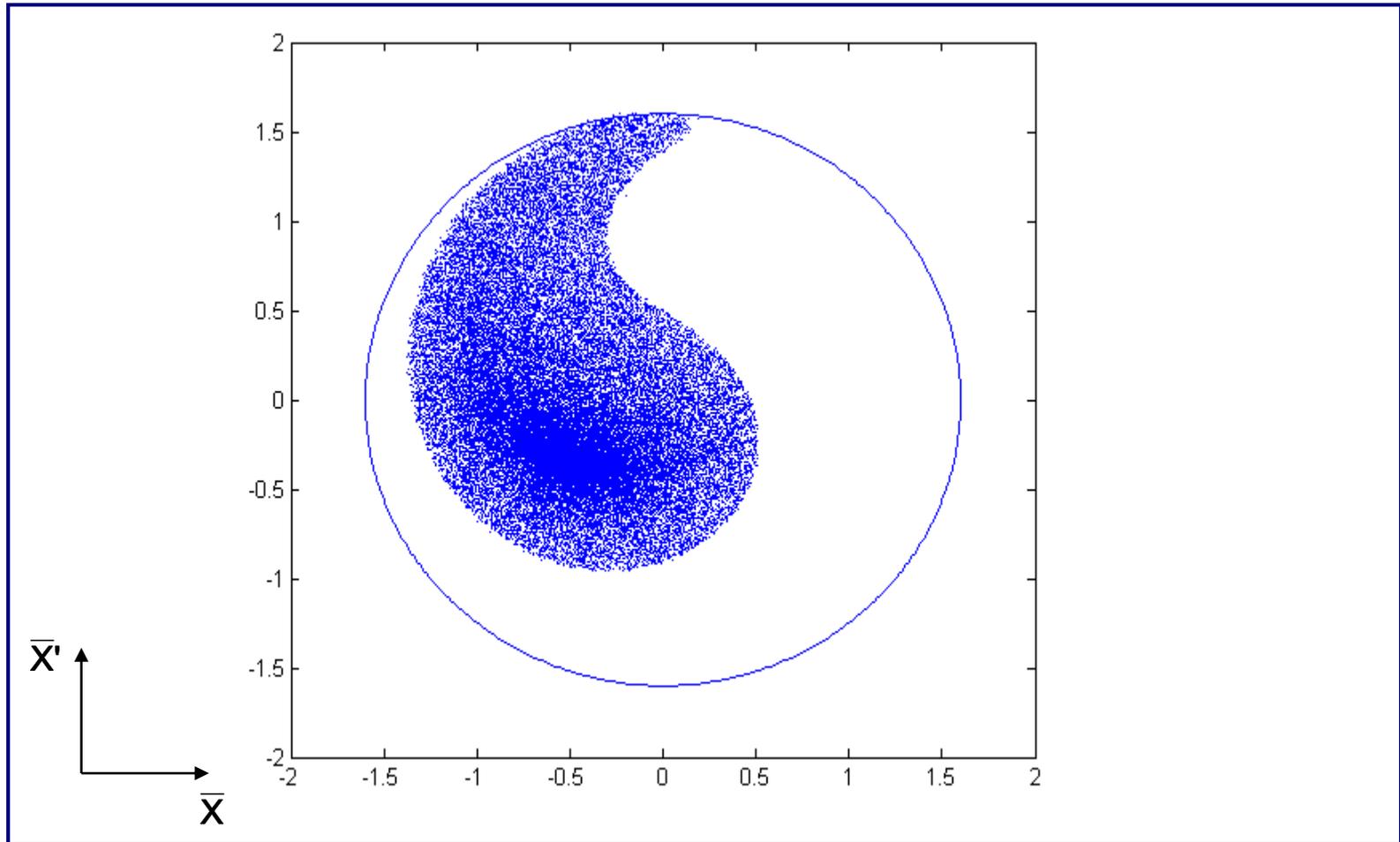
After 3 turns etc...



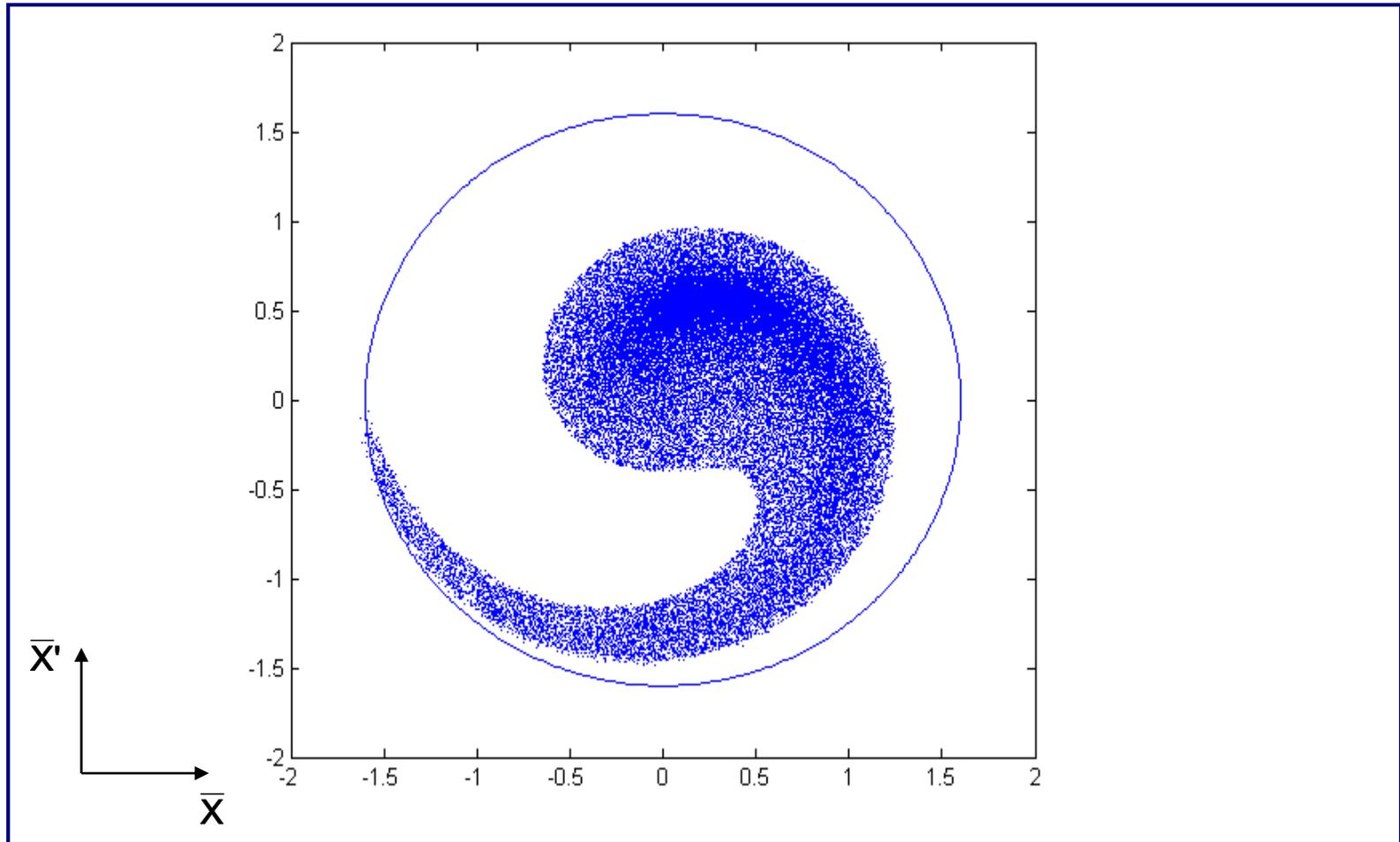
Filamentation



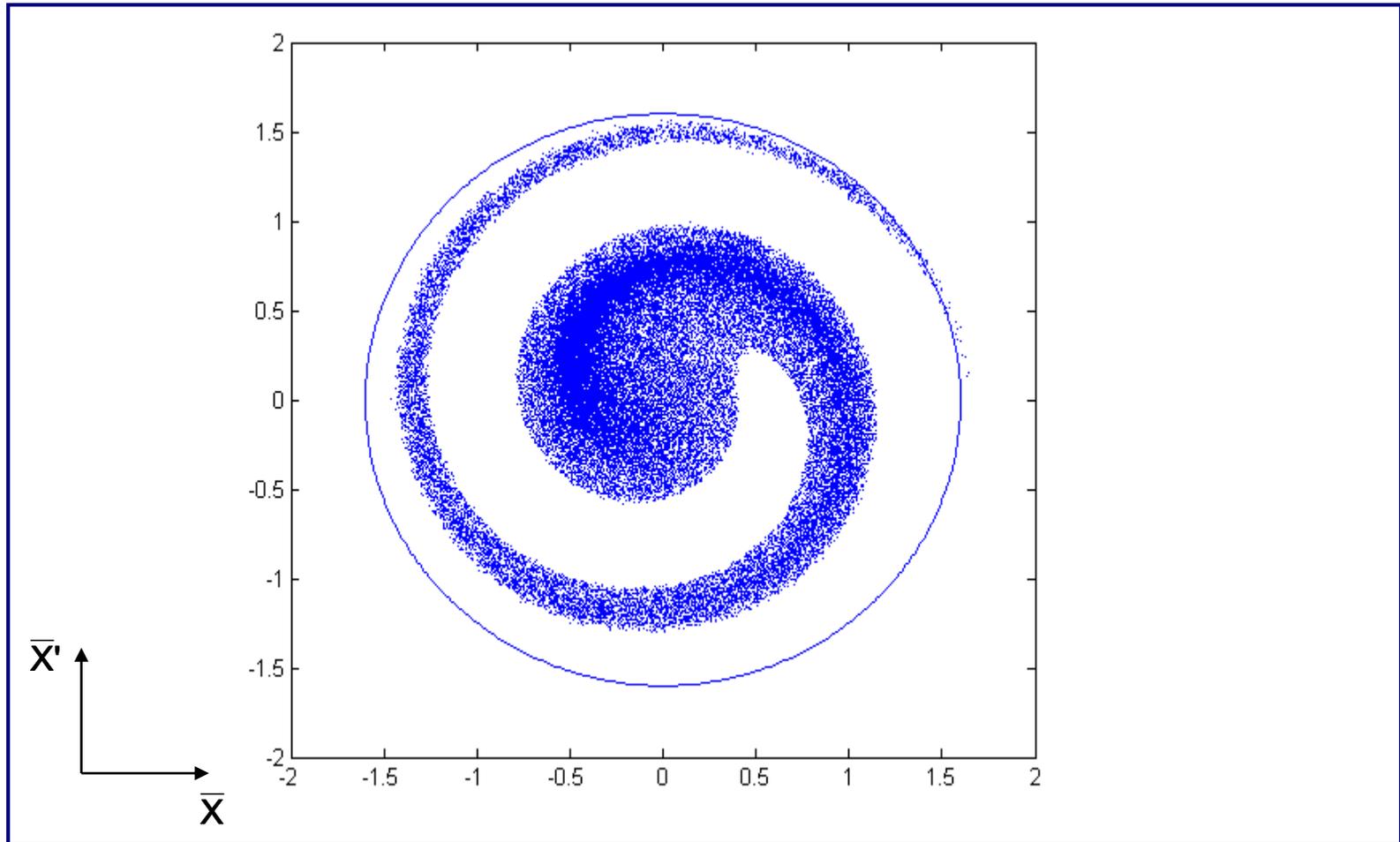
Filamentation



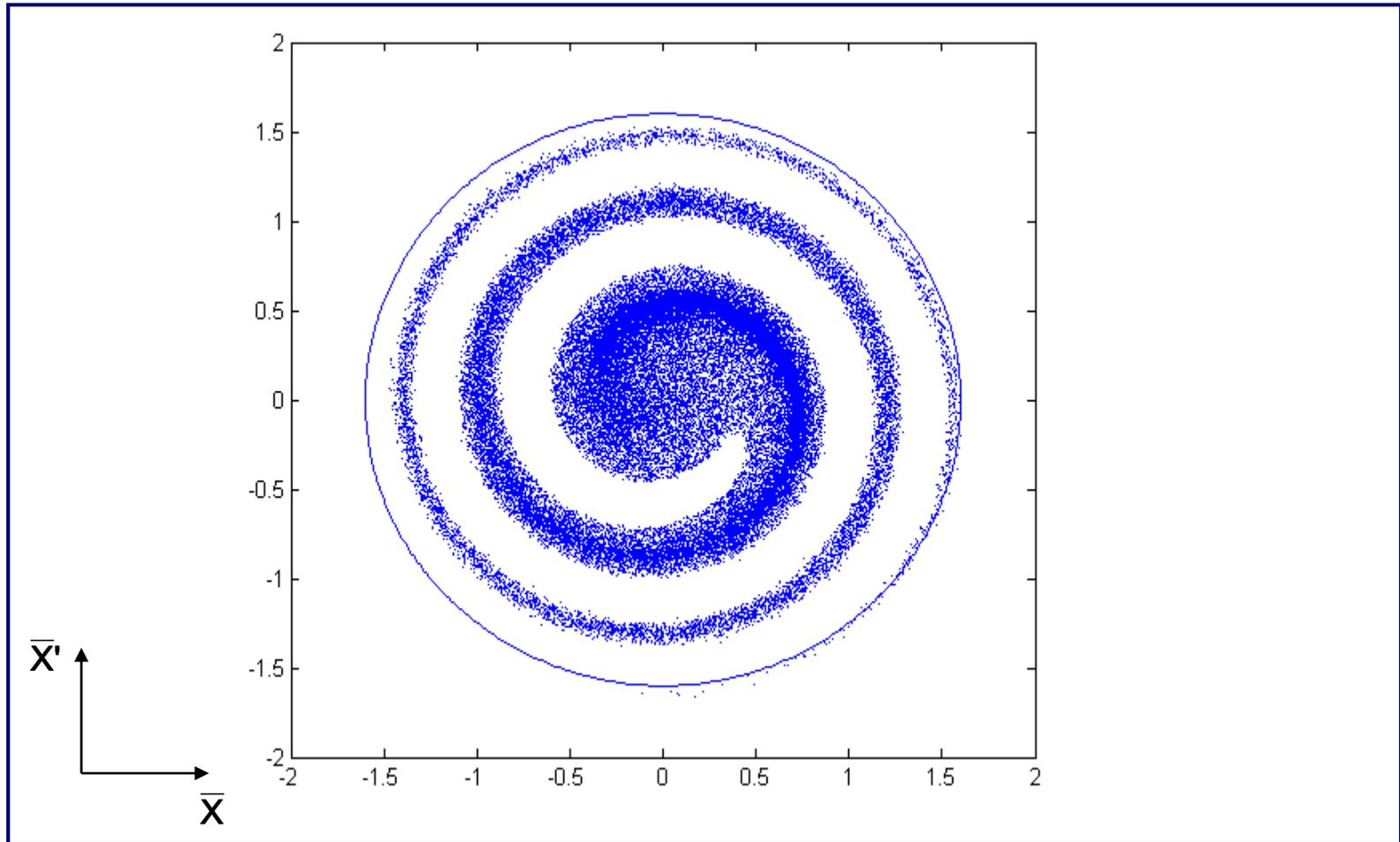
Filamentation



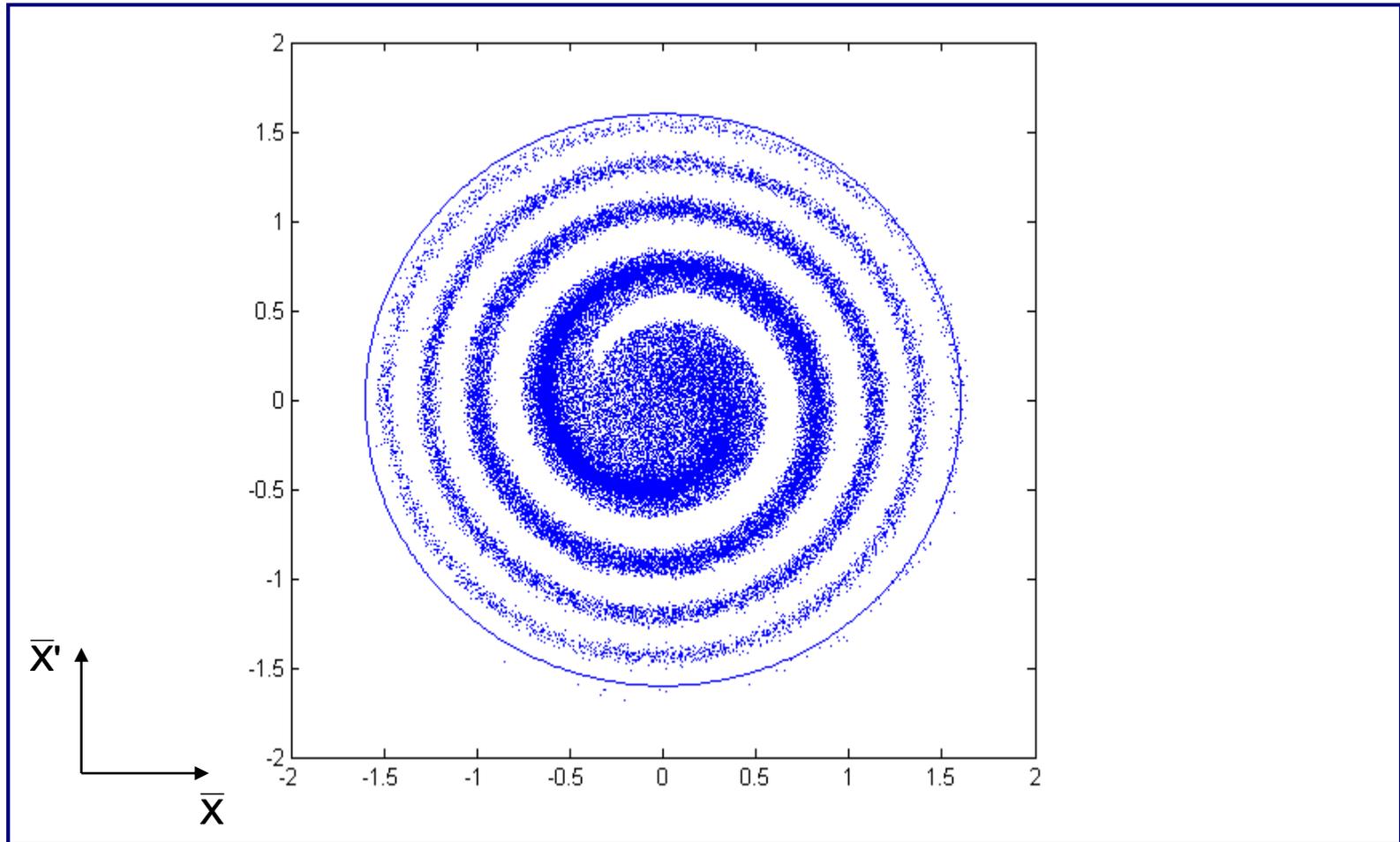
Filamentation



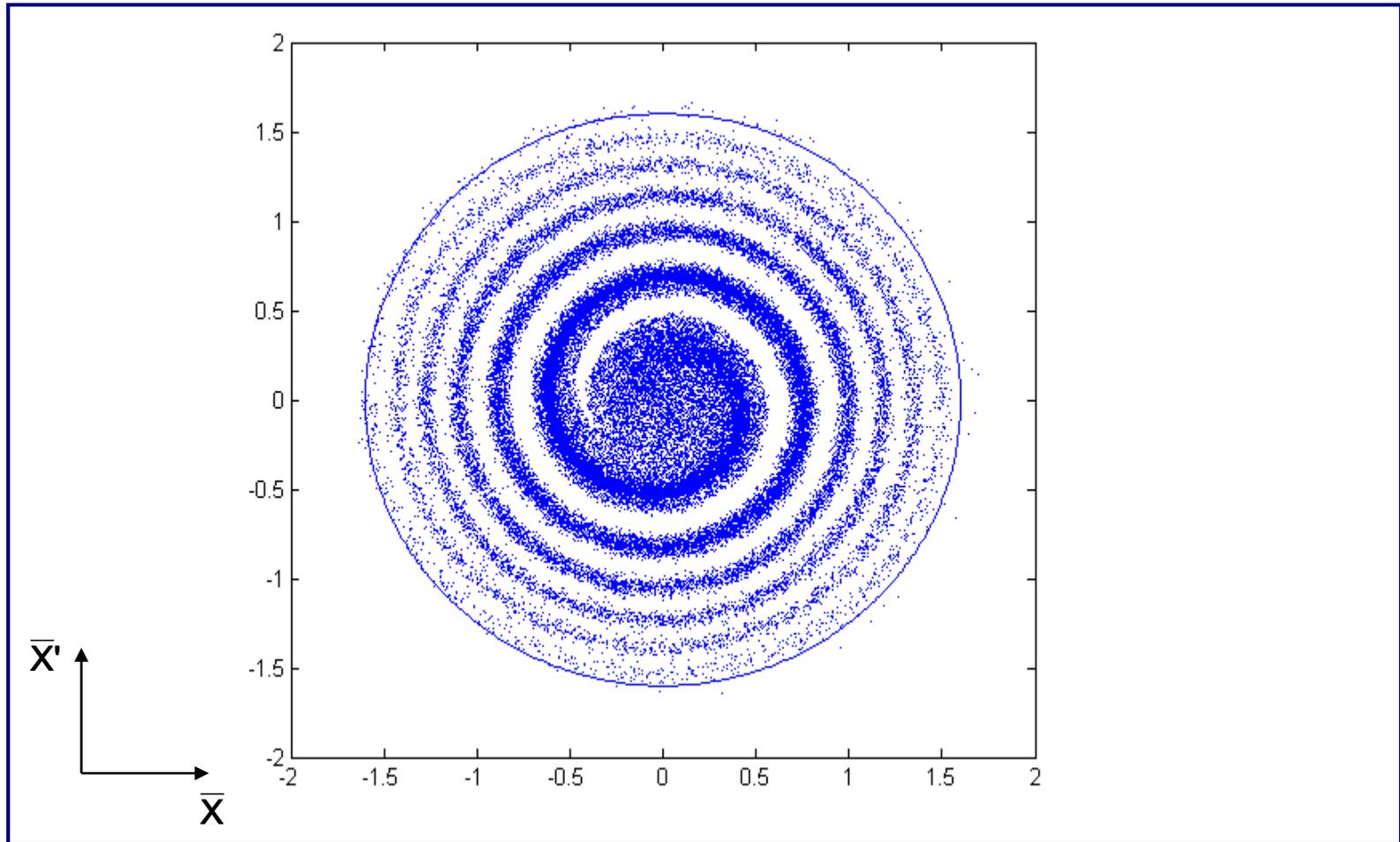
Filamentation



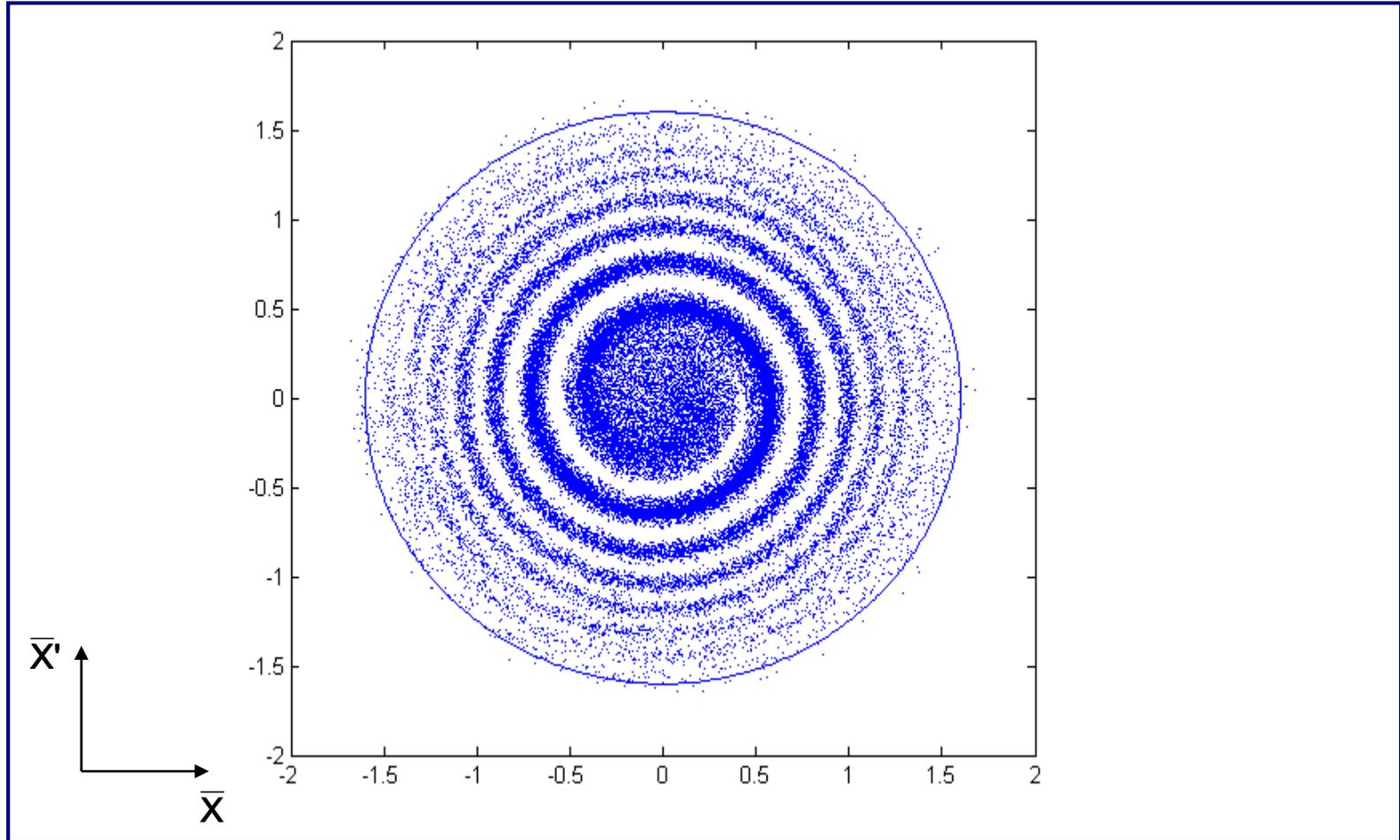
Filamentation



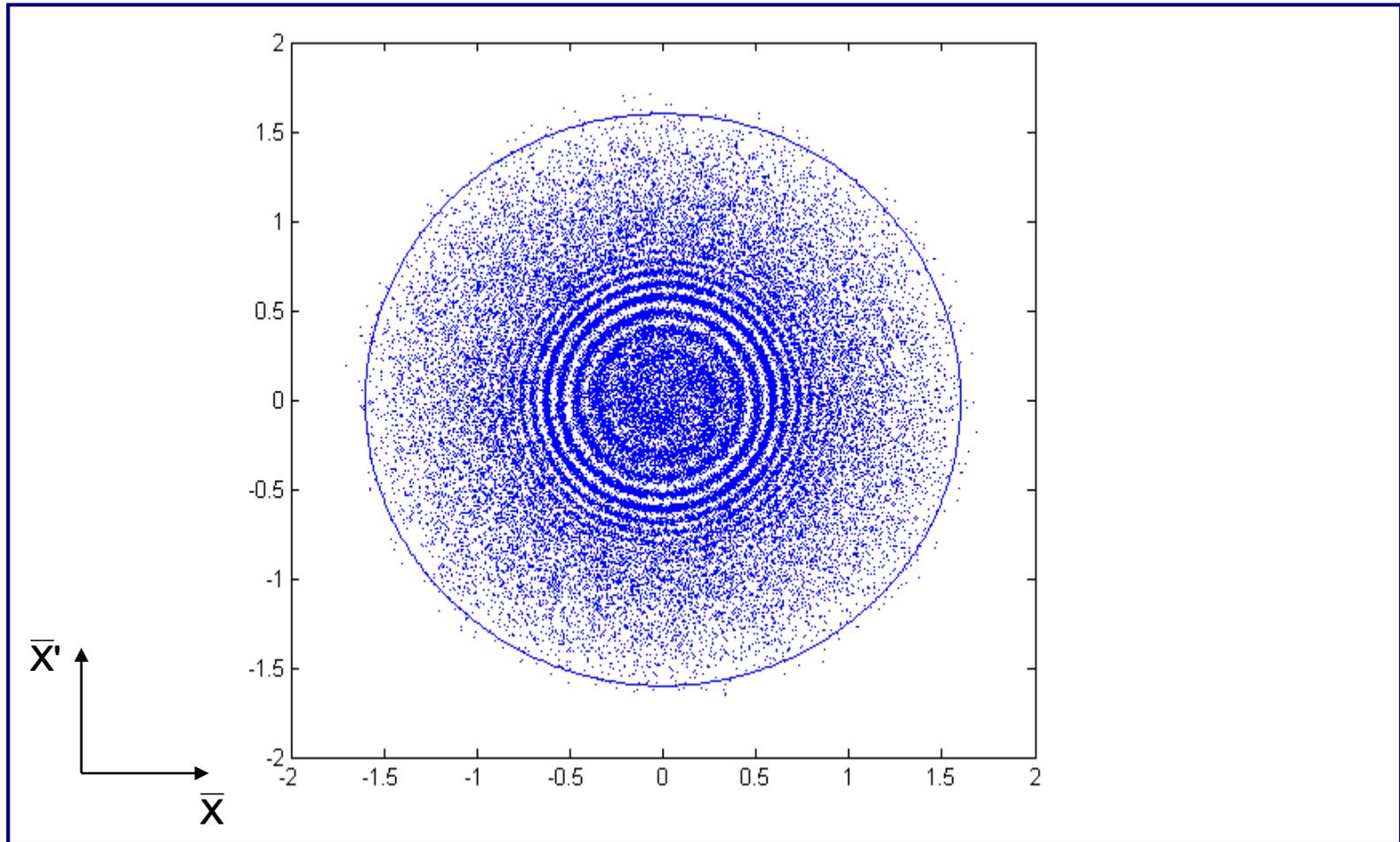
Filamentation



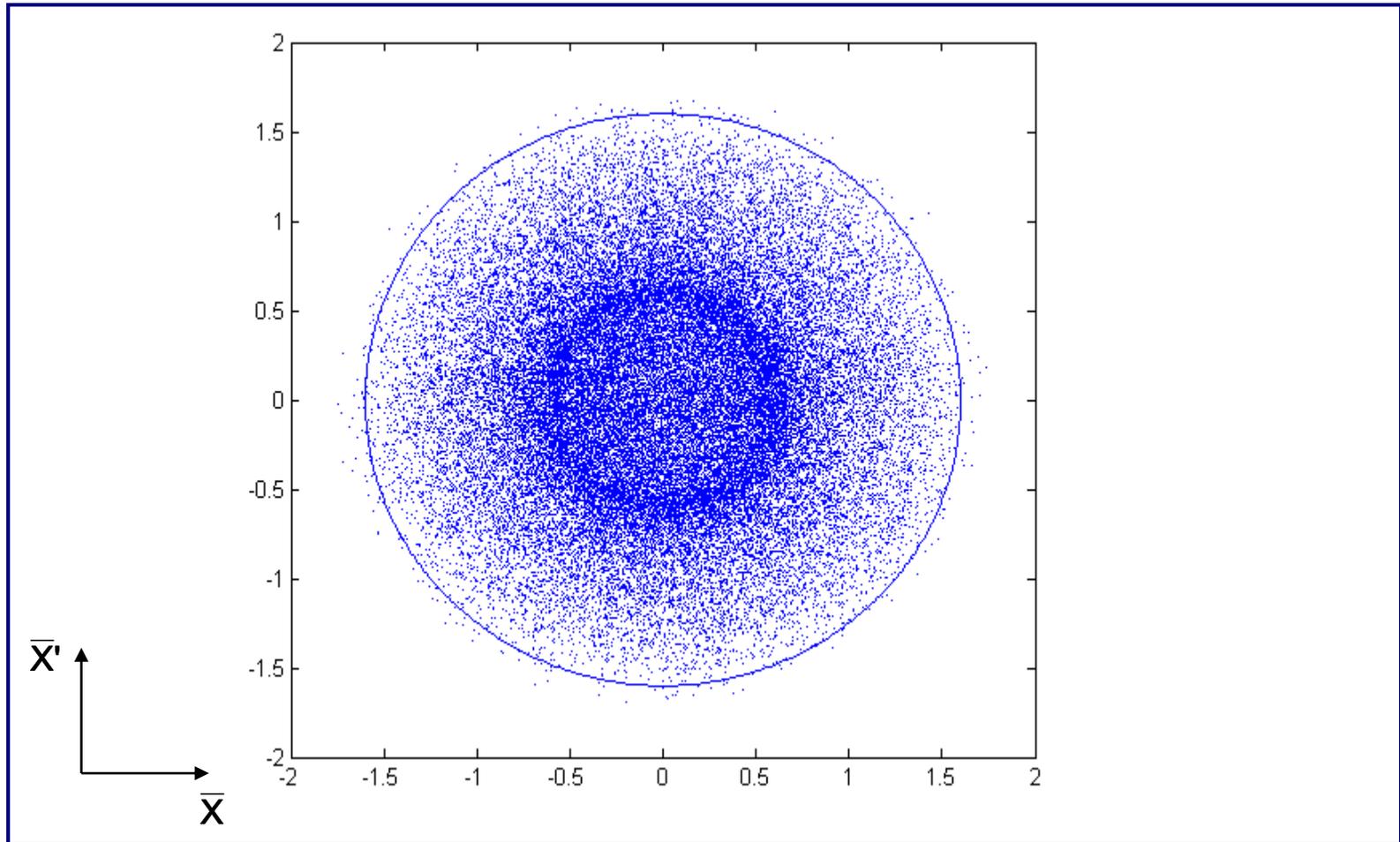
Filamentation



Filamentation

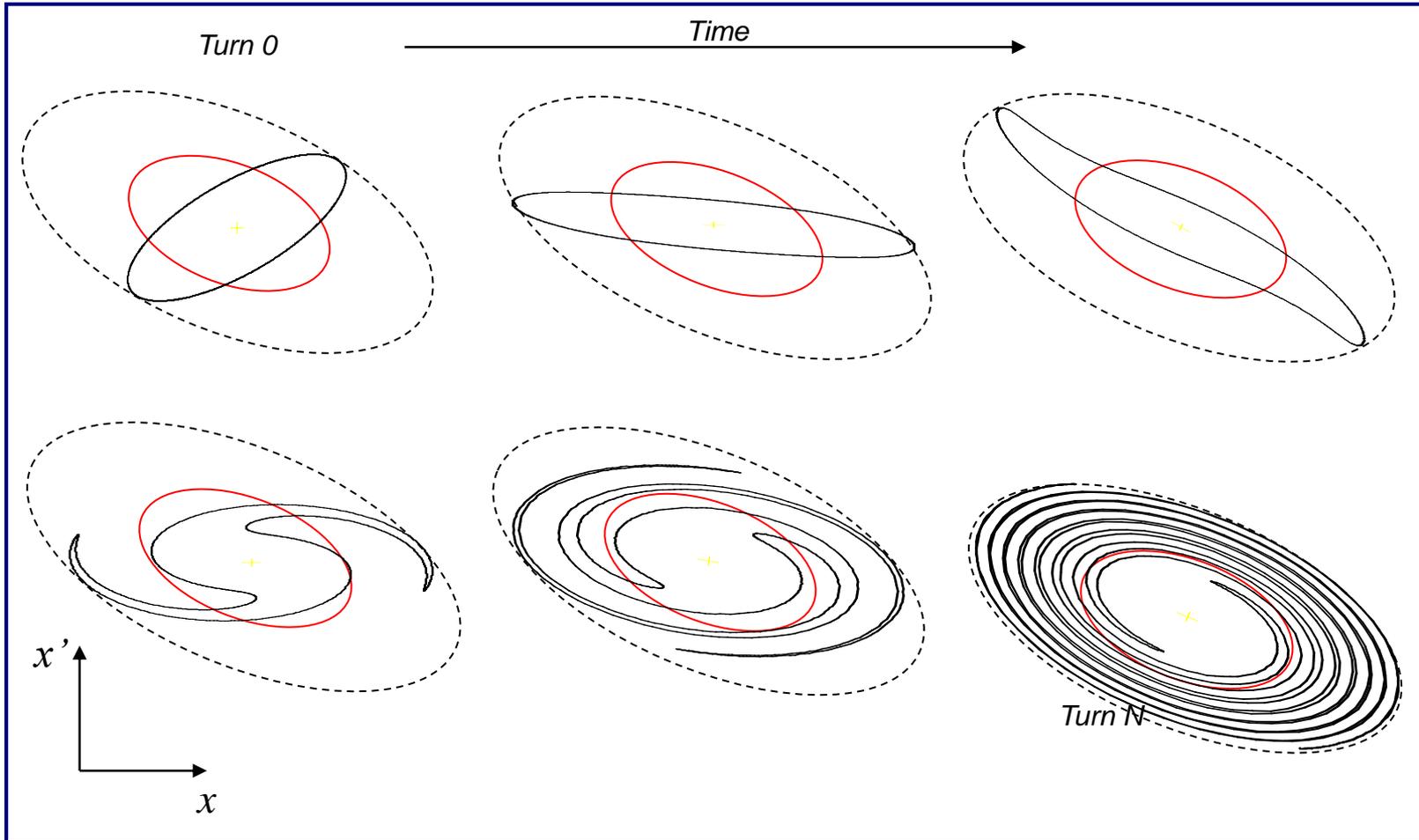


Filamentation



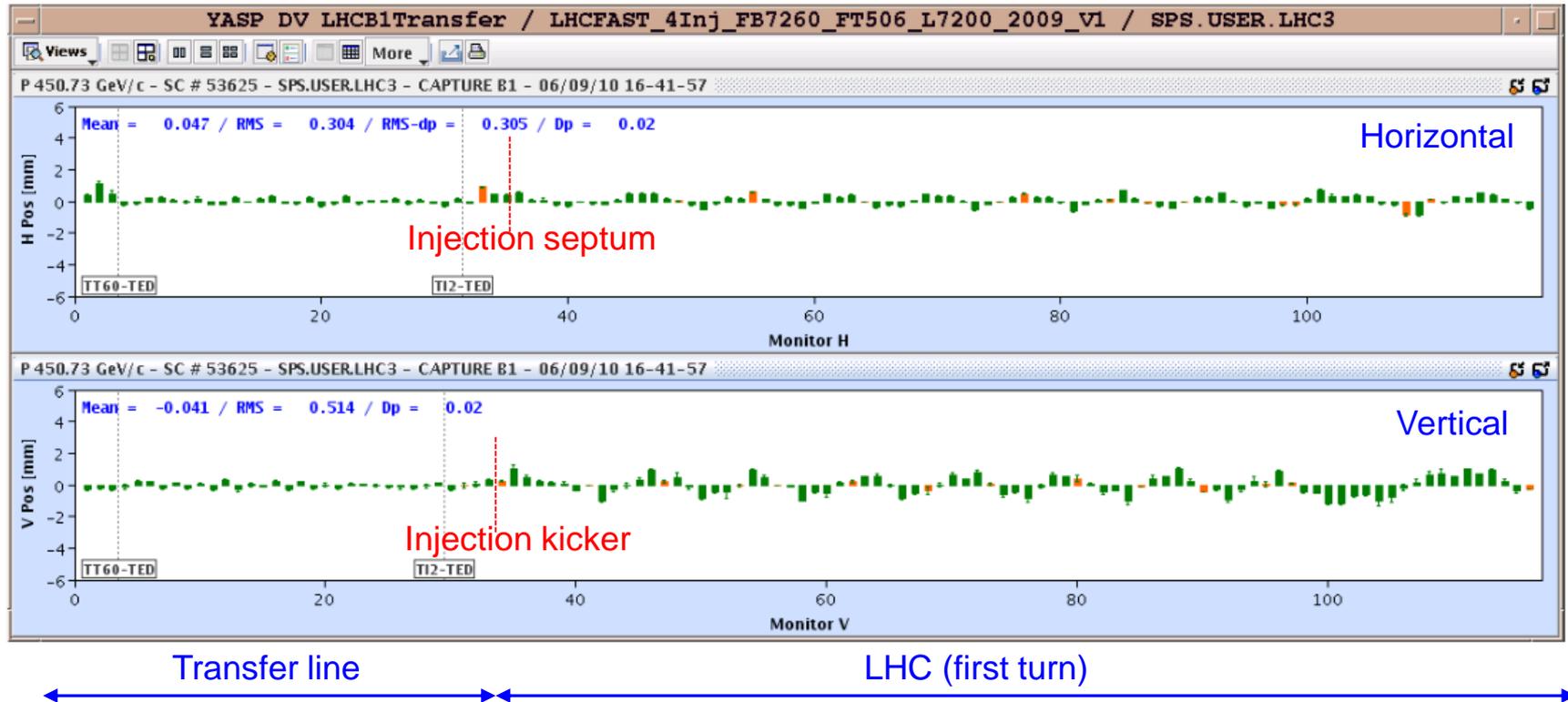
Optical Mismatch at Injection

- Filamentation fills larger ellipse with same shape as matched ellipse



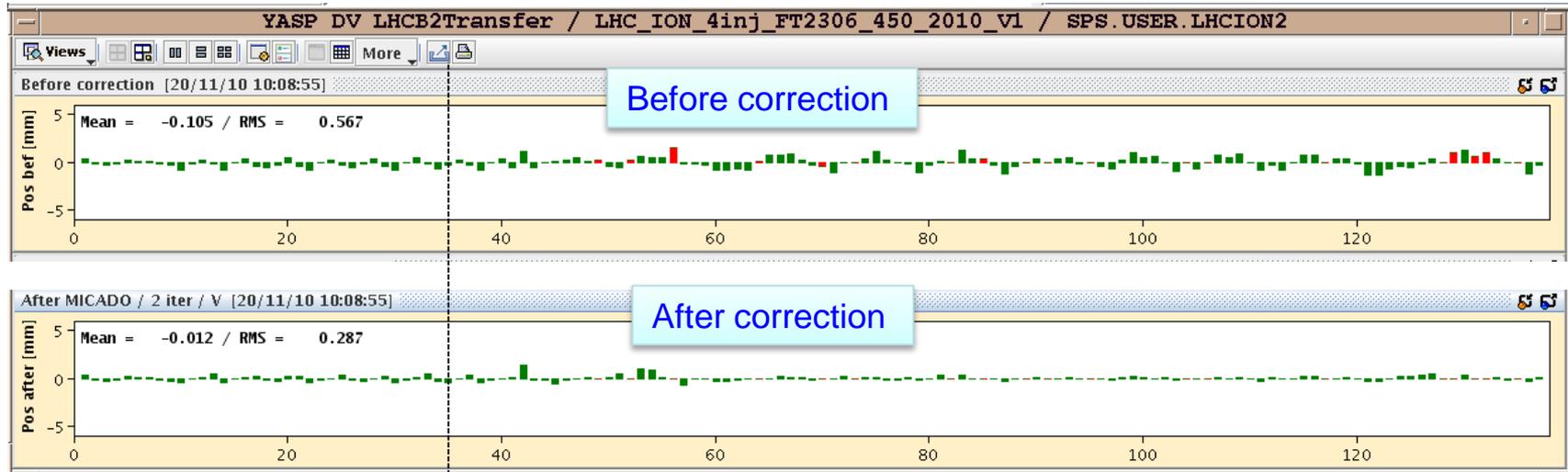
Injection oscillations

- Betatron oscillations with respect to the Closed Orbit:



Injection oscillation correction

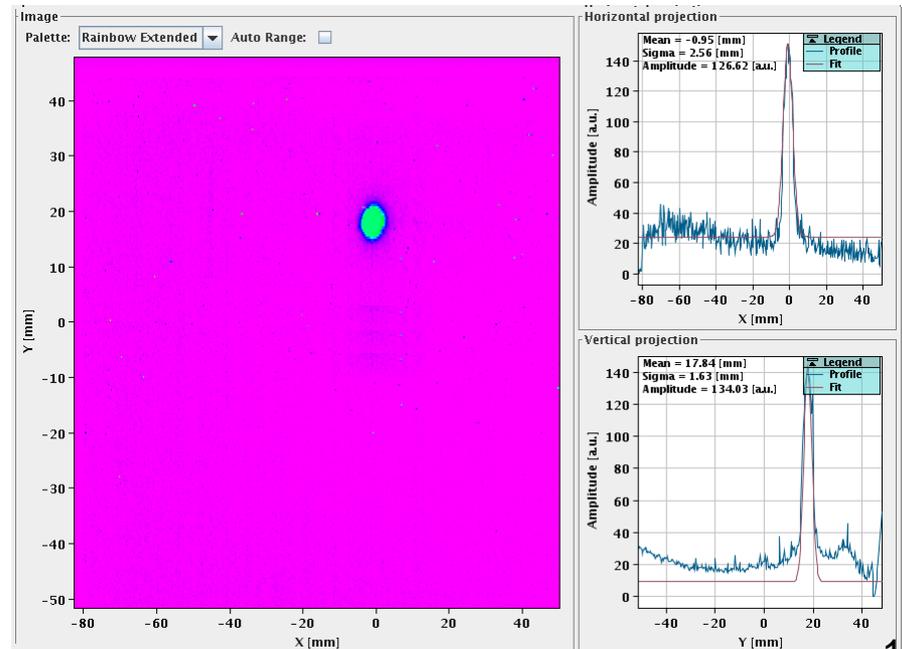
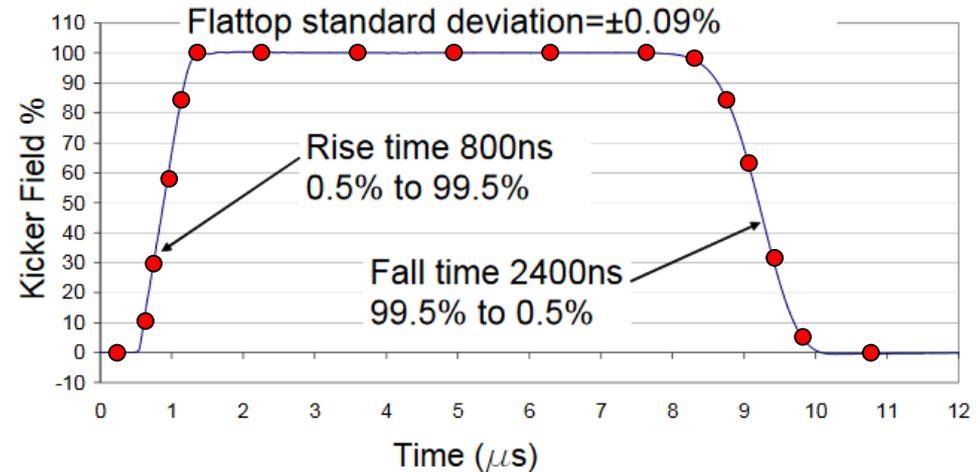
- Injection oscillations due to steering errors in the transfer line can be mitigated by a correct steering of the line (needed corrections calculated from BPM reading)



Injection kicker waveform

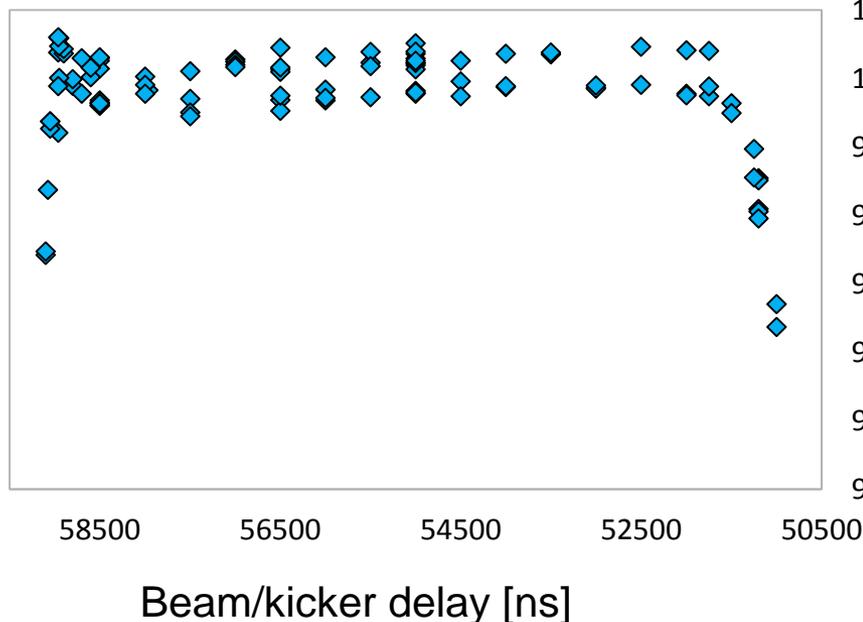
We can measure the kick seen by the beam by varying the delay of the kicker wrt the beam (1 bunch) and measuring the position of the beam on a downstream screen

Ideal waveform

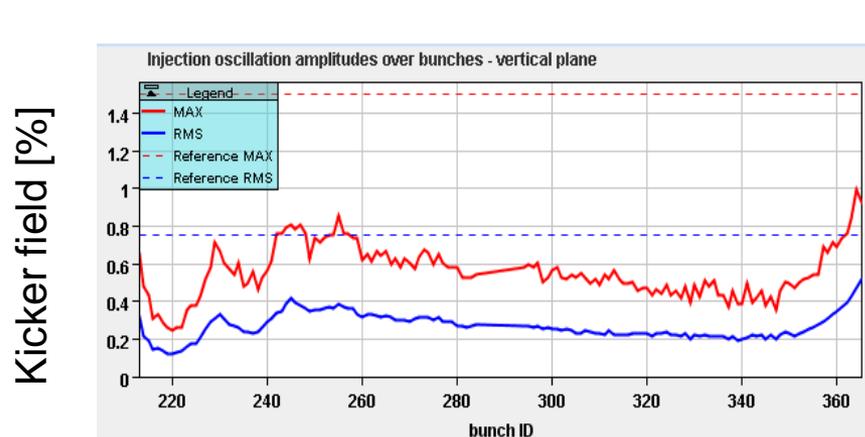
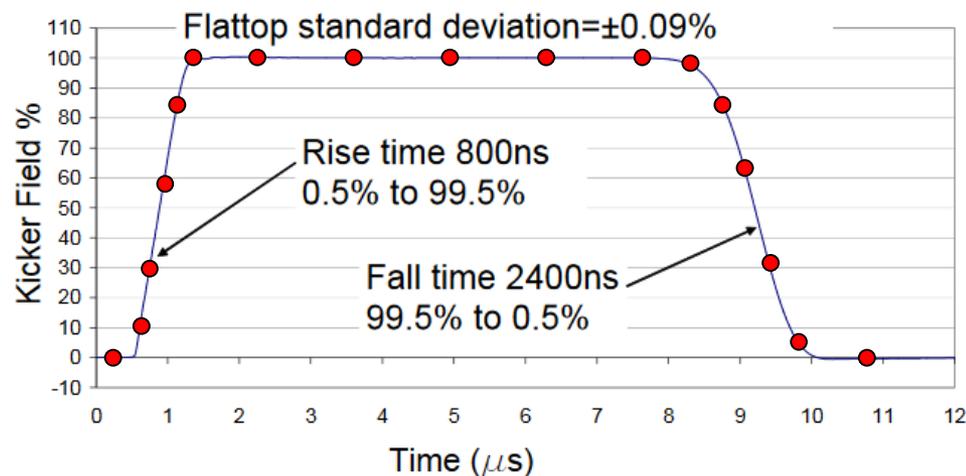


Injection kicker waveform

We can measure the kick seen by the beam by varying the delay of the kicker wrt the beam (1 bunch) and measuring the position of the beam on a downstream screen



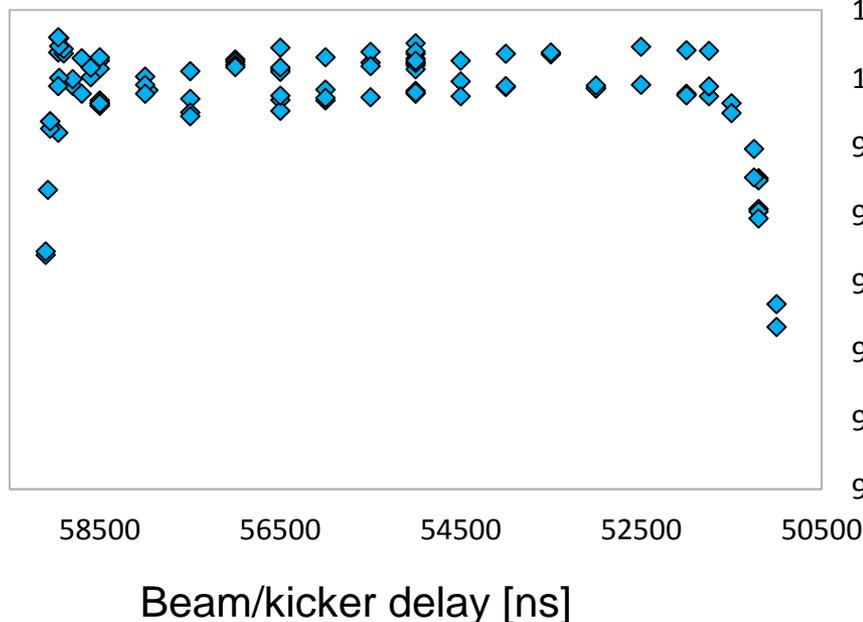
Ideal waveform



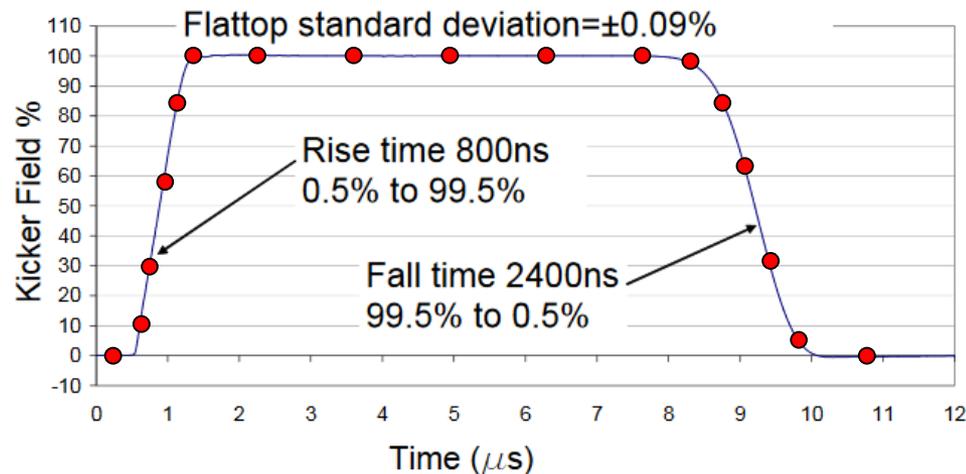
Multi-bunch injection: bunch-by-bunch position wrt closed orbit

Injection kicker waveform

We can measure the kick seen by the beam by varying the delay of the kicker wrt the beam (1 bunch) and measuring the position of the beam on a downstream screen



Ideal waveform



101

100

99

98

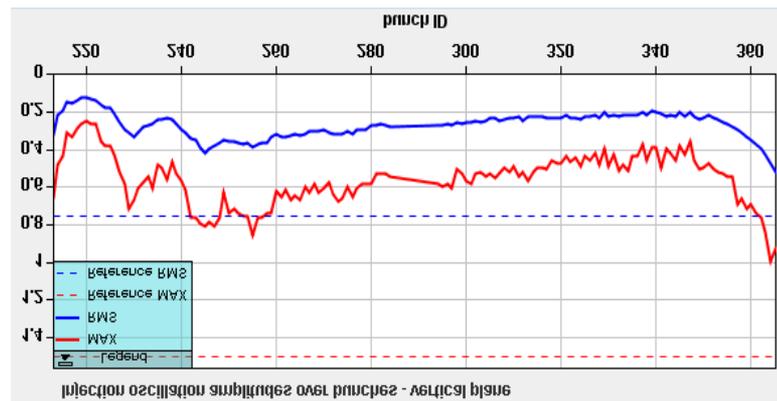
97

96

95

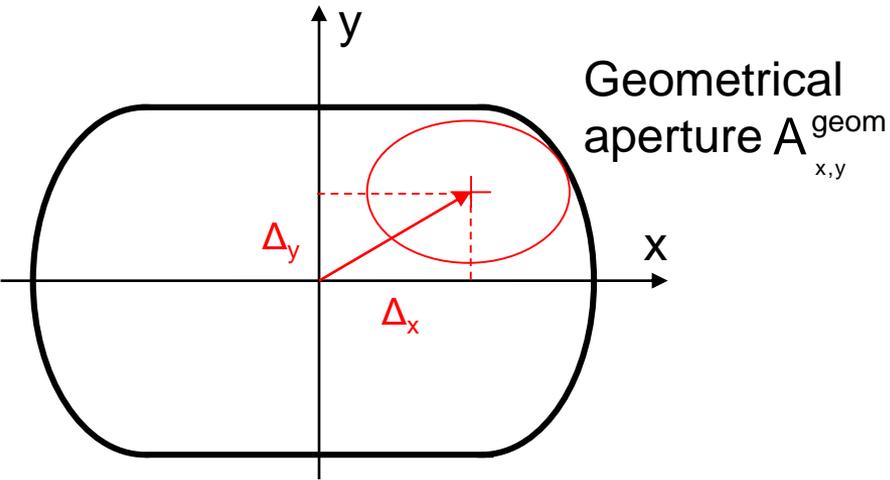
94

Kicker field [%]



Multi-bunch injection: bunch-by-bunch position wrt closed orbit

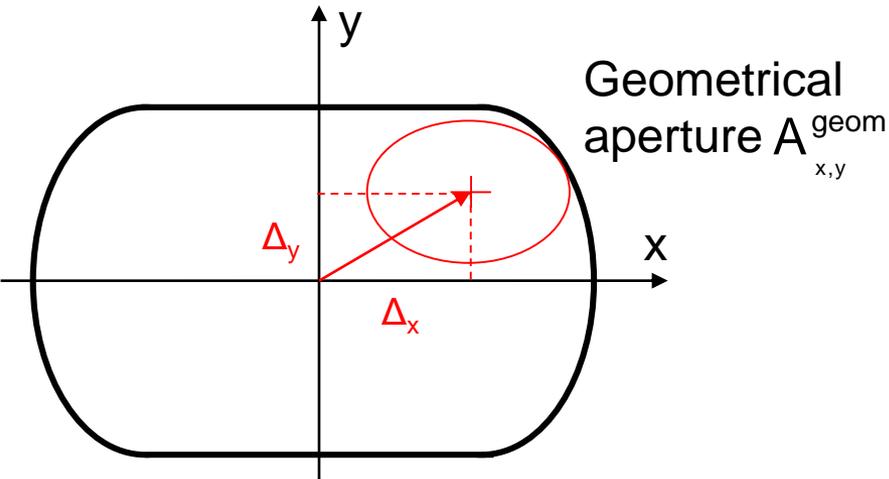
Aperture



Maximum beam displacement:

$$\Delta_{x,y}(s) = CO_{x,y}^{peak} \sqrt{\frac{\beta_{x,y}(s)}{\beta_{x,y}^{max}(s)}} + [\delta_{x,y}^{mech}(s) + \delta_{x,y}^{align}(s)] + k_{\beta} D_{x,y}(s) \delta_p + d_{x,y}^{sep}(s) + d_{x,y}^{inj}(s) + d_{x,y}^{axis}(s)$$

Aperture



Orbit displacement due to crossing and separation
Injection oscillations
Magnet axis offset wrt ring axis

Maximum beam displacement:

$$\Delta_{x,y}(s) = \underbrace{CO_{x,y}^{\text{peak}} \sqrt{\frac{\beta_{x,y}(s)}{\beta_{x,y}^{\text{max}}(s)}}}_{\text{Closed orbit}} + \underbrace{[\delta_{x,y}^{\text{mech}}(s) + \delta_{x,y}^{\text{align}}(s)]}_{\text{Mechanical and alignment tolerances}} + \underbrace{k_{\beta} D_{x,y}(s) \delta_p}_{\text{Dispersive term}} + \underbrace{d_{x,y}^{\text{sep}}(s) + d_{x,y}^{\text{inj}}(s) + d_{x,y}^{\text{axis}}(s)}_{\text{Taking into account parasitic coupling:}}$$

Closed orbit

Mechanical and
alignment
tolerances

Dispersive term

δ_p = momentum offset + max. momentum spread

k_{β} = β -beating correction

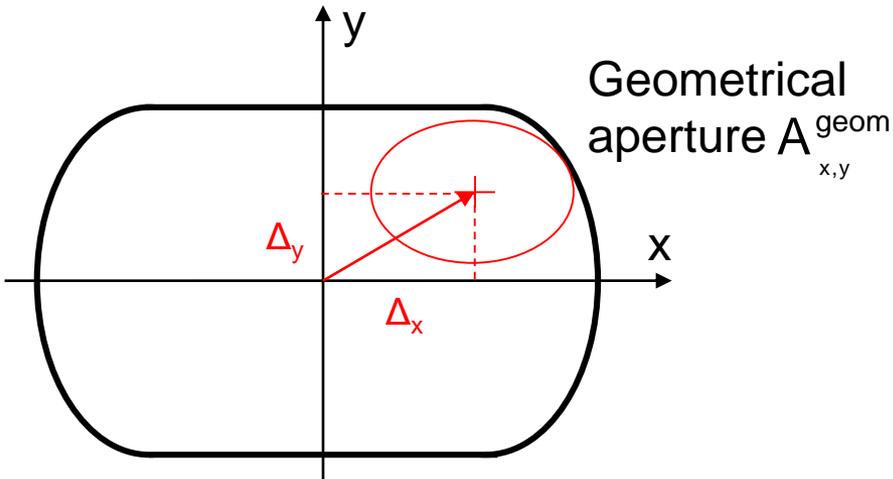
Taking into account parasitic coupling:

$$D_{x,y}(s) = D_{x,y}^{\text{linear}}(s) + k_D \sqrt{\frac{\beta_{x,y}(s)}{\beta_{\text{QF},x}}} D_{\text{QF}}$$

k_D = coupling coefficient

D_{QF} = peak linear dispersion

Aperture



Maximum beam displacement:

$$\Delta_{x,y}(s) = CO_{x,y}^{peak} \sqrt{\frac{\beta_{x,y}(s)}{\beta_{x,y}^{max}(s)}} + [\delta_{x,y}^{mech}(s) + \delta_{x,y}^{align}(s)] + k_{\beta} D_{x,y}(s) \delta_p + d_{x,y}^{sep}(s) + d_{x,y}^{inj}(s) + d_{x,y}^{axis}(s)$$

Betatron beam envelope (maximum oscillation amplitude of the beam particles):

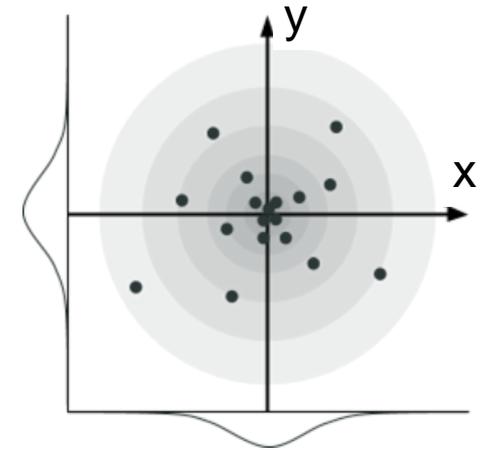
$$\delta_{x,y}(s) = n_{x,y} \sigma_{\beta_{x,y}}(s) \quad \text{where} \quad \sigma_{\beta_{x,y}}(s) = k_{\beta} \sqrt{\epsilon_{x,y} \beta_{x,y}(s)} \quad \text{betatron beam size}$$

How many $n_{x,y}$ have to fit in the aperture? It depends on the stored beam energy and the aperture which has to be protected. The higher the stored energy the larger $n_{x,y}$ (special care for superconductive machines!)

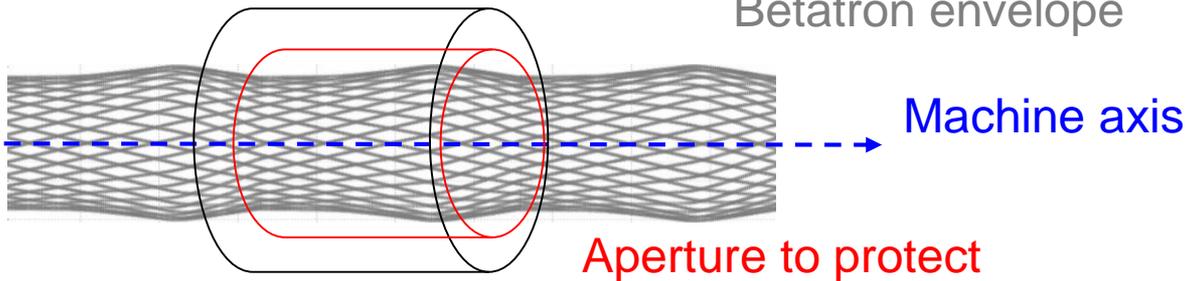
Aperture

Stored energy: $E_{\text{stored}}(x, y) = n_{\text{part}}(x, y) E_{\text{part}}^{\text{tot}}$

for a Gaussian (in general non uniform) distribution
 E_{stored} varies with the amplitude



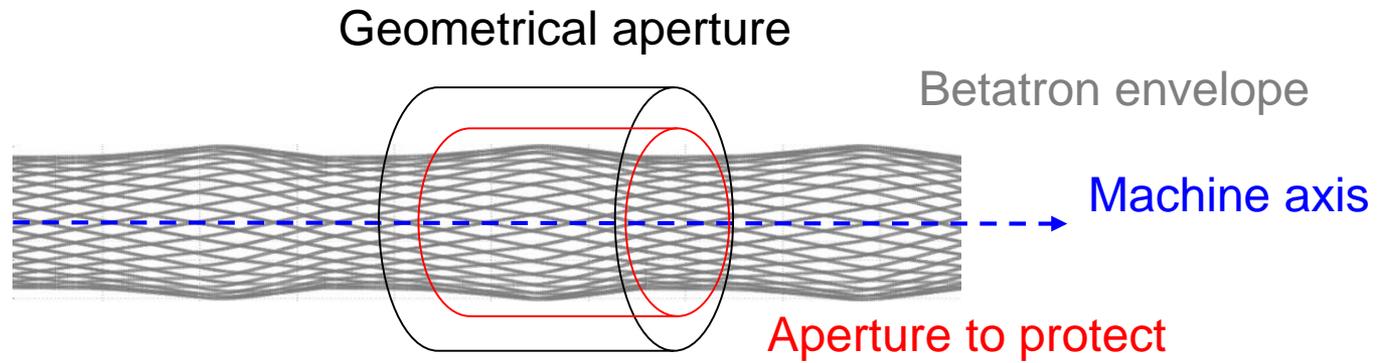
Geometrical aperture



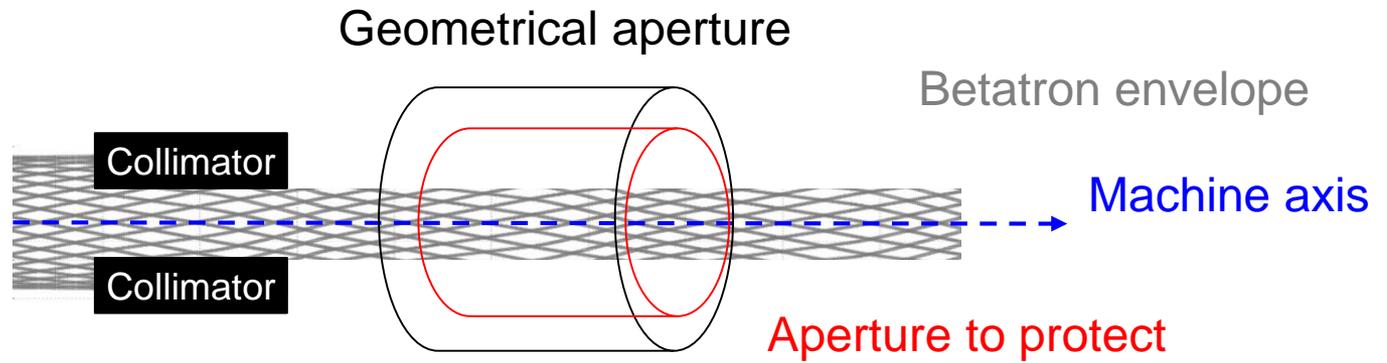
$$n_{x,y} \leq A_{x,y}^{\text{prot}} [\sigma_{\beta_{x,y}}] \leq \min_{s \in [0, L]} \left(\frac{A_{x,y}^{\text{geom}}(s) - \Delta_{x,y}(s)}{\sigma_{\beta_{x,y}}(s)} \right)$$

How can we ensure that this condition is fulfilled?

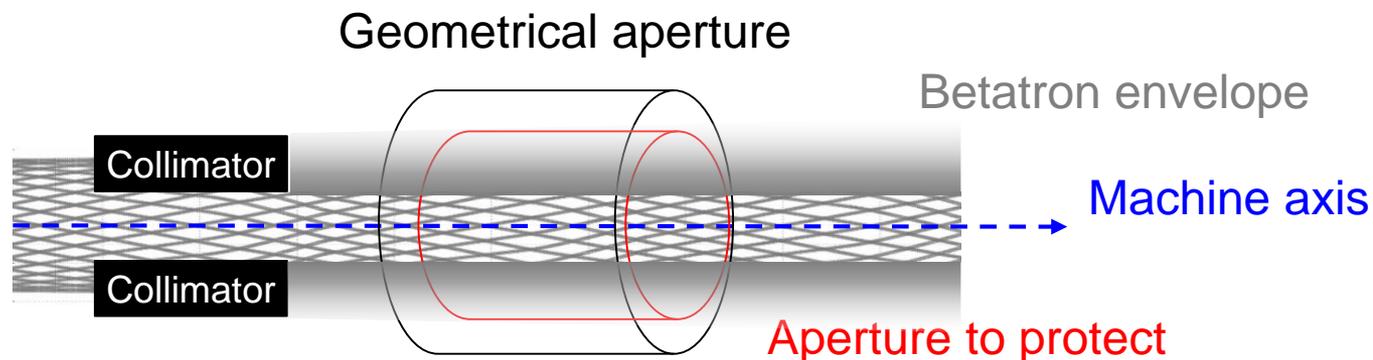
Protection Devices



Protection Devices

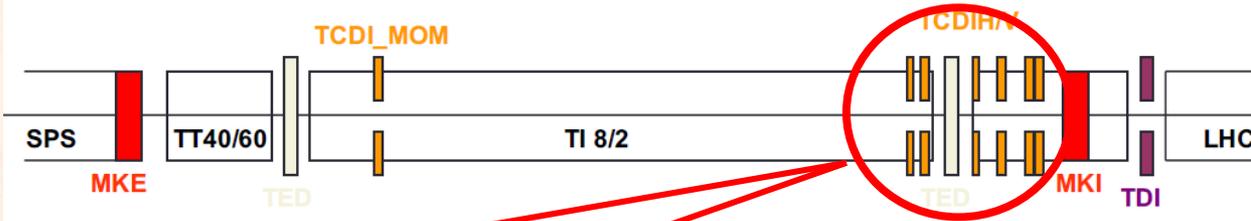
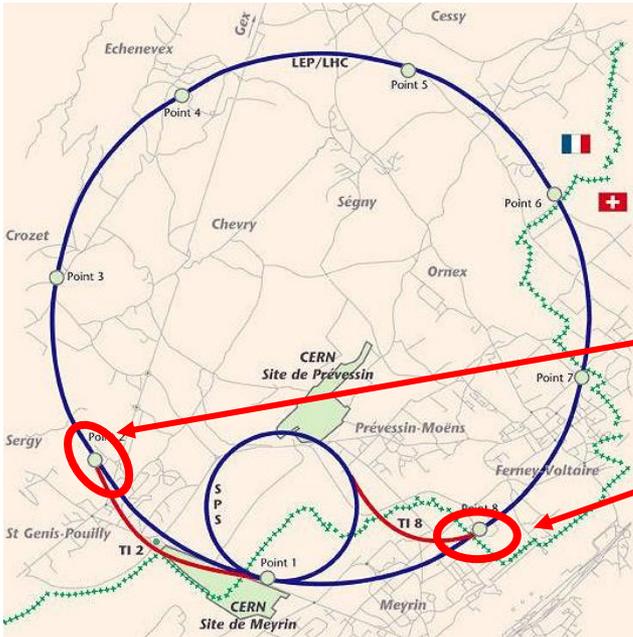


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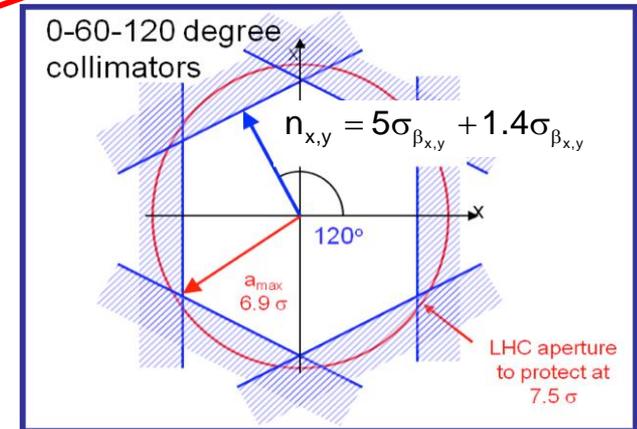
- Passive protection devices are designed to dilute and absorb beam energy safely
- They have to withstand direct beam impacts and reduce the energy deposition on the downstream elements due to secondary showers to below the damage limit
- Failures associated with beam transfer equipment are typically very fast and difficult to catch by active systems (e.g. interlocks, magnet current monitors etc.)

LHC transfer line collimators

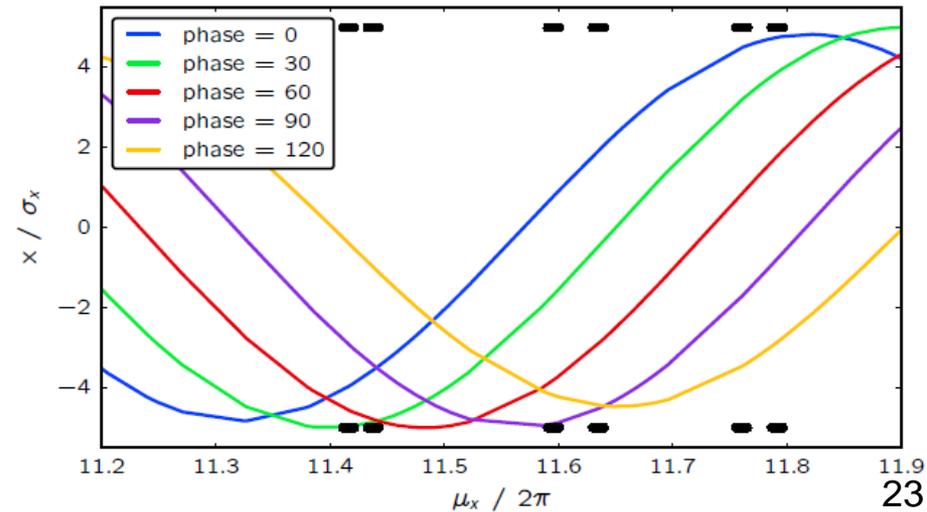


$$a_{\max} = \frac{n_{x,y}}{\cos\left(\frac{\Delta\mu_{x,y}}{2}\right)}$$

$$n_{x,y} = n_{\text{coll}} + n_{\text{error}}$$

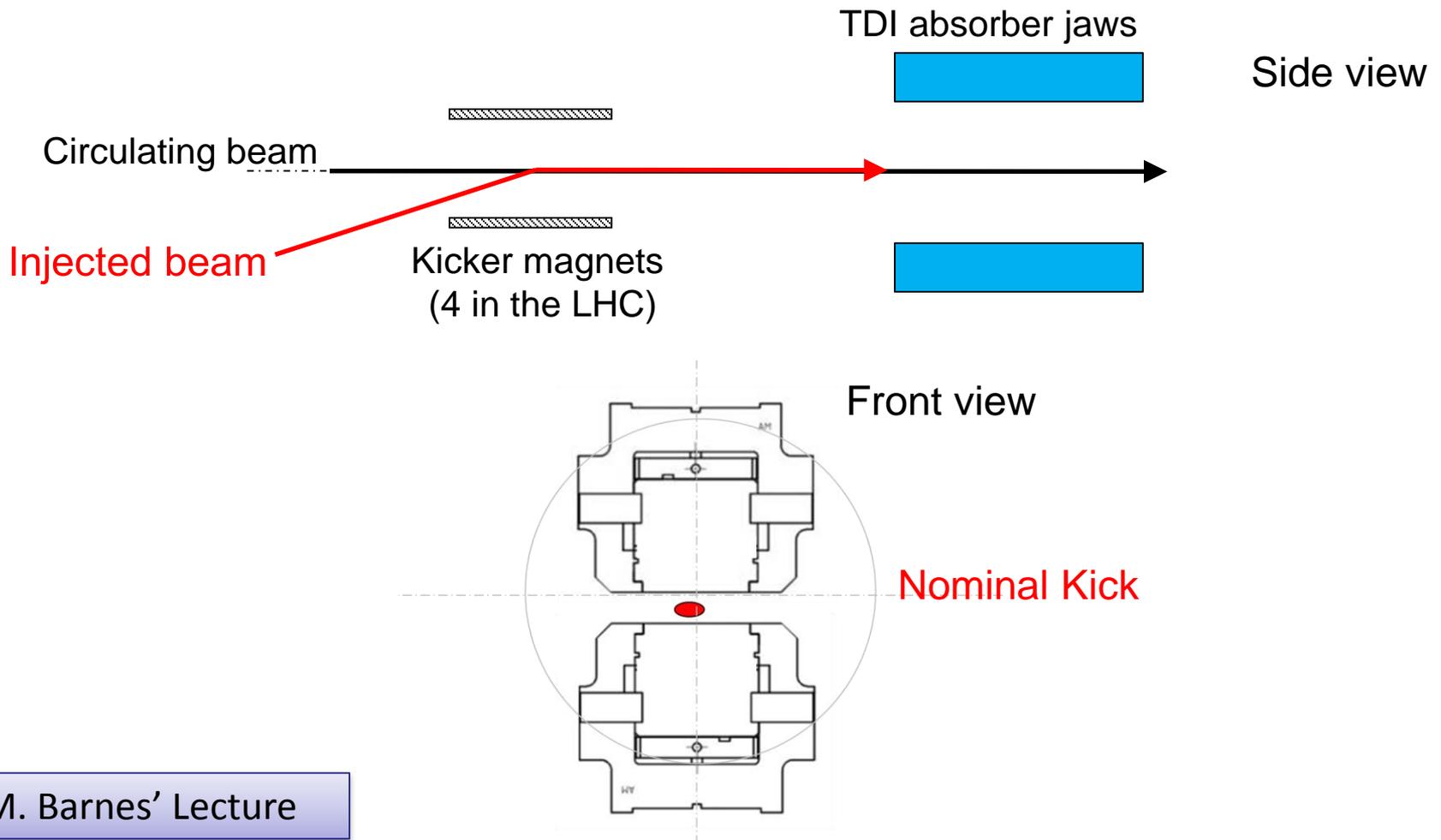


A system of 3 collimators (2 jaws each) per plane at 60° phase advance from each other is installed at the end of the SPS-to-LHC TLs [3] → optimized ratio: provided protection/number of collimators. Intercept large amplitude oscillations and attenuate escaping intensity below damage limit of LHC aperture → collimator geometry (material and active length)



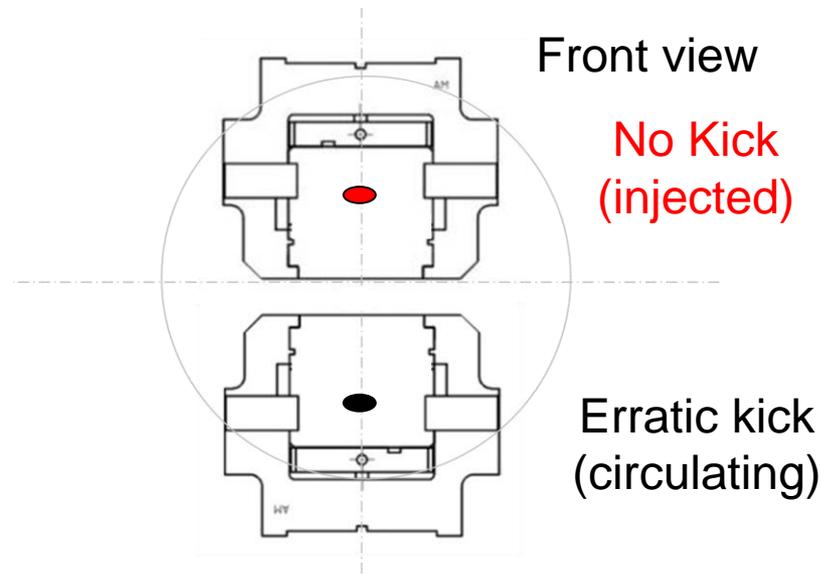
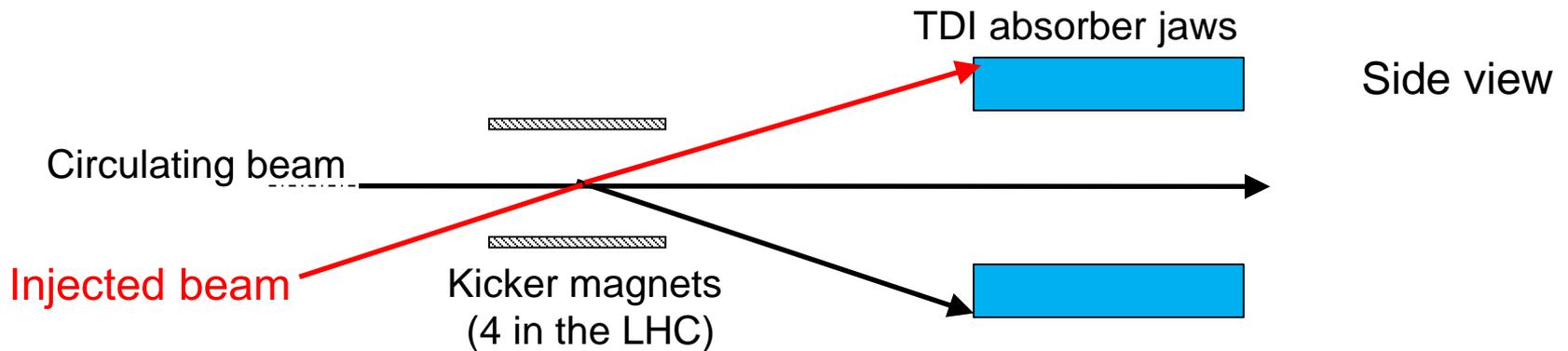
LHC injection protection

- LHC has a dedicated injection dump (TDI [3]) to protect against fast failures of the injection kicker



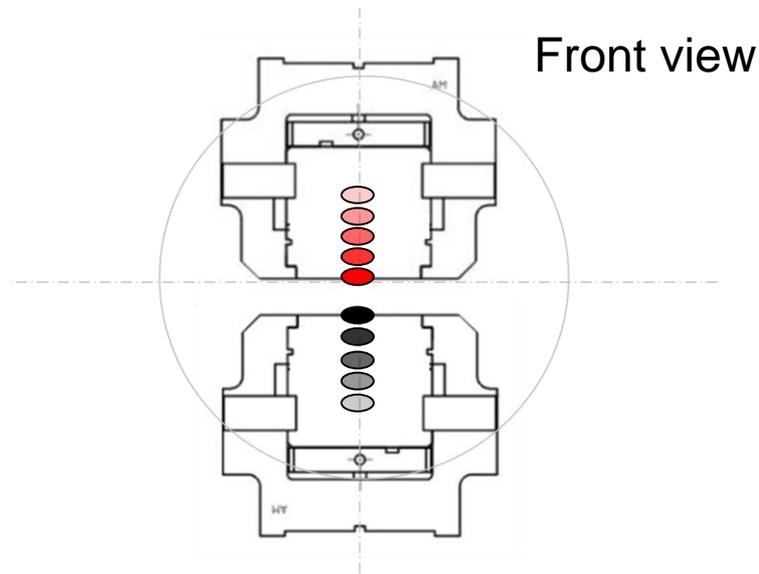
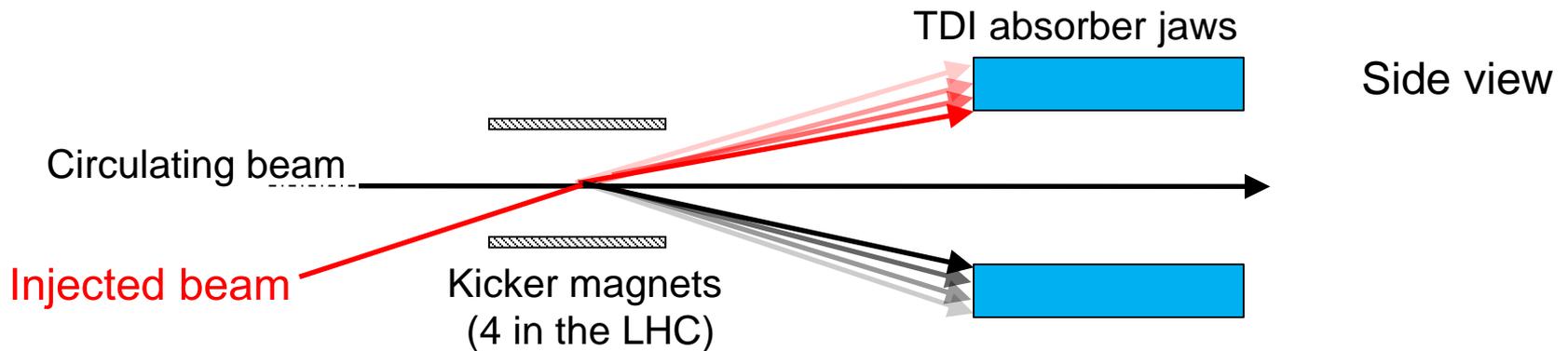
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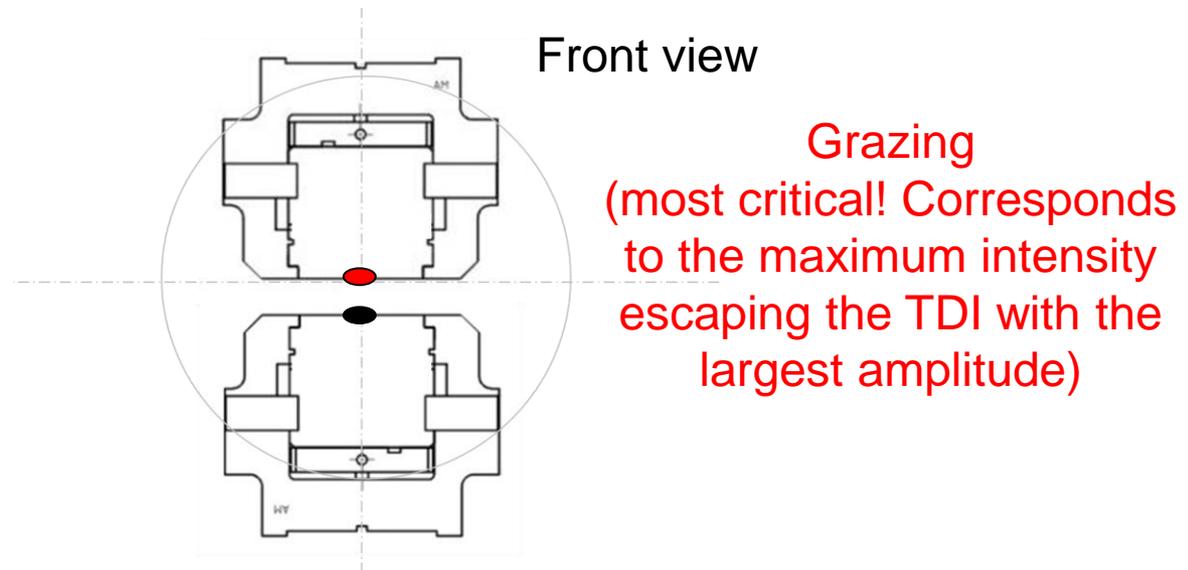
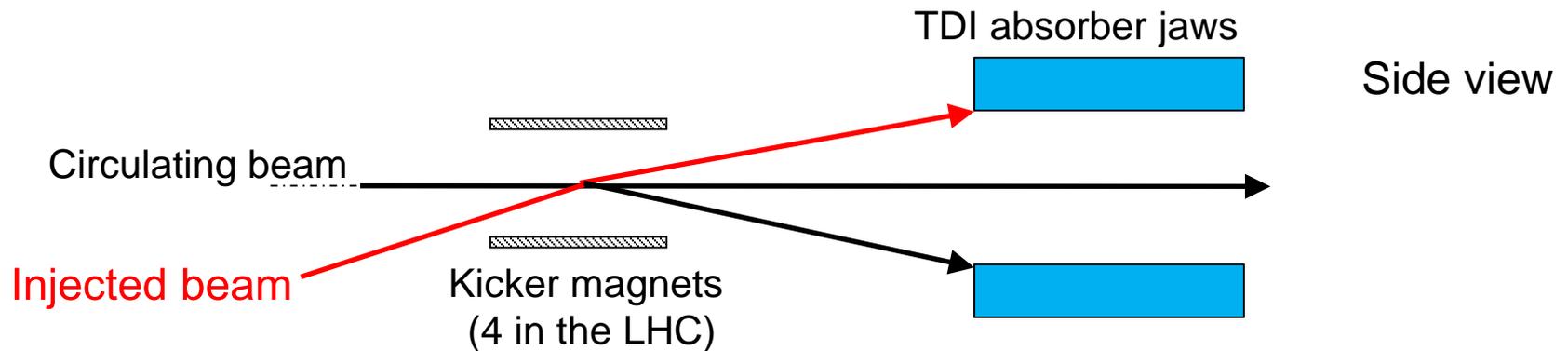
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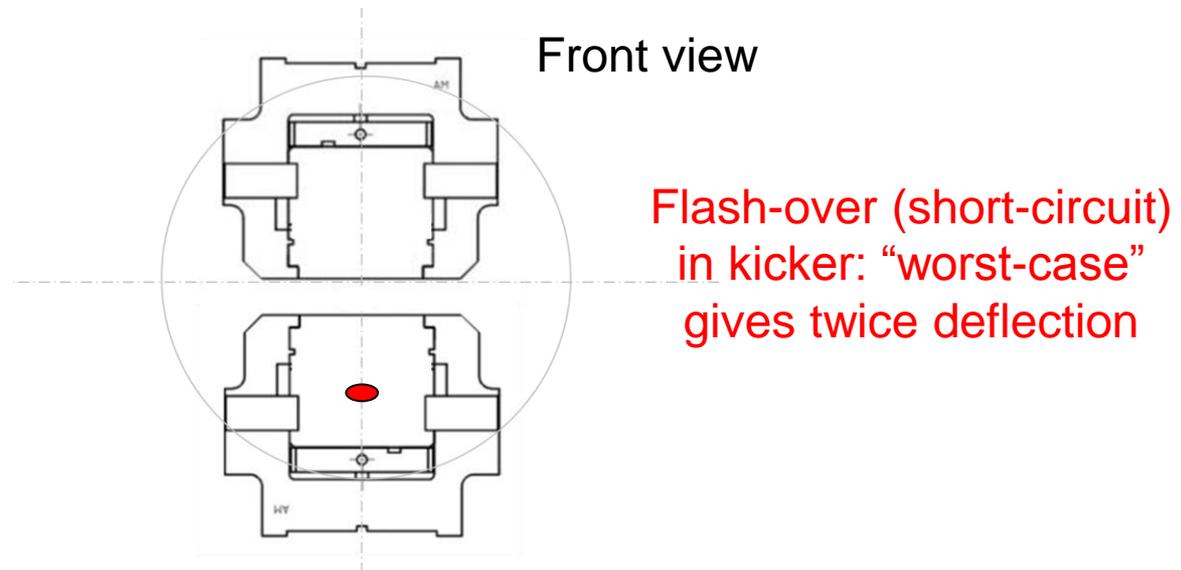
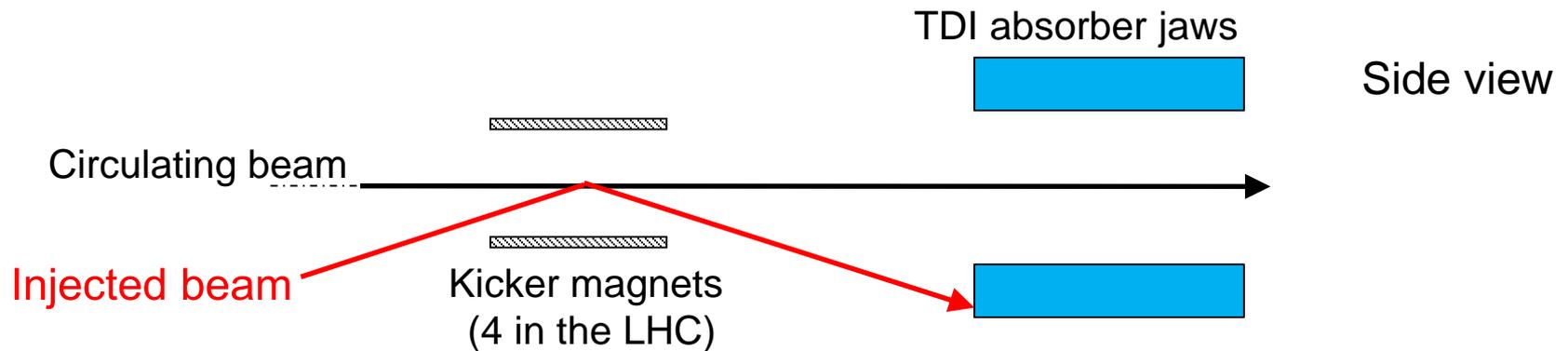
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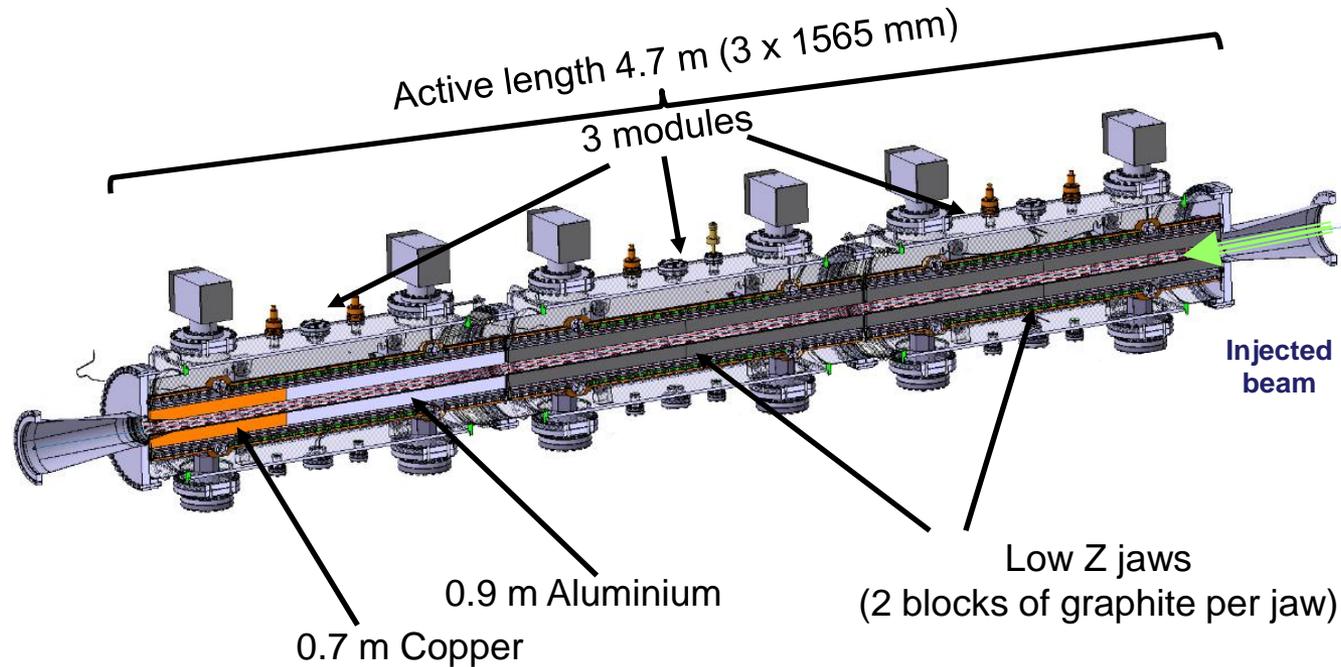
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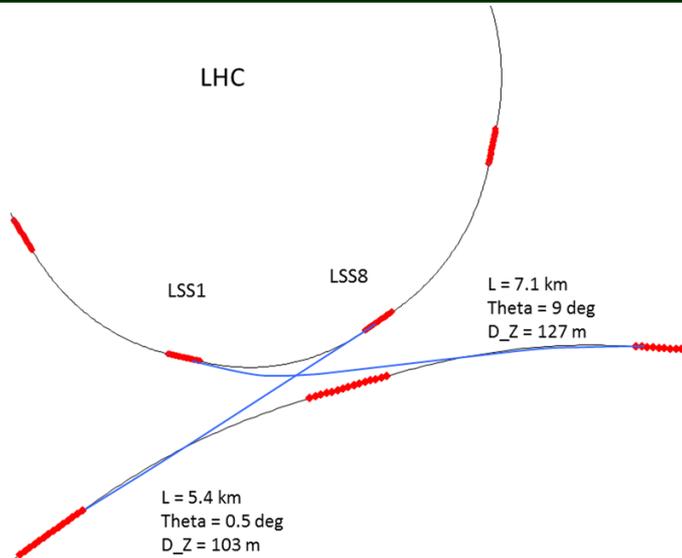
Injection protection collimators for HL-LHC

TDIS: 4.75 m active length, 2 low Z material modules + 1 high Z material module to absorb secondary showers



5 MJ stored energy per injection → already at **the limit of what materials can deal (robustness and transmission)**.

Injection into FCC



Assumptions:

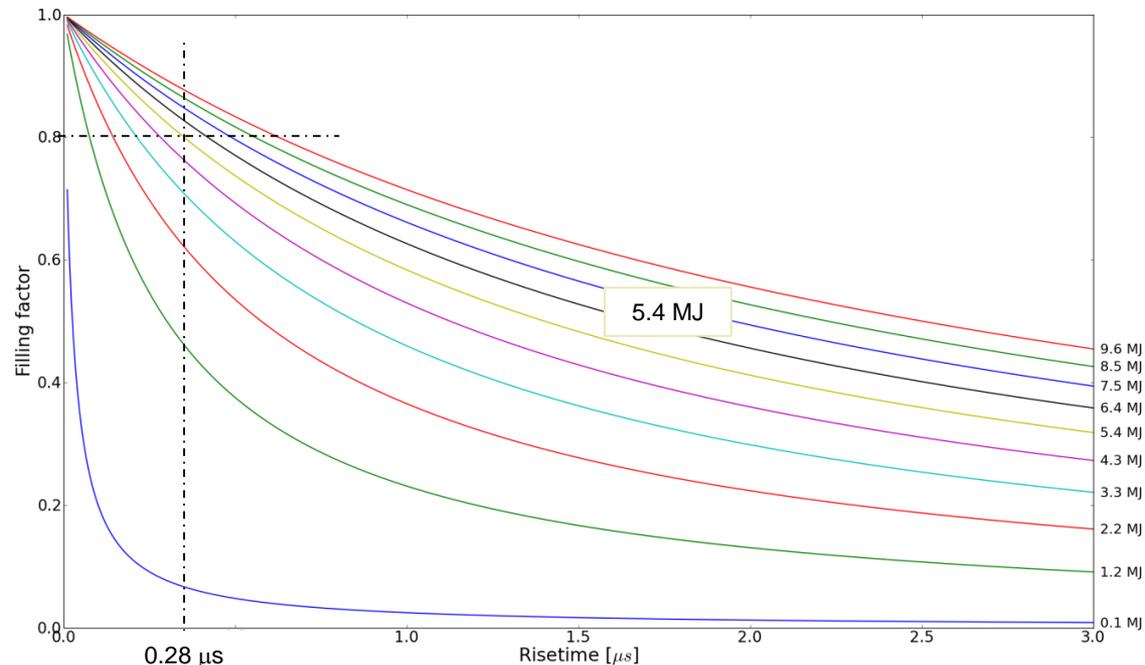
LHC used as injector for the FCC

Injection energy = 3.3 TeV (x7 HL-LHC)

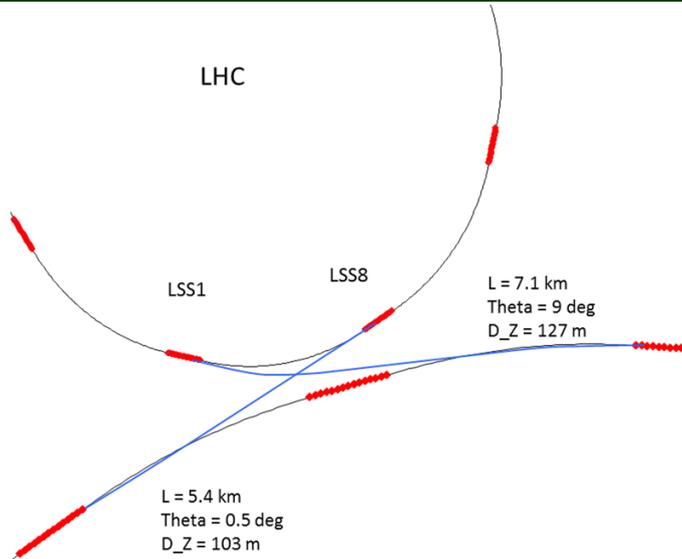
Bunch population = $1.1E11$ ppb

LHC can be filled with ~ 2800 bunches but, in order to limit the stored energy of the injected beam into the FCC to **5 MJ** one can **extract/inject 90 bunches at the time** \rightarrow **reduce kicker flattop!** [6]

t_{rise} injection kicker defined by the target filling factor assuming a target filling factor of 80% $\rightarrow 0.28 \mu\text{s}$



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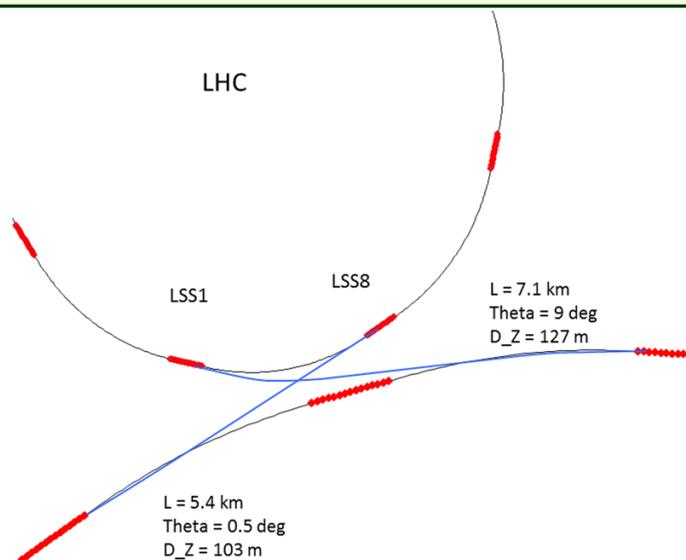
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The LHC is emptied to fill FCC by pulsing the extraction/injection kickers at high frequency. Required kicker recharging time driven by synchronisation of two machines to common frequency:



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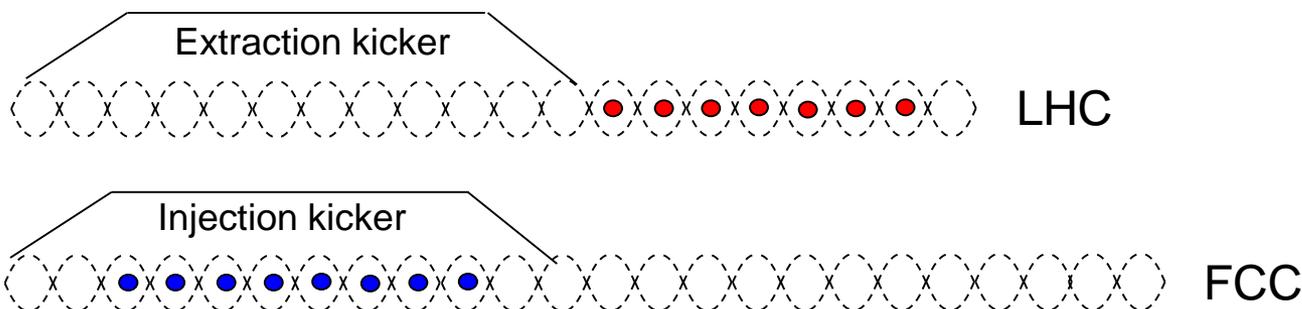
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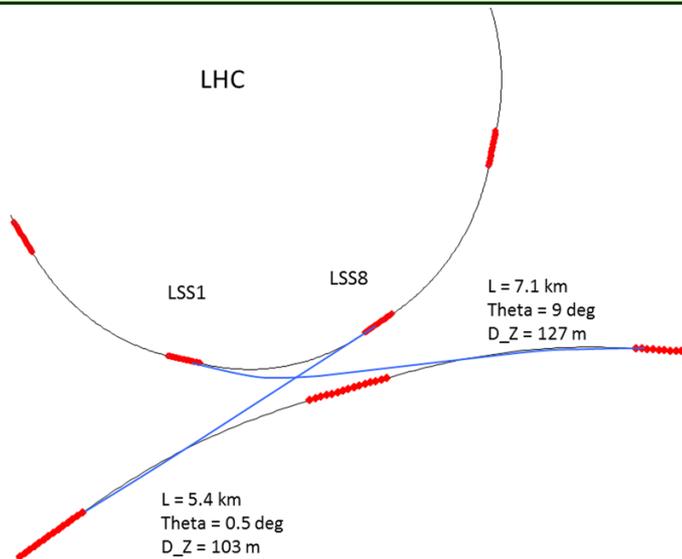
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$$f_c = \frac{f_{\text{rev}}^{\text{FCC}}}{C_{\text{LHC}}}$$

Recharging at **115 Hz**
(0.03 Hz for LHC injection kickers)

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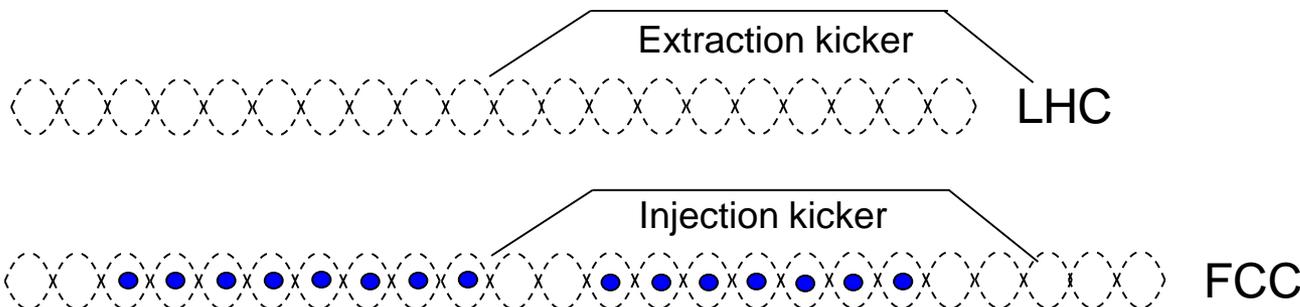
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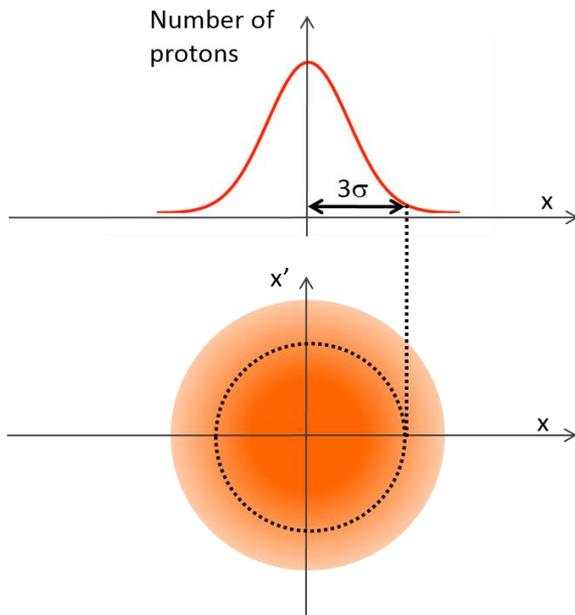
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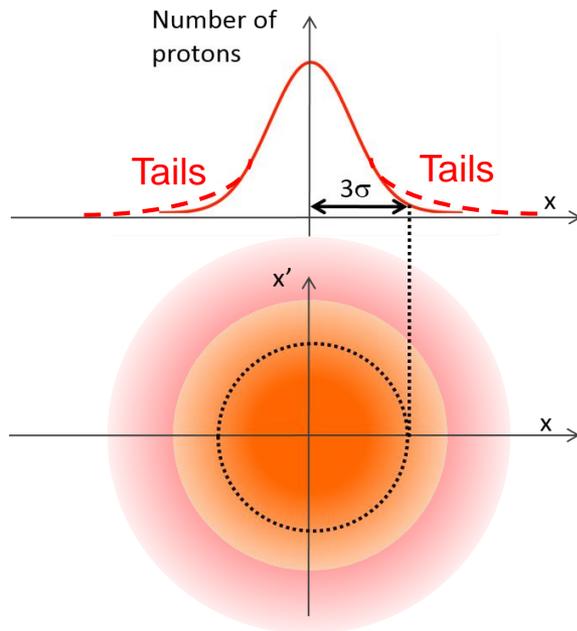
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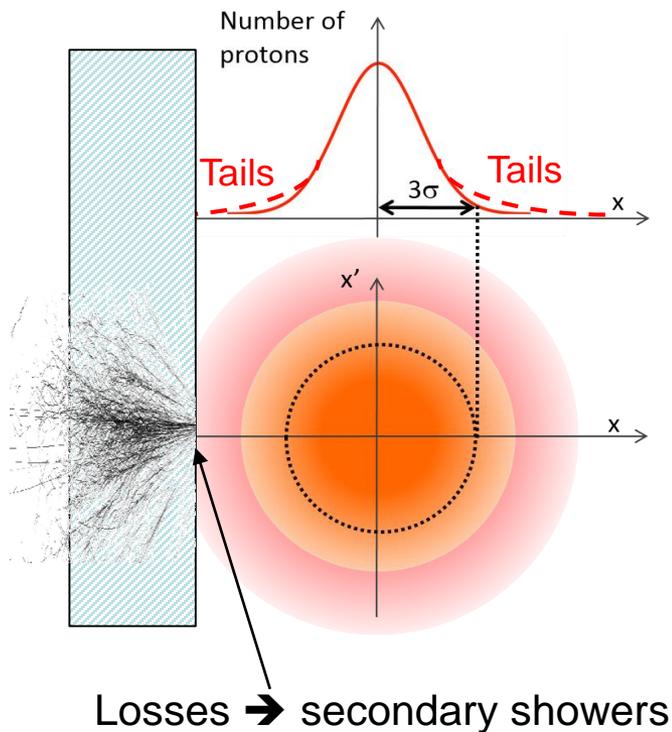
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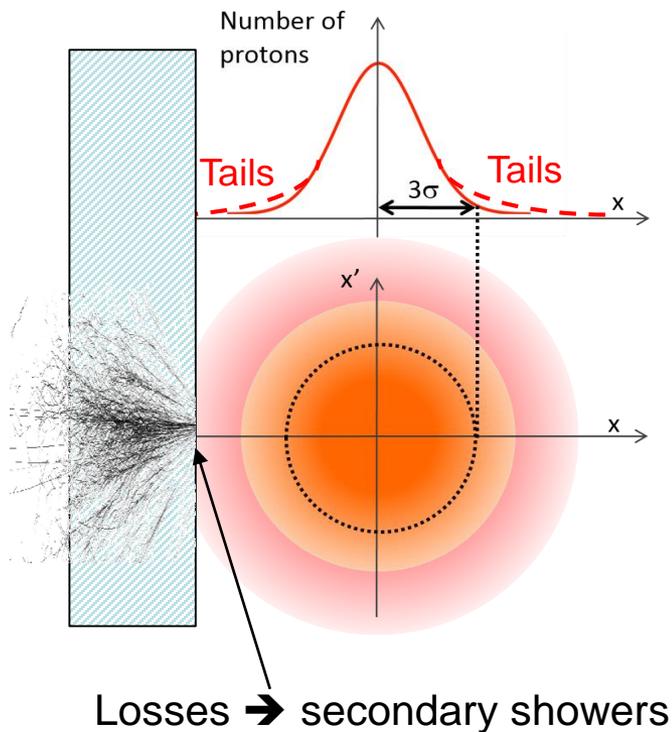
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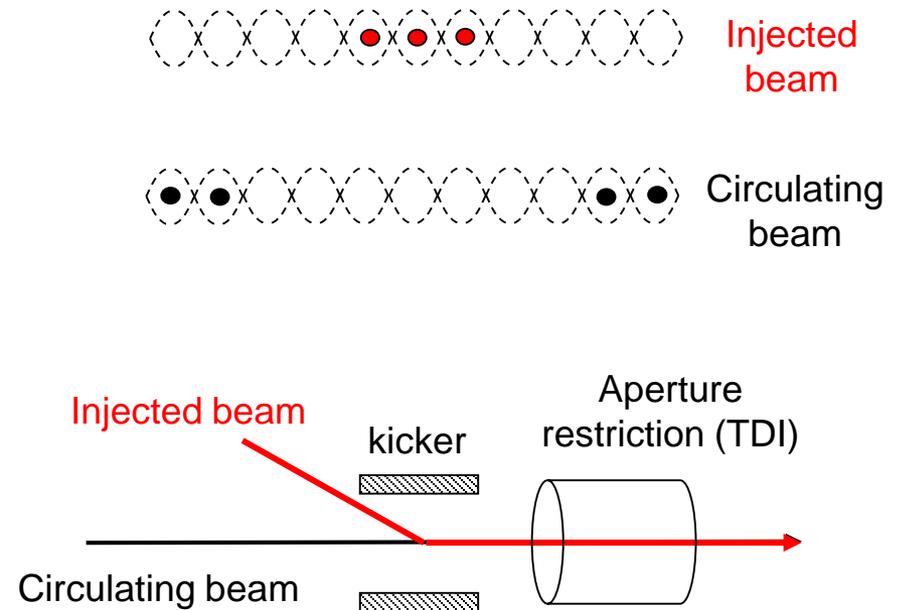
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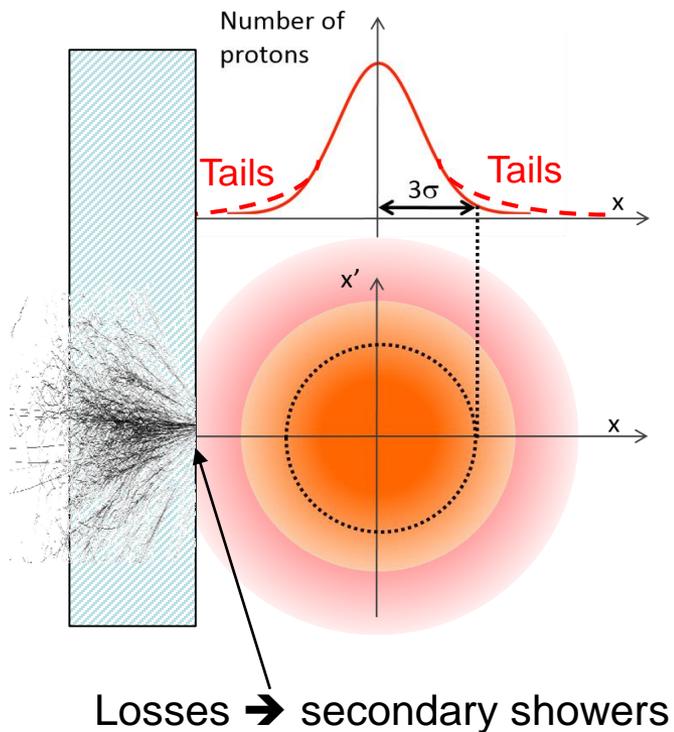
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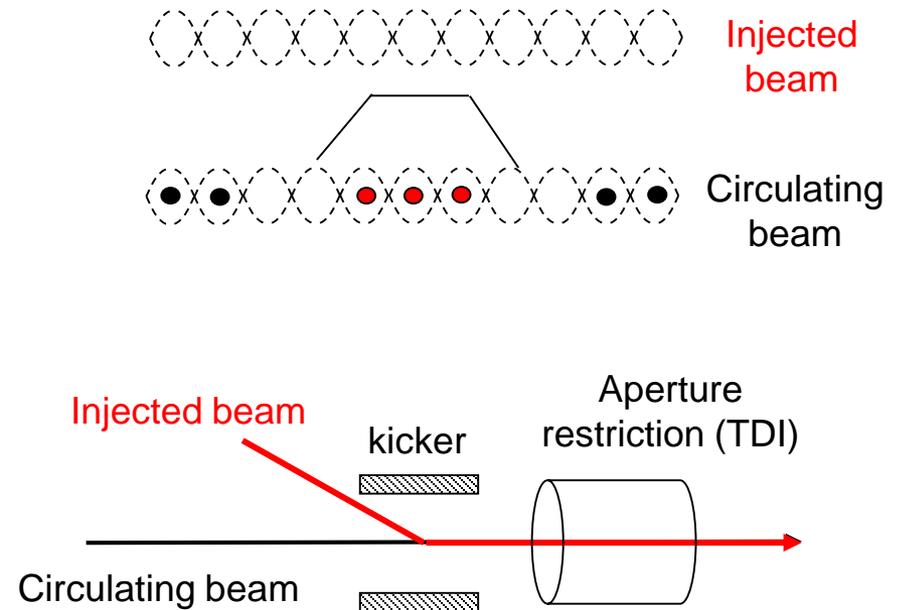
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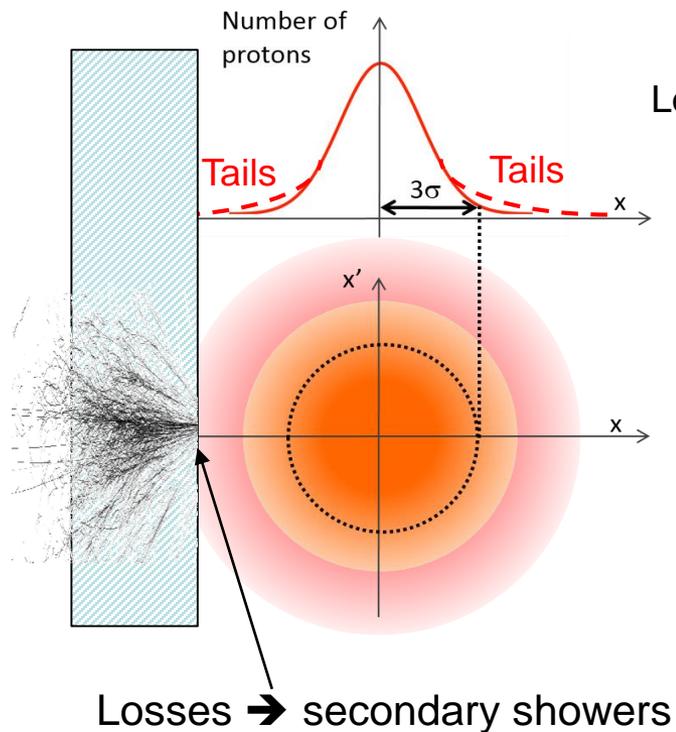
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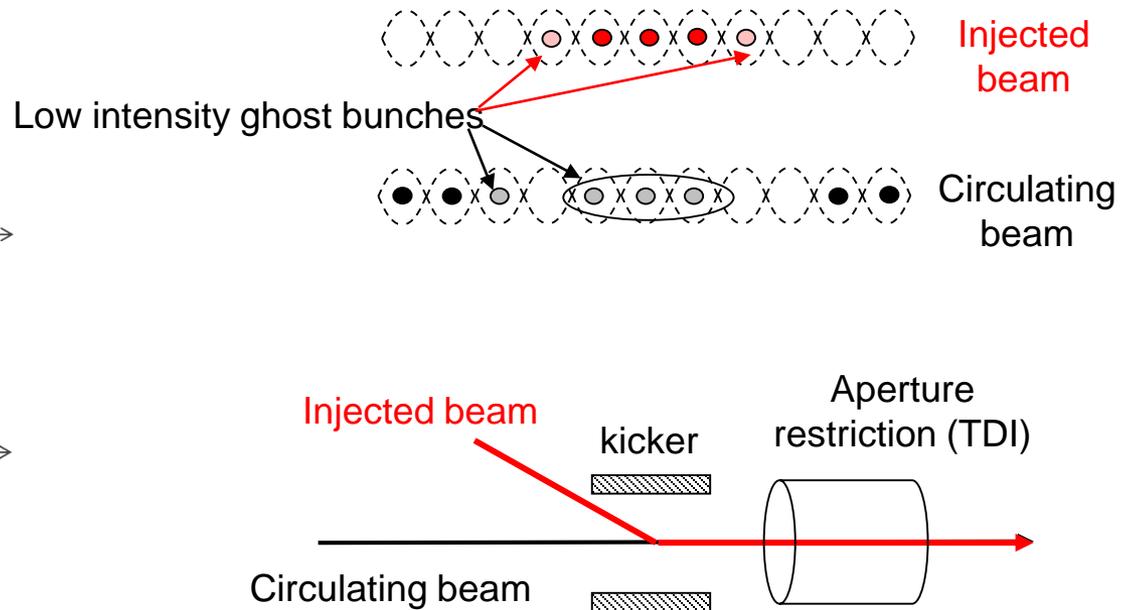
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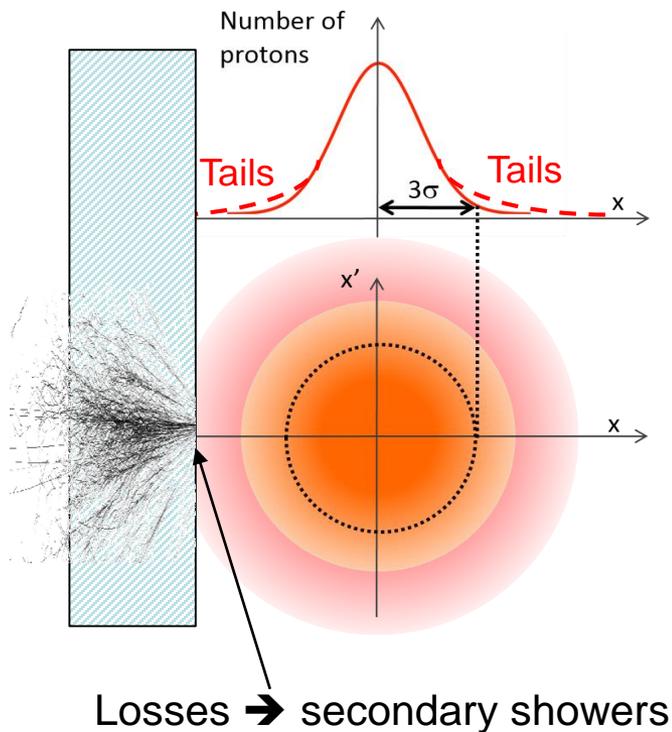
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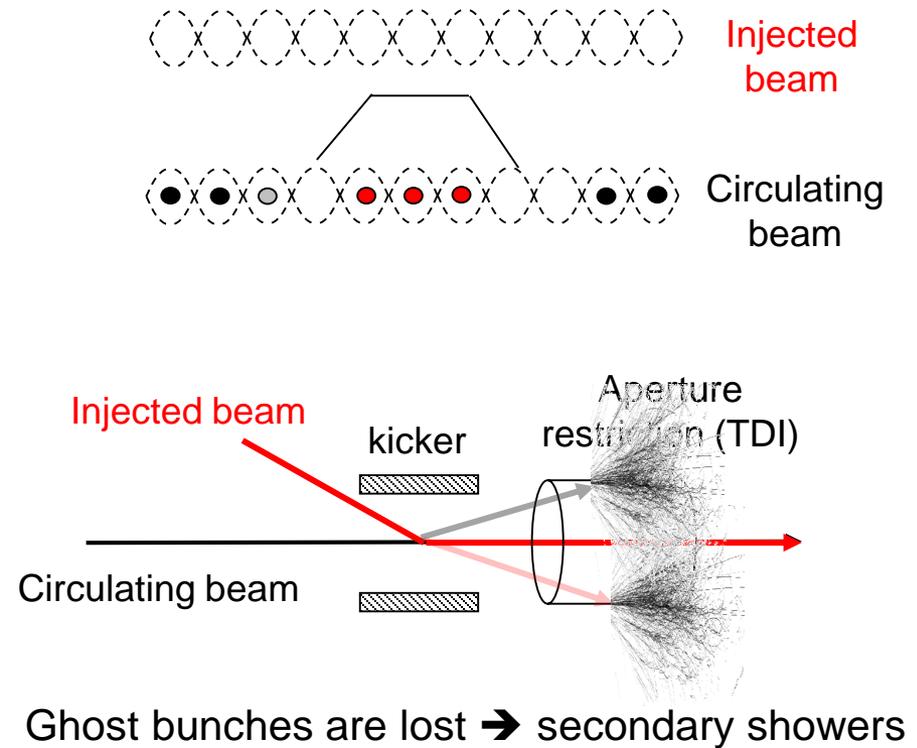
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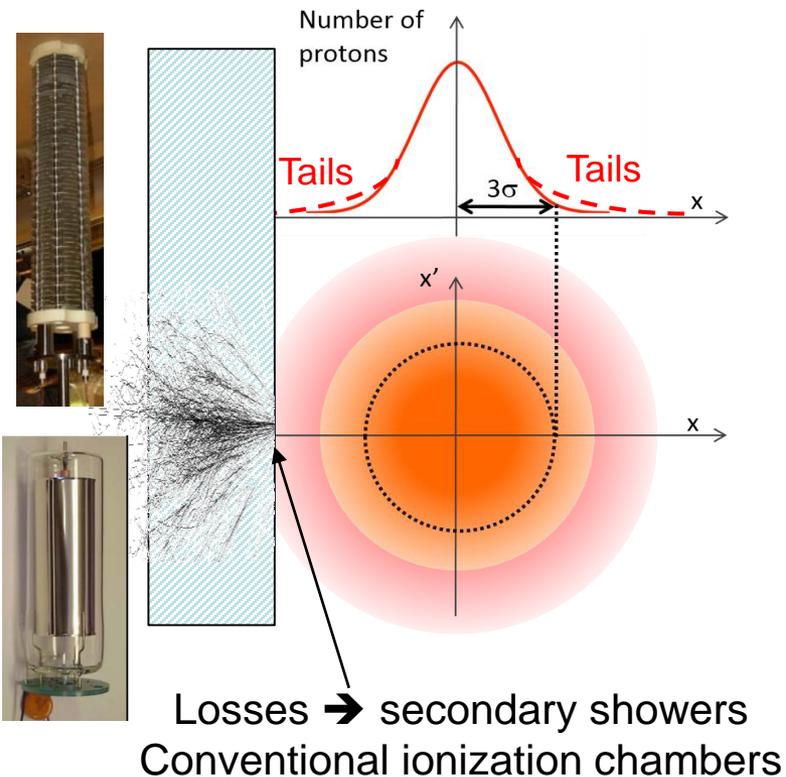
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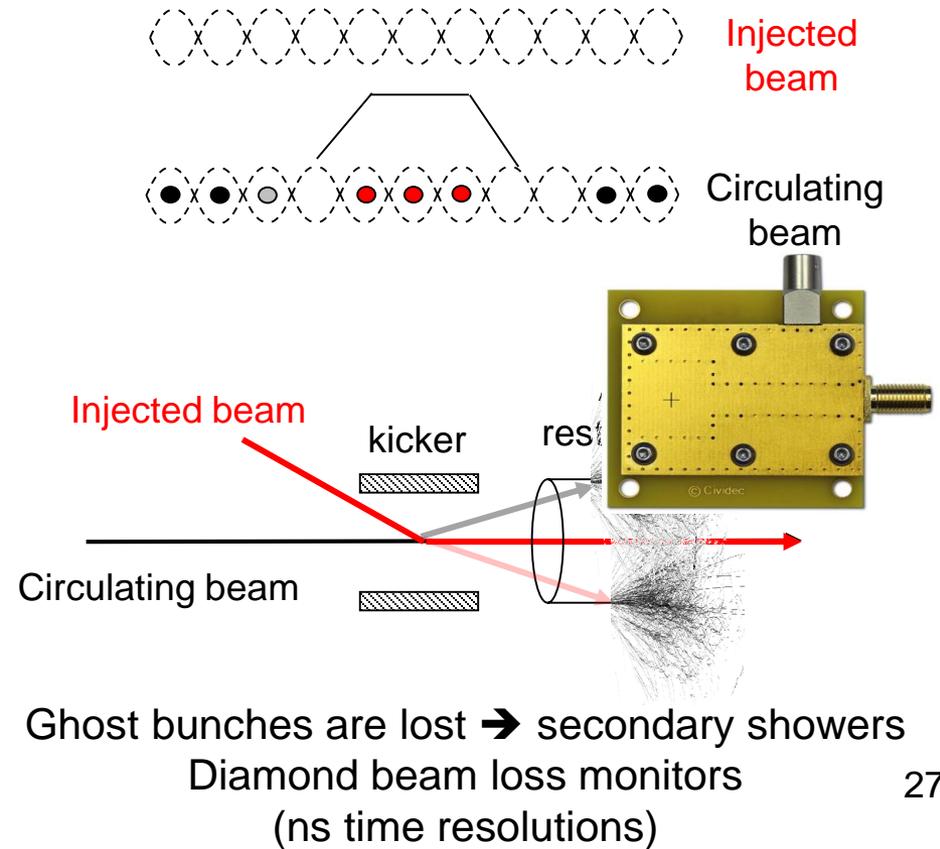
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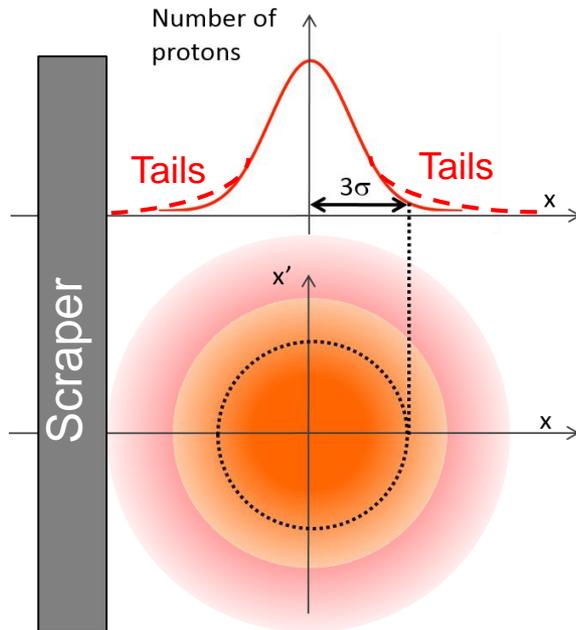
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Mitigations for losses

Optimize steering and RF settings!

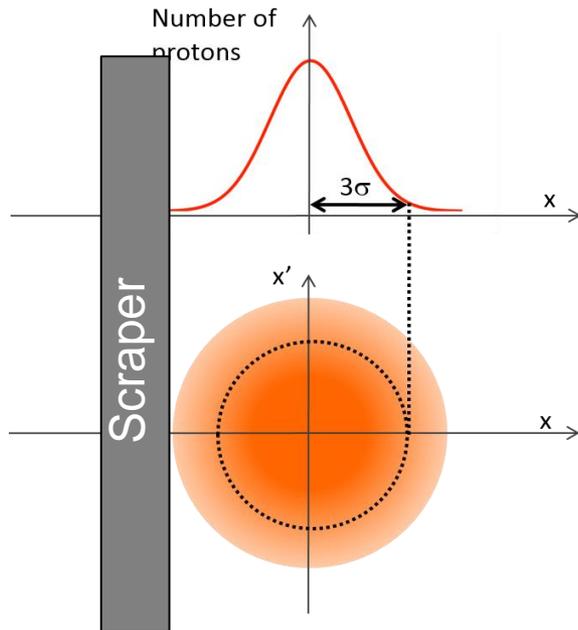
Transverse beam scraping in pre-injector
to remove tails before transfer and
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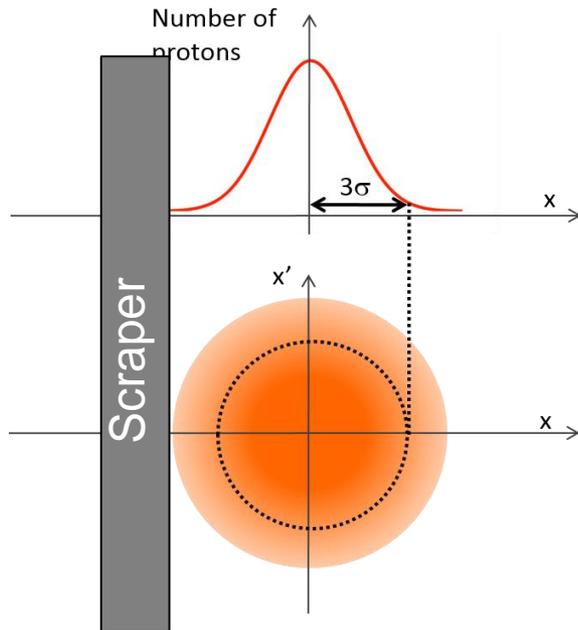
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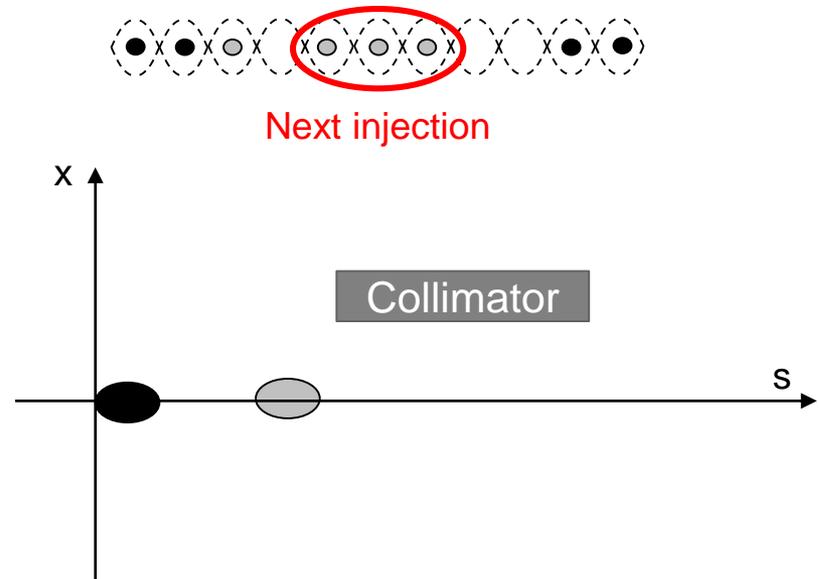
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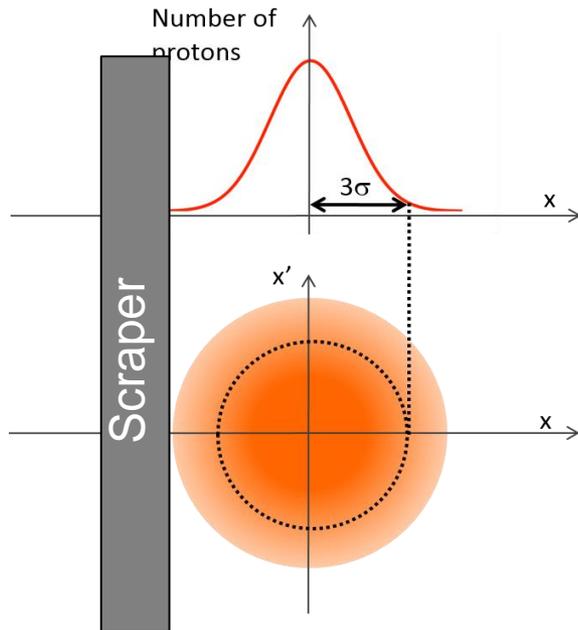
Injection cleaning: excite betatron oscillations (with transverse damper) of ghost bunches occupying the buckets where the beam has to be injected \rightarrow ghosts lost in cleaning insertions



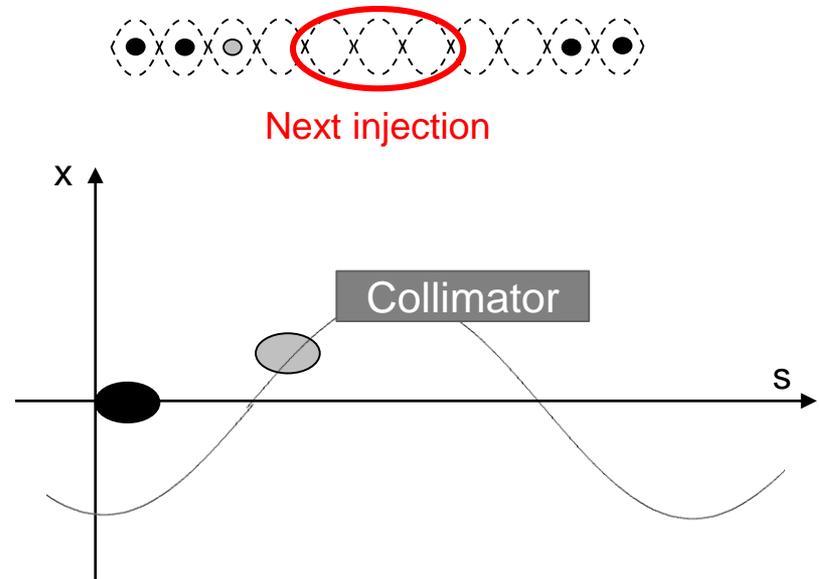
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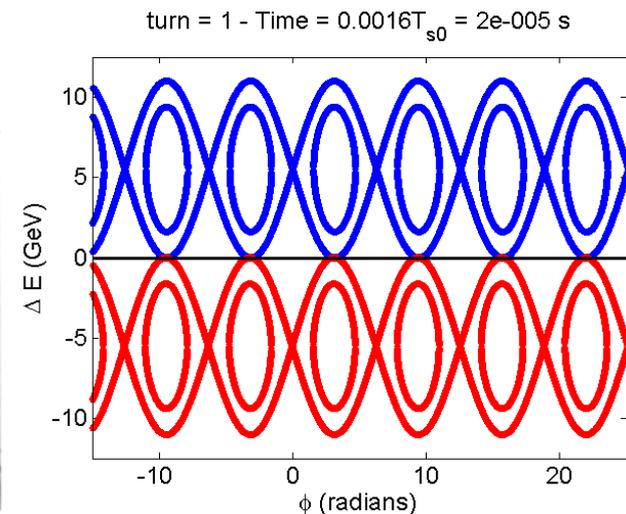
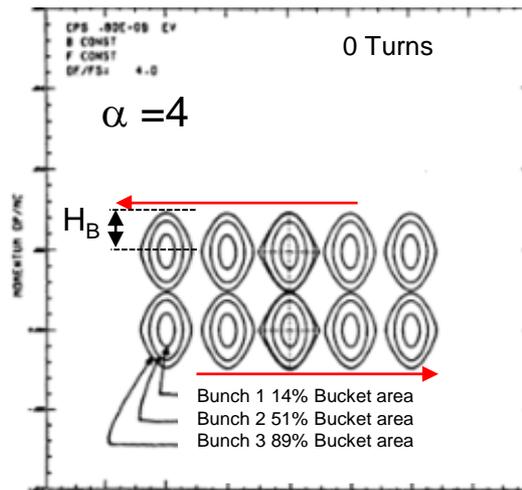
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Slip stacking

- It is a configuration used to store (and accelerate) particle beams with different momenta in the same accelerator. Demonstrated for the first time at the CERN SPS to increase the production of anti-protons for p-pbar physics [10]
- The two beams (b_1 and b_2) are longitudinally focused by two RF cavities with a small frequency difference ($\Delta f = f_2 - f_1$)
- Each beam is synchronized to one RF cavity ($b_1 \rightarrow f_1$ and $b_2 \rightarrow f_2$) and perturbed by the other ($f_1 \nrightarrow b_2$ and $f_2 \nrightarrow b_1$)
 - Slip-stacking parameter $\alpha = \Delta f / f_s = 2 \Delta E / H_B$ large perturbation when $\alpha \rightarrow 1$

Stationary buckets:
 $\alpha > 4$ to keep buckets independent, still emittance growth [11]



T. Argyropoulos

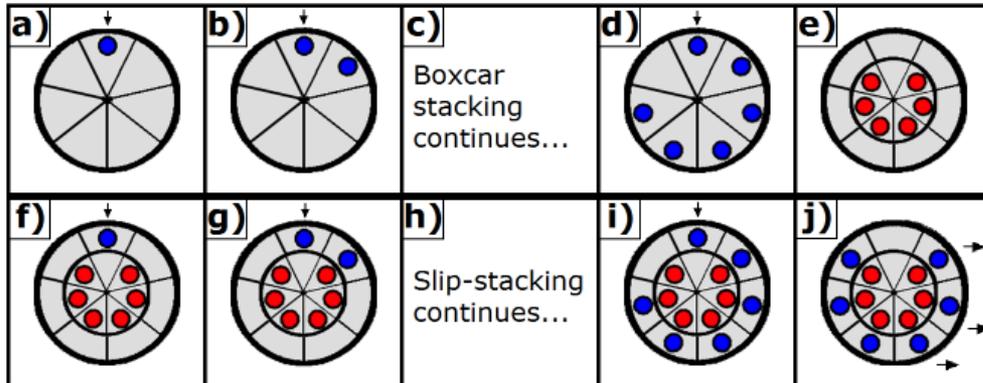
- The bunches rotate at different frequencies and will periodically coincide azimuthally producing high local line density. These bunches can be directly “used” or being combined in a large bucket with frequency $(f_2 + f_1) / 2$

Slip-Stacking at Fermilab

Fermilab used slip-stacking initially for \bar{p} production in the main injector, then in the Recycler to double the power of the proton beam. Aim: **increase the power** for target physics in particular **neutrino physics** (from 700 kW up to 1.2 MW and beyond)

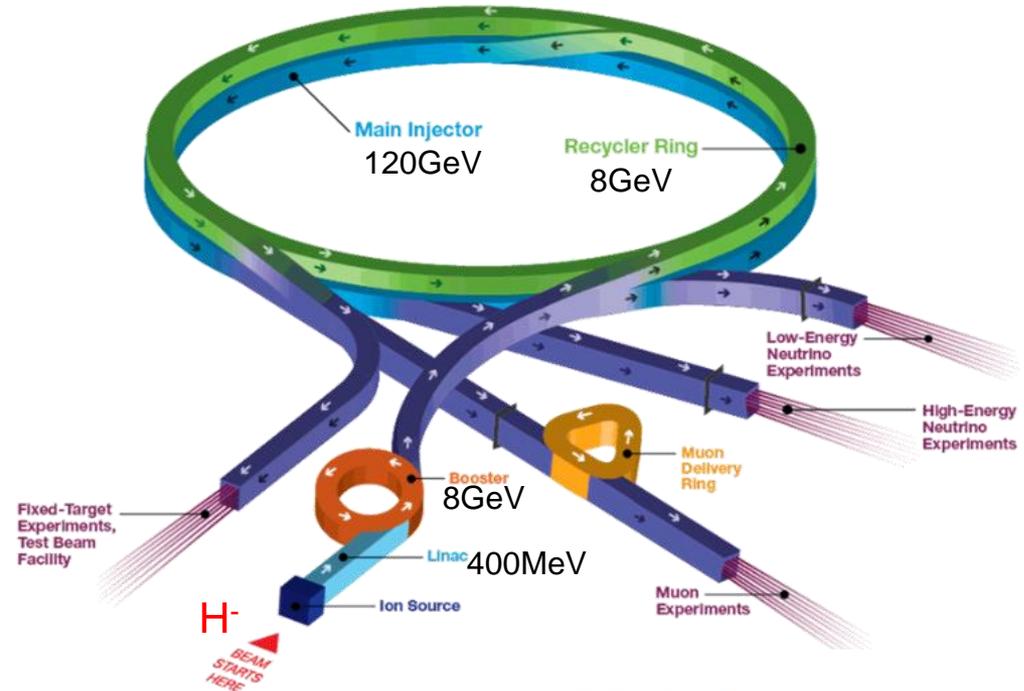
12 batches are accumulated in the recycler by overlapping azimuthally two beam with different momenta

[12]

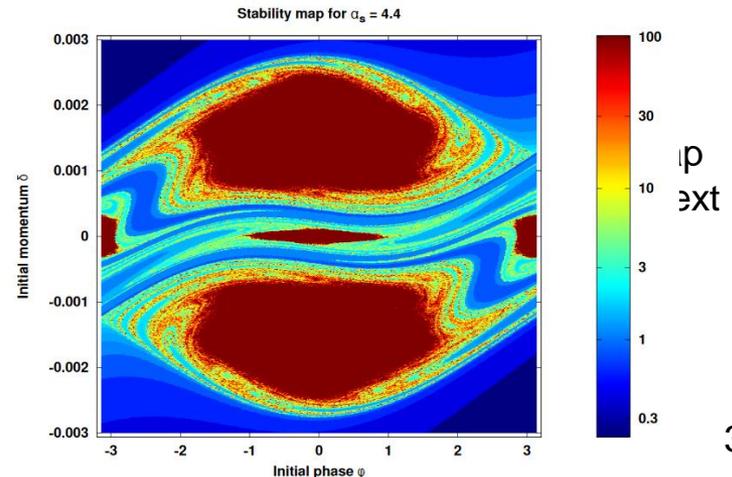


*Booster harmonic number and cycle rate

Fermilab Accelerator Complex



e) f) g) h) i) j) k) l) m) n) o) p) q) r) s) t) u) v) w) x) y) z) aa) ab) ac) ad) ae) af) ag) ah) ai) aj) ak) al) am) an) ao) ap) aq) ar) as) at) au) av) aw) ax) ay) az) ba) bb) bc) bd) be) bf) bg) bh) bi) bj) bk) bl) bm) bn) bo) bp) bq) br) bs) bt) bu) bv) bw) bx) by) bz) ca) cb) cc) cd) ce) cf) cg) ch) ci) cj) ck) cl) cm) cn) co) cp) cq) cr) cs) ct) cu) cv) cw) cx) cy) cz) da) db) dc) dd) de) df) dg) dh) di) dj) dk) dl) dm) dn) do) dp) dq) dr) ds) dt) du) dv) dw) dx) dy) dz) ea) eb) ec) ed) ee) ef) eg) eh) ei) ej) ek) el) em) en) eo) ep) eq) er) es) et) eu) ev) ew) ex) ey) ez) fa) fb) fc) fd) fe) ff) fg) fh) fi) fj) fk) fl) fm) fn) fo) fp) fq) fr) fs) ft) fu) fv) fw) fx) fy) fz) ga) gb) gc) gd) ge) gf) gg) gh) gi) gj) gk) gl) gm) gn) go) gp) gq) gr) gs) gt) gu) gv) gw) gx) gy) gz) ha) hb) hc) hd) he) hf) hg) hh) hi) hj) hk) hl) hm) hn) ho) hp) hq) hr) hs) ht) hu) hv) hw) hx) hy) hz) ia) ib) ic) id) ie) if) ig) ih) ii) ij) ik) il) im) in) io) ip) iq) ir) is) it) iu) iv) iw) ix) iy) iz) ja) jb) jc) jd) je) jf) jg) jh) ji) jj) jk) jl) jm) jn) jo) jp) jq) jr) js) jt) ju) jv) jw) jx) jy) jz) ka) kb) kc) kd) ke) kf) kg) kh) ki) kj) kk) kl) km) kn) ko) kp) kq) kr) ks) kt) ku) kv) kw) kx) ky) kz) la) lb) lc) ld) le) lf) lg) lh) li) lj) lk) ll) lm) ln) lo) lp) lq) lr) ls) lt) lu) lv) lw) lx) ly) lz) ma) mb) mc) md) me) mf) mg) mh) mi) mj) mk) ml) mm) mn) mo) mp) mq) mr) ms) mt) mu) mv) mw) mx) my) mz) na) nb) nc) nd) ne) nf) ng) nh) ni) nj) nk) nl) nm) nn) no) np) nq) nr) ns) nt) nu) nv) nw) nx) ny) nz) oa) ob) oc) od) oe) of) og) oh) oi) oj) ok) ol) om) on) oo) op) oq) or) os) ot) ou) ov) ow) ox) oy) oz) pa) pb) pc) pd) pe) pf) pg) ph) pi) pj) pk) pl) pm) pn) po) pp) pq) pr) ps) pt) pu) pv) pw) px) py) pz) qa) qb) qc) qd) qe) qf) qg) qh) qi) qj) qk) ql) qm) qn) qo) qp) qq) qr) qs) qt) qu) qv) qw) qx) qy) qz) ra) rb) rc) rd) re) rf) rg) rh) ri) rj) rk) rl) rm) rn) ro) rp) rq) rr) rs) rt) ru) rv) rw) rx) ry) rz) sa) sb) sc) sd) se) sf) sg) sh) si) sj) sk) sl) sm) sn) so) sp) sq) sr) ss) st) su) sv) sw) sx) sy) sz) ta) tb) tc) td) te) tf) tg) th) ti) tj) tk) tl) tm) tn) to) tp) tq) tr) ts) tt) tu) tv) tw) tx) ty) tz) ua) ub) uc) ud) ue) uf) ug) uh) ui) uj) uk) ul) um) un) uo) up) uq) ur) us) ut) uu) uv) uw) ux) uy) uz) va) vb) vc) vd) ve) vf) vg) vh) vi) vj) vk) vl) vm) vn) vo) vp) vq) vr) vs) vt) vu) vv) vw) vx) vy) vz) wa) wb) wc) wd) we) wf) wg) wh) wi) wj) wk) wl) wm) wn) wo) wp) wq) wr) ws) wt) wu) wv) ww) wx) wy) wz) xa) xb) xc) xd) xe) xf) xg) xh) xi) xj) xk) xl) xm) xn) xo) xp) xq) xr) xs) xt) xu) xv) xw) xx) xy) xz) ya) yb) yc) yd) ye) yf) yg) yh) yi) yj) yk) yl) ym) yn) yo) yp) yq) yr) ys) yt) yu) yv) yw) yx) yy) yz) za) zb) zc) zd) ze) zf) zg) zh) zi) zj) zk) zl) zm) zn) zo) zp) zq) zr) zs) zt) zu) zv) zw) zx) zy) zz)

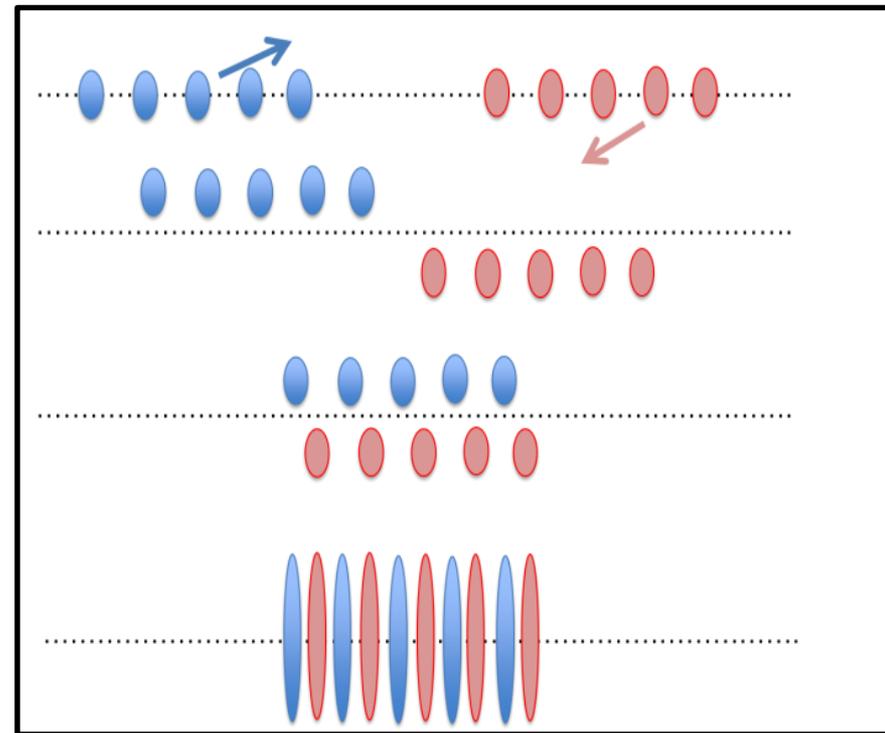


Slip-stacking at CERN

Momentum slip-stacking will be used in the SPS to increase the number of bunches for HL-LHC ion physics

T. Argyropoulos

- Two super-batches (24 bunches separated by 100 ns) injected into the SPS from the PS
- The two super-batches are captured by two pairs of 200 MHz cavities (independent control)
- RF frequency variation to accelerate the first batch and decelerate the second
- Let the batches slip
- Bring them back by decelerating the first and accelerating the second
- Once the bunches are interleaved they are recaptured at average RF frequency

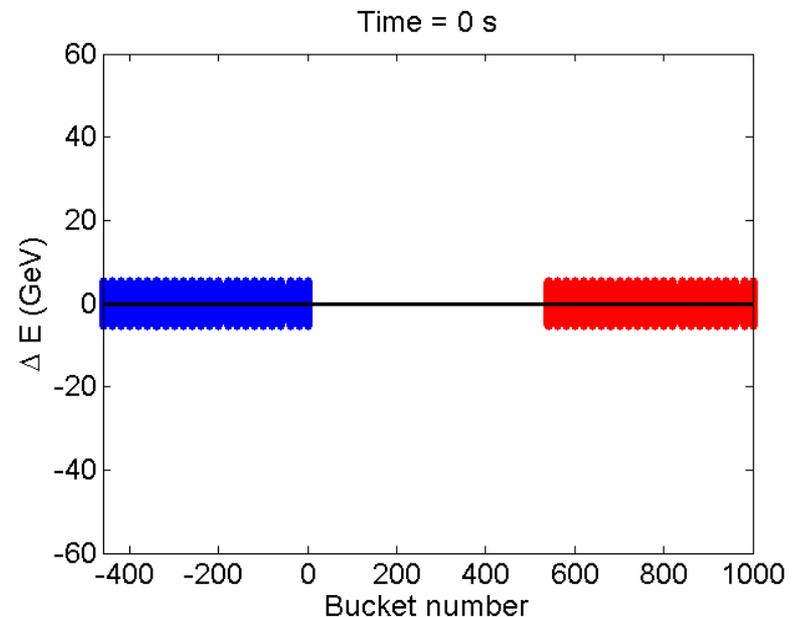


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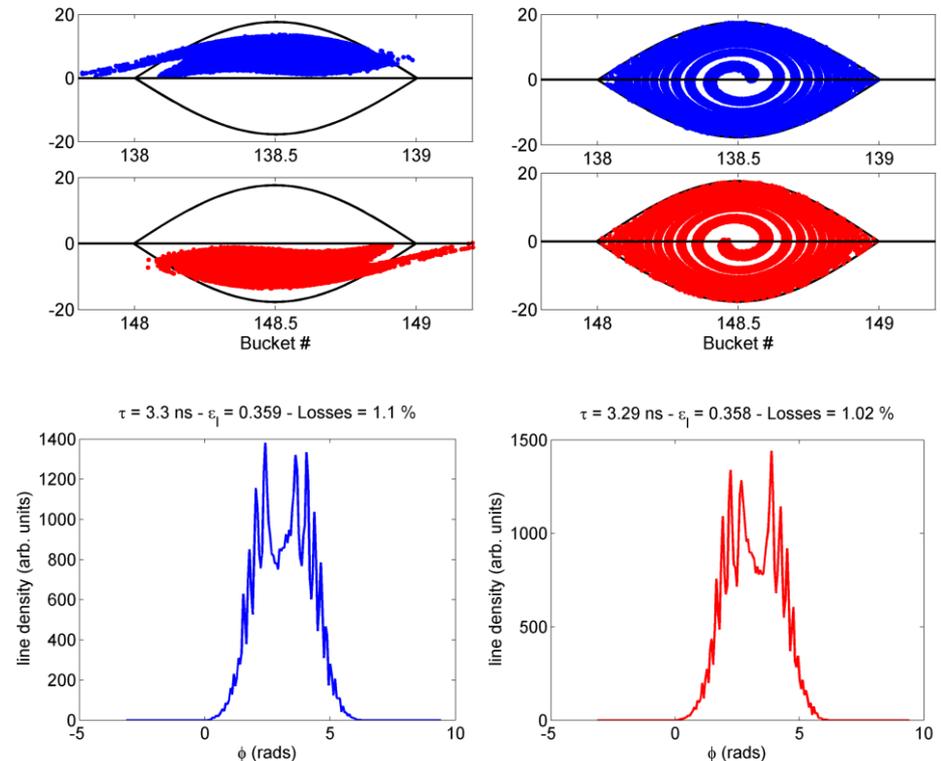


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X3 emittance blowup after recapture

Multi-turn injection

- For hadrons the beam density at injection can be limited either by space charge effects or by the injector capacity
- If we cannot increase charge density, we can sometimes fill the horizontal phase space to increase overall injected intensity.
 - If the acceptance of the receiving machine is larger than the delivered beam emittance we can accumulate intensity
- Proton and heavy-ion distributions are function of the details of the multi-turn process and of the space charge level achieved.

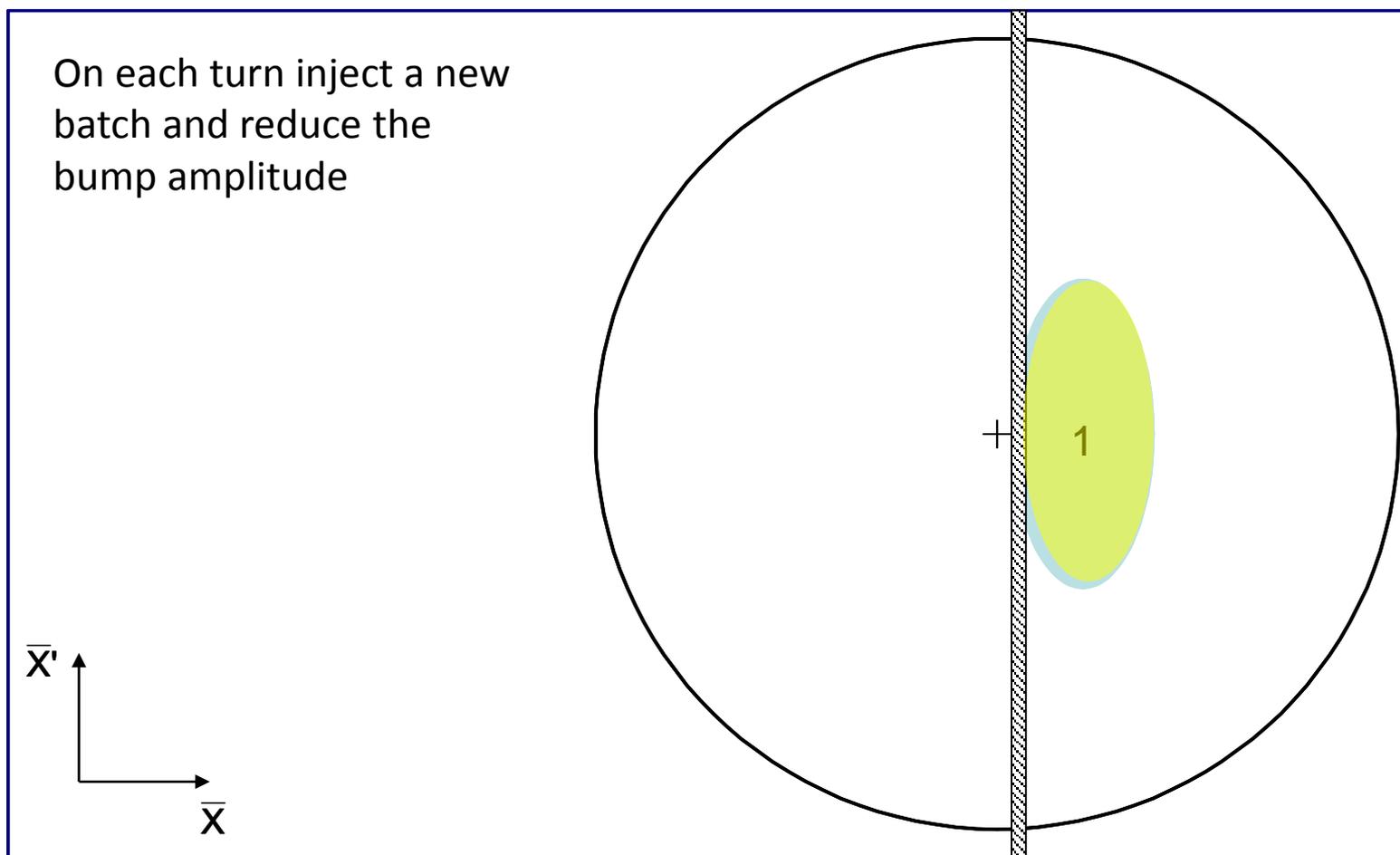
Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 1

On each turn inject a new batch and reduce the bump amplitude



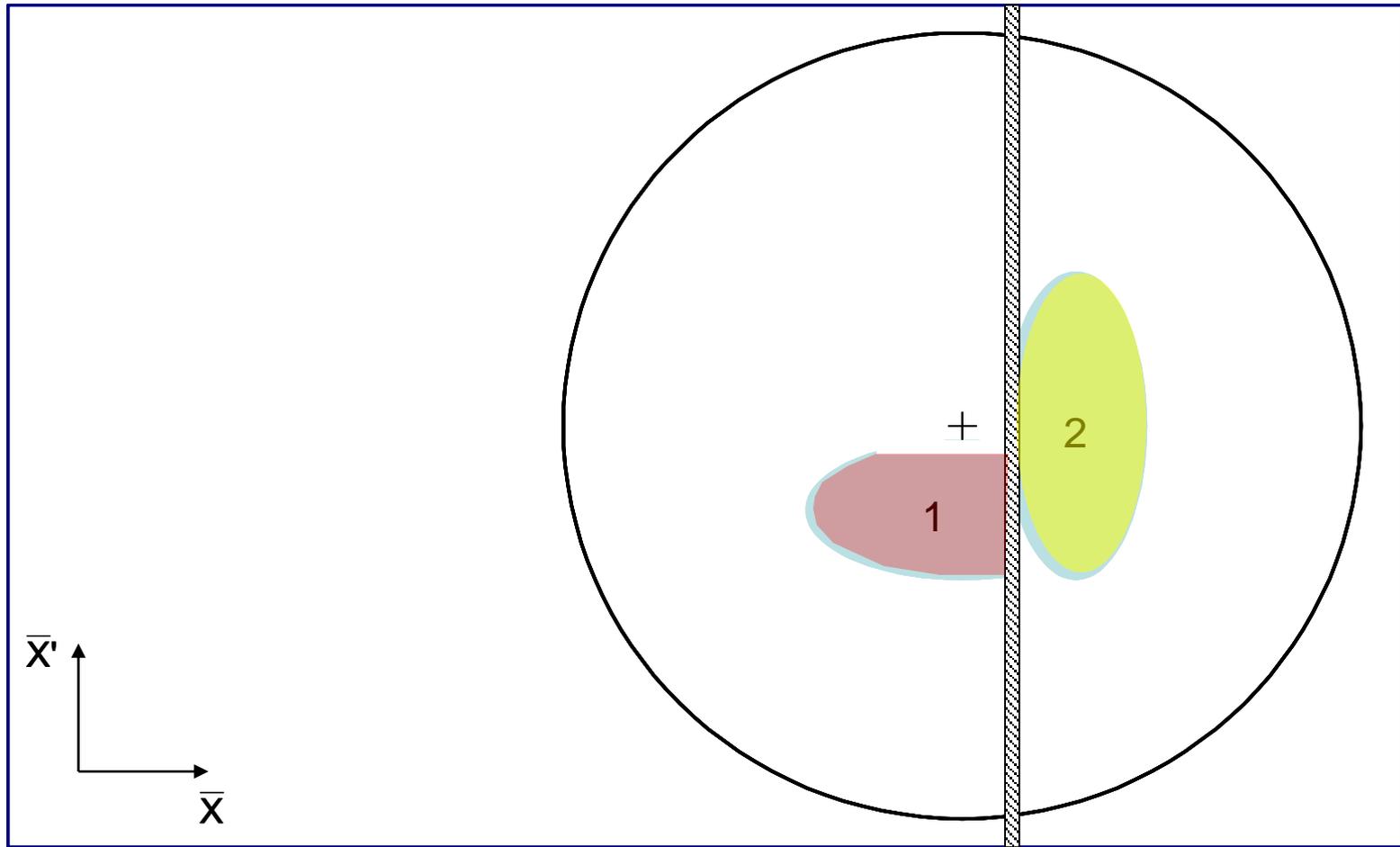
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 2



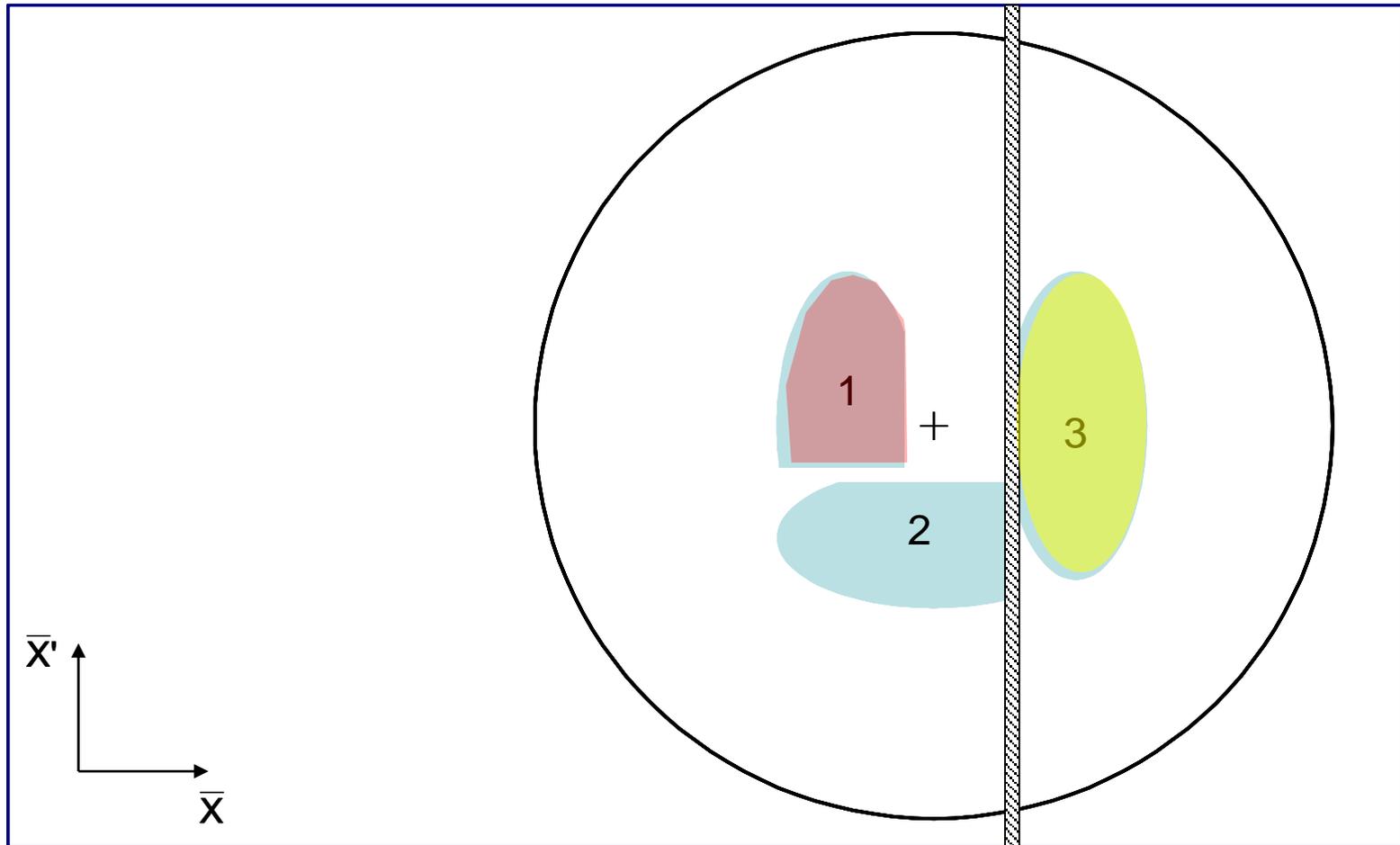
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 3



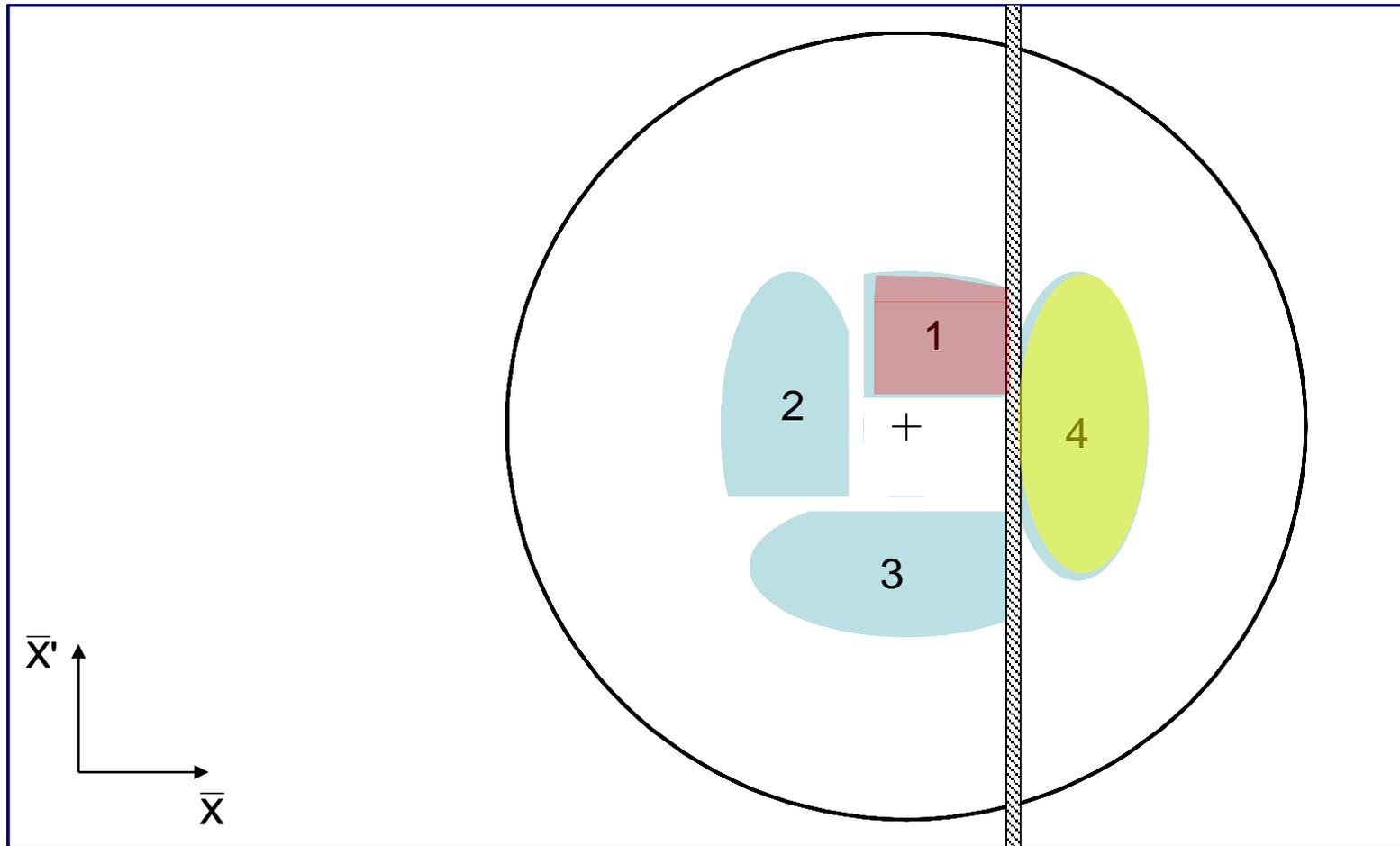
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 4



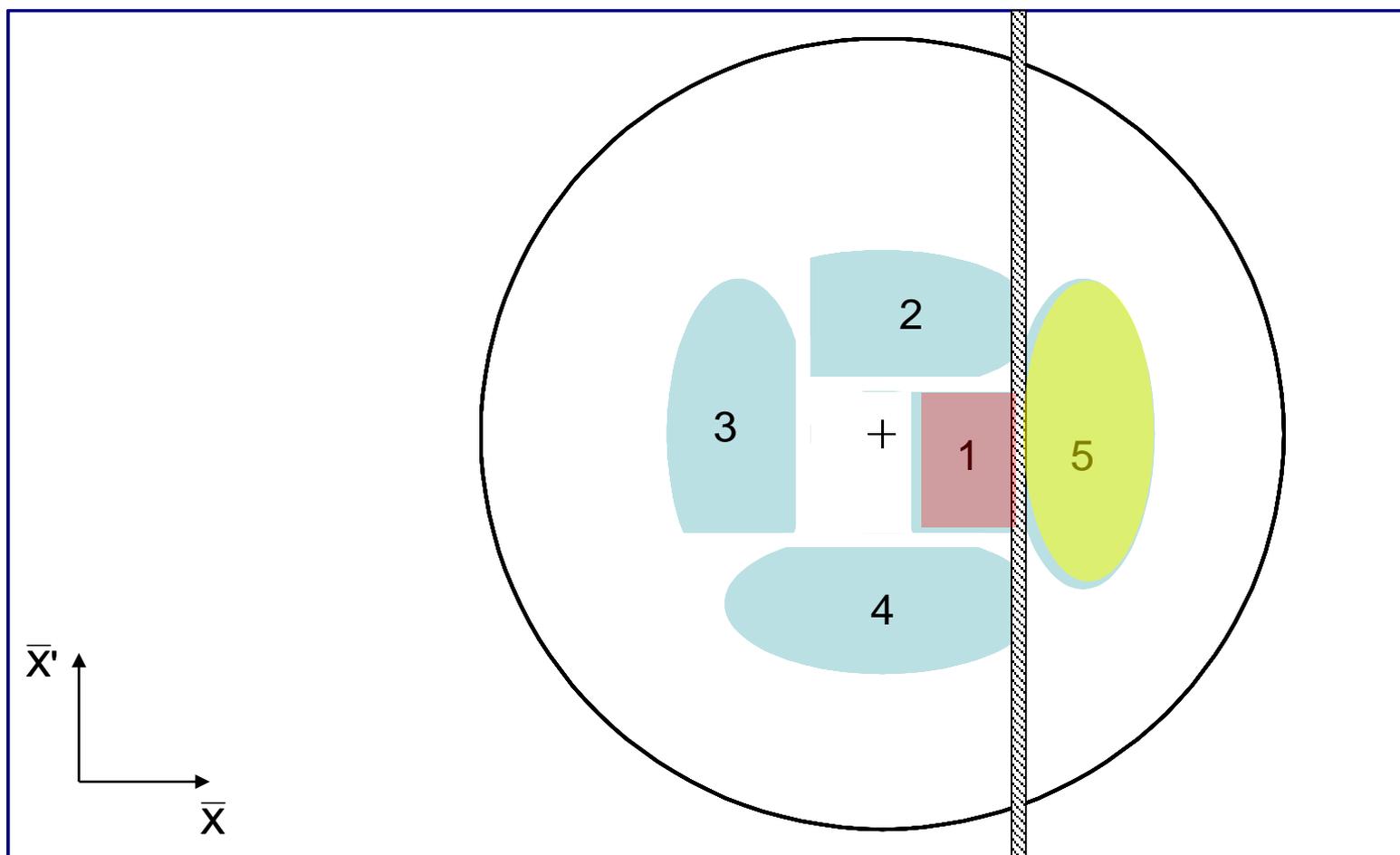
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 5



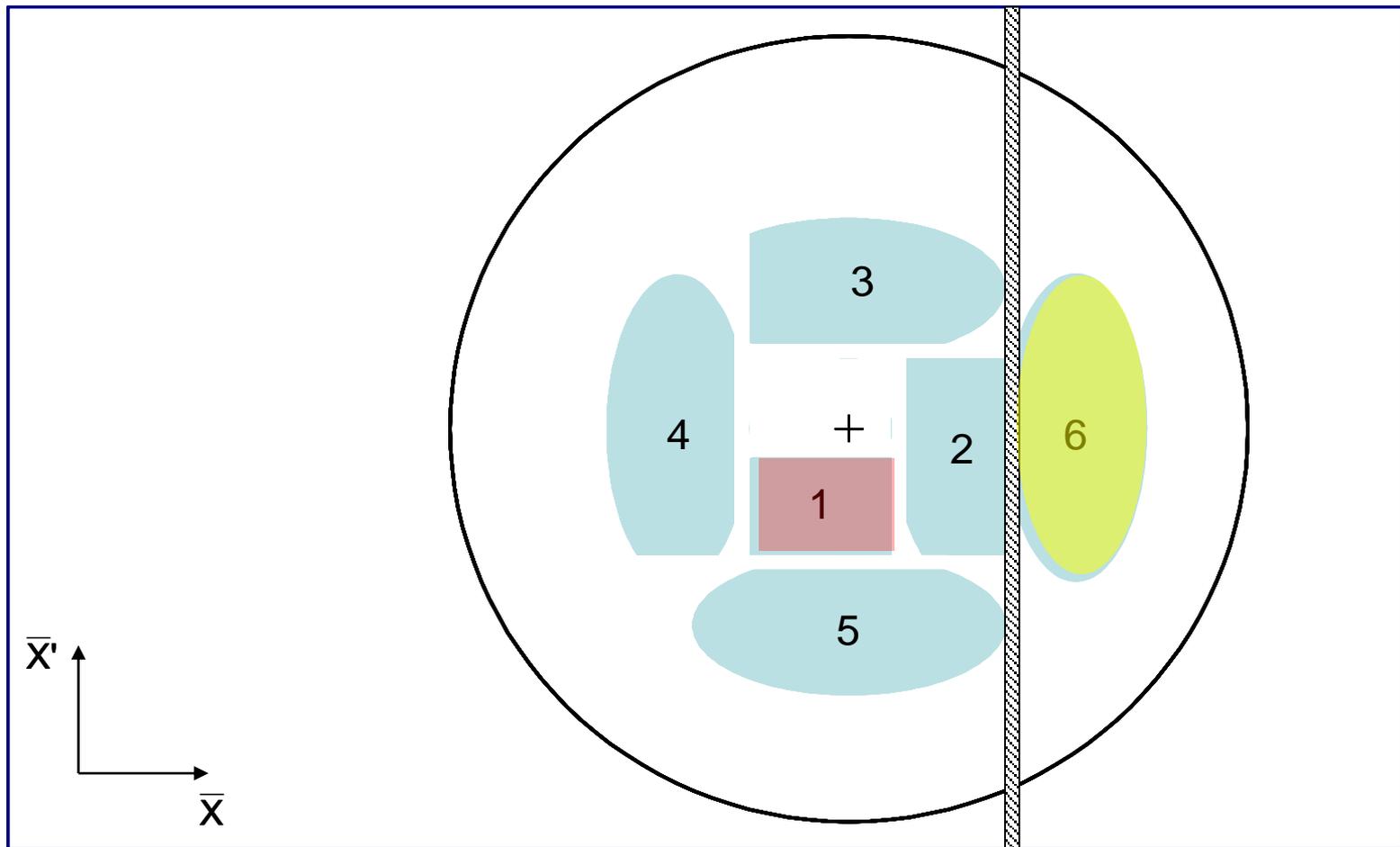
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 6



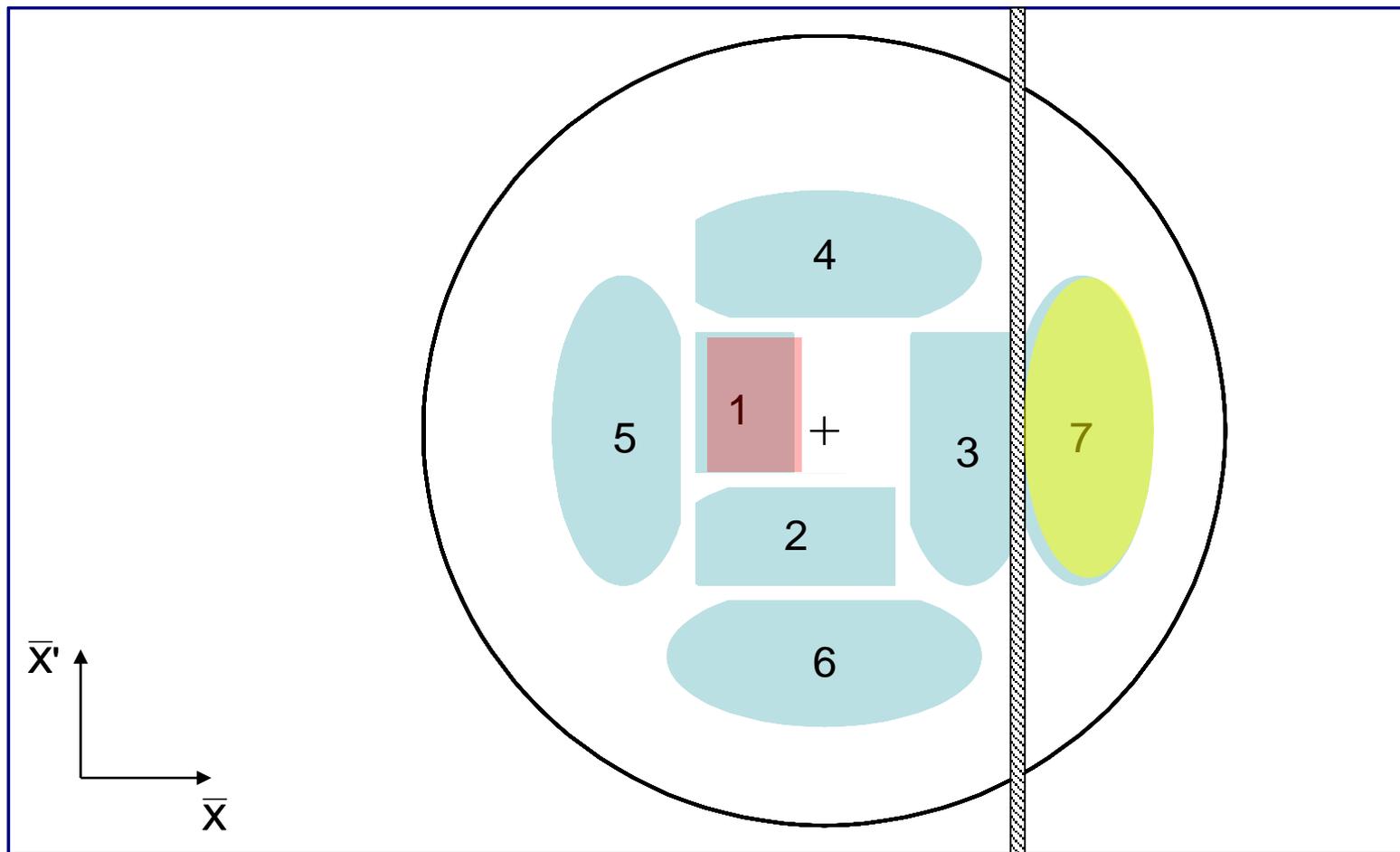
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 7



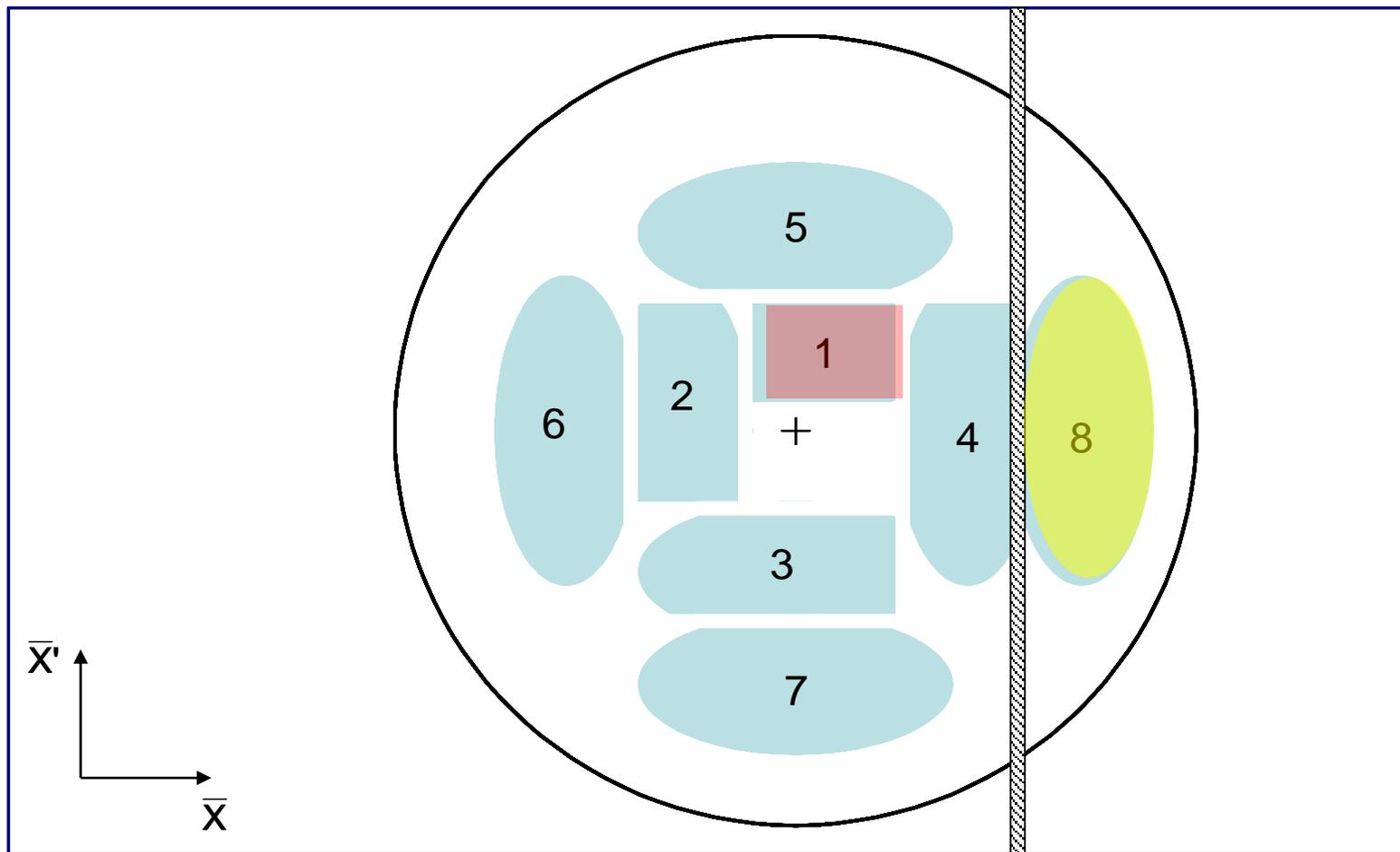
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 8



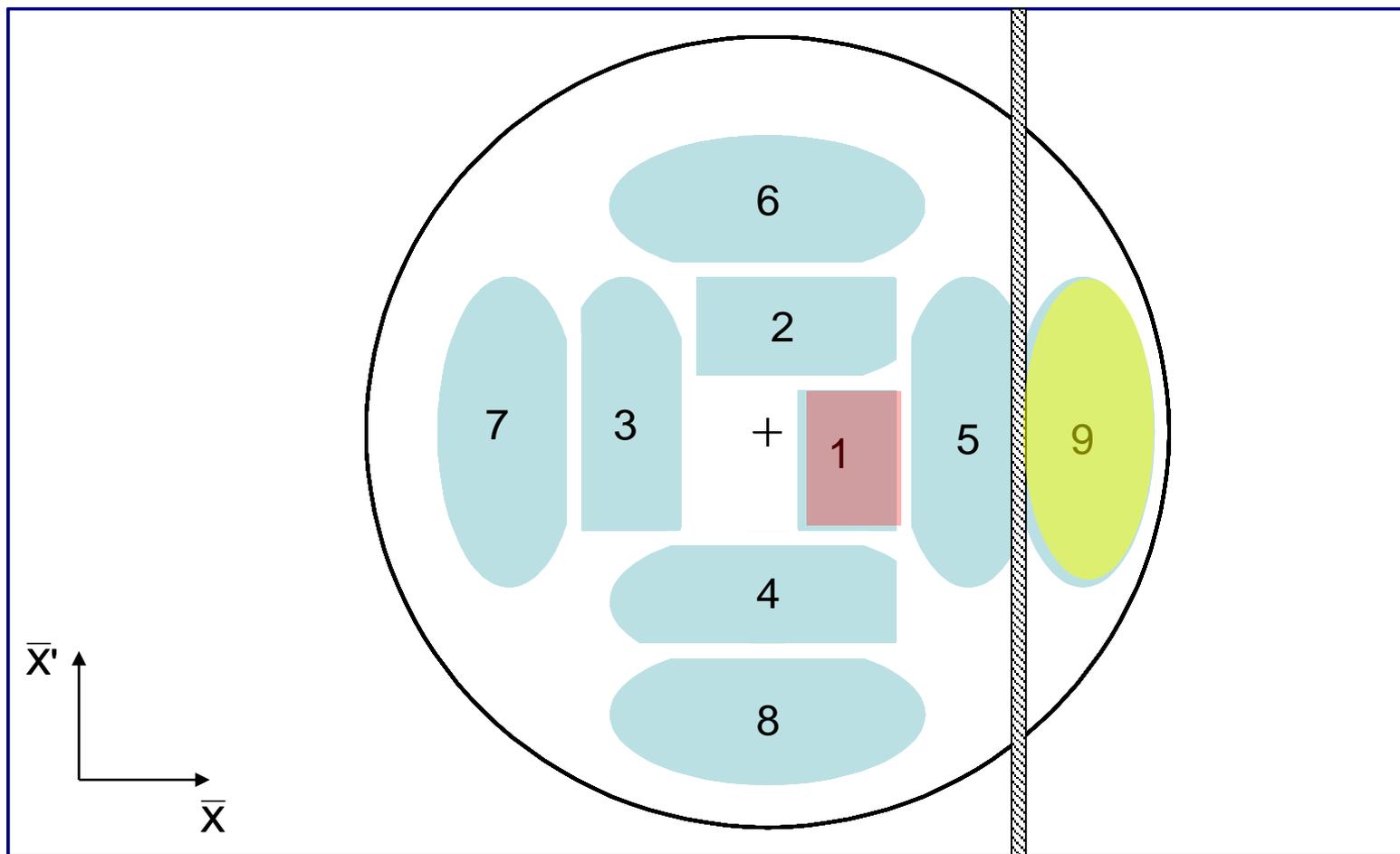
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 9



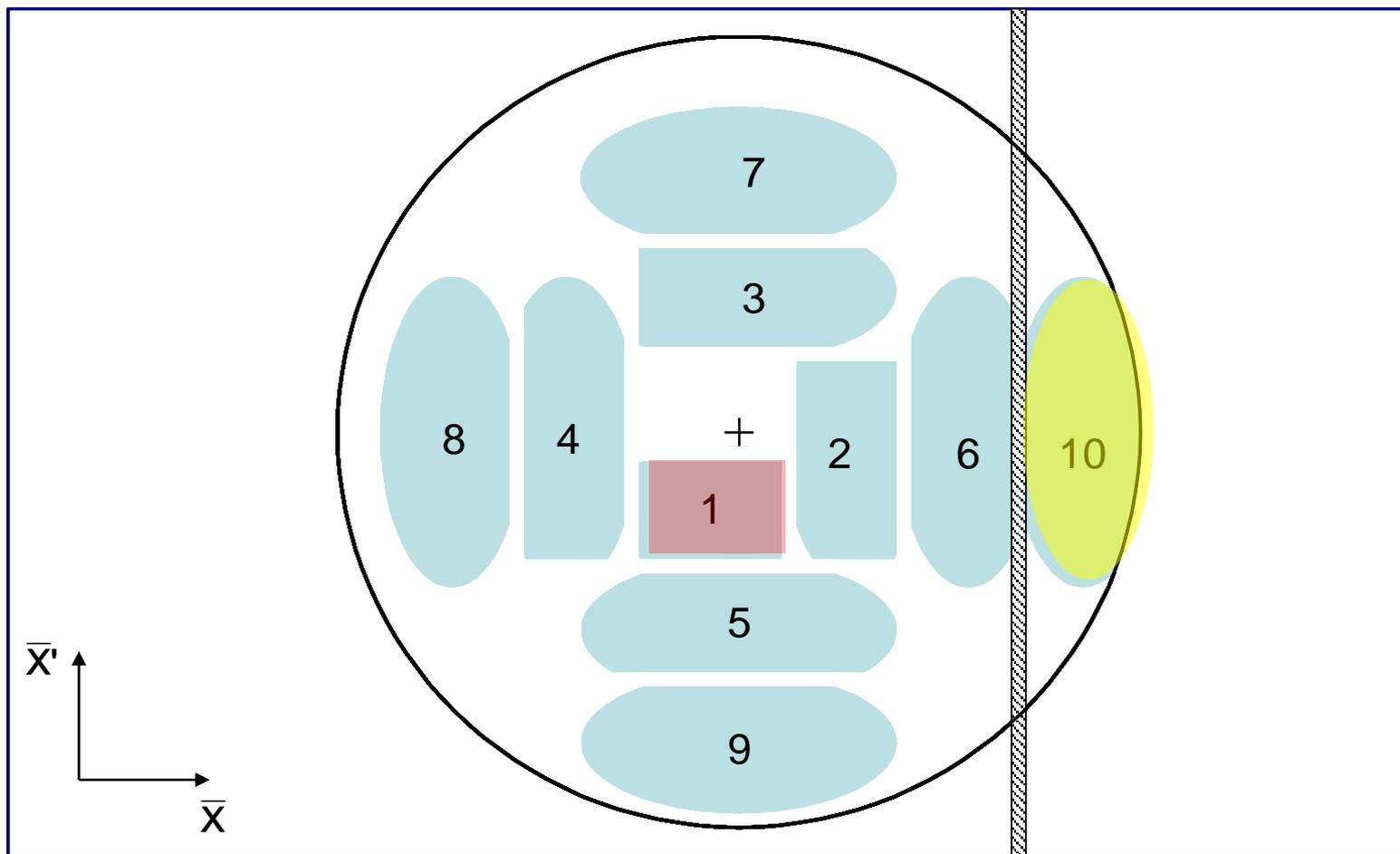
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 10



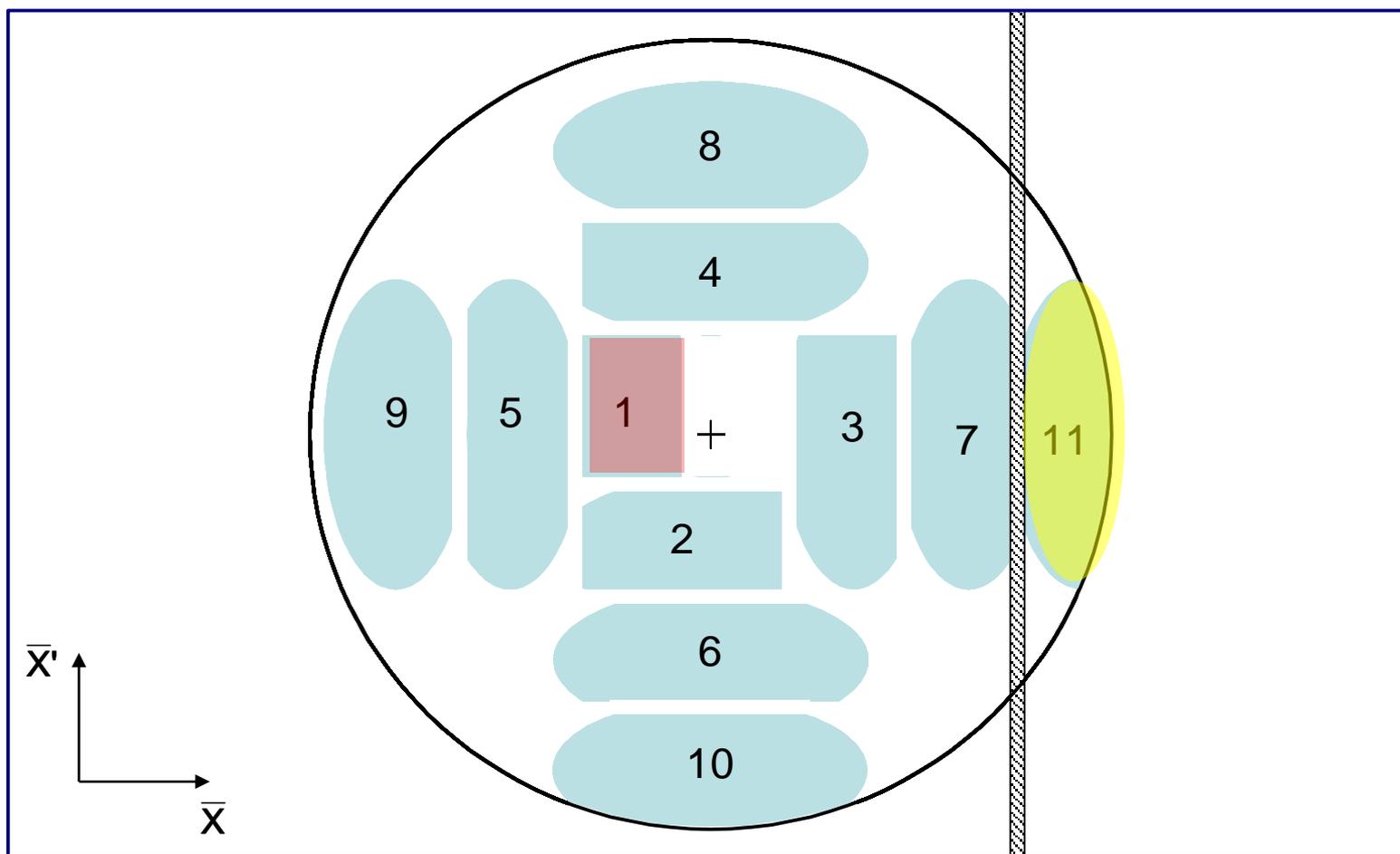
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 11



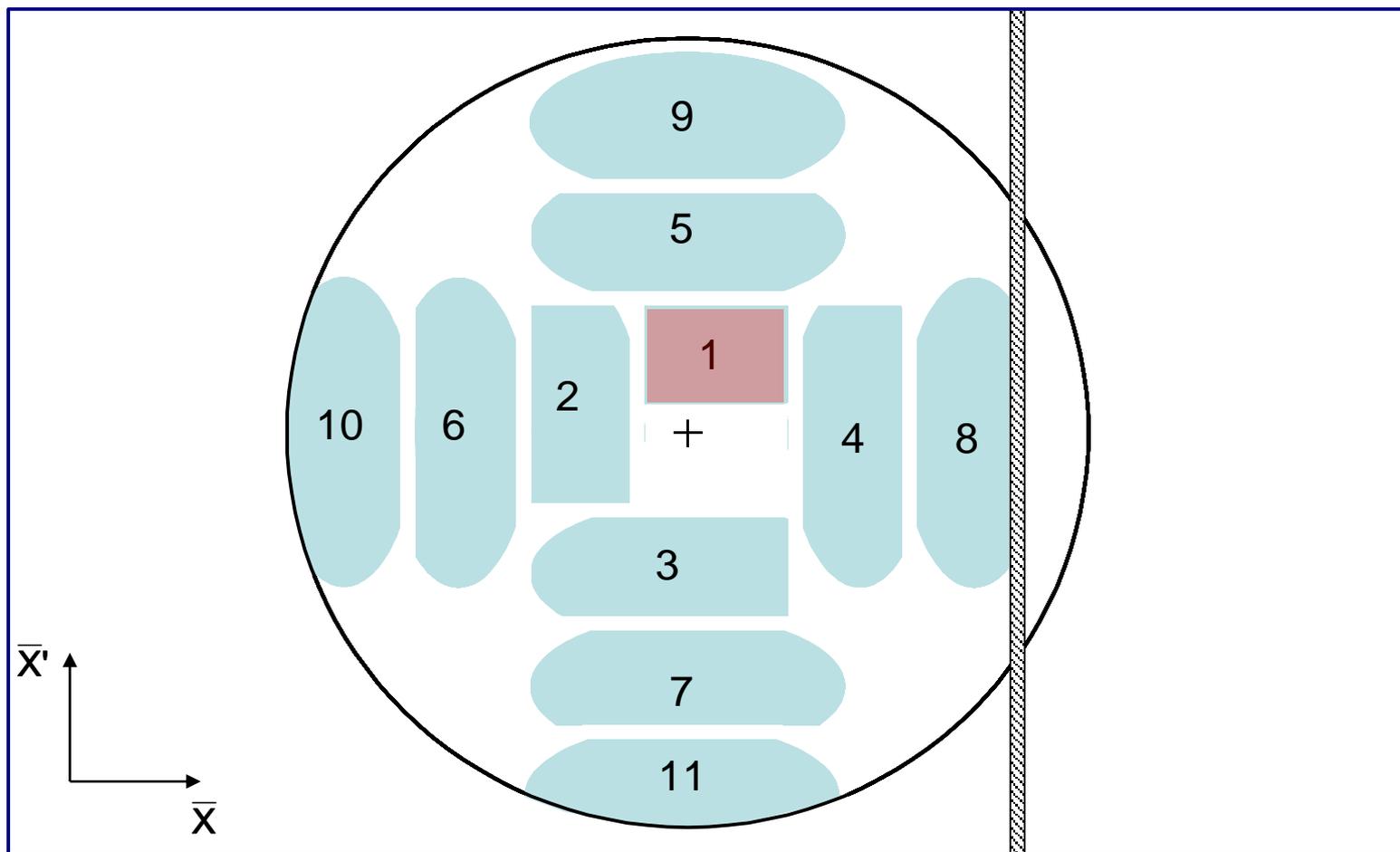
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 12



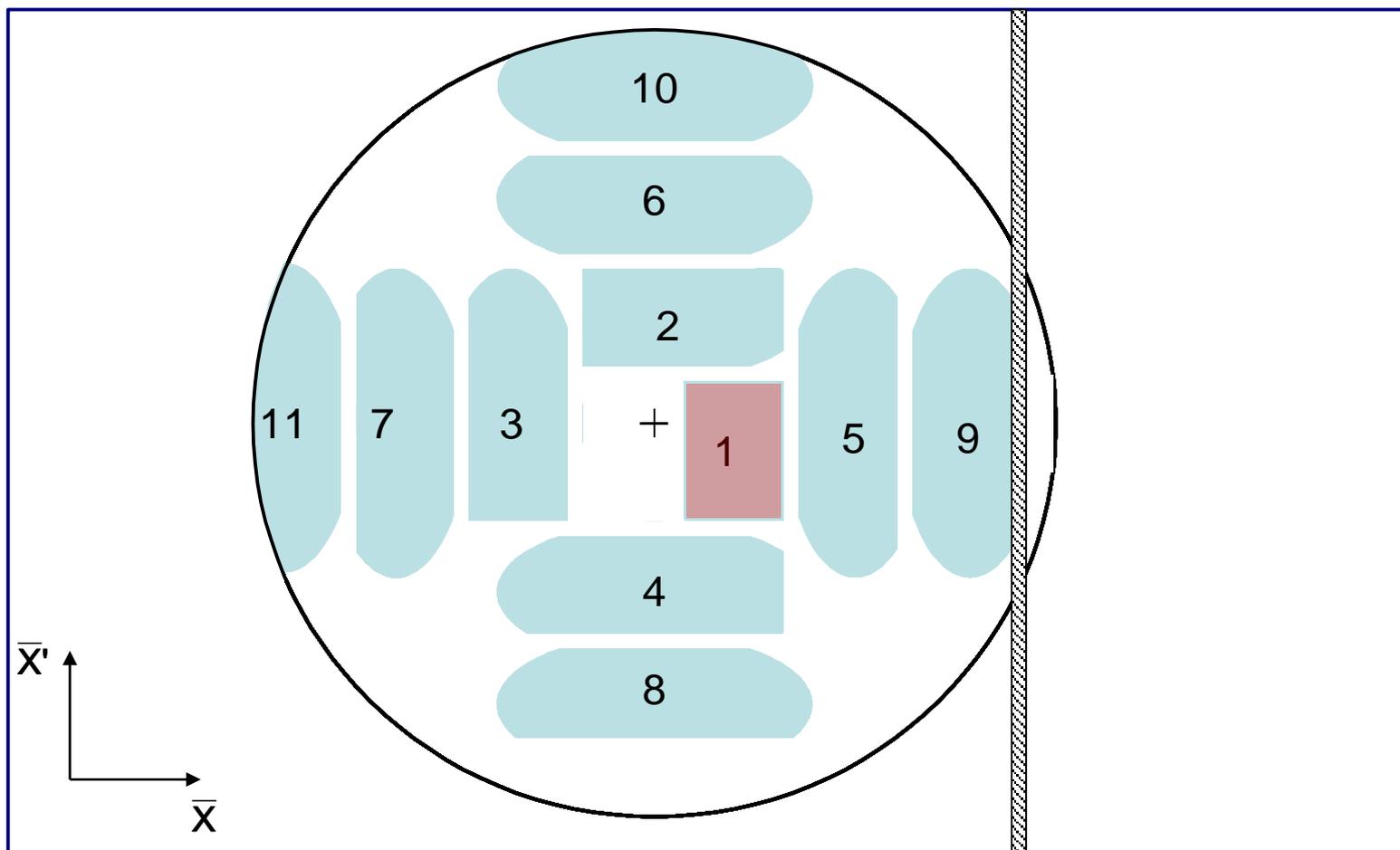
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 13



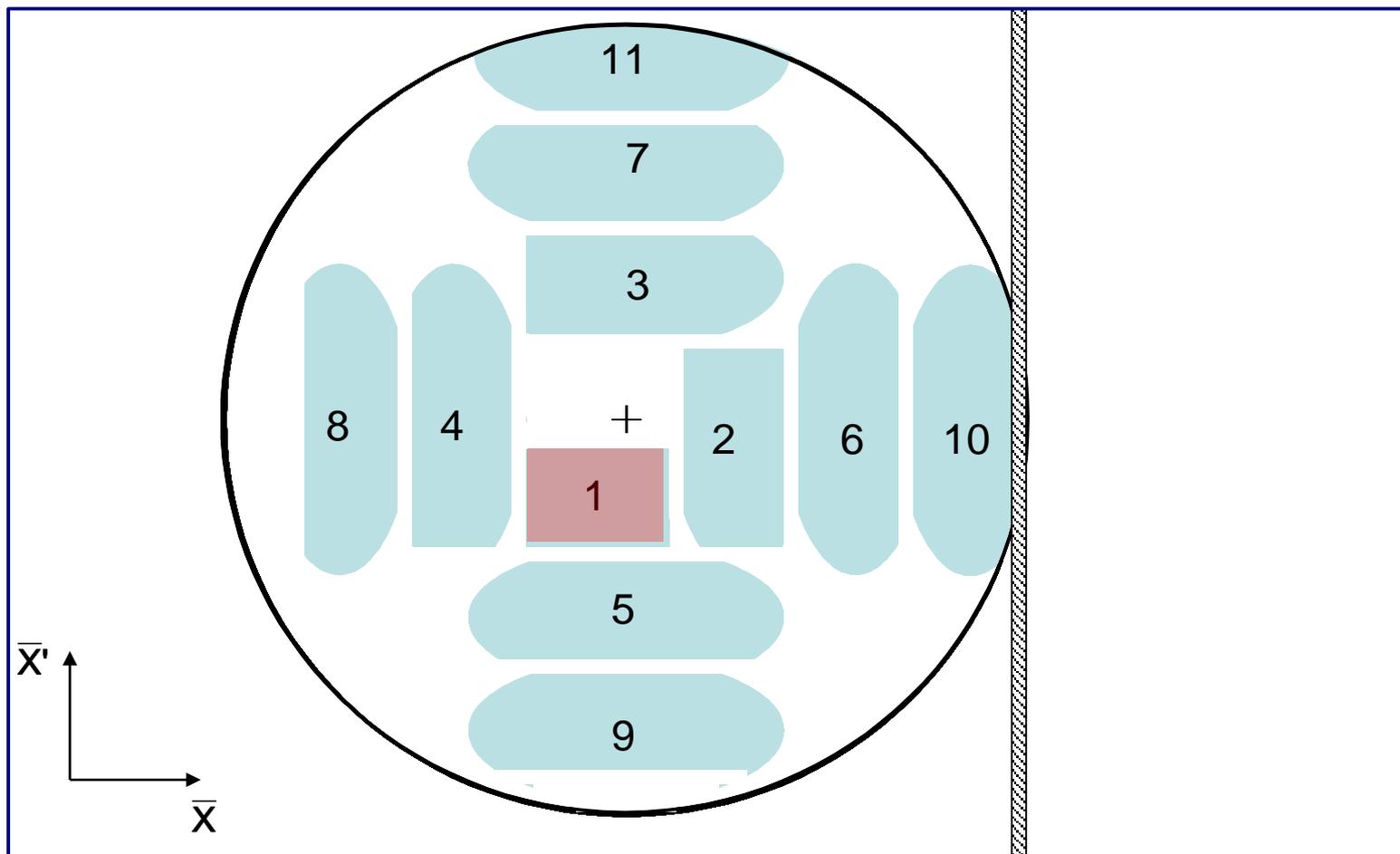
Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates $\pi/2$ per turn in phase space

Turn 14

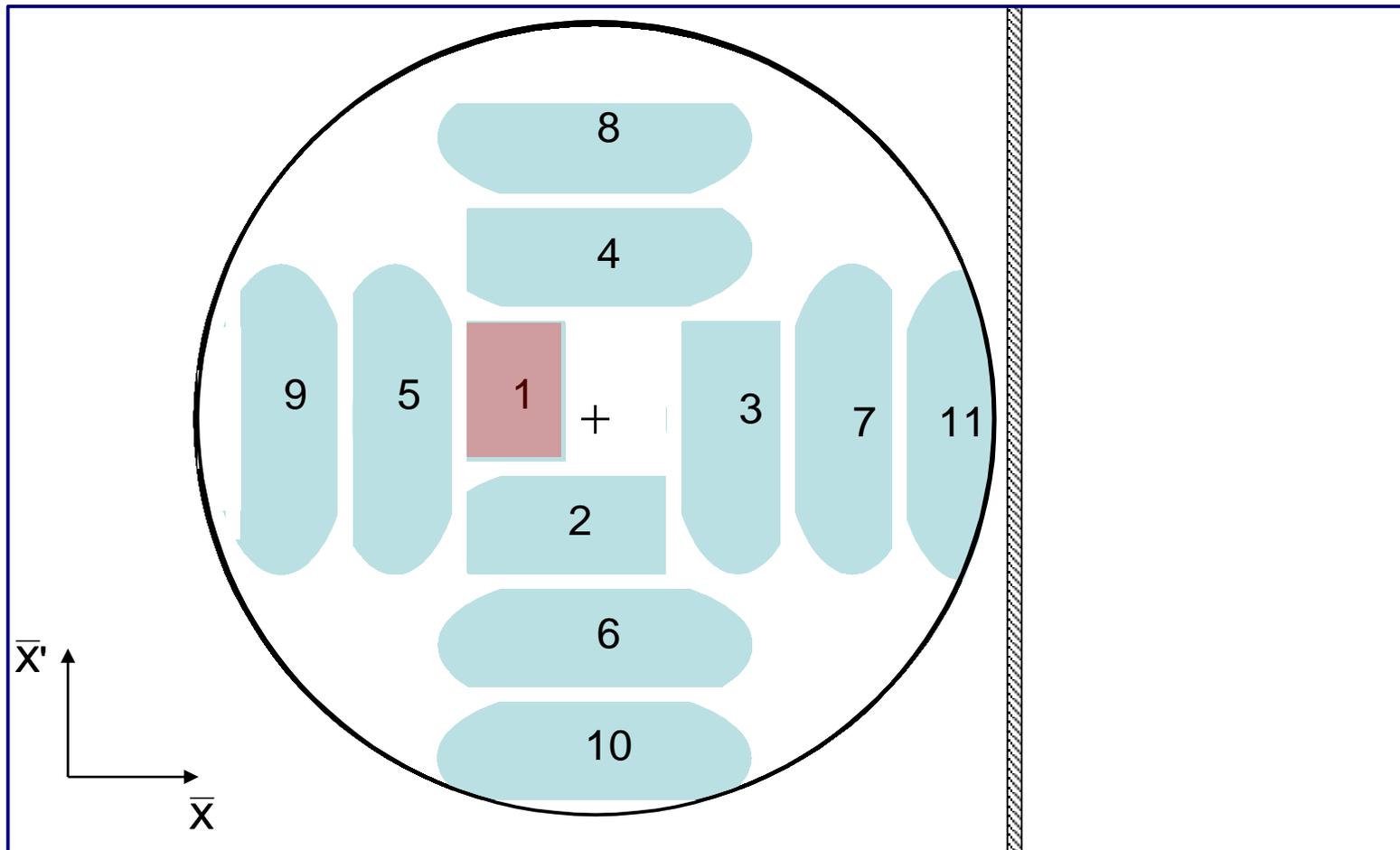


Septum

Multi-turn injection for hadrons

Phase space has been “**painted**”

Turn 15

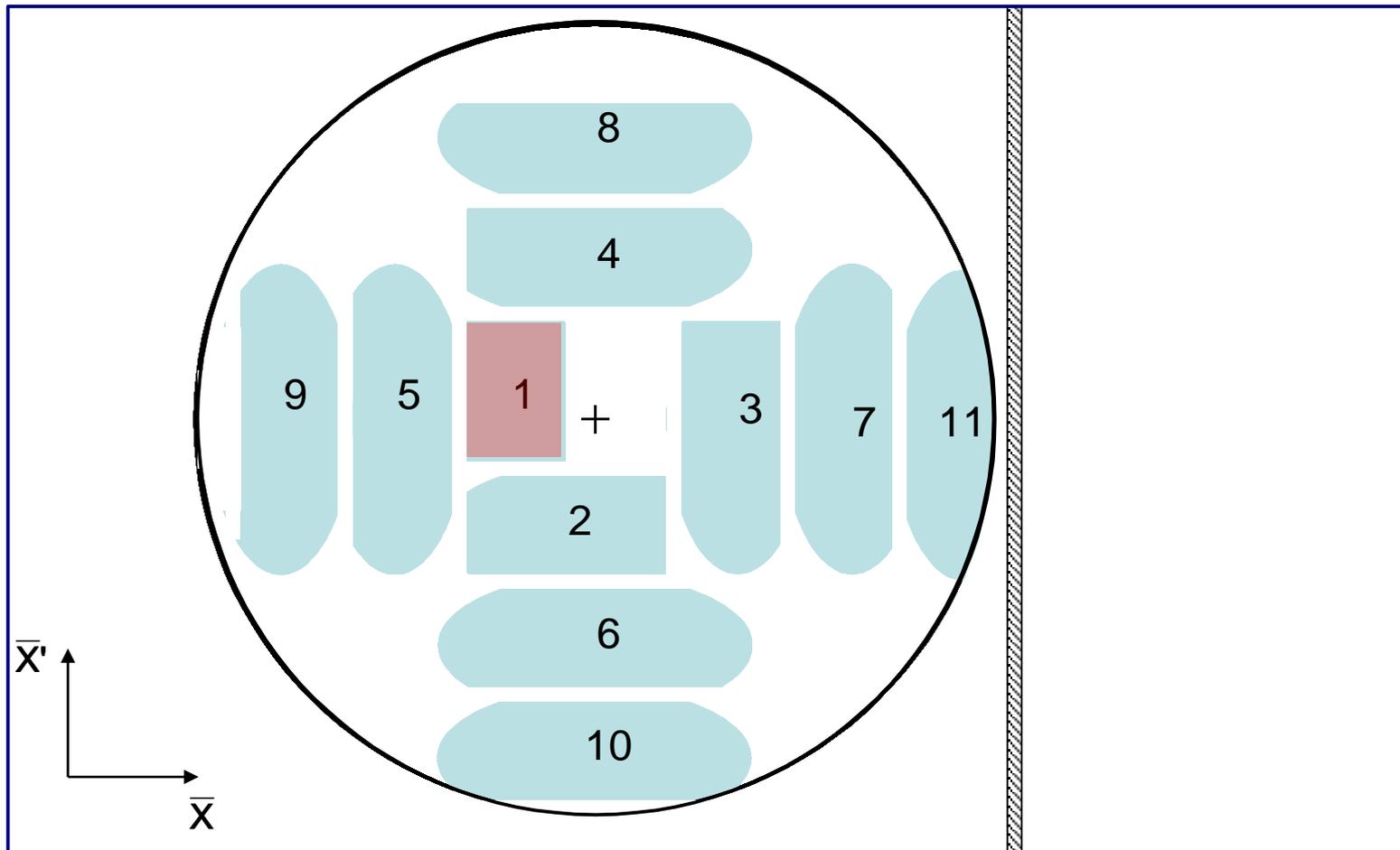


In reality, filamentation (often space-charge driven) occurs to produce a quasi-uniform beam

Multi-turn injection for hadrons

Phase space has been “**painted**”

Turn 15



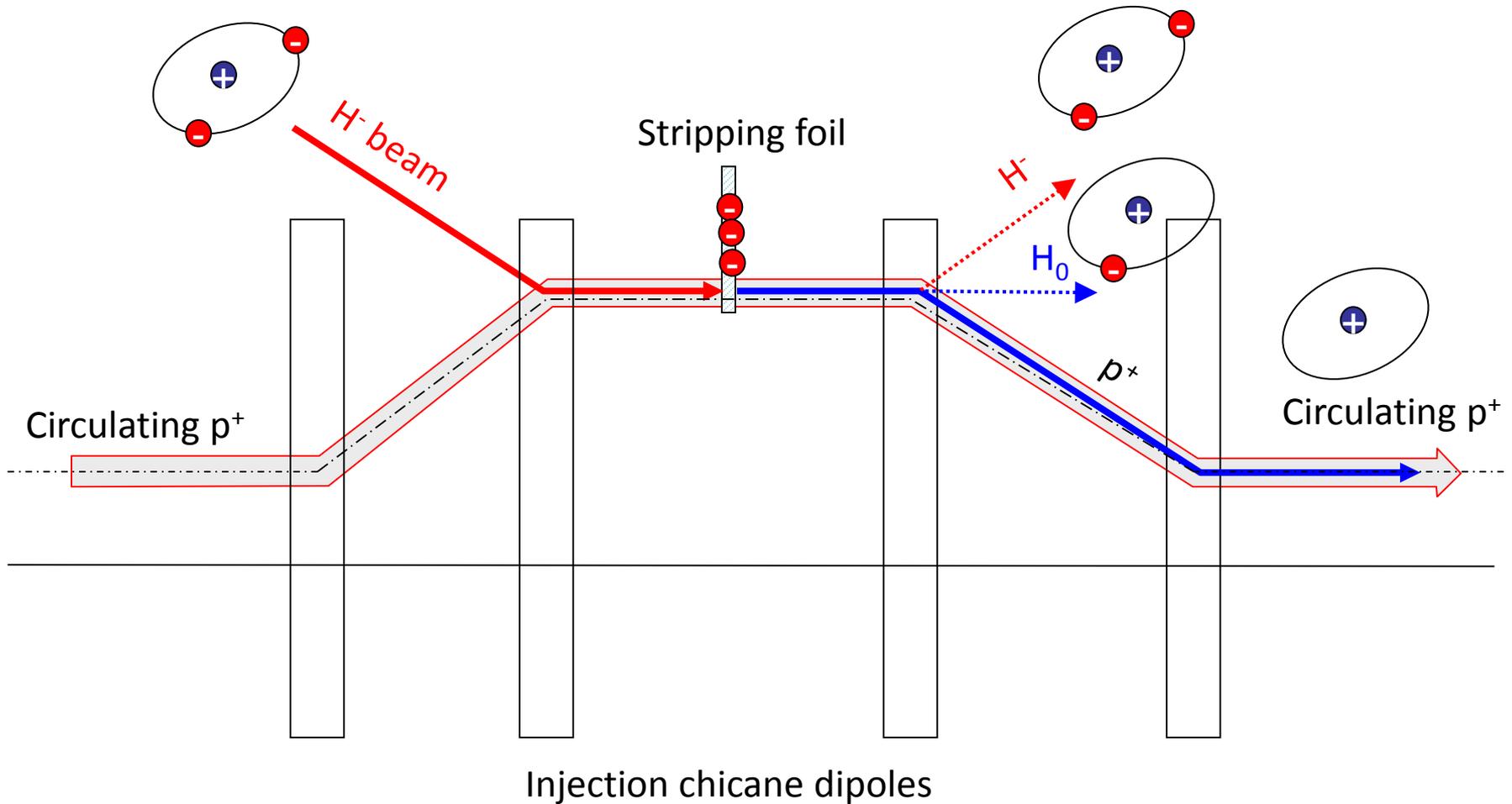
In reality, filamentation (often space-charge driven) occurs to produce a quasi-uniform beam

Charge exchange H- injection

- Multi-turn injection is essential to accumulate high intensity
- Disadvantages inherent in using an injection septum:
 - Width of several mm reduces aperture
 - **Beam losses from circulating beam hitting septum:**
 - typically 30 – 40 % for the CERN PSB injection at 50 MeV
 - Limits number of injected turns to 10 - 20
- Charge-exchange injection provides elegant alternative
 - Convert H^- to p^+ using a thin stripping foil, allowing injection into the same phase space area

Charge exchange H- injection

Start of injection process

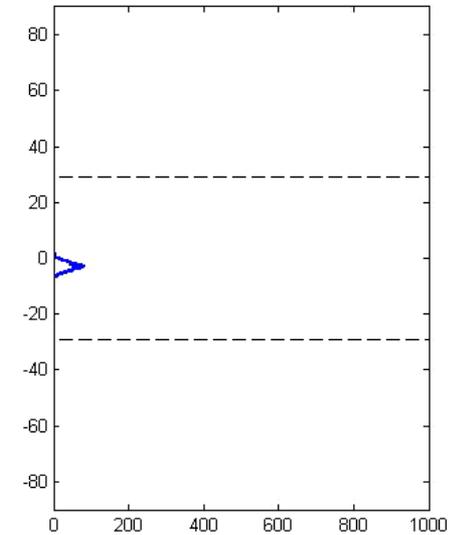
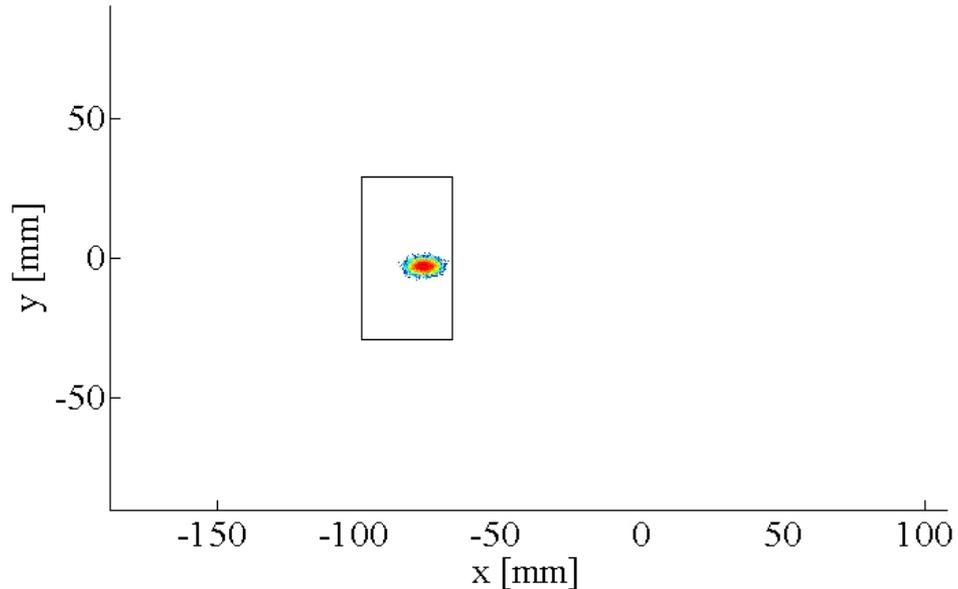
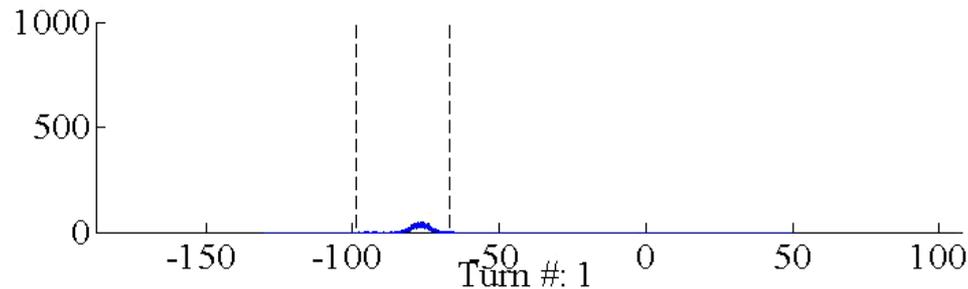


Charge exchange H- injection

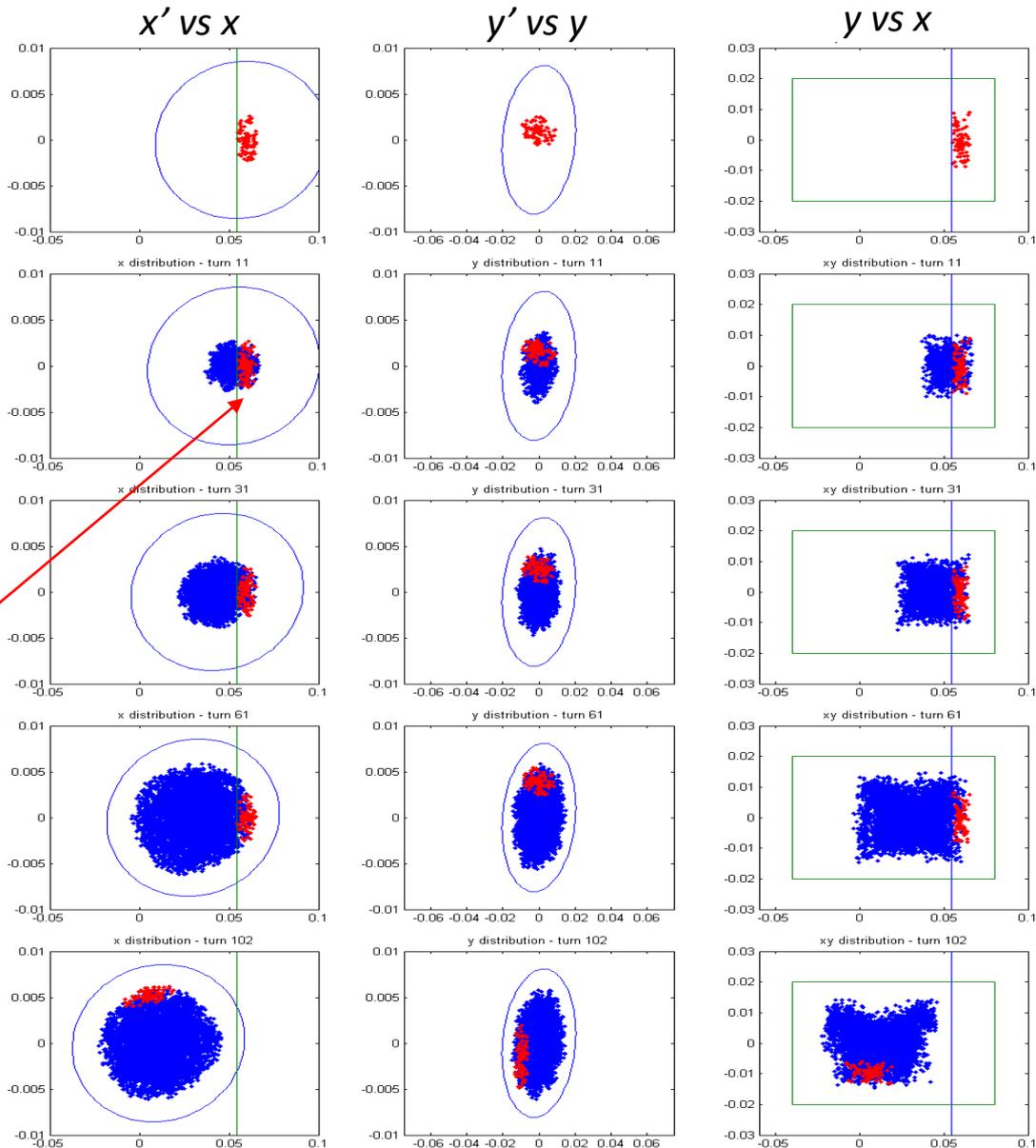
- Paint uniform transverse phase space density by modifying closed orbit bump and steering injected beam
- Foil thickness calculated to double-strip most ions ($\approx 99\%$)
 - 50 MeV – $50 \mu\text{g}\cdot\text{cm}^{-2}$
 - 800 MeV – $200 \mu\text{g}\cdot\text{cm}^{-2}$ ($\approx 1 \mu\text{m}$ of C!)
- Carbon foils generally used – very fragile
- Injection chicane reduced or switched off after injection, to avoid excessive foil heating and beam blow-up
- Longitudinal phase space can also be painted turn-by-turn:
 - Variation of the injected beam energy turn-by-turn (linac voltage scaled)
 - Chopper system in linac to match length of injected batch to bucket

Accumulation process on foil

- Linac4 connection to the PS booster at 160 MeV:
 - H^- stripped to p^+ with an estimated efficiency $\approx 98\%$ with C foil $200 \mu\text{g}\cdot\text{cm}^{-2}$ [14]



H- injection - painting

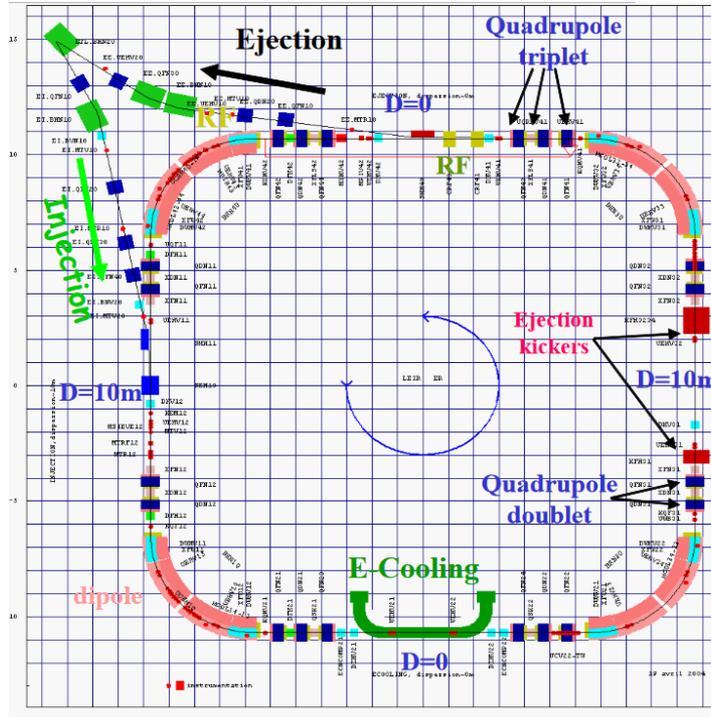


Note injection into same phase space area as circulating beam

Combined longitudinal and transverse multi-turn injection

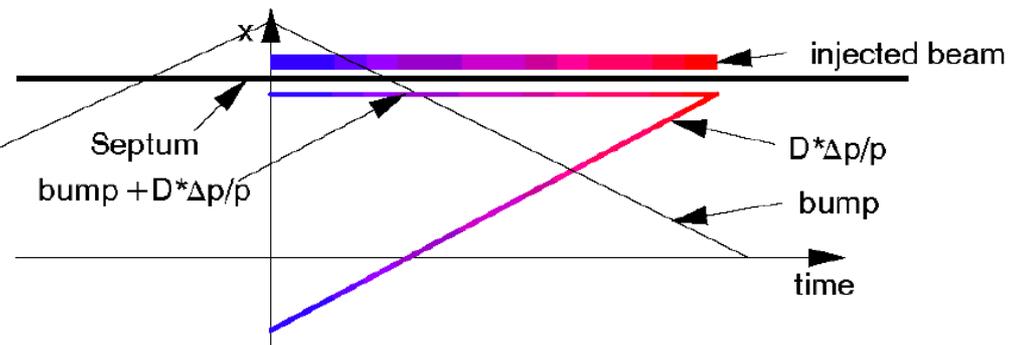
Used at CERN LEIR to increase intensity of ion bunches

4.2 MeV/nucleon \rightarrow 72 MeV/nucleon



Combined multi-turn injection and stacking by electron cooling [15].

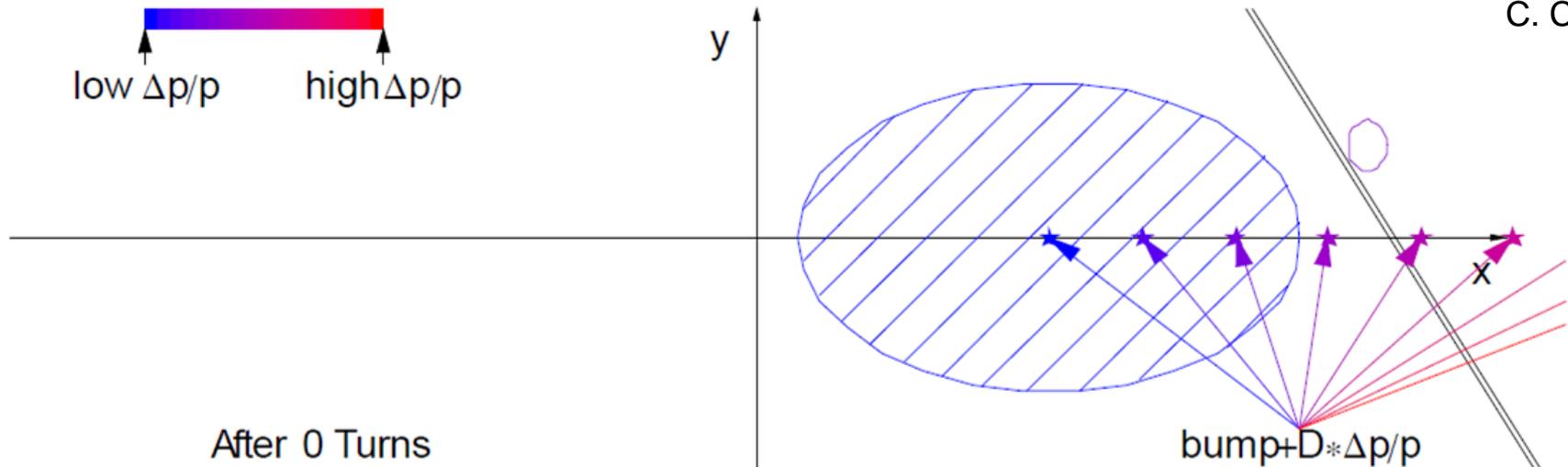
- Injection occurs in a section with large normalised dispersion
- Decrease closed orbit bump
- Ramp momentum of the linac such that the orbit (bump + $D \Delta p/p$) stays constant at injection (same betatron amplitude but stacking in momentum)
- After injection the ions cooled and stacked before the new pulse arrives



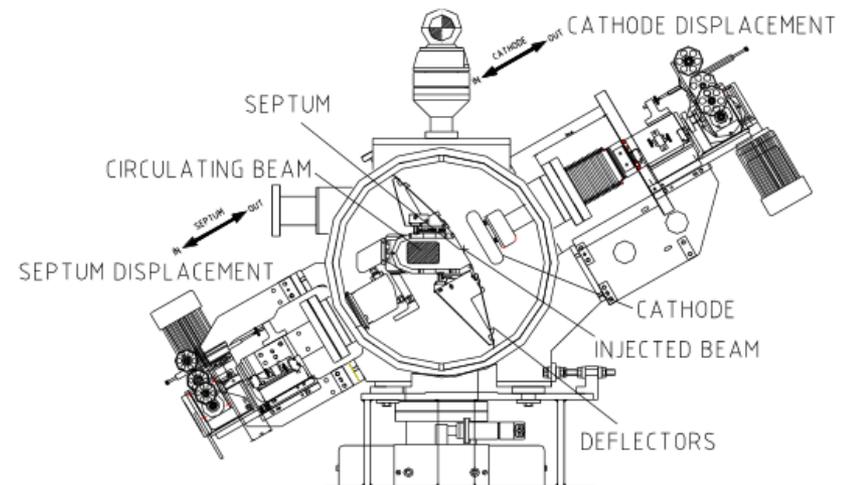
Horizontal Positions during injection

Combined longitudinal and transverse multi-turn injection

C. Carli

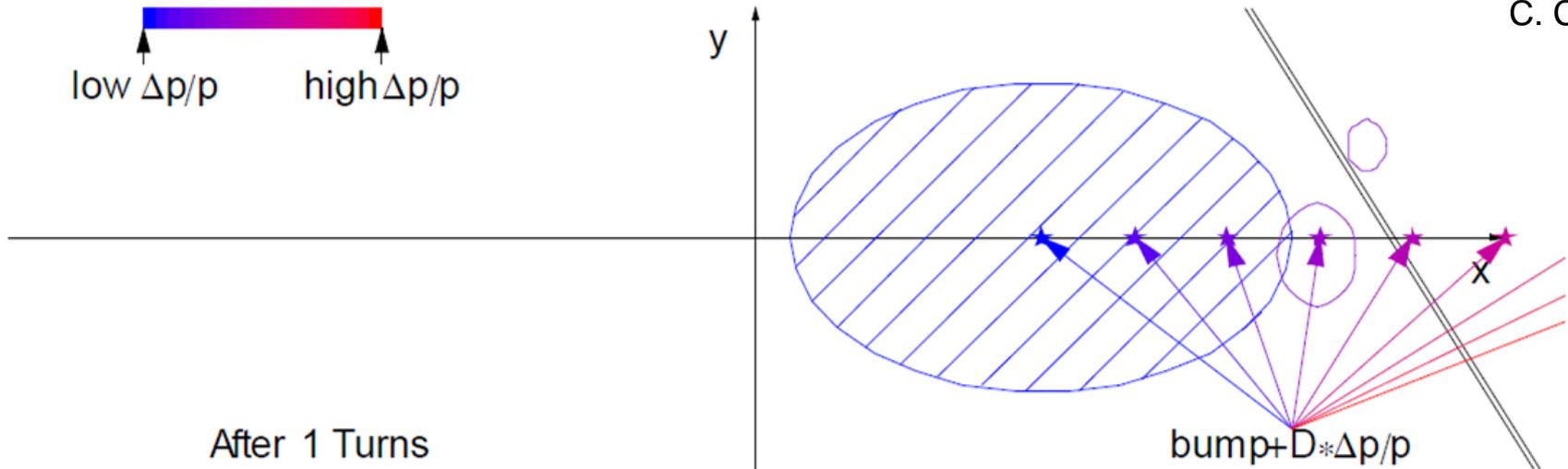


- Stacking in both horizontal and vertical plane requires an inclined electrostatic septum
- The tune has to be optimised to avoid touching the septum blade at each turn
- Injection of 1 pulse from the linac takes $\sim 200 \mu\text{s}$ (76 turns)
- One injection every 200 ms to allow for cooling (100-150 ms). In total 7 injections to fill LEIR.
- After each injection the momentum offset is reduced from 4‰ down to 1‰ \rightarrow cooling \rightarrow move orbit of stack beam to leave room for next injection
- Efficiency: 50- 70%

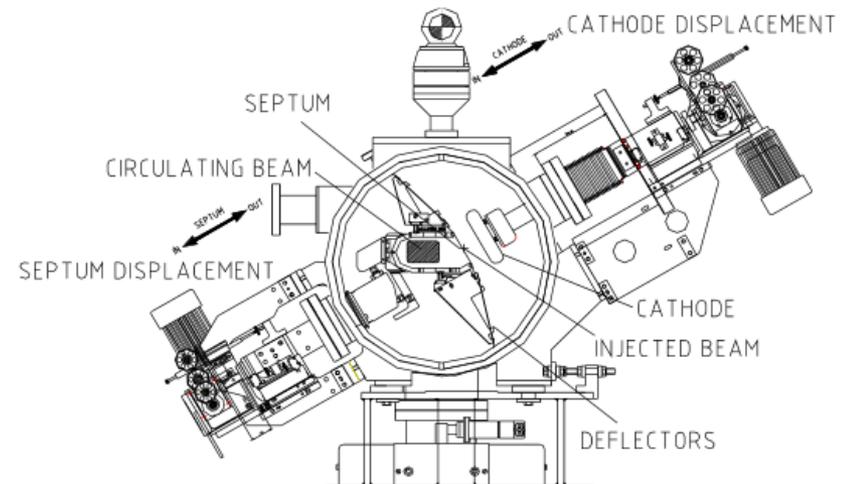


Combined longitudinal and transverse multi-turn injection

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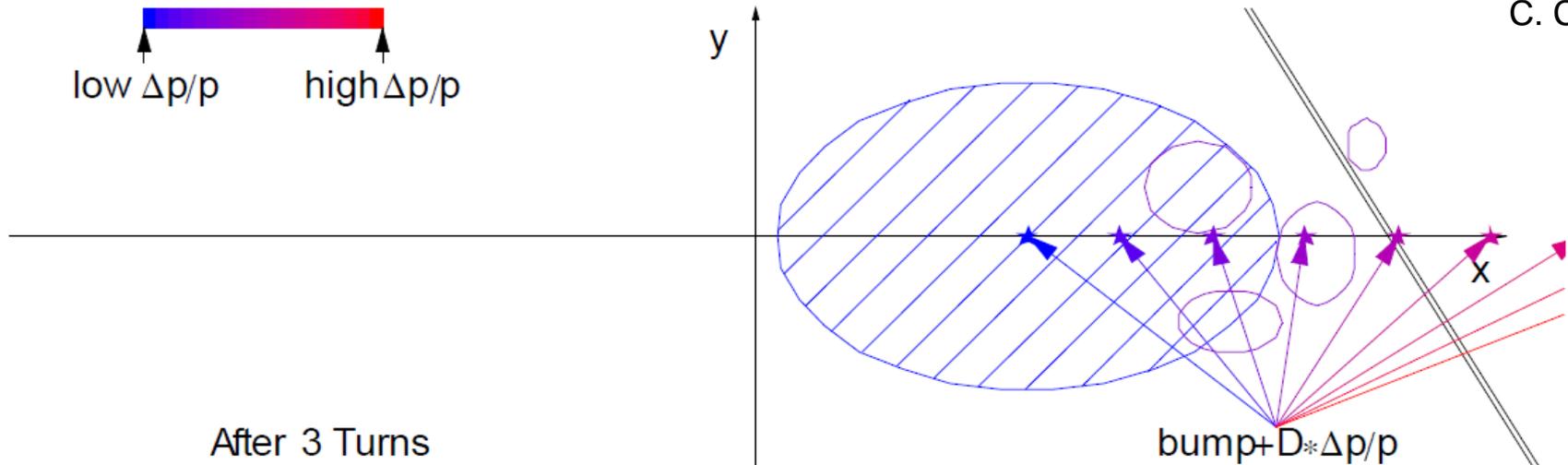


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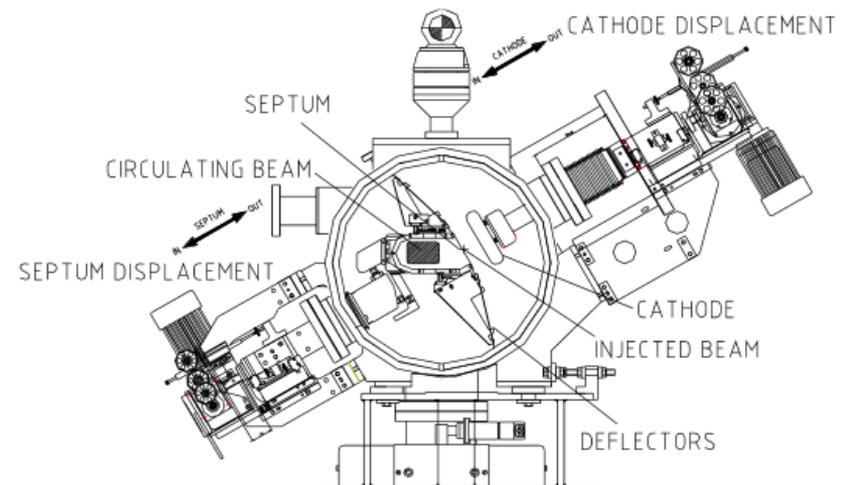


Combined longitudinal and transverse multi-turn injection

C. Carli

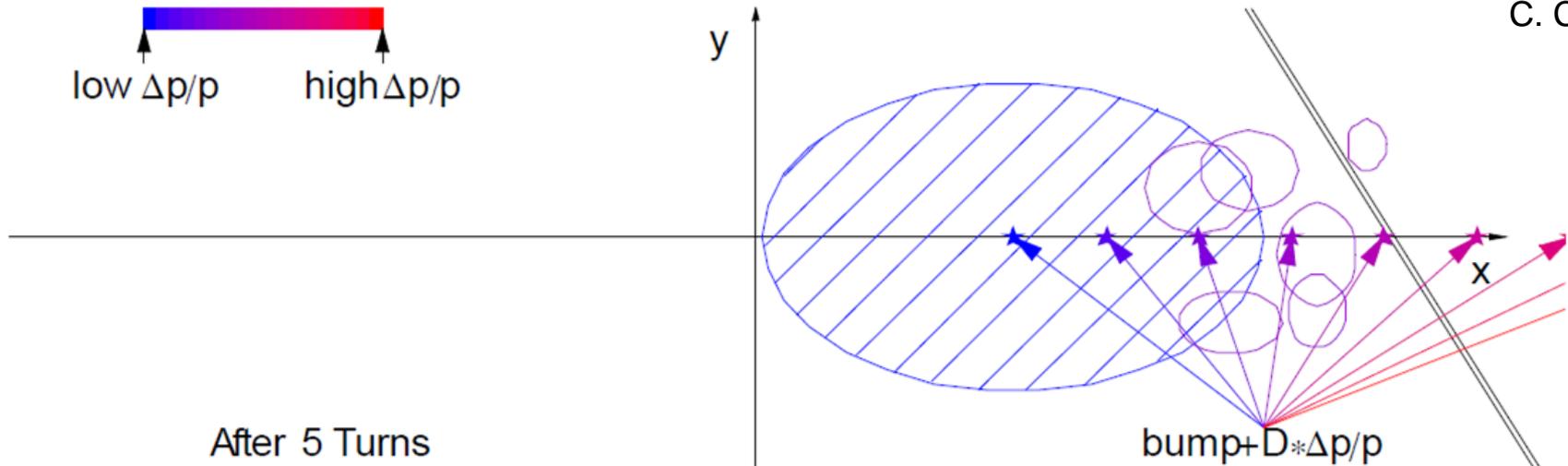


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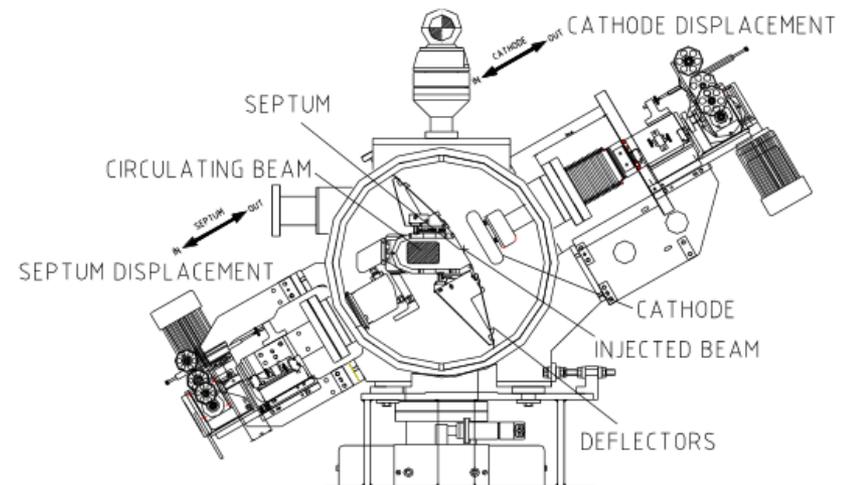


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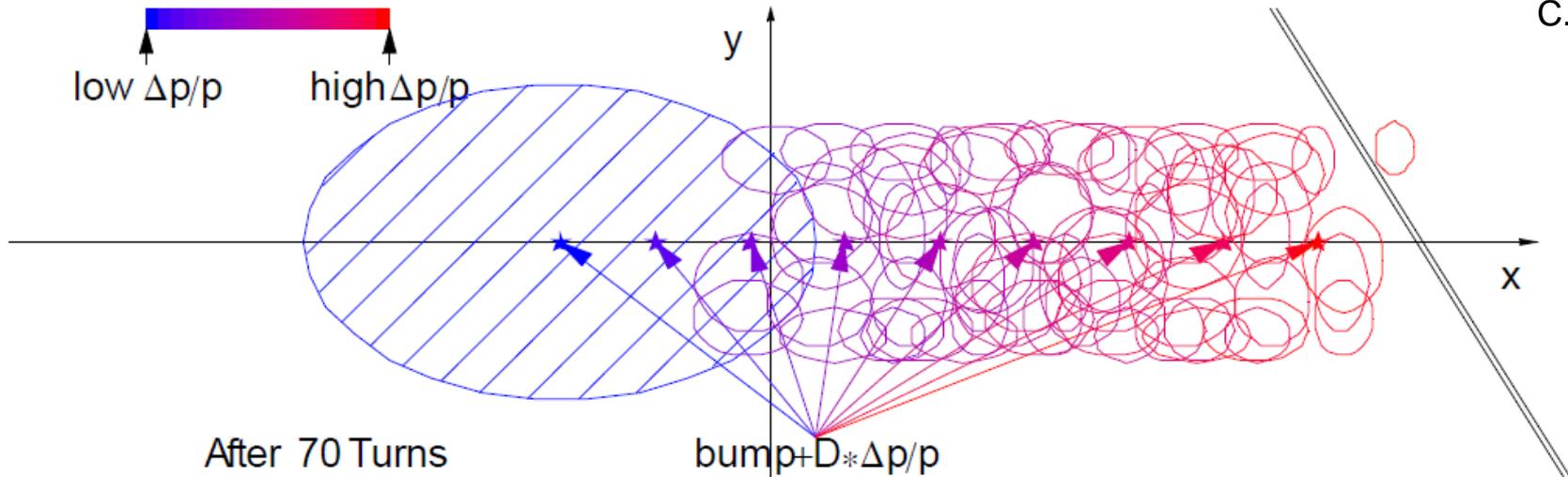


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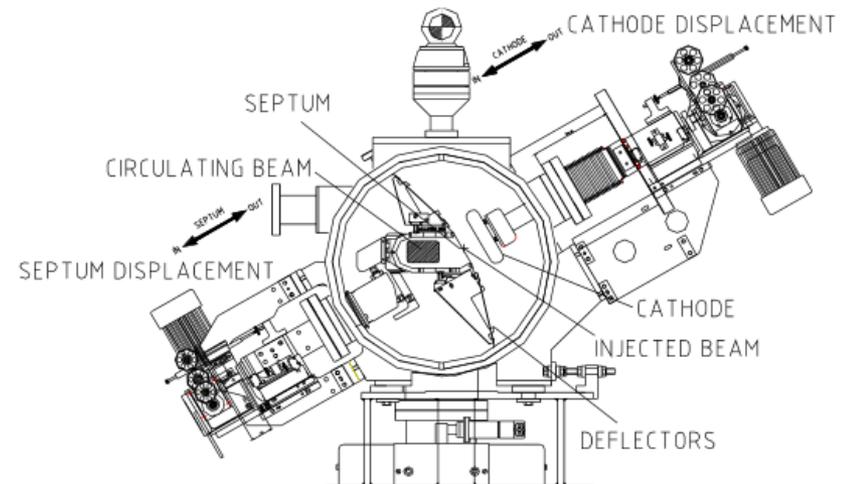


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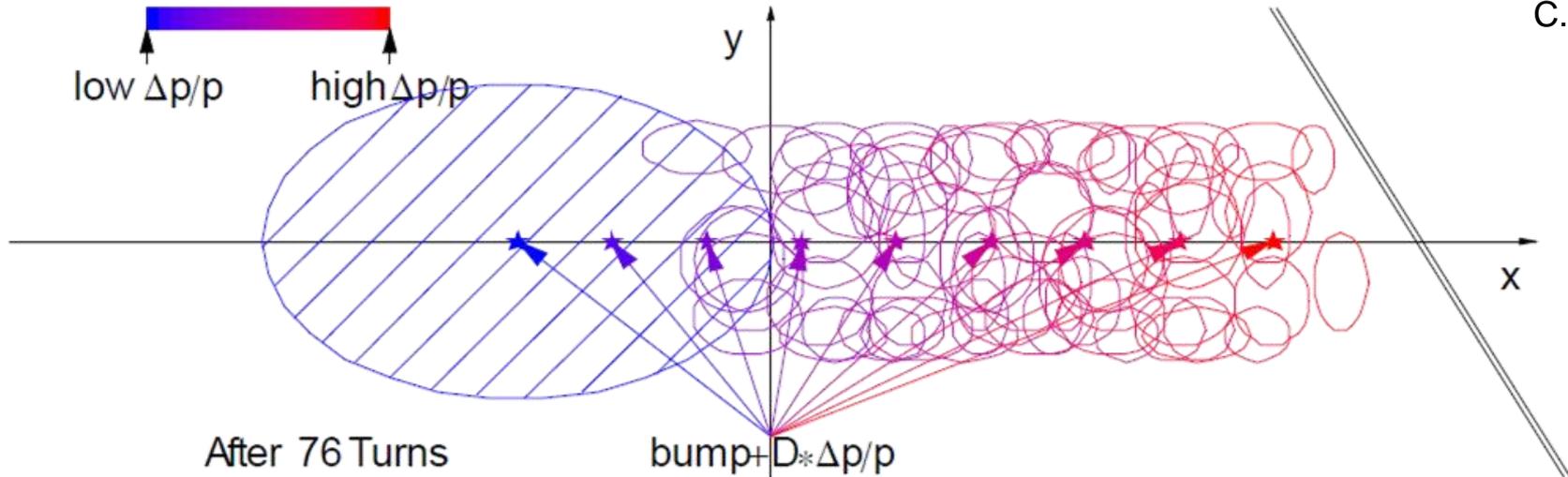


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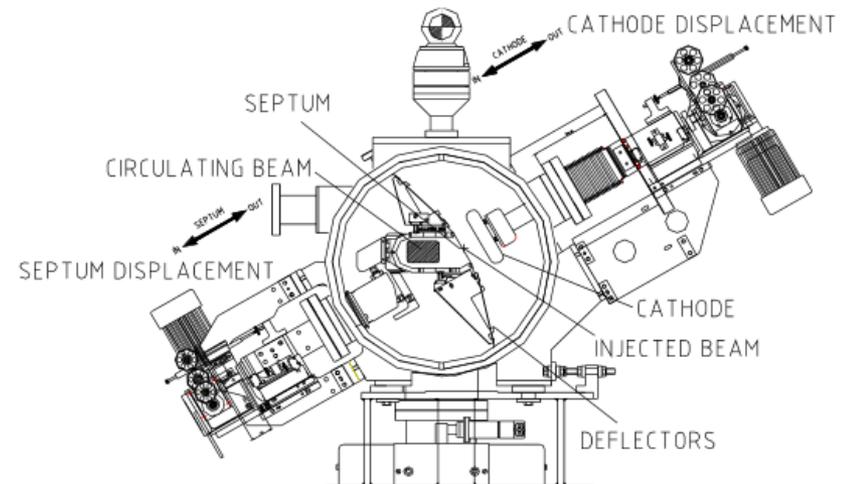


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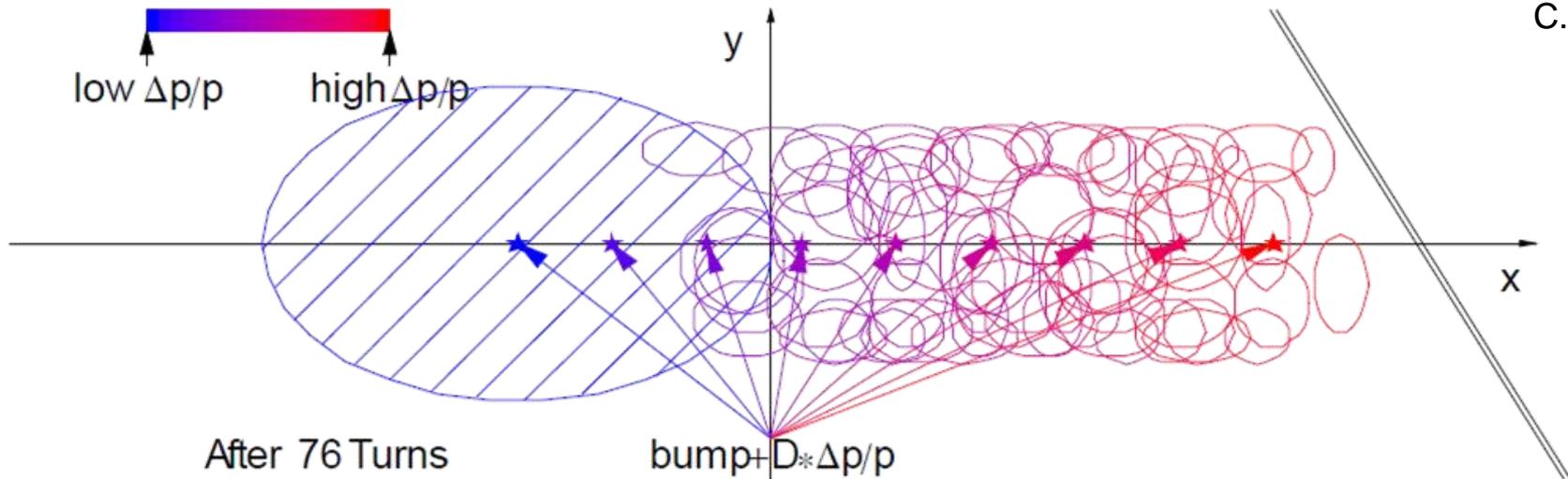


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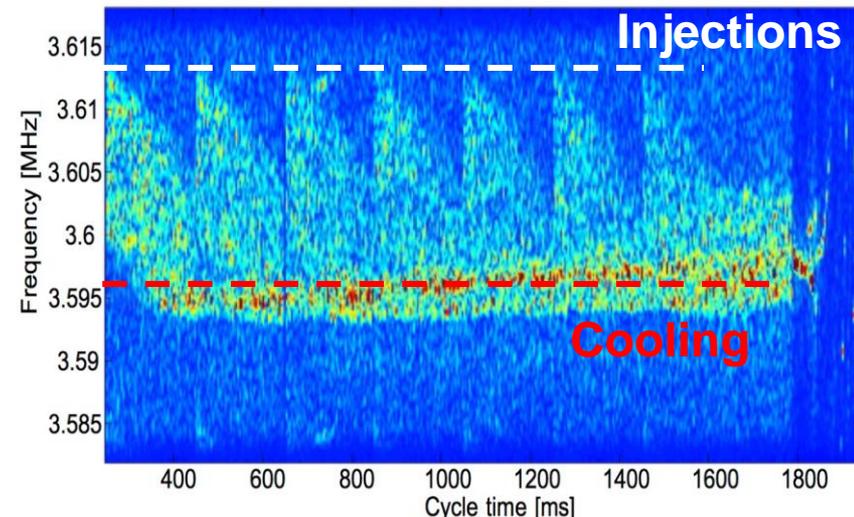


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- Injection of 1 pulse from the linac takes $\sim 200 \mu\text{s}$ (76 turns)
- One injection every 200 ms to allow for cooling (100-150 ms). In total 7 injections to fill LEIR.
- After each injection the momentum offset is reduced from 4‰ down to 1‰ \rightarrow cooling \rightarrow move orbit of stack beam to leave room for next injection
- Efficiency: 50- 70%



Lessons learnt

Low energy: dominated by space charge → challenging storing high intensity and high brightness beams

- Conventional multi-turn injection with phase space painting: emittance growth, limited by number of turns and injection losses at the septum → charge exchange H^- injection allows to overcome these limitations

High energy: fast process involving handling of high power beams → machine protection concerns!

- Minimize mismatch and injection errors (optimize HW, steering, optics) → reduce losses, injection oscillations and thus emittance blow up
- Need for passive protection elements: aperture, materials and length such to minimize energy deposition on the machine elements and avoid damage while limiting activation

Alternative injection methods:

- Slip-stacking: increasing intensity by accelerating particle beams with different momenta in the same accelerator
- Combined 3-planes multi-turn injection, electron cooling and accumulation

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Back-up slides

Normalised phase space

- Transform real transverse coordinates (x, x', s) to normalized co-ordinates (\bar{x}, \bar{x}', μ) where the independent variable becomes the phase advance μ :

$$\begin{bmatrix} \bar{x} \\ \bar{x}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta(s)}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha(s) & \beta(s) \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cos[\mu(s) + \mu_0]$$

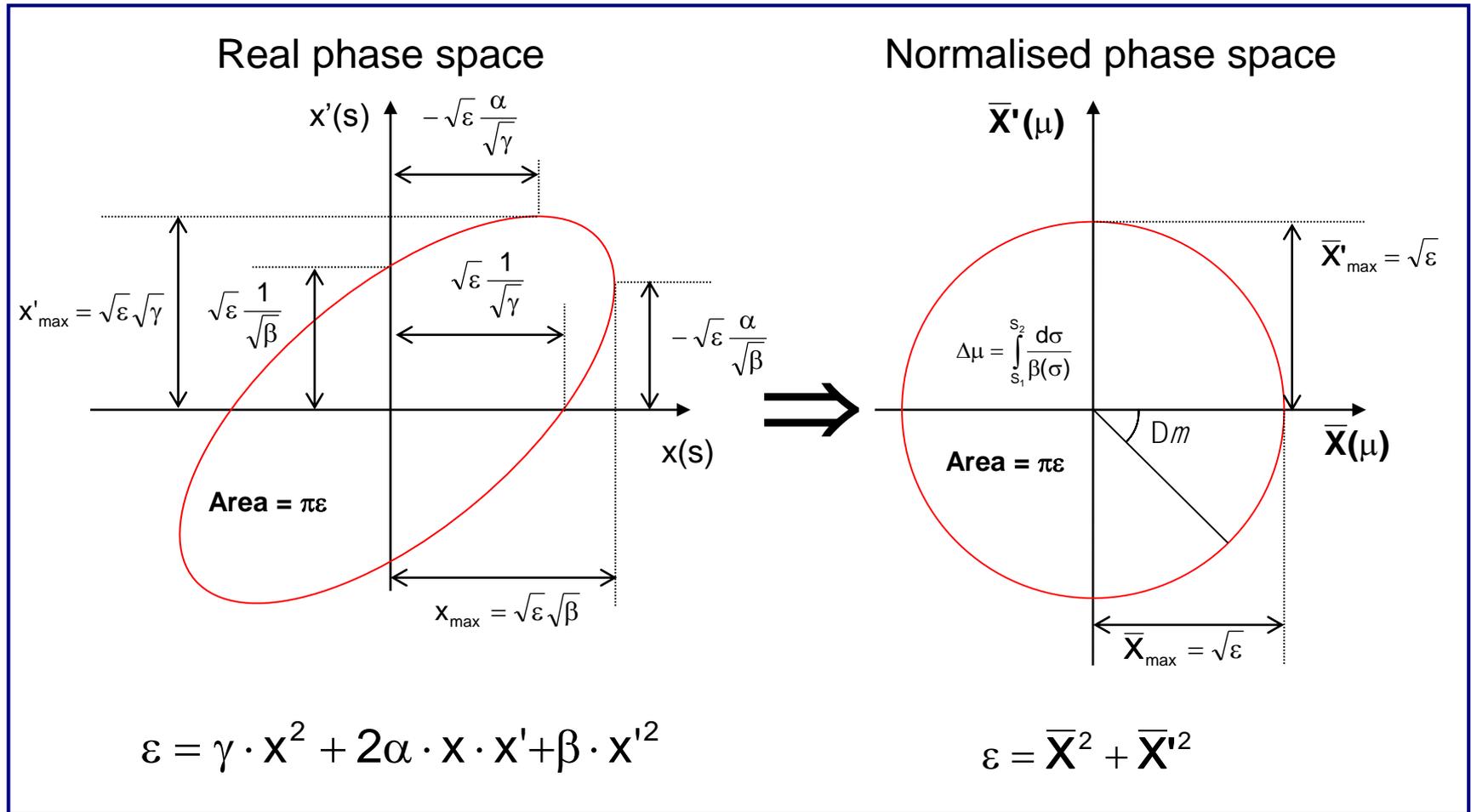
$$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}} [\alpha(s) \cos(\mu(s) + \mu_0) + \sin(\mu(s) + \mu_0)]$$

$$\mu(s) = \int_0^s \frac{d\sigma}{\beta(\sigma)}$$

$$\bar{x}(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot x = \sqrt{\varepsilon} \cos[\mu + \mu_0]$$

$$\bar{x}'(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot \alpha(s)x + \sqrt{\beta(s)}x' = -\sqrt{\varepsilon} \sin[\mu + \mu_0] = \frac{d\bar{x}}{d\mu}$$

Normalised phase space



Beam position and angle

At the kicker, where $\beta(s) = \beta_k$, we want $x(s) = 0$ and $x'(s) = -\theta_{\text{kicker}}$, being μ the phase advance between the septum and the kicker:

$$x(s) = \sqrt{\varepsilon} \sqrt{\beta_k} \cos(\mu + \mu_0) = 0 \quad \longrightarrow \quad \mu + \mu_0 = \frac{\pi}{2}$$

$$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta_k}} [\alpha(s) \cos(\mu + \mu_0) + \sin(\mu + \mu_0)] = -\theta_{\text{kicker}} \quad \longrightarrow \quad \sqrt{\varepsilon} = \theta_{\text{kicker}} \sqrt{\beta_k}$$

At the septum:

$$x_s = \sqrt{\varepsilon} \sqrt{\beta_s} \cos(\mu_0) = \theta_{\text{kicker}} \sqrt{\beta_k \beta_s} \cos\left(\mu - \frac{\pi}{2}\right) = \theta_{\text{kicker}} \sqrt{\beta_k \beta_s} \sin(\mu)$$

$$x'_s = -\frac{\theta_{\text{kicker}} \sqrt{\beta_k}}{\sqrt{\beta_s}} [\alpha_s \sin(\mu) + \cos(\mu)] = -\frac{x_s}{\beta_s} [\alpha_s + \cot g(\mu)]$$

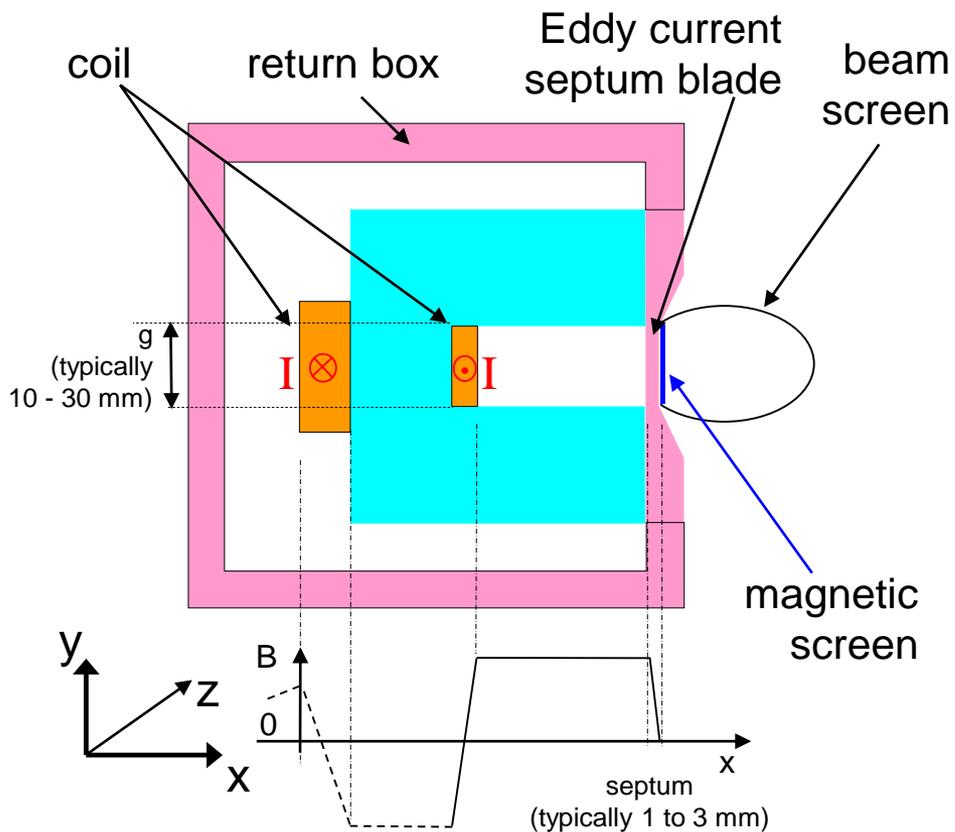
$$\theta_{\text{kicker}} = \frac{x_s}{\sqrt{\beta_k \beta_s} \sin(\mu)}$$

A small θ_{kicker} means a reduced cost \rightarrow we want μ as close as possible to $\pi/2$ and large β_k

Septum leakage field

Stray field loss into the “field-free” region.

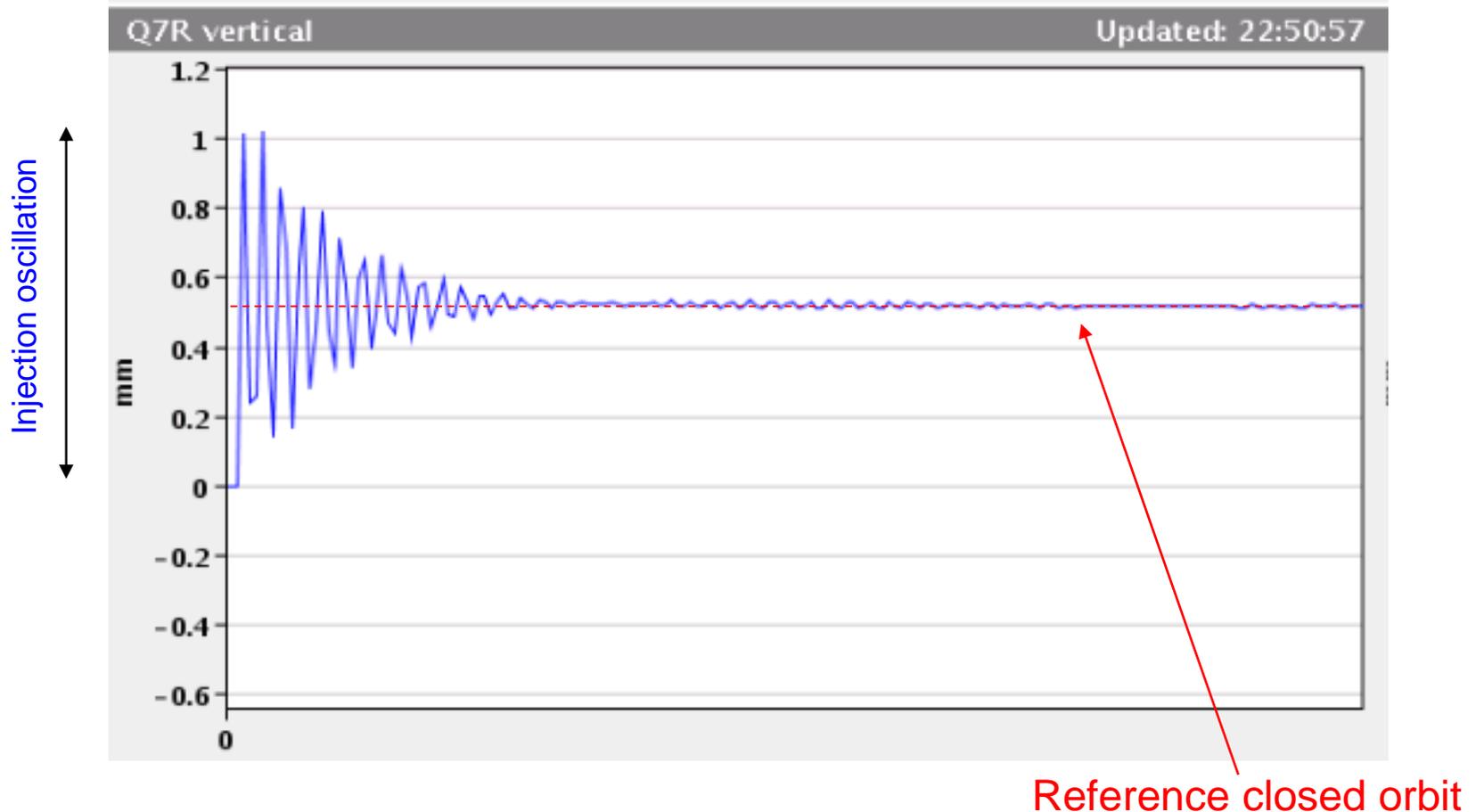
Particularly critical for low injection energy → adequate shielding



- Coil removed from septum and placed behind C-core yoke:
 - Coil dimension not critical
 - Very thin septum blade
- Magnetic field pulse induces eddy currents in septum blade
- Eddy currents shield the circulating beam from magnetic field
- Return box and magnetic screen reduce fringe field seen by circulating beam

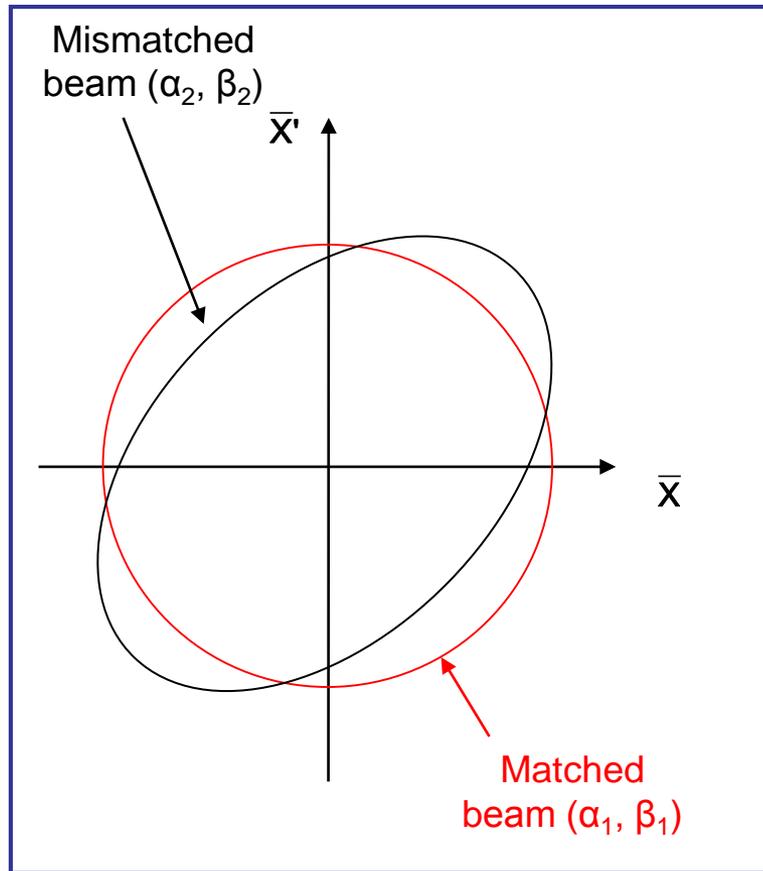
Filamentation

- The residual transverse oscillations lead to an *effective* emittance blow-up through filamentation:



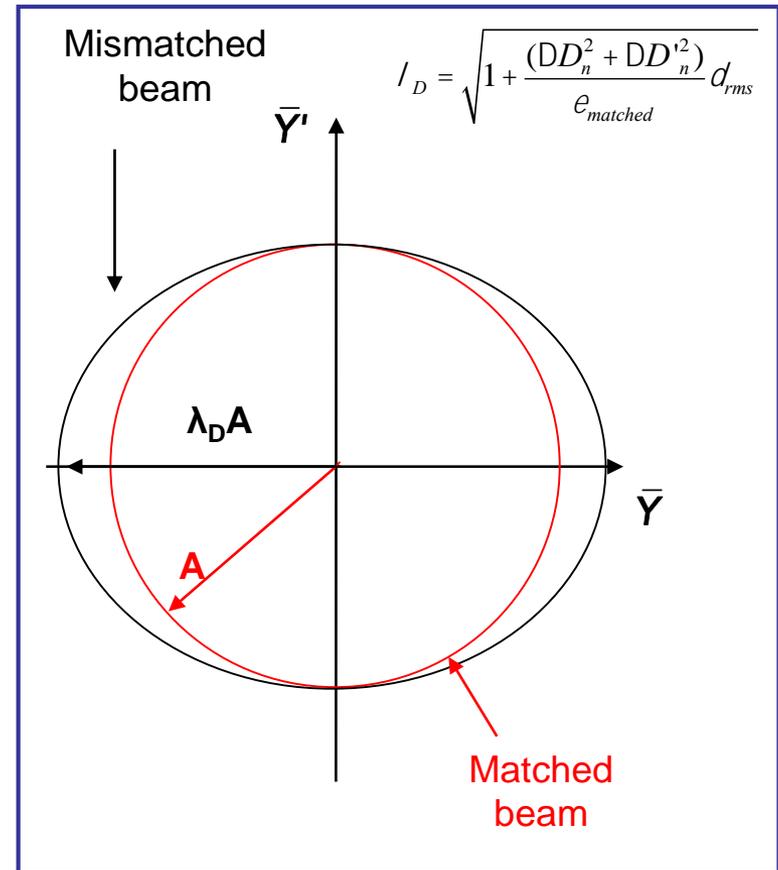
Blow-up from optics mismatch

Betatron mismatch



$$\epsilon_{\text{diluted}} = \frac{1}{2} \left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 + \frac{\beta_2}{\beta_1} \right) \epsilon_{\text{matched}}$$

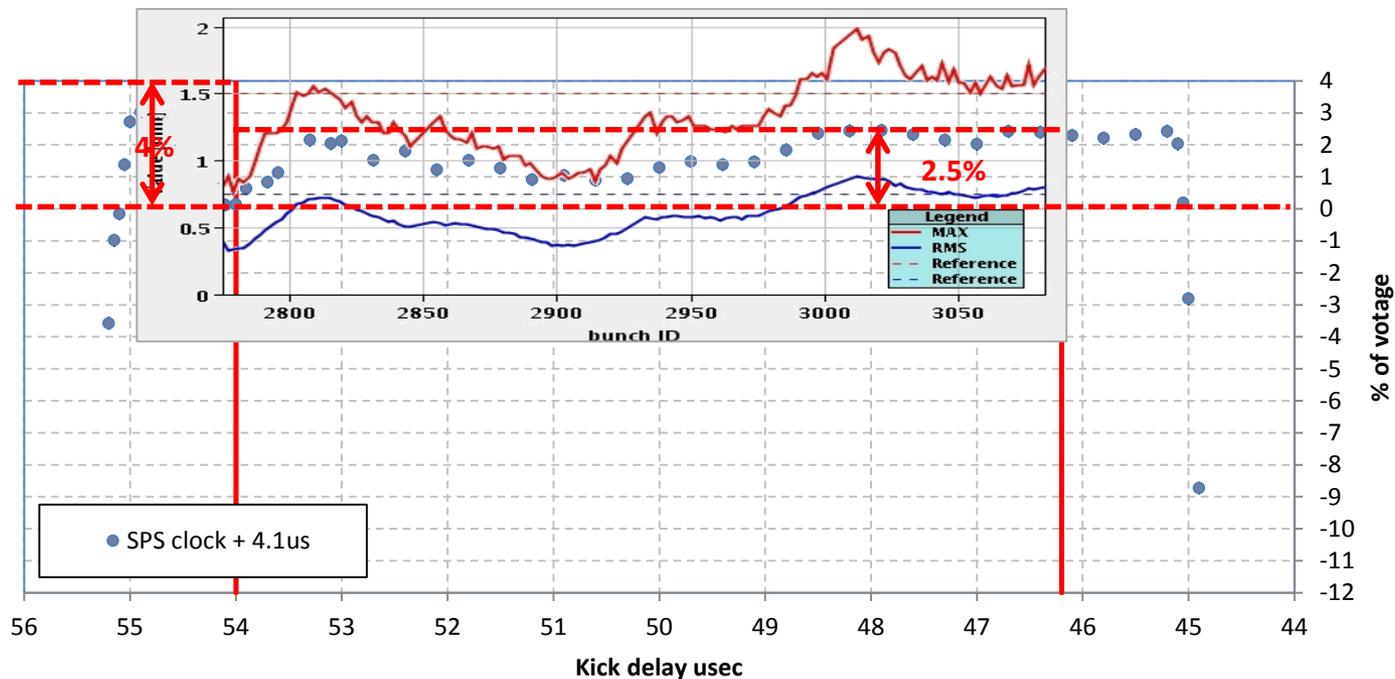
Dispersion mismatch



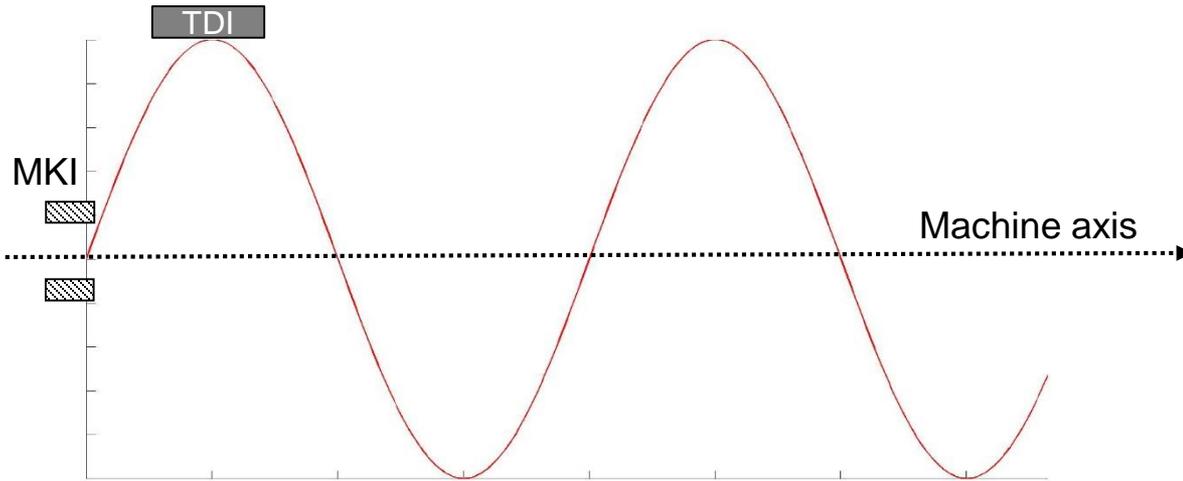
$$\epsilon_{\text{diluted}} = \epsilon_{\text{matched}} + \frac{\Delta D_n^2 + \Delta D_n'^2}{2} \delta_{\text{rms}}^2$$

SPS extraction kicker waveform

- The kicker waveform shows a ripple varying up to **2.5%** of kick (max. to min.) at the flat-top (**4%** at initial overshoot)
- Mitigations:
 - Improve the kicker flat-top ripple
 - Change the delay to move the beam to a flatter part of the waveform (if enough space)

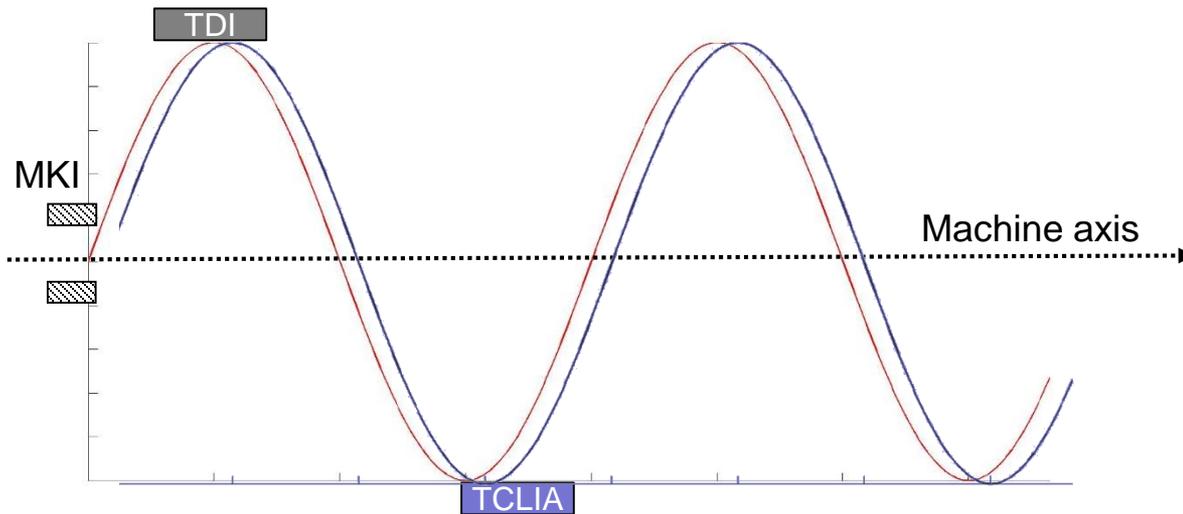


Phase advance for Inj. Protection



90° phase advance MKI → TDI

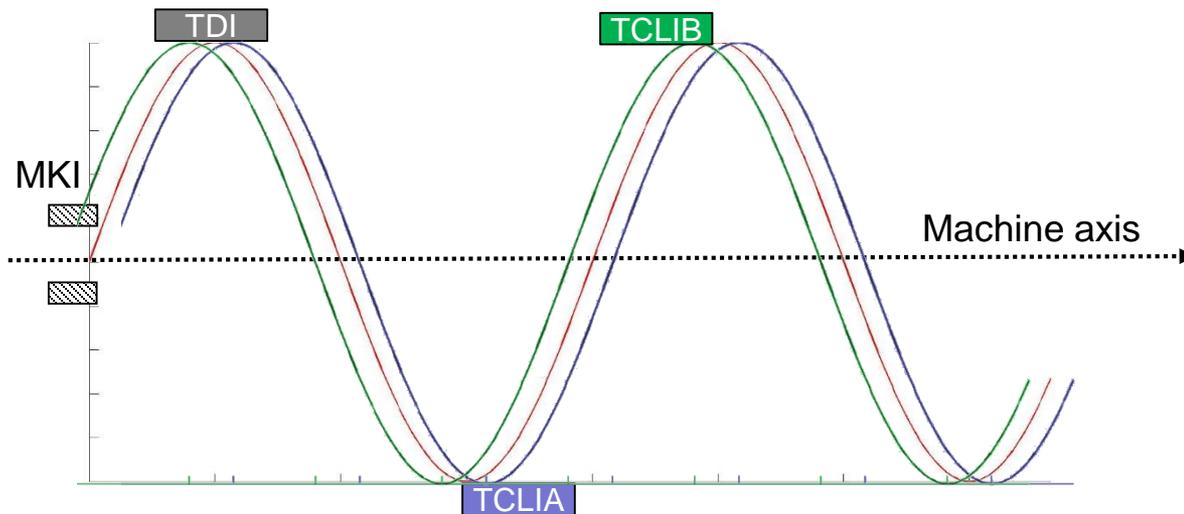
Phase advance for Inj. Protection



90° phase advance MKI → TDI

Two auxiliary collimators are installed at $180^\circ + 20^\circ$ (TCLIA)

Phase advance for Inj. Protection

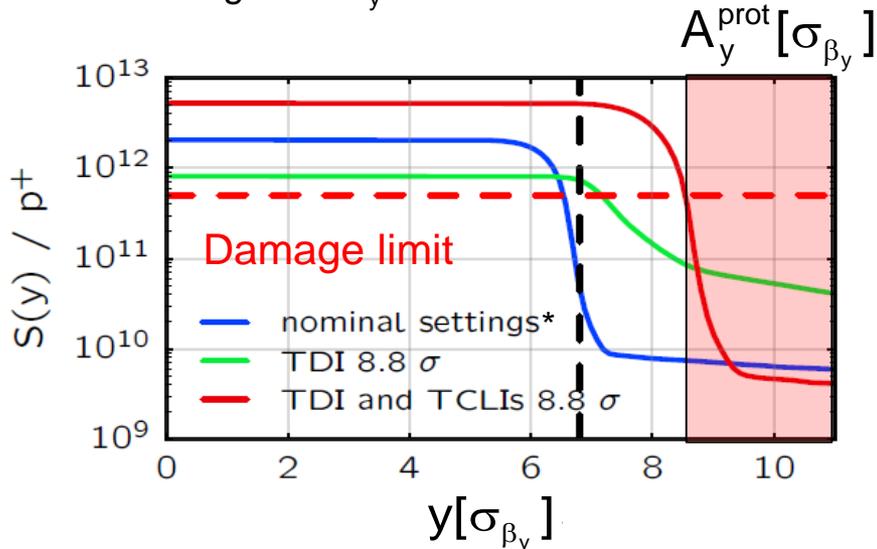


90° phase advance MKI → TDI

Two auxiliary collimators are installed at $180^\circ + 20^\circ$ (TCLIA) and $360^\circ - 20^\circ$ (TCLIB) wrt TDI to take into account possible phase-advance errors [3].

Transmission and multi-turns effects

*Nominal settings: $6.8 \sigma_y$



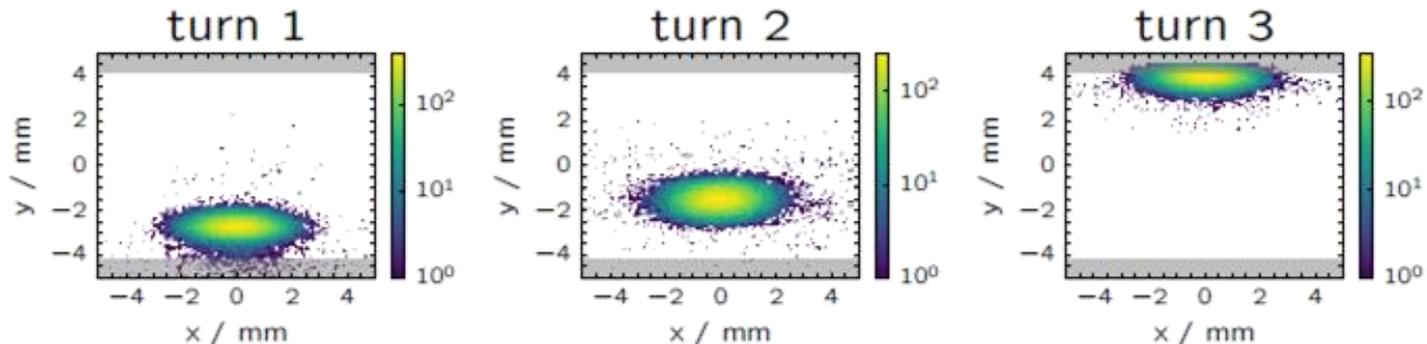
Survival function $S(y)$ normalised to the beam intensity N_p : number of protons escaping the protection devices as a function of the amplitude [4]

$$S(y) \equiv N_p \int_A^{\infty} f(y) dy$$

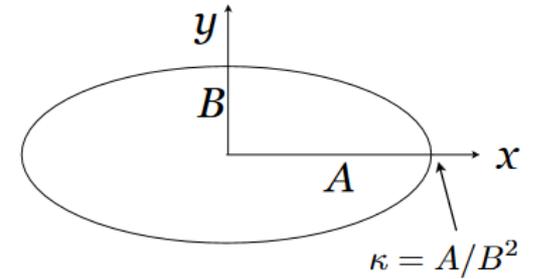
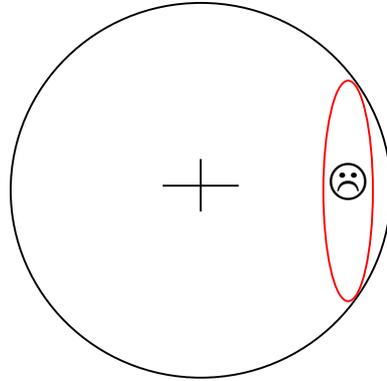
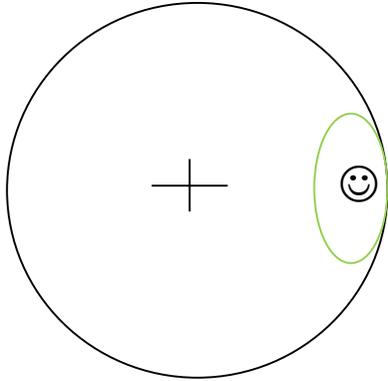
Where $f(y)$ is the probability density function of the kicked beam

Multi-turn effects: mis-kicked beam escaping the TDI (misaligned by 1σ) will intercept the TCLIB at the 3rd turn (vertical tune 59.31). Three turns are needed between high loss detection and beam dump [5]!

TCLIB



Optimum optics condition for injected beam



Curvature of the ellipse in the ring phase space and normalised coordinates:

$$k = \frac{1}{\sqrt{\varepsilon}} \quad \text{Where} \quad \bar{x}^2 + \bar{x}'^2 = \varepsilon$$

The ellipse curvature for the injected beam (assuming upright ellipse) is:

$$k_i = \left(\frac{\beta_i}{\beta} \right)^{\frac{3}{2}} \frac{1}{\sqrt{\varepsilon_i}}$$

Conditions for optimum injection into phase space [13]:

$$\frac{\alpha_i}{\beta_i} = \frac{\alpha}{\beta} = -\frac{x_i' - x_0'}{x_i - x_0} \quad \text{and} \quad \frac{\beta_i}{\beta} \geq \left(\frac{\varepsilon_i}{\varepsilon} \right)^{\frac{1}{3}}$$

(x_0, x_0') = Closed orbit

(x_i, x_i') = Centre of injected beam