TRANSFER LINES – PAPER STUDIES

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CAS, Erice, March 2017

Outline

- Introduction
  - What is a transfer line?
- Paper studies
  - Geometry – estimate of bend angles
  - Optics – estimate of quadrupole gradients and apertures
  - Error estimates and tolerances on fields

1st hour

- Examples of using MADX for
  - Optics and survey matching
  - Achromats
  - Final focus matching
  - Error and correction studies

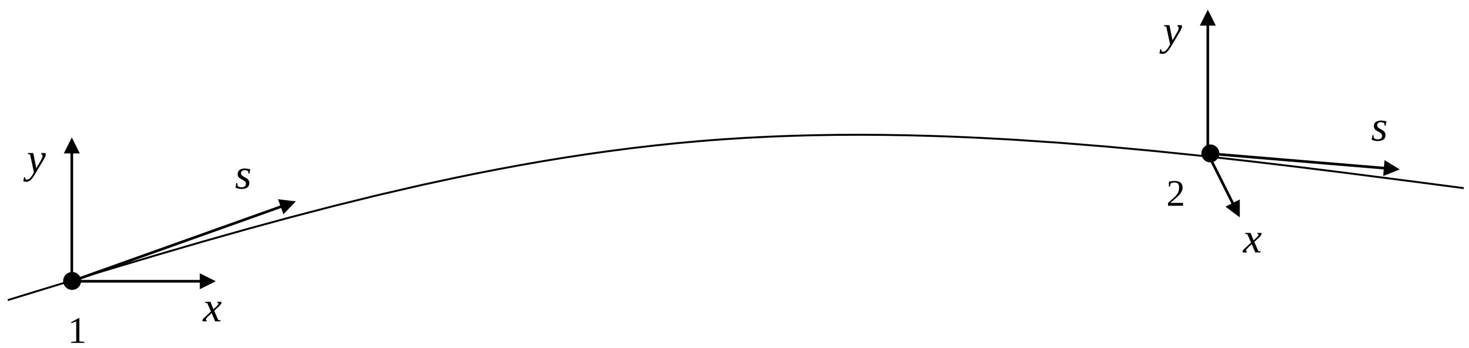
2nd hour

- Special cases in transfer lines
  - Particle tracking
  - Plane exchange
  - Tilt on a slope
  - Gantry matching
  - Dilution
  - Stray fields

3rd hour

General beam transport

…moving from s_1 to s_2 through *n* elements, each with transfer matrix M_i



$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

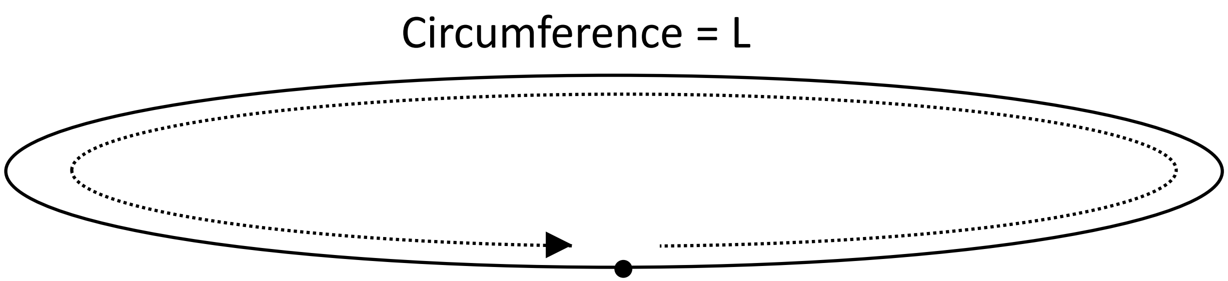
$$\mathbf{M}_{1\to 2} = \prod_{i=1}^{n} \mathbf{M}_n$$

Twiss parameterisation

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1 - \alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

[1] B. Goddard, CAS (2004), URL http://cas.web.cern.ch/cas/Baden/PDF/transfer.pdf

Circular Machine



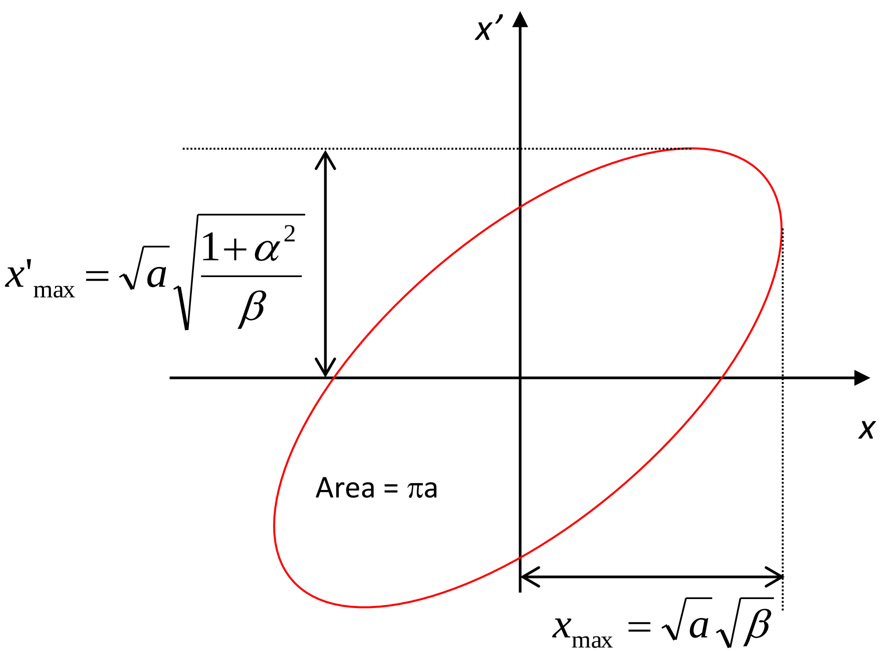
One turn

$$\mathbf{M}_{1\to 2} = \mathbf{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\frac{1}{\beta}\left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1=\alpha_2$, $\beta_1=\beta_2$, $D_1=D_2$
- This condition *uniquely* determines $\alpha(s)$, $\beta(s)$, $\mu(s)$, $D(s)$ around the whole ring

Circular Machine

- Periodicity of the structure leads to regular motion
  - Map single particle coordinates on each turn at any location
  - Describes an ellipse in phase space, defined by one set of a and b values ⇒ Matched Ellipse (for this location)

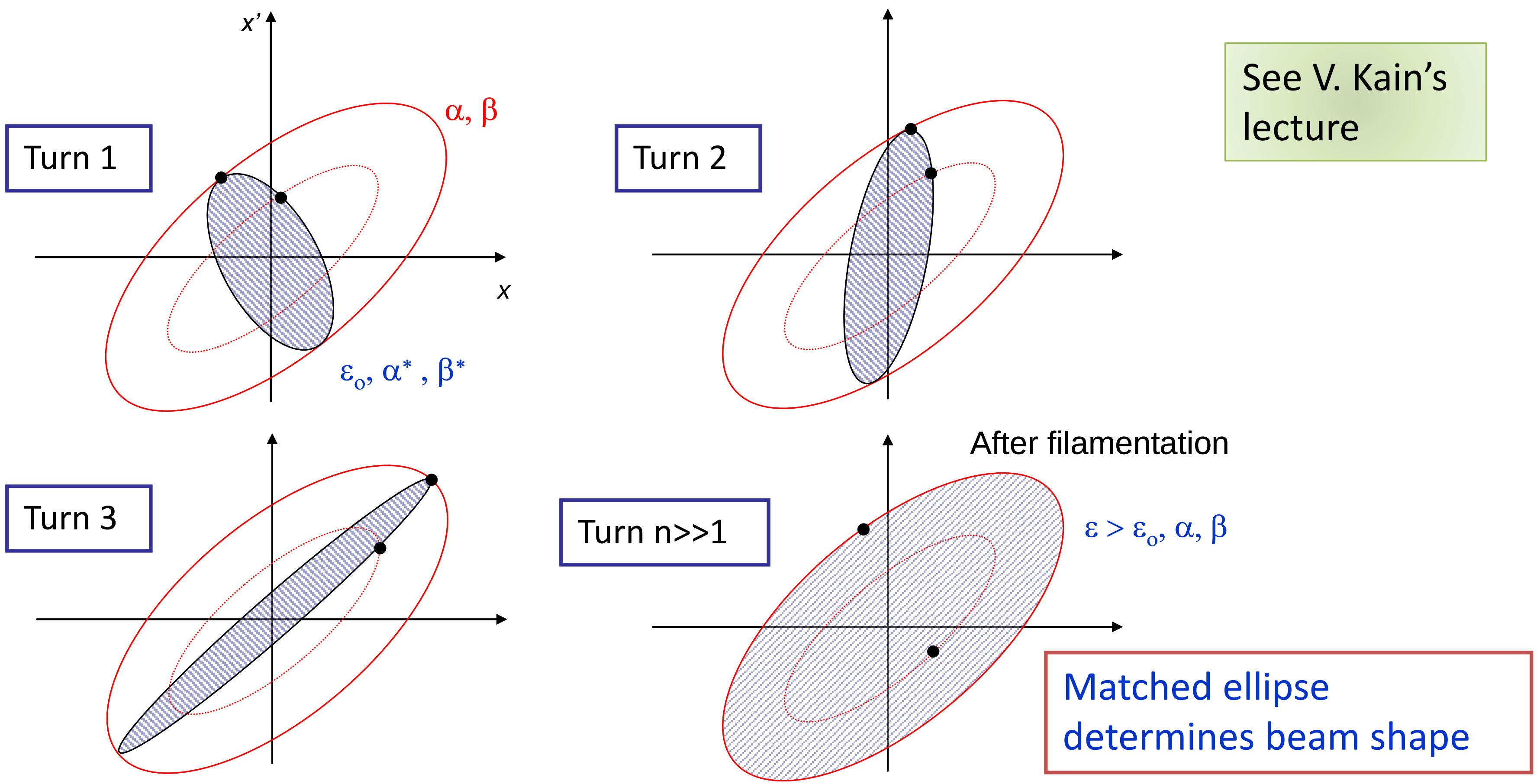


$$a = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

Circular Machine

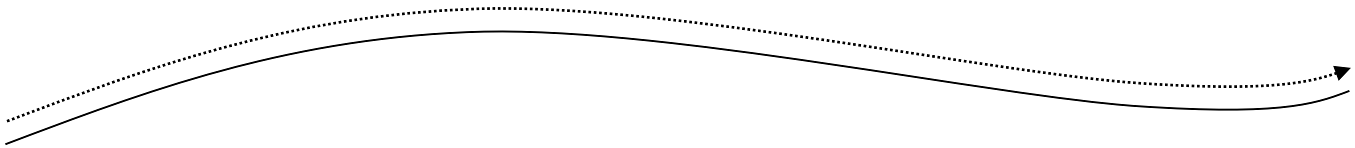
- For a location with matched ellipse (a, b), an injected beam of emittance e, characterised by a different ellipse (a^*, b^*) generates (via filamentation) a large ellipse with the original a, b, but larger e



Transfer line

Single pass: $$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$



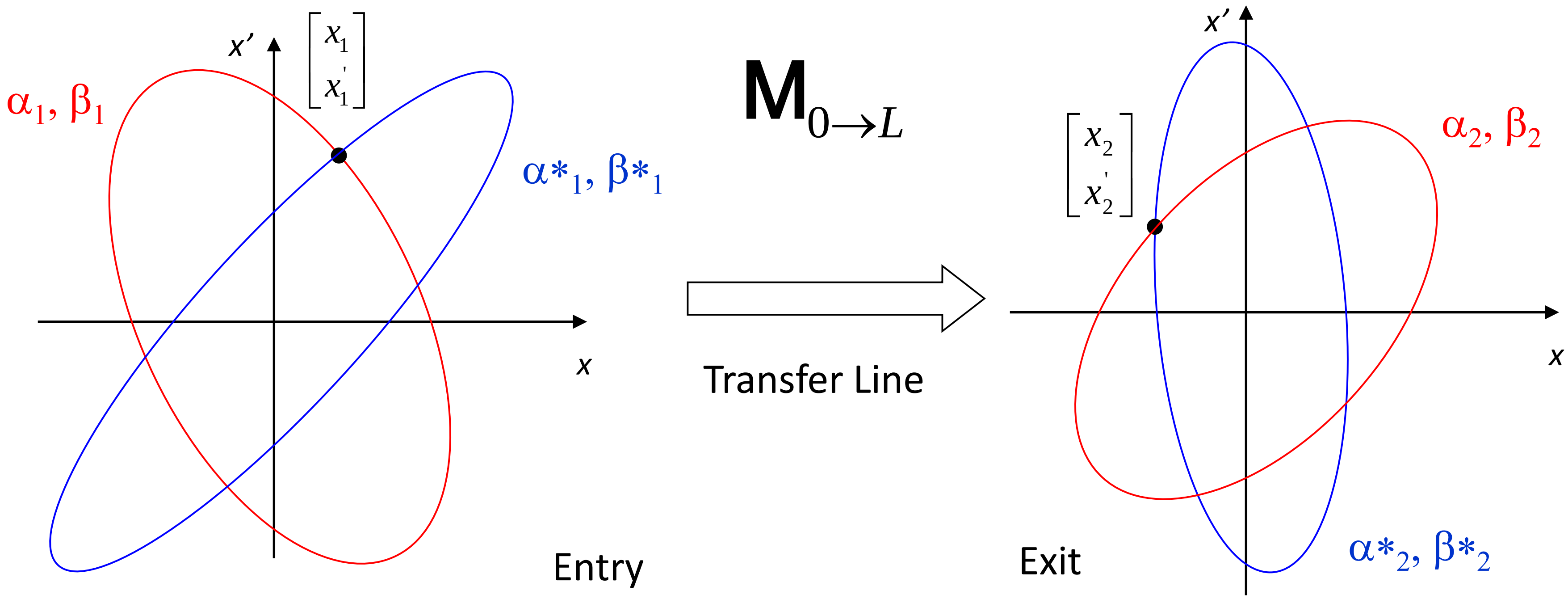
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2 / \beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\,\sin\Delta\mu \\ \sqrt{1 / \beta_1\beta_2}\left[\left(\alpha_1 - \alpha_2\right)\cos\Delta\mu - \left(1 + \alpha_1\alpha_2\right)\sin\Delta\mu\right] & \sqrt{\beta_1 / \beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, $\alpha(s)$ $\beta(s)$ are functions of α_1 β_1

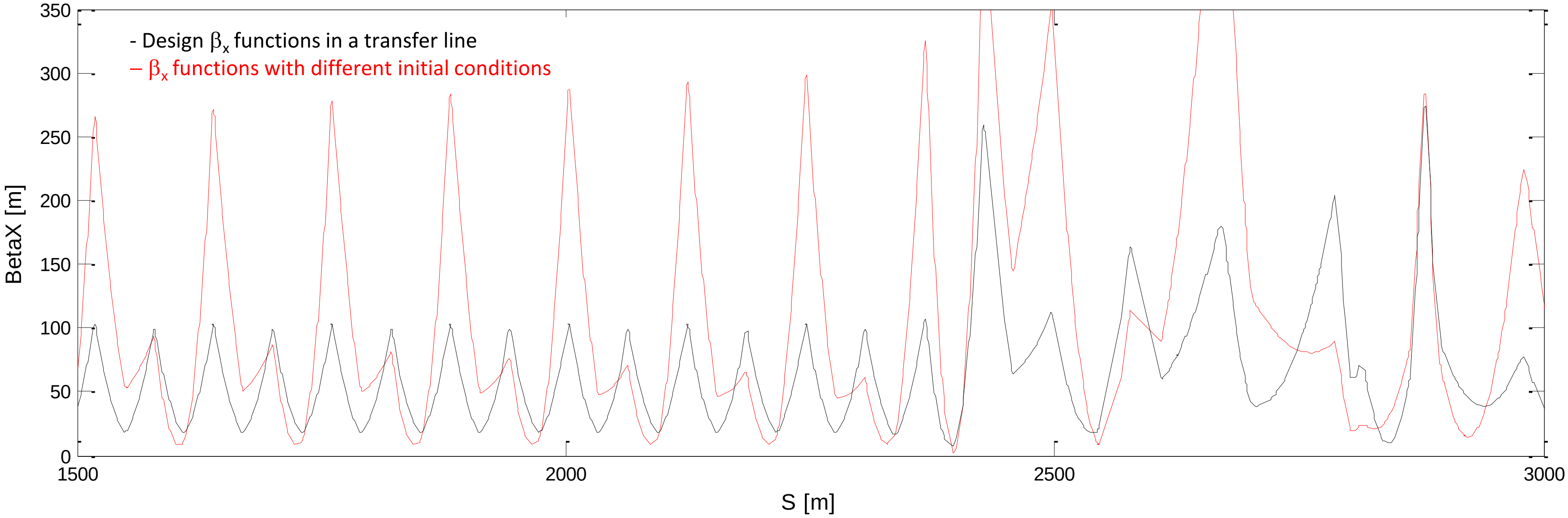
Transfer line

- On a single pass there is no regular motion
  - Map single particle coordinates at entrance and exit.
  - Infinite number of equally valid possible starting ellipses for single particle ……transported to infinite number of final ellipses…



Transfer Line

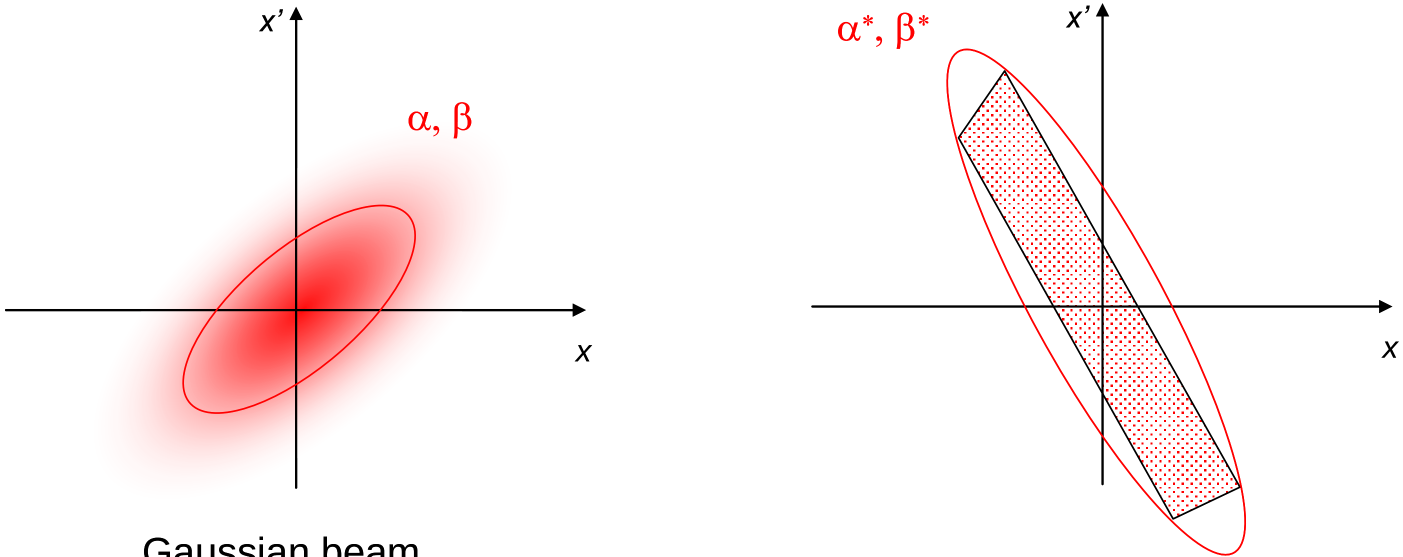
- The optics functions in the line depend on the initial values



- Same considerations are true for Dispersion function:
  - Dispersion in ring defined by periodic solution → ring elements
  - Dispersion in line defined by initial D and D' and line elements

Transfer Line

- Initial a, b defined for transfer line by beam shape at entrance



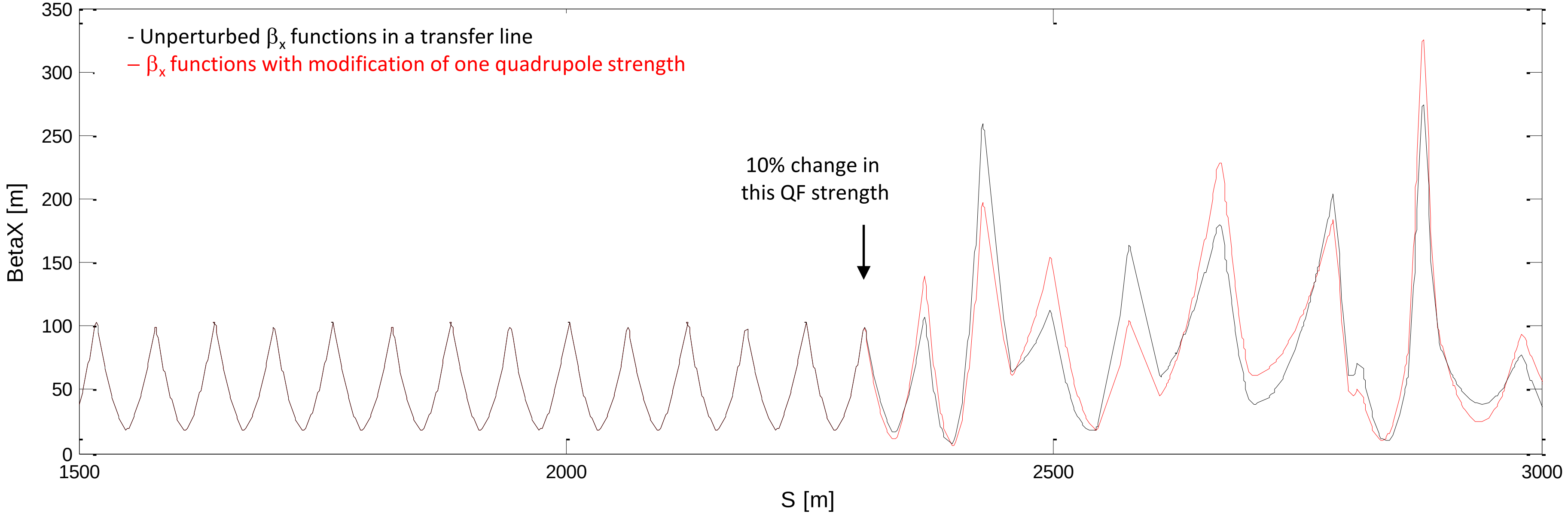
Gaussian beam

Non-Gaussian beam
(e.g. slow extracted)

- Propagation of this beam ellipse depends on line elements
- [A transfer line optics is different for different input beams]()

Transfer Line

- Another difference….unlike a circular ring, a change of an element in a line affects *only* the downstream Twiss values (including dispersion)

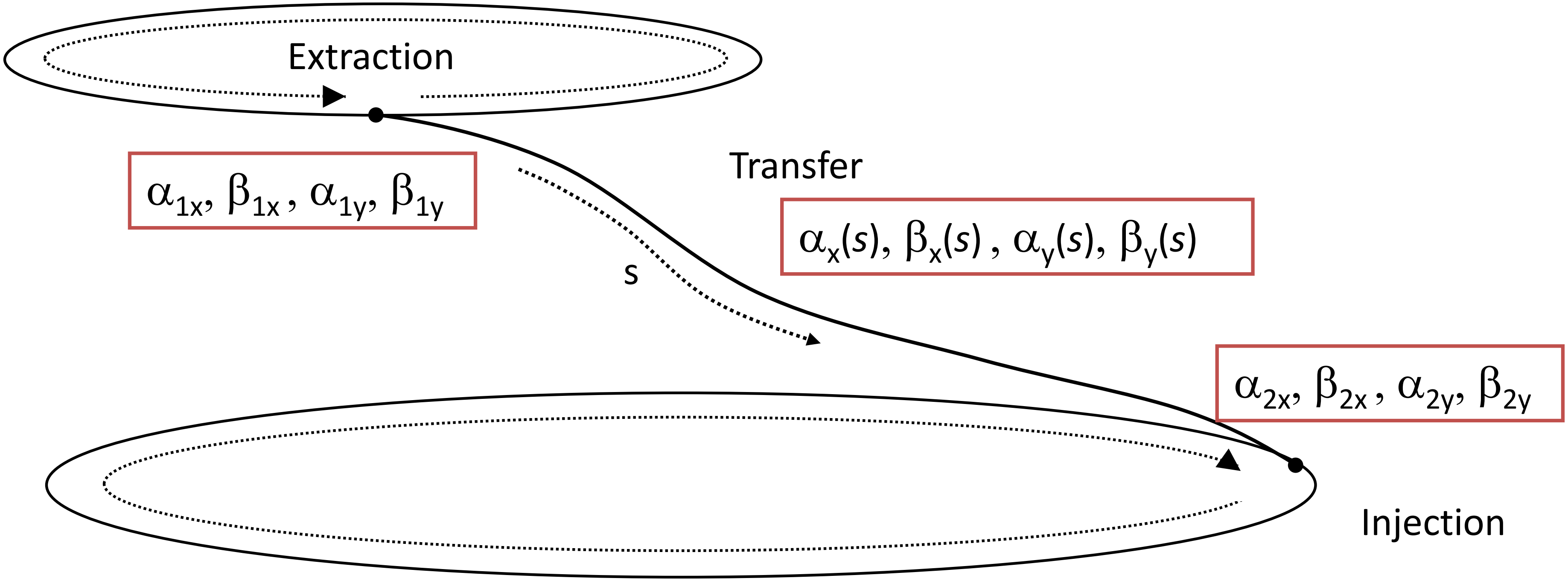


Linking Machines

- Beams have to be transported from extraction of one machine to injection of next machine
  - Trajectories must be matched, ideally in all 6 geometric degrees of freedom (x,y,z,theta,phi,psi)

- Other important constraints can include
  - Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology

Linking Machines



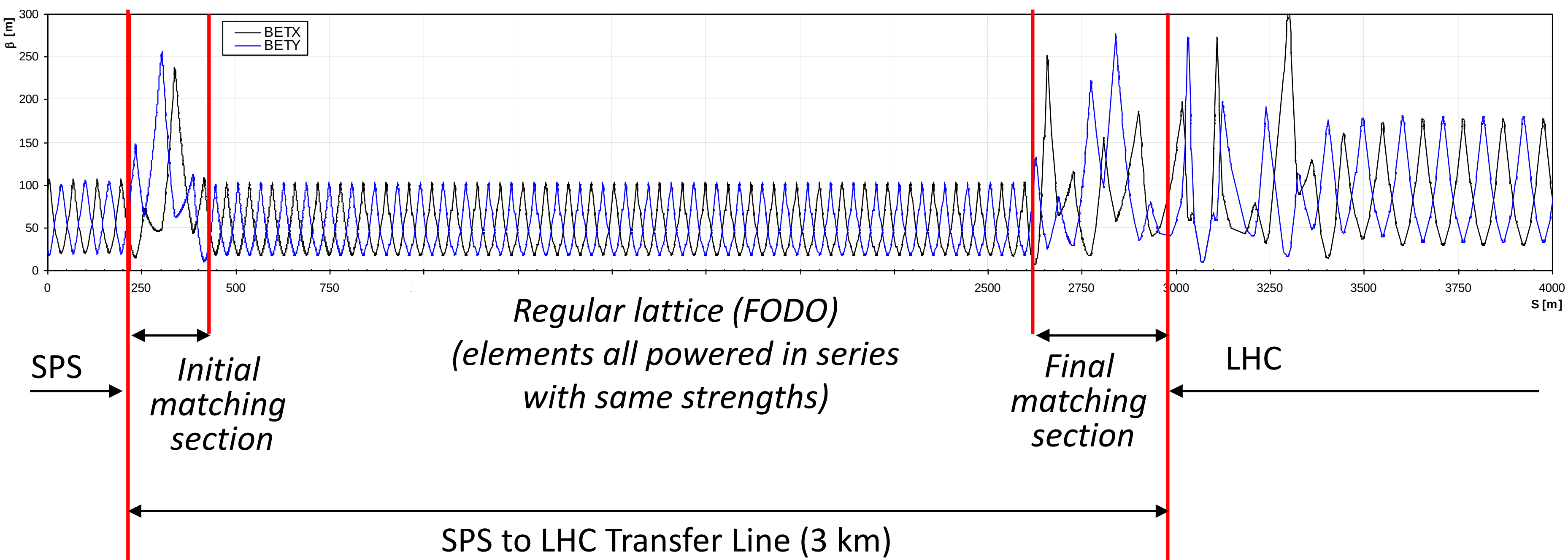
The Twiss parameters can be propagated when the transfer matrix M is known

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

Linking Machines

- For long transfer lines we can simplify the problem by designing the line in separate sections
  - Regular central section – e.g. FODO or doublet, with quads at regular spacing, (+ bending dipoles), with magnets powered in series
  - Initial and final matching sections – independently powered quadrupoles, with sometimes irregular spacing.



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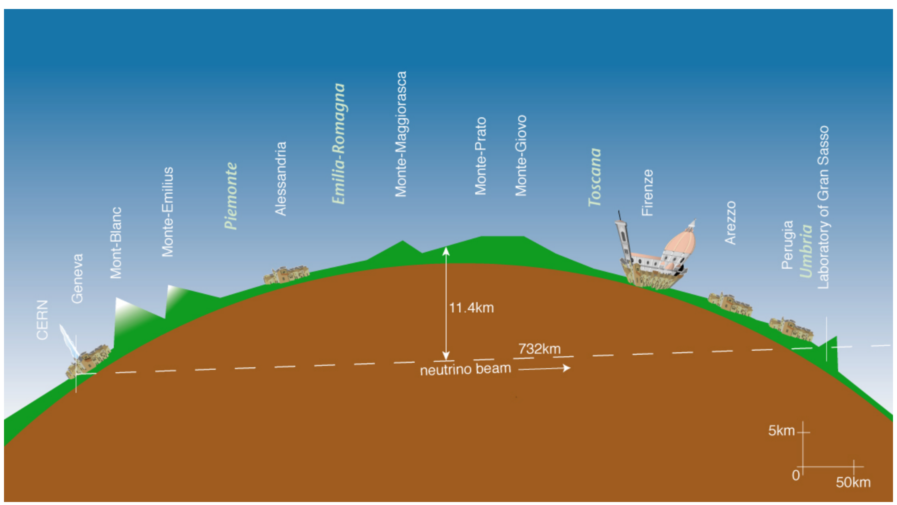
1st hour

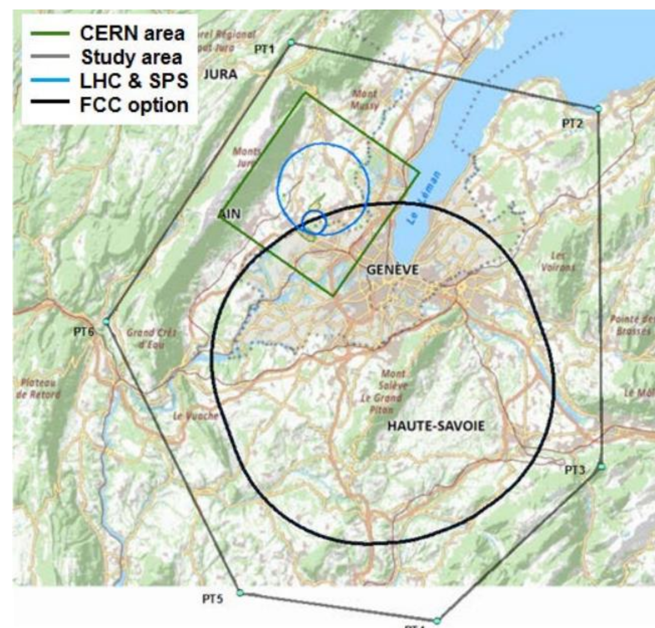
2nd hour

3rd hour

Survey

- Need coordinates and angles of points to be linked in a common coordinate system
- Linking CNGS to Gran Sasso in Italy the CERN reference frame had to be connected to the global systems of Switzerland and Italy – small rotations seen but negligible
- FCC study covers an area ten times bigger than existing installations

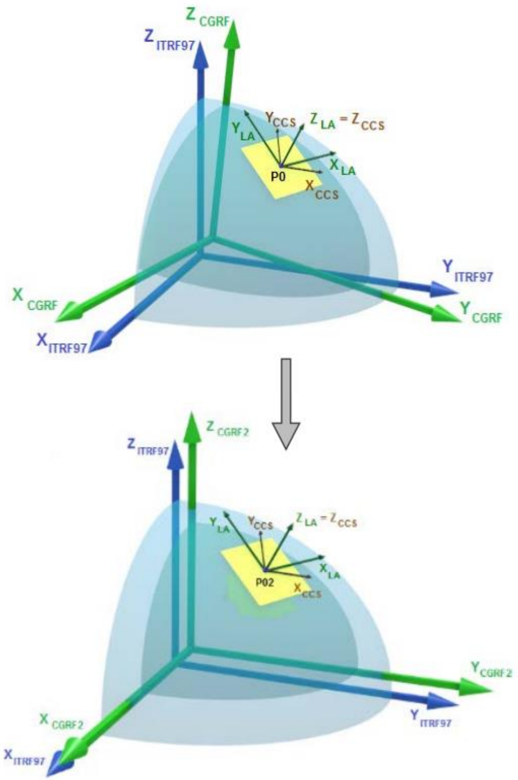


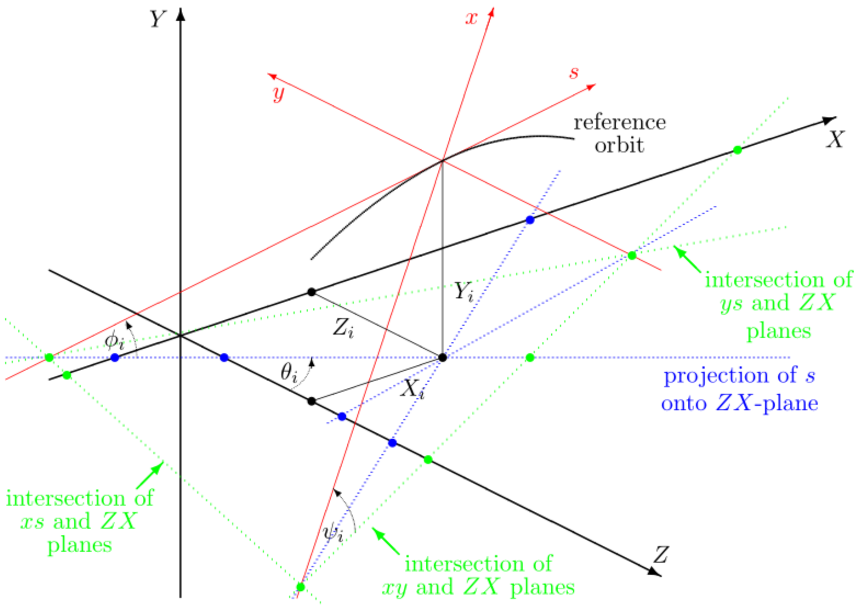


[2] N. Ibarrola and M. Jones, International Workshop of Accelerator Alignment (2016).

Survey vs MADX

- Clear definition of coordinate system with survey colleagues is essential!





MADX reference system

[2] N. Ibarrola and M. Jones, International Workshop of Accelerator Alignment (2016).

Bending fields

- Magnetic and electric rigidity:

$$B\rho = \frac{p}{q} \quad \Longrightarrow \quad B\rho\,[Tm] = 3.3356\,\frac{A}{n}\,p\,[GeV/c]$$

A … atomic mass number
n … charge state
p … average momentum per nucleon

$$E\rho = \frac{pv}{q} \quad \Longrightarrow \quad E\rho\,[kV] = \frac{\gamma+1}{\gamma}\,\frac{A}{n}\,T\,[keV]$$

T … average kinetic energy per nucleon

- Deflection angle: $\theta = \frac{Bdl}{B\rho}$ or $\frac{Edl}{E\rho}$

Where is the limit between electric and magnetic?

- Electric devices are limited by the applied voltage – one can assume several 10s of kV as limit for reasonable accelerator apertures

- Magnets are limited by the field quality at low fields
  - Strong dependence on material properties
  - Remnant fields become important
  - Measuring the field becomes a challenge

- Example
  - 100 keV antiprotons
  - Electrostatic quadrupoles with 60 mm diameter require applied voltages of below 10 kV
  - Electrostatic bends of up to 30 kV

If you are in the energy grey zone…how to choose between magnetic and electric?

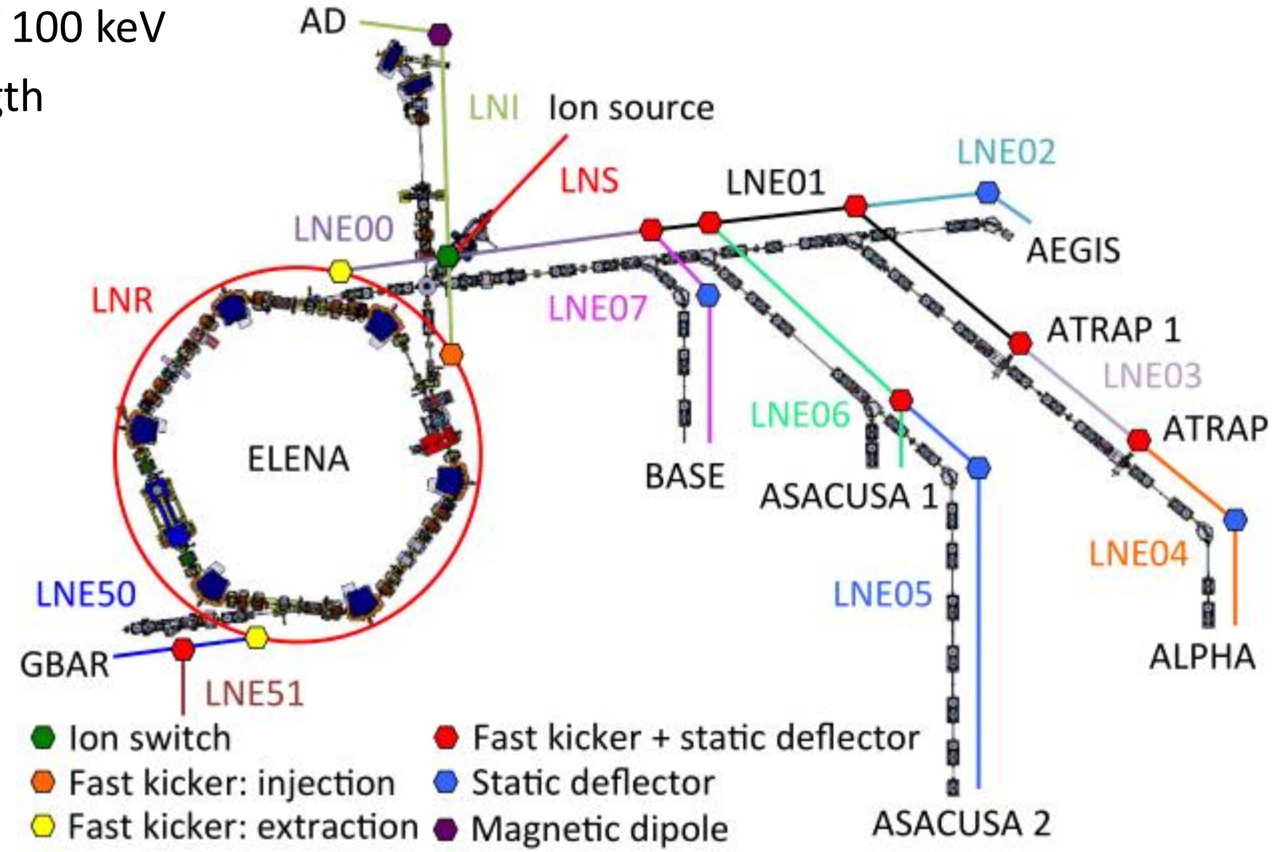
Pros and cons of electrostatic beam lines:

- Cheap element production
- Cheap power supplies and cabling
- Mass-independent
- No hysteresis effects (easy operation)
- No power consumption – no cooling
- Transverse field shape easy to optimize

- Difficult to measure field shape – effective length
- Diagnosis of bad connections
  - Inside vacuum
    - Large outgassing surface area
    - Vulnerable to dirt inside vacuum
    - Requires vacuum interlock for sparking and safety
    - Repair requires opening the vacuum
    - Limited choice of vacuum and bake-out compatible insulators

2D geometry

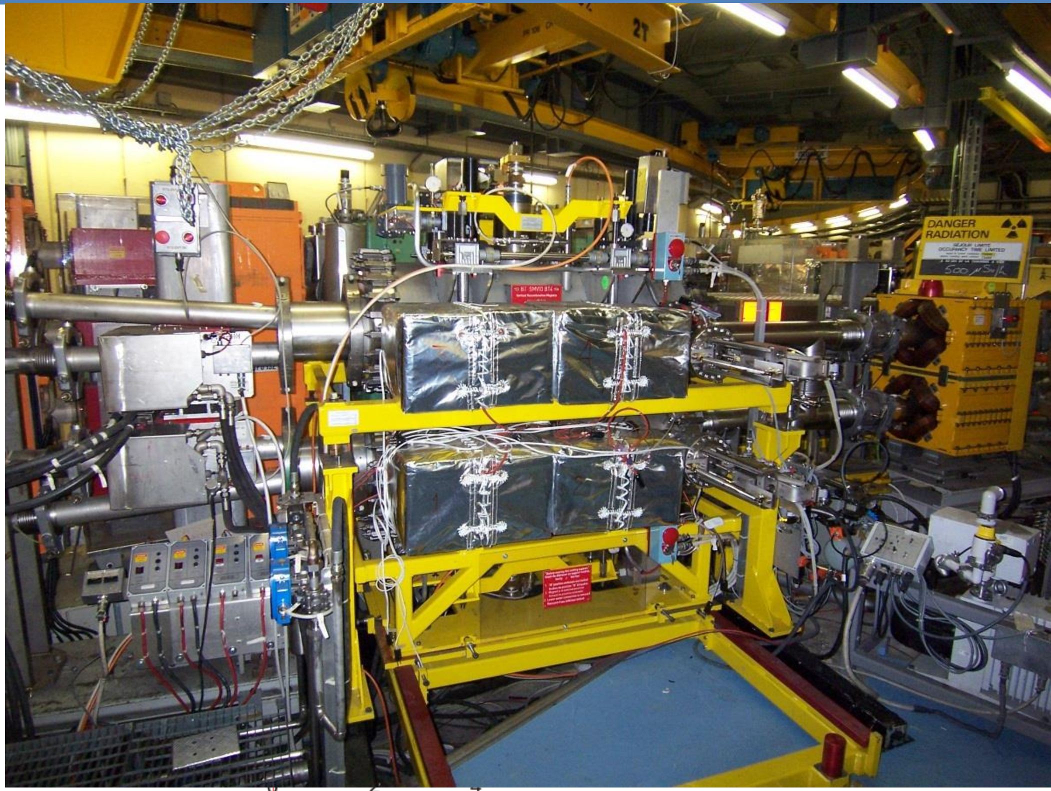
- Very low energy of 100 keV
- Short bending length

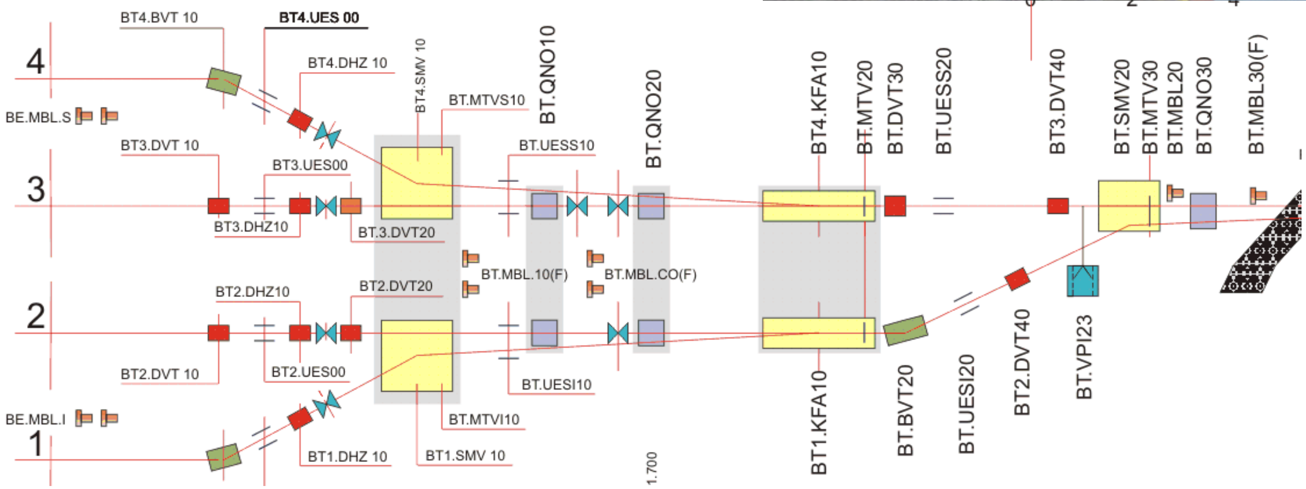


[3] M Fraser et al. Beam Dynamics Studies of the ELENA Electrostatic Transfer Lines. (CERN-ACC-2015-340):MOPJE044. 4 p, 2015.

2D geometry

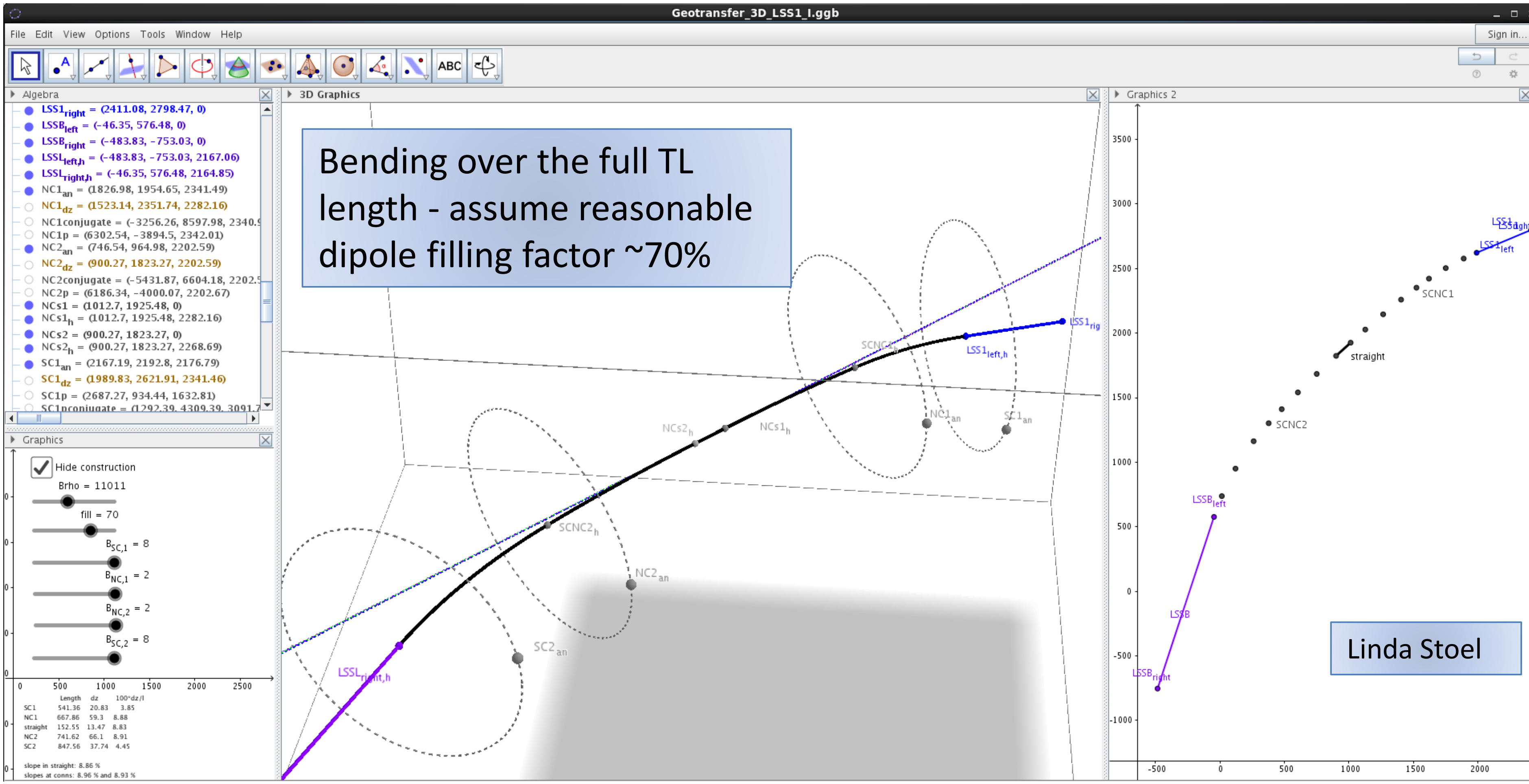
- 36 cm vertical height difference over several m
- 1-2 GeV





More complex 3D geometry

Several 100 m vertical, several km length, 3.3 TeV…distributed bending



Bending field limits

- So far we considered the bending fields in transfer lines limited solely by hardware
- A few 10 kV on electrostatic devices to avoid sparking
- 2 T for normal conducting magnets
- Something like existing LHC dipole reach 9-9.5 T for superconducting magnets

- But is there anything else which might limit the bending field?

Lorentz Stripping

- Transfer of H^- ions
- Extra electron binding energy is 0.755 eV
- A moving ion sees the magnetic field in its rest frame – Lorentz transform gives electric field as

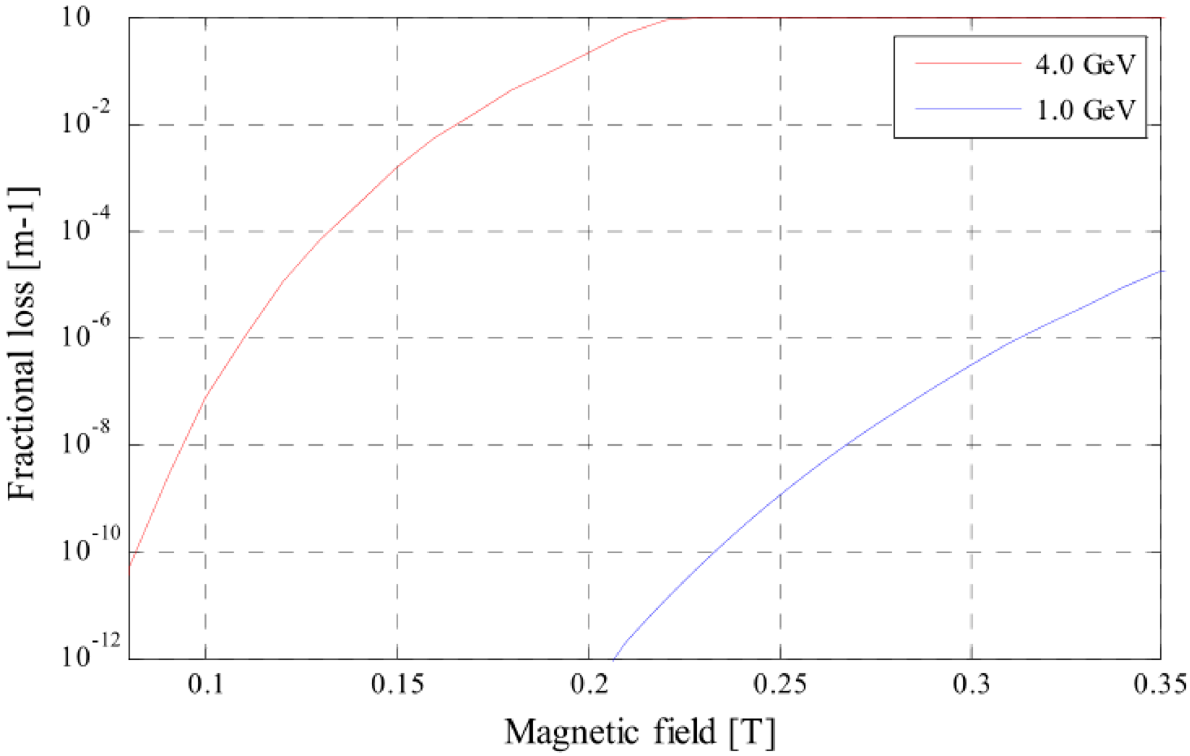
$$E\ \left[\frac{MV}{cm}\right] = 3.197 \cdot p[\frac{GeV}{c}] \cdot B[T]$$

- Lifetime

$$\tau = \frac{A}{E} e^{(\frac{C}{E})} \qquad A = 7.96 \cdot 10^{-14}\ \mathrm{s\ MV/cm},\ C = 42.56\ \mathrm{MV/cm}$$

Example of PS2

- 4 GeV injection
- Fractional loss below 1e-4 limits magnetic field to 0.13 T

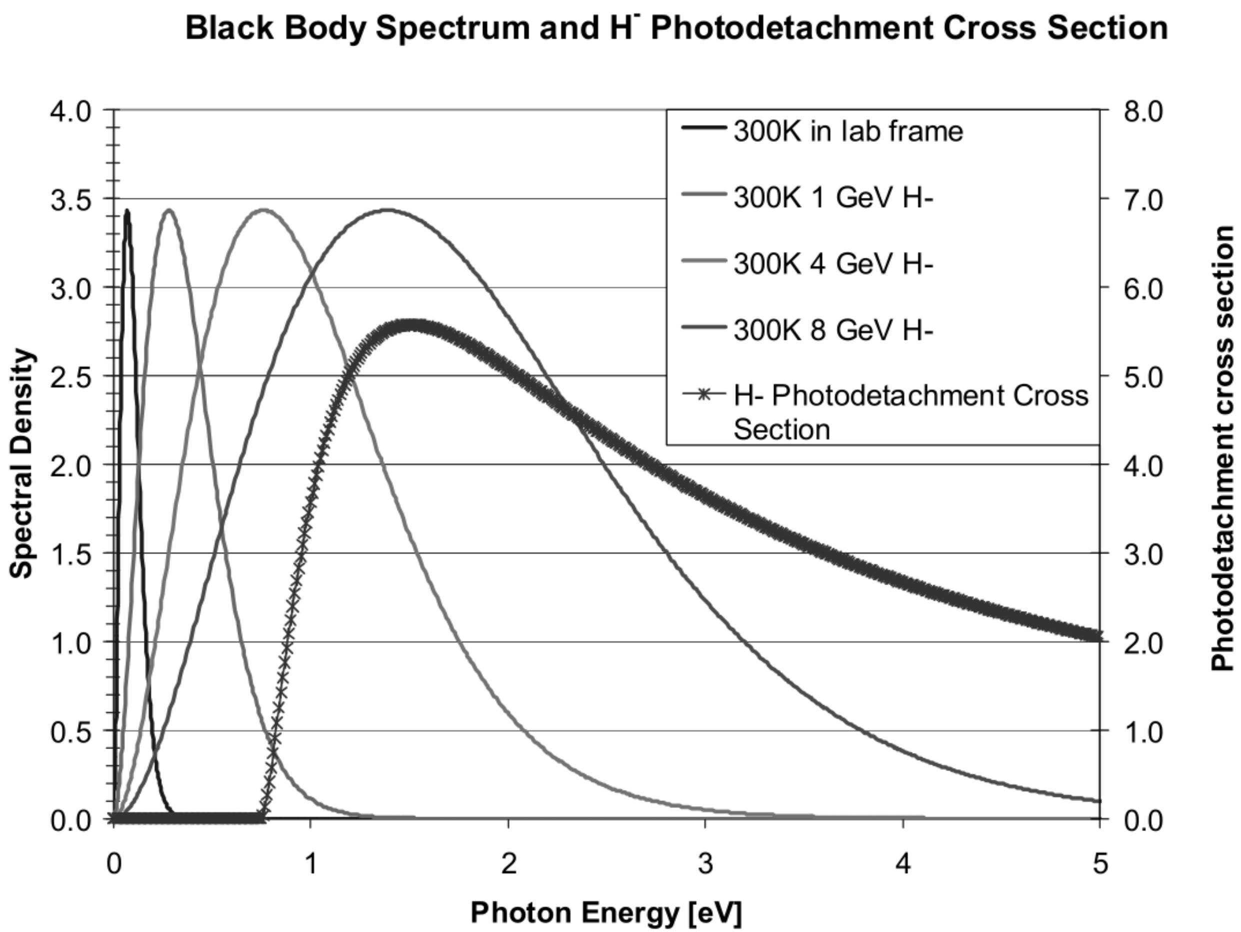


Example of Fermilab Project-X

- Was considered as proton source with **8 GeV H-** into recycler ring for neutrino program
- Usual power loss limit in lines of ~1W/m
- Activation was found to be not acceptable for 8 GeV ions
- Reduction to 0.05 W/m power loss to meet radioprotection requirements
- In this regime also other loss processes become relevant…

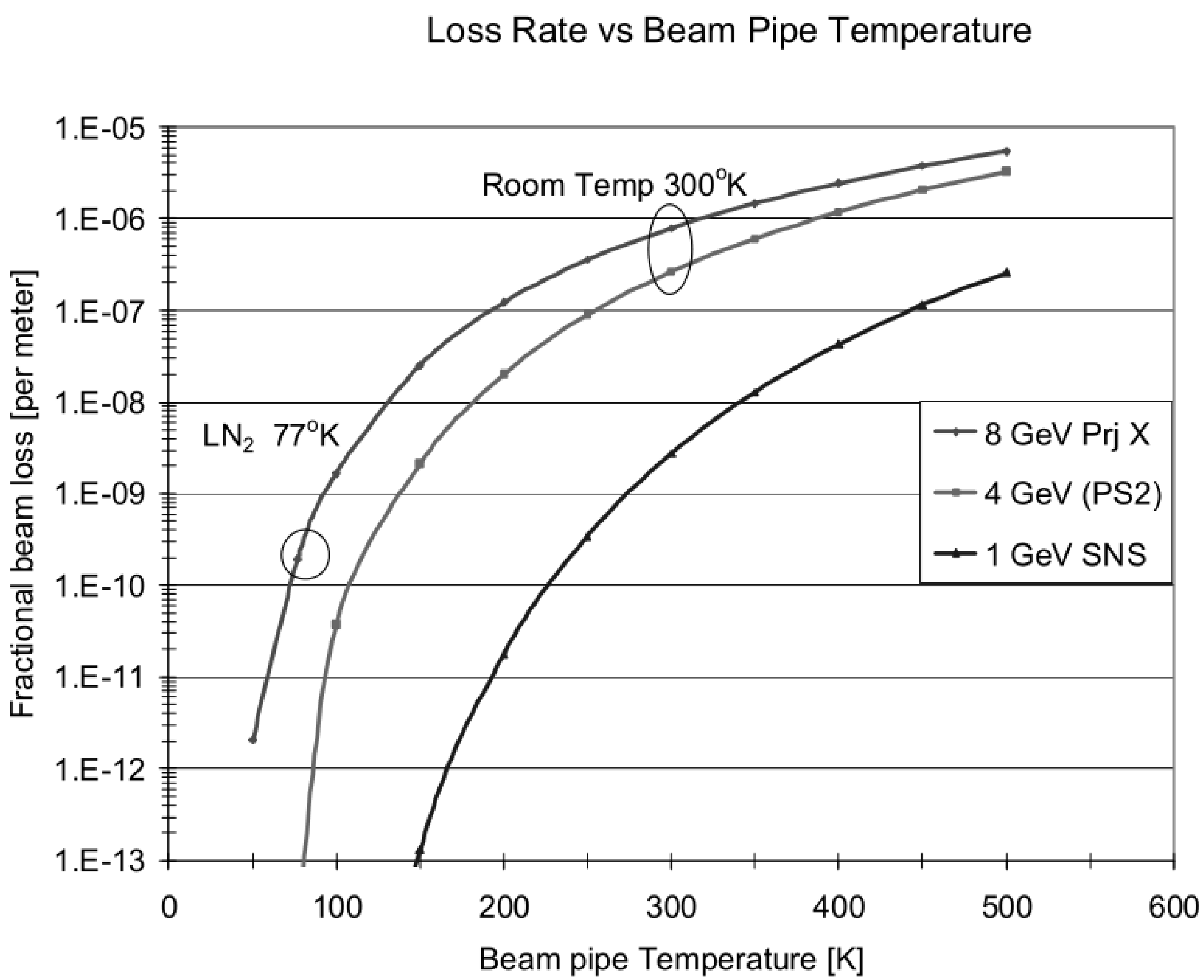
[4] D. Johnson. Challenges Associated with 8 GeV H- Transport and Injection for FERMILAB PROJECT-X *. (Proceedings of Hadron Beam 2008), 2008.

Black body radiation



[6] D. Johnson. Challenges Associated with 8 GeV H- Transport and Injection for FERMILAB PROJECT-X *. (Proceedings of Hadron Beam 2008), 2008.

Black body radiation



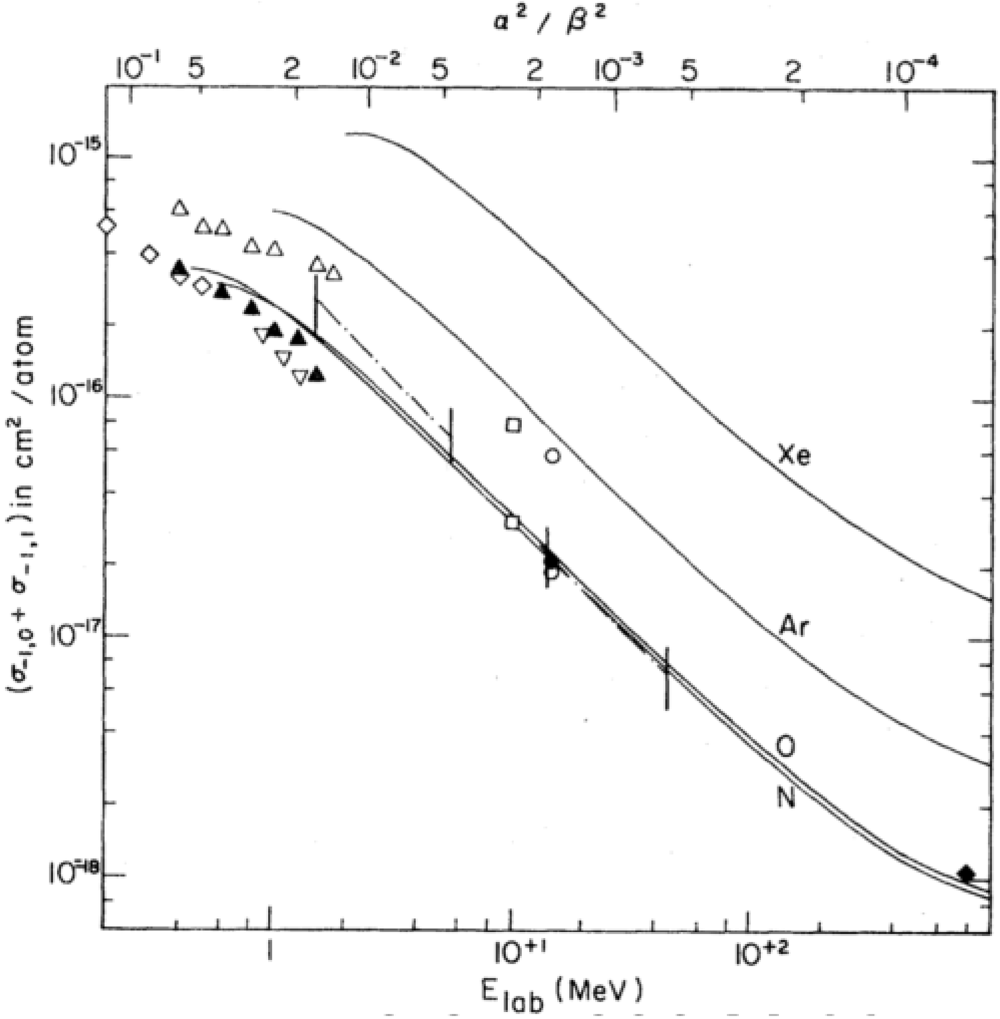
Rest gas stripping

- Power loss due to stripping on rest gas per length l

$$P = E\ I\ \frac{d\sigma}{d\Omega}\ \rho\ l$$

Beam energy and intensity

Gas density …fct of T, p



$$\frac{d\sigma}{d\Omega} = \frac{7{\cdot}10^{-19}}{\beta^2} cm^2 \text{ per atom of nitrogen or oxygen}$$

$$\frac{d\sigma}{d\Omega} = \frac{1{\cdot}10^{-19}}{\beta^2} cm^2 \text{ per atom of hydrogen}$$

$$\text{Electron loss} \propto \frac{1}{\beta^2}$$

Example Fermilab Project X

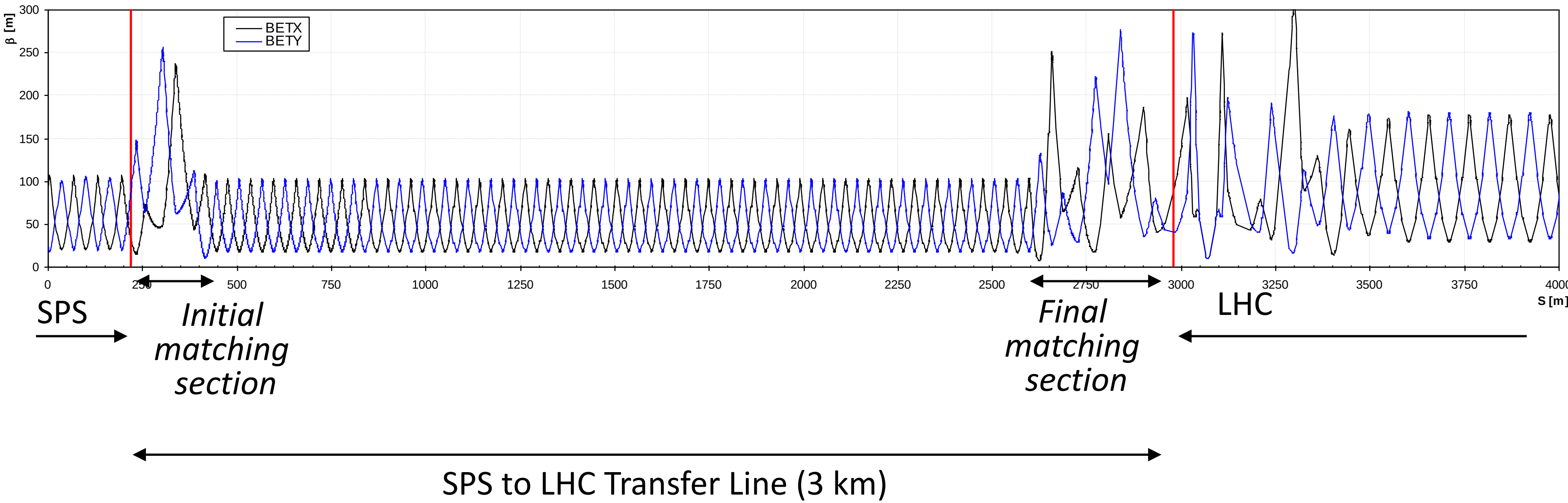
- Loss rate from black body radiation at 300 K not acceptable

- Installing a cool beam screen (77 K)
  - Reduces black body radiation by factor ~16
  - Improves vacuum pumping (better than 1e-8 Torr)

- Lorentz stripping limits dipole fields to 0.05 T

Focussing structure

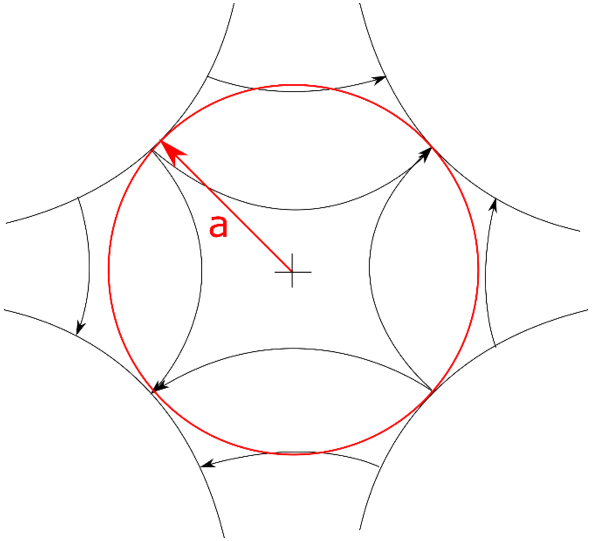
- Cell length optimised for dipole filling at extraction energy
- Can assume this as a good starting point for our transfer line
- For transfer lines often a 90 deg FODO structure is chosen
  - Good ratio of max/min in beta function
  - Same aperture properties
  - Provides good locations for trajectory correctors and instrumentation
  - Good phase advance for injection/extraction and protection equipment



Quadrupole field

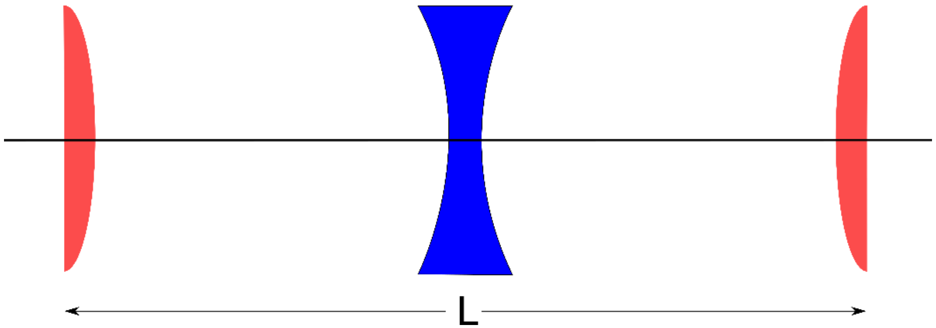
- What is needed to specify the quadrupole pole tip field:

$$B = g \cdot x \qquad B_{poletip} = g \cdot a$$



- Need to define quadrupole gradient g [T/m] and pole radius a [m]

FODO cell



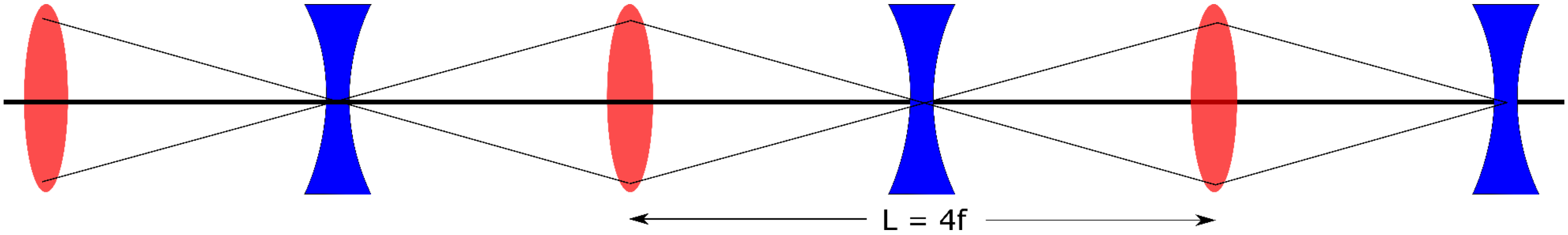
$$R = \begin{pmatrix} 1 & 0 \\ 1/2f & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/2f & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - L^2/8f^2 & L - L^2/4f \\ -L/4f^2 - L^2/16f^3 & 1 - L^2/8f^2 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

FODO stability

Stability for: $f > \frac{L}{4}$



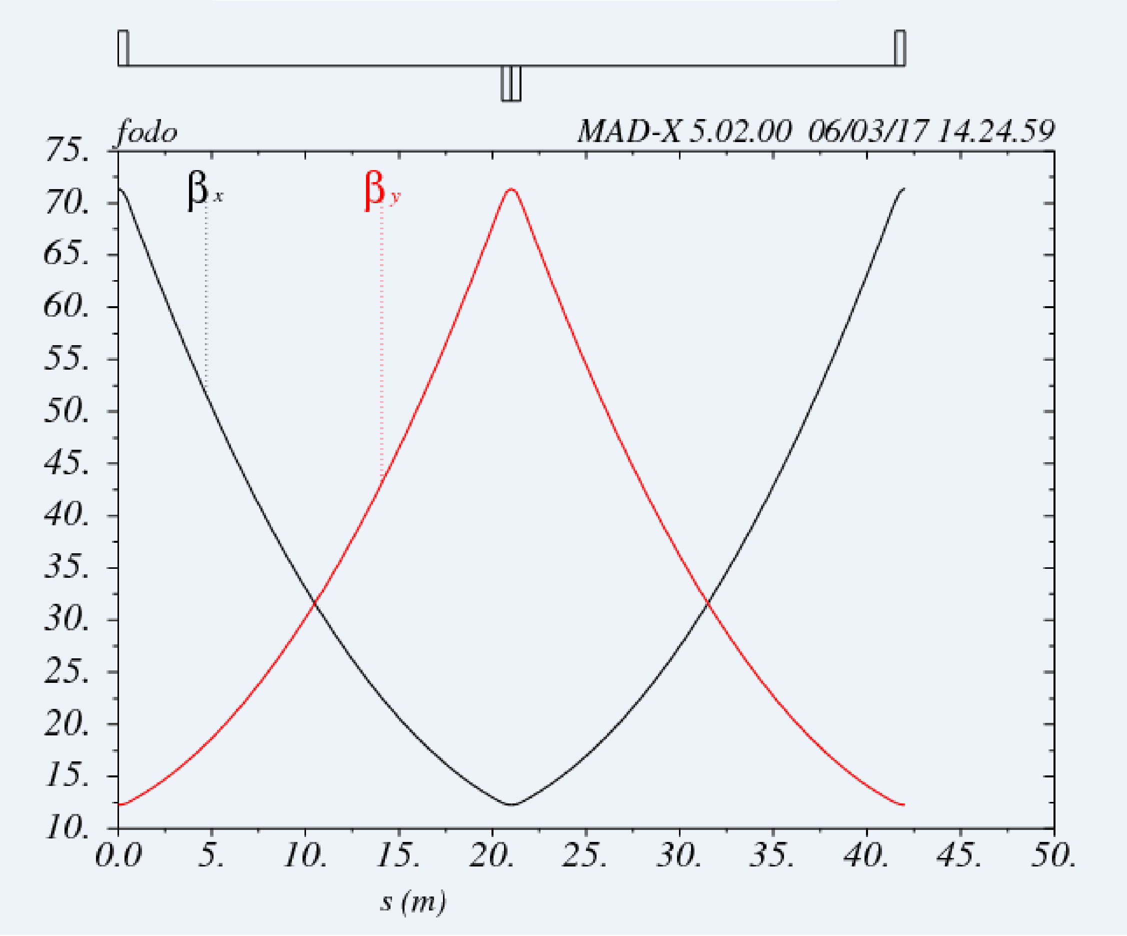
$$\frac{L}{f} = 4 \sin \frac{\mu}{2}$$

Estimate required gradient of quadrupoles

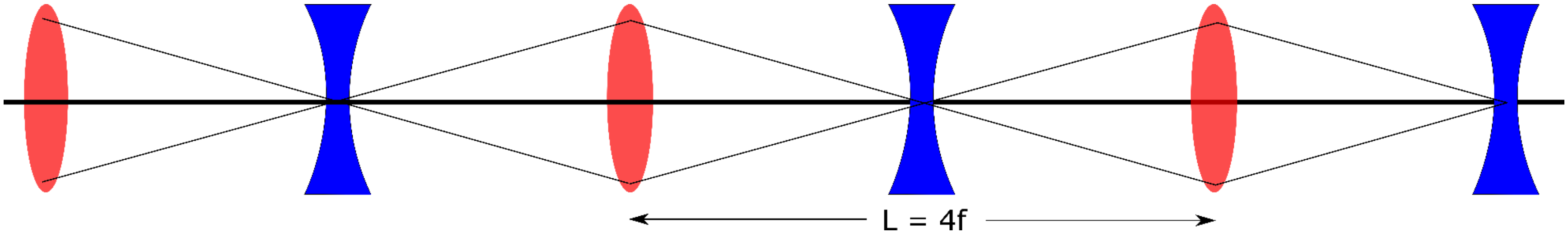
[7] J. Rossbach and P. Schmueser. Basic Course on Accelerator Optics. (CAS), 1992.

FODO optics

$$\mu = 90^o \Rightarrow f = \frac{L}{2\sqrt{2}}$$



FODO stability



$$\frac{L}{f} = 4\sin\frac{\mu}{2}$$

Estimate required gradient of quadrupoles

$$\beta = \left(L + \frac{L^2}{4f}\right) / \sin\mu$$

Use maximum betatron function to estimate beam size and pole tip field of quadrupoles

[7] J. Rossbach and P. Schmueser. Basic Course on Accelerator Optics. (CAS), 1992.

Apertures

- Arbitrary choice and depends on beam energy (destructive?)
- While in a collider have O(10 sig + few mm)
- Less in TLs
- $A_{x,y} = \pm n_{sig} \cdot \sqrt{k_\beta \cdot \beta_{x,y} \cdot \frac{\epsilon_{x,y}}{\beta\gamma}} \pm D_{x,y} \cdot k_\beta \cdot \frac{\Delta p}{p} \pm CO \cdot \sqrt{\frac{\beta_{x,y}}{\beta_{xmax,ymax}}} \pm$ align

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Optics uncertainty in TLs vs rings

Conservative approach
But be aware when you specify minimum beam sizes

$$x = \theta \cdot \sqrt{\beta_1 \beta_2} \cdot \sin(\mu_2 - \mu_1)$$

Apertures

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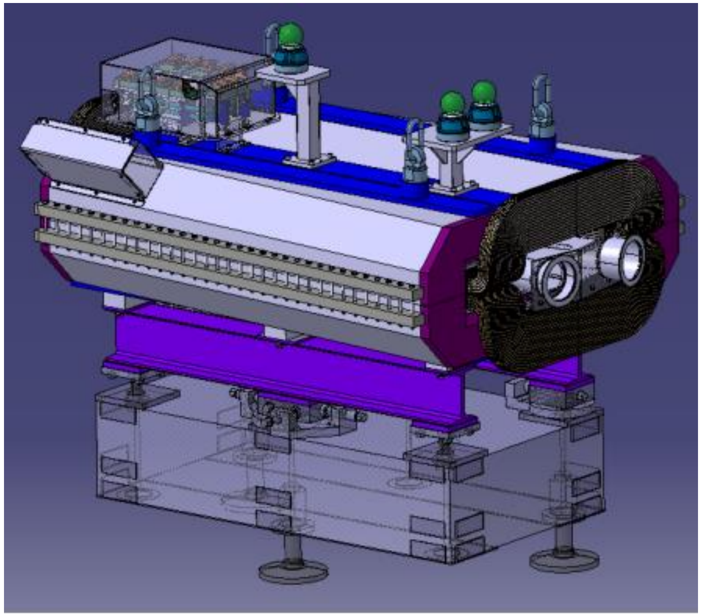
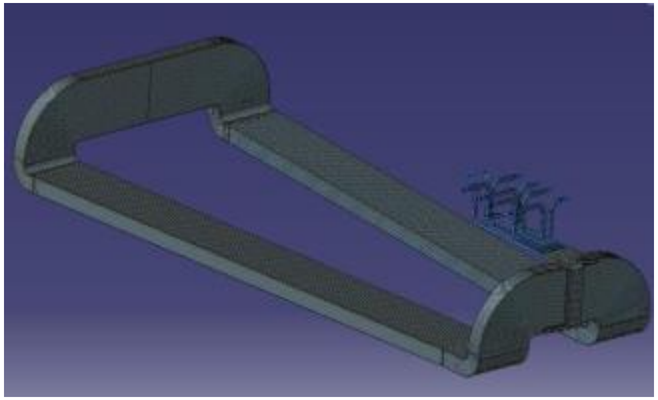




Aperture calculation examples

- Low energy transfer lines (100 keV, ELENA)
  - 10 mm trajectory variation
  - 4 mm alignment

$$\Sigma_{x,y}(95\%) = \sqrt{(\beta_{x,y}\kappa_{x,y}\epsilon_{x,y}(95\%))} + \left(D_{x,y}\kappa_{x,y}\frac{\delta p}{p}(95\%)\right)$$

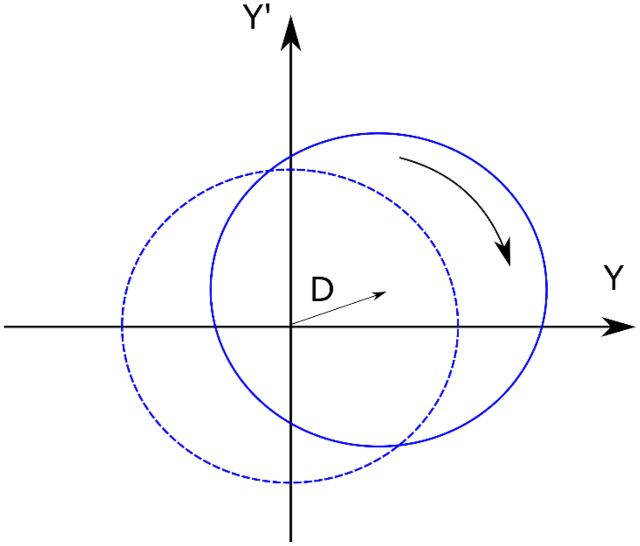
$$\kappa_{x,y} = 1.2$$

- Medium energy (1-2 GeV, PS Booster)
  - $A_{x,y} = \pm n_{sig} \cdot \sqrt{k_\beta \cdot \beta_{x,y} \cdot \frac{\epsilon_{x,y}}{\beta\gamma}} \pm D_{x,y} \cdot k_\beta \cdot \frac{\Delta p}{p} \pm CO \cdot \sqrt{\frac{\beta_{x,y}}{\beta_{xmax,ymax}}} \pm \text{align}$
  - $n_{sig} = 3$, k_β = 1.2, CO = 3 mm, align = 0 (usually ~2 mm)

- High energy transfer lines (0.45 – 3 TeV, LHC, FCC)
  - $n_{sig} = 6$, k_β = 1.0, CO = 1.5 mm

First estimate of field error specification

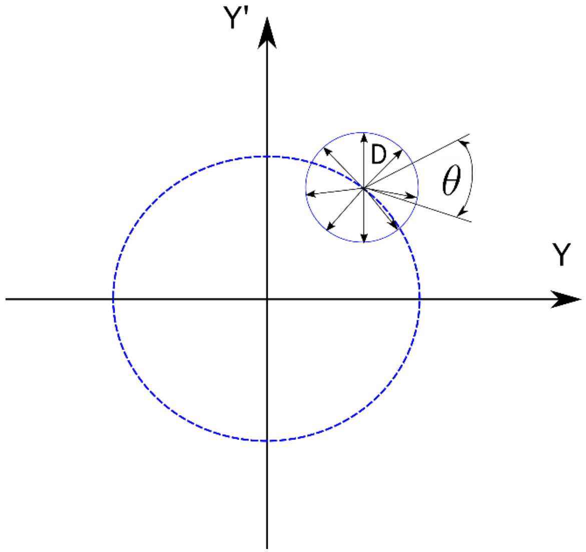
- Impact of field errors on aperture requirements should be negligible
- Beam quality is the constraint – emittance growth



$$Y_2 = Y_1 + D\cos\theta$$

$$Y_2^2 = Y_1^2 + Y_1 D\cos\theta + D^2\cos^2\theta$$

$$\langle Y_2^2\rangle = \langle Y_1^2\rangle + 2\langle Y_1 D\cos\theta\rangle + \langle D^2\cos^2\theta\rangle$$



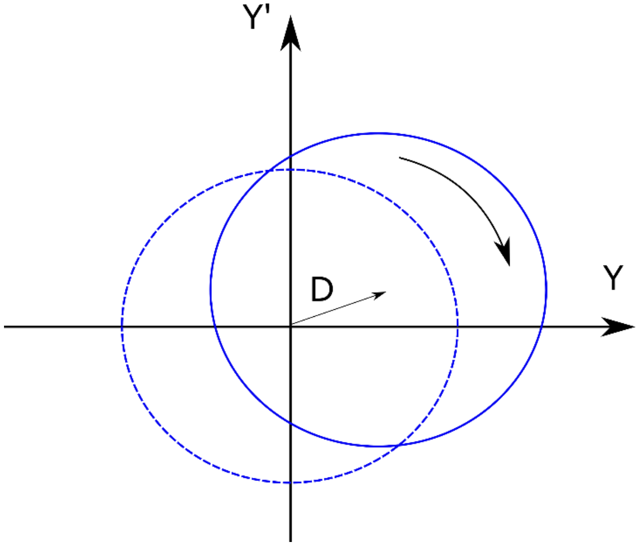
[5] P. Bryant. Beam Transfer Lines. (CAS), 1992.

First estimate of field error specification

$$2\langle Y_1 D\cos\theta\rangle = 2\langle Y_1\rangle\langle D\cos\theta\rangle$$

D and Y_1 are uncorrelated

And averaging over the constant D gives 0

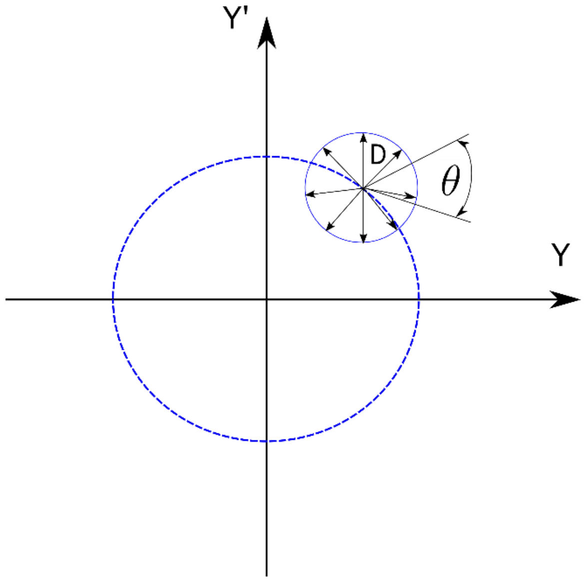


$$\langle Y_2^2\rangle = \langle Y_1^2\rangle + 2\langle Y_1 D\cos\theta\rangle + \langle D^2\cos^2\theta\rangle$$

$$\langle Y_2^2\rangle = \langle Y_1^2\rangle + \frac{1}{2}D^2$$

This is valid for any point P on any circle…

$$\epsilon_2 = \epsilon_1 + \frac{1}{2}D^2$$



First estimate of field error specification

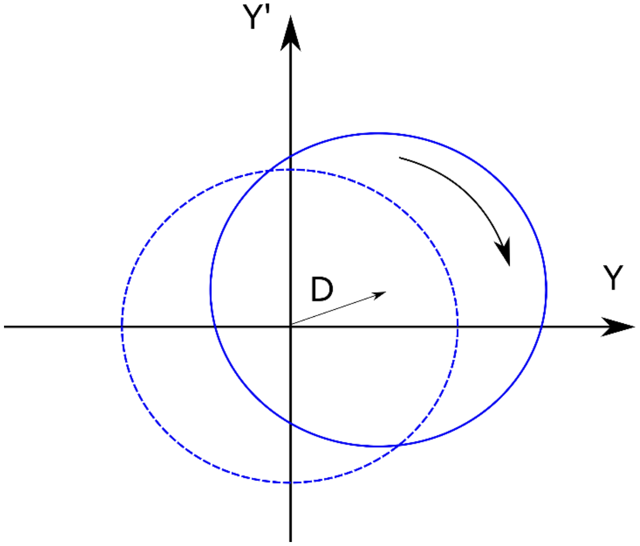
$$D^2 = (\Delta Y)^2 + (\Delta Y'^2) = (\Delta y)^2 \frac{1+\alpha^2}{\beta} + (\Delta y')^2 \beta$$

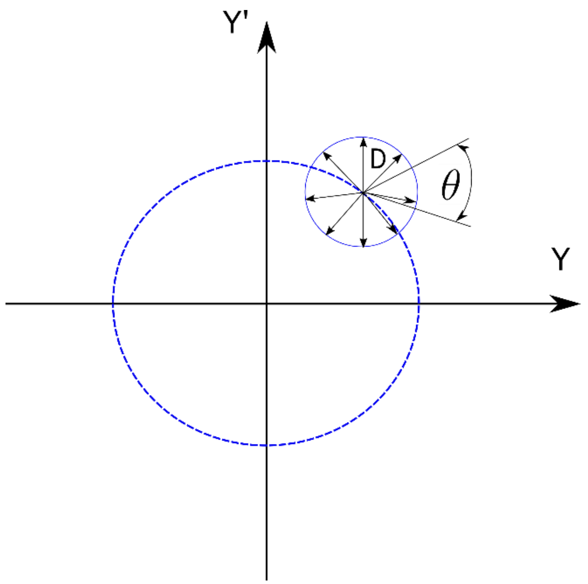
$$\epsilon_2 = \epsilon_1 + \frac{1}{2}\left((\Delta y)^2 \frac{1+\alpha^2}{\beta} + (\Delta y')^2 \beta\right)$$

Magnet misalignment

Dipole field error

$$\Delta y' = \frac{\Delta B l}{B\rho}$$

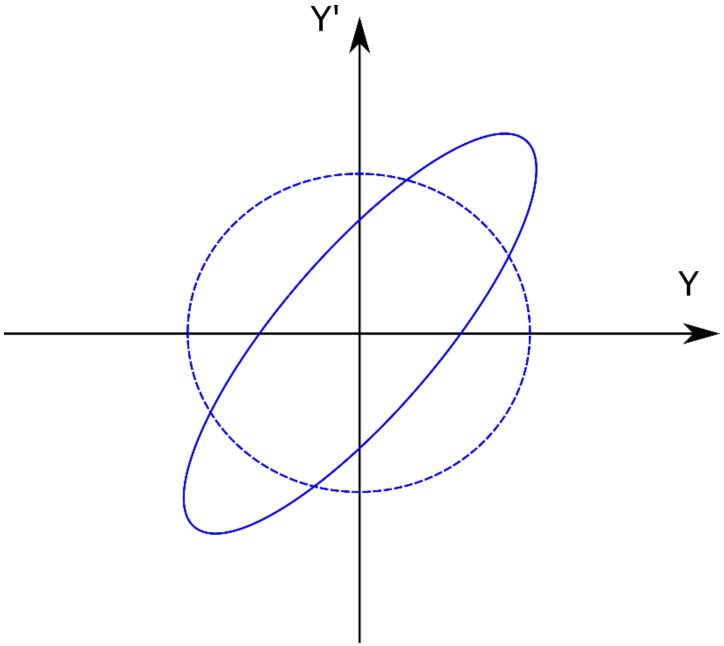


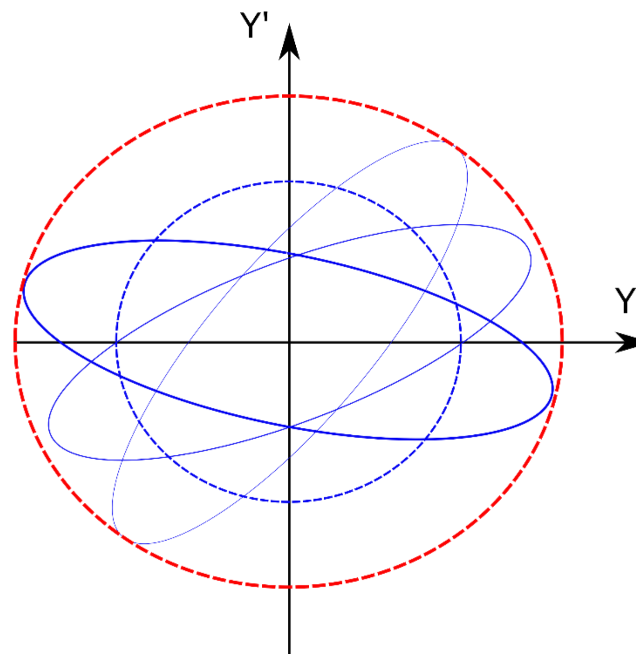


Gradient errors

$$k = -\frac{\Delta G l}{B\rho}$$

$$\epsilon_2 = \frac{1}{2}\left(k^2\beta^2 + 2\right)\epsilon_1$$





[5] P. Bryant. Beam Transfer Lines. (CAS), 1992.

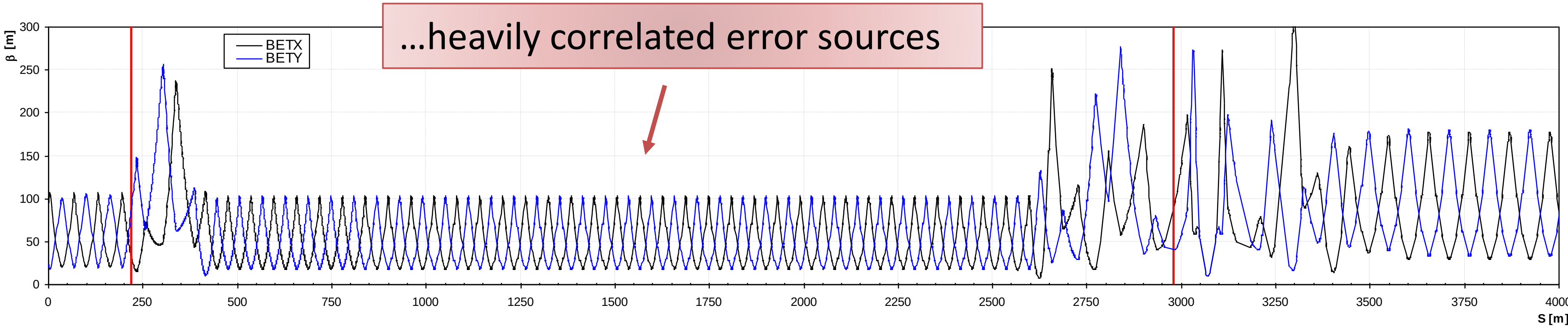
Combining errors

- Averaging over a distribution of uncorrelated errors

$$\langle y^2 \rangle = \frac{\beta(s)}{2} \sum_n \beta_n \langle \delta_n^2 \rangle \qquad \delta_n = \frac{\Delta B l}{B_0 \rho_0} \qquad and \qquad \delta_n = -lk\Delta y$$

$$\langle y^2 \rangle = \frac{\beta_{aver} n \delta_{rms}}{2}$$

- But here we have to be very careful…



Combining errors example

Not correlated at all but dominated by a single error source

Typical specifications from correction studies

- Number of monitors and required resolution
  - Every ¼ betatron wavelength
  - Grid resolution: ~3 wires/sigma
- Number of correctors and strength
  - Every ½ betatron wavelength H - same for V
  - Displace beam by few betatron sigma per cell
- Dipole and quadrupole field errors
  - Integral main field known to better than 1-10e-4
  - Higher order field errors < 1-10e-4 of the main field
- Dynamic errors from power converter stability
  - 1-10e-5
- Alignment tolerances
  - 0.1-0.5 mm
  - 0.1-0.5 mrad

Take values with caution!

They can strongly vary depending on energy, intensity, machine purpose, etc.

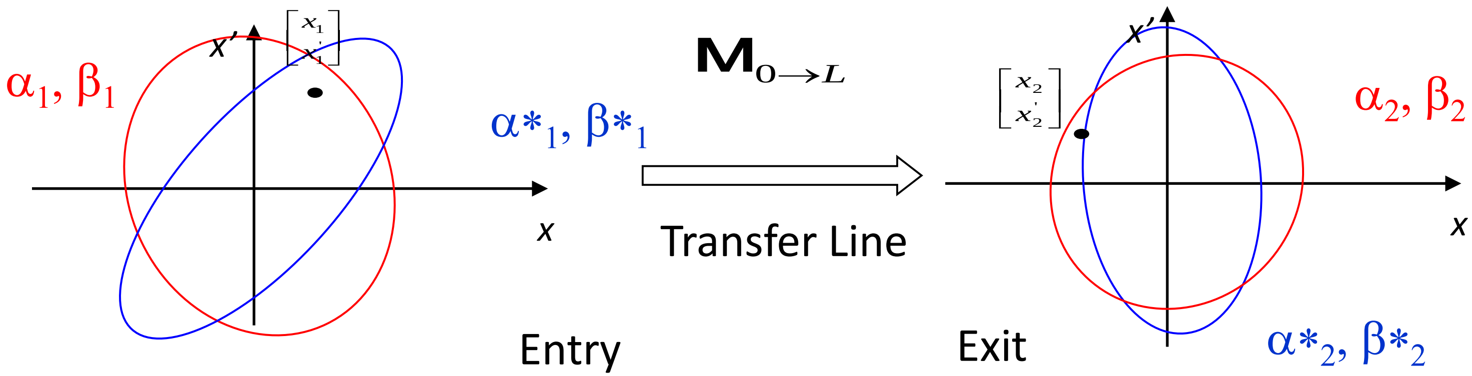
Summary

Before switching on a computer we can define for a transfer line:

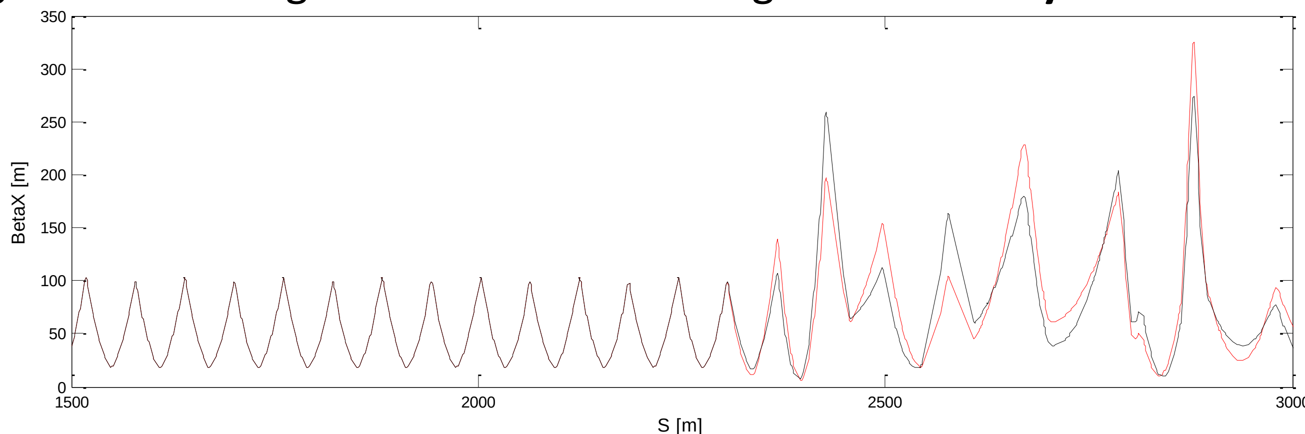
- Number of dipoles and quadrupoles, correctors and monitors
- Dipole field and quadrupole pole tip field
- Aperture of magnets and beam instrumentation
- Rough estimate of required field quality and alignment accuracy

Wrap up

- Optics in a ring is defined by ring elements and **periodicity** – optics in a transfer line is dependent line elements and **initial conditions**



- **Changes** of the strength of a transfer line magnet **affect only downstream optics**



Wrap up

- Geometry calculations require a set of coordinates in a common reference frame

- **Bending fields** are defined by geometry and the **magnetic or electric rigidity**:

$$B\rho\ [Tm] = 3.3356\ \frac{A}{n}\ p\ [GeV/c] \qquad\qquad E\rho\ [kV] = \frac{\gamma+1}{\gamma}\ \frac{A}{n}\ T\ [keV]$$

$$\theta = \frac{Bdl}{B\rho} \text{ or } \frac{Edl}{E\rho}$$

- The choice between magnetic and electric depends mainly on the beam energy

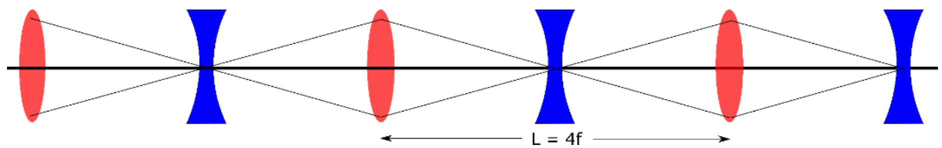
- If you are in the grey zone, consider: field design and measurement, power consumption, vacuum, interlocking

- For the estimates of bending radii in lines remember to take into account the filling factor (~70%) and Lorentz-Stripping in case of H^- ions

Wrap up

- **Quadrupole gradients and apertures** can be estimated in case of simple focussing structure like FODO cells

$\frac{L}{f} = 4 \sin \frac{\mu}{2}$ ⇦ Stability $f > \frac{L}{4}$ ⇨



$\beta = \left(L + \frac{L^2}{4f}\right) / \sin \mu$ ⇨ Defines beam size and quadrupole pole tip field

- **Aperture specifications** require safety factors for the optics and constant contributions for trajectory variations and alignment errors
  - $A_{x,y} = \pm n_{sig} \cdot \sqrt{k_\beta \cdot \beta_{x,y} \cdot \frac{\epsilon_{x,y}}{\beta\gamma}} \pm D_{x,y} \cdot k_\beta \cdot \frac{\Delta p}{p} \pm CO \cdot \sqrt{\frac{\beta_{x,y}}{\beta_{xmax,ymax}}}$ ± alignment

Wrap up

- **Estimating tolerances** from emittance growth:

- Dipole field and alignment:

$$\epsilon_2 = \epsilon_1 + \frac{1}{2}\left((\Delta y)^2 \frac{1+\alpha^2}{\beta} + (\Delta y')^2 \beta\right)$$

Magnet misalignment

Dipole field error

$$\Delta y' = \frac{\Delta B l}{B\rho}$$

- Gradient errors:

$$\epsilon_2 = \frac{1}{2}\left(k^2\beta^2 + 2\right)\epsilon_1 \qquad k = -\frac{\Delta G l}{B\rho}$$

Thank you for your attention

And many thanks to my colleagues for helpful input:

R. Baartman, D. Barna, M. Barnes, C. Bracco, P. Bryant, F. Burkart, V. Forte, M. Fraser, B. Goddard, C. Hessler, D. Johnson, V. Kain, T. Kramer, A. Lechner, J. Mertens, R. Ostojic, J. Schmidt, L. Stoel, C. Wiesner

[1] B. Goddard. Transfer lines. *CAS*, 2004.

[2] N. Ibarrola and M. Jones. Analysis and Evaluation of the CERN Reference Systems. *International Workshop of Accelerator Alignment*, 2016.

[3] M Fraser et al. Beam Dynamics Studies of the ELENA Electrostatic Transfer Lines. (CERN-ACC-2015-340):MOPJE044. 4 p, 2015.

[4] D. Johnson. Challenges Associated with 8 GeV H- Transport and Injection for FERMILAB PROJECT-X *. (Proceedings of Hadron Beam 2008), 2008.

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[9] Werner Herr. Mathematical and Numerical Methods for Non Linear Beam Dynamics in Rings. CAS 2013.