Some Basic Superconductivity for Accelerator Builders – Lecture 1 – Reversible Properties

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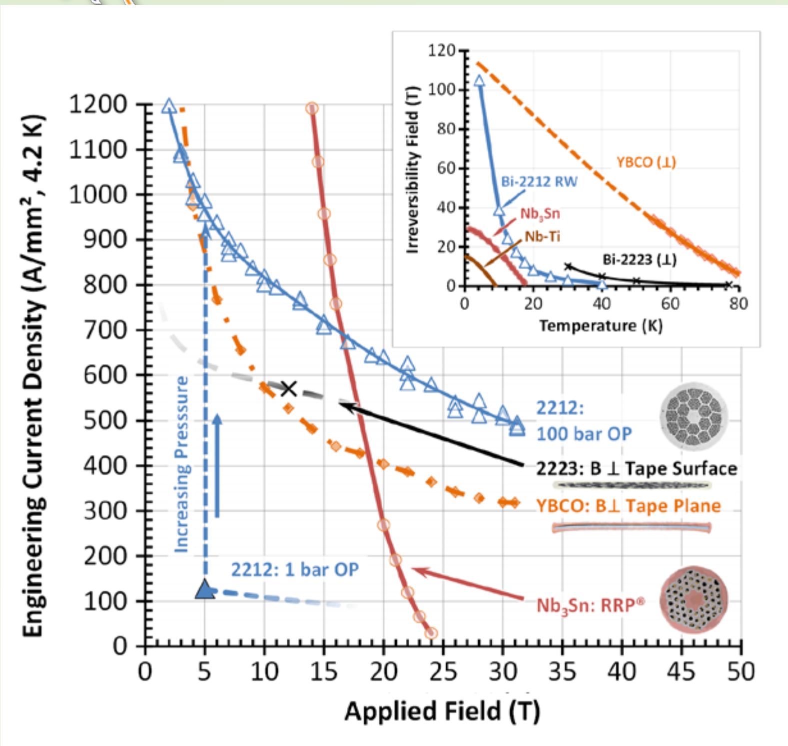
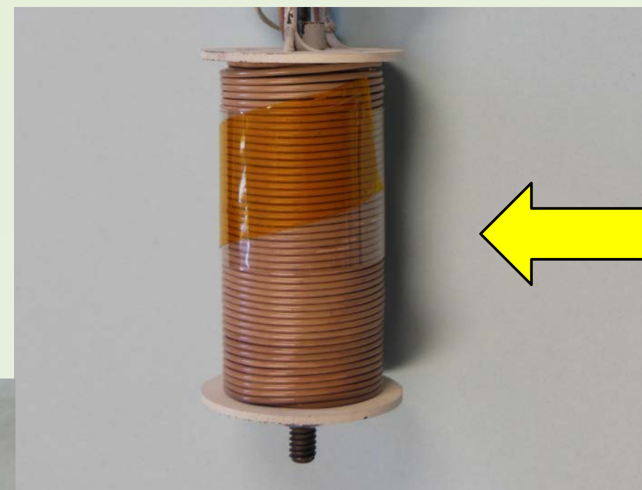
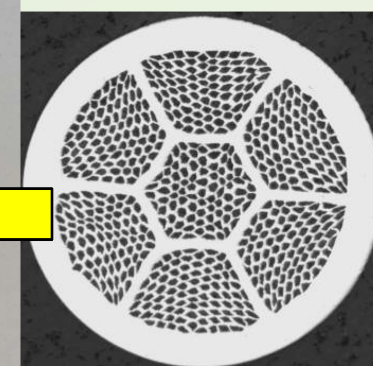
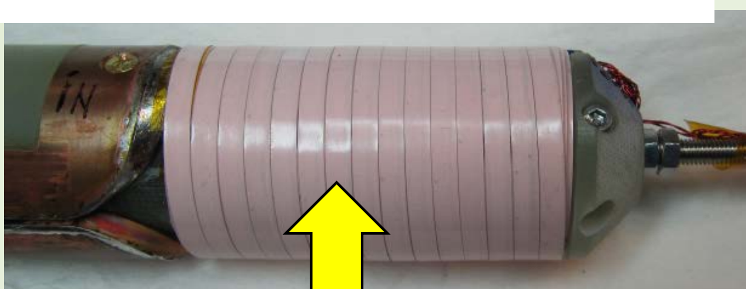


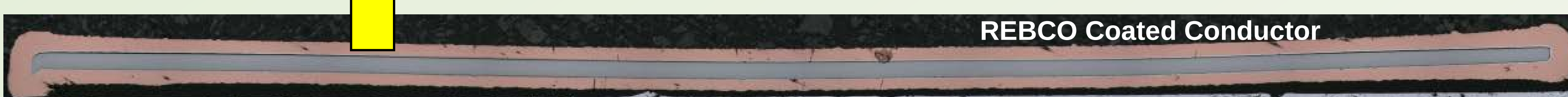




34T (in 31T) – Bi-2212



35T (in 31T) – REBCO coated conductor



My point of view

- The “killer app” for superconductors is magnets –
  - Onnes described this clearly in 1913 (in Chicago)
  - Only by accident did the path to magnet conductors emerge.. (Kunzler *et al.* Bell Labs 1960)
- Magnet builders want:
  - Conductors of varying I_c
  - Small lab magnets can operate at 100 A, big ones like ITER or LHC may need 20-60 kA – requires cables of many strands
  - Conductors with many small filaments to minimize charging losses, field errors and to avoid single-defect flaws
  - Conductors with good normal metal around each filament
- Magnet builders need high conductor current density (J_e)
  - Demands strong vortex pinning for high J_c, high H_{irr} and high H_{c2}
  - High J_e demands either exceptional J_c or sc fill factors of 20-40vol.%
- High strength, km lengths, affordability ($/kA.m), ….

Transparent grain boundaries are critical for all above requirements…..

What is it essential to know?

- Superconductors are not just perfect conductors
  - Persistent currents leading to error fields
- High current density is not thermodynamically stable
  - Flux jumps, flux creep
- High field (and high temperature) superconductors have short coherence lengths
  - Strong sensitivity to defects leading to both strong pinning and high J_c and current blocking at GBs and low J_c
- Few materials have been made useful for magnets
  - Thousands of superconductors, so far only 5-7 useful conductors
- Superconductivity can exist up (so far) up to well over 100 T and over 100 K
  - Material presently used for accelerator magnets has limits of about 15 T and 9 K
- Extended Ginzburg-Landau (GLAG) is much more useful for applied scientists than BCS
  - BCS is a theory of Tc, while GLAG handles vortices and the mixed state

The first 50 years

- Zero resistivity: Onnes - 1911
  - The vision of a 10 T magnet in 1913
- Perfect diamagnetism: Meissner and Ochsenfeld 1933
  - An explanation by the Londons 1935
- The type I-II transition in Pb alloys: Shubnikov – 1936
  - Mendelsohn (wrongly) explains it away as a sponge - 1937
- A phenomenological theory: Ginzburg and Landau – 1950
  - Vortices at high κ: Abrikosov – 1953/1957
- The mechanism of superconductivity – Bardeen Cooper and Schrieffer 1957
- Experiment finally shows high field superconductivity is possible –Nb_3Sn superconducts well at 88 kgauss – Kunzler *et al.* 1960 – finally!

It is truly extraordinary that theory, experiment and Onnes' original dream never effectively connected until Kunzler's exploratory experiment showed that high J_c was possible in high fields (10^5 A/cm^2 at 88kgauss)

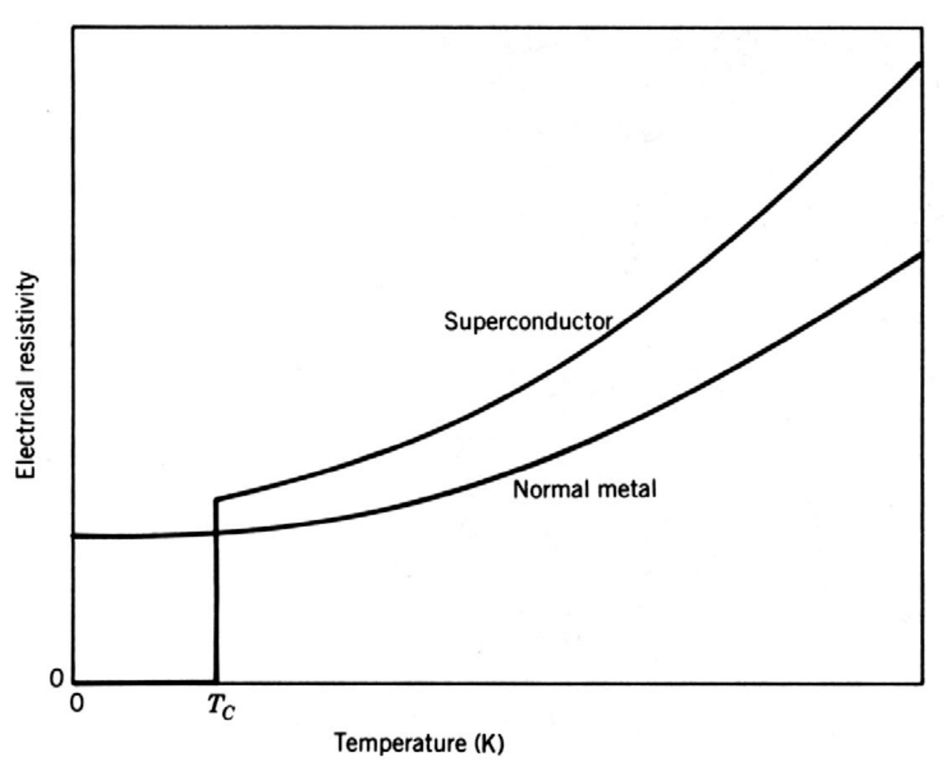
Zero Resistivity – Onnes 1911

- **Non-Superconducting Metals**
  - $\rho = \rho_o + aT$ for $T > 0$ K*
  - $\rho = \rho_o$ Near $T = 0$ K

  *Recall that $\rho(T)$ deviates from linearity near $T = 0$ K

- **Superconducting Metals**
  - $\rho = \rho_o + aT$ for $T > T_c$
  - $\rho = 0$ for $T < T_c$

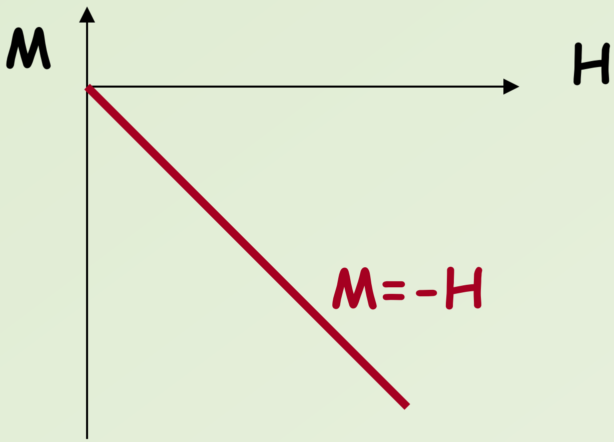
- **Superconductors are more resistive in the normal state than good conductors such as Cu**



Onnes's dream of a 10 T magnet (1913) was soon dashed by his discovery that <0.1 T destroyed the superconducting state in his Pb and Hg wires

1933 - A 2nd property of the superconducting state: Perfect Diamagnetism

- $\chi_m = -1$

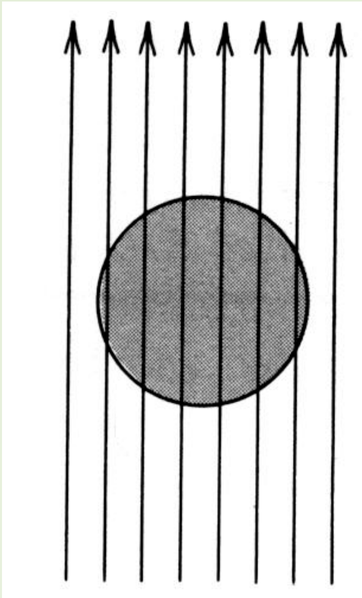


- The internal flux density:

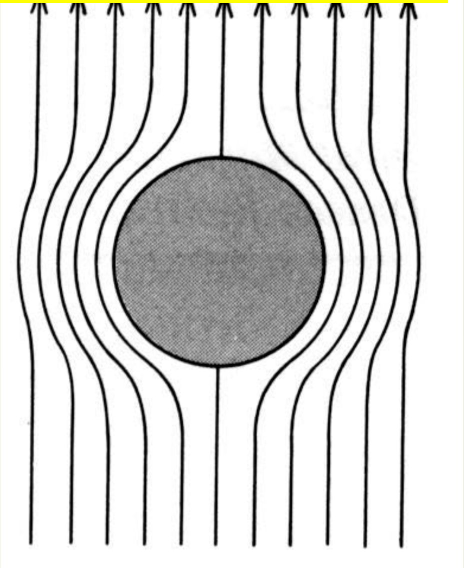
$$B = \mu_o(H + M)$$
$$B = \mu_o(H + \chi_m H)$$
$$B = 0$$

Key point: B = 0 shows that the S ↔ N transformation is reversible allowing thermodynamics to be applied

Meissner and Ochsenfeld 1933



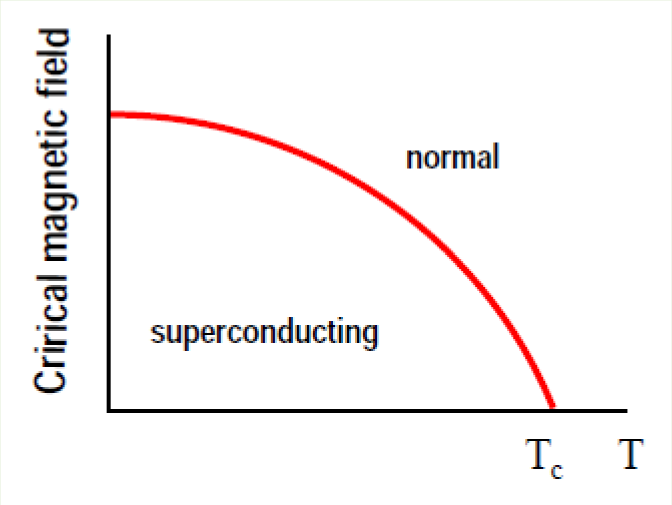
Normal Metal



Superconductor

Flux is excluded from the bulk by surface supercurrents which maintain B = 0 internally

Thermodynamic treatment



Empirical behavior of pure metals (type I) seen first by Onnes

- **Meissner effect then allows:**

  $G_s(H) = G_s(0) + ½.\mu_0 H^2$

  While for the normal phase:

  $G_n(H) = G_n(0) = G_n$ since normal state magnetization is effectively zero

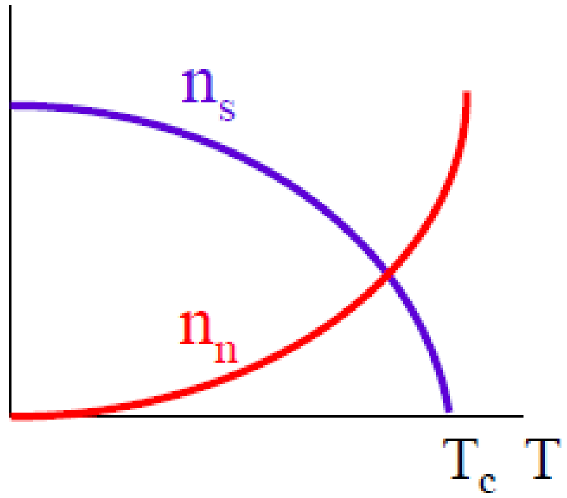
  And of course $G_s(H_c) = G_n(H_c)$

  Meaning that $G_n - G_s = ½.\mu_0 H_c^2$

- **Usual thermodynamic manipulation brings out many quantities including**
  - Latent heat L is found at $L = - \mu_0 H_c\, {}_dH_c/dT$ which is zero at $H = 0$ and $H = H_c$ – 2nd order transition with a specific hear discontinuity

London equations (1935)

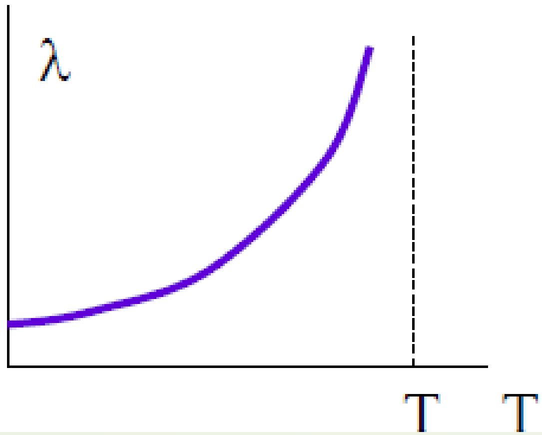
- Two-fluid model: two coexisting SC and N "liquids" with densities $n_s(T) + n_n(T) = n$.
- Electric field E accelerates only the SC component, the N component is short circuited.
- Second Newton law for the SC component: $mn_s dv_s/dt = en_s E$. Substituting $J_s = en_s v_s$ yields the first London equation:



$$dJ_s/dt = (e^2 n_s/m)E \qquad (1)$$

- Substituting the Maxwell equations, $\nabla\times E = -\mu_0 \partial H/\partial t$ and $\nabla\times H = J_s$ into (1), and assuming that weak J_s does not affect n_s, we obtain the second London equation:

$$\lambda^2 \nabla H - H = 0 \qquad (2)$$

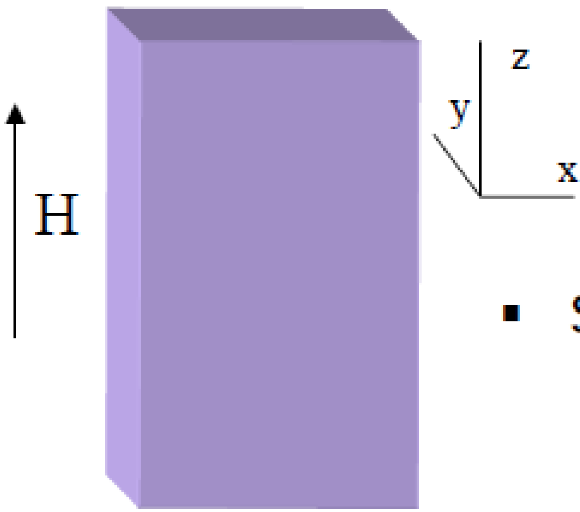


- London penetration depth:

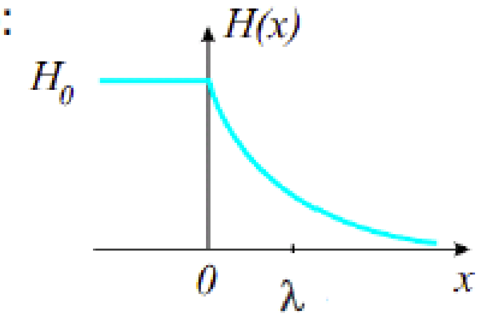
$$\boxed{\lambda = \left(\frac{m}{e^2 n_s(T) \mu_0} \right)^{1/2}}$$

White background slides like this from UW Applied Superconductivity Course – portion taught by Alex Gurevich

London equation explains the Meissner effect



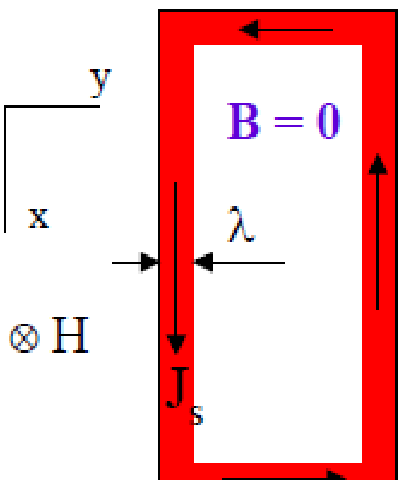
- magnetic field penetration into a slab:



$$\lambda^2 \frac{\partial^2 H_z}{\partial y^2} - H_z = 0$$

- Screening surface current density $J_s(y)$:

$$\boxed{H(y) = H_0 e^{-y/\lambda}, \qquad J_s(y) = \frac{H_0}{\lambda} e^{-y/\lambda}}$$



- Supercurrents completely screen the external field H_0
- Meissner effect: no magnetic induction B in the bulk.
- Surface current density cannot exceed the depairing current density J_d :

$$\boxed{J_d = \frac{H_c(T)}{\lambda(T)} \cong J_0 \left(1 - \frac{T^2}{T_c^2}\right)^{3/2}}$$

Problems with the London electrodynamics

- the linear London equations

$$\frac{\partial \vec{J}_s}{\partial t} = -\frac{\vec{E}}{\lambda^2 \mu_0}, \qquad \lambda^2 \vec{H} - \vec{H} = 0$$

along with the Maxwell equations, $\nabla \times H = J_s$ and $\nabla \times E = -\mu_0 \partial H/\partial t$ describe the electrdynamics of SC at all T provided that:

- current density J_s is much smaller than the depairing current density J_d
- the superfluid density n_s is spatially uniform

- Lots of important phenomena in SC occur because n_s is nonuniform
- Generalization of the London equations to account for nonlinear problems
- Phenomenological Ginzburg-Landau (GL) theory (1950, Nobel prize 2003) was developed before the microscopic BCS theory (1957).
- GL theory is one of the most widely used theories of superconductivity

Basic ideas behind G-L equations

- The condensation energy is $1/2\mu_0 Hc^2$
- All sc electrons have the same wave function Ψ where $n_s = |\psi|^2$
- In presence of H, n_s can vary and the free energy is written as a Taylor expansion
- The solutions come from minimizing Ψ and the vector potential A over all space

G-L is a very general treatment that is used widely today even though only presented at the time as a treatment of superconductivity in pure metals where there was a positive surface energy

Detailed treatments of G-L: Goodman Rep Progr Phys. **29**, 445 (1966)
Blatter and Geshkenbein Vortex Matter in Physics of Superconductors 2003, ed. by KH Bennemann and JB Ketterson (Springer, Berlin 2003)

Notes here taken from Alex Gurevich's treatment in team-taught graduate class in Applied Superconductivity at U Wisconsin and Florida State U

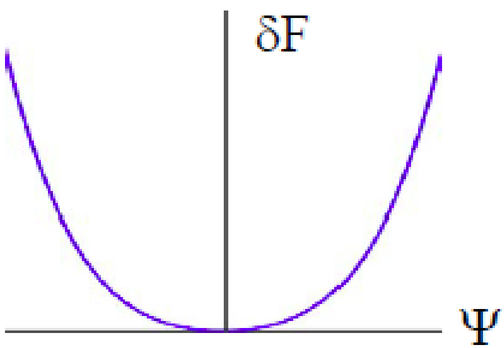
GL free energy

- Complex superconducting order parameter $\Psi = (n_s/2)^{1/2}\exp(i\theta)$ (envelope wave function of the Cooper pair).
- Let $T \approx T_c$ so Ψ is small, and the free energy F can be expanded in Taylor series in Ψ:

$$\boxed{F = F_n + \int dV \left[\alpha(T)\,|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{\hbar^2}{2m^*}\left|\left(\nabla + \frac{2\pi i\vec{A}}{\phi_0}\right)\Psi\right|^2 + \frac{\mu_0 H^2}{2} \right]}$$

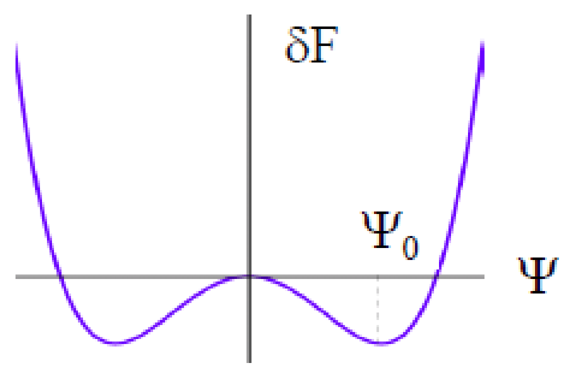
nonlinear **inhomogeneity** **magnetic**

- The coefficient $\alpha(T) = \alpha_0(T - T_c)/T_c$ changes sign at T_c



Normal state

$T > T_c$, $\Psi = 0$



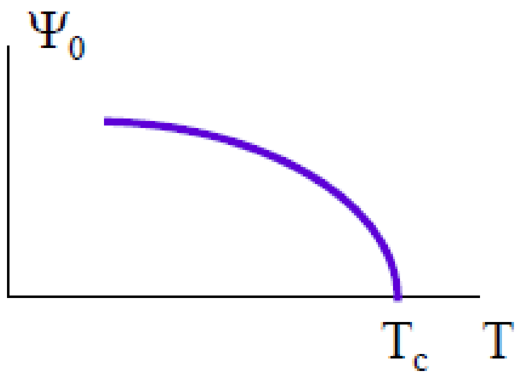
Superconducting state

$T < T_c$, $\Psi_0 = (|\alpha|/\beta)^{1/2}$

Equilibrium order parameter and H_c

- Minimization of F gives the spontaneous uniform order parameter $\Psi_0 = [n_s/2]^{1/2}$ below T_c:

$$\boxed{\Psi_0 = \sqrt{\frac{\alpha_0 (T_c - T)}{\beta T_c}}}$$

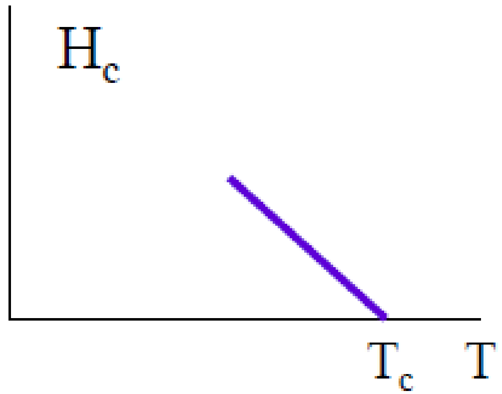


- Energy gain defines the thermodynamic critical field H_c:

$$F_n - F_s = V\frac{\alpha^2(T)}{2\beta} = V\frac{\mu_0 H_c^2(T)}{2}$$

- Linear temperature dependence of $H_c(T)$ near T_c:

$$\boxed{H_c(T) = \frac{\alpha_0}{\sqrt{\beta\mu_0}}\frac{(T_c - T)}{T_c}}$$



in accordance with the empirical relation $H_c(T) = H_0\,[1 - (T/T_c)^2]$

GL equations for nonuniform $\Psi(r)$ and $A(r)$

- Energy minimization conditions $\delta F/\delta\Psi^* = 0$ and $\delta F/\delta A = 0$ yield the GL equations for the dimensionless order parameter $\psi = \Psi/\Psi_0$

$$\boxed{\begin{aligned} &\xi^2\left(\nabla + \frac{2\pi i}{\phi_0}\vec{A}\right)^2\psi + \psi - \psi\,|\psi|^2 = 0, \\ &\nabla \times \nabla \times \vec{A} = \vec{J}_s = -\frac{|\psi|^2}{\lambda^2\mu_0}\left(\frac{\phi_0}{2\pi}\nabla\theta + \vec{A}\right) \end{aligned}}$$

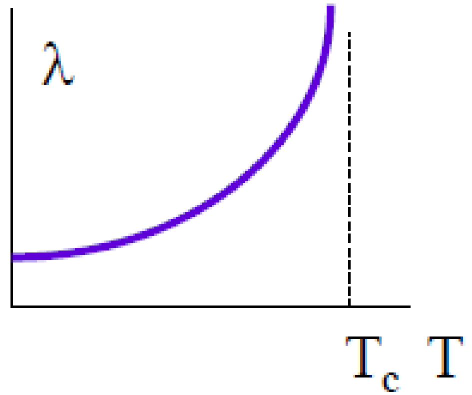
- Two coupled complex nonlinear PDE for the pair wave function $\psi(r)$ and the magnetic vector-potential $A(r)$.
- Two fundamental lengths ξ and λ
- Boundary condition between a superconductor and vacuum $J_s = 0$:

$$\boxed{\left(\nabla + \frac{2\pi i}{\phi_0}\vec{A}\right)\psi\vec{n} = 0}$$

Two fundamental lengths λ and ξ and the GL parameter κ = λ/ξ

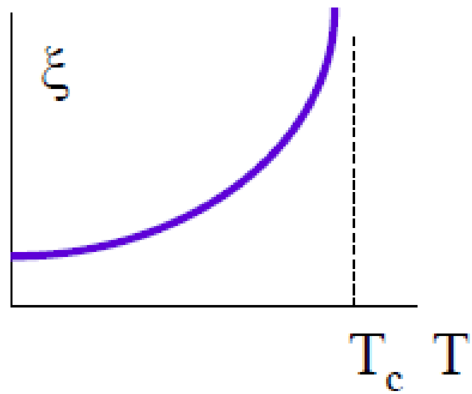
- Magnetic London penetration depth:

$$\boxed{\lambda(T) = \left(\frac{m\beta}{2e^2\mu_0\alpha_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c - T}}}$$



- Coherence length – a new scale of spatial variation of the superfluid density $n_s(r)$ or superconducting gap $\Delta(r)$:

$$\boxed{\xi(T) = \left(\frac{\hbar^2}{4m\alpha_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c - T}}}$$



- The GL parameter κ = λ/ξ is independent of T.
- Critical field $H_c(T)$ in terms of λ and ξ:

$$\boxed{H_c(T) = \frac{\phi_0}{2\sqrt{2}\pi\xi(T)\lambda(T)}}$$

Depairing current density

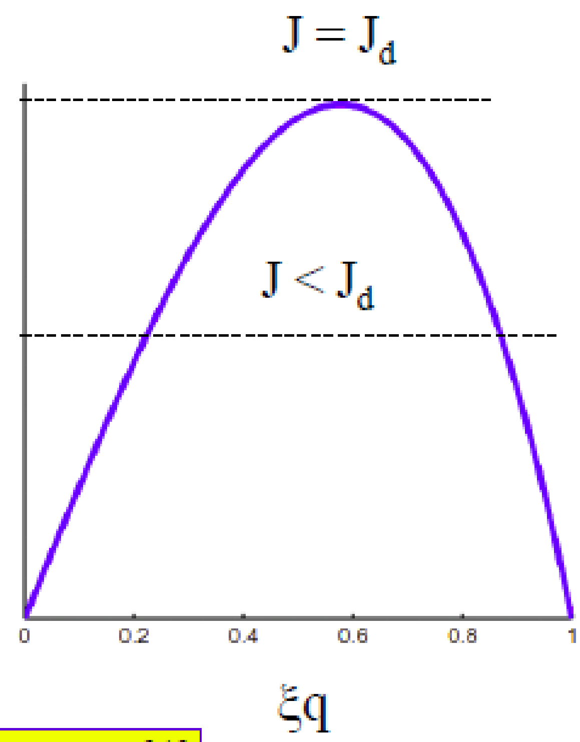
- What maximum current density J can a superconductor carry?
- Consider a current-carrying state with $\psi = \psi_0 \exp(-iqx)$, in a thin filament, where q is proportional to the velocity of the Cooper pairs. The GL equations give:

$$\psi_0^2 = 1 - \xi^2 q^2, \qquad J = \frac{\psi_0^2 \phi_0 q}{2\pi\lambda^2\mu_0}$$

- Current density as a function of q:

$$J = \frac{\phi_0 q}{2\pi\lambda^2\mu_0}(1 - \xi^2 q^2)$$

Suppression of ψ by current



- Maximum J at $\xi q = 1/\sqrt{3}$ yields the depairing current density:

$$J_d = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\lambda^2\xi} \cong 0.54\frac{H_c}{\lambda} \propto \left(1 - \frac{T}{T_c}\right)^{3/2}$$

My years with Landau

The discoverer of "type-II" superconductivity lets us in on the excitement of an important time for low-temperature physics

Physics Today, p 56 January 1973

A. A. Abrikosov

In 1950 Vitali L. Ginzburg and Landau wrote their well known paper[1] on superconductivity. Without the microscopic theory, developed later by John Bardeen, Leon Cooper and J. Robert Schrieffer,[2] the meaning of several quantities entering the Ginzburg–Landau work remained unclear, above all the meaning of the "superconducting electron wave function" itself. Nevertheless this theory was the first to explain such phenomena as the surface energy of electrons at the superconducting–normal phase boundary and the dependence of the critical field and current in thin films on temperature and thickness.

1953 work.........

After that I tried to investigate the magnetic behavior of bulk type-II superconductors. The solution of the Ginzburg–Landau equation in the form of an intinitesimal superconducting layer in a normal sea of electrons was already contained in their paper. Starting from this solution I found that below the limiting critical field, which is the stability limit of every superconducting nucleation, a new and very peculiar phase arose, with a periodic distribution of the ψ function, magnetic field and current. I called it the "mixed state."

- A complex story
  - Landau utterly disapproved of vortices until they were discovered in rotating He
  - Abrikosov only published in 1957, the same year as BCS
  - All the publicity went to BCS and only after extensions of BCS by the Landau Group (Ginzburg-Landau-Abrikosov-Gorkov – GLAG) did the value of G-L become fully clear

Upper critical field H_{c2}

- Let us calculate the maximum uniform magnetic field H_{c2} above which the GL equation has no superconducting solutions.
- For a uniform field H along the z-axis, the GL equation for small ψ takes the form

$$\boxed{\xi^2 \nabla^2 \psi + [1 - (2\pi B x \xi / \phi_0)^2]\psi = 0}$$

- Similar to the Schrodinger equation for a harmonic oscillator:

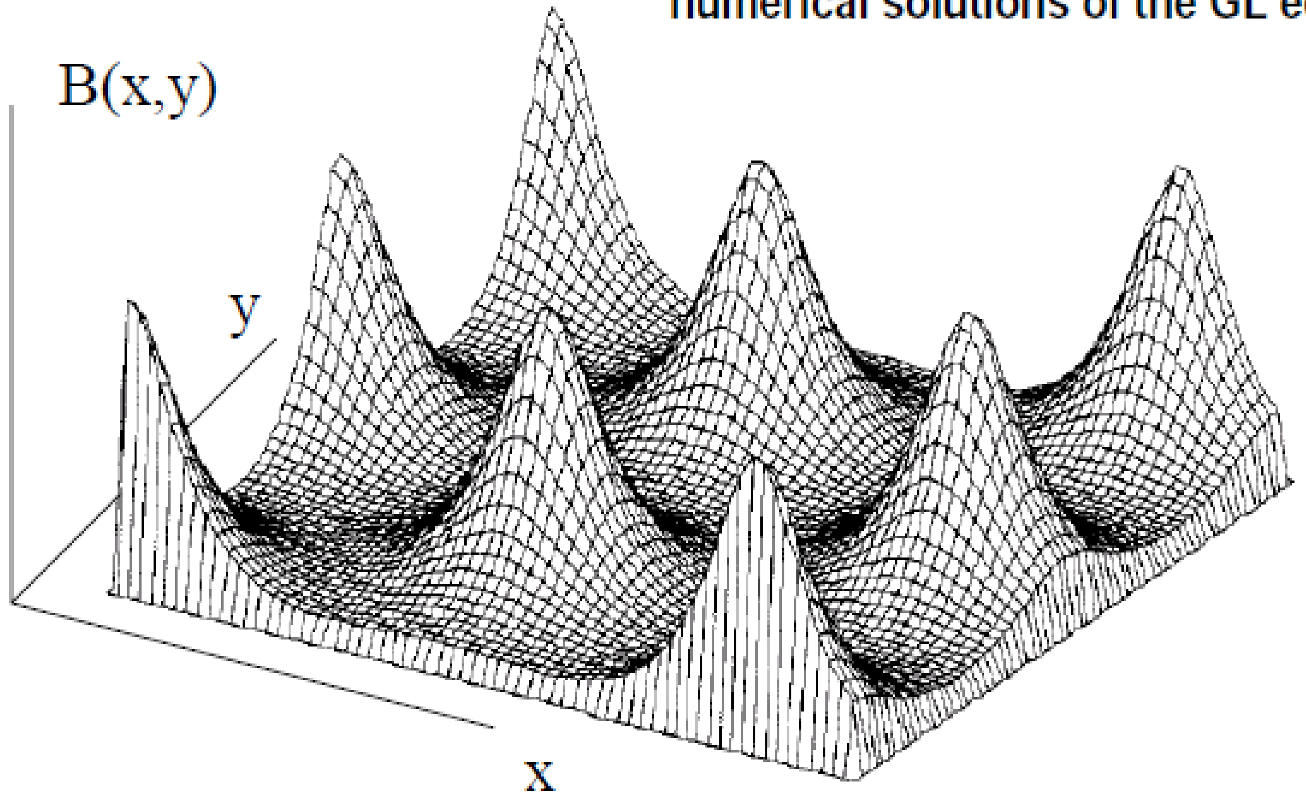
$$\frac{\hbar^2}{2M}\nabla^2\psi + \left(E - \frac{M\omega^2 x^2}{2}\right)\psi = 0: \qquad \frac{\hbar^2}{2M} \to \xi^2, \qquad E \to 1, \qquad \sqrt{M}\omega \to \frac{2^{3/2}\pi H \xi}{\phi_0}$$

- The oscillator energy spectrum $E = \hbar\omega(n + ½)$ for $n = 0$, then gives H_{c2} below which bulk superconductivity exists (surface SC can exist at even higher $H_{c3} = 1.69H_{c2}$)

$$B_{c2}(T) = \frac{\phi_0}{2\pi\xi^2(T)} = \frac{\phi_0}{2\pi\xi_0^2}\left(1 - \frac{T}{T_c}\right)$$

Vortex lattice at $H_{c1} < H < H_{c2}$ (Abrikosov 1956, Nobel prize, 2003)

numerical solutions of the GL equations



- Hexagonal lattice of vortices, each carrying the flux quantum ϕ_0
- Vortex density n defines the magnetic induction $n\phi_0 = B$
- Spacing between vortices: $a = (\phi_0/B)^{1/2}$

How can H_{c2} be higher than H_c?

- Compare B_c and B_{c2} obtained from the GL theory:

$$B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}, \qquad B_{c2} = \frac{\phi_0}{2\pi\xi^2}$$

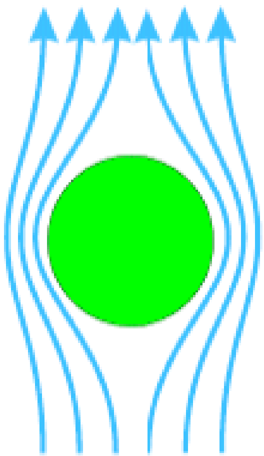
- Type-I superconductors: $B_c > B_{c2}$, or $\kappa = \lambda/\xi < 1/\sqrt{2}$: mostly simple metals
- Type-II superconductors: $B_c < B_{c2}$, or $\kappa = \lambda/\xi > 1/\sqrt{2}$: 100 (HTS), 40(MgB_2)

- There are many type-II superconductors with the GL parameter $\kappa = \lambda/\xi >> 1$, which can be further increased by alloying with nonmagnetic impurities.

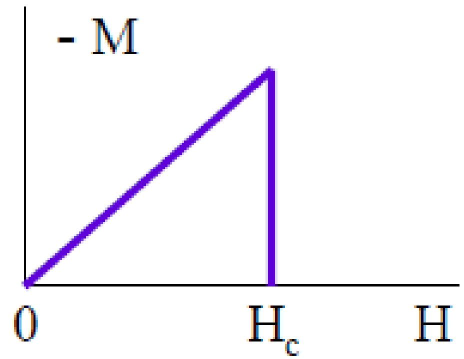
Dirty SC with the electron mean-free path $\ell < \xi_0$: the penetration depth $\lambda \cong \lambda_0(\xi_0/\ell)^{1/2}(1 - T/T_c)^{-1/2}$ increases as ℓ decreases, but the coherence length $\xi = (\xi_0\ell)^{1/2}(1 - T/T_c)^{-1/2}$ decreases as ℓ decreases. Thus, H_c does not change, but H_{c2} increases proportionally to the residual resistivity ρ

$$B_{c2} \cong \frac{\phi_0}{2\pi\xi_0 l}\left(1 - \frac{T}{T_c}\right) \propto \rho$$

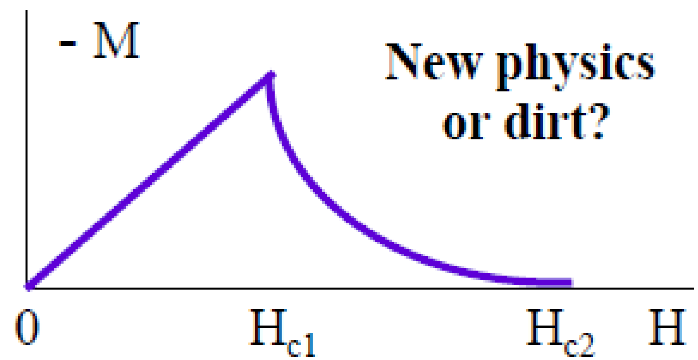
Type-I and type-II superconductors



- Measurements of magnetization M(H) have shown partial Meissner effect in many superconducting compounds and alloys Shubnikov, 1935.



Complete Meissner effect in type-I superconductors

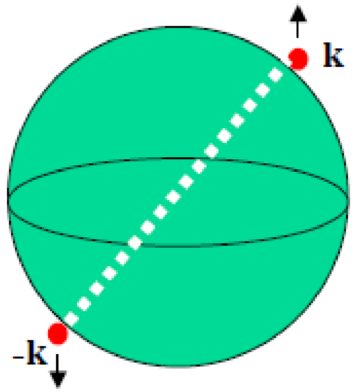


High-field partial Meissner effect in type-II superconductors

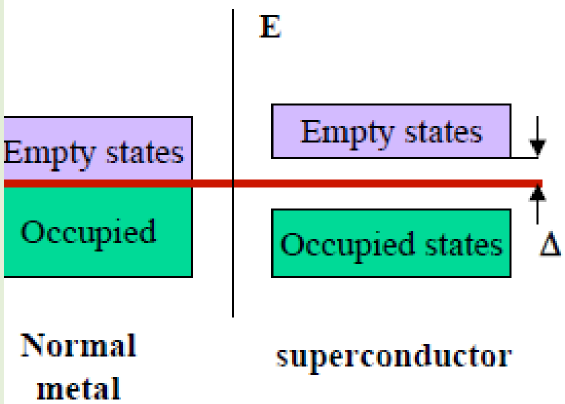
- Type-I: Meissner state $B = H + M = 0$ for $H < H_c$; normal state at $H > H_c$
- Type-II: Meissner state $B = H + M = 0$ for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$
- Lower and upper critical fields H_{c1} and H_{c2}.
- High field superconductivity with $H_{c2} \sim 100$ Tesla

Cooper pairs and BCS theory of superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957). Nobel prize in 1972



Cooper pair on the Fermi surface



- Attraction between electrons with antiparallel momenta k and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs into a coherent superconducting state.
- Superconducting gap Δ on the Fermi surface
- Critical temperature: $k_B T_c \approx 1.13\hbar\omega_D \exp(-1/\gamma)$, $\gamma \approx 0.1\text{-}1$ is a dimensionless coupling constant

$2\hbar\Delta = 3.52 k_B T_c$, $T_c << T_D \sim 300K$

For applications people the most important consequence of BCS may be that it challenged the Landau group to reconcile BCS with GL – GLAG theory – which was extensively studied and proved in the 1960s

Extensive tests of GLAG

- **See for example:**
  - Fietz and Webb, "Magnetic properties of Type II alloys near H_{c2}", Phys Rev 161 (1967)
  - Hake, "Paramagnetic superconductivity in extreme type II superconductors", Phys Rev 158, 356 (1967)
  - Orlando et al., "Critical fields and Pauli paramagnetic limiting in Nb_3Sn and V_3Si", Phys Rev B20, 4545 (1979)

Moving to extreme type II (κ 50-100, H_{c2} 5-30 T) brings in the need to account for the energy of the normal state in G_n and the scattering introduced by alloying or disorder that greatly reduces ξ. The extensions of GLAG by Maki, Werthamer, Helfand etc. have been so helpful – the classic expression:

$$H_{c2}(0) = 0.69\ T_c\ dH_{c2}/dT|_{Tc}$$

is always the first to be used to estimate H_{c2} in any new material

PHYSICAL REVIEW VOLUME 161, NUMBER 2 10 SEPTEMBER 1967

Magnetic Properties of Some Type-II Alloy Superconductors near the Upper Critical Field*

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(Received 23 March 1967)

Parameters pertinent to the magnetic properties of type-II superconductors near the upper critical field H_{c2} [namely, the generalized Ginzburg-Landau parameters κ_1 and κ_2, and the functions $h^* = H_{c2}/(-dH_{c2}/dt)_{t=1}$] have been obtained from magnetization measurements on a series of niobium-titanium alloys. The range of electron-transport mean free paths, from $0.1\xi_0$ to about $15\xi_0$ (where ξ_0 is the coherence length in pure Nb), effectively spans the range from the clean to the dirty limit, with annealed and cold-worked specimens at temperatures between $0.13T_c$ and T_c. It was found that both κ_1 and κ_2 increased with decreasing temperature in all alloys and that the magnitude of the increase was 20–50% higher than expected from existing theory. The experimental value of the parameter $H_{c2}/(dH_{c2}/dt)$ at $T=T_c$ various with impurity roughly as expected in Ginzburg-Landau theory. Defects generated by cold work enhanced the increase of κ_1 at low temperatures.

TABLE IV. Pertinent parameters of the alloys studied. See text. Estimated errors of experiments are given in top row and previously published data on pure niobium are given in the bottom two rows of the table (Refs. 21 and 23). Parentheses indicate that the estimated error is double the column estimate.

1 Specimen	2 T_c meas. (K°)	3 ρ_n (ohm cm)	4 γ (erg cm^{-3} deg^{-2})	5 κ_0 calc. Eq. 6	6 ρ calc. Eq. 9	7 κ calc. Eq. 5	8 κ meas.	9 ρ calc. Eq. 10	10 ρ calc. Eq. 7	11 $H_c(0)$ calc. (Oe)	12 $H_c(0)$ meas. (Oe)	13 $(dH_{c2}/dt)_{t=1}$ meas. (kOe)
Estim. Error	±0.1	±10%	±8%	···	···	···	±8%	···	···	···	−0 +6%	±10%
Annealed												
Nb	9.2	7.3×10^{-8}	7.0×10^{3}	0.80	0.06	0.84	0.9	0.06	0.13	1860	2120	6.1
Nb-0.5 Ti	9.1	4.9×10^{-7}	7.1×10^{3}	0.83	0.38	1.1	1.3	0.41	0.6	1870	2160	8.3
Nb-1.5 Ti	9.1	1.4×10^{-6}	7.2×10^{3}	0.85	1.12	1.7	2.1	0.93	1.6	1890	2240	13.3
Nb-4.5 Ti	9.15	3.5×10^{-6}	7.5×10^{3}	0.91	3.03	3.4	4.2	1.5	3.9	1950	2330	26.6
Nb-9.0 Ti	9.2	8.6×10^{-6}	7.9×10^{3}	0.97	7.15	7.3	7.4	5.7	7.3	2000	2360	52.0
Worked												
Nb	9.1	5.9×10^{-7}	7.0×10^{3}	0.80	0.47	1.2	1.1	0.66	0.39	···	···	7.4
Nb-0.5 Ti	9.0	1.0×10^{-6}	7.1×10^{3}	0.83	0.80	1.5	1.6	0.83	1.0	···	···	9.9
Nb-1.5 Ti	9.0	1.9×10^{-6}	7.2×10^{3}	0.85	1.50	2.0	2.5	1.2	2.2	···	···	15.9
Nb-4.5 Ti	9.1	4.6×10^{-6}	7.5×10^{3}	0.91	3.75	4.0	4.4	3.0	4.2	···	···	30.0
Nb-9.0 Ti	9.2	8.9×10^{-6}	7.9×10^{3}	0.97	7.45	7.6	7.7	5.5	7.7	···	···	60.0
Nb-12.5Ti	9.2	(12.3×10^{-6})	8.3×10^{3}	1.0	9.9	10.	12.4	10.	12.5	···	···	84.0
Ms	9.23	2.8×10^{-8}	8.0×10^{3}	···	0.03	···	0.85	···	···	···	2040	4.9
FSS	9.25	7×10^{-9}	7.3×10^{3}	···	0.006	···	0.78	···	···	···	1990	4.8

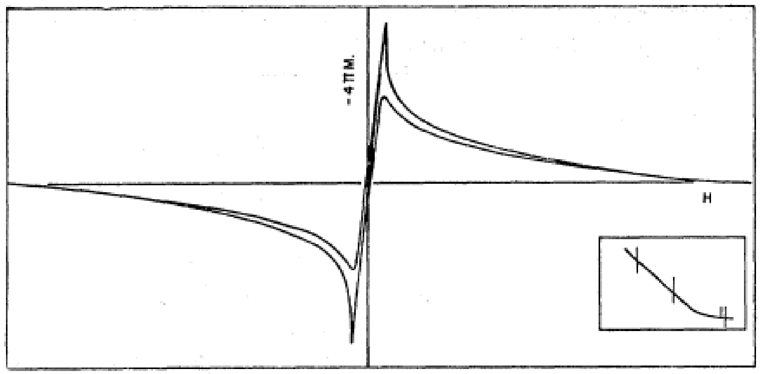

FIG. 1. An example of magnetization data obtained by electronic integration of induced signals proportional to the time rate of change of M and H during field sweeping. The inset shows a section of the curve near H_{c2} with the vertical scale expanded by a factor of 10. In the expanded section, the normal-state susceptibility may be determined from the slope of the curve above H_{c2}.

Extraction of all GLAG parameters in not too dirty alloys

Single phase alloys can be reasonably reversible, allowing extraction of the condensation energy and H_c, H_{c1}, H_{c2} and κ

$H_{c2}(t) = \sqrt{2}.\ \kappa_1(t)\ H_c(t)$, where $t = T/T_c$

$-(dM/dH)_{Hc2} = [1.16(2\kappa_2^2(t) - 1]^{-1}$

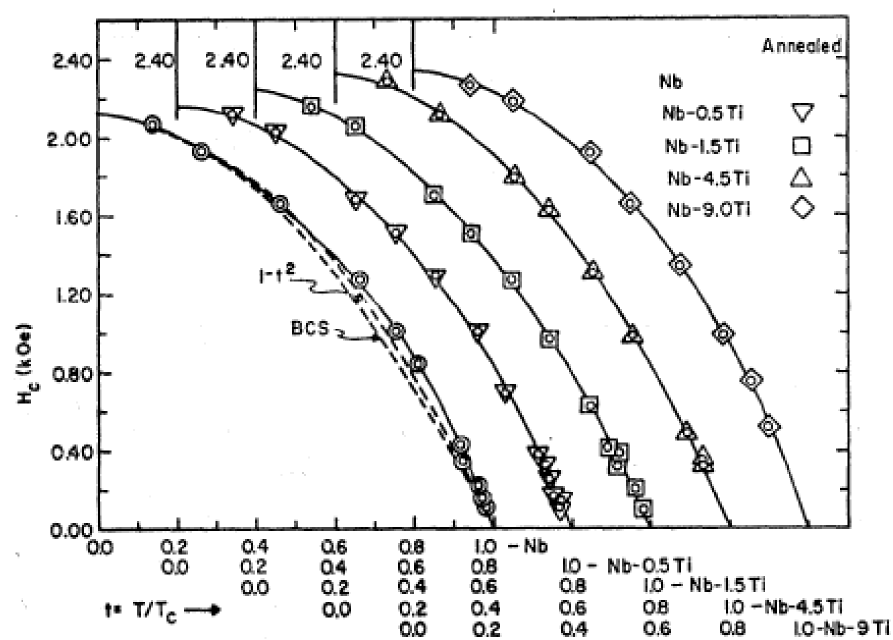

FIG. 2. The theromdynamic critical fields $H_c(t)$ determined from measurements of the areas under the magnetization curves recording during field increases. For clarity the horizontal scale for each curve has been displaced. The higher dashed curve has the form $1-t^2$ and the lower one is the BCS function.

Hake shows explicitly how normal state energy changes the

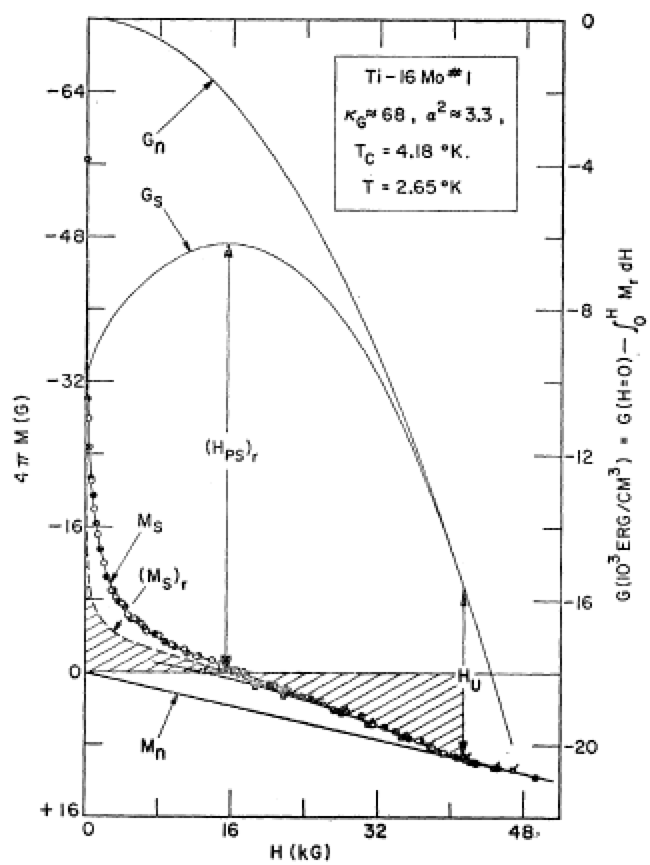

FIG. 3. Magnetization M versus applied magnetic field H for Ti(16 at.% Mo) No. 1 at $T = 2.65$°K. Black and white data points were taken on different days. Square points with ticks were taken on the return decreasing-H cycle. The reversible superconducting magnetization curve $(M_s)_r$ and the Gibbs free energies $G_s(H)$ and $G_n(H)$ are constructed with the help of the specific-heat data of Ref. 45 as explained in the text. The upper critical field H_u (2.65°K) is determined by the contact point of the $G_s(H)$ and $G_n(H)$ curves.

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Paramagnetic Superconductivity in Extreme Type-II Superconductors

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- Strong effect of the normal state paramagnetism in lowering the energy of the normal state
- Spin flip scattering can counter this so that for example pure Nb-Ti has lower H_{c2} than Nb-Ti-Ta

Extensive set of key GLAG equations – see also Orlando

APPENDIX

The formulas used to calculate or estimate the electronic properties of Table II are listed below in terms of experimentally convenient parameters and units: the low-temperature normal-state electrical resistivity $\rho_n(\Omega\text{ cm})$, the normal-state electronic-specific-heat coefficient γ (erg cm^{-3}°K^{-2}), the superconducting transition temperature T_c(°K), the conduction electron density $n(\text{cm}^{-3})$, and the ratio S/S_f of the free Fermi surface area S to that of a free-electron gas of density n. Other symbols and units are the free Fermi surface in wave-vector space $S_k(\text{cm}^{-2})$, the BCS half-energy gap at zero temperature Δ_{00}(erg), the Bohr magneton μ_B (erg G^{-1}), Planck's constant h or $\hbar = h/2\pi$ (erg sec), Boltzmann's constant k_B(erg °K^{-1}), the electron charge e (esu), and the velocity of light c (cm sec^{-1}).

1. Average Fermi velocity[105]:

$$\langle V_F\rangle \geq \langle 1/V\rangle_F^{-1} = k_B^2 S_k (6h\gamma)^{-1}$$
$$= 5.76\times10^{-5} n^{2/3}(S/S_f)\gamma^{-1}\text{ cm/sec}, \qquad \text{(A1)}$$

where the equality holds for a spherical Fermi surface.

2. Electron mean free path[105]:

$$l = 6\pi^2 h[e^2 S_k \rho_n]^{-1}$$
$$= 1.27\times10^4[\rho_n n^{2/3}(S/S_f)]^{-1}\text{ cm}, \qquad \text{(A2)}$$

where the first ρ_n is in esu and the second ρ_n is in Ω cm.

3. Thermal effective electron-mass ratio:

$$(m^*/m)_t \equiv \gamma/\gamma\text{ (free electron)} = 6.21\times10^4 \gamma n^{-1/3}. \qquad \text{(A3)}$$

4. Transport scattering time:

$$\tau_{tr} \approx l\langle 1/V\rangle_F = 2.21\times10^8\gamma[\rho_n n^{4/3}(S/S_f)^2]^{-1}\text{ sec}, \qquad \text{(A4)}$$

using Eqs. (A1) and (A2).

5. Density of states of one spin direction:

$$N = \gamma(\tfrac{2}{3}\pi^2 k_B^2)^{-1} = 8.0\times10^{30}\gamma\text{ erg}^{-1}\text{ cm}^{-3}$$
$$= 0.212\gamma\text{ ev}^{-1}\text{ atom}^{-1}, \qquad \text{(A5)}$$

where the last γ only is in units of [mJ mole^{-1}(°K)$^{-2}$] and mole means Avogadro's number of *atoms*.

6. Pauli spin susceptibility:

$$\chi_P(N) = 2\mu_B^2 N = 3\mu_B^2\gamma(\pi^2 k_B^2)^{-1}$$
$$= 1.37\times10^{-9}\gamma\text{ emu cm}^{-3}. \qquad \text{(A6)}$$

7. BCS coherence length[96]:

$$\xi_0 = \hbar\langle V_F\rangle(\pi\Delta_{00})^{-1} = 0.180\hbar\langle V_F\rangle(k_B T_c)^{-1}$$
$$\approx 7.93\times10^{-17} n^{2/3}(S/S_f)(\gamma T_c)^{-1}\text{ cm}, \qquad \text{(A7)}$$

using Eq. (A1) and assuming $\langle V_F\rangle \approx \langle 1/V\rangle_F^{-1}$.

8. Ginzburg-Landau coherence length $(\xi_0 \gg l)$[106,107]:

$$\xi_G \approx (\xi_0 l)^{1/2}(1-t)^{-1/2}$$
$$\approx 1.0\times10^{-6}(\rho_n\gamma T_c)^{-1/2}(1-t)^{-1/2}\text{ cm}, \qquad \text{(A8)}$$

using Eqs. (A2) and (A7), with $t \equiv T/T_c$.

9. Electromagnetic coherence length (0°K)[106,108]:

$$\xi_e \approx (\xi_0^{-1} + l^{-1})^{-1}$$
$$= \{1.26\times10^{16}\gamma T_c[n^{2/3}(S/S_f)]^{-1}$$
$$+ 7.87\times10^{-5}\rho_n n^{2/3}(S/S_f)\}^{-1}\text{ cm}, \qquad \text{(A9)}$$

using Eqs. (A2) and (A7).

10. London penetration depth (0°K)[16,109,110]:

$$\lambda_{L0} = 3ch\gamma^{1/2}\pi^{1/2}(ek_B S_k)^{-1}$$
$$= 1.33\times10^8\gamma^{1/2}[n^{2/3}(S/S_f)]^{-1}\text{ cm}. \qquad \text{(A10)}$$

11. Penetration depth (0°K, $\lambda \gg l$, $\xi_0 \gg l$)[106,108]:

$$\lambda_0 \approx \lambda_{L0}(\xi_0/l)^{1/2} = 1.05\times10^{-2}(\rho_n/T_c)^{1/2}\text{ cm}, \qquad \text{(A11)}$$

using Eqs. (A2), (A7), and (A10).

12. Thermodynamic critical field (BCS)[96]:

a. $$H_c = H_{c0}(1-t^2) + D_{\mathrm{BCS}}(t)H_{c0}, \qquad \text{(A12a)}$$

b. $$H_{c0} = 2.42\gamma^{1/2}T_c\text{ G}, \qquad \text{(A12b)}$$

where $D_{\mathrm{BCS}}(t) \equiv$ BCS deviation function[111].

13. Gor'kov-Goodman–calculated Ginzburg-Landau parameter κ_G[1,3,16,20,22,112]:

a. intrinsic:

$$\kappa_0 = 0.96\lambda_{L0}\xi_0^{-1} = 1.61\times10^{24}\gamma^{3/2}T_c[n^{4/3}(S/S_f)^2]^{-1}, \qquad \text{(A13a)}$$

using Eqs. (A7) and (A10);

b. extrinsic:

$$\kappa_l = ec\gamma^{1/2}\rho_n(k_B\pi^3)^{-1}[21\zeta(3)/2\pi]^{1/2} = 7500\rho_n\gamma^{1/2}, \qquad \text{(A13b)}$$

where $\zeta(3) = 1.202$;

c. total:

$$\kappa_G = \kappa_0 + \kappa_l, \qquad \text{(A13c)}$$

to within 6% for all $\xi_0 l^{-1}$ and to within 2.5% for the present $\xi_0 l^{-1} > 38)$[3].

14. Ginzburg-Landau parameter $\kappa_1(T_c)$:

$$\kappa_1(T_c) = (dH_u/dT)_{T_c}/[\sqrt{2}(dH_c/dT)_{T_c}]$$
$$= (6.0\gamma^{1/2})^{-1}(-dH_u/dT)_{T_c}, \qquad \text{(A14)}$$

assuming, from BCS,[97] $(dH_c/dT)^2_{T_c} = 18.0\gamma$.

15. Lower critical field $(\xi_0 \gg l)$[27]:

$$H_{c1}(t) = \sqrt{2}H_c(t)\{[\ln\kappa_3(t)]/[2\kappa_3(t)]\} \qquad \text{(A15)}$$

where $\kappa_3(t) = \kappa_3^*(t)\kappa_G$, $\kappa_3^*(t) \equiv \kappa_3(t)/\kappa_3(t=1)$ is given graphically by Maki, $\kappa_3^*(t=0) = 1.53$, and κ_G is given by Eq. (A13c).

16. Neo-GLAG[9–14] nonparamagnetically limited upper critical field $(\xi_0 \gg l$, 0°K):

a. $$H_{c20}^* = \sqrt{2}[\kappa_1(0°\text{K})/\kappa_1(T_c)]\kappa_1(T_c)H_{c0}; \qquad \text{(A16a)}$$

b. $$H_{c20}^* \approx 3.06\times10^4\rho_n\gamma T_c G, \qquad \text{(A16b)}$$

by substitution of[12] $\kappa_1(0°\text{K})/\kappa_1(T_c) = 1.195$, $\kappa_1(T_c) \approx \kappa_l = 7500\rho_n\gamma^{1/2}$, and $H_{c0}(\text{BCS}) = 2.42\gamma^{1/2}T_c$ in Eq. (A16a)].

c. From upper critical field slope[28,29] [see Eq. (5)],

$$H_{c20}^* = 0.693T_c(-dH_u/dT)_{T_c}. \qquad \text{(A16c)}$$

17. Clogston upper-critical-field limit (0°K)[23]:

$$\tfrac{1}{2}\chi_P H_{p0}^2 = H_{c0}^2/8\pi = \tfrac{1}{2}N\Delta_{00}^2; \qquad \text{(A17a)}$$

$$H_{p0} = \Delta_{00}(\sqrt{2}\mu_B)^{-1} = 1.84\times10^4 T_c G, \qquad \text{(A17b)}$$

substituting $\chi_P = 2\mu_B^2 N$ of Eq. (A6) into Eq. (A17a).

18. Maki paramagnetic limitation parameter[28]:

a. $$\alpha \equiv \sqrt{2}H_{c20}^*/H_{p0}; \qquad \text{(A18a)}$$

b. $$\alpha = 2.35\rho_n\gamma, \qquad \text{(A18b)}$$

substituting Eqs. (A16b) and (A17b) into Eq. (A18a), $\xi_0 \gg l$ is assumed;

c. $$\alpha = 5.33\times10^{-5}(-dH_u/dT)_{T_c}, \qquad \text{(A18c)}$$

substituting Eqs. (A16c) and (A17b) into Eq. (A18a).

19. Spin-flip scattering time[29]:

$$\tau_{so} = \hbar(3\pi k_B T_c\lambda_{so})^{-1} = 8.11\times10^{-13}(T_c\lambda_{so})^{-1}\text{ sec}. \qquad \text{(A19)}$$

PHYSICAL REVIEW B VOLUME 19, NUMBER 9 1 MAY 1979

Critical fields, Pauli paramagnetic limiting, and material parameters of Nb_3Sn and V_3Si

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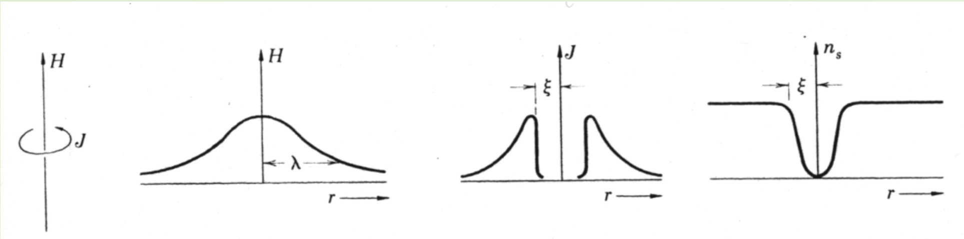
(Received 9 November 1978)

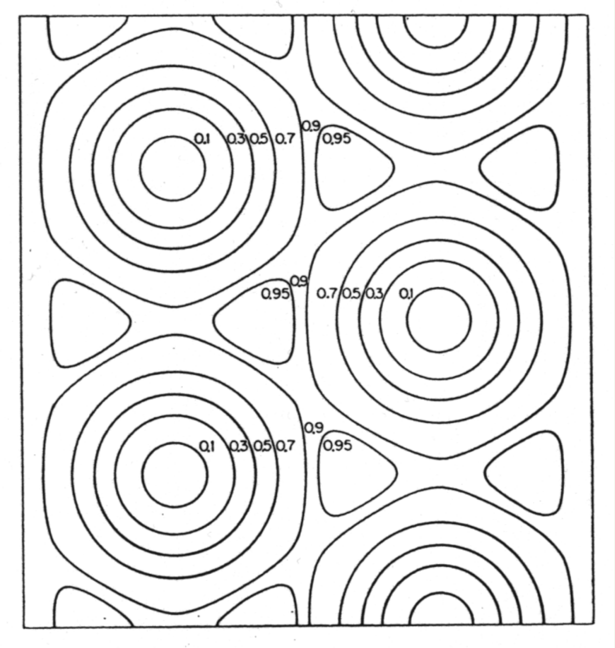
The upper-critical-field behavior of Nb_3Sn and V_3Si is studied as a function of residual resistivity. The results are analyzed in the framework of the Ginzburg-Landau-Abrikosov-Gor'kov theory of type-II superconductivity including the effects of the electron-phonon interaction. The importance of the electron-phonon interaction on the Pauli paramagnetic limiting process is stressed and it is found that inclusion of the electron-phonon corrections (most importantly the electron-phonon renormalization of the normal-state parameters) is needed to sensibly fit the data. For Nb_3Sn failure to include these effects leads to too high spin-orbit scattering rates. The critical-field data are also used to determine the density of states of these materials as well as several other superconducting and normal-state parameters.

Summary

- These 3 papers give examples of studies of the intrinsic properties of interesting high field materials in the absence – so far as possible – of explicitly added pinning centers
  - Quite visibly true for Fietz and Webb and Hake since they measured M(H)
- To compare measured J_c values to fundamental limits set by intrinsic properties
  - How close can J_c be to J_d? $J_d \sim H_c/\lambda$

Let's return to the mixed state



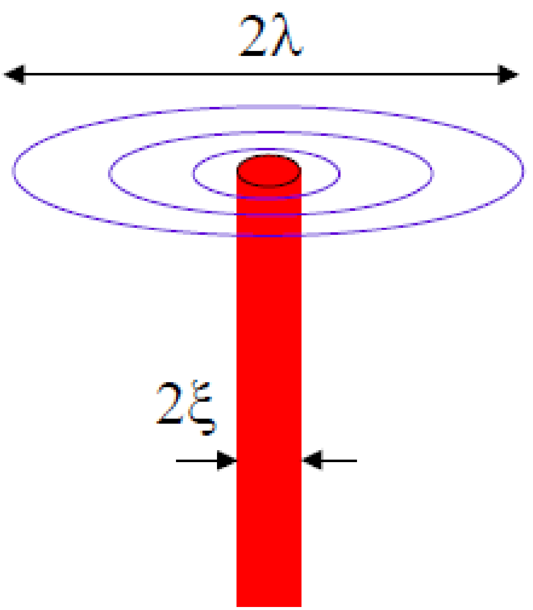


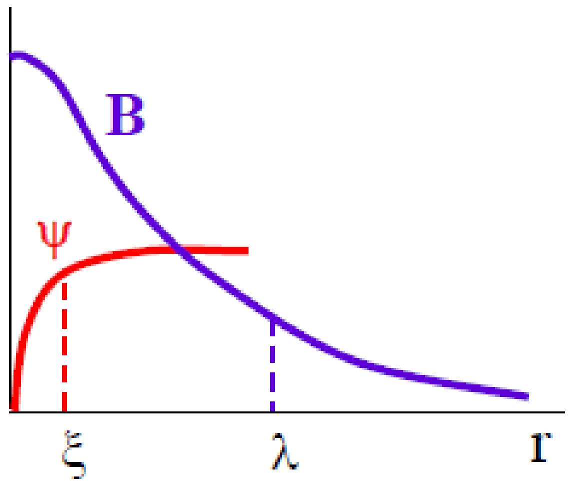
- **Two characteristic lengths**
  - coherence length ξ, the pairing length of the superconducting pair
  - penetration depth λ, the length over which the screening currents for the vortex flow
- **Vortices have defined properties in superconductors**
  - normal core dia, ~2ξ
  - each vortex contains a flux quantum ϕ_0 currents flow at J_d over dia of 2λ
  - vortex separation $a_0 = 1.08(\phi_0/B)^{0.5}$

$H_{c2} = \phi/2\pi\xi^2$

ϕ_0 = h/2e = 2.07 x 10^{-15} Wb

Single vortex line





- Small core region $r < \xi$ where $\Delta(r)$ is suppressed
- Region of circulating supercurrents, $r < \lambda$.
- Each vortex carries the flux quantum ϕ_0

Distributions of $\Delta(r)$ and $J(r)$ for $r < \lambda$

$$\Delta(r) \cong \frac{r\Delta_0}{\sqrt{2\xi^2 + r^2}}, \qquad J(r) \cong \frac{\phi_0}{2\pi\mu_0\lambda^2 r}$$

Suppression of $\Delta(r)$ in the core $r < \xi$ occurs because $J(r)$ reaches the depairing current density.

For $\kappa >> 1$, the London equation yields

$$B - \lambda^2 \nabla^2 B = \phi_0 \delta(\vec{r}),$$

$$B = \frac{\phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right),$$

$$B \cong \frac{\phi_0}{2\pi\lambda^2} \ln\left(\frac{\lambda}{r}\right), \qquad \xi \le r \le \lambda,$$

$$B \cong \frac{\phi_0}{2\lambda^{3/2}\sqrt{2\pi r}} \exp\left(-\frac{r}{\lambda}\right), \qquad r \ge \lambda$$

Why are vortices energetically favorable?

- Each vortex carries the paramagnetic flux quantum, so its energy in a magnetic field H is reduced by $H\phi_0$. Thermodynamic potential G per unit length of a single vortex:

$$G = \varepsilon - H\phi_0, \qquad \varepsilon = \frac{1}{2\mu_0}\int[\lambda^2(\nabla B)^2 + B^2]dS$$

Vortex self energy

Magnetic dipole in field

Kinetic energy of supercurrents

Energy of local fields

- Vortices are energetically favorable for G < 0, above the lower critical field $H_{c1} = \varepsilon/\phi_0$
  - Let us estimate ε $\Rightarrow$

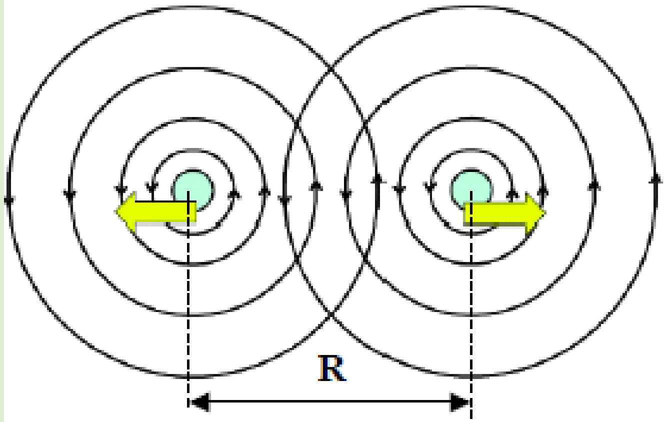
$$\varepsilon \cong \frac{\lambda^2}{2\mu_0}\left(\frac{\phi_0}{2\pi\lambda^2}\right)^2\int_\xi^\lambda \frac{2\pi r}{r^2}dr = \frac{\phi_0^2}{4\pi\mu_0\lambda^2}\ln\frac{\lambda}{\xi}$$

- Detailed calculations with the account of the vortex core structure give:

$$H_{c1} = \frac{\phi_0}{4\pi\mu_0\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right)$$

$H_{c1} \sim H_c/\kappa \sim H_{c2}/\kappa^2$, thus
$H_{c1} \ll H_c \ll H_{c2}$ for $\kappa \gg 1$

Interaction between vortices



- Energy of two vortices

$$U = \frac{\phi_0}{2}\left[H(r_1) + H(r_2)\right], \qquad H(r) = H_0 + H_{12}(R)$$

H_0 is the self-field in the core, $H_{12}(R)$ is the field produced at the position of the other vortex:

- Interaction energy $U_i(R) = \phi_0 H_{12}(R)$ and force $f = -\partial U_i/\partial R$:

$$U = 2\varepsilon + \phi_0 H_{12}(R), \qquad U_{\text{int}} = \frac{\phi_0^2}{2\pi\mu_0\lambda^2} K_0\left(\frac{R}{\lambda}\right), \qquad f_y = -\phi_0 \frac{\partial H_{12}}{\partial R} = \phi_0 J_x$$

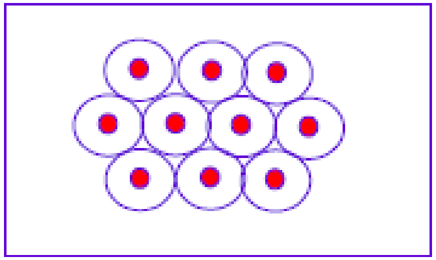
- Vortices repel each other, vortex and antivortex attract each other .
- General current-induced Lorentz force acting on a vortex

$$\vec{f} = \phi_0[\vec{J} \times \hat{n}]$$

- vortex is pushed perpendicular to the local current density J at the vortex core
- ň is the unit vector along the vortex line

David Larbalestier, CERN Accelerator School, Erice Italy April 25 – May 3, 2013

Hexagonal vortex lattice and equilibrium magnetization



- Above H_{c1} vortices form a hexagonal lattice because it provides maximum spacing between repelling vortices for a given vortex density $n = B/\phi_0$

- Calculate the equilibrium vortex density, B/ϕ_0 at a given H from the minimum of G:

$$\frac{\partial G}{\partial B} = \frac{\partial}{\partial B}\left[\frac{B\varepsilon}{\phi_0} + \sum_{i>j} U(R_{ij}) - BH\right] = 0$$

Self energy

Interaction energy

Magnetic energy

Minimization of G yields H(B) in the form

$$H = H_{c1} + \frac{\phi_0^2}{2\pi\mu_0\lambda^2}\frac{\partial}{\partial B}\sum_{i>j} K_0\left(\frac{R_{ij}}{\lambda}\right)$$

$$a_\Delta = \left(\frac{2}{\sqrt{3}}\right)^{1/2}\left(\frac{\phi_0}{B}\right)^{1/2} \approx 1.07\left(\frac{\phi_0}{B}\right)^{1/2}$$

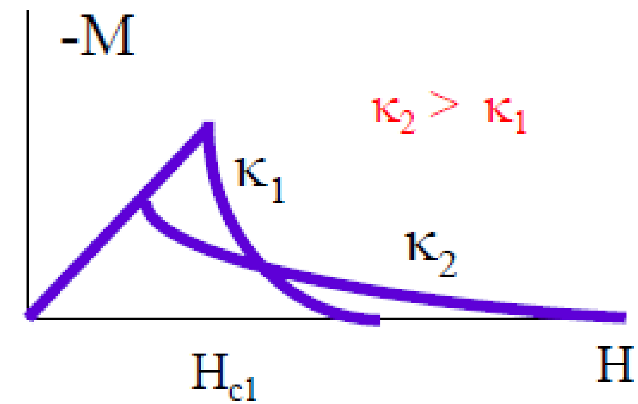
Here R_{ij} is the spacing between i-th and j-th vortices in the hexagonal lattice

Intermediate fields, $H_{c1} << H << H_{c2}$

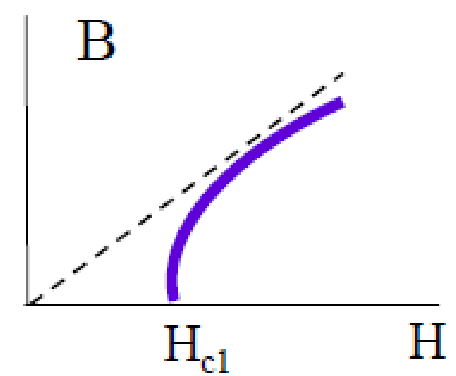
- For a << λ, and κ >> 1, the field H(B) and the magnetization M(H) are

$$H \approx \frac{B}{\mu_0} + H_{c1}\frac{\ln(B_{c2}/B)}{2\ln\kappa}, \qquad M \cong -H_{c1}\frac{\ln(H_{c2}/H)}{2\ln\kappa}$$

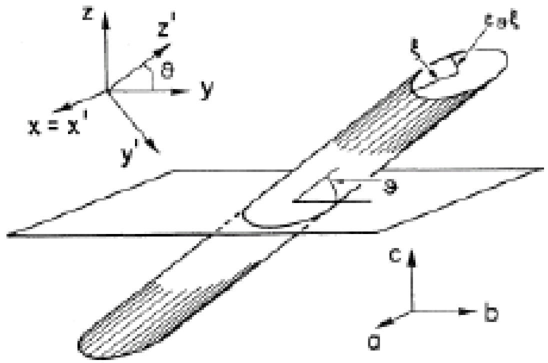
Superconductivity disappears at $B_{c2} = \phi_0/2\pi\xi^2$
because nonsuperconducting vortex cores overlap

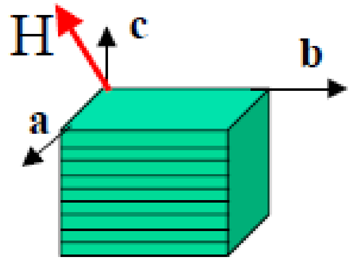


Material	T_c (K)	λ(0), nm	ξ(0), nm	H_{c2} (T)
Nb-Ti	9.5	240	4	13
Nb-N	16	200	5	15
Nb_3Sn	18	65	3	30
MgB_2 (dirty)	32-39	140	6	35
$YBa_2Cu_3O_7$	92	150	1.5	>100
Bi-2223	108	200	1.5	>100

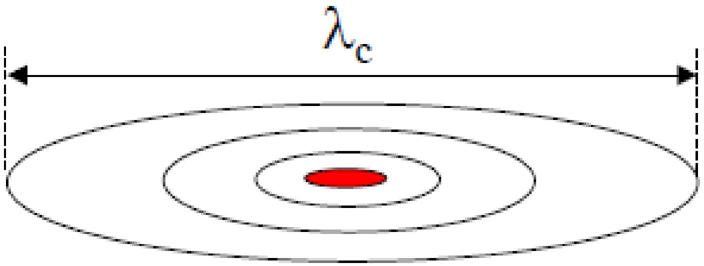


Vortices in anisotropic superconductors





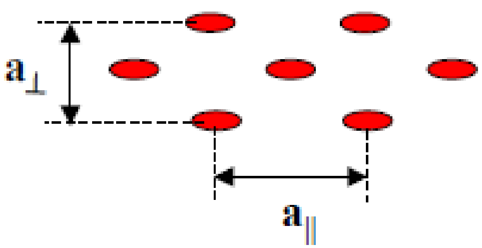
- Uniaxial supereconductor (HTS, MgB_2, etc.)
- Different penetration depths: λ_{ab} - field along c, and λ_c - field along ab plane $\gamma = \lambda_c/\lambda_{ab} > 1$
- Different coherence lengths: ξ_{ab} in the ab plane and ξ_c along the c-axis ($\xi_{ab} > \xi_c$)



Elliptical current streamlines and vortex core H||ab:

$$H(x,y)=\frac{\phi_0}{2\pi\mu_0\lambda_c\lambda_{ab}}K_0\left[\left(\frac{x^2}{\lambda_c^2}+\frac{z^2}{\lambda_{ab}^2}\right)^{1/2}\right]$$

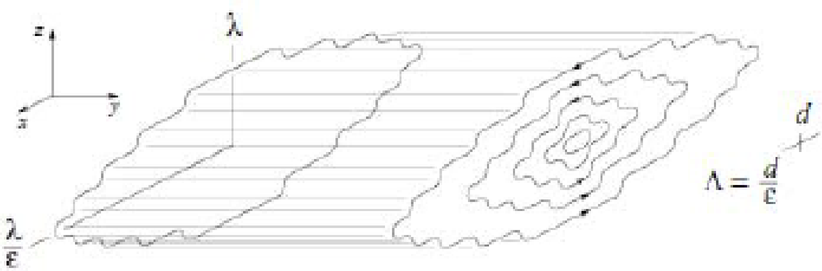
Anisotropic London theory



- Squeezed hexagonal lattice: $a_\parallel a_\perp B = 2\phi_0$, $a_\perp\lambda_c = \sqrt{3}\lambda_{ab}a_\parallel$

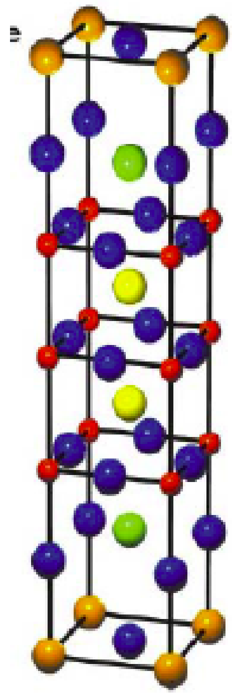
$$a_\parallel = (2\gamma\phi_0/\sqrt{3}B)^{1/2}, \qquad a_\perp = (2\phi_0/\gamma B)^{1/2}$$

Vortices in layered high-T_c superconductors



Weakly coupled ab planes
In layered HTS

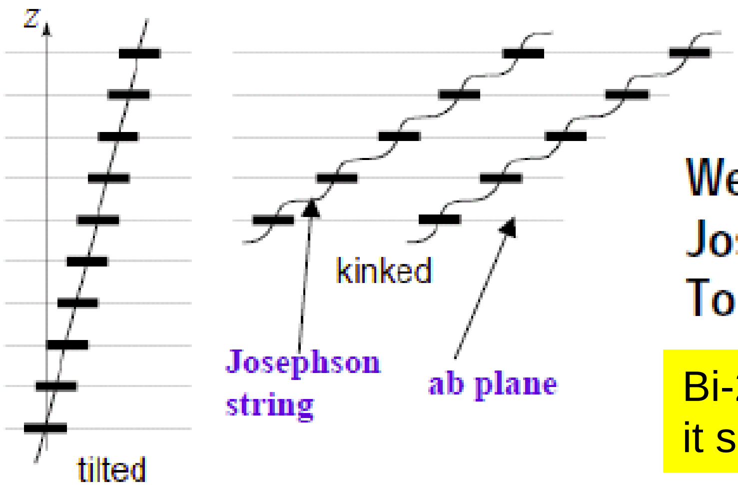
The mass ratio: $\varepsilon = m/M \sim 100$



Bi-2212

No normal core in the strongly anisotropic Josephson vortex parallel to the ab planes

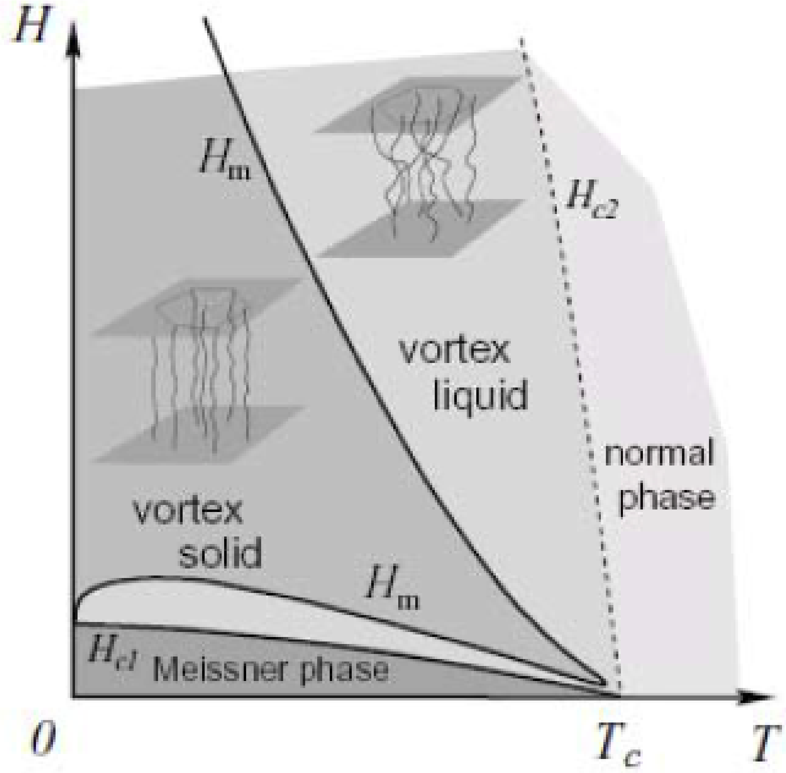
Stack of 2D pancake vortices on different ab planes



Weak magnetic and
Josephson coupling between pancakes due
To magnetic and Josephson interaction

Bi-2212 is the classic HTS layered sc – but actually it seems now to develop high Jc fine at 4 K

Melting of the vortex lattice



Strong thermal fluctuations of "soft" vortices in layered HTS.

Vortex lattice melts if the amplitude of thermal vibrations of vortices u is comparable to the vortex spacing a.

Lindemann criterion:

$$u(T,B) = c_L a(B)$$

$c_L \approx 0.3$ is the Lindemann number.

The melting field $H_m(T)$ in Bi-2212 can be well below $H_{c2}(T)$

The great silence: 1914-1961

- An interesting talk was put together some years ago by Dick Hake who summarized how complex it was as a SCIENCE from the viewpoint of someone studying superconductivity BEFORE any applications seemed feasible…………..

The Dick Hake Story (U. of Indiana and Atomics International)

EARLY HISTORY
OF
HIGH FIELD
SUPERCONDUCTIVITY
1930–1967 AD

A Tragicomedy in Twelve Acts
R.R. Hake (borrowing heavily from ref. 1)
I.U. Condensed Matter Playhouse 2/3/89 (slight revision 7/89)

REFERENCES

1. T.G. Berlincourt "Type II Superconductivity: Quest for Understanding" [H. Kamerlingh Onnes Symposium on the Origins of Applied Superconductivity] IEEE MAG 23, 403 (1987)

2. J.E. Kunzler "Recollection of Events Associated with the Discovery of High Field-High Current Superconductivity," ibid., p. 396.

3. G.B. Yntema, "Niobium Superconducting Magnets," ibid., p. 390.

4. A.B. Pippard, "Early Superconducting Research (Except Leiden)," ibid., p. 371.

ACT I. PURE OR SPONGE?

W.J. de Haas & J. Voogd, Commun. Phys. Lab U. Leiden #208b (1930); ibid. #214b (1931)

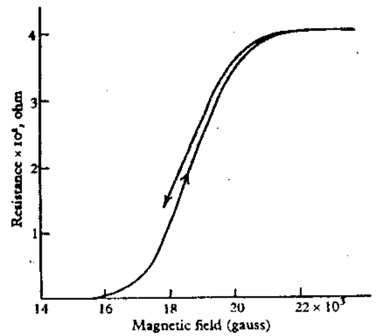


Fig. 14. Restoration of resistance of Pb-Bi eutectic by a magnetic field at 4·2° K. (de Haas and Voogd, 1930).

[Attempt technolo applicat producti H-fields conducti solenoids (1935) on (1933)

Early Ideas on High-Field Superconducti

I. Could be bulk property of HOMOGENEOU (**PURE**) materials associated with negativ interphase surface energy:

H. London, Proc. Roy. Soc. (London) A152, 65
C.J. Gorter, Physica 2, 449 (1935)
(says $H_{max} \simeq \frac{\lambda}{\xi} H_c$) Thermodynamic Cr

Gorter's "minimu superconductor" i same as GL coher

II K. Mendellsohn Proc. Roy Soc. (London) A152, 34

"We think that all experimental results so far obta on IMPURE (our caps + underline) metals and on a can be explained by their INHOMOGENEITY (us causes the formation of a **SPONGE** of higher value."

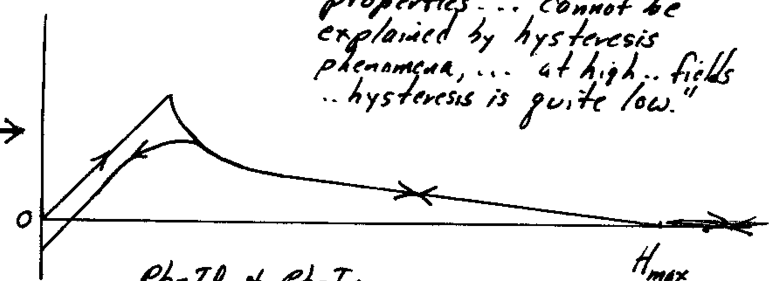
III. THE CRUCIAL EXPERIMENT.

(5)

L.V. Shubnikov, V.I. Khotkevich, J.D. Shepelev, J.N. Rjabinin, J. Exptl. Theoret. Phys. (USSR) 7, 221 (1937) [Portions were reported in English!: J.N. Rjabinin and L.V. Shubnikov, Nature 135, 581 (1935); Phys. Z. Sowjet 7, 122 (1935)]

"Such unusual magnetic properties ... cannot be explained by hysteresis phenomena, ... at high.. fields ..hysteresis is quite low."

This work ignored for 20 YEARS until Abrikosov compared this data with his theory in 1957!!



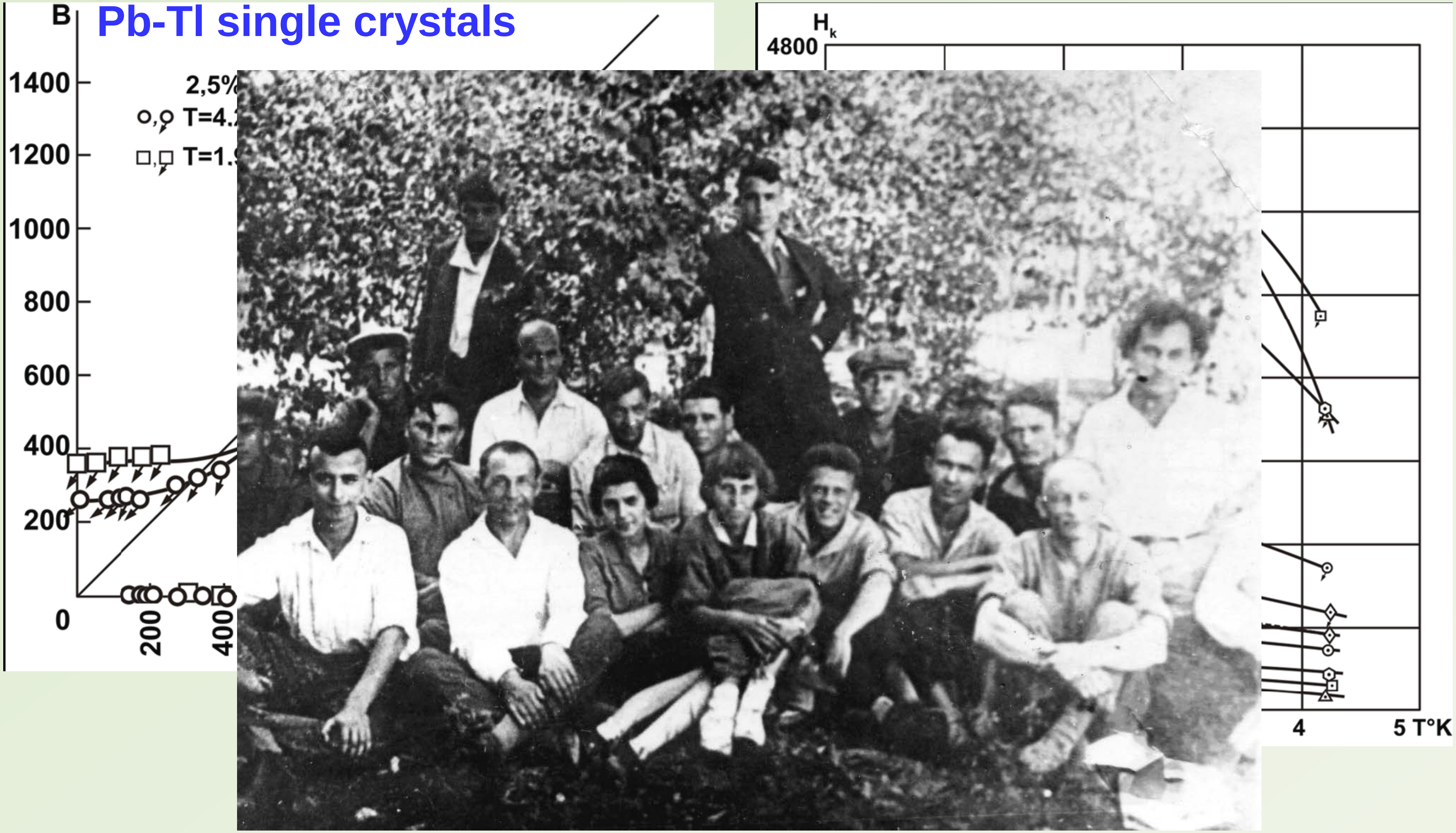
Pb-Tl + Pb-In
Single Phase Single Crystals

Shubnikov et al. said:

1. $\int M dH$ = Superconducting state condensation energy
2. Even though Hmax exceeds H_c of pure metals, the condensation energies are are comparable and depend on T in the same way
3. The zero-field specific heat jump in an alloy superconductor should be comparable to that of a pure superconductor, (and not have gigantic value, expected if complete flux expulsion existed up to Hmax)

BUT SHUBNIKOV et al. "FAILED TO EXPLOIT THEIR NEWFOUND UNDERSTANDING"... (making) "no mention of the Gorter-H. London theory ... "nor of the Mendelssohn SPONGE..." T.G. Berlincourt

1936: Type II Superconductivity discovered – and unappreciated



Shubnikov returned to Kharkov from Leiden to start single crystal alloy studies – persistence of superconductivity beyond the Meissner state - then imprisoned and shot

ACT III. RUSSIAN SLOTHS IGNORE RUSSIA

V.L. Ginzburg and L.D. Landau, Zh. Eks
i Teor. Fiz. 20, 1064 (1950).

"It has not been necessary to investiga
the nature of the state which occurs
when $\kappa > 1/\sqrt{2}$, since from the
experimental data . . . . it follows .
$\kappa \ll 1$." [Apparently oblivious o
Schubnikov et al.!!

K. Mendelssohn to T.G. Berlincourt 1

"It was extremely nice of you to send me a copy of your own paper, as well as a translation of Shubnikov's paper published in 1937. This is indeed of considerable help in assessing the earlier developments. At that time the Stalin Purge was only beginning, and I was very puzzled at the blanks I drew in trying to get in touch with Shubnikov. In 1957 Landau introduced me in Moscow to Shubnikov's widow, Olga Trapeznikova, who also is a physicist. She told me that her husband had just been exonerated posthumously from all charges. This made it possible for Abrikosov to refer to Shubnikov's papers, since up to then Soviet etiquette required that anyone who had disappeared in the purges had never lived."

(According to Balabekyan,(1966) Sh
was unjustly arrested in 19
sentenced to 10 years impri
and died in 1945.)

ACT IV. PIPPARD PIDDLES WHILE GINZBURG SQUIRMS

9

In 1951-53 Pippard used intuitive ideas to explain that a short electron mean free path would lead to negative surface energy. He was aware of GL-theory and the Gorter - H. London ideas.

PIPPARD IS VERY SMART!

WHY DIDN'T PIPPARD PUT IT ALL TOGETHER?

"So in the early 1950's there was a certain amount of conflict which wasn't helped, incidentally, by the fact that Ginzburg kept on writing small papers in which he said it would be much better if we interpreted the electronic charge as not being exactly e, but e times a small numerical factor which might be as large as 2! He didn't say it was exactly 2; instead he wanted to introduce a fudge factor of (say) 1.6, and Landau kept on telling him he couldn't just put in arbitrary numbers, and muttered darkly about gauge invariance going wrong if you did."

A.B. Pippard in
"Historical Context of Josephson's Discovery"
in SQUIDS + Machines (Plenum, 1977) p.1.

SLOTH = SoLid State TheOrist

ACT V. THE KID AND GEEZER SLOT
TEAM UP FOR SOME BREAK
BCS AND GLAG

NOBEL PRIZE WINNING MICROSCOPIC THEORY OF SUPERCO

J. Bardeen, L.N. Cooper, J.R. Schrieffe
Phys. Rev. 108, 1175 (1957)

L.P. Gor'kov, Zh. Eksperim. i Teor. Fiz
1918 (1959); Sov. Phys. JETP 9,
1364 (1959)
[In "dirty limit" $H_{c2}(T=0) = (\text{const.})$

GLAG: Ginzburg, Landau, Abrikosov

The basic theory of high-field
Superconductivity (except for the
paramagnetic limitation) is in
place in 1959 but virtually
ignored until 1962!

ACT VII. NUTTY GEORGE

(14)

G.B. Yntema, Phys. Rev. 98, 1197 (1955)
Also (unaware of Yntema): S.H. Autler, Bull. Am. Phys. Soc. 4, 413 (1959)

FIRST SUPERCONDUCTING-WIRE MAGNET

0.71 Tesla

Cold-drawn
Nb wire

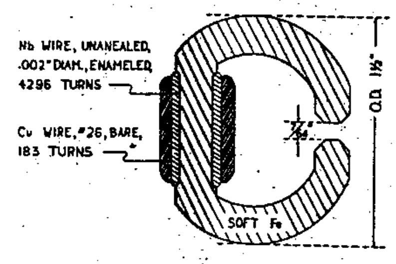


Figure 2. Electromagnet with superconducting niobium windings, horizontal cross-section. Magnet constructed at University of Illinois in 1954.

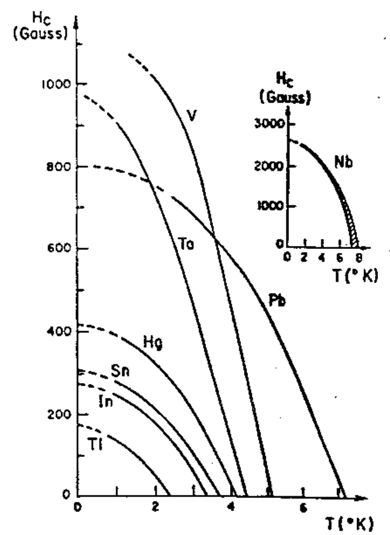


Figure 1. Critical fields as functions of temperature. Traced from figure compiled by D. Shoenberg, 1952 (Ref. 1).

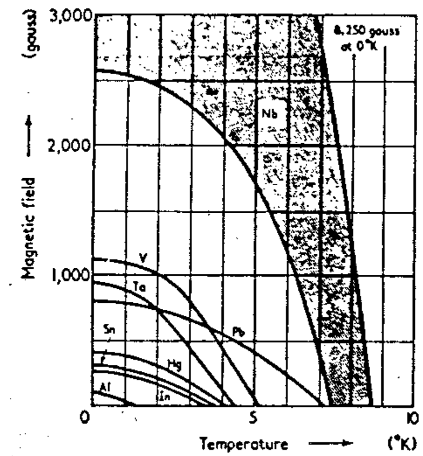


Figure 8. Critical fields as functions of temperature. The shaded area shown for niobium illustrates the variation in reported values. Compiled by V. D. Arp and R. H. Kropschot, 1960 (Ref. 18).

Almost there in July 1960……

VOLUME 5, NUMBER 4 PHYSICAL REV

CRITICAL FIELD FOR SUPERCO

R. M. Bozorth, A. J. Will
Bell Telephone Laboratories,
(Received July

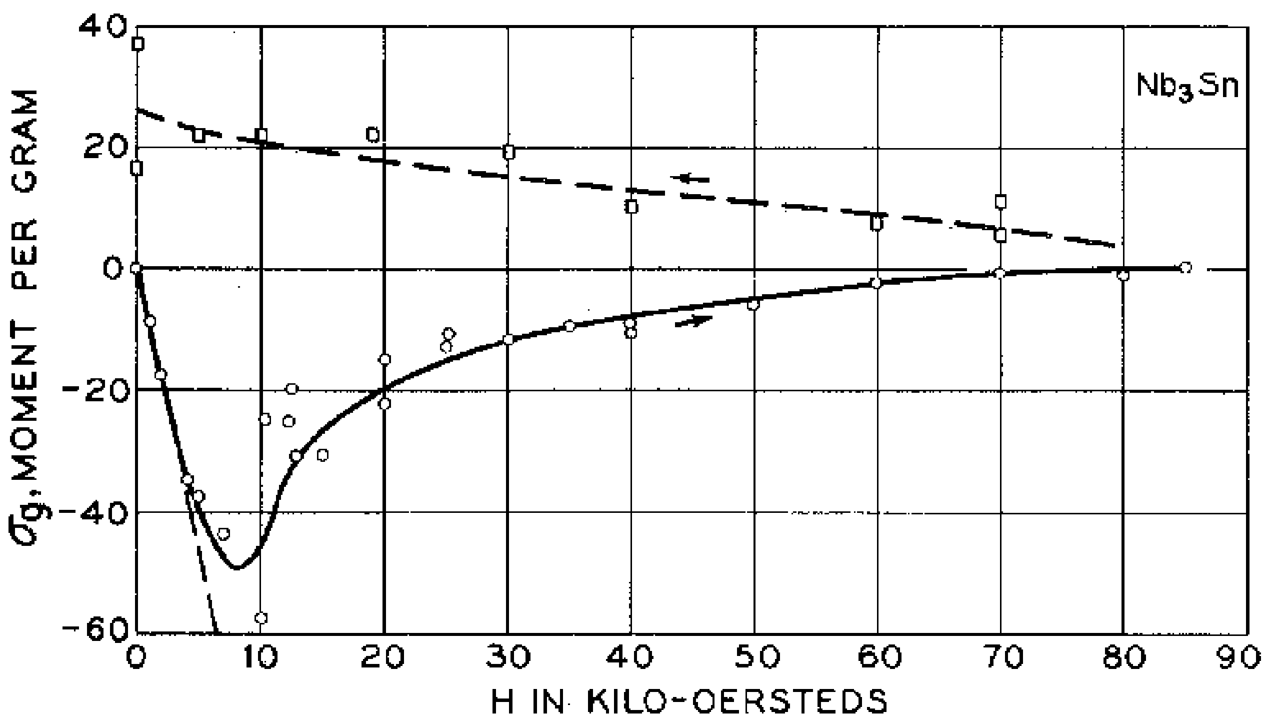
It is well known[1] that Nb_3Sn is a superconductor with a high critical temperature, 18°K. The measurements here reported show that it has also an exceptionally high critical field, about 70 000 oersteds at 4.2°K, necessary for the suppression of all superconductivity.

The material was prepared by melting together niobium and tin in the argon arc, and the button so obtained was formed by grinding into a rod about 2 cm long and 4 mm in diameter, with rounded ends. The magnetic moment per gram, σ_g, was measured by pulling the specimen from one search coil to another in a constant field, the two search coils being connected in series opposition to a ballistic galvanometer. Calibration was with nickel of high purity.

Measurements were made in increasing fields, after cooling in zero field to liquid helium temperature. Results are shown in Fig. 1. The initial points (circles) follow accurately the line for $B=0$ ($H=-4\pi\sigma_g d$, where d is the density, 8.9), and then begin to deviate at about 4000 to 5000 oersteds. The variations in the readings in fields from 5000 to 20 000 oersteds reflect the well-known irregular changes in magnetization resulting from changes in domain structure in the intermediate state, as observed by Schawlow et al.[2] and others. The general shape of the magnetization curve is that observed in a hard superconductor. Polishing, or annealing the specimen at 1100°C for several hours, made no essential change in the character of the curve.

When the field was decreased from its maximum value (points marked with squares) some of the flux was frozen in, and irregularities were again observed.

The authors are indebted to E. Corenzwit for preparation of the material, to W. E. Henry of the Naval Research Laboratory for details of the method of measurement, and to H. W. Dail for assistance with the experiment. The field was produced in a Bitter coil excited with a motor generator with a nominal power rating of one megawatt.



[1]B. T. Matthias and T. H. Geballe, Phys. Rev. 95, 1435 (1954).
[2]A. L. Schawlow, G. E. Devlin, and J. K. Hulm, Phys. Rev. 116, 626 (1959).

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A one page PRL – but no Bean Model yet, no way to relate magnetization hysteresis to Jc

Decisive experiment only in late 1960

SUPERCONDUCTIVITY IN Nb_3Sn AT HIGH CURRENT DENSITY IN A MAGNETIC FIELD OF 88 kgauss

J. E. Kunzler, E. Buehler, F. S. L. Hsu, and J. H. Wernick
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received January 9, 1961)

We have observed superconductivity in Nb_3Sn at average current densities exceeding 100 000 amperes/cm^2 in magnetic fields as large as 88 kgauss. The nature of the variation of the critical current (the maximum current at a given field for which there is no energy dissipation) with magnetic field shows that superconductivity extends to still higher fields. Existing theory does not account for these observations. In addition to some remarkable implications concerning superconductivity, these observations suggest the feasibility of constructing superconducting solenoid magnets capable of fields approaching 100 kgauss, such as are desired as laboratory facilities and for containing plasmas for nuclear fusion reactions.[1,2]

The highest values of critical magnetic fields previously reported for high current densities

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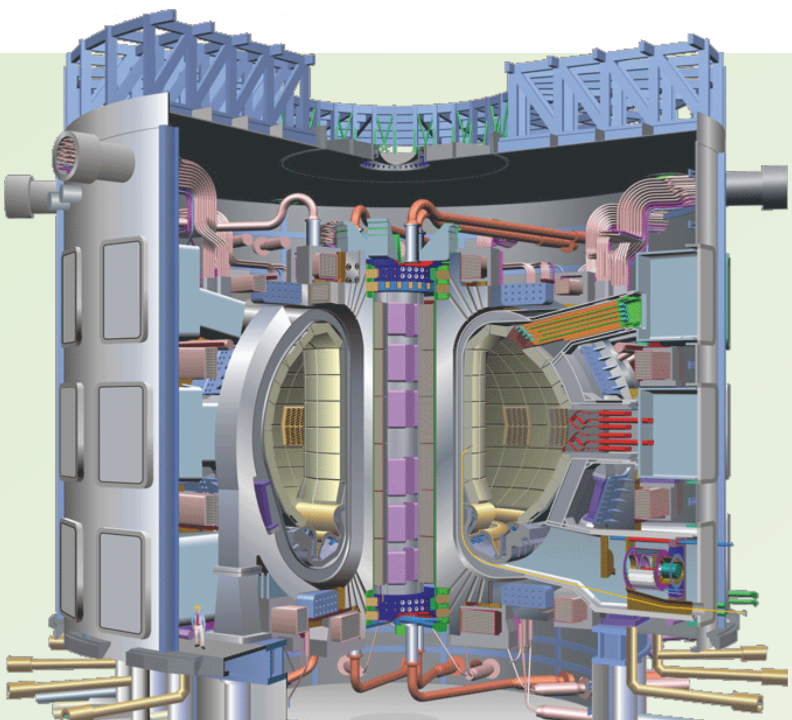

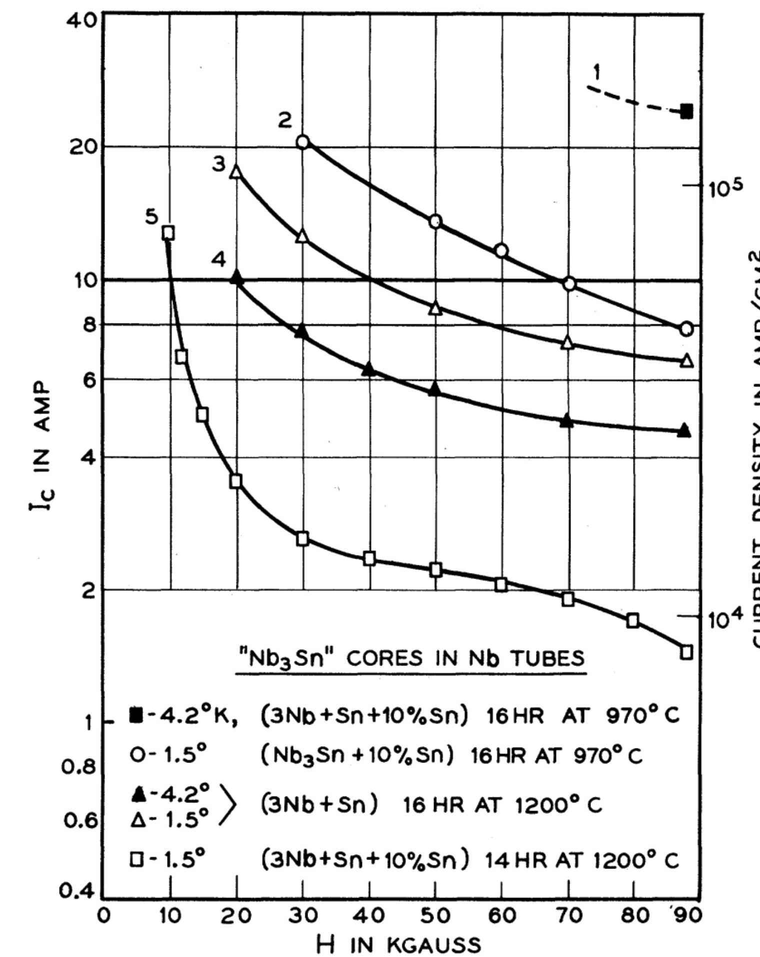

ITER uses 600 tonnes of Nb_3Sn

Phys Rev Letts **6**, 89 (1961), submitted **January 9, 1961**, published **February 1, 1961**!

The November1961 magnet Technology Conference at MIT

…

BRIT. J. APPL. PHYS., 1962, VOL. 13

International Conference on High Magnetic Fields, Massachusetts Institute of Technology, November 1961

Who	Field	Material	Bore
Bell	6.9 T	Nb_3Sn	0.25”
Atomics International	5.9 T	Nb25Zr	0.5”
Westing house	5.6 T	Nb25Zr	0.15”

Concluding remarks

After any conference of this type it is often asked if there should be another. The argument against conferences in which the common factor linking sessions is a technique is that they cover far too wide a field or multiplicity of fields. This can be true but is a factor under the control of the organizers. With this particular conference the 'net' was perhaps too widely spread. However, the conference could hardly avoid being a success owing to the sessions involved with high critical field superconductors which are fairly new in their application to the generation of high fields and on which a very great deal of active work is in progress. This topic was wisely left to the last, after review of all the other fields of application and methods of generating high fields.

In applying steady high magnetic fields to physical experiments and in equipment there have seemed to be two barriers. The first is a cost barrier at which fields easily achievable with iron cooled magnets are passed (about 30 kG); the second is the barrier set by the strength of materials, which at present seems to be at about 250 to 300 kG. The first of these is being finally swept away with the advent of superconducting solenoids and the second will soon be approached in several laboratories, probably simultaneously.

Ministry of Aviation,
Royal Radar Establishment,
St. Andrews Road,
Great Malvern,
Worcs.

D. H. PARKINSON
20*th June* 1962

Horst Rogalla and Peter H. Kes

100 Years of Superconductivity

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed in the United States of America on acid-free paper
Version Date: 20110811

International Standard Book Number: 978-1-4398-4946-0 (Hardback)

Chapter 11: Wires and Tapes

Editor: David Larbalestier

For more on the history and recent developments too – see here……….

Summary of lecture 1

- Technology depends on the science, even if as applied scientists or engineers we often ignore the science
- Superconductivity science is not easy – so knowing where to go for answers is important to keep the technology developing
- In lecture 2:
  - Irreversible effects and development of high Jc by vortex pinning
  - Grain boundary effects and their profound impact on conductor choices