X-ray Optics

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Synchrotron Radiation & Free Electron Lasers

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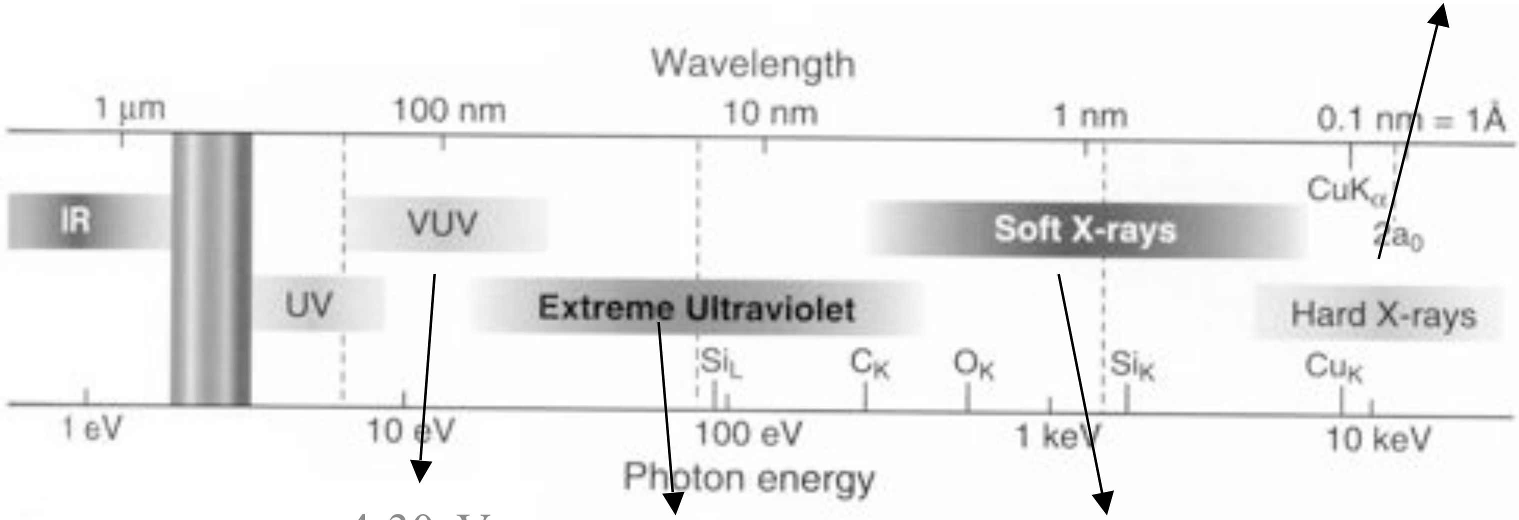
Majorana Center for Scientific Culture, Erice, Sicily

Joint US-CERN-Japan-Russia School on Particle Accelerators

Energy regions

few keV - 50-100 keV



4-30eV
300-40nm

30-250eV
40-5nm

250eV - several keV

These regions are very interesting because are characterized by the presence of the absorption edges of most low and intermediate Z elements → photons with these energies are **a very sensitive tool** for elemental and chemical identification
But… these regions are difficult to access.

Refraction Index

refractive index $\mu = 1 - \delta - i\beta$

$$\delta = (e^2\lambda^2/2\pi mc^2)|N + \Sigma_H N_H [\lambda/\lambda_H]^2 \ln[\lambda_H^2/\lambda^2 - 1]|$$

δ (unit decrement) related to the speed in the medium

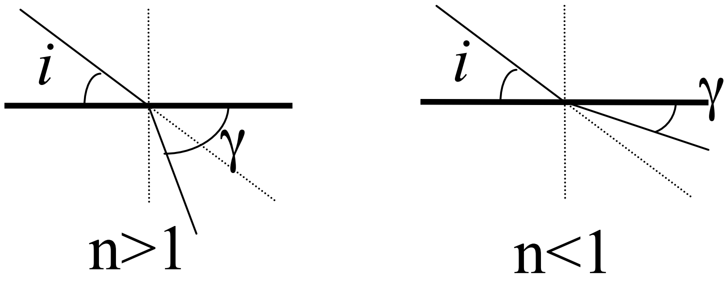
β related to the absorption

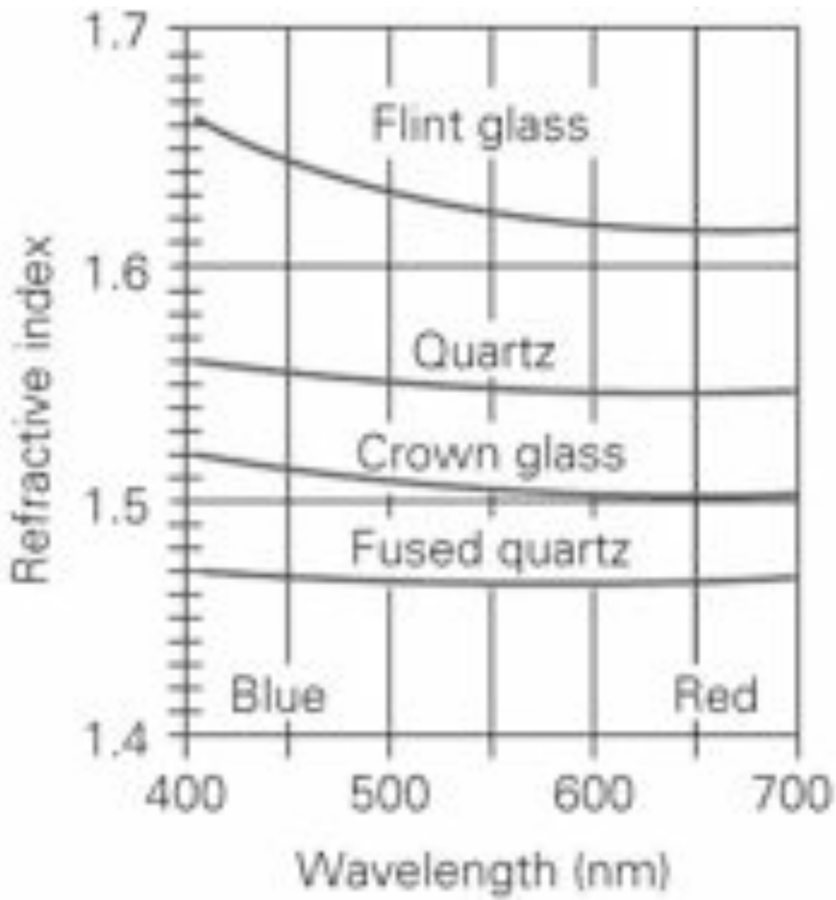
N=electron density (10^{23}-10^{24} el./cm^3)
λ_H=adsorption edge's wavelength

λ far from $\lambda_H \Rightarrow \delta = Ne^2\lambda^2/2\pi mc^2$

$\beta = \lambda\mu_l/4\pi$ μ_l=linear absorption coefficient

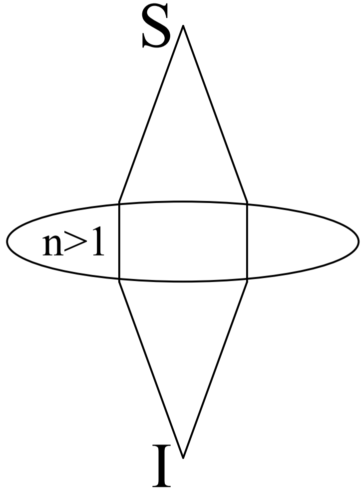
Snell Law

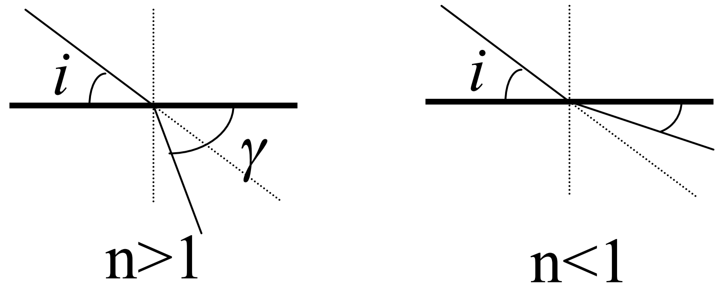


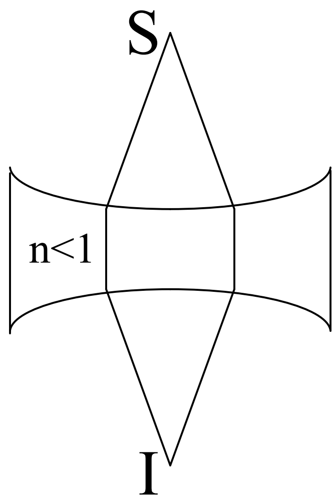


Snell's law: $n_1 \cos\gamma = n_2 \cos i$

Snell Law







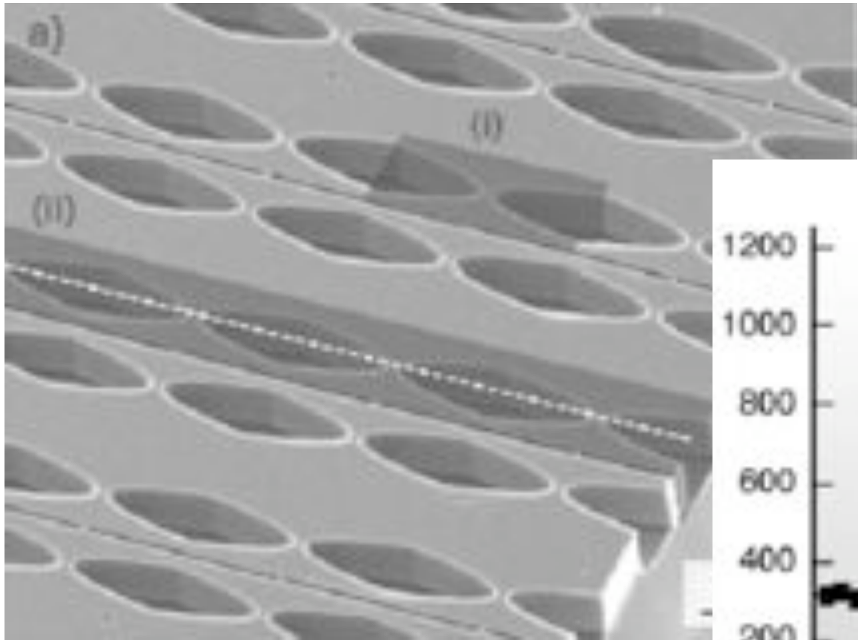
Fermat's principle

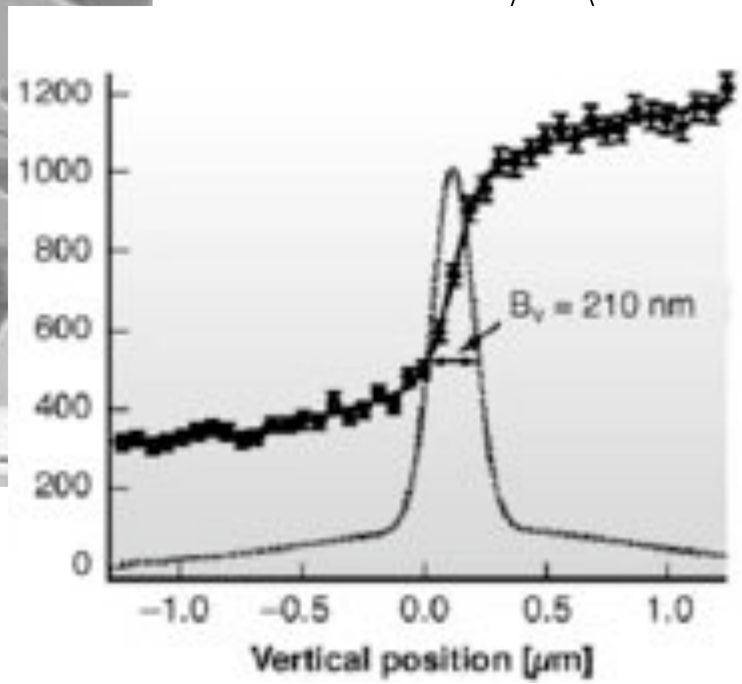
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \approx (1-\delta-1)\cdot\frac{2}{R} < 0$$

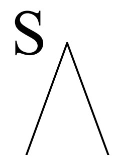
$$\delta = \frac{Ne^2\lambda^2}{2\pi mc^2} \approx 10^{-2} - 10^{-4}$$

$$\delta \approx 10^{-4} \quad HXR \Rightarrow f \approx 1m \quad if \quad R \approx 1mm$$

X-ray Lenses

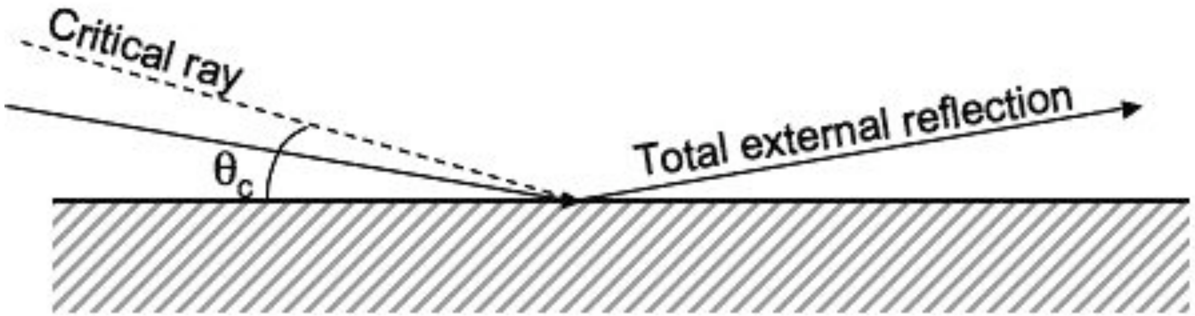






$$\delta \approx 10^{-4} \quad HXR \Rightarrow f \approx 1m \quad if \quad R \approx 1mm$$

Snell law - Total external reflection



Snell's law: $\cos\gamma = \cos i / n$

$\gamma = 0 \quad n = \cos i_c$

i_c critical angle: total external reflection

$\sin i_c = \lambda (e^2 N / \pi m c^2)^{1/2}$

$\lambda_c(\min) = 3.333 \cdot 10^{-13} N^{-1/2} \sin i_c$

Material	Density (g/cm^3)	N ($electron/cm^3$)	λ_{min} nm
Pentadecane (oil)	0.77	7×10^{22}	$64.1 \sin i$
Glass	2.6	78×10^{22}	$37.9 \sin i$
Aluminum oxide	3.9	115×10^{22}	$31.2 \sin i$
Gold	19.3	466×10^{22}	$15.4 \sin i$

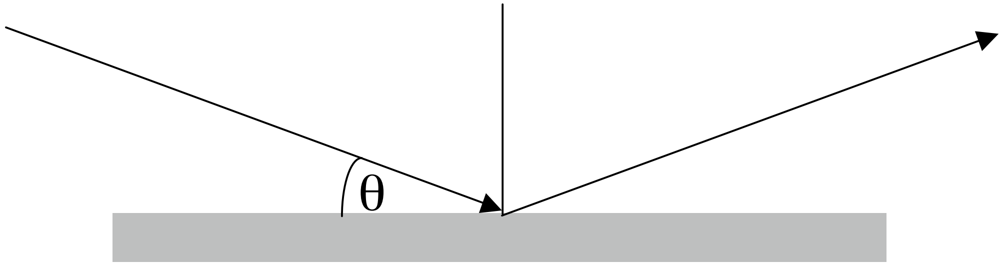
$i = 5^o$: λ_{min}glass=3.3nm=375 eV

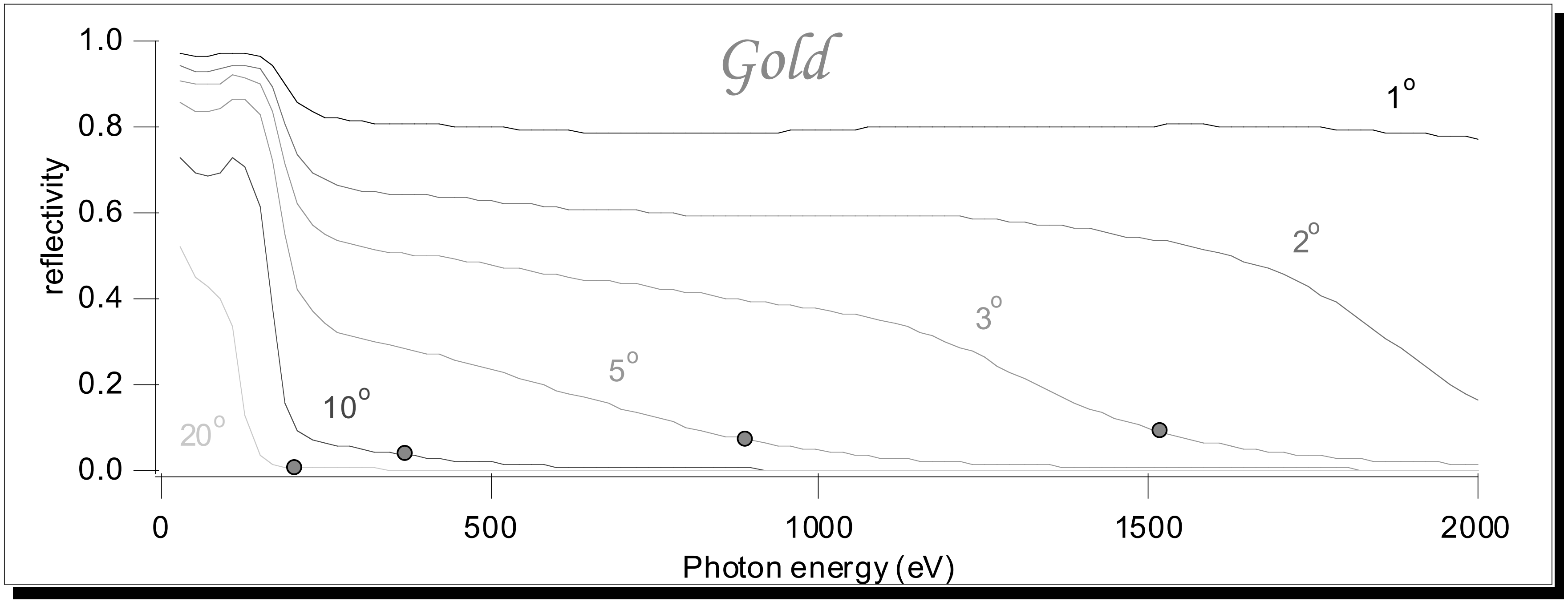
λ_{min}gold=1.34nm=923eV

shorter wavelength needs smaller angles of incidence

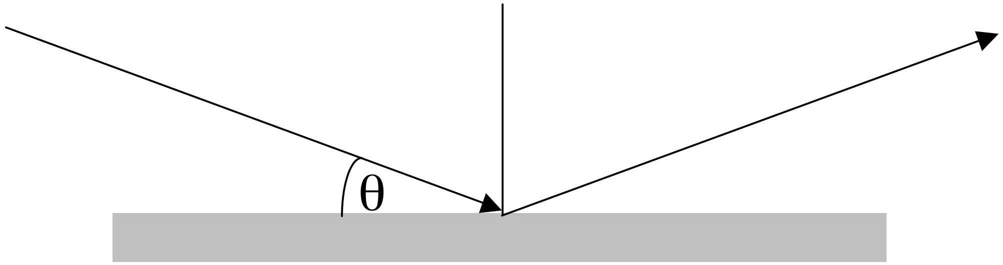
Materials with higher density (i.e. higher atomic weight) have higher reflectivity

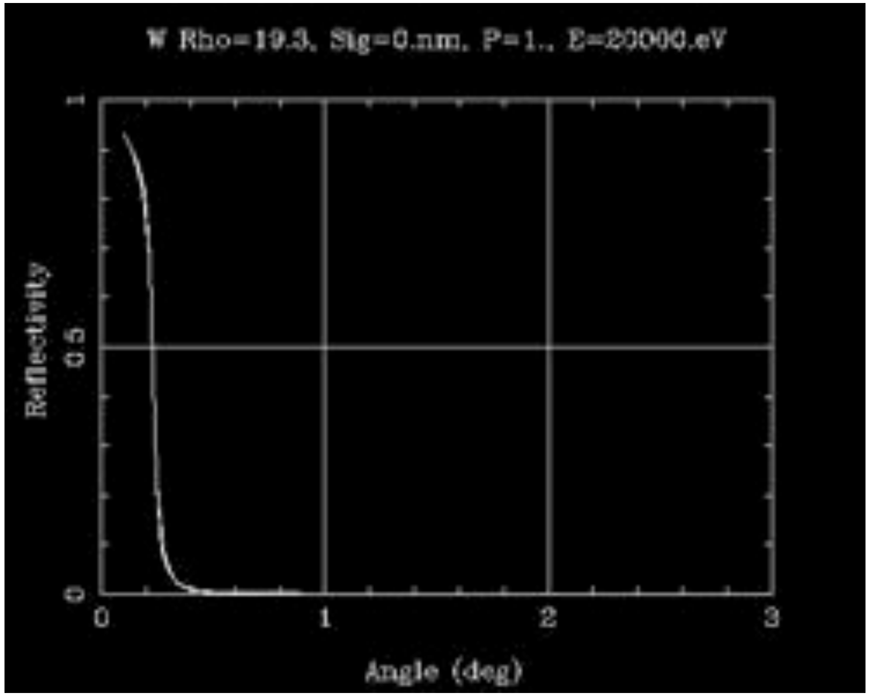
Grazing incidence mirror reflectivity

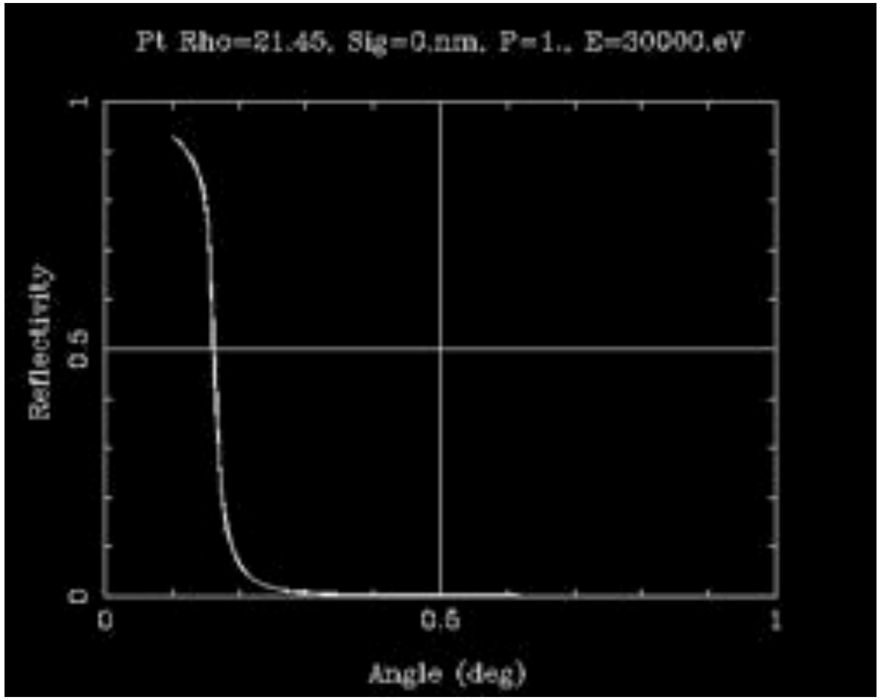




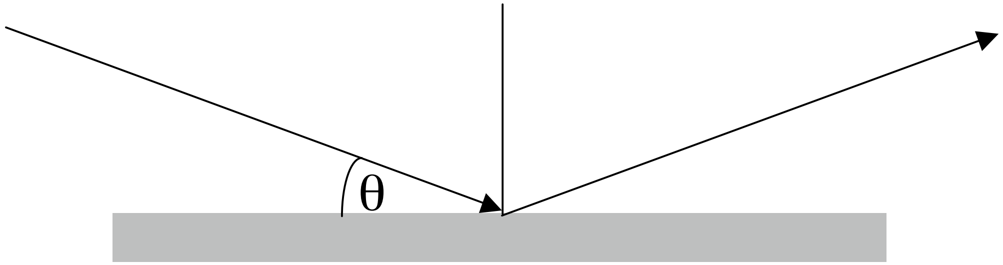
Hard X-ray reflectivity



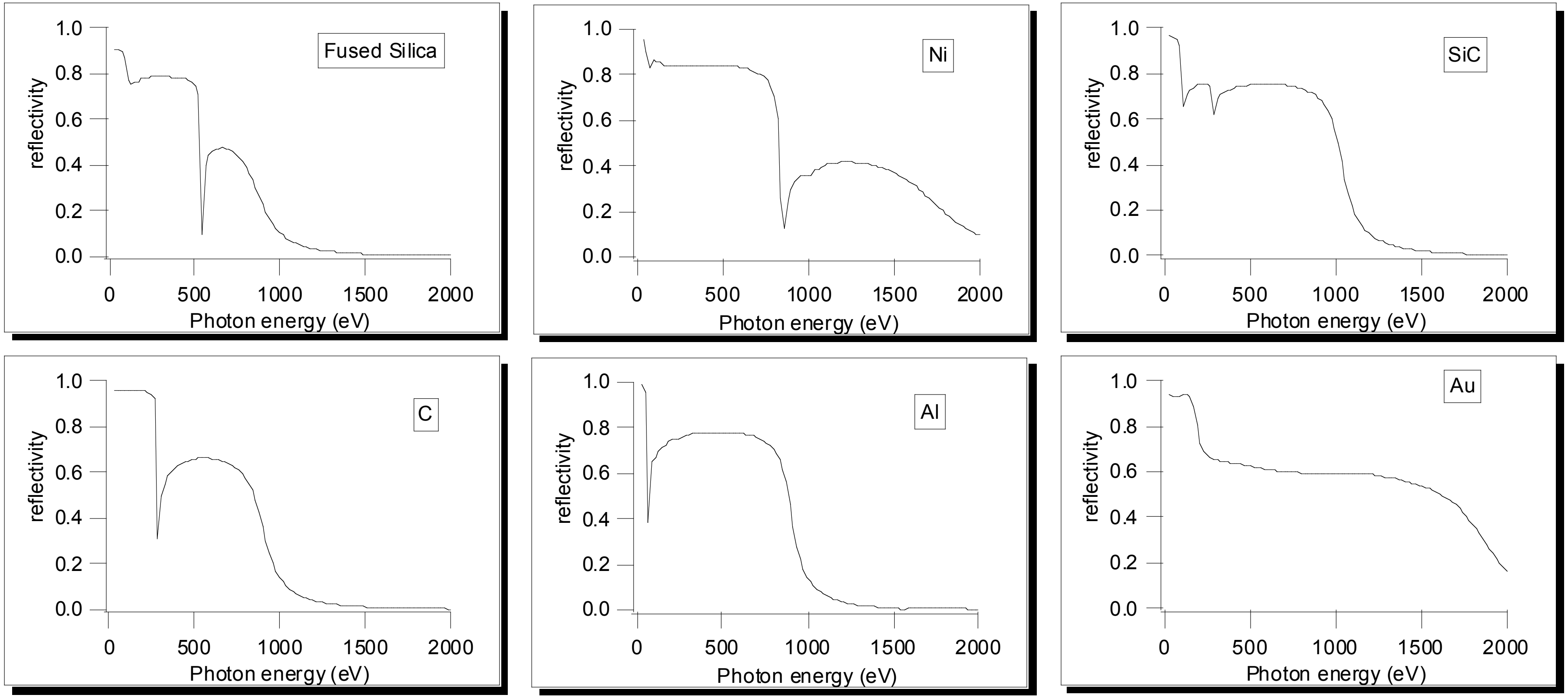




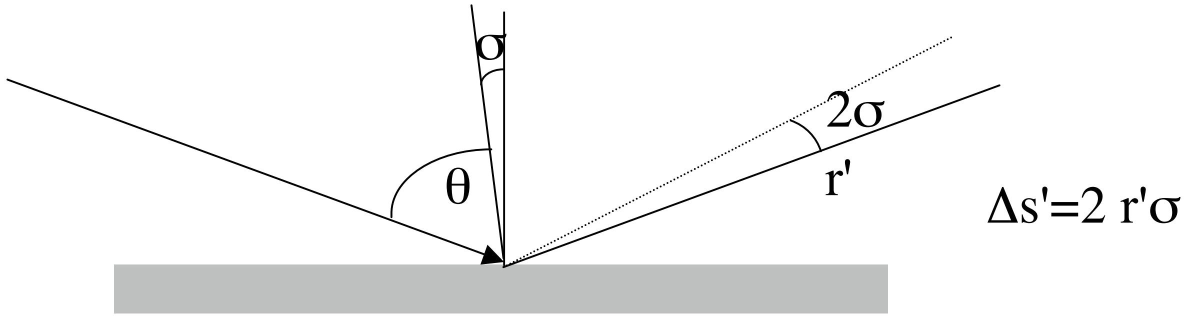
Other coatings

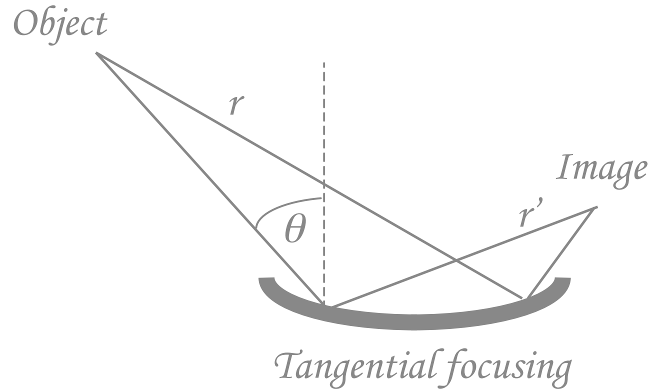


$\theta = 2^o$



Effect of Defects (slope errors)





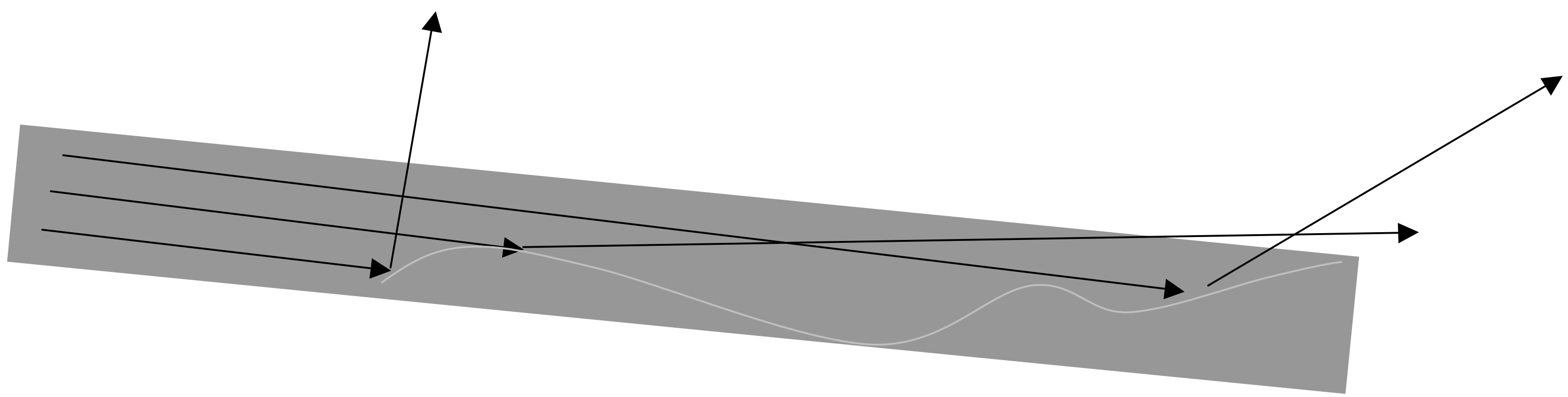
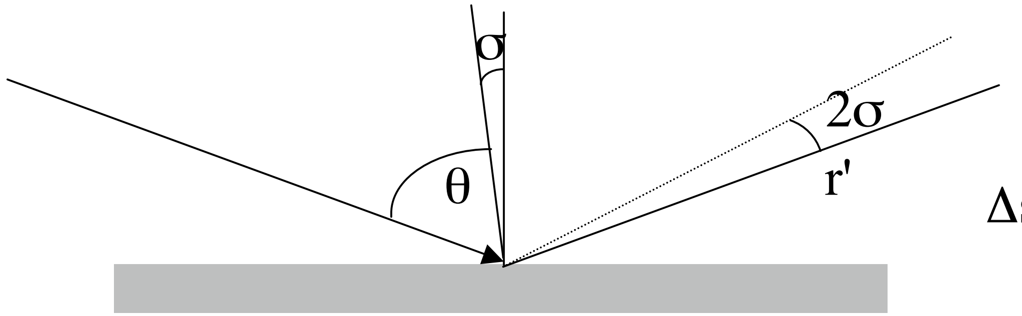
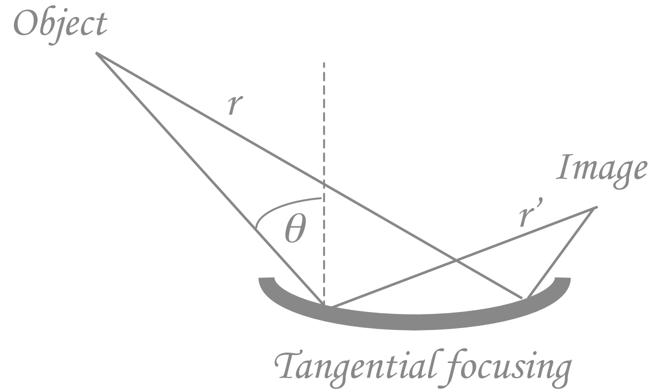
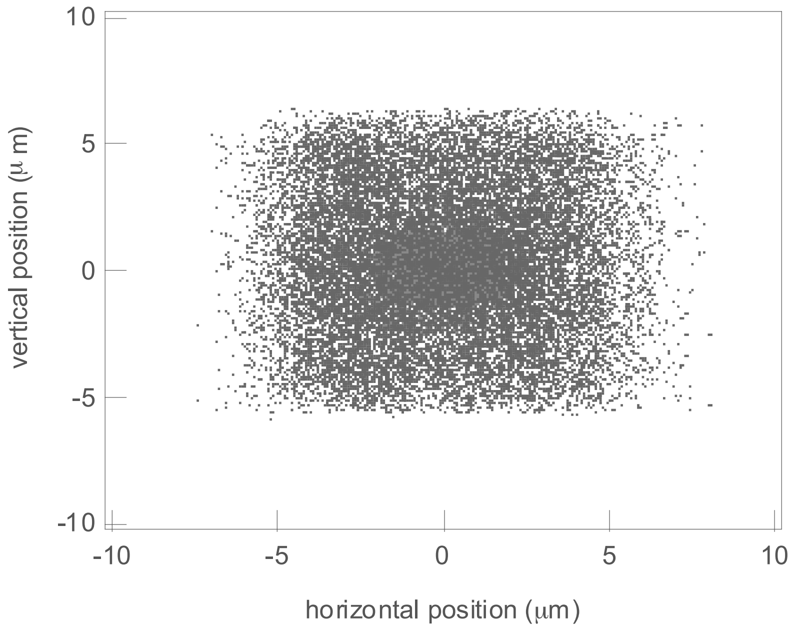


Image (spot) enlargement



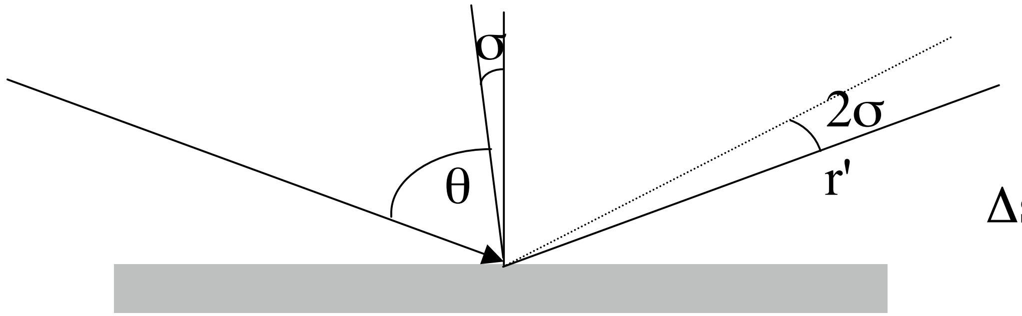
$\Delta s' = 2\, r'\sigma$



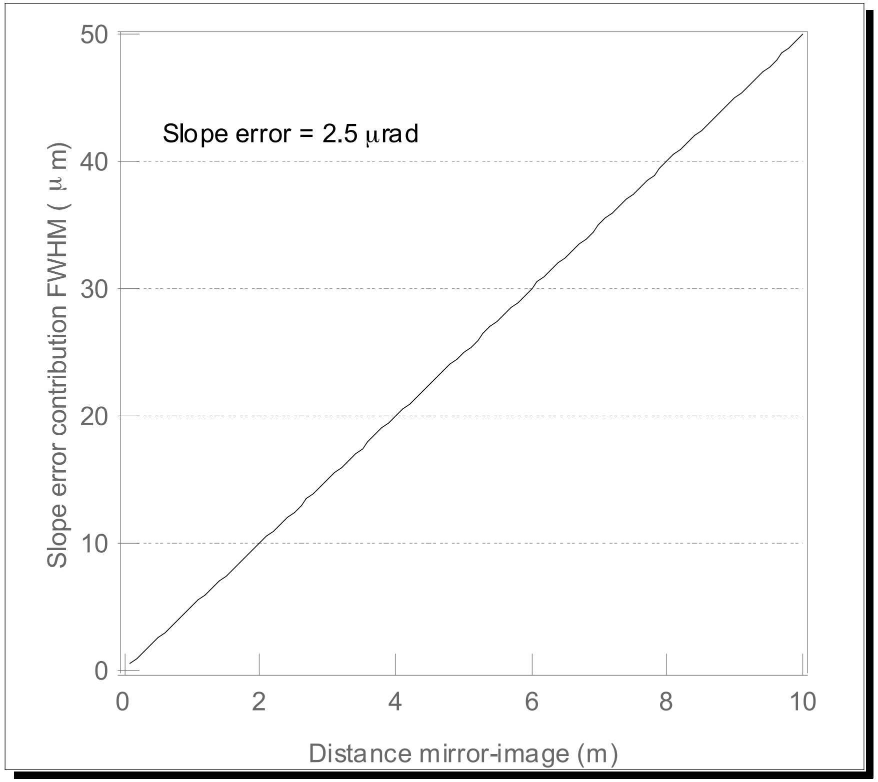


$$s' = \sqrt{(Ms)^2 + (2\, r'\sigma)^2}$$

Image (spot) enlargement



$\Delta s' = 2\, r' \sigma$



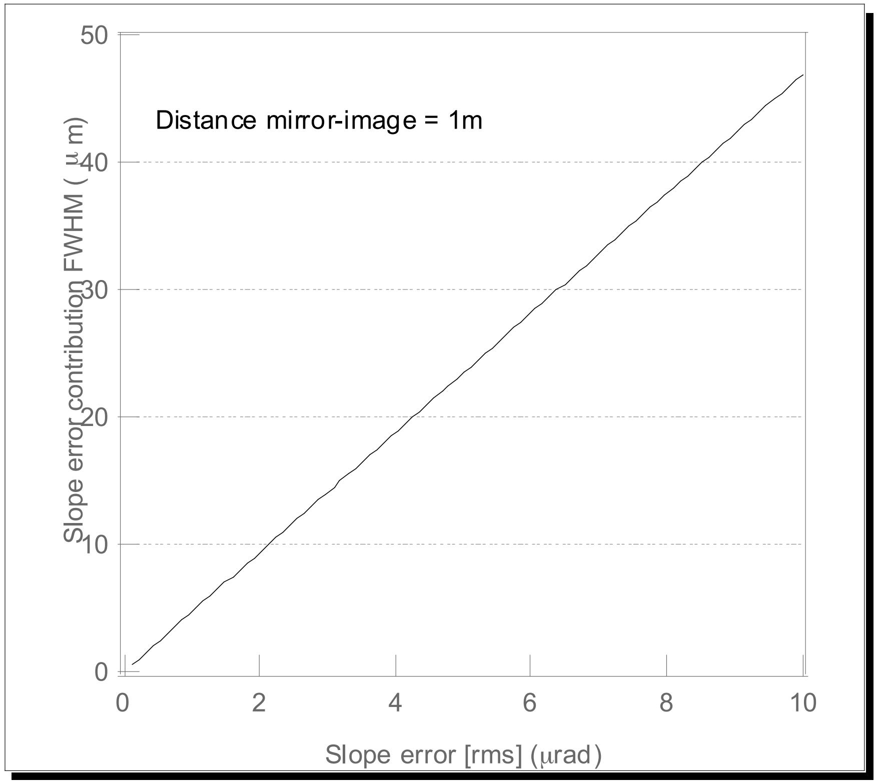
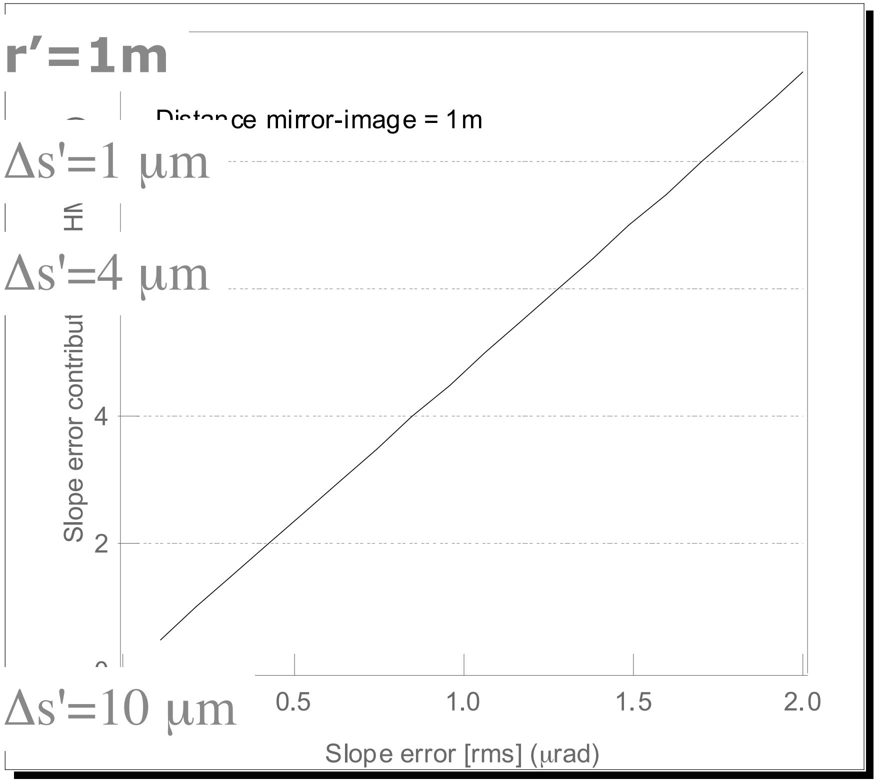


Image (spot) enlargement

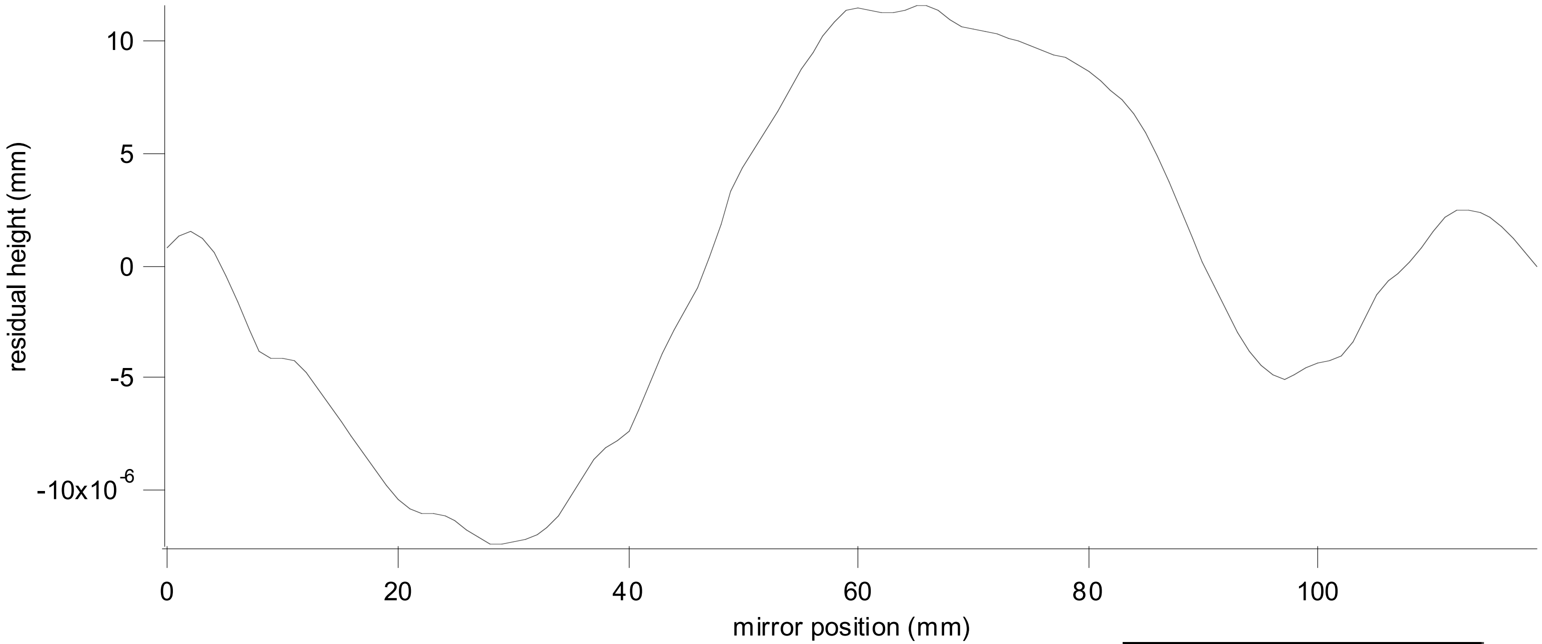
Typical manufacturer capabilities *(SESO, ZEISS, Winlight, Jobin Yvon)*

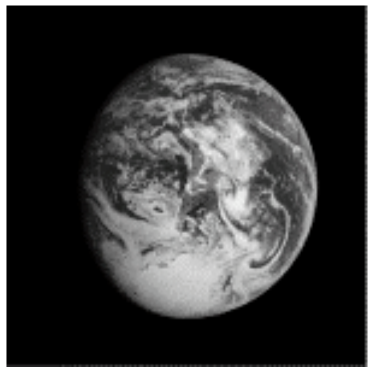
Shape	Length	rms errors
Spherical/flat	Up to 500 mm	< 0.5 μrad
Spherical/flat	> 500 mm	1 μrad
Toroidal	Up to 500 mm	≥ 1 μrad
Toroidal	> 500 mm	≥ 1-2 μrad
Aspherical	Up to 500 mm	≥ 1-2 μrad
Aspherical	> 500 mm	≥ 2 μrad



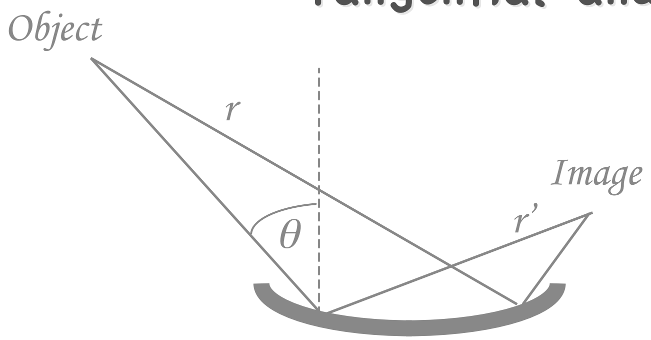
Mirror profile precision

Typical manufacturer capabilities *(SESO, ZEISS, Winlight, Jobin Yvon)*

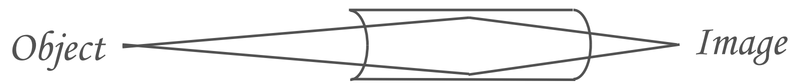




Tangential and Sagital focusing geometries



Tangential focusing



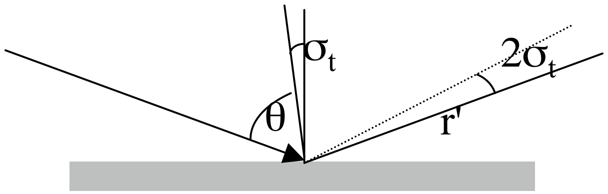
Sagital focusing

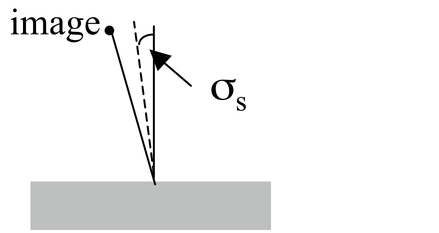
Term F_{20} of the optical path function

$(1/r+1/r')\cos\theta/2=1/R$ *spherical mirror*

Term F_{02} of the optical path function

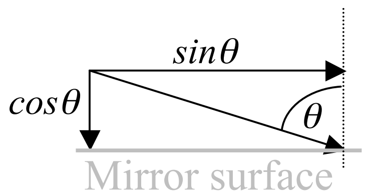
$(1/r+1/r')/(2\cos\theta)=1/R$ *cylindrical/toroidal mirror*



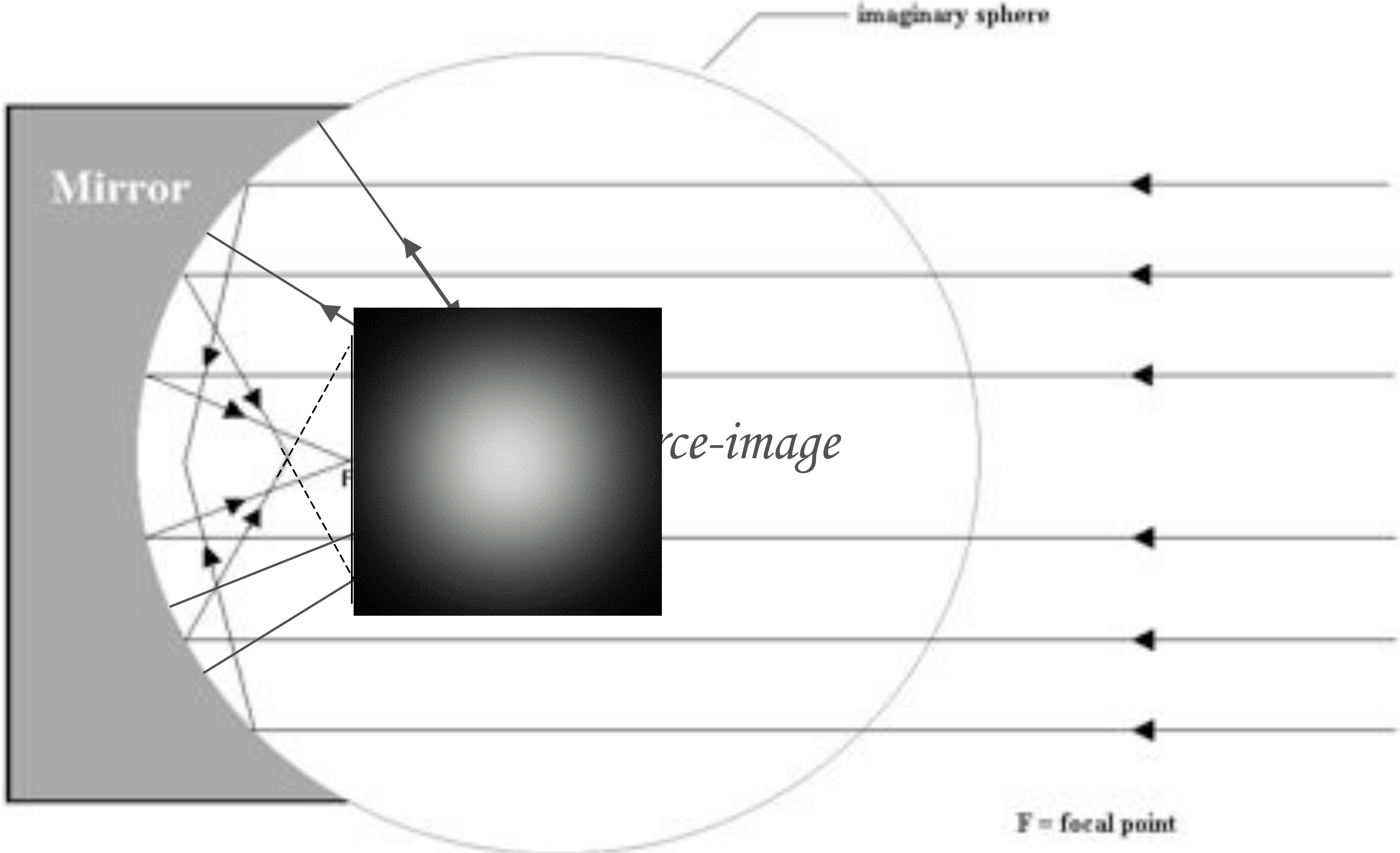


$$\Delta s'_t=2\,r'\sigma_t$$

$$\Delta s'_s=2\,r'\cos\theta\,\sigma_s$$



Solution: work in 1:1 configuration



Spherical mirror suffer of spherical aberration

Deviations from perfect imaging are called aberrations

Torodial mirror

The bicycle tyre toroid is generated by rotating a circle of radius ρ in an arc of radius R. In general, two non-coincident focii are produced: one in the meridional plane and one in the sagittal plane

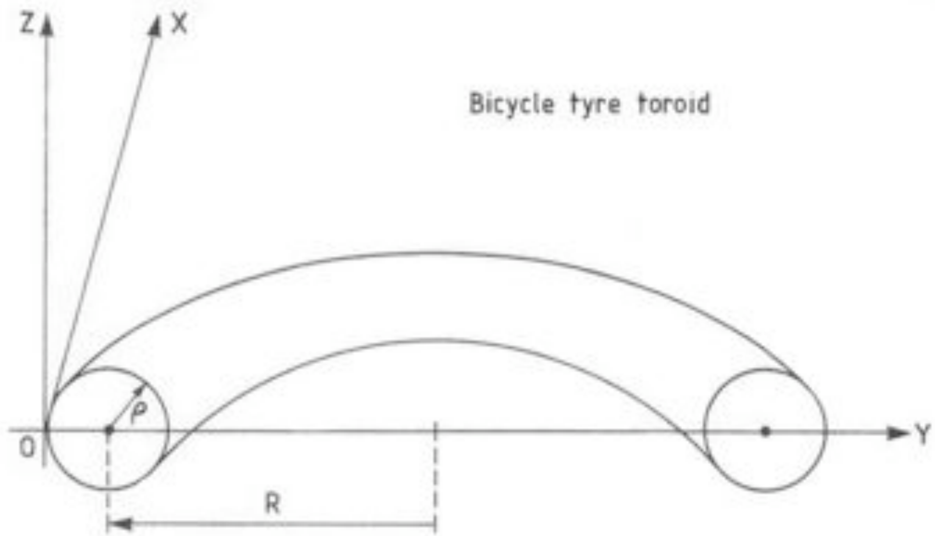
Tangential focus:

$$\left(\frac{1}{r}+\frac{1}{r'}\right)\frac{\cos\vartheta}{2}=\frac{1}{R}$$

Sagittal focus:

$$\left(\frac{1}{r}+\frac{1}{r'}\right)\frac{1}{2\cos\vartheta}=\frac{1}{\rho}$$

Stigmatic image: $$\frac{\rho}{R}=\cos^2\vartheta$$



J.B. West and H.A. Padmore, Optical Engineering, 1987

Toroidal mirror focal properties

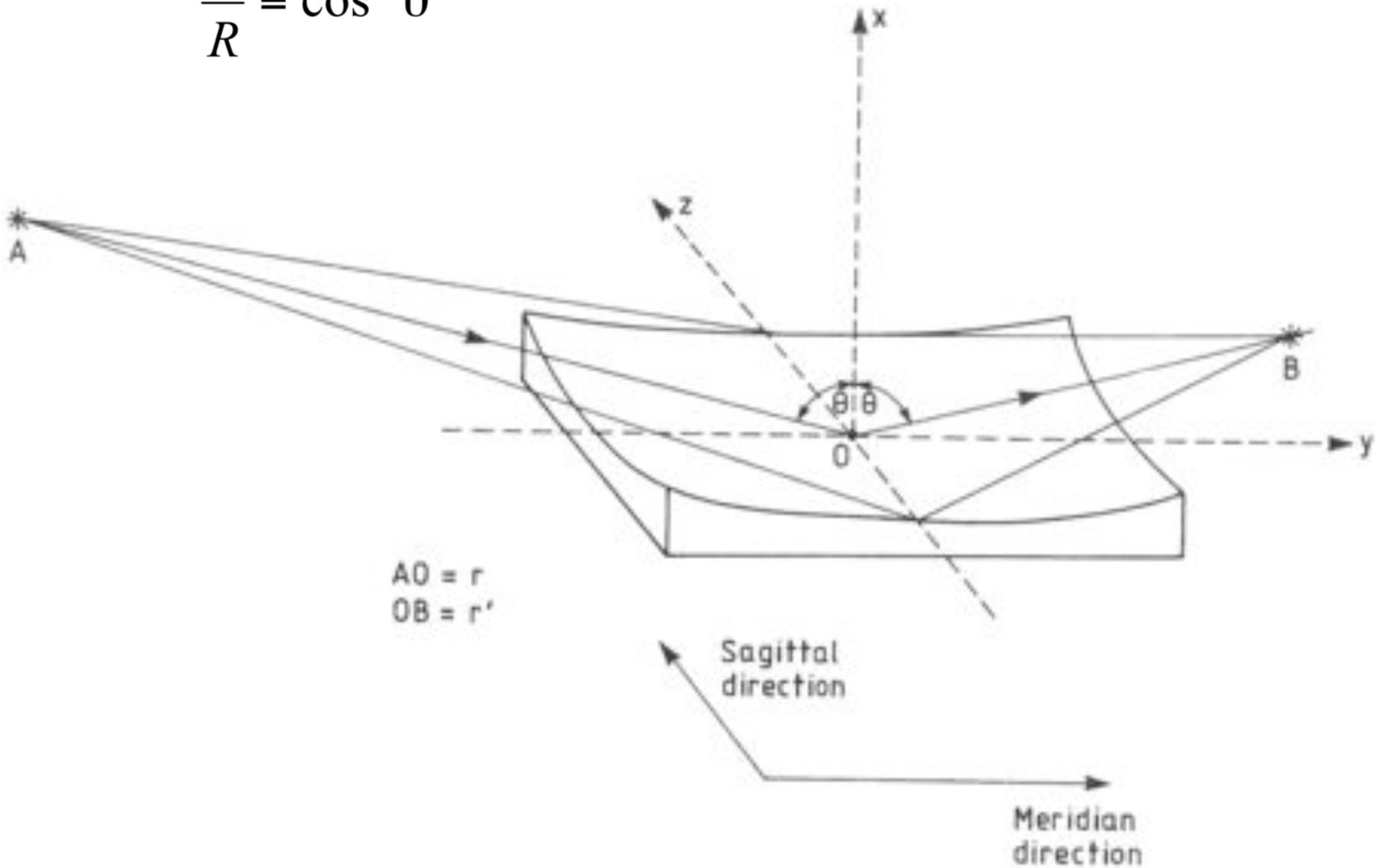
Tangential focus:

$$\left(\frac{1}{r}+\frac{1}{r'}\right)\frac{\cos\vartheta}{2}=\frac{1}{R}$$

Sagittal focus:

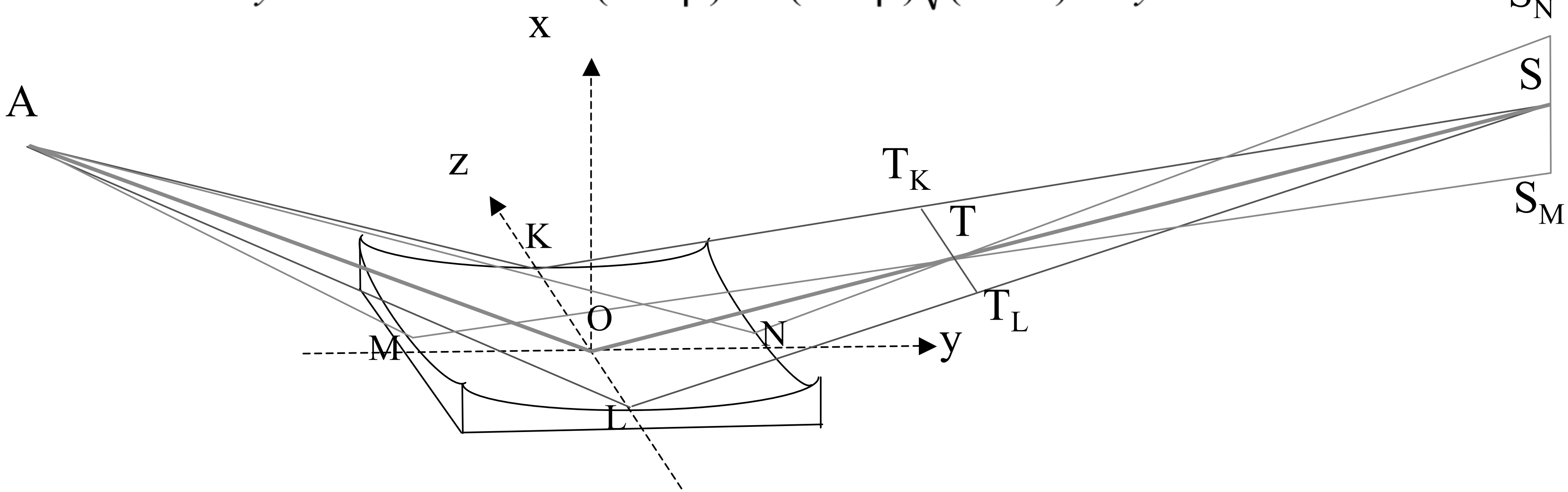
$$\left(\frac{1}{r}+\frac{1}{r'}\right)\frac{1}{2\cos\vartheta}=\frac{1}{\rho}$$

Stigmatic image: $$\frac{\rho}{R}=\cos^2\vartheta$$



Toroidal mirror focal properties

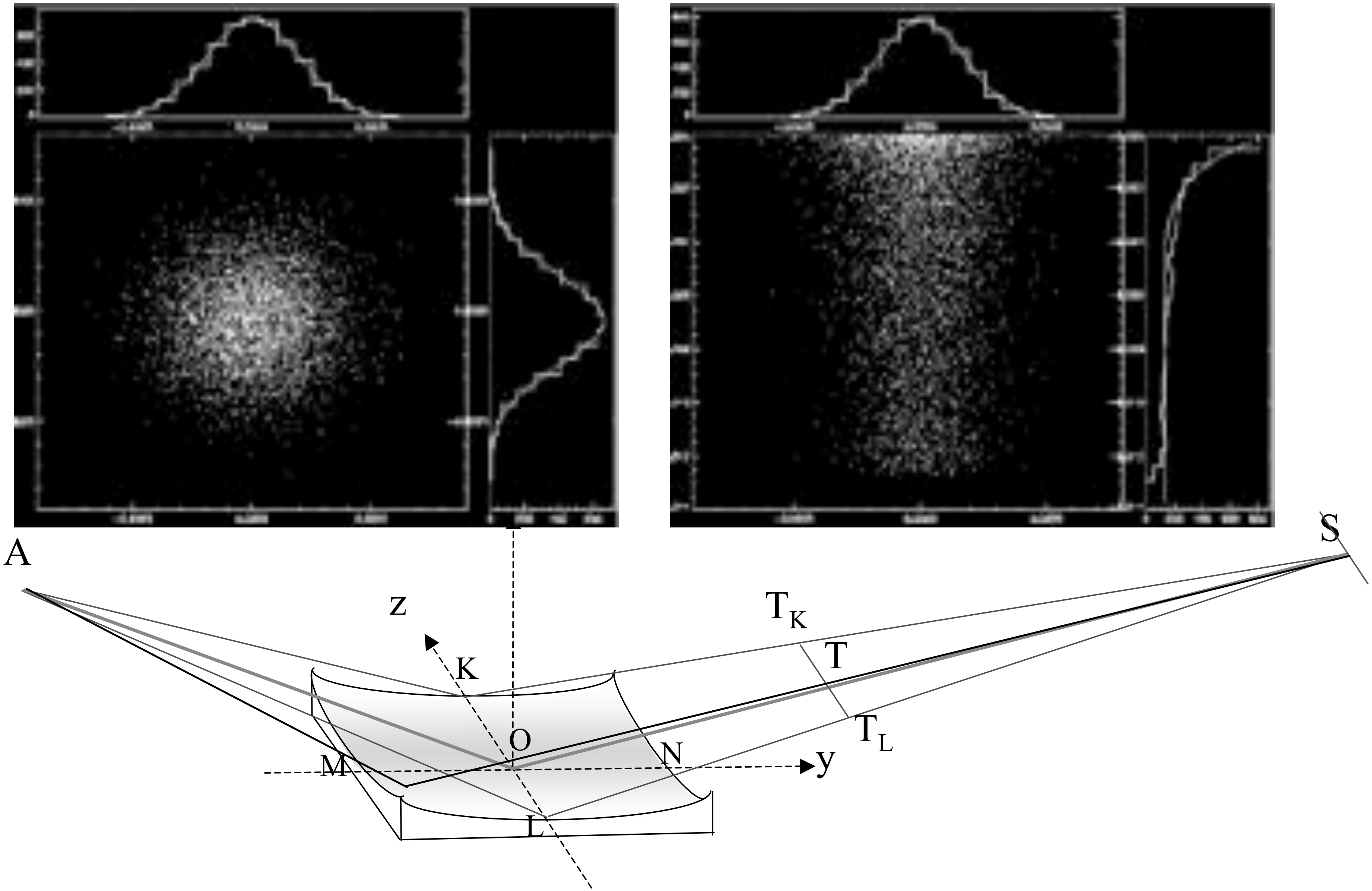
$$x^2 + y^2 + z^2 = 2Rx - 2R(R-\rho) + 2(R-\rho)\sqrt{(R-x)^2 + y^2}$$



For ρ=R → spherical mirror

A stigmatic image can only be obtained at normal incidence.
For a vertical deflecting spherical mirror at grazing incidence the horizontal sagittal focus is always further away from the mirror than the vertical tangential focus. The mirror only weakly focusses in the sagittal direction.

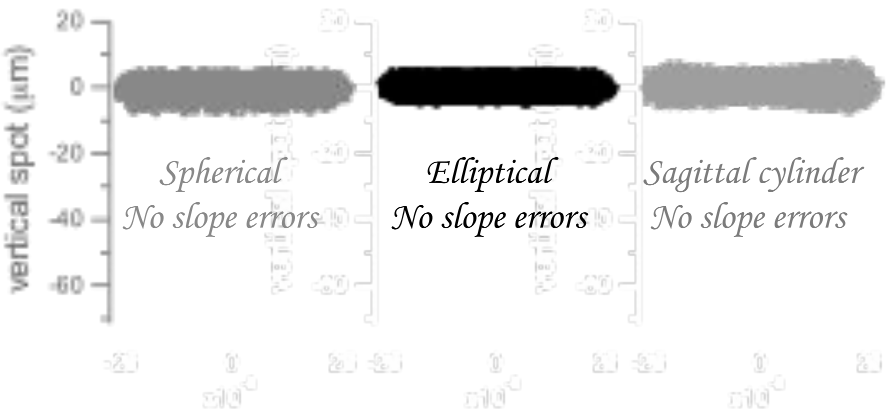
Toroidal mirror Tangential and Sagital focus

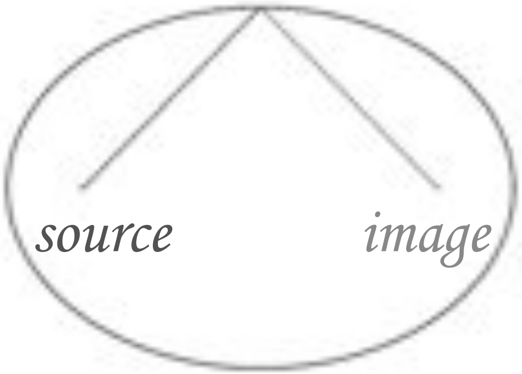


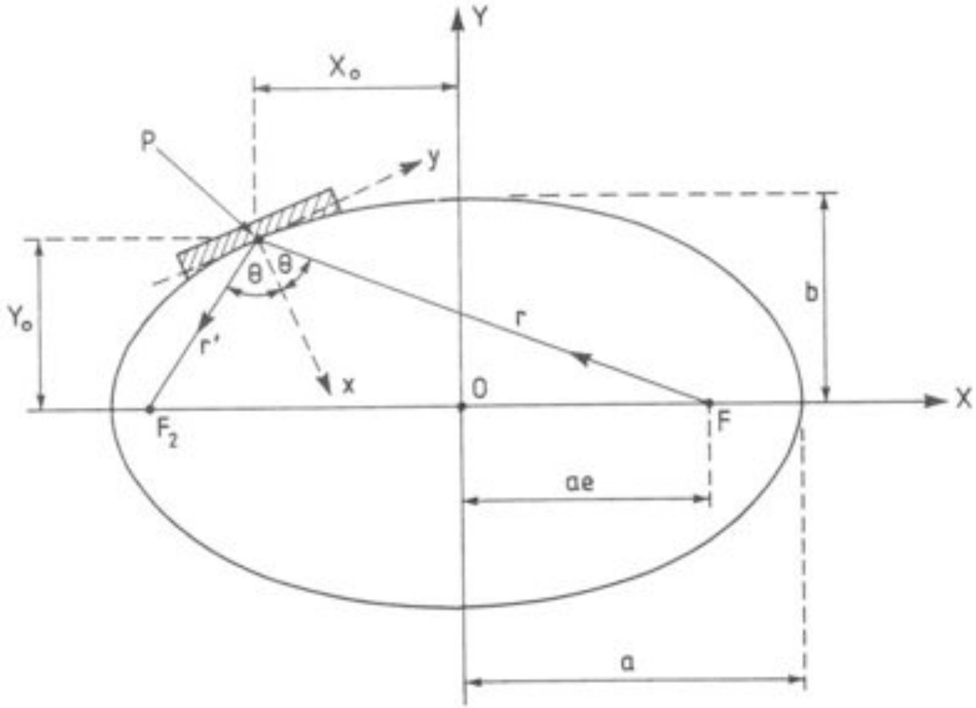
Other focusing geometries

source 80 μm vertical; r=4000 mm r´=400 mm (10:1) θ=88º

Beam divergence 100X100 μrad

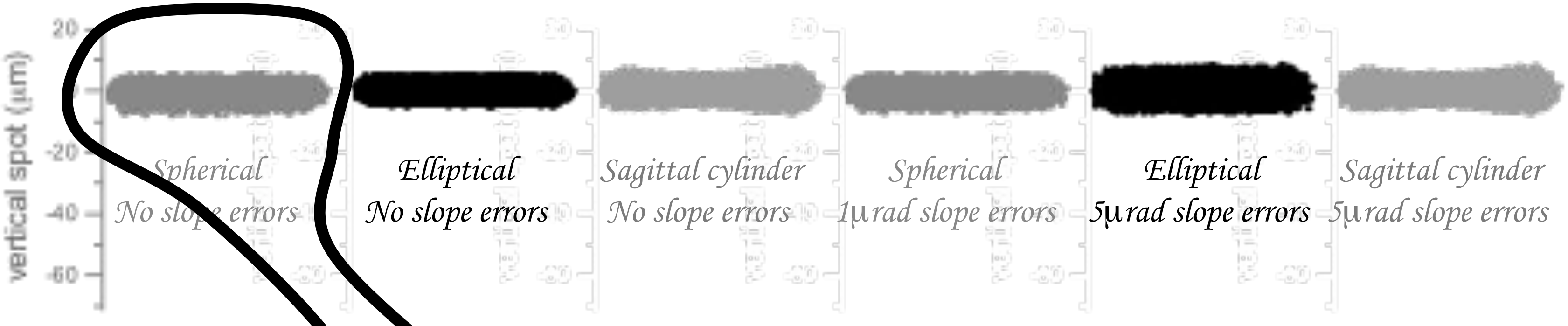




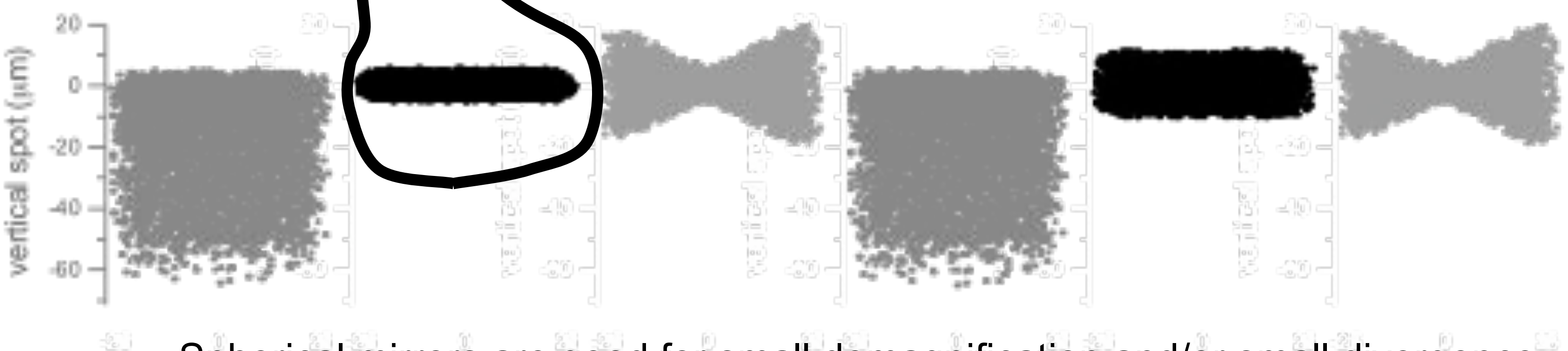


In search of the perfect focusing geometry

Beam divergence 100X100 μrad



Beam divergence 500X500 μrad



Spherical mirrors are good for small demagnification and/or small divergence
Elliptical mirrors are better for very large demagnifications and larger divergence but..
the slope errors have to be small
Toroidal / parabolic mirrors are perfect if the induced aberration are acceptable

Paraboloids

Rays traveling parallel to the symmetry axis OX are all focused to a point A.
Conversely, the parabola collimates rays emanating from the focus A.

Line equation: $Y^2 = 4aX$

Paraboloid equation: $Y^2 + Z^2 = 4aX$

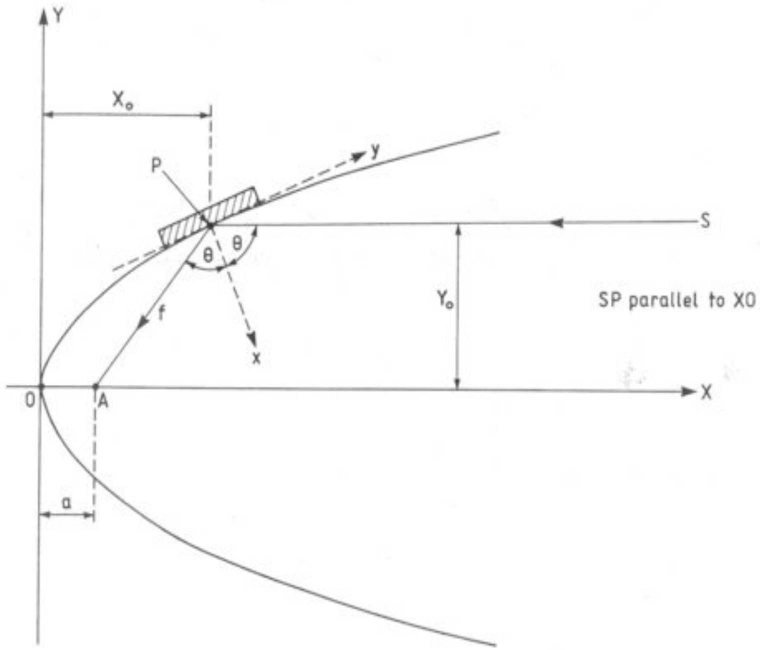
where: $a = f\cos^2\vartheta$

Position of the pole P:

$$X_o = a\tan^2\vartheta$$
$$Y_o = 2a\tan\vartheta$$

Paraboloid equation:

$$x^2\sin^2\vartheta + y^2\cos^2\vartheta + z^2 - 2xy\sin\vartheta\cos\vartheta - 4ax\sec\vartheta = 0$$



J.B. West and H.A. Padmore, Optical Engineering, 1987

Other mirror defect - Roughness

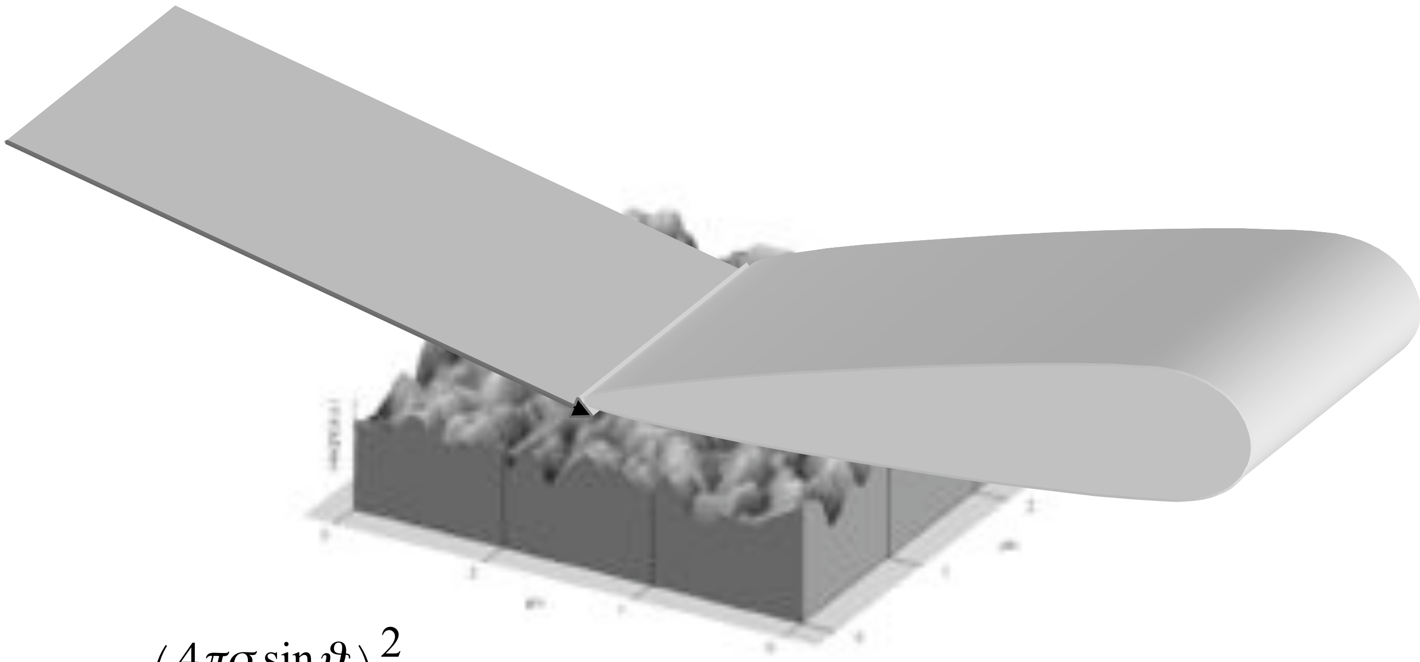
Slope errors = every deviation from the ideal surface with period larger then ~ 1,2 mm

Typical definition is μrad or arcsec rms.
Alternative definition is λ/10 or λ/20 and so on… P-V or rms
used for normal incidence mirror or "poorer" quality mirrors

Roughness = every deviation from the ideal surface with period smaller then ~ 0.5-1 mm

Typical definition is Å rms.
Alternative definition is surface quality 20-10 or 10-5 (scratch-dig)
used for normal incidence mirror or "poorer" quality mirrors
A dig is nearly equal in terms of its length and width. A scratch could be much longer then width
20-10 means 20/1000 of mm max scratch width 10/100 mm max dig dimension

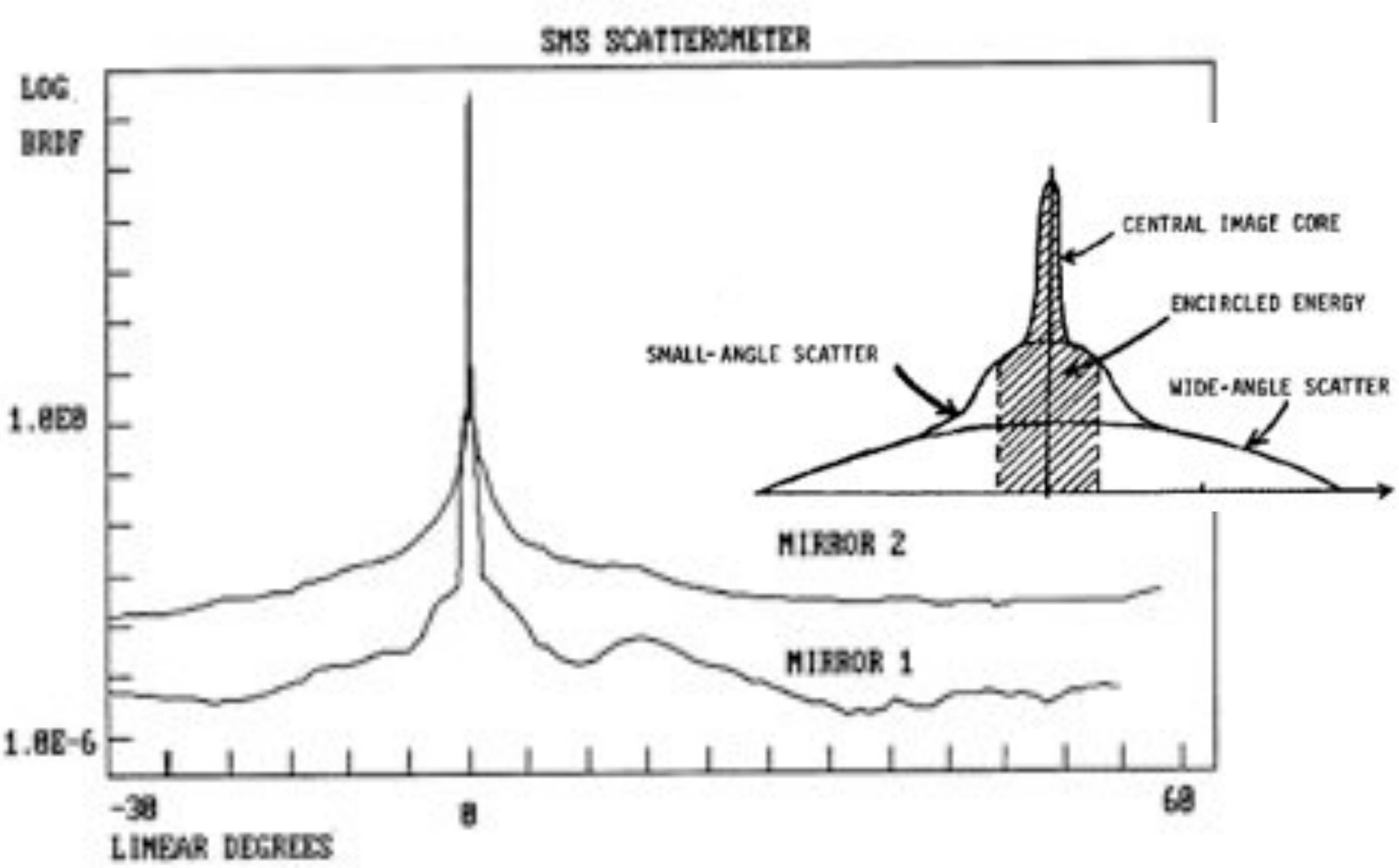
Roughness



$$I = I_0 e^{-\left(\frac{4\pi\sigma\sin\vartheta}{\lambda}\right)^2}$$

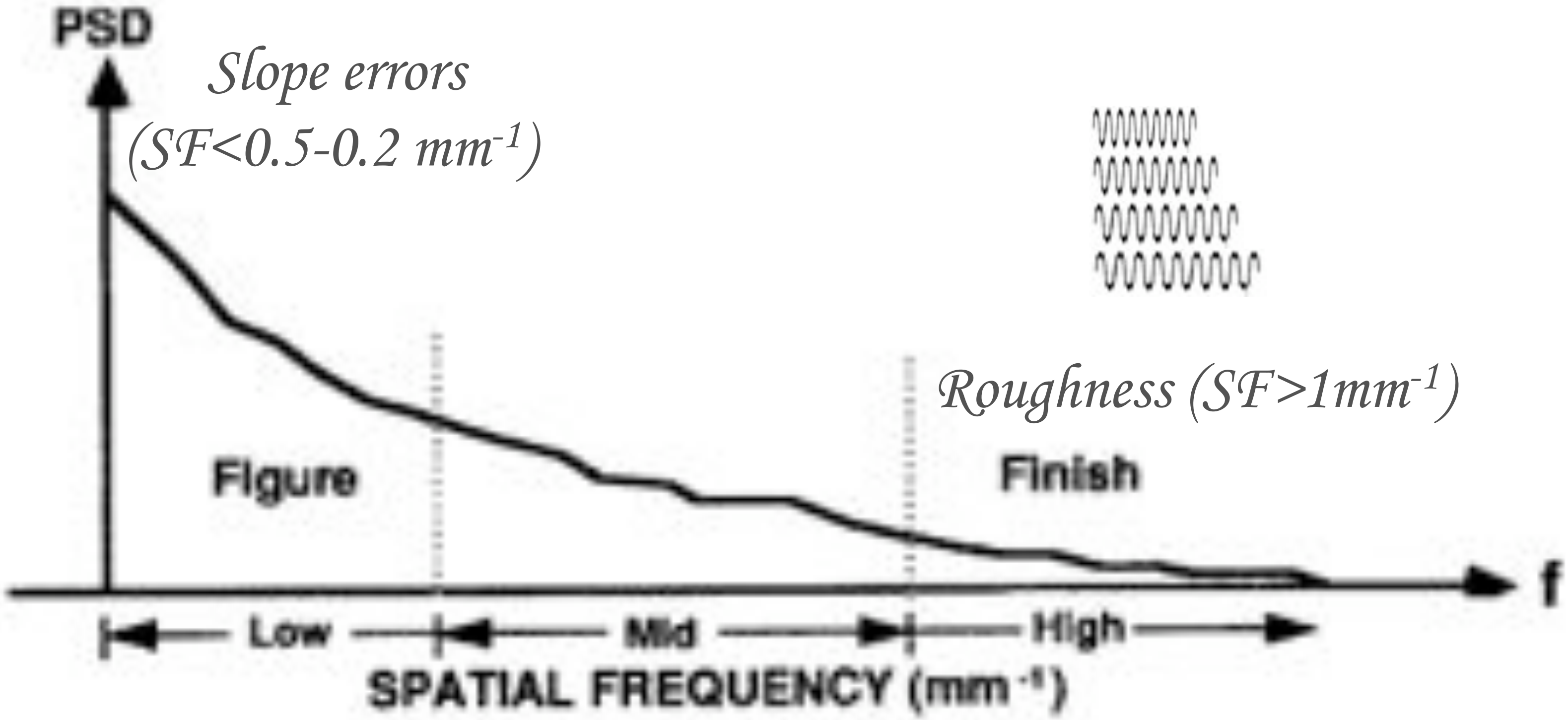
$$\sigma = \sqrt{\frac{1}{n}\sum_{x=0}^{n}\left[s(x) - \overline{s(x)}\right]^2}$$

Roughness

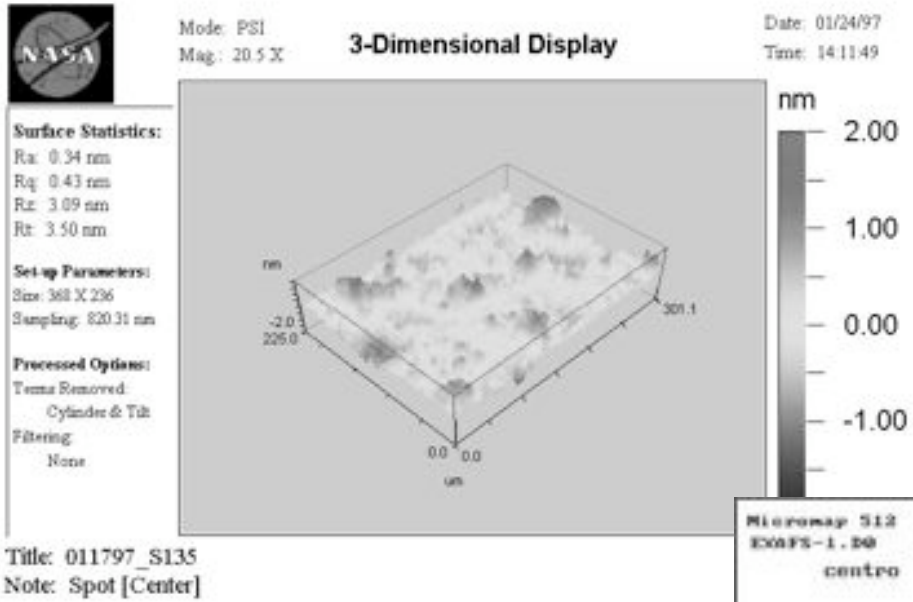


SMS SCATTEROMETER
LOG
BRDF
1.0E0
1.0E-6
-30
0
60
LINEAR DEGREES
MIRROR 2
MIRROR 1
CENTRAL IMAGE CORE
ENCIRCLED ENERGY
SMALL-ANGLE SCATTER
WIDE-ANGLE SCATTER

Power spectral density



Roughness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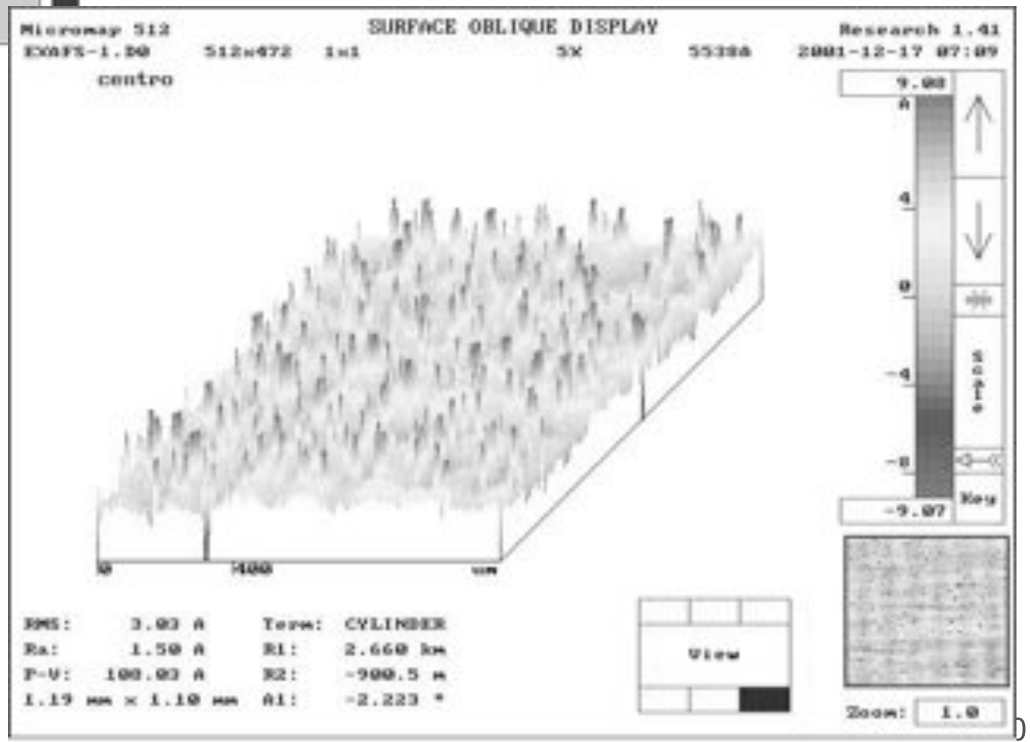
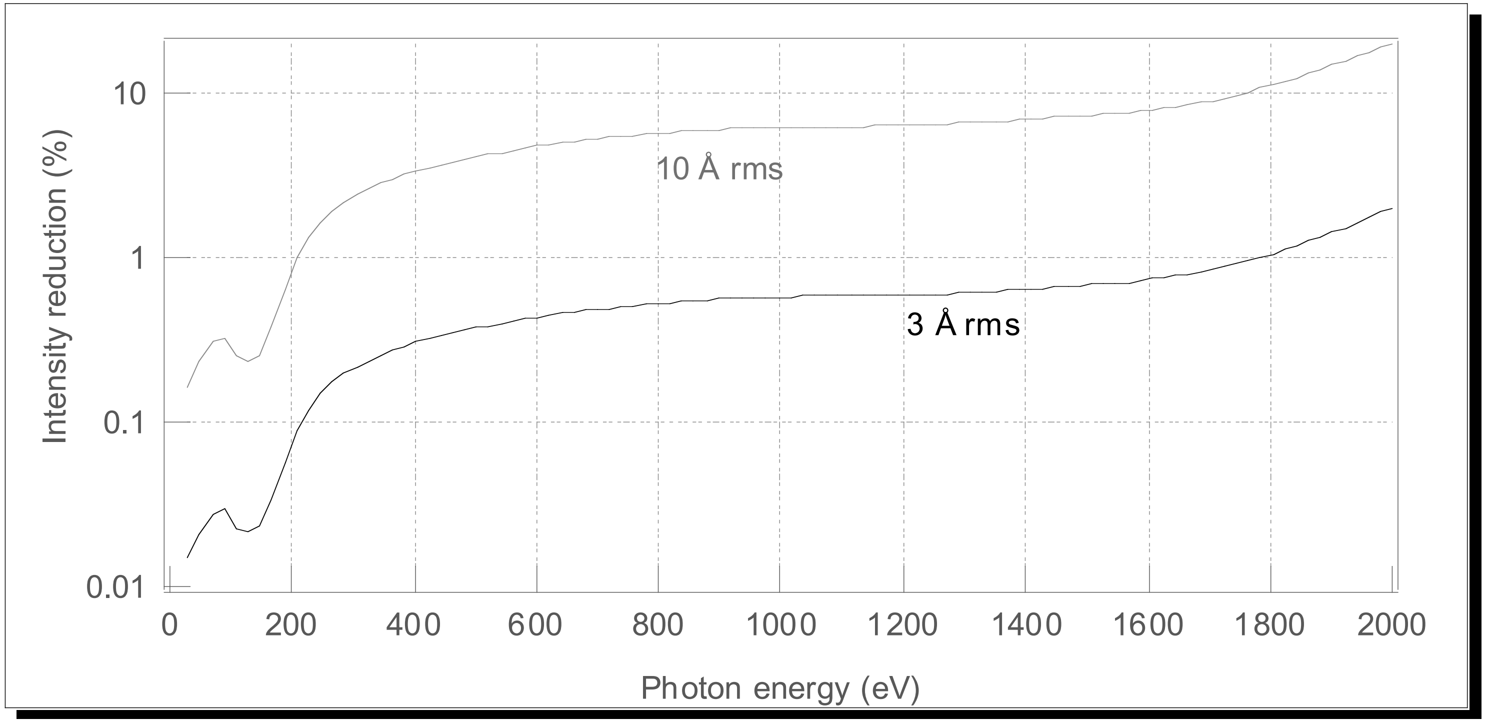
Mag.: 20.5 X
3-Dimensional Display
Date: 01/24/97
Time: 14:11:49
Surface Statistics:
Ra: 0.34 nm
Rq: 0.43 nm
Rz: 3.09 nm
Rt: 3.50 nm
Set-up Parameters:
Size: 368 X 236
Sampling: 820.31 nm
Processed Options:
Terms Removed:
Cylinder & Tilt
Filtering:
None
nm
2.00
1.00
0.00
-1.00
Title: 011797_S135
Note: Spot [Center]


SURFACE OBLIQUE DISPLAY
Micromap 512
Research 1.41
centro
9.08
-9.07
Key
View
RMS: 3.03 A
Ra: 1.50 A
P-V: 100.03 A
Term: CYLINDER
Zoom: 1.0

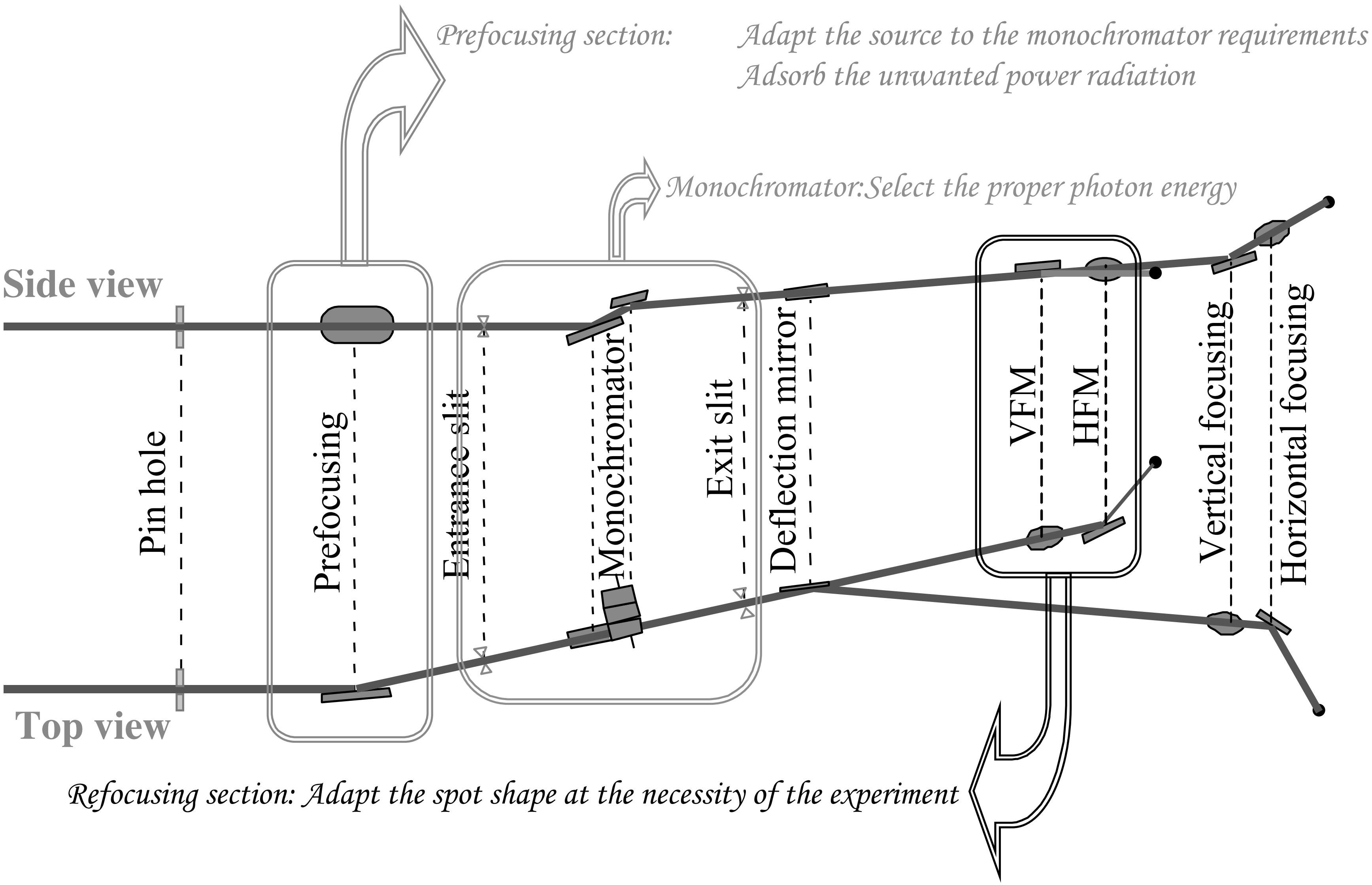
Flux reduction



Shape	Spherical/Flat	Toroidal/aspherical
Roughness (Å)	3 standard 1 best	5 standard 3 best (1-2 if very lucky)

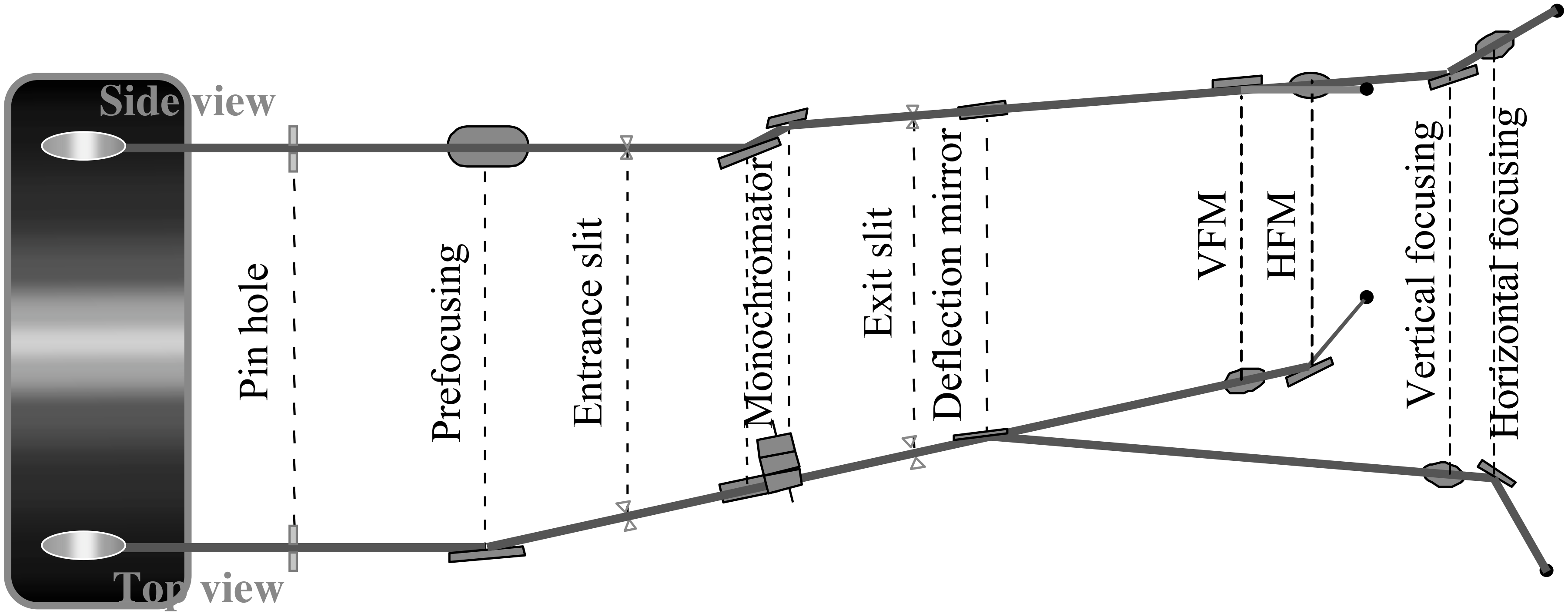
$$I = I_0 e^{-\left(\frac{4\pi\sigma\sin\vartheta}{\lambda}\right)^2}$$

Synchrotron Radiation Beamlines

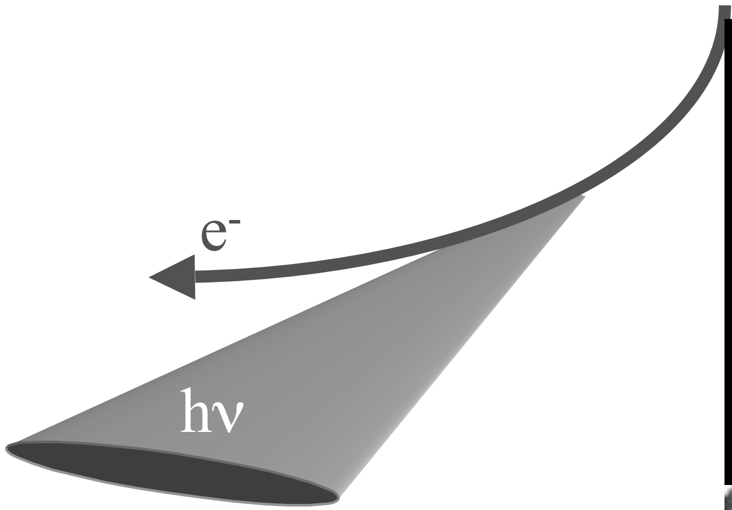


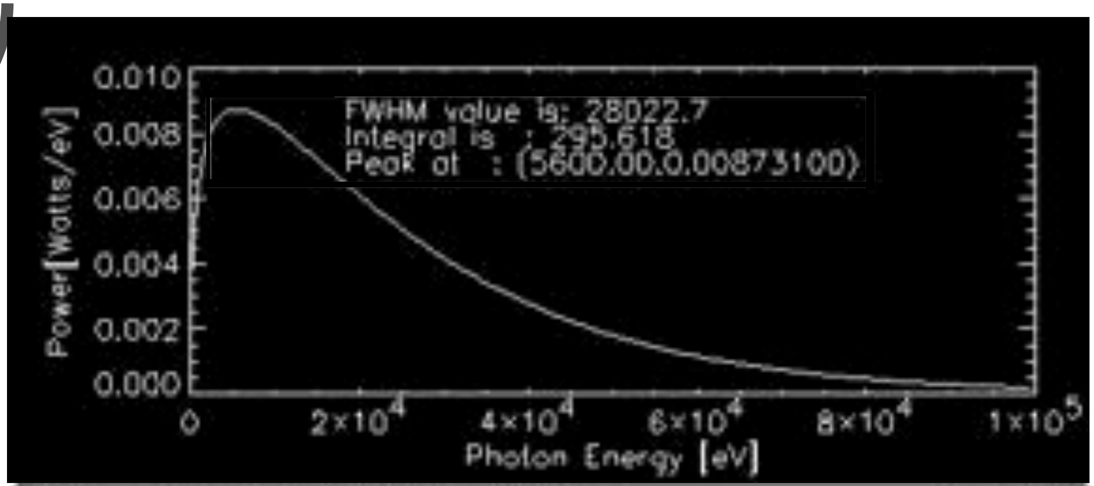
Synchrotron Radiation Beamlines

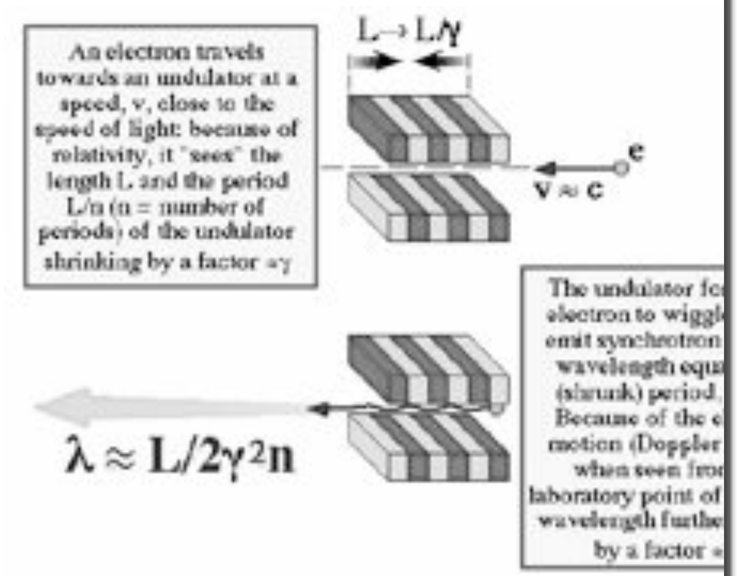
There are several reasons to choose a mirror substrate, one is the power arriving on it

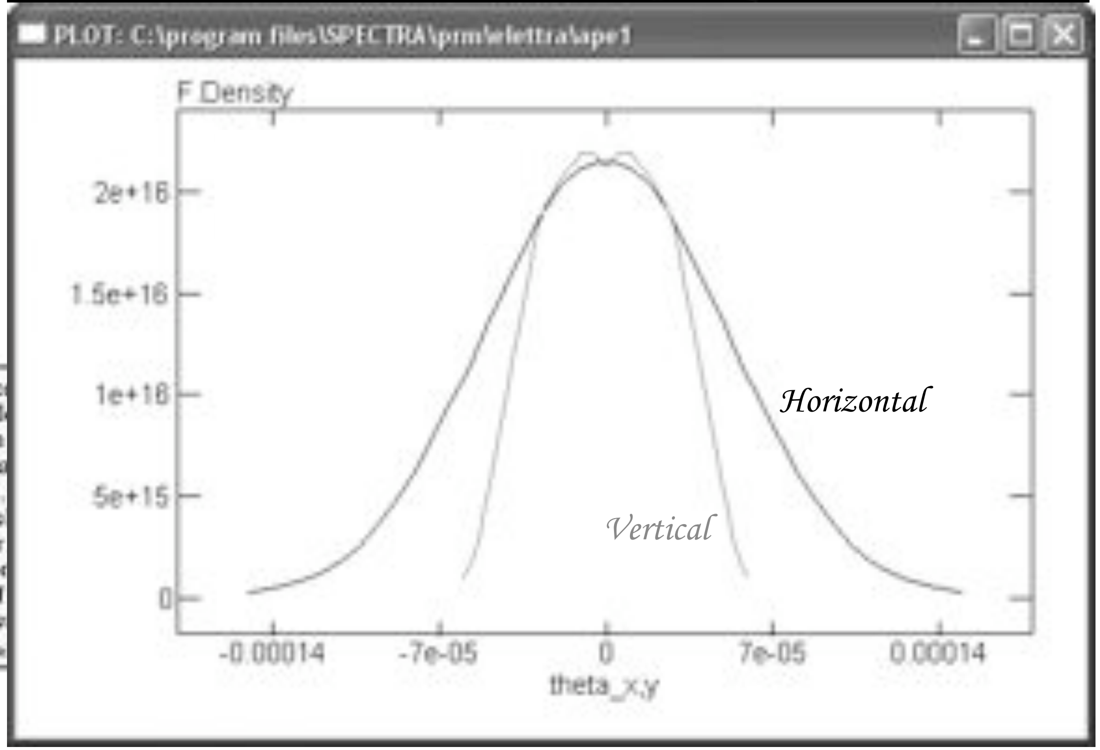


SR sources

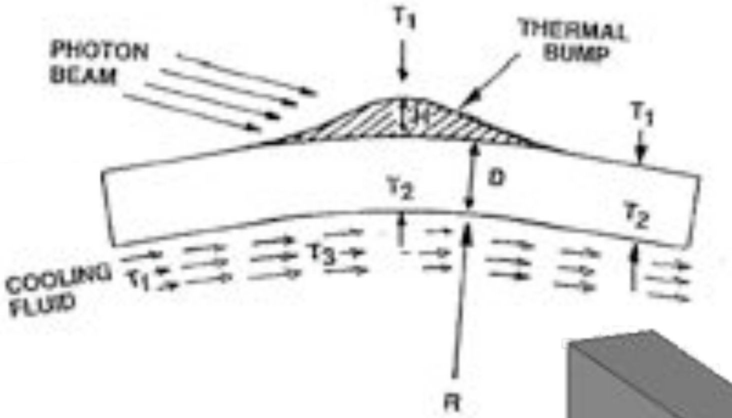

e-
hν

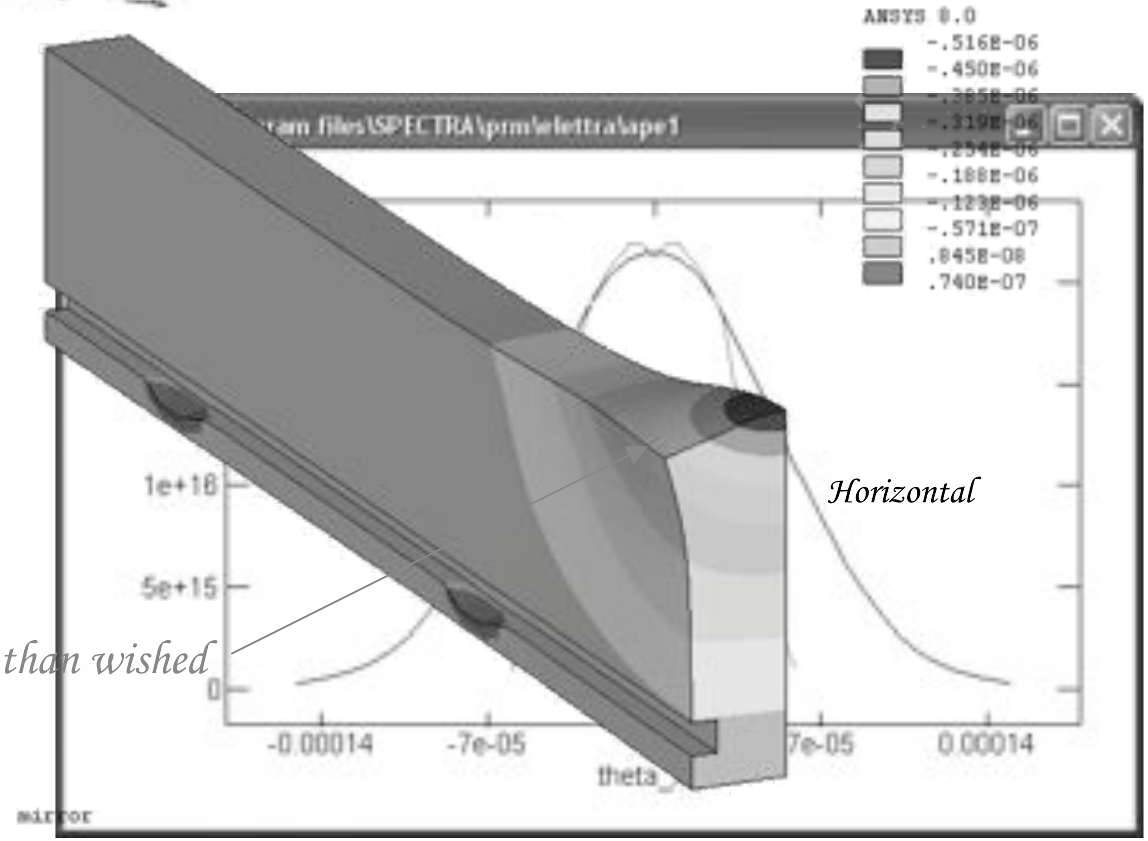

FWHM value is: 28022.7
Integral is : 295.618
Peak at : (5600.00,0.00873100)
Power [Watts/eV]
0.010
0.008
0.006
0.004
0.002
0.000
0
2×10^4
4×10^4
6×10^4
8×10^4
1×10^5
Photon Energy [eV]


An electron travels towards an undulator at a speed, v, close to the speed of light: because of relativity, it "sees" the length L and the period L/n (n = number of periods) of the undulator shrinking by a factor ≈γ
L → L/γ
e
v ≈ c
λ ≈ L/2γ²n


PLOT: C:\program files\SPECTRA\prm\elettra\ape1
F.Density
2e+16
1.5e+16
1e+16
5e+15
0
-0.00014
-7e-05
0
7e-05
0.00014
theta_x,y
Horizontal
Vertical

Thermal deformations

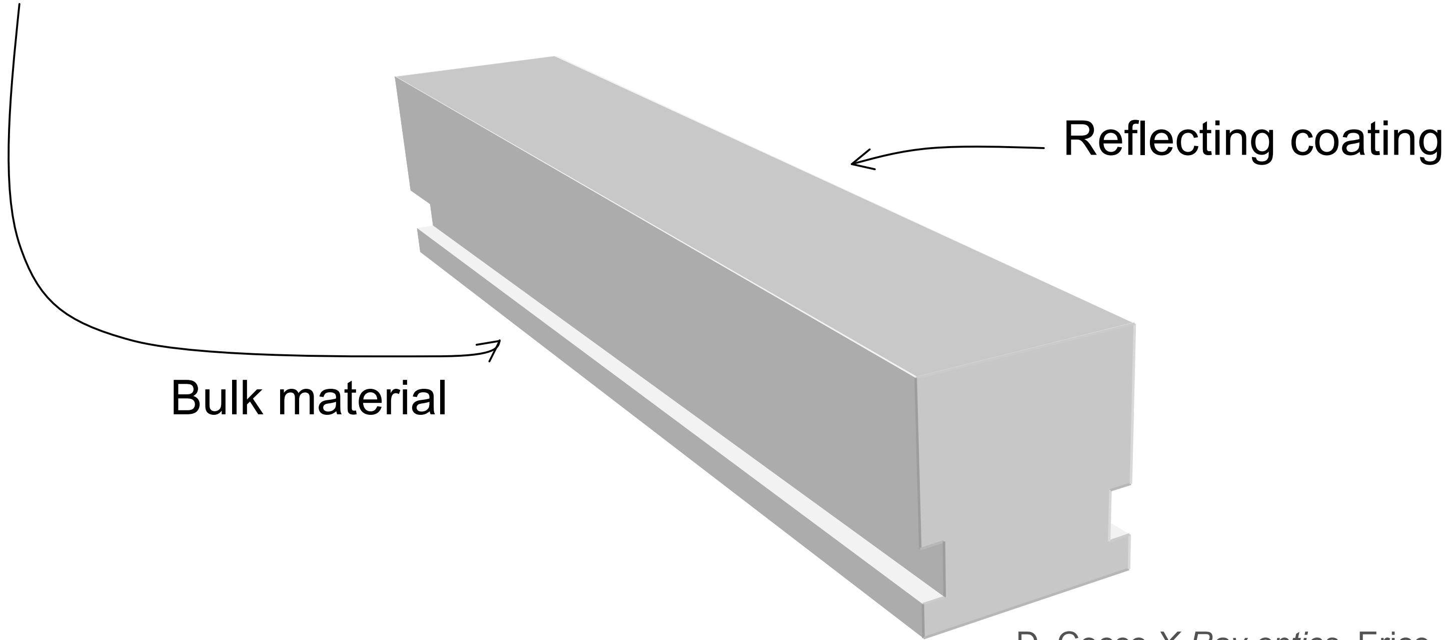




400W → 1μm,; 10 to 100 time larger than wished

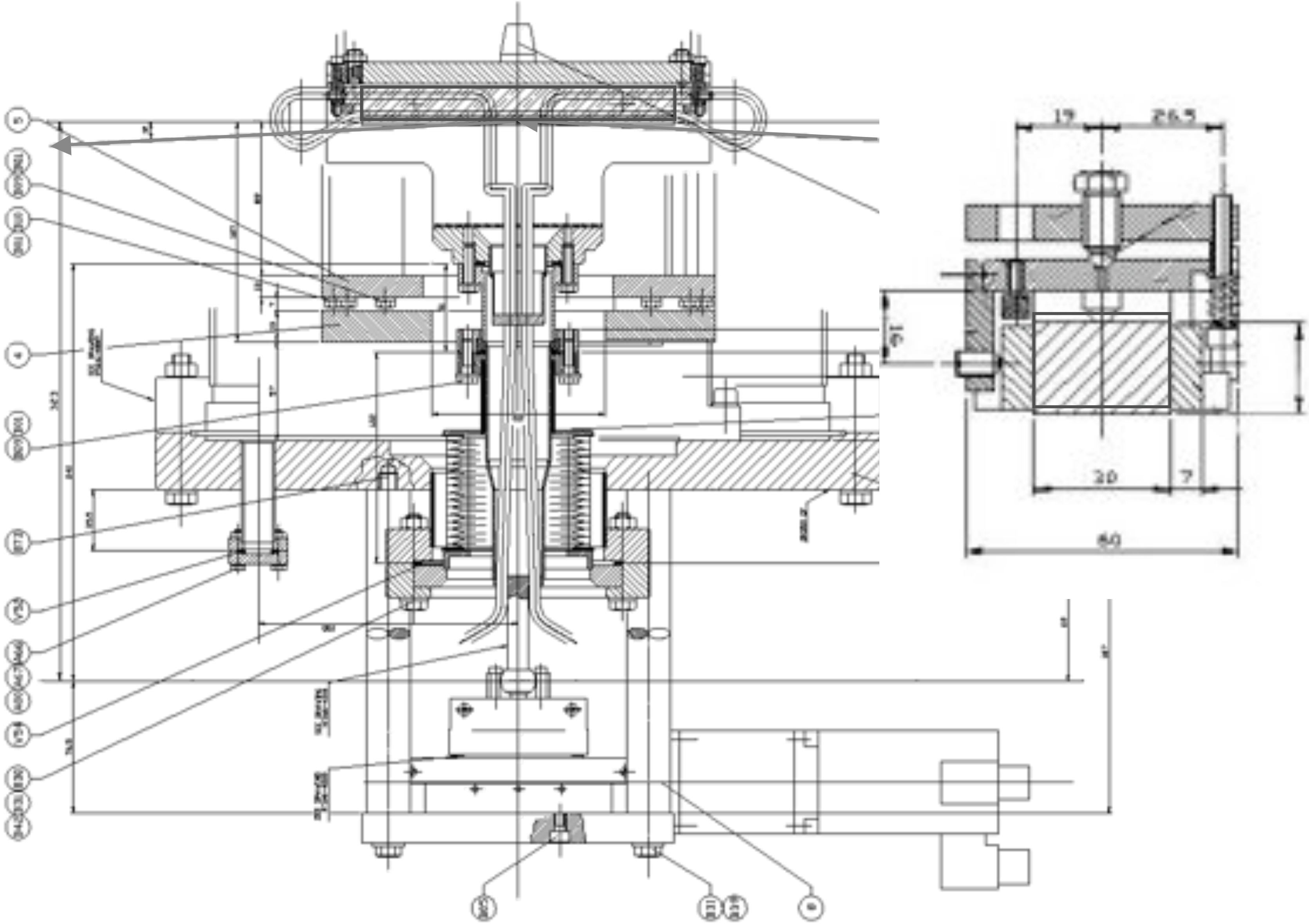
Properties of typical mirror materials

	Density gm/cc	Young's modulus GPa	Thermal expansion (α) ppm/^{o}C	Thermal conductivity (k) W/m/^{o}C	Figure of merit k/α
Fused silica	2.19	73	0.50	1.4	2.8
Zerodur	2.53	92	0.05	1.60	32
Silicon	2.33	131	2.60	156	60
SiC CVD	3.21	461	2.40	198	82
Aluminum	2.70	68	22.5	167	7.42
Copper	8.94	117	16.5	391	23.7
Glidcop	8.84	130	16.6	365	22
Molybdenum	10.22	324.8	4.80	142	29.6

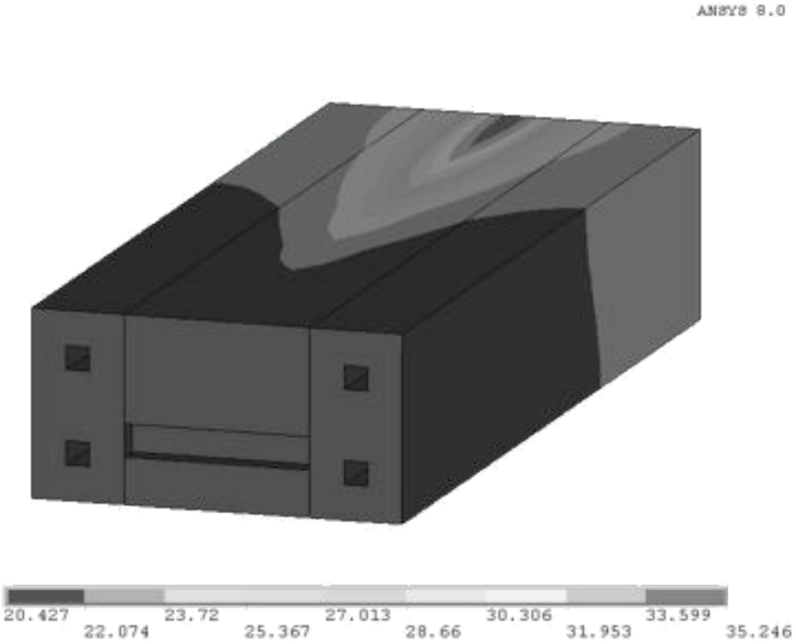


Silicon bulk mirrors

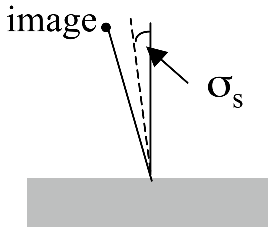
Direct side cooling



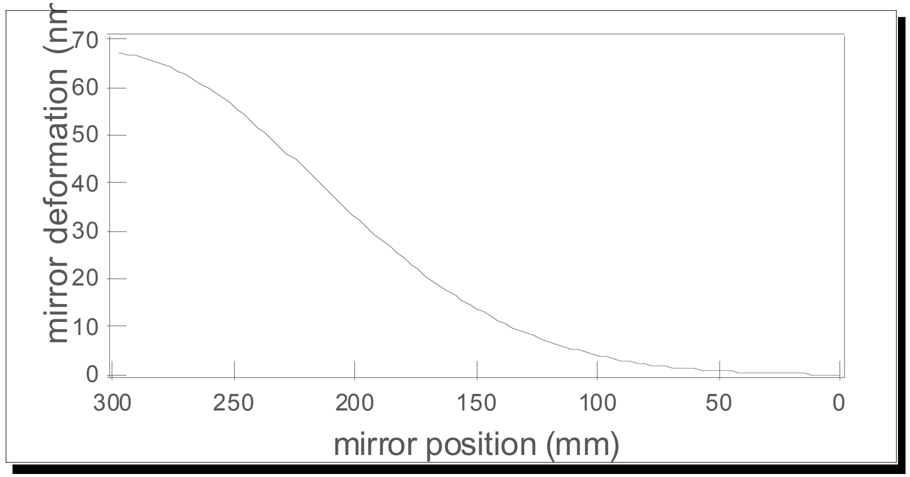
Direct side cooling



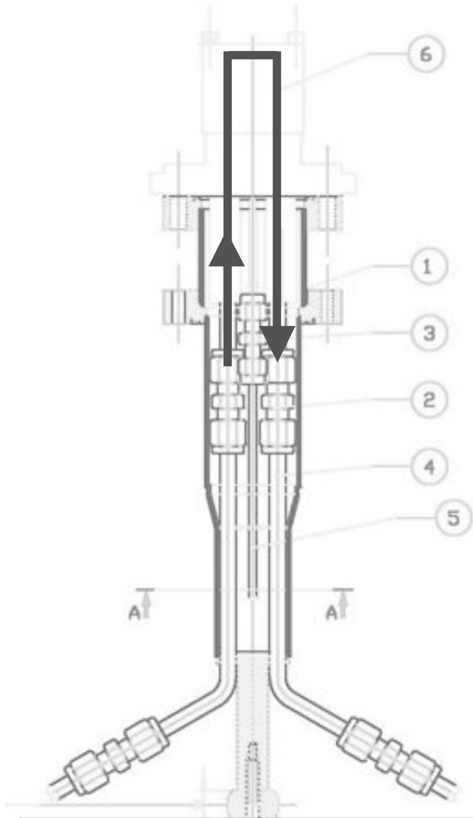
1^{st} *mirror sagittally oriented*

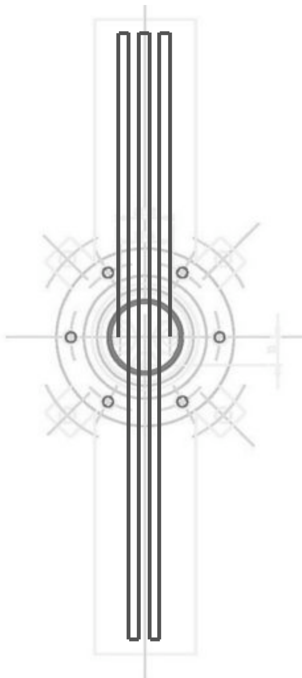


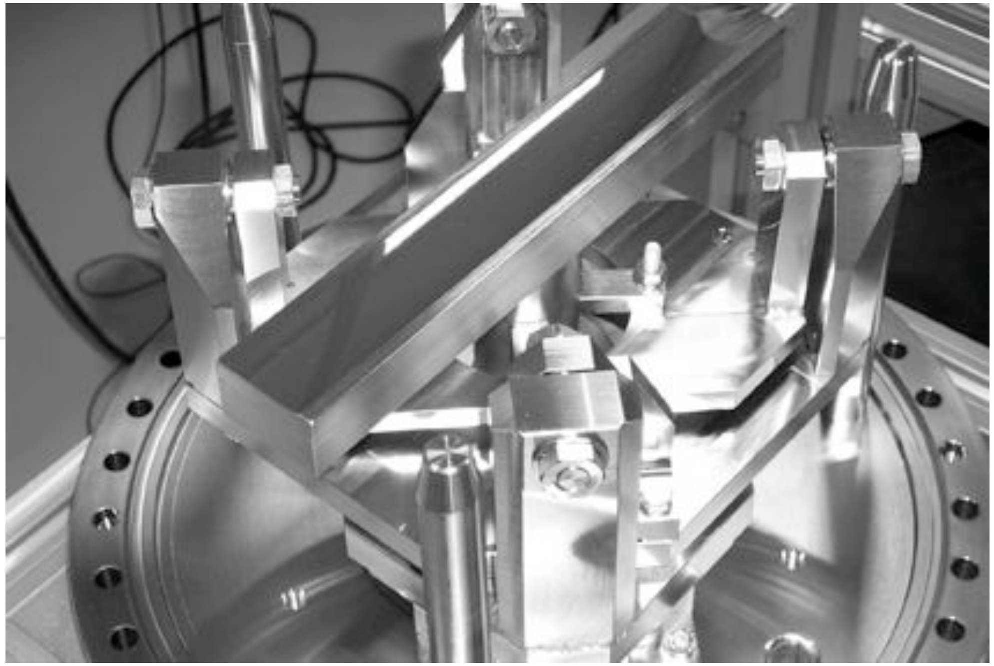
$$\Delta s'_s = 2\, r' \cos\theta\, \sigma_s$$



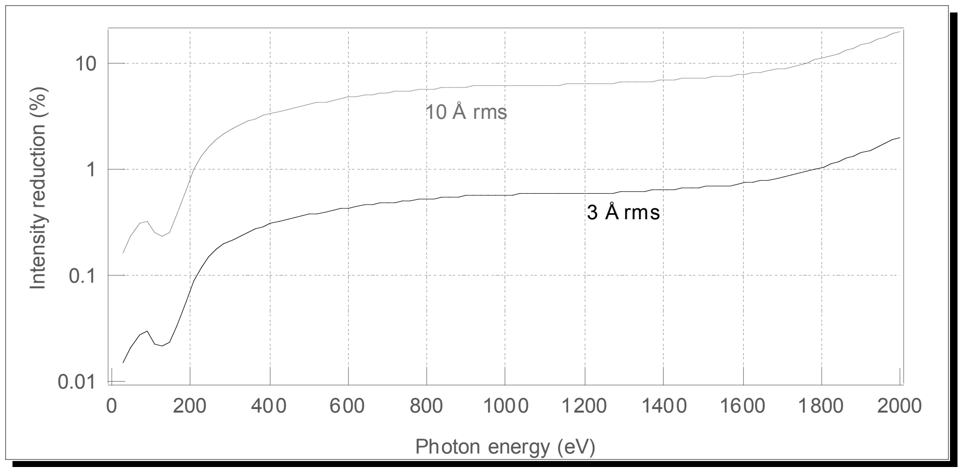
Internally cooled mirrors







Shape	Spherical/Flat
Roughness (Å) Glass/Silicon	3 standard 1 best
Roughness (Å) Metallic	5 standard 2-3 best

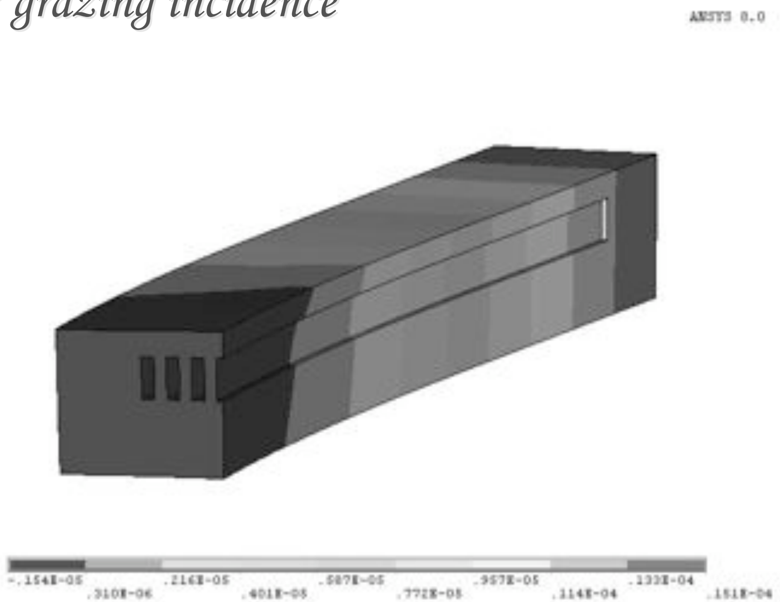


Internally cooled mirrors (Glidcop)

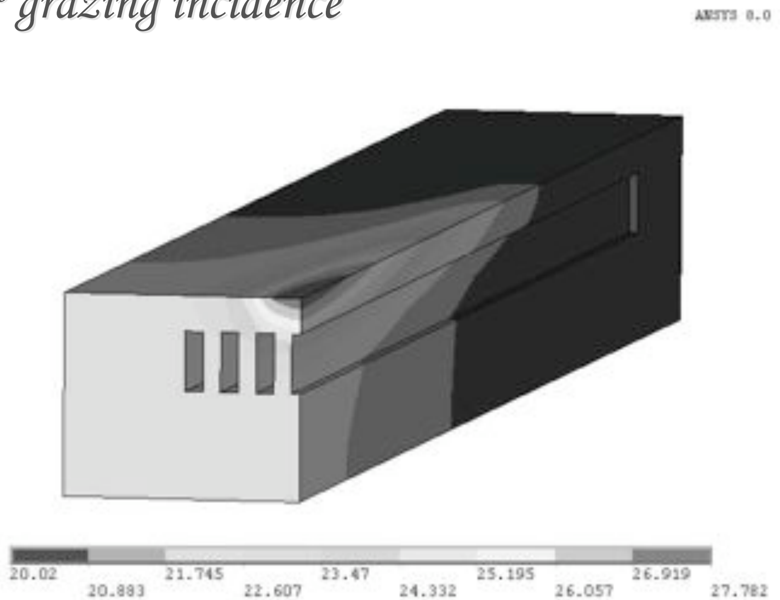
3GeV Synchrotron source
6.6 cm period undulator K_{max}=5.7
BL6.1

glidcop

1.5° grazing incidence | *1.5° grazing incidence*



Δh=17µm slope 26 µrad



ΔT=7.7°

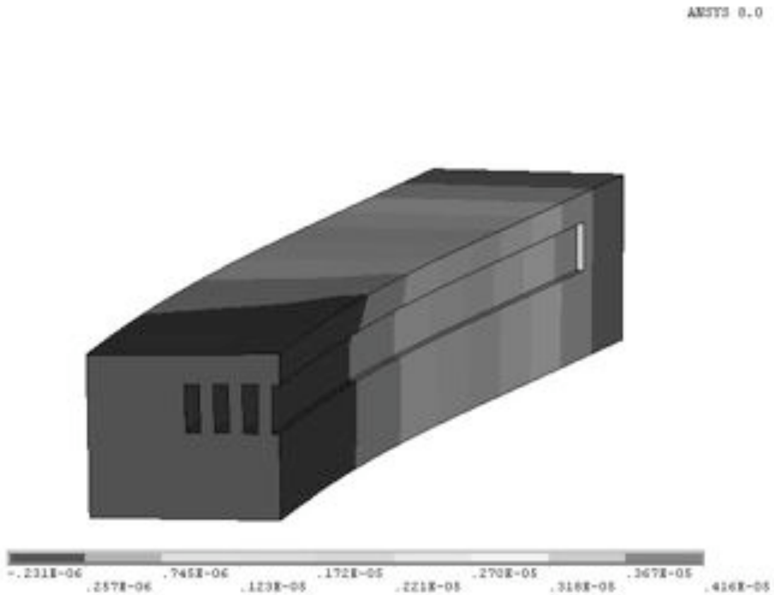
Invar & SuperInvar

	Density gm/cc	Young's modulus GPa	Thermal expansion (α) ppm/°C	Thermal conductivity (k) W/m/°C	Figure of merit k/α
Silicon	2.33	131	2.60	156	60
SiC CVD	3.21	461	2.40	198	82
Aluminum	2.70	68	22.5	167	7.42
Copper	8.94	117	16.5	391	23.7
Glidcop	8.84	130	16.6	365	22
Molybdenum	10.22	324.8	4.80	142	29.6
Invar 36	9.05	141	0.5	10.4	20.8
SuperInvar	8.13	145	0.06	10.5	210

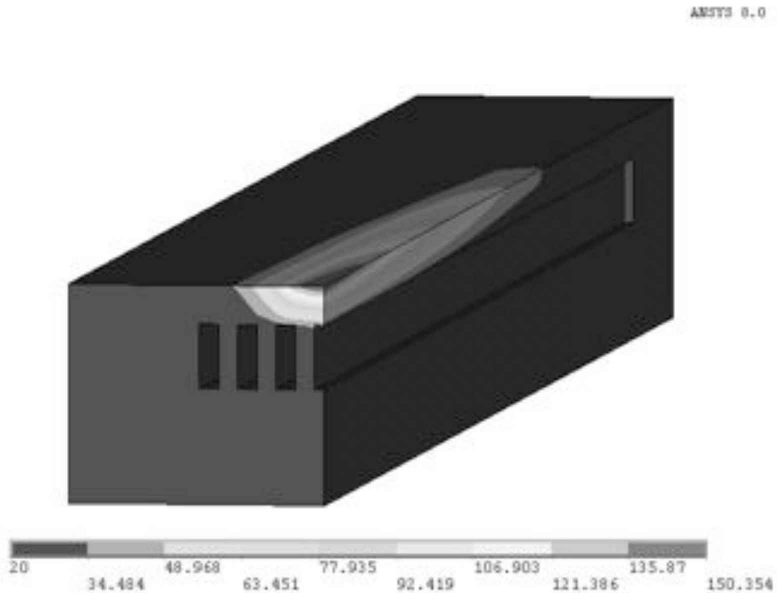
INVAR®
Carpenter Technology Inc.
Alloy 36 iron-nickel(36%) alloy with carbon (0.02%), manganese (0.35%), Silicon (0.2%)
Supernvar: iron-nickel(32%) alloy with carbon (0.02%), manganese (0.40%), Silicon (0.25%), Cobalt (5.5%)

SuperInvar

	Density gm/cc	Young's modulus GPa	Thermal expansion (α) ppm/°C	Thermal conductivity (k) W/m/°C	Figure of merit k/α
Glidcop	8.84	130	16.6	365	22
Molybdenum	10.22	324.8	4.80	142	29.6
SuperInvar	8.13	145	0.06	10.5	210



$\Delta h = 6 \mu m$



$\Delta T = 130^o$

Carbon Contamination

Contamination process:

Hydrocarbons adsorbed by the surface
Cracking induced by the incoming radiation
Formation of graphitic carbon layer (mixed C coumpond)

Effect of the contamination:

Strong adsorption at the carbon edge (≈270 eV)
Reduction of reflectivity due to enanchment of the surface roughness
general deterioration of the surface

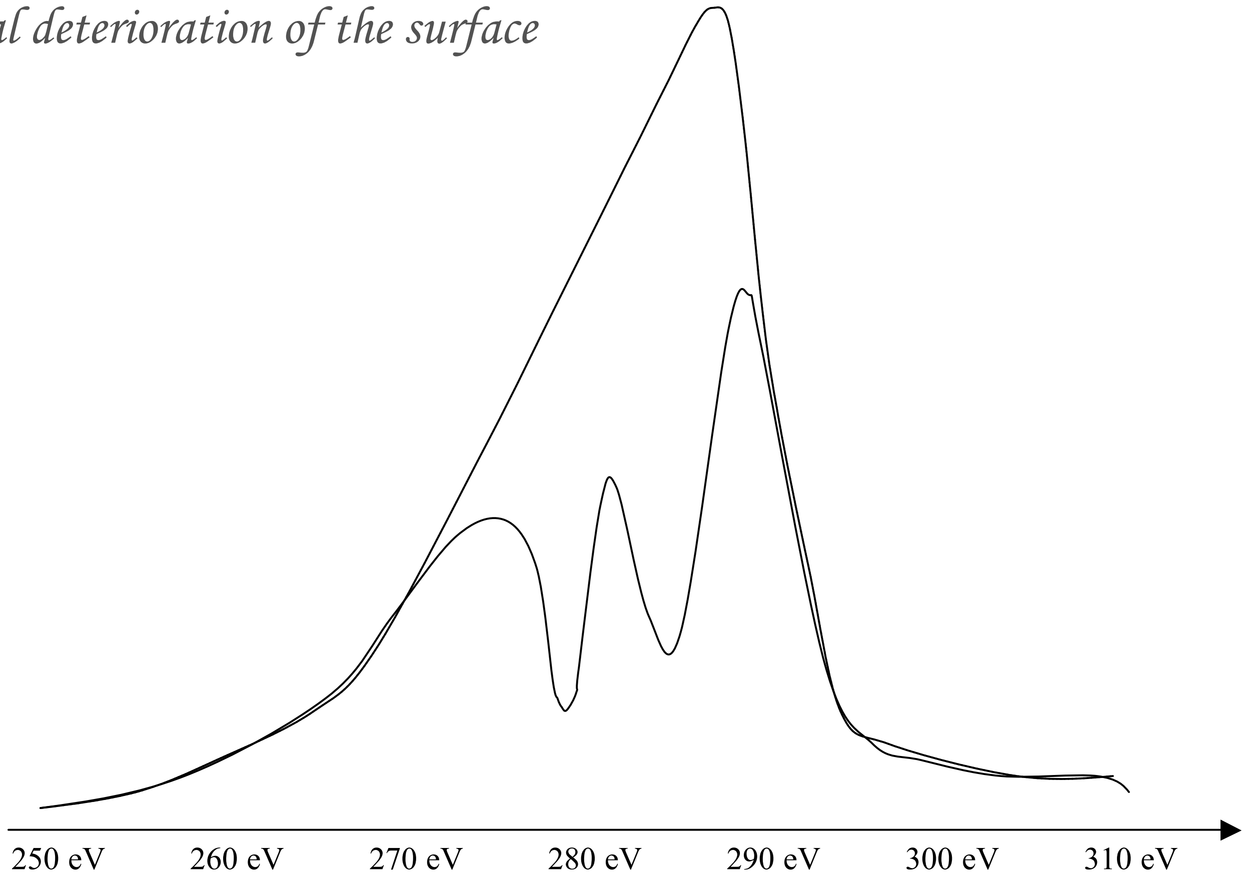
Carbon Contamination

Effect of the contamination:

Strong adsorption at the carbon edge (≈270 eV)

Reduction of reflectivity due to enanchment of the surface roughness

general deterioration of the surface



Carbon Contamination and Cleaning

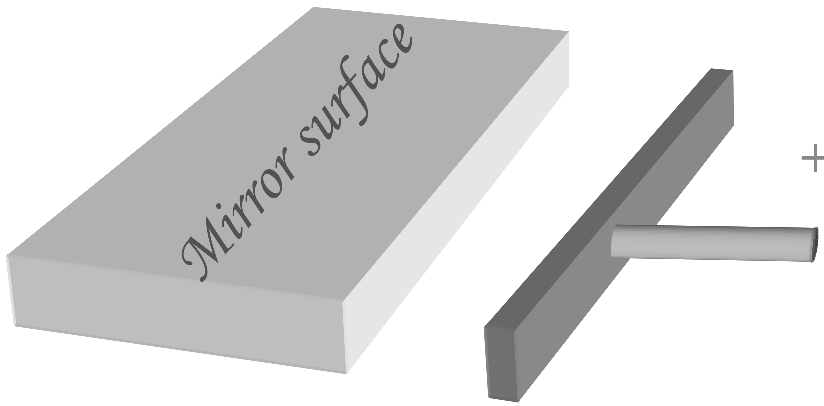
Contamination process:

Hydrocarbons adsorbed by the surface
Cracking induced by the incoming radiation
Formation of graphitic carbon layer (mixed C coumpond)

Effect of the contamination:

Strong adsorption at the carbon edge (≈270 eV)
Reduction of reflectivity due to enanchment of the surface roughness
general deterioration of the surface

UV lamp discharge



+ 300-500 V (DC)

I 100 mA-1A
P 0.5-1 mbar O_2

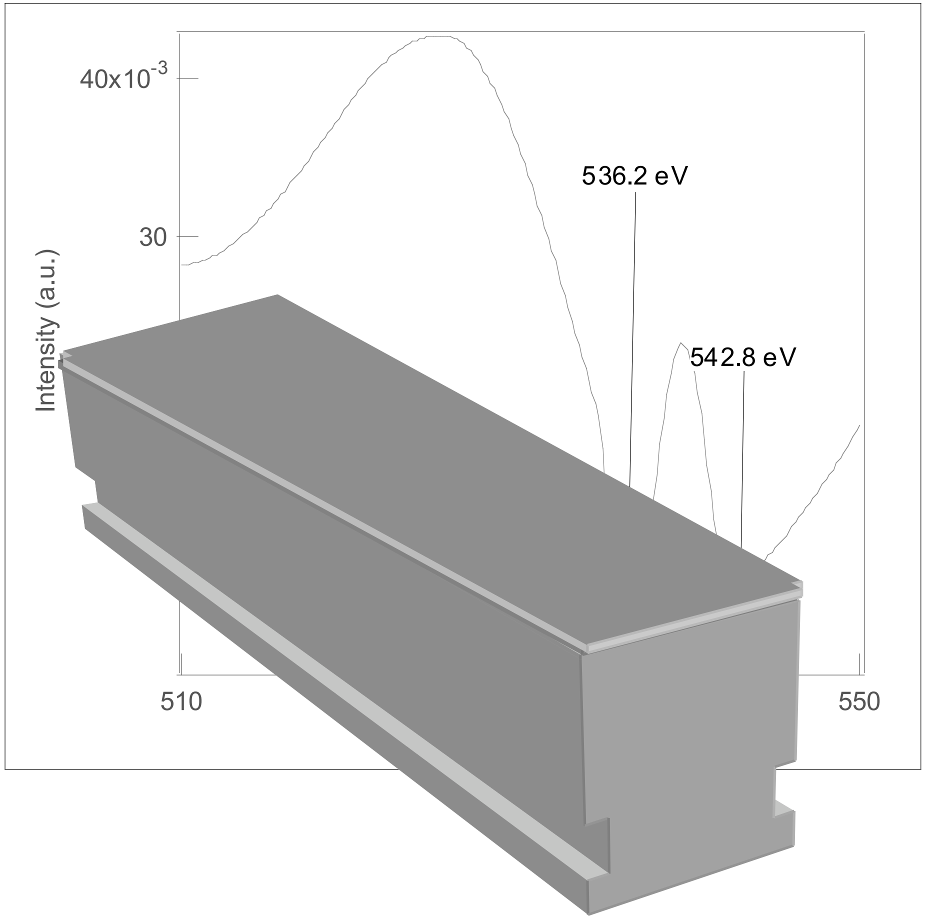
Other Contamination

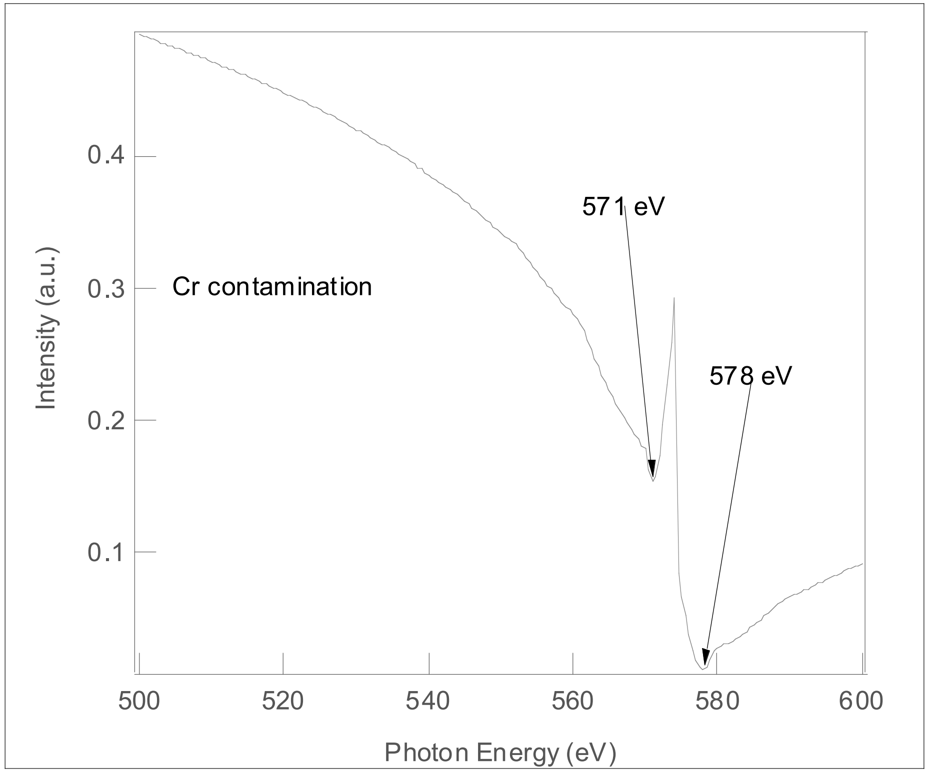
Effect of the contamination:

Strong adsorption at the O/Cr edge

Reduction of reflectivity due to enanchment of the surface roughness

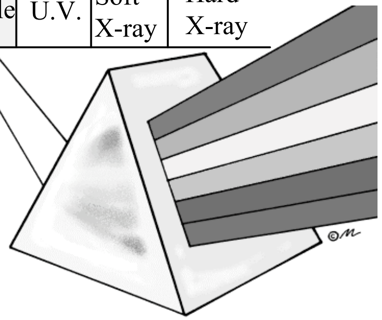
general deterioration of the surface

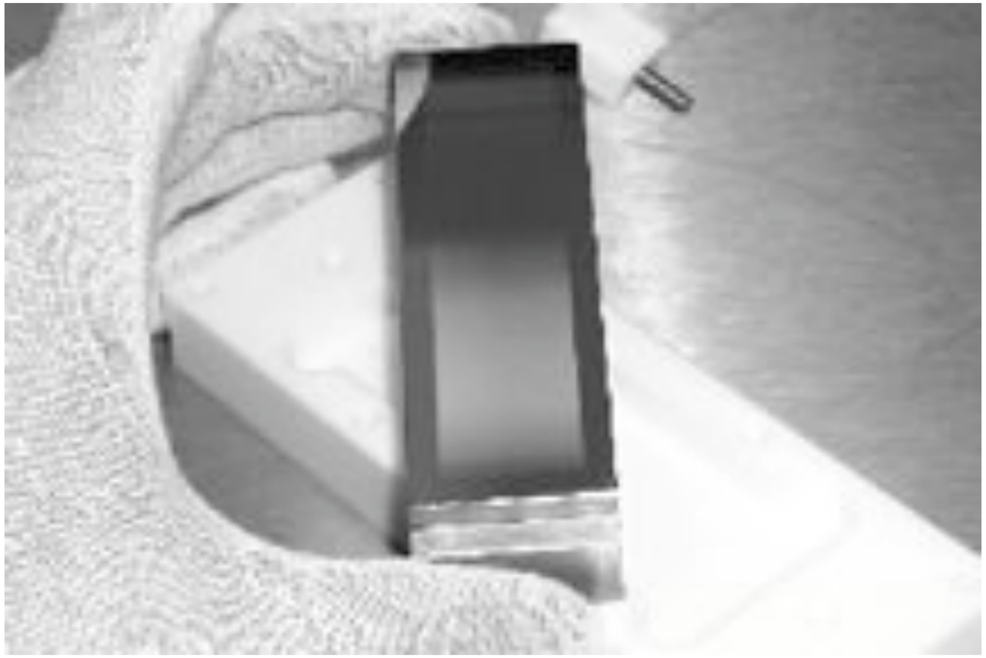




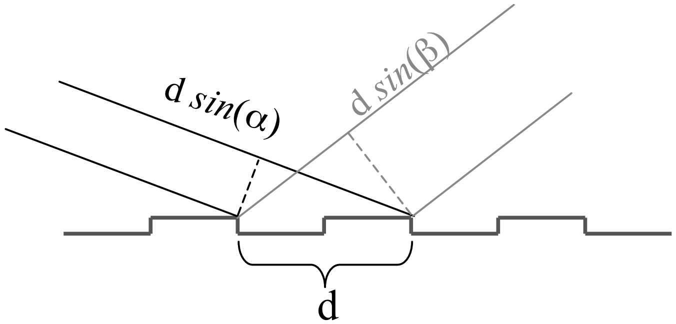
Dispersive elements

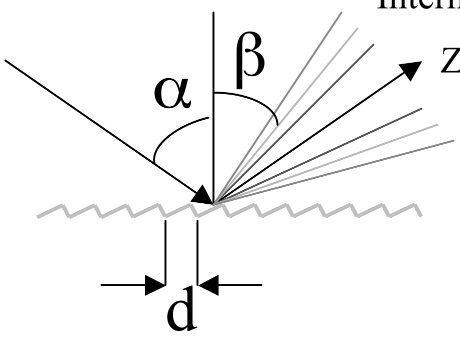
Micro wave	I.R.	Visible	U.V.	Soft X-ray	Hard X-ray





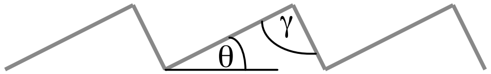
Micro wave	I.R.	Visible	U.V.	Soft X-ray	Hard X-ray

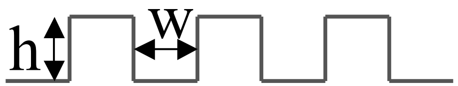




$$n\lambda = d\left(\sin(\alpha) - \sin(\beta)\right)$$

Grating's profiles


Blaze profile

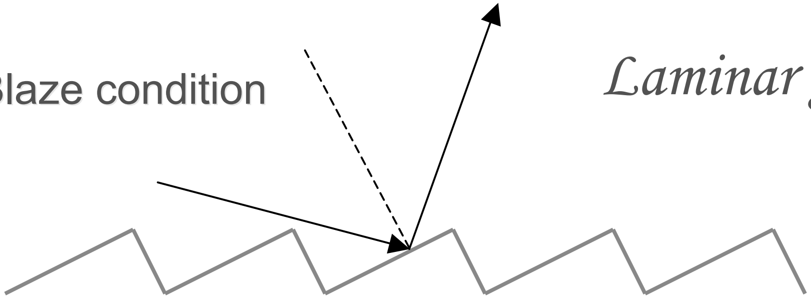


Laminar profile

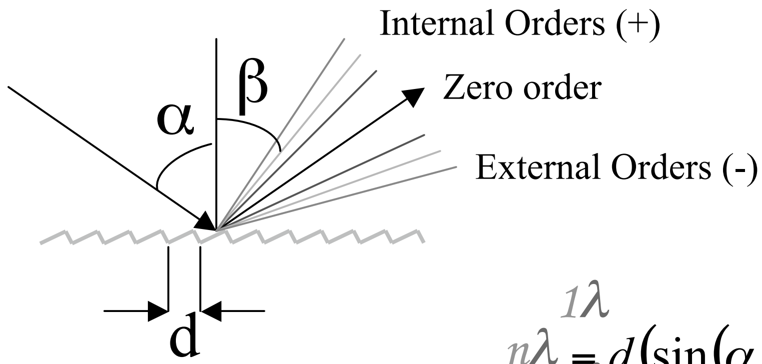
Blaze gratings: *higher efficiency*

Laminar gratings: *Higher spectral purity*
Higher resolution

Blaze condition



Blaze angle=(α+β)/2

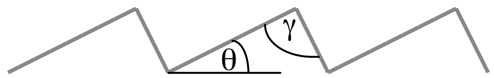


1λ

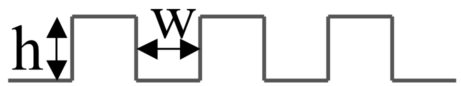
$n\lambda = d\left(\sin(\alpha) - \sin(\beta)\right)$

$2\frac{\lambda}{2}$

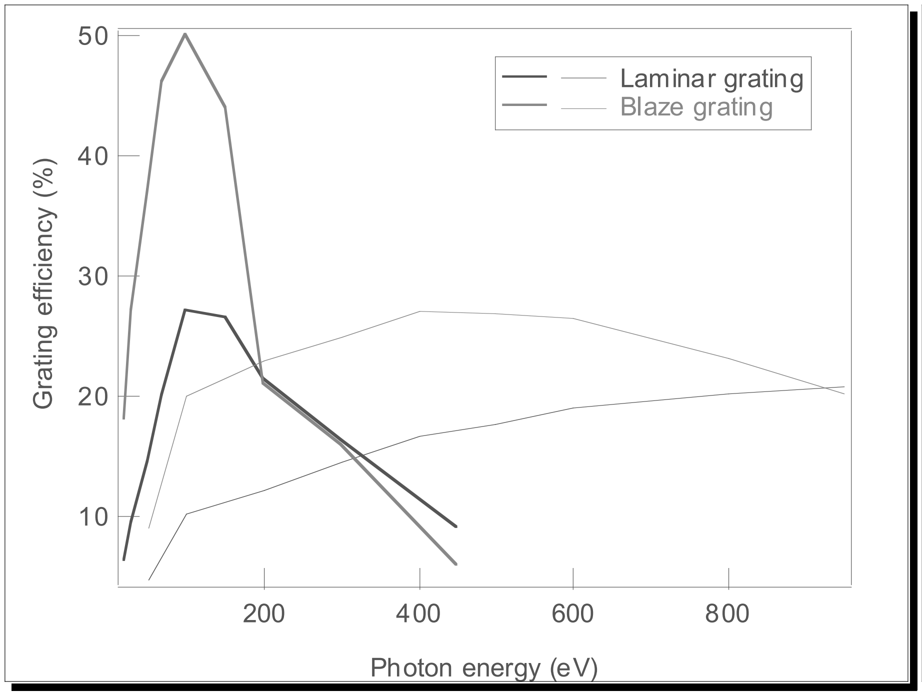
Grating's efficiency

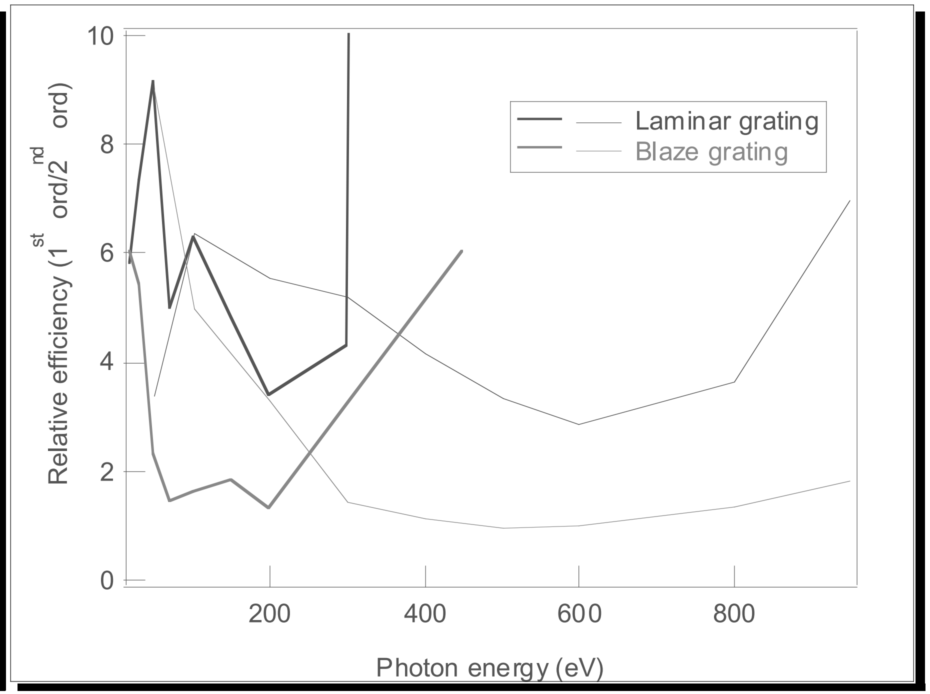


Blaze profile



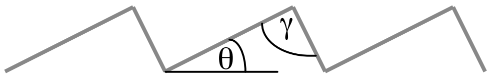
Laminar profile



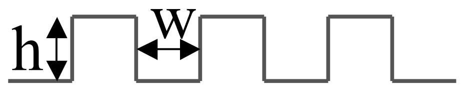


$$n\lambda = d\left(\sin(\alpha) - \sin(\beta)\right)$$

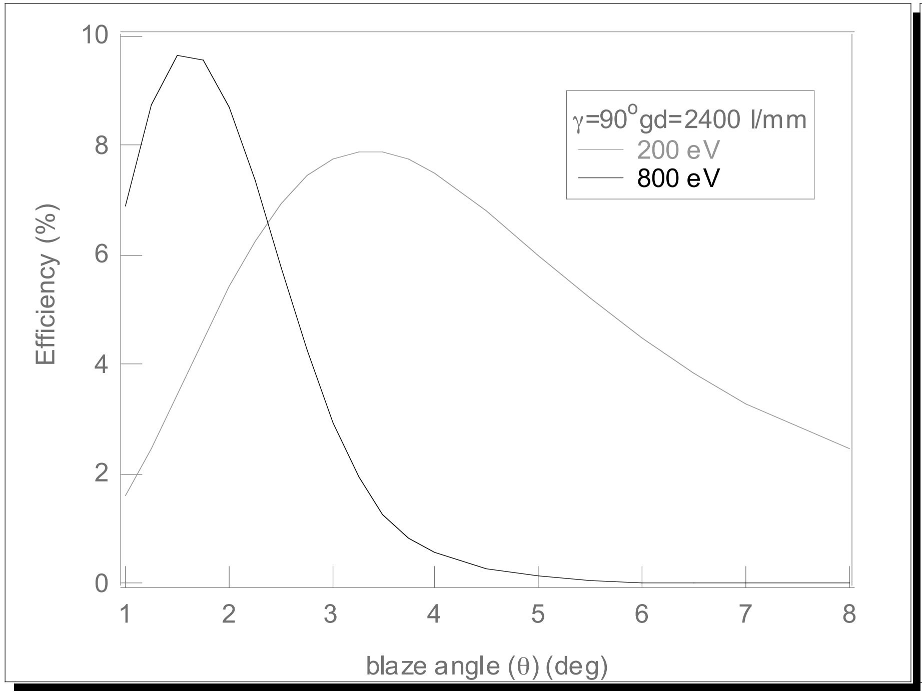
Grating's efficiency

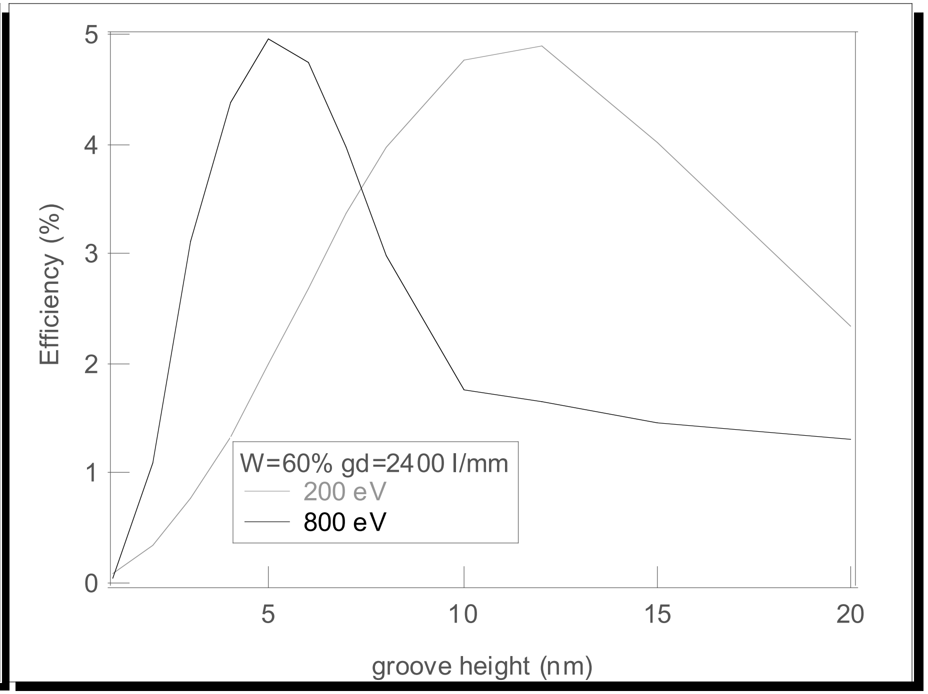


Blaze profile



Laminar profile

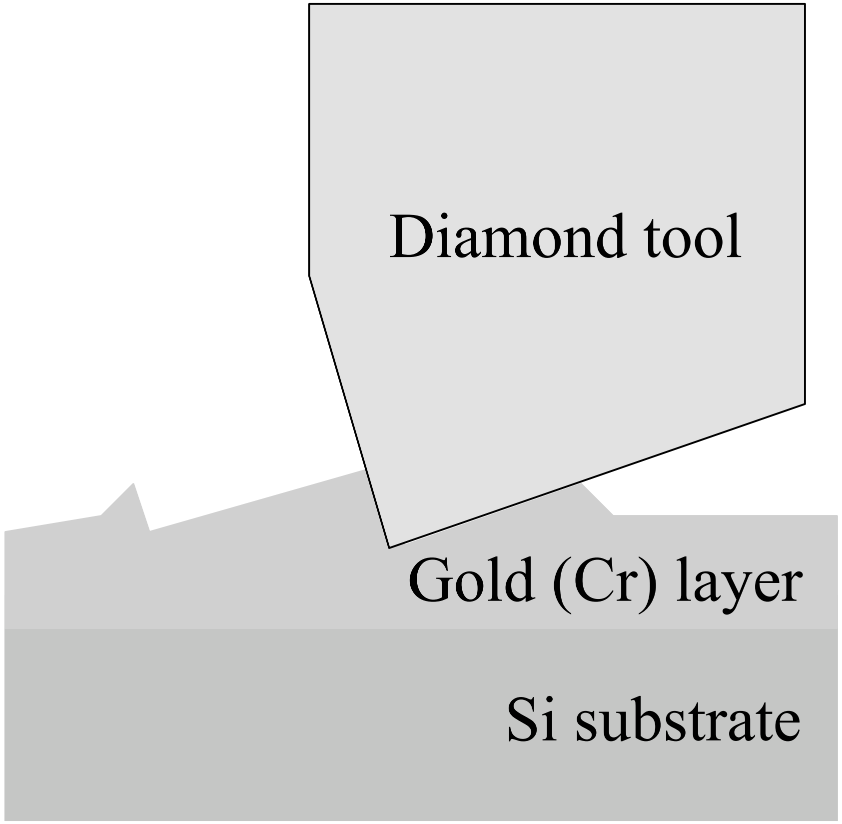


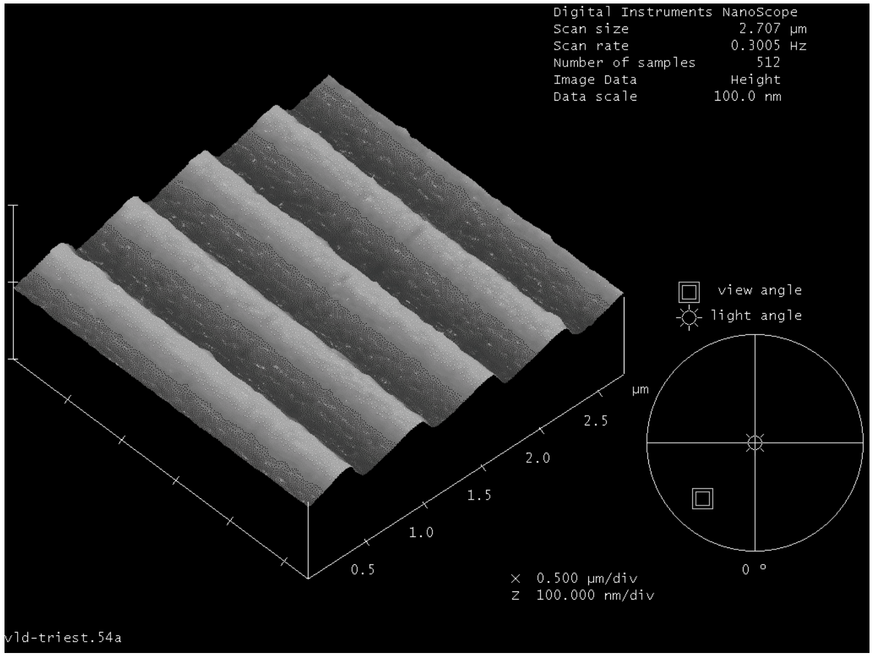


Grating's production

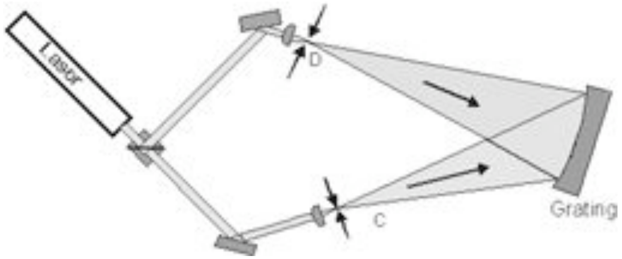
Mechanical ruling *blaze profile* → *smaller blaze angles; higer efficiency*

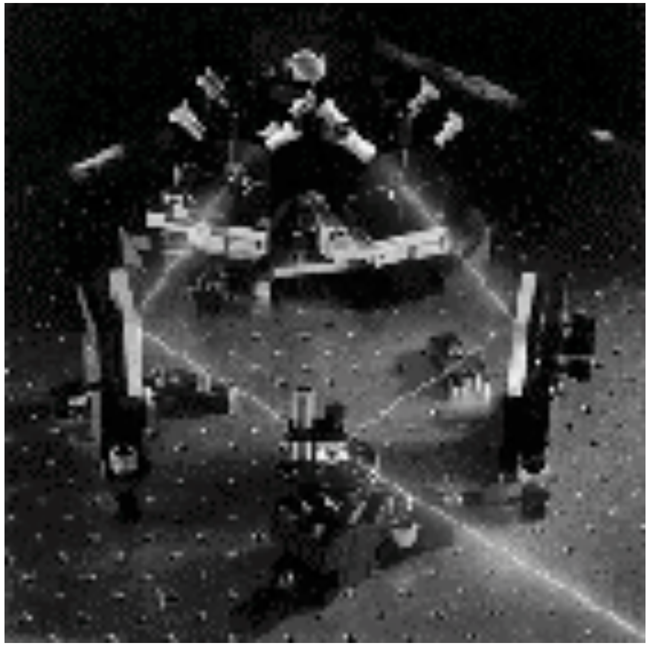
Holographically recording *laminar and blaze profile (large blaze angle)*
→ *higher groove density; lower spacing disomogeneity*

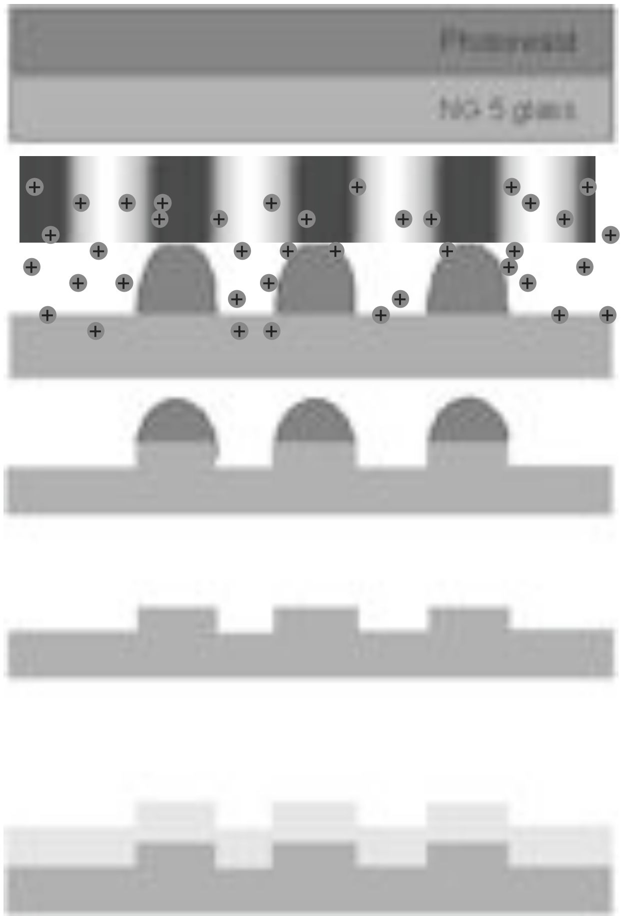




Holographic Recorded Gratings





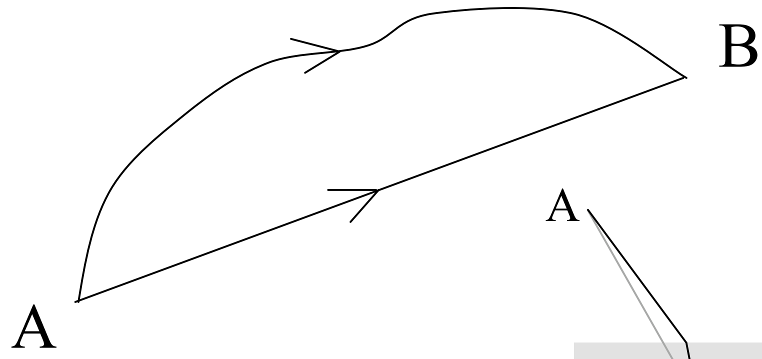


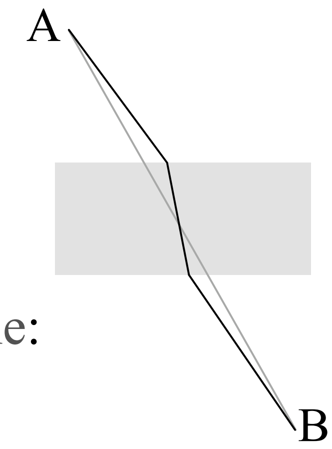
Fermat's principle

Light rays choose their paths to minimize the optical length

$$\int_A^B n(\vec{r})\,dl$$

where $n(\vec{r})$ is the index of refraction of the medium and dl is the line segment along the path



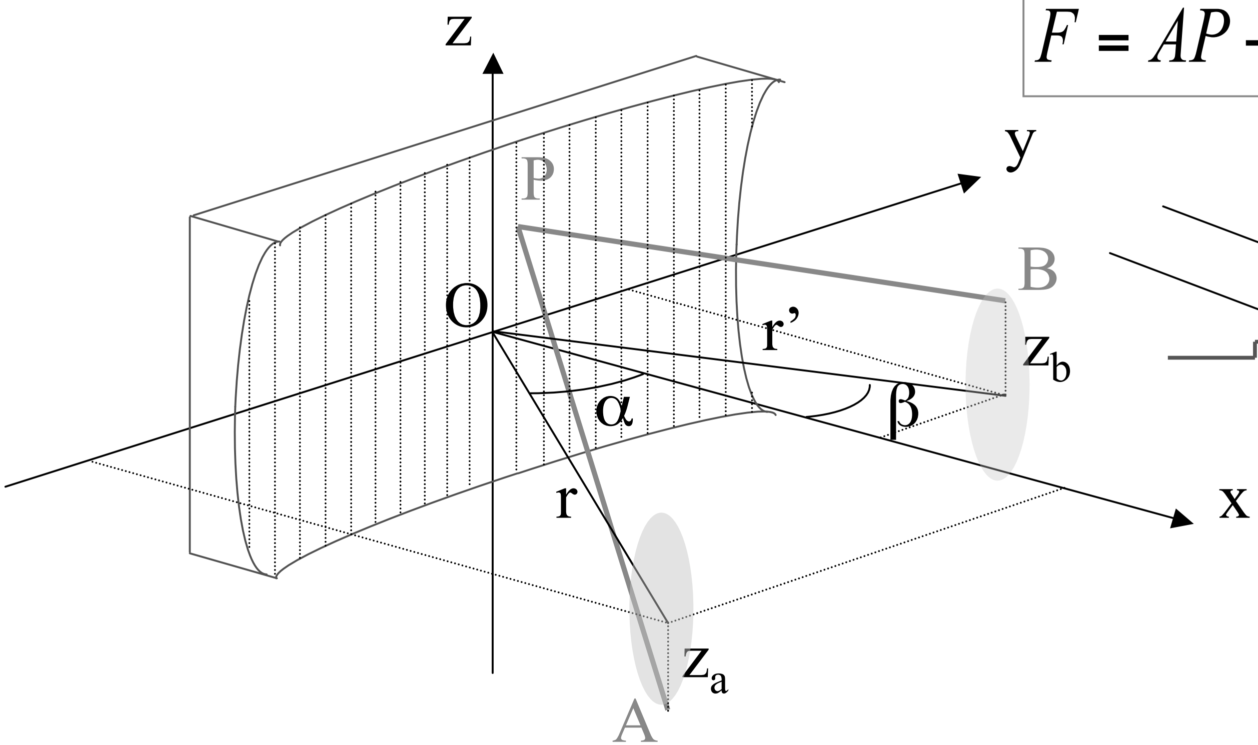


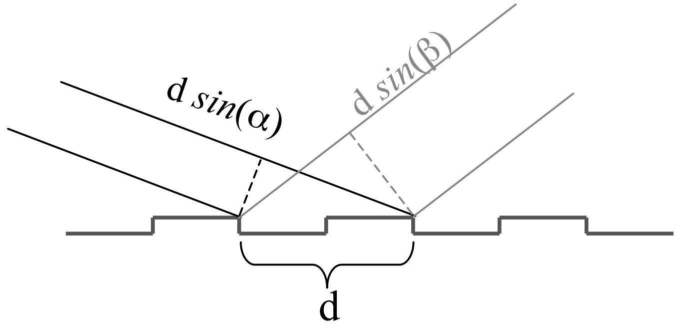
Fermat's principle is also known as the principle of least time:

$$\int_A^B n(\vec{r})\,dl = \int_A^B \frac{c}{v}\,dl = c\int_A^B dt$$

For a classical grating with rectilinear grooves parallel to z with constant spacing d, the optical path length is:

$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

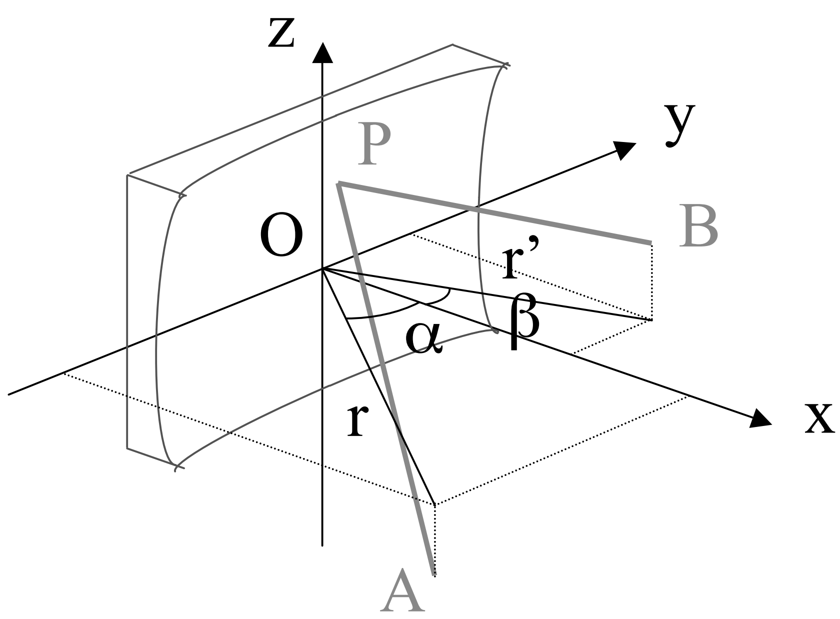




where λ is the wavelength of the diffracted light, k is the order of diffraction (±1,±2,...), N=1/d is the groove density

Optical Path - Focal condition

Let us consider some number of light rays starting from A and impinging on the grating at different points P. Fermat's principle states that if the point A is to be imaged at the point B, then all the optical path lengths from A via the grating surface to B will be the same.



B is the point of a perfect focus if:

$$\frac{\partial F}{\partial y} = 0 \qquad \frac{\partial F}{\partial z} = 0$$

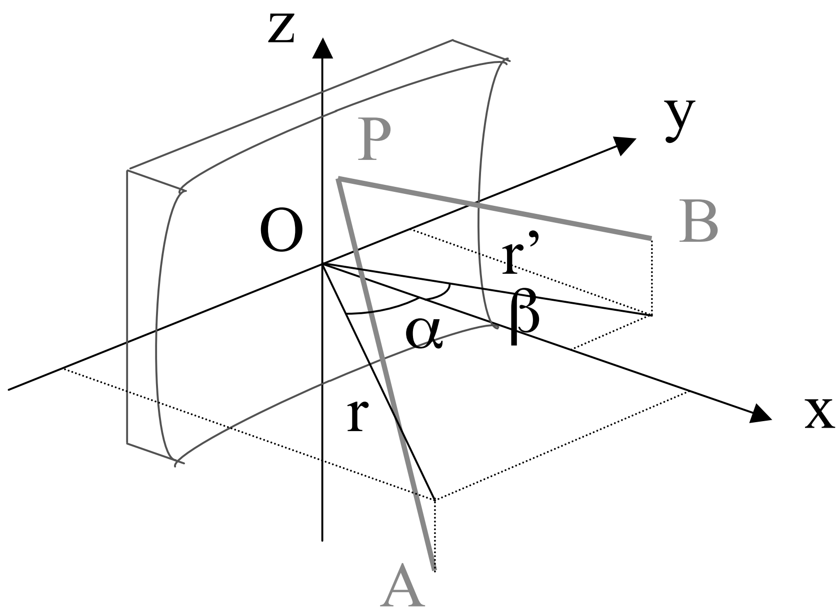
for any pair of (y,z)

Equations:

$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

+

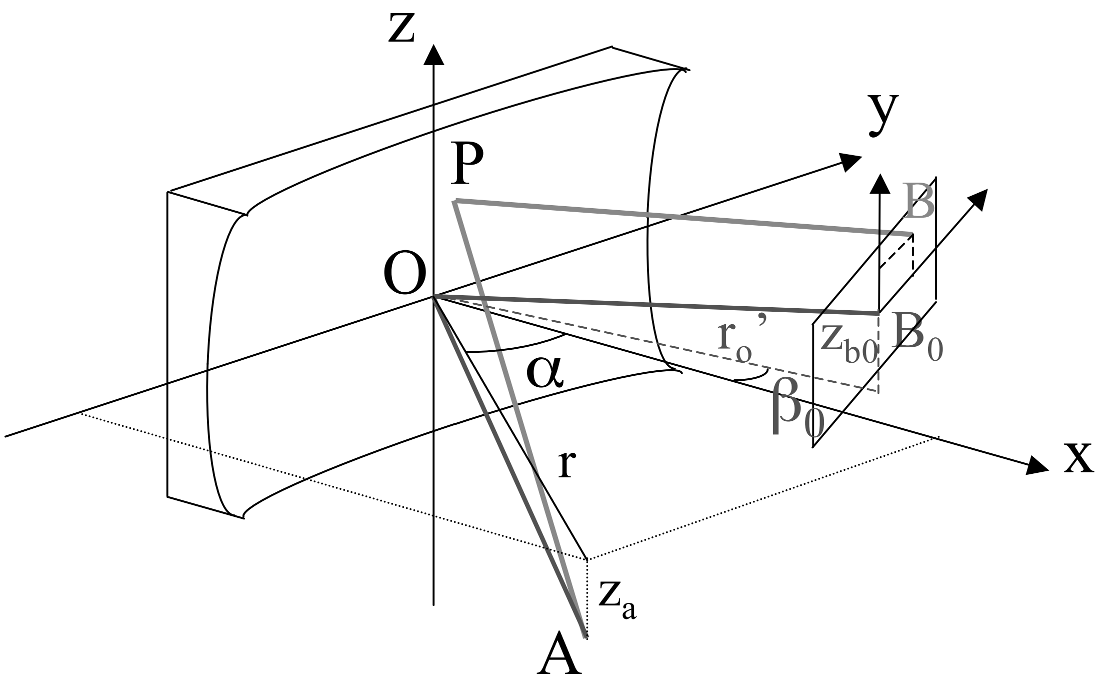
$$\frac{\partial F}{\partial y} = 0 \qquad \frac{\partial F}{\partial z} = 0 \quad \textit{for any pair of } (y,z)$$



can be used to decide on the required characteristics of the diffraction grating, in particular the shape of the surface, the grooves density, the object and image distances.

In general, $\frac{\partial F}{\partial y}$ and $\frac{\partial F}{\partial z}$ are functions of y and z and can not be made zero for any y,z

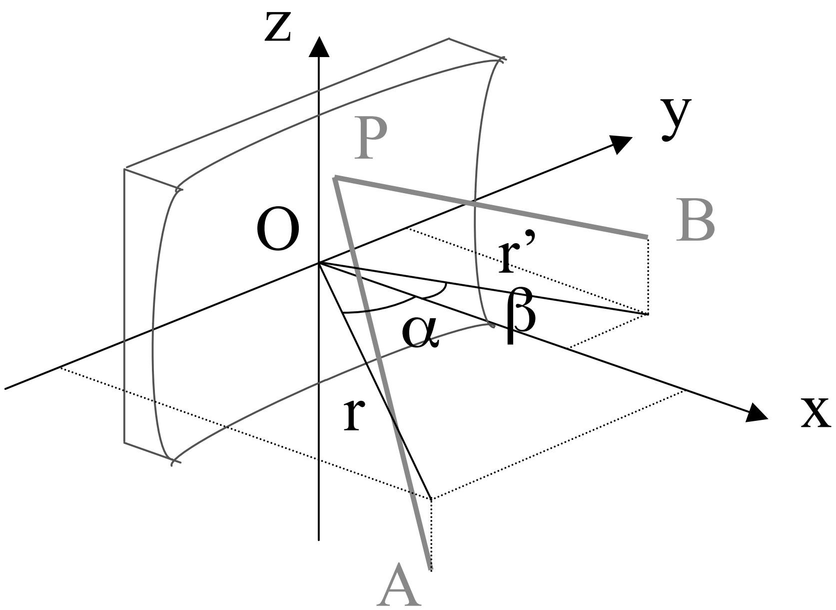
→ when the point P wanders over the grating surface, diffracted rays fall on slightly different points on the focal plane and an aberrated image is formed



Goal: produce simple expressions for the intersection points in the image plane produced by the rays diffracted from different points on the grating surface

Grating surface

The grating surface may in general be described by a series expansion:



$$x = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{ij} y^i z^j$$

$a_{00} = a_{10} = a_{01} = 0$ because of the choice of origin

j = even if the xy plane is a symmetry plane

Giving suitable values to the coefficients a_{ij}'s we obtain the expressions for the various geometrical surfaces.

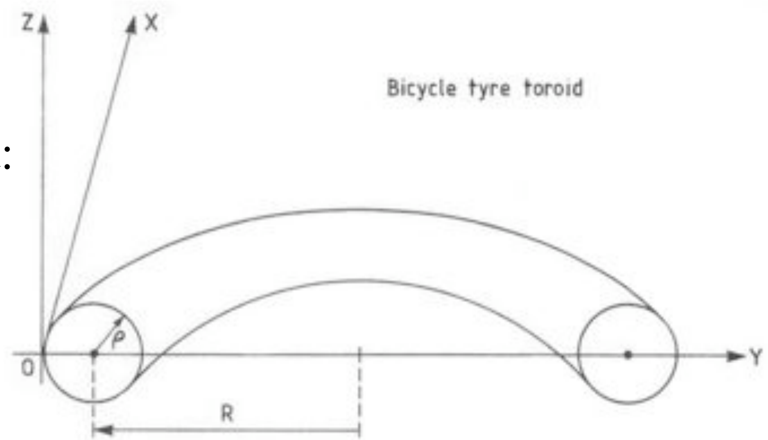
Toroid

$$a_{02} = \frac{1}{2\rho}; \quad a_{20} = \frac{1}{2R}; \quad a_{22} = \frac{1}{4R^2\rho}; \quad a_{40} = \frac{1}{8R^3};$$

$$a_{04} = \frac{1}{8\rho^3}; \quad a_{12} = 0; \quad a_{30} = 0$$

Sphere, cylinder and plane are special cases of toroid:

- R=ρ → sphere
- R= ∞ → cylinder
- R=ρ= ∞ → plane

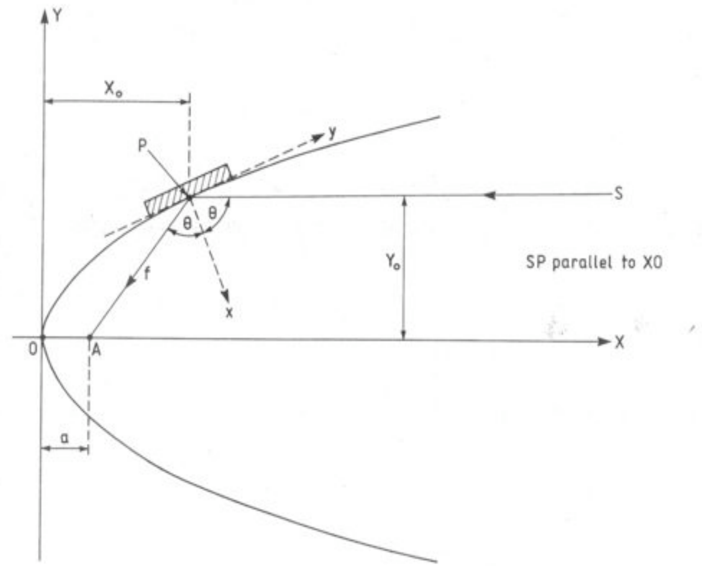


Paraboloid

$$a_{02} = \frac{1}{4f\cos\vartheta}; \quad a_{20} = \frac{\cos\vartheta}{4f}; \quad a_{22} = \frac{3\sin^2\vartheta}{32f^3\cos\vartheta};$$

$$a_{12} = -\frac{\tan\vartheta}{8f^2}; \quad a_{30} = -\frac{\sin\vartheta\cos\vartheta}{8f^2}$$

$$a_{40} = \frac{5\sin^2\vartheta\cos\vartheta}{64f^3}; \quad a_{04} = \frac{\sin^2\vartheta}{64f^3\cos^3\vartheta}$$



Ellipsoid

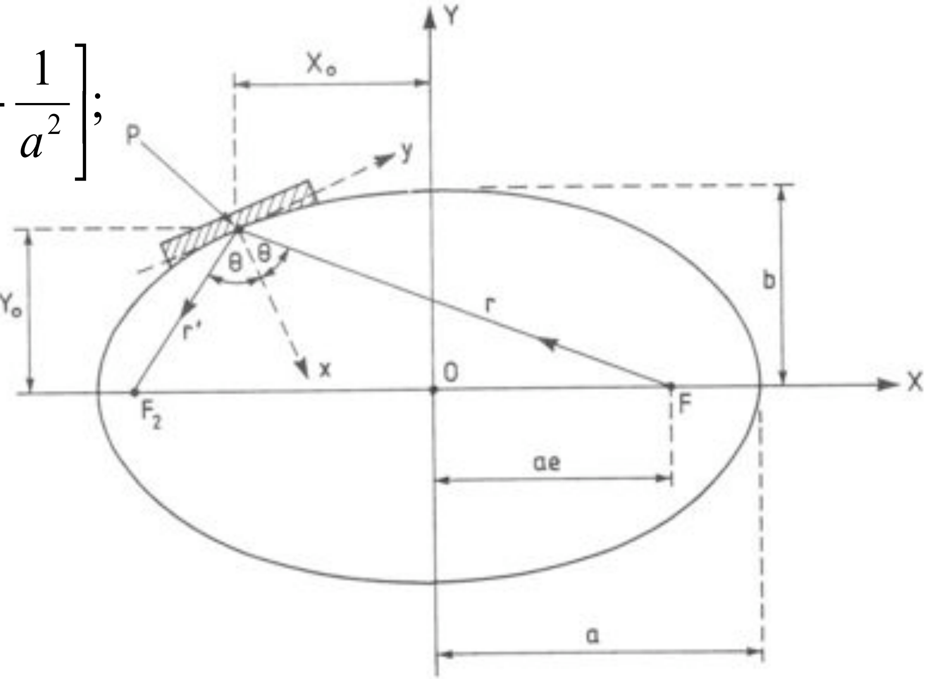
$$a_{02}=\frac{1}{4f\cos\vartheta};\quad a_{20}=\frac{\cos\vartheta}{4f};\quad a_{04}=\frac{b^2}{64f^3\cos^3\vartheta}\left[\frac{\sin^2\vartheta}{b^2}+\frac{1}{a^2}\right];$$

$$a_{12}=\frac{\tan\vartheta}{8f^2\cos\vartheta}\sqrt{e^2-\sin^2\vartheta};\quad a_{30}=\frac{\sin\vartheta}{8f^2}\sqrt{e^2-\sin^2\vartheta}\;;$$

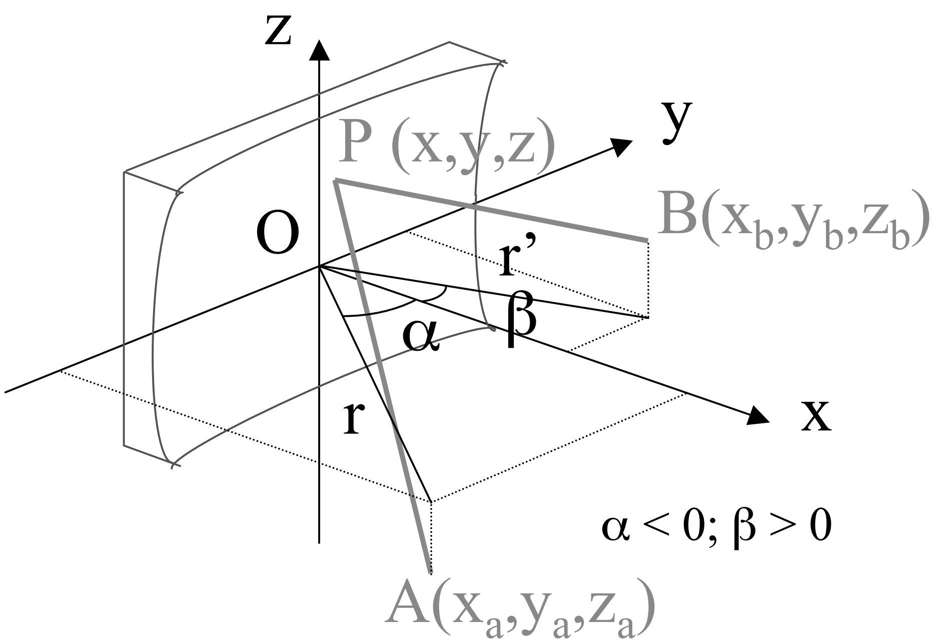
$$a_{40}=\frac{b^2}{64f^3\cos^3\vartheta}\left[\frac{5\sin^2\vartheta\ \cos^2\vartheta}{b^2}-\frac{5\sin^2\vartheta}{a^2}+\frac{1}{a^2}\right];$$

$$a_{22}=\frac{\sin^2\vartheta}{16f^3\cos^3\vartheta}\left[\frac{3}{2}\cos^2\vartheta-\frac{b^2}{a^2}\left(1-\frac{\cos^2\vartheta}{2}\right)\right]$$

where $f=\left[\frac{1}{r}+\frac{1}{r'}\right]^{-1}$



Optical Path Function



$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

$$\overline{AP} = \sqrt{(x_a - x)^2 + (y_a - y)^2 + (z_a - z)^2}$$

$$\overline{PB} = \sqrt{(x_b - x)^2 + (y_b - y)^2 + (z_b - z)^2}$$

$$x_a = r\cos\alpha \qquad y_a = r\sin\alpha$$

$$x_b = r'\cos\beta \qquad y_b = r'\sin\beta$$

$$F = \sum_{ijk} F_{ijk} y^i z^j$$

$$= F_{000} + yF_{100} + zF_{011} + \frac{1}{2}y^2F_{200} + \frac{1}{2}z^2F_{020} + \frac{1}{2}y^3F_{300}$$
$$+ \frac{1}{2}yz^2F_{120} + \frac{1}{8}y^4F_{400} + \frac{1}{4}y^2z^2F_{220} + \frac{1}{8}z^4F_{040}$$
$$+ yzF_{111} + \frac{1}{2}yF_{102} + \frac{1}{4}y^2F_{202} + \frac{1}{2}y^2zF_{211} + \dots$$

Perfect focal condition

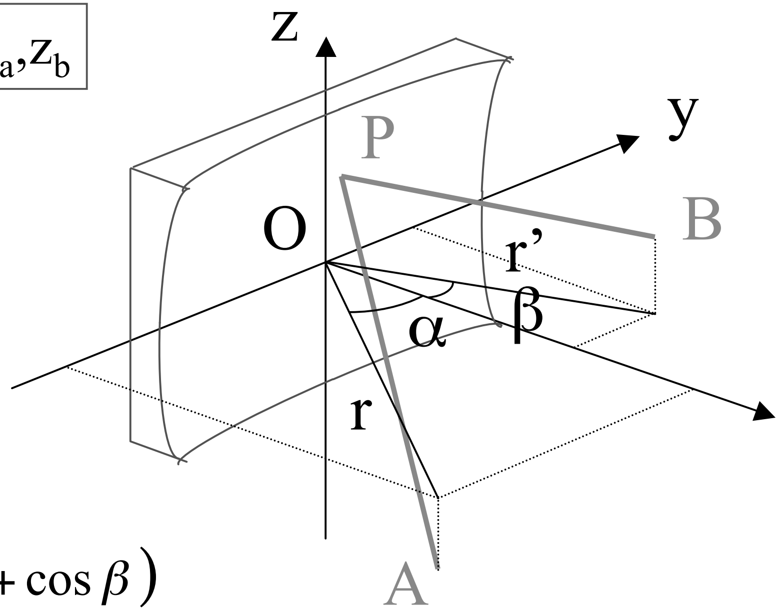
$$\frac{\partial F}{\partial y} = 0 \qquad \frac{\partial F}{\partial z} = 0 \quad \text{for any pair of } (y, z)$$

$$F_{ijk} = 0 \qquad \text{for all } ijk \neq (000)$$

Each term $F_{ijk} y^i z^j$ in the series (except F_{000} and F_{100}) represents a particular type of aberration

Aberrations Terms

for r,r' >> z_a,z_b



$$F_{000} = r + r'$$

$$F_{100} = Nk\lambda - (\sin\alpha + \sin\beta)$$

$$F_{200} = \left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta}{r'}\right) - 2a_{20}\left(\cos\alpha + \cos\beta\right)$$

$$F_{020} = \frac{1}{r} + \frac{1}{r'} - 2a_{02}\left(\cos\alpha + \cos\beta\right)$$

$$F_{300} = \left[\frac{T(r,\alpha)}{r}\right]\sin\alpha + \left[\frac{T(r',\beta)}{r'}\right]\sin\beta - 2a_{30}\left(\cos\alpha + \cos\beta\right)$$

$$F_{120} = \left[\frac{S(r,\alpha)}{r}\right]\sin\alpha + \left[\frac{S(r',\beta)}{r'}\right]\sin\beta - 2a_{12}\left(\cos\alpha + \cos\beta\right)$$

where $T(r,\alpha) = \frac{\cos^2\alpha}{r} - 2a_{20}\cos\alpha$ and $S(r,\alpha) = \frac{1}{r} - 2a_{02}\cos\alpha$

and analogous expressions for $T(r',\beta)$ and $S(r',\beta)$

Aberrations Terms

$F_{100} = 0 \Longrightarrow \sin\alpha + \sin\beta_0 = Nk\lambda$ grating equation

Most important imaging errors:

F_{200}	defocus
F_{020}	astigmatism
F_{300}	primary coma (aperture defect)
F_{120}	astigmatic coma
F_{400} F_{220} F_{040}	spherical aberration

There is an ambiguity in the naming of the aberrations in the grazing incidence case!

Focal conditions

The tangential focal distance r'_0 is obtained by setting:

$$F_{200} = 0 \Longrightarrow \left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta_0}{r'_0}\right) - 2a_{20}(\cos\alpha + \cos\beta_0) = 0$$ tangential focusing

The sagittal focal distance r'_0 is obtained by setting:

$$F_{020} = 0 \Longrightarrow \frac{1}{r} + \frac{1}{r'} - 2a_{02}(\cos\alpha + \cos\beta) = 0$$ sagittal focusing

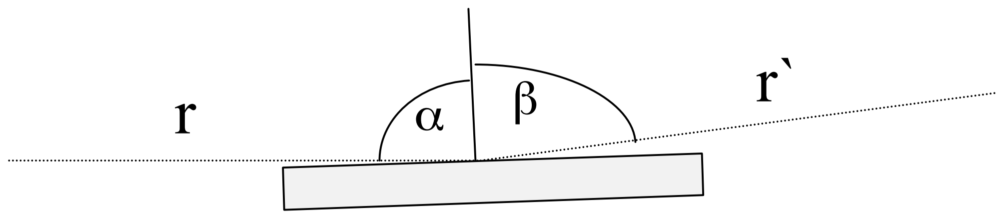
Example: toroidal mirror

Substituting $a_{02} = \frac{1}{2\rho}; \quad a_{20} = \frac{1}{2R}$ in $F_{200} = 0; \quad F_{020} = 0$

and imposing $\alpha = -\beta = \theta$

$$\Longrightarrow \left(\frac{1}{r} + \frac{1}{r_t'}\right)\frac{\cos\vartheta}{2} = \frac{1}{R} \qquad \left(\frac{1}{r} + \frac{1}{r_s'}\right)\frac{1}{2\cos\vartheta} = \frac{1}{\rho}$$

Spherical Gratings



Optical path function

$$F_{100} = -n\lambda D_0 + (\sin\alpha - \sin\beta)$$ *grating equation*

$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right)$$ *tangential focus*

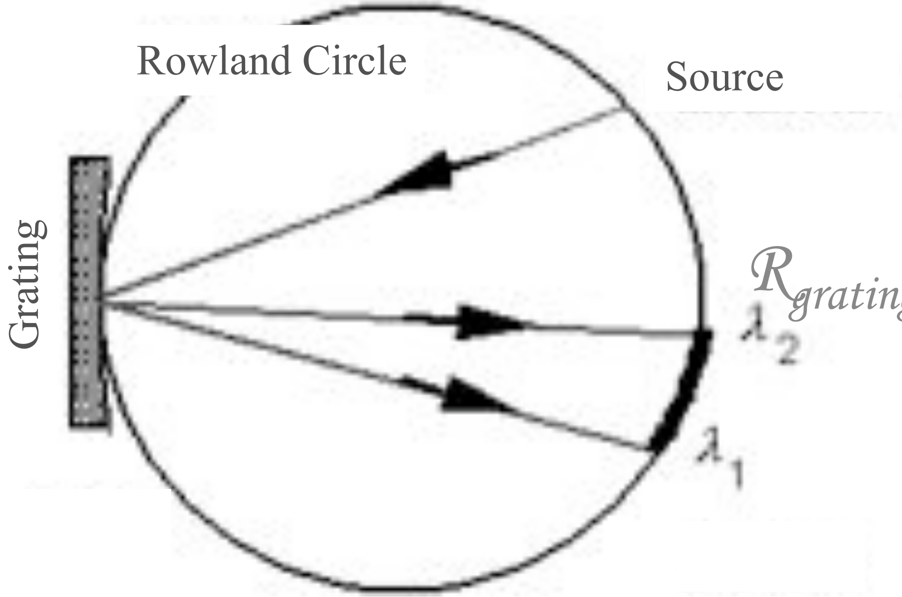
$$F_{300} = \left[\left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} \right) \frac{\sin\alpha}{r} + \left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right) \frac{\sin\beta}{r'} \right]$$ *primary coma*

Spherical Gratings

$$\mathcal{F}_{200} = \mathcal{F}_{300} = 0$$

$$r' = R\cos\beta$$

$$r = R\cos\alpha$$



$$\mathcal{R}_{grating} = 2\mathcal{R}_{rowland\ circle}$$

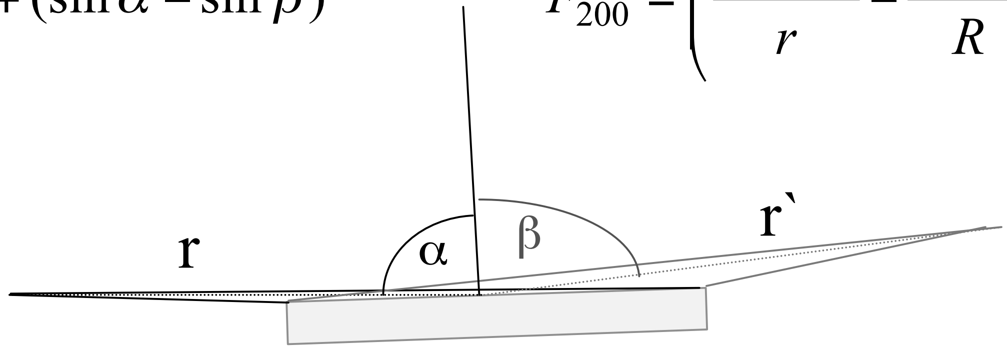
$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R}\right) + \left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R}\right)$$ *tangential focus*

$$F_{300} = \left[\left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R}\right)\frac{\sin\alpha}{r} + \left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R}\right)\frac{\sin\beta}{r'}\right]$$ *primary coma*

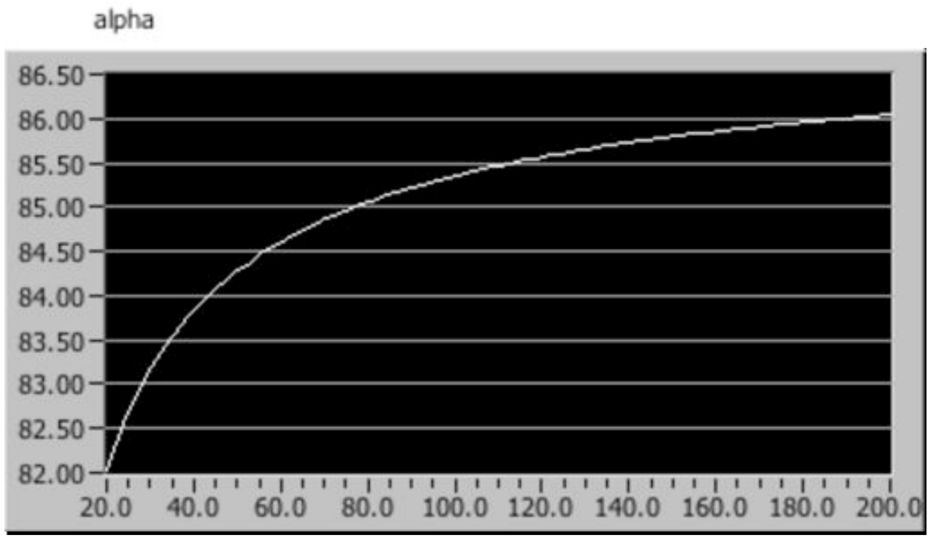
Variable Included Angle Spherical Grating Monochromator

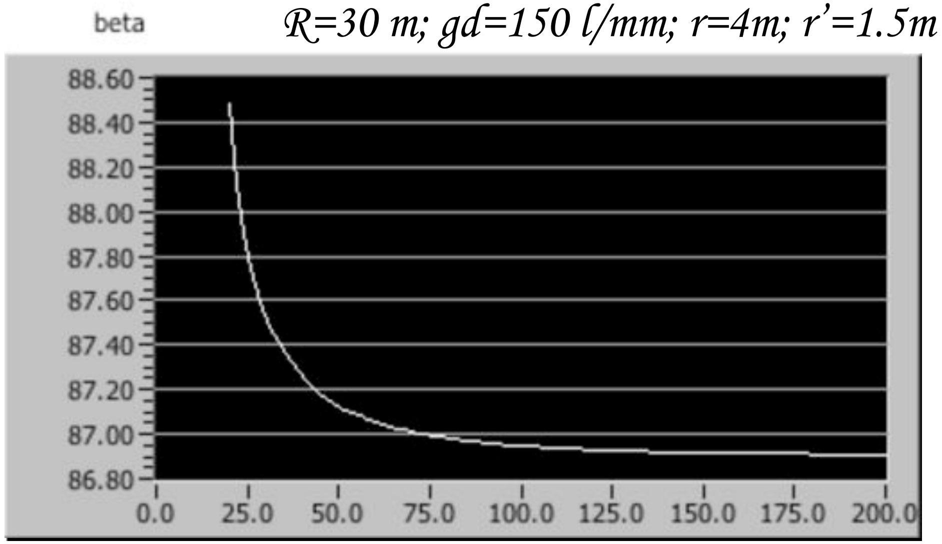
$$F_{100} = -n\lambda D_0 + (\sin\alpha - \sin\beta)$$

$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right)$$



R=30 m; gd=150 l/mm; r=4m; r'=1.5m



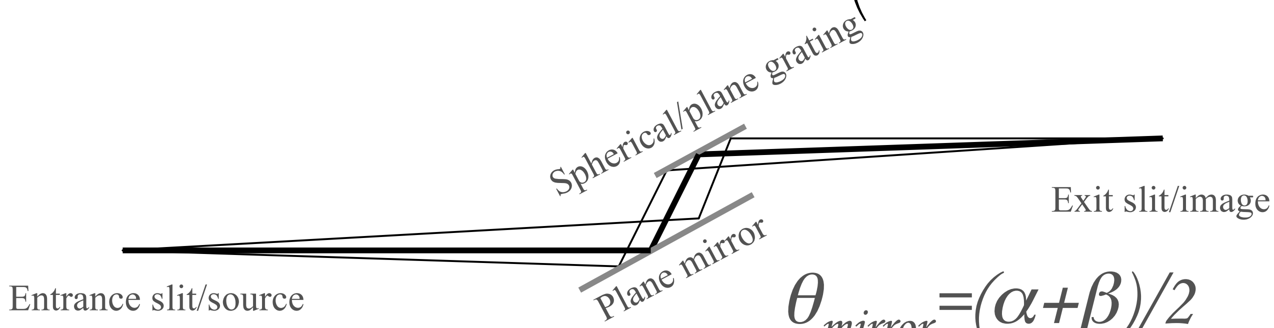


Mantain fixed source and image in position and direction

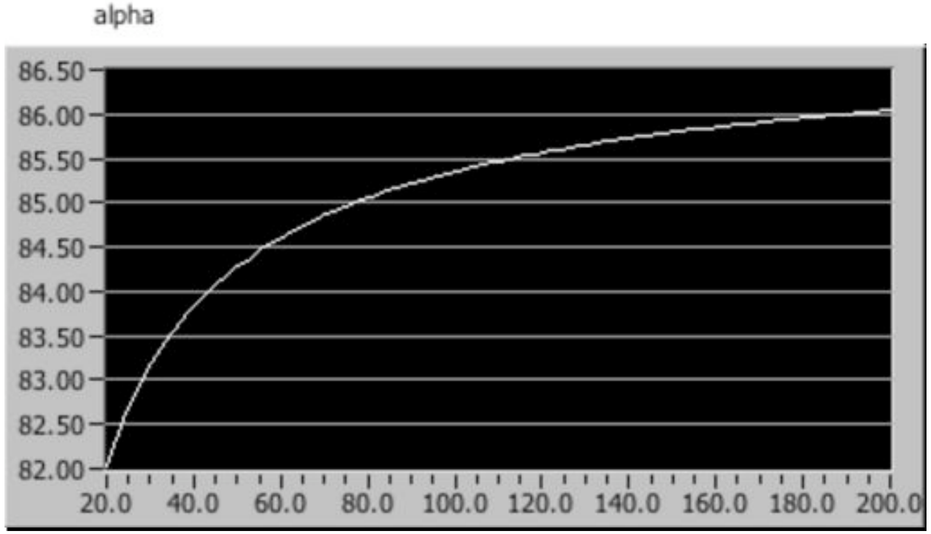
Variable Included Angle Spherical Grating Monochromator

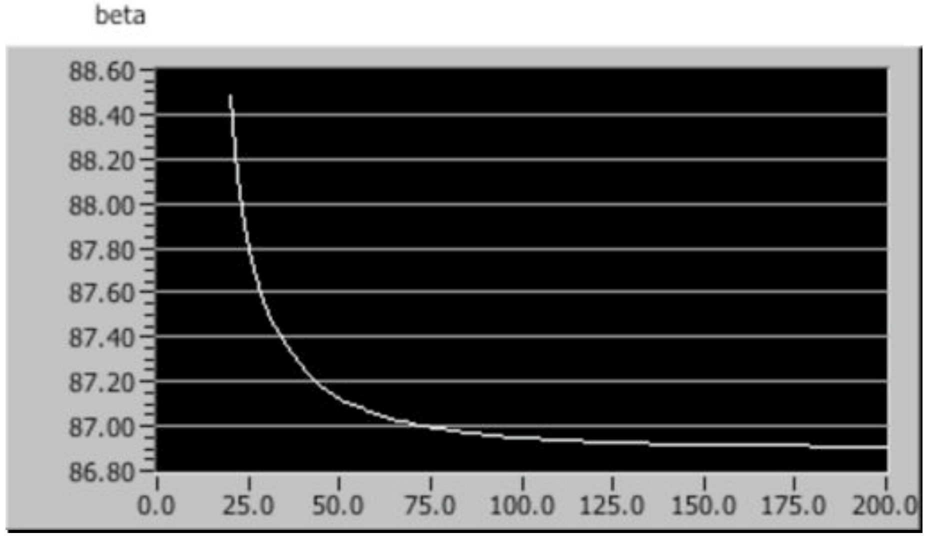
$$F_{100} = -n\lambda D_0 + (\sin\alpha - \sin\beta)$$

$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right)$$

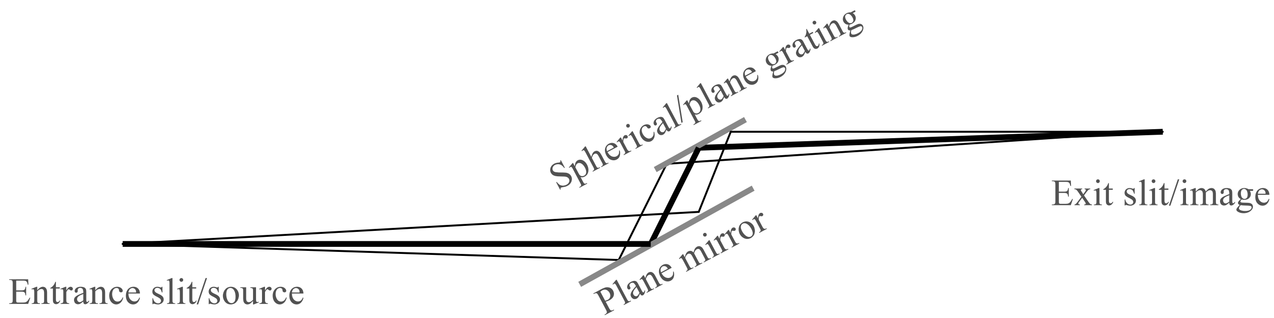


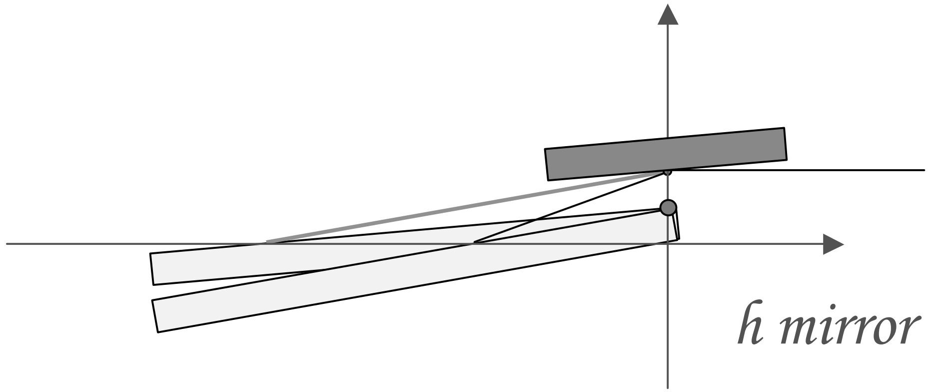
$$\theta_{mirror} = (\alpha+\beta)/2$$





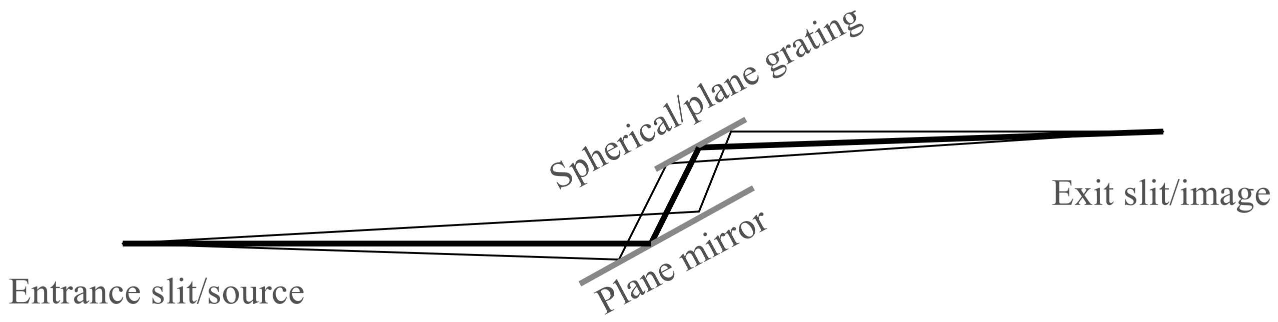
Variable Included Angle Spherical Grating Monochromator

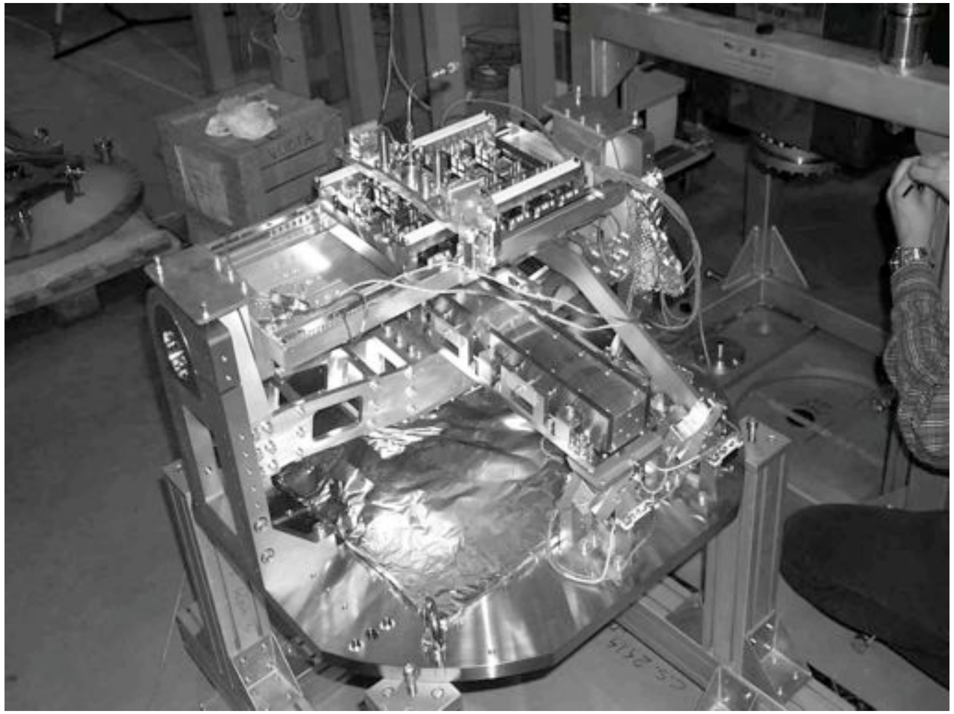


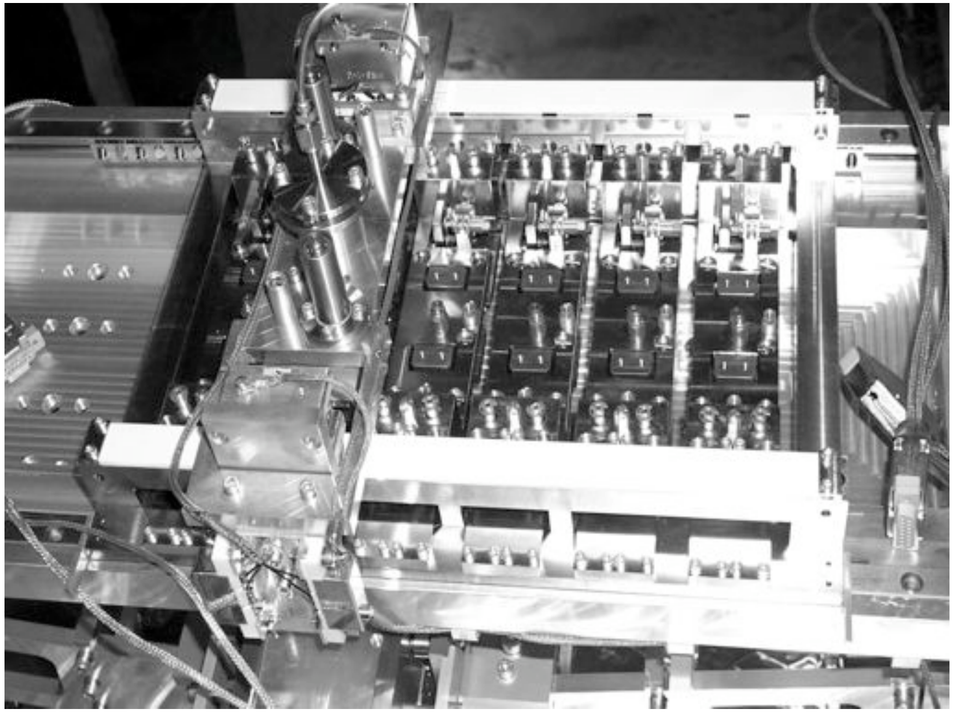


h mirror axis ~ h/2 grating center

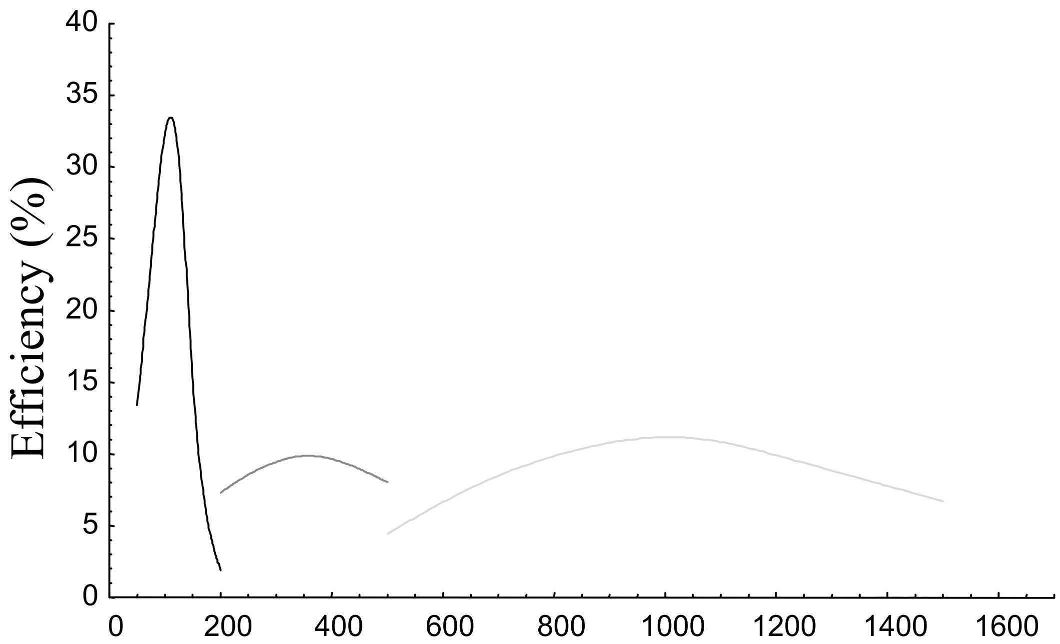
Variable Included Angle Spherical Grating Monochromator

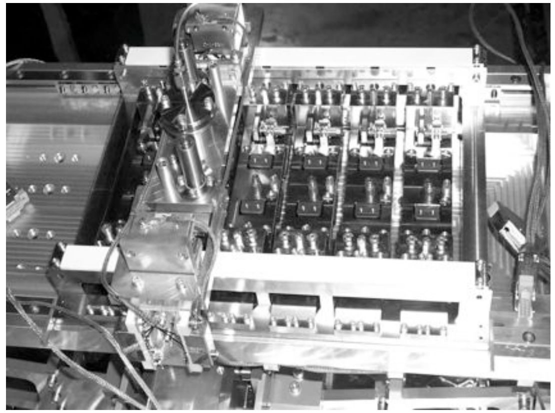


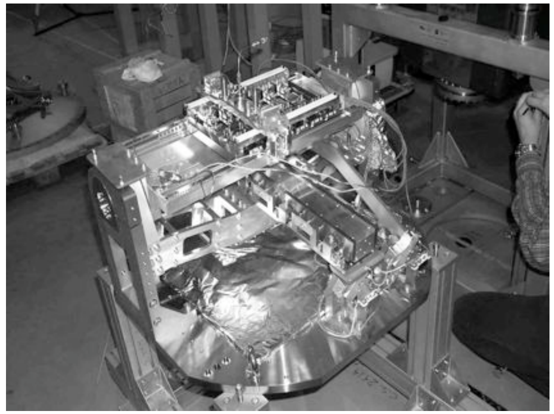




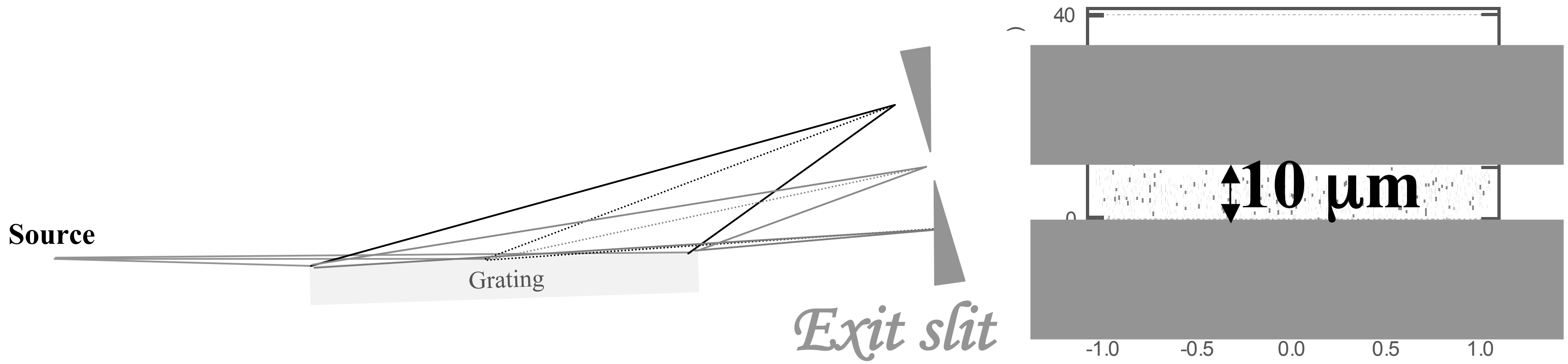
Efficiency Curves

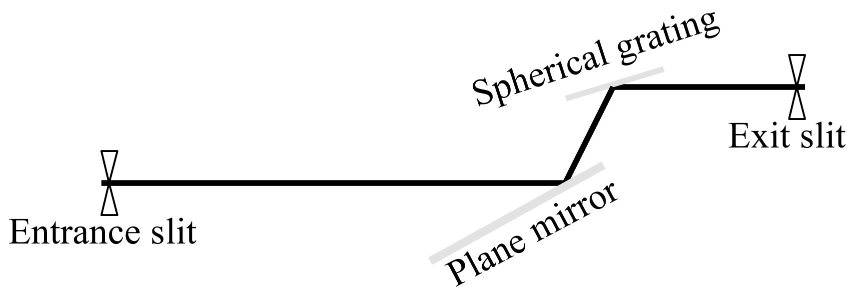


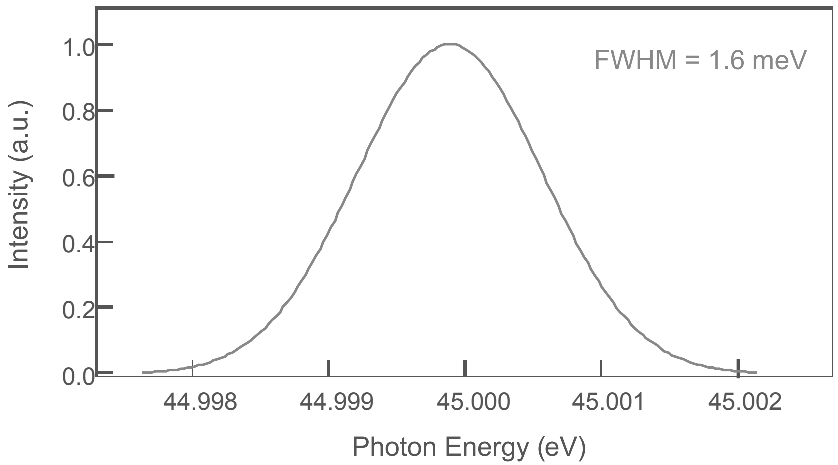




Resolving Power







Resolving Power

smaller are s and s′,
smaller will be the bandpass

$$Nk\lambda = \sin(\alpha) - \sin(\beta)$$

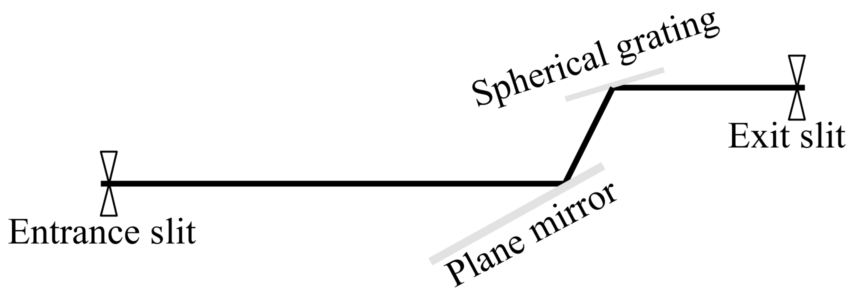
$$\left(\frac{\partial\lambda}{\partial\alpha}\right) = \frac{\cos(\alpha)}{Nk} \qquad \Delta\alpha = \frac{s}{r} \qquad \Delta\lambda_{entrance} = \frac{s\cdot\cos(\alpha)}{Nkr}$$

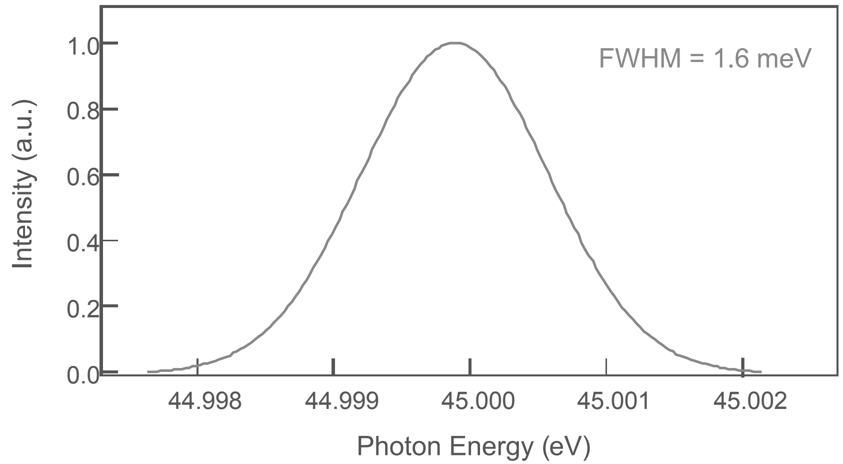
entrance slit contribution

$$\left(\frac{\partial\lambda}{\partial\beta}\right) = \frac{\cos(\beta)}{Nk} \qquad \Delta\beta = \frac{s'}{r'} \qquad \Delta\lambda_{exit} = \frac{s'\cdot\cos(\beta)}{Nkr'}$$

exit slit contribution

$$\text{Resolving power} = \lambda/\Delta\lambda = E/\Delta E$$

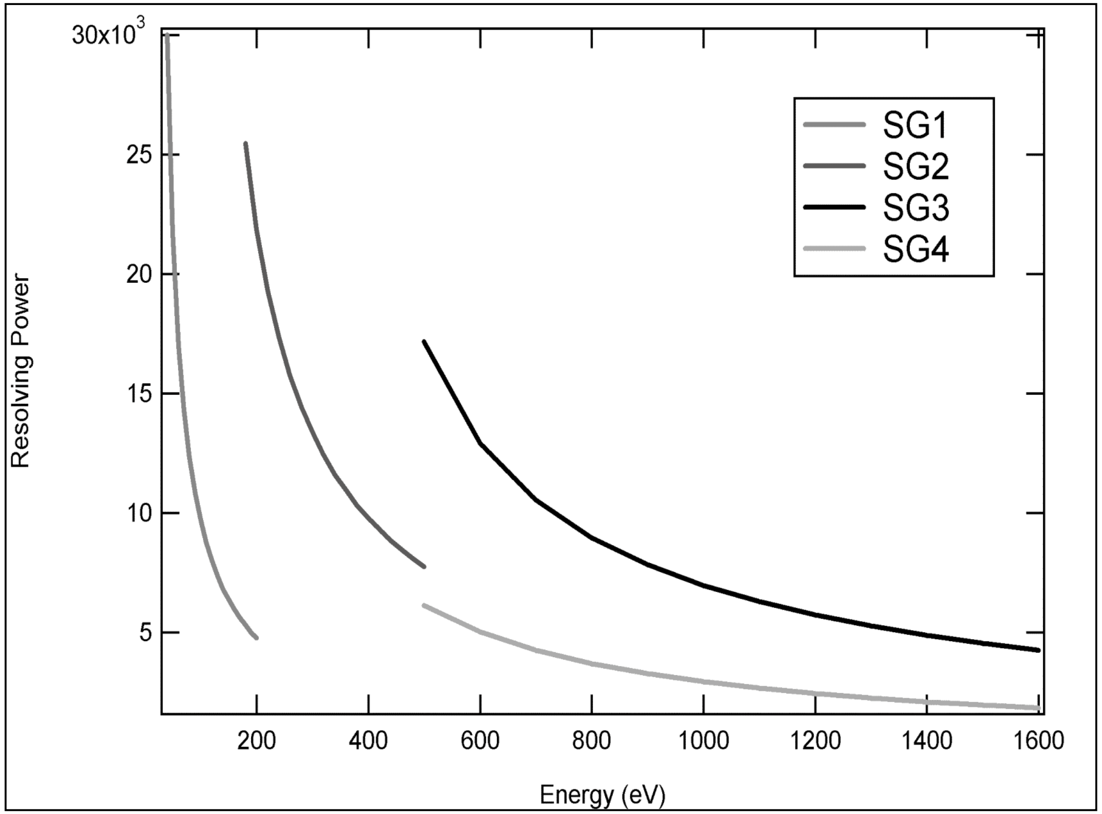


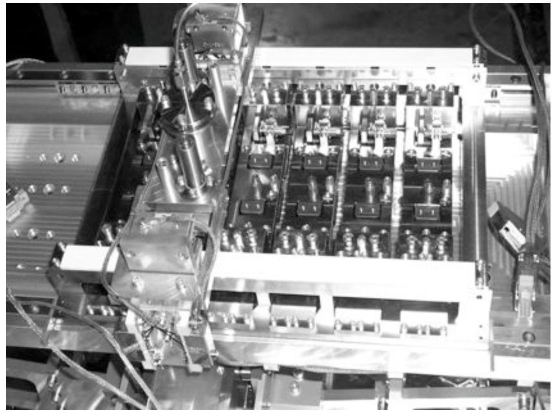


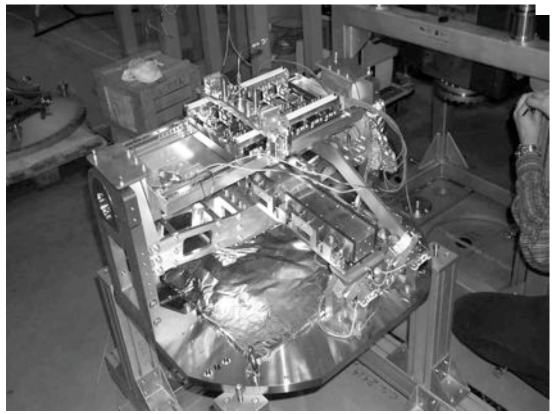
$45/0.0016 \approx 28000$

Resolving Power

Typical Spherical grating monochromator resolving power



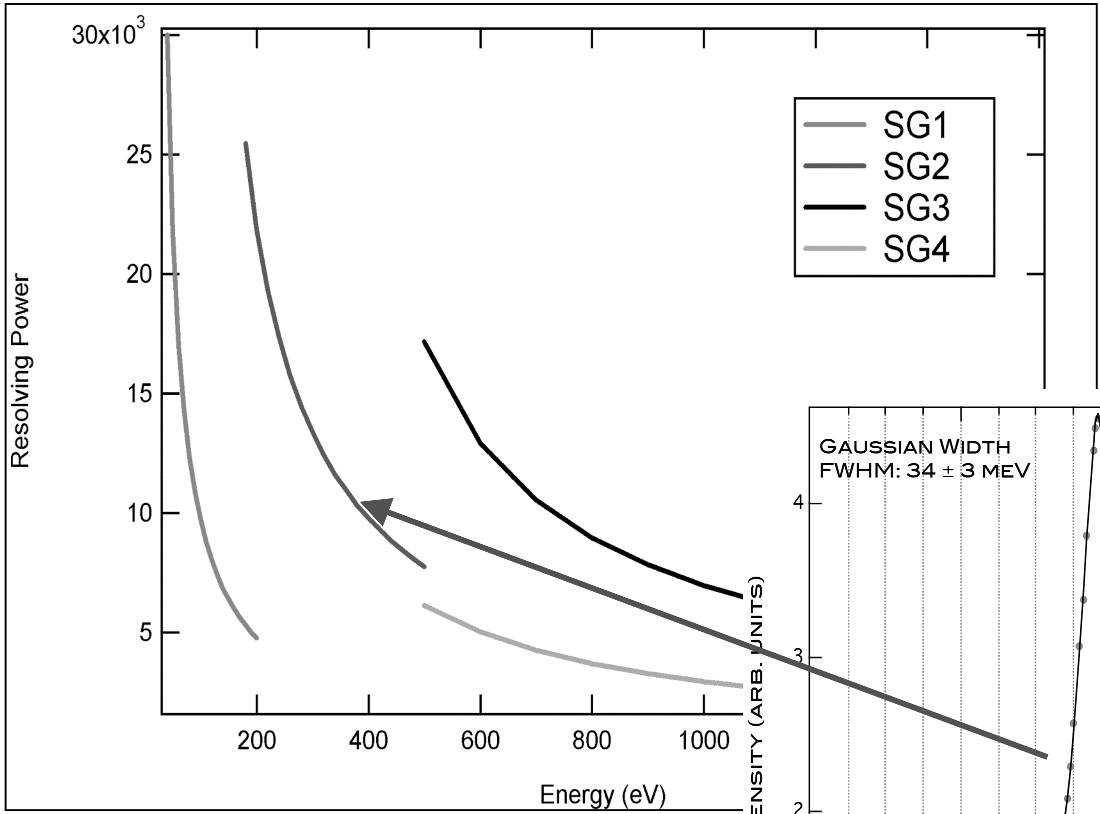


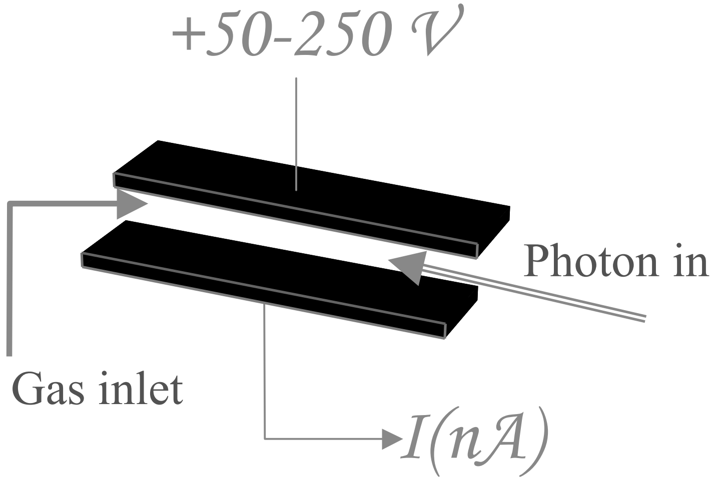


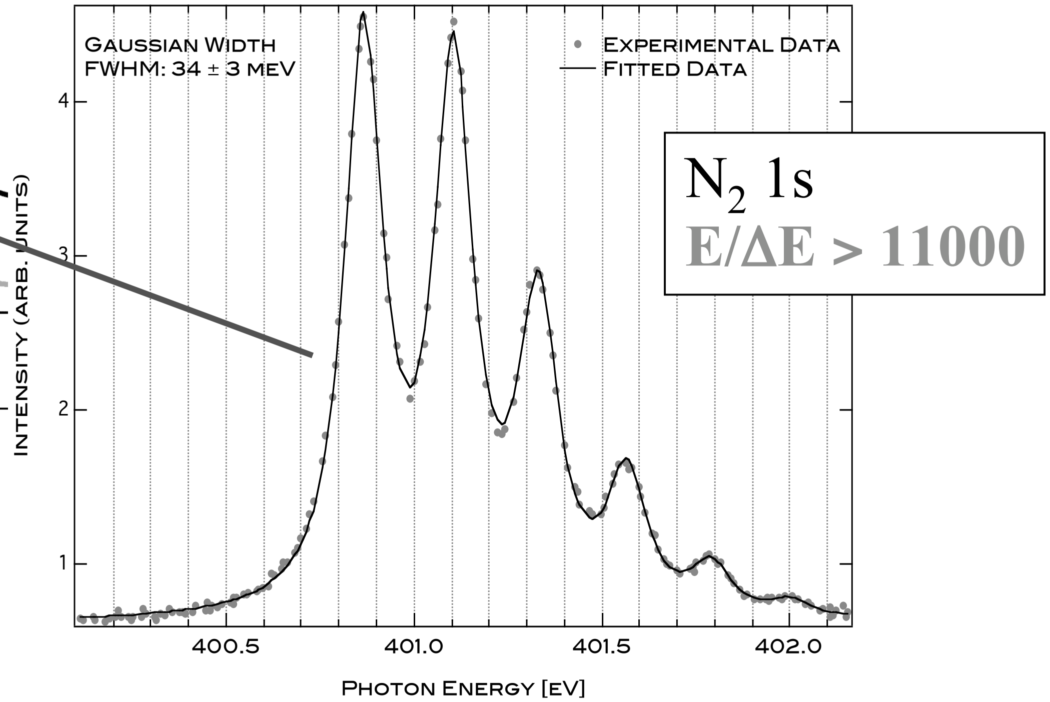
$$\frac{E}{\Delta E} = \frac{\lambda}{\Delta\lambda} = \frac{\lambda N k r'}{s' \cdot \cos(\beta)}$$

Resolving Power measurement

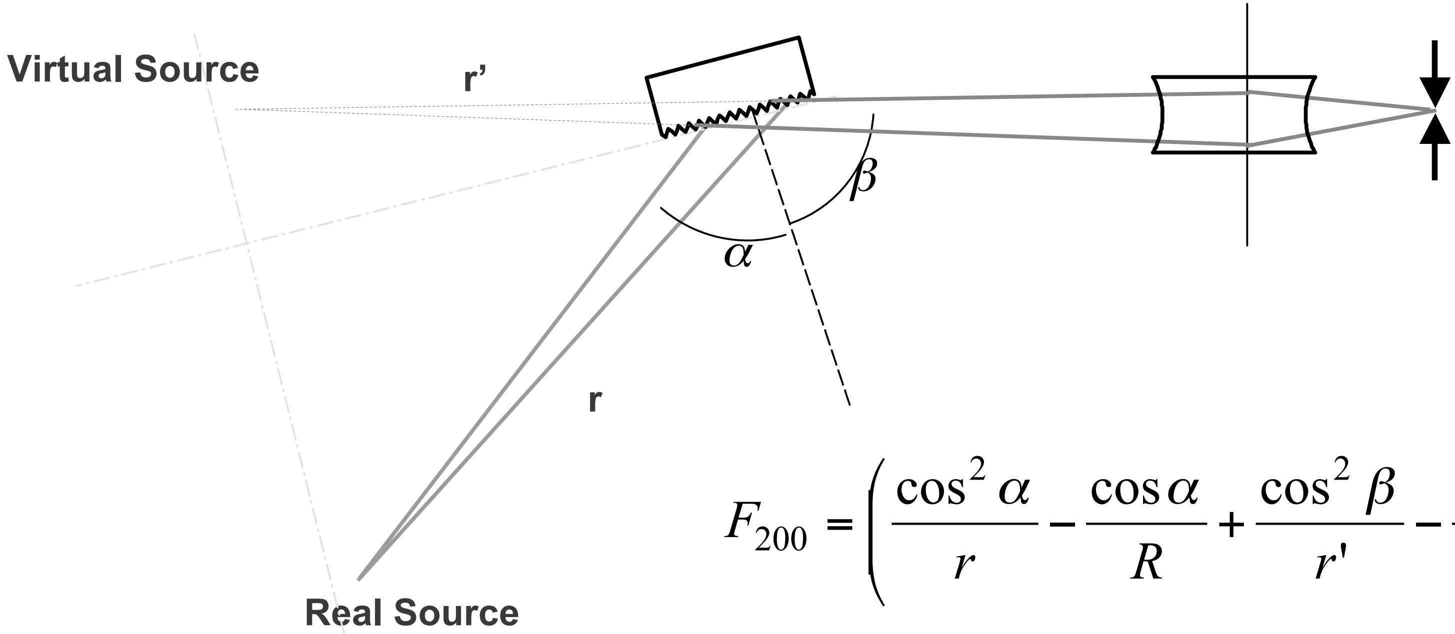
Typical Spherical grating monochromator resolving power







Plane Grating



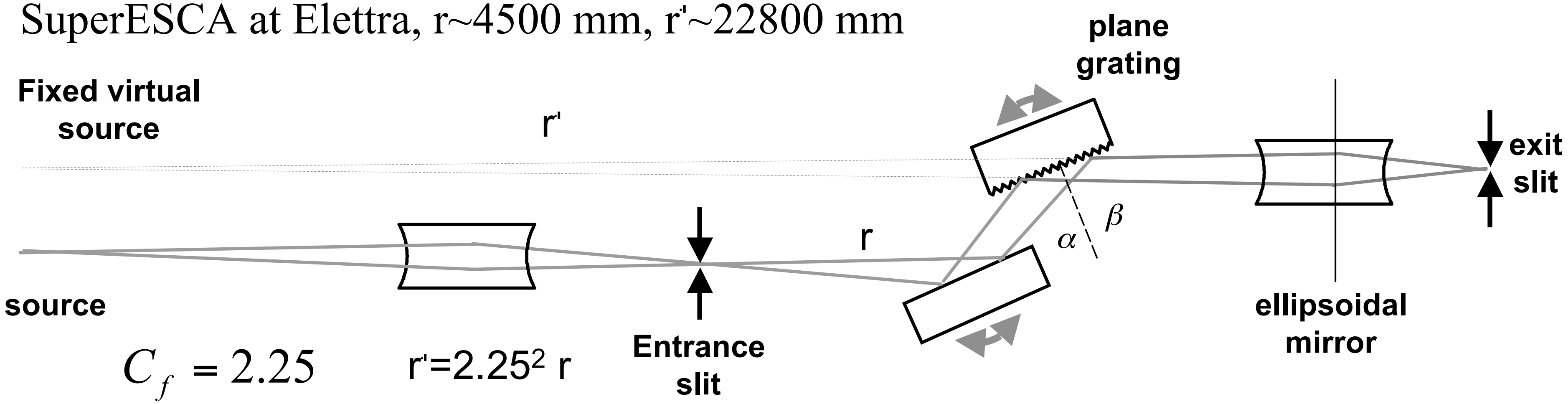
$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right)$$

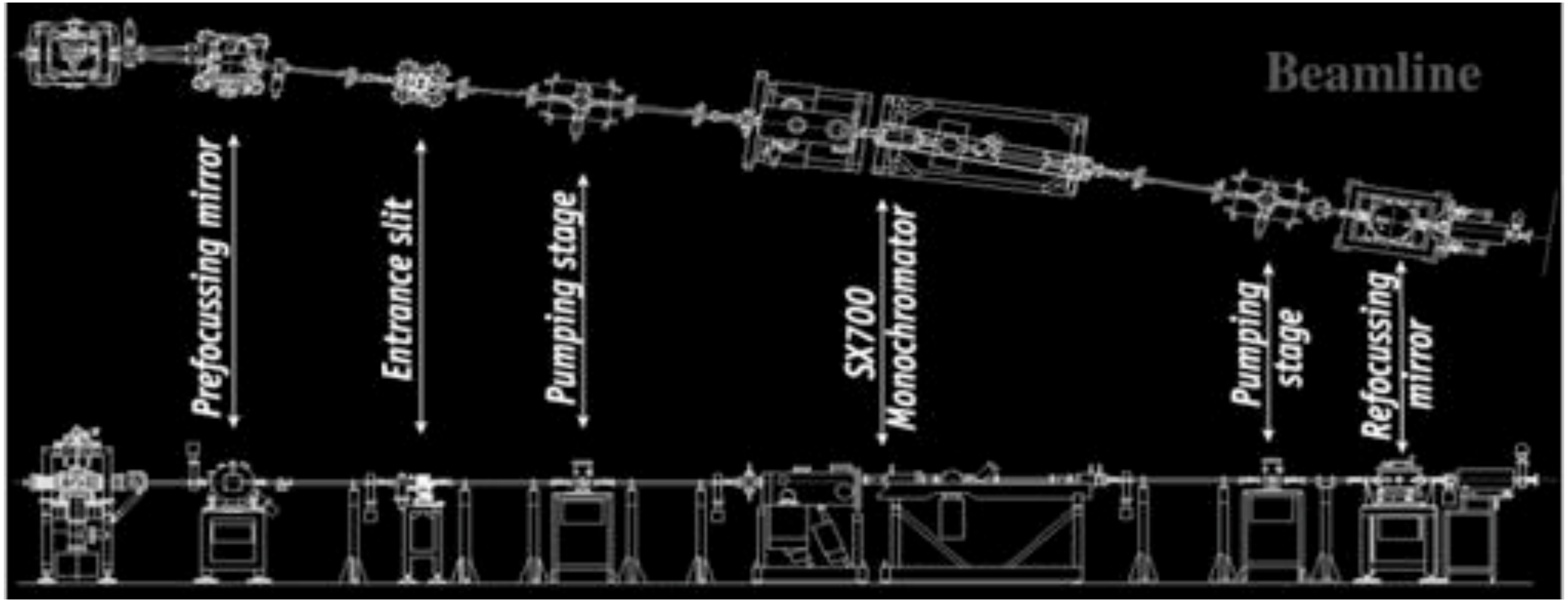
$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{\infty} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{\infty} \right) = 0 \qquad \frac{\cos^2\alpha}{r} + \frac{\cos^2\beta}{r'} = 0 \qquad r' = -r\frac{\cos^2\beta}{\cos^2\alpha}$$

$$C_f = \frac{\cos\beta}{\cos\alpha} \qquad |r'| = r\,C^2{}_f$$

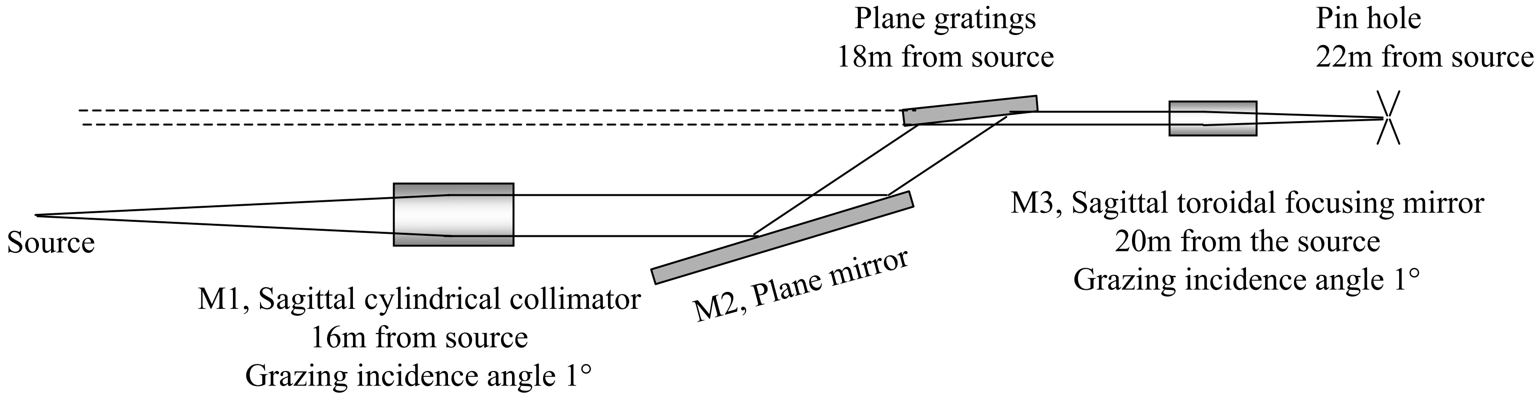
SX 700

SuperESCA at Elettra, r~4500 mm, r'~22800 mm





Collimated light SX 700

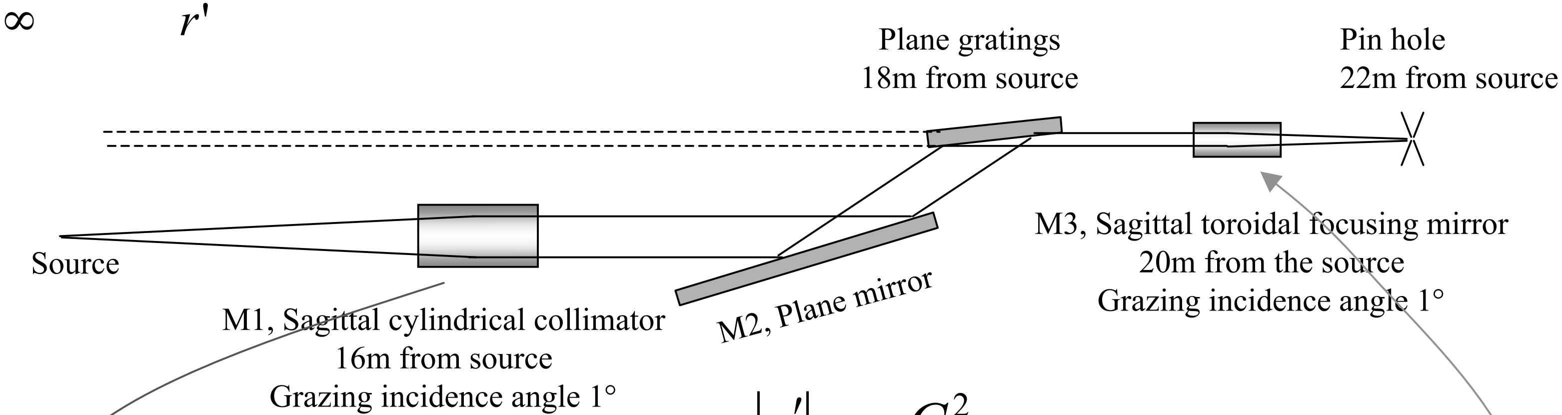


$$F_{200} = \left(\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{\infty} + \frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{\infty} \right) = 0 \qquad \frac{\cos^2 \alpha}{\infty} + \frac{\cos^2 \beta}{r'} = 0 \quad \Rightarrow \quad r' = \infty$$

One can select to work in:
High resolution mode (accept to loose some flux)
High efficiency mode (accept a reduction of resolution)
High order suppression mode (with a typical appreciable reduction of flux)

Collimated Light SX 700

$$\frac{\cos^2\alpha}{\infty}+\frac{\cos^2\beta}{r'}=0 \quad \Rightarrow \quad r'=\infty$$



$$|r'| = r\, C^2{}_f$$

In principle one can work with any C_f value, higher or lower than 1 but…

This mirror do not produce a perfectly collimated light (NEVER)

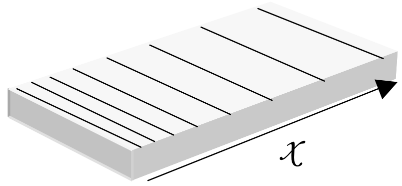
→ divergence changes with C_f

This mirror is no more able to focus the radiation

Problem amplified for C_f value lower than 1

Variable groove desnity gratings

Groove density D varies along the grating surface: $\mathcal{D}(x) = \mathcal{D}_0 + \mathcal{D}_1 x + \mathcal{D}_2 x^2 + \mathcal{D}_3 x^3 + \cdots$



$$F_{200} = \frac{1}{2}\left(-n\lambda\mathcal{D}_1 + \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} + \frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R}\right)\right)$$

$$F_{300} = -\frac{1}{3}n\lambda\mathcal{D}_2 + \frac{1}{2}\left[\left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R}\right)\frac{\sin\alpha}{r} + \left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R}\right)\frac{\sin\beta}{r'}\right]$$

$$F_{200} = \frac{1}{2}\left(-n\lambda\mathcal{D}_1 + \left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta}{r'}\right)\right)$$

A plane grating can focus!

$$F_{100} = -n\lambda D + (\sin\alpha - \sin\beta)$$

$$\sin\beta = \sin\alpha - n\lambda D$$