



# Instabilities Part I: Introduction – multiparticle systems, macroparticle models and wake functions

Giovanni Rumolo and Kevin Li

We will look conceptually into **collective effects** and their **impact on beams**. We will first introduce **multiparticle systems** and investigate **multiparticle effects**. This will be important to effectively describe collective effects. We will then introduce the concept of **wake fields** as one very important collective effect.

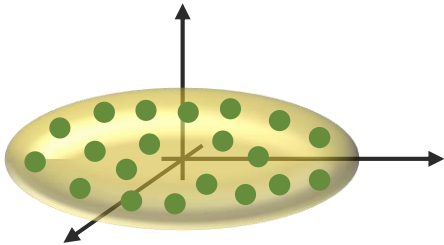
- Part 1: Introduction – multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - Basic concepts
    - Particles and macroparticles – macroparticle distributions
    - Beam matching
    - Multiparticle effects – filamentation and decoherence
    - Wakefields as sources of collective effects

# What are collective effects?

- We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.

# What are collective effects?

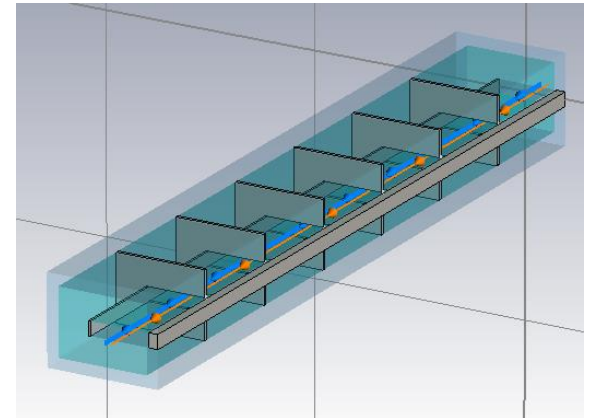
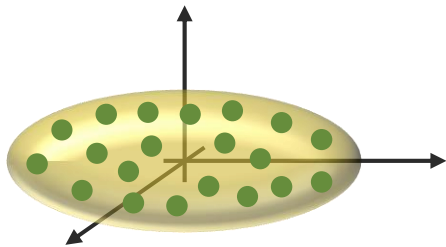
- A charged particle beam is generally described as a **multiparticle system** via the **generalized coordinates** and **canonically conjugate momenta** of all of its particles – this makes up a distribution in the 6-dimensional beam phase space which can be described by a **particle distribution function**.
- Hence, we will study the **evolution of the beam phase space** (or particle distribution function):



$$\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s)$$

# What are collective effects?

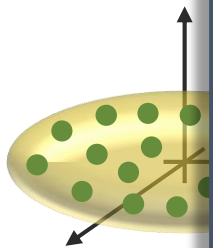
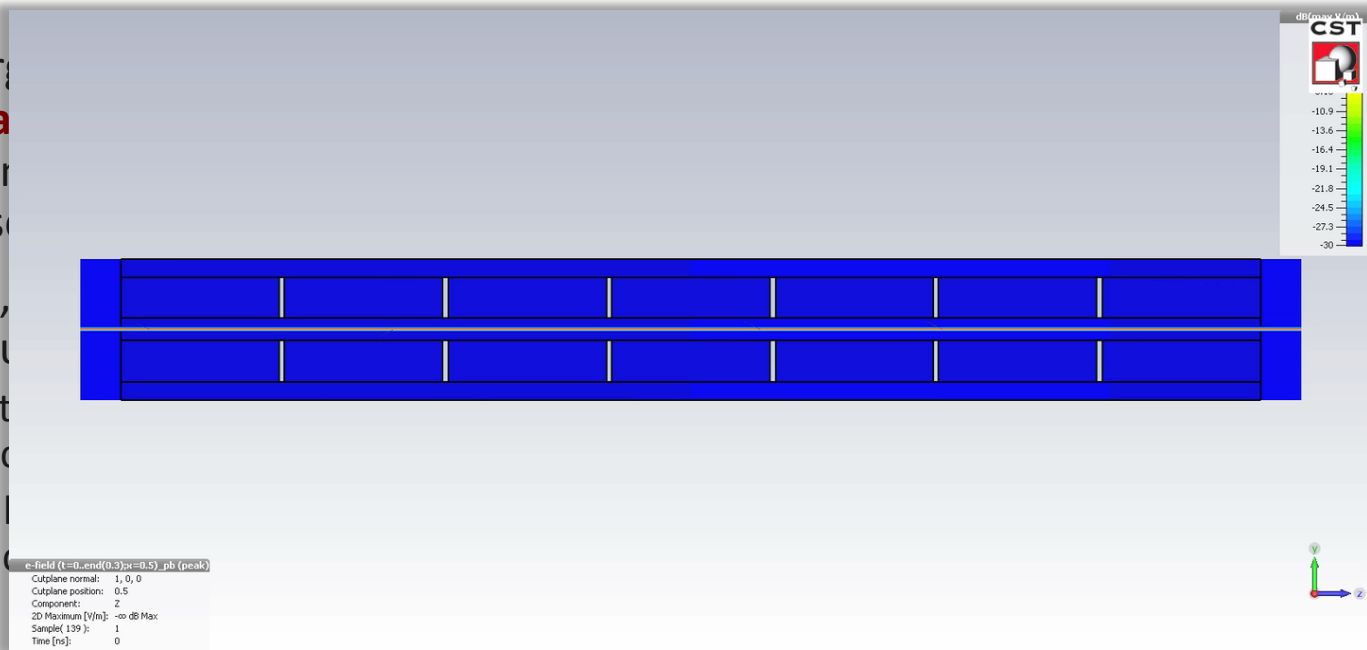
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- Hence, we will study the **evolution of the beam phase space** (or particle distribution function):
  - Optics defined by the machine lattice provides the **external force fields** (magnets, electrostatic fields, RF fields), e.g. for guidance and focusing
  - Collective effects add to this **distribution dependent force fields** (space charge, wake fields)



$$\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s) \propto f (F_{\text{extern}} + F_{\text{coll}} (\psi))$$

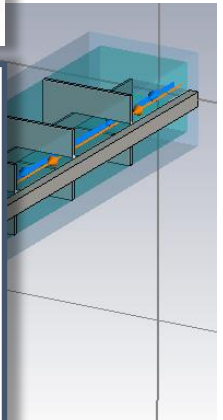
# What are collective effects?

- A charge distribution is **generated** by the particles – this can be described by a **macro-particle**
- Hence, the **macro-particle** distribution is affected by the **fields** of the magnets, **wake fields**, etc.



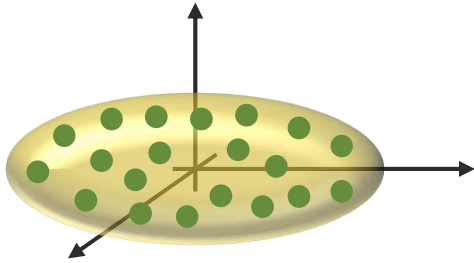
- For a **multiparticle system** this self-consistency equation becomes arbitrarily complex and practically **impossible to solve**
- Obtaining the **multiparticle dynamics** very often requires **computer simulation codes**

$$\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s) \propto f (F_{\text{extern}} + F_{\text{coll}} (\psi))$$



# What is a beam instability?

- A beam becomes unstable when a **moment of its distribution** exhibits an **exponential growth** (e.g. mean positions, standard deviations, etc.), resulting into beam loss or emittance growth!



$$N = \int \psi(x, x', y, y', z, \delta) dx dx' dy dy' dz d\delta$$

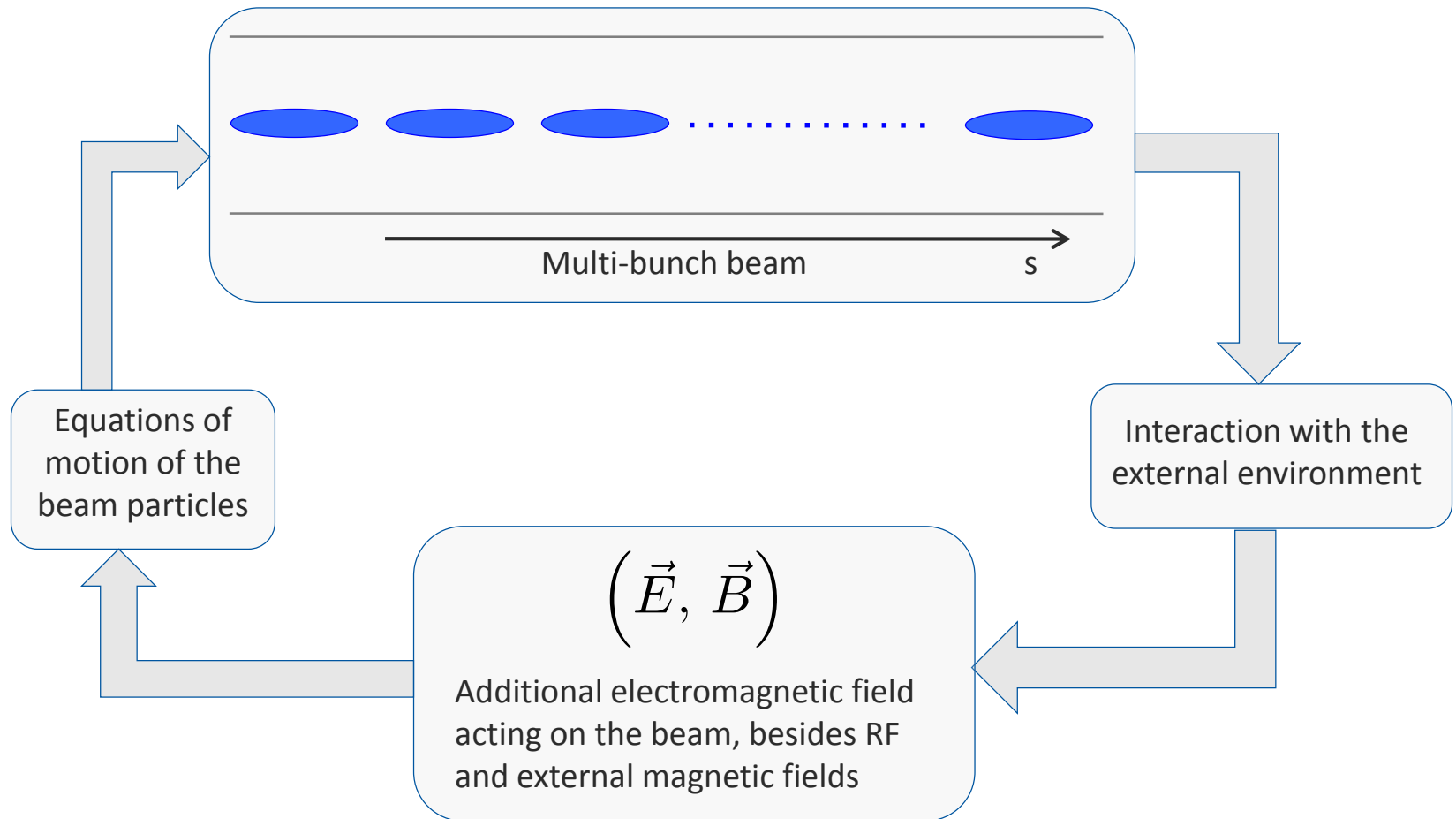
$$\langle x \rangle = \frac{1}{N} \int x \cdot \psi(x, x', y, y', z, \delta) dx dx' dy dy' dz d\delta$$

$$\sigma_x^2 = \frac{1}{N} \int (x - \langle x \rangle)^2 \cdot \psi(x, x', y, y', z, \delta) dx dx' dy dy' dz d\delta$$

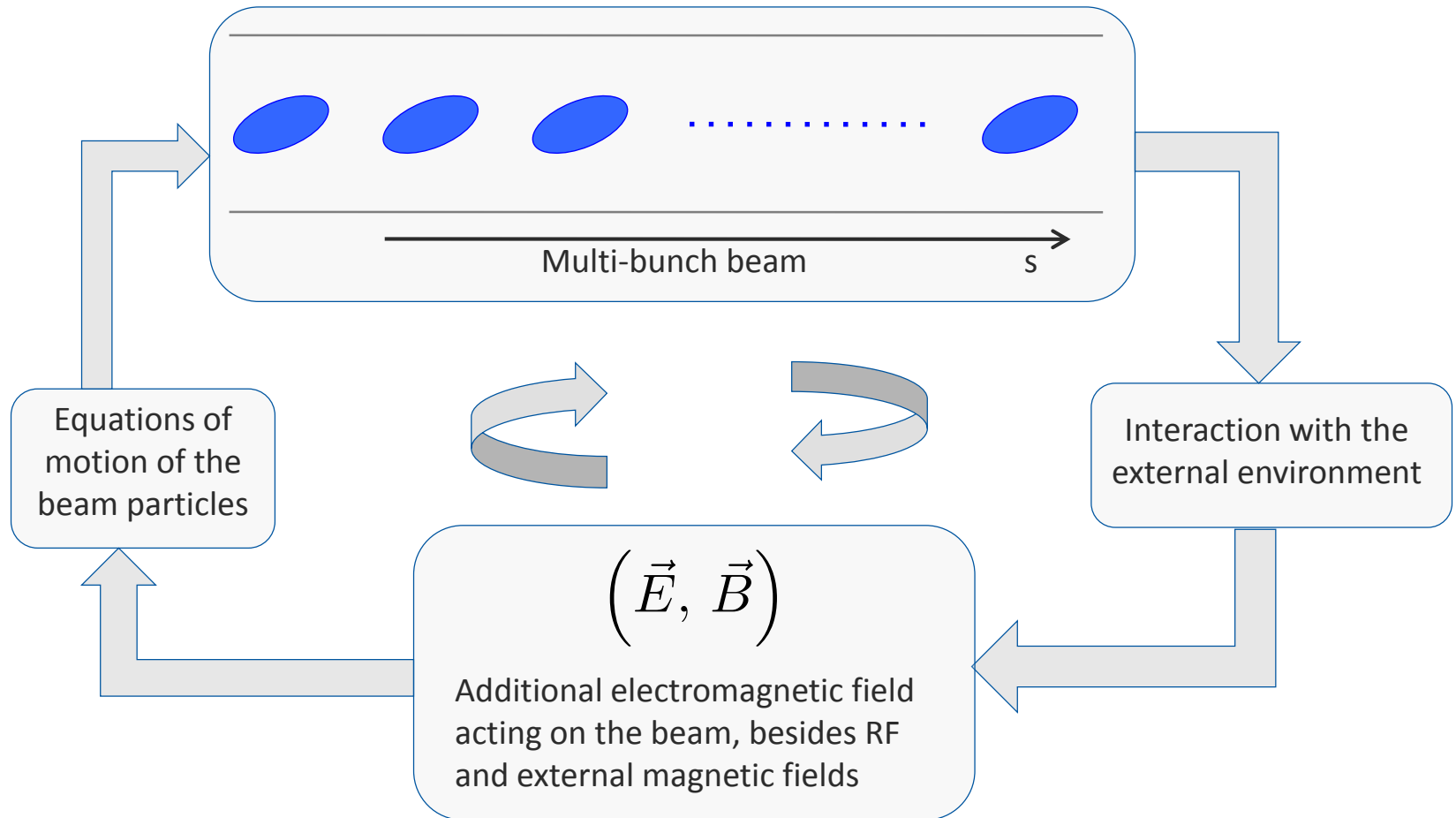
and similar definitions for  $\langle y \rangle, \sigma_y, \langle z \rangle, \sigma_z$



# The instability loop

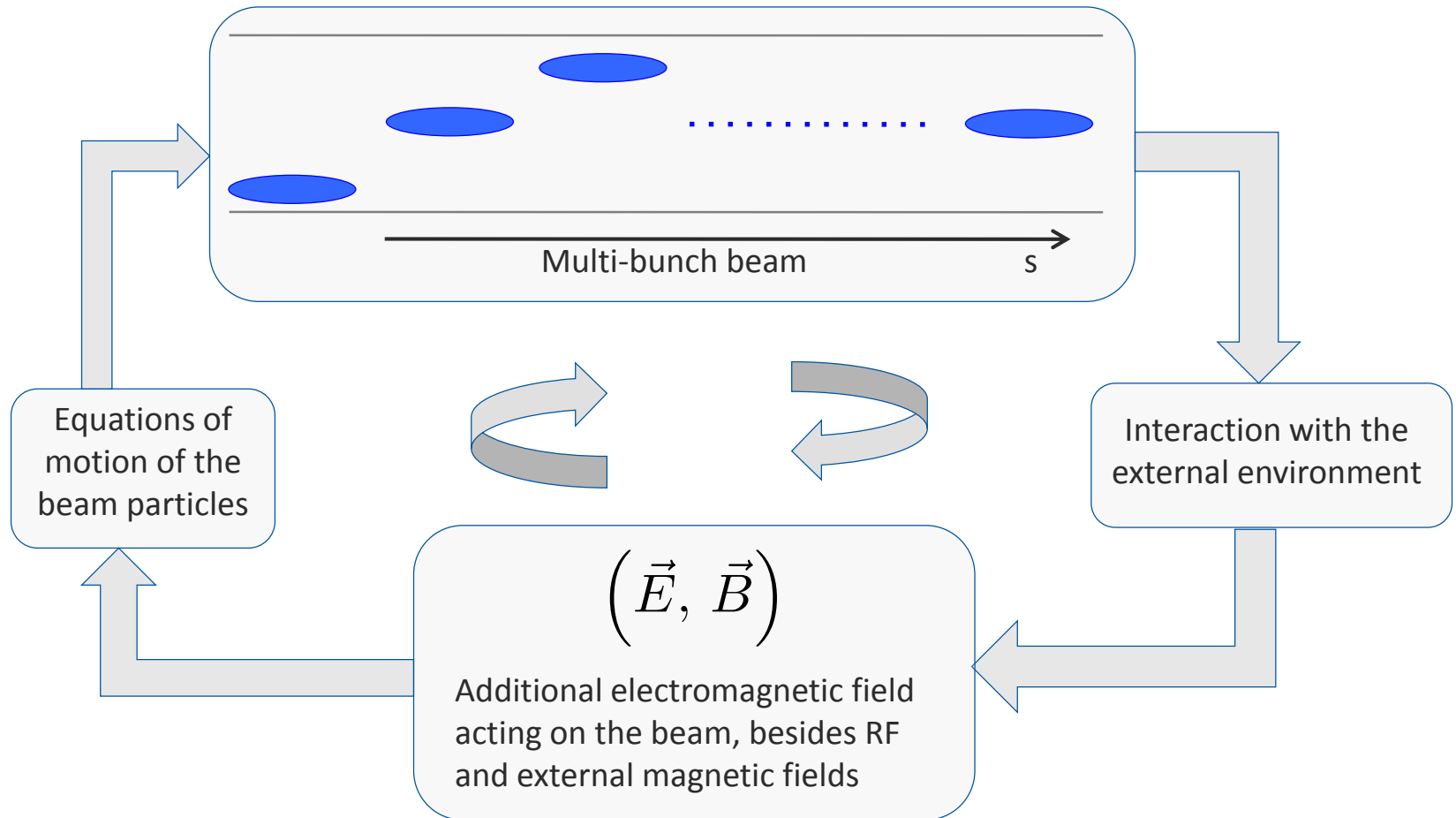


# The instability loop



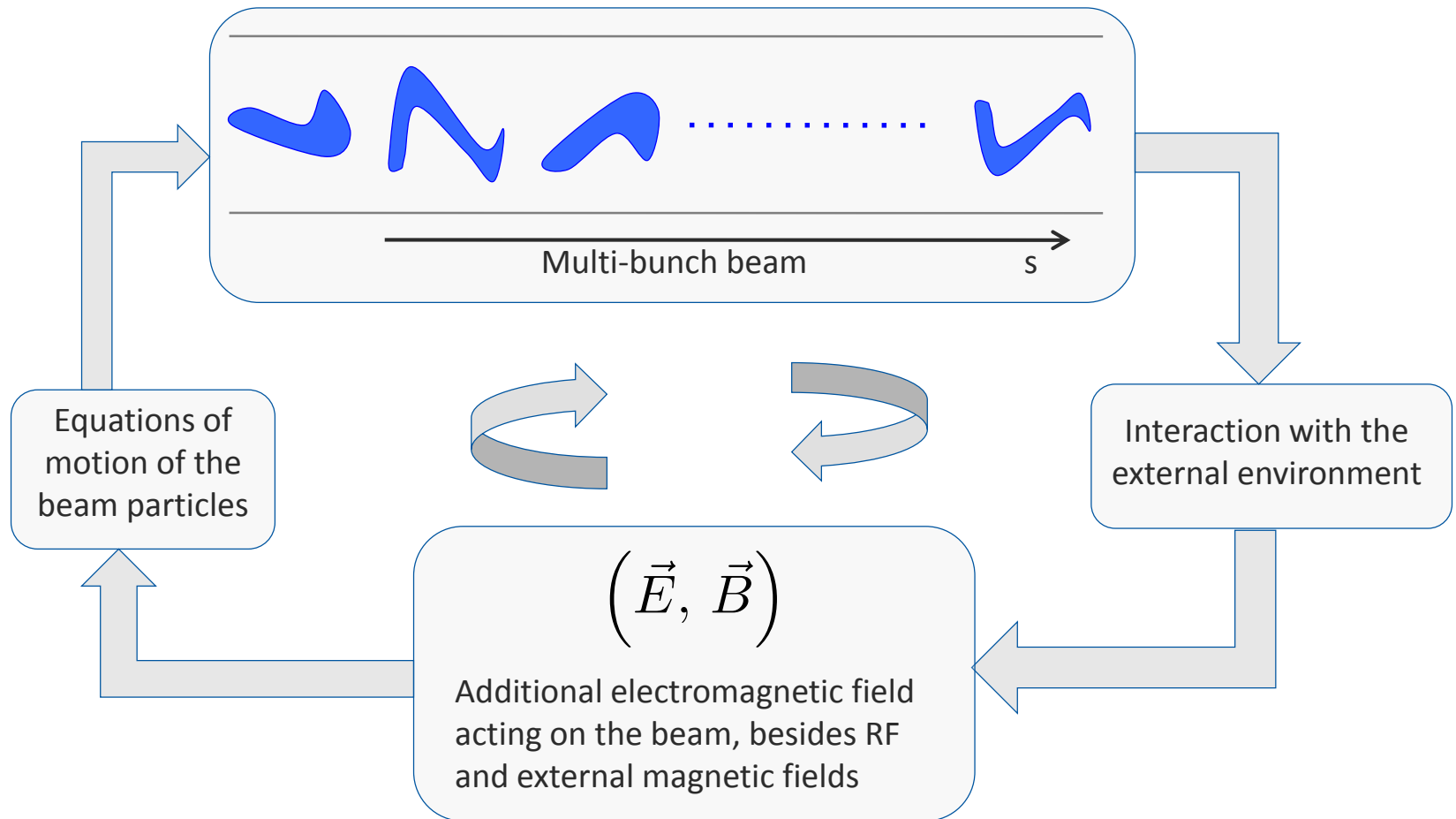
When the loop closes, either the beam will find a new stable equilibrium configuration ...

# The instability loop



... or it might develop an instability along the bunch train ...

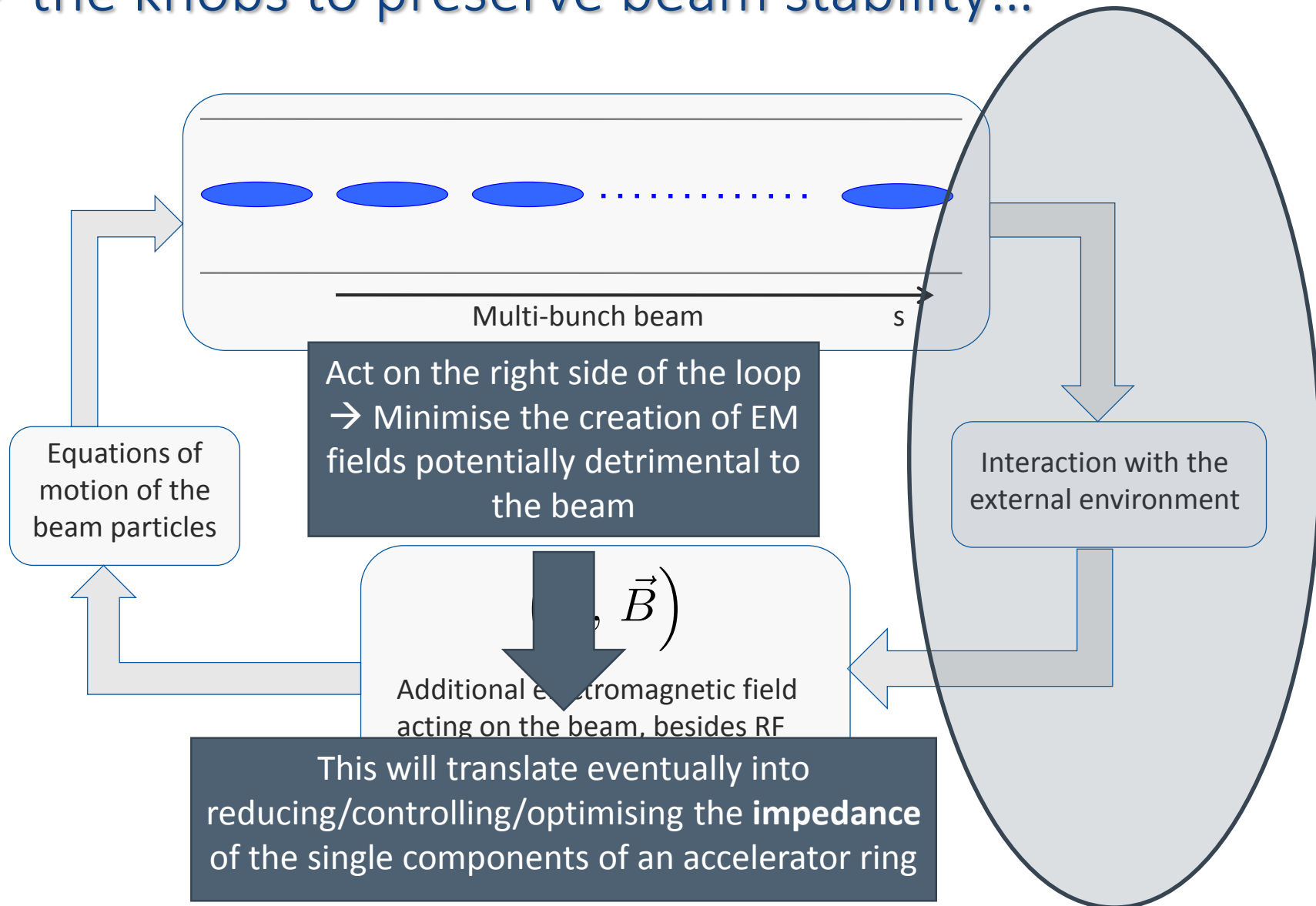
# The instability loop



... or also an instability affecting different bunches independently of each other

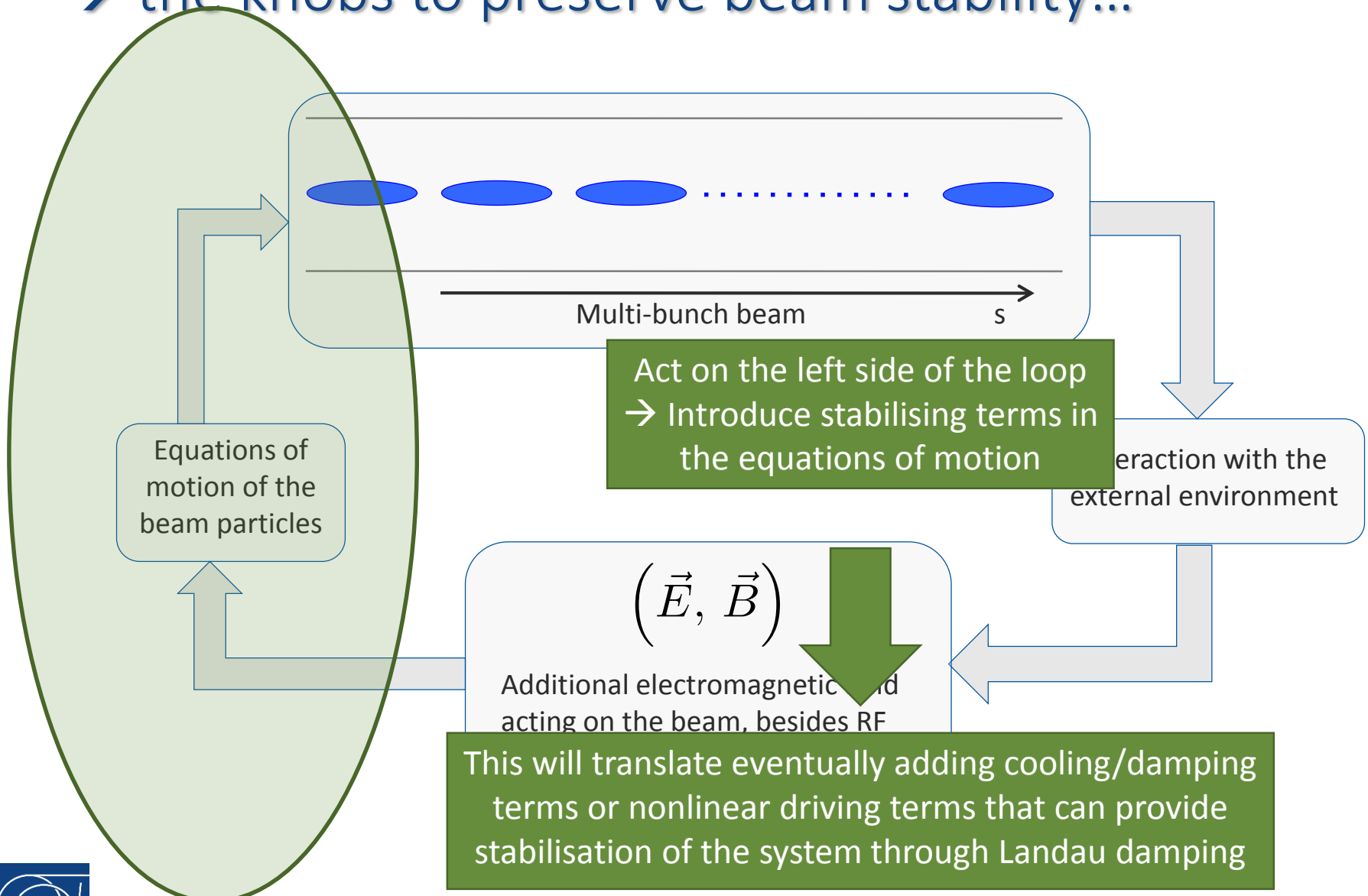
# The instability loop

→ the knobs to preserve beam stability...



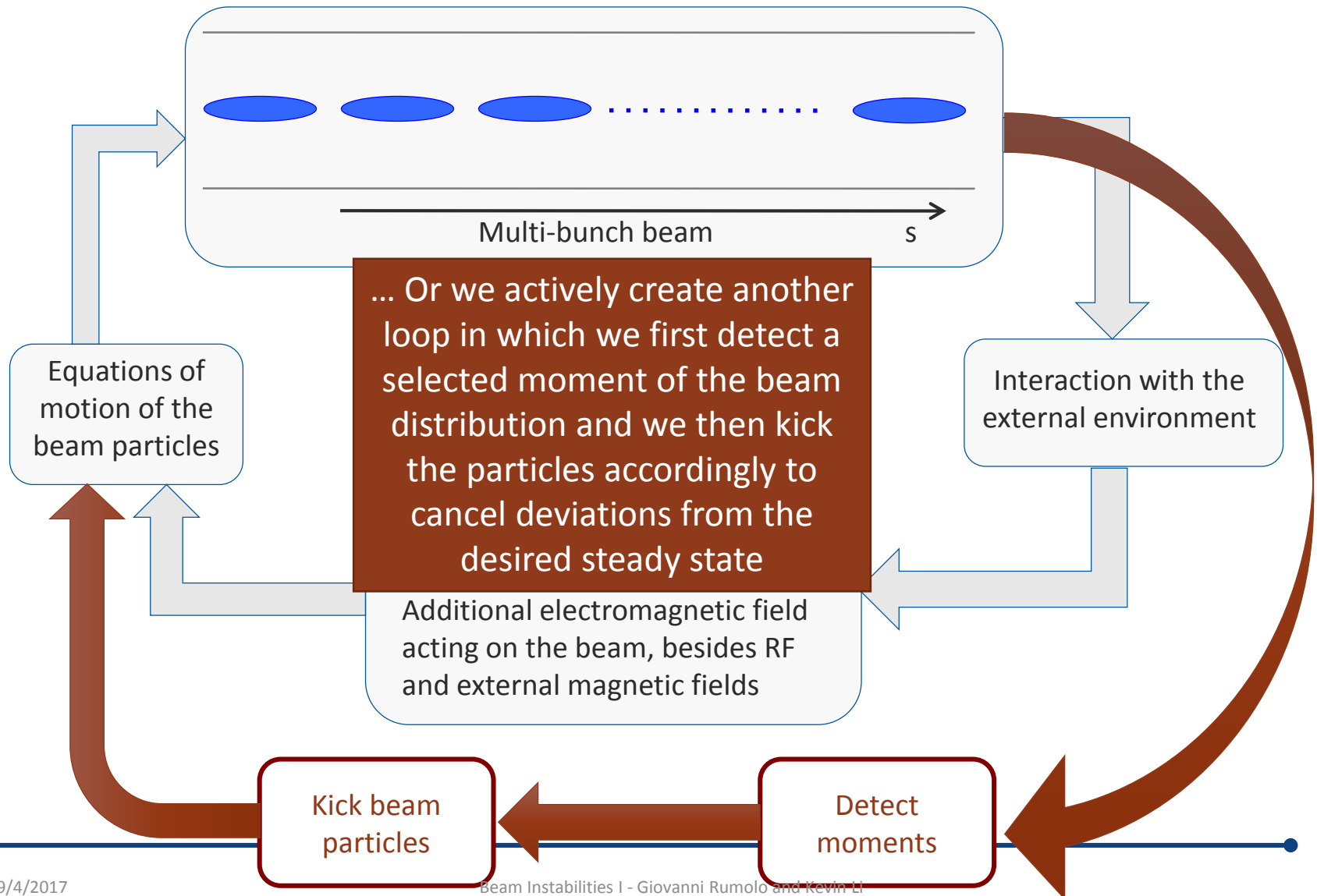
# The instability loop

→ the knobs to preserve beam stability...



# The instability loop

→ the knobs to preserve beam stability...



# Formal description of a beam instability

- Formally, instead of investigating the full set of equations for a multiparticle system, we typically instead describe the latter by a **single particle distribution function**:

$$\psi = \psi(x, x', y, y', z, \delta, s)$$

where

$$d\mathbf{N}(s) = \psi(x, x', y, y', z, \delta, s) dx dx' dy dy' dz d\delta$$

- The accelerator environment constitutes a **Hamiltonian system** for which:

$$\frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'}, \quad \frac{\partial x'}{\partial s} = -\frac{\partial H}{\partial x}, \quad \frac{d}{ds}\psi = 0$$

Vlasov equation

- It follows for the evolution of this **particle distribution function**:

$$\begin{aligned} \frac{d}{ds}\psi &= \frac{\partial\psi}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial\psi}{\partial x'} \frac{\partial x'}{\partial s} + \frac{\partial}{\partial s}\psi \\ &= \underbrace{\frac{\partial\psi}{\partial x} \frac{\partial H}{\partial x'} - \frac{\partial\psi}{\partial x'} \frac{\partial H}{\partial x}}_{[\psi, H]} + \frac{\partial}{\partial s}\psi = 0 \end{aligned}$$

$[\psi, H]$  Poisson bracket



# Formal description of a beam instability

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s} \psi = [\mathbf{H}, \psi]$$

- With the Hamiltonian composed of **an external** and **a collective part**, and the particle distribution function decomposed into **an unperturbed part** and **a small perturbation** one can write

$$\frac{\partial}{\partial s} \psi = [\mathbf{H}_0 + \mathbf{H}_1, \psi_0 + \psi_1]$$

- This becomes to **first order**

$$\frac{\partial}{\partial s} \psi_1 = [\mathbf{H}_0, \psi_1] + [\mathbf{H}_1(\psi_0 + \psi_1), \psi_0]$$

Linearization in  $\psi_1$ :  $\dots \propto \hat{\Lambda} \psi_1 = -i \frac{\Omega}{\beta c} \psi_1$

$$\implies \psi_1(s) = \exp\left(-i \frac{\Omega}{\beta c} s\right) \psi_1(0)$$

We are looking for the EV of the evolution  
→ **becomes an EV problem!**

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- With the Hamiltonian composed of an external and a collective part, and the particle distribution function decomposed into an unperturbed part and a perturbation...

We call these distinct eigenvalues  $\psi_1$  **a bunch or a beam mode**.

The mode and thus for example also an instability is fully characterized by a single number:

**the complex tune shift  $\Omega$**

- This becomes to first order

$$\frac{\partial}{\partial s} \psi_1 = [H_0, \psi_1] + [H_1(\psi_0 + \psi_1), \psi_0]$$

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# Formal description of a beam instability

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s} \psi = [\mathbf{H}, \psi]$$

Remark:

- The stationary distribution  $\psi_0$  is the distribution where

$$\frac{\partial}{\partial s} \psi = [\mathbf{H}_0, \psi_0] = 0$$

- In particular, a distribution is always stationary if

$$\psi_0 = \psi_0(\mathbf{H}_0), \quad \text{as} \quad [\mathbf{H}_0, \psi_0(\mathbf{H}_0)] = 0$$

Solving for or finding the stationary solution for a given  $H_0$  (which in fact represents the machine ,potential‘) will be later referred to as **matching**.

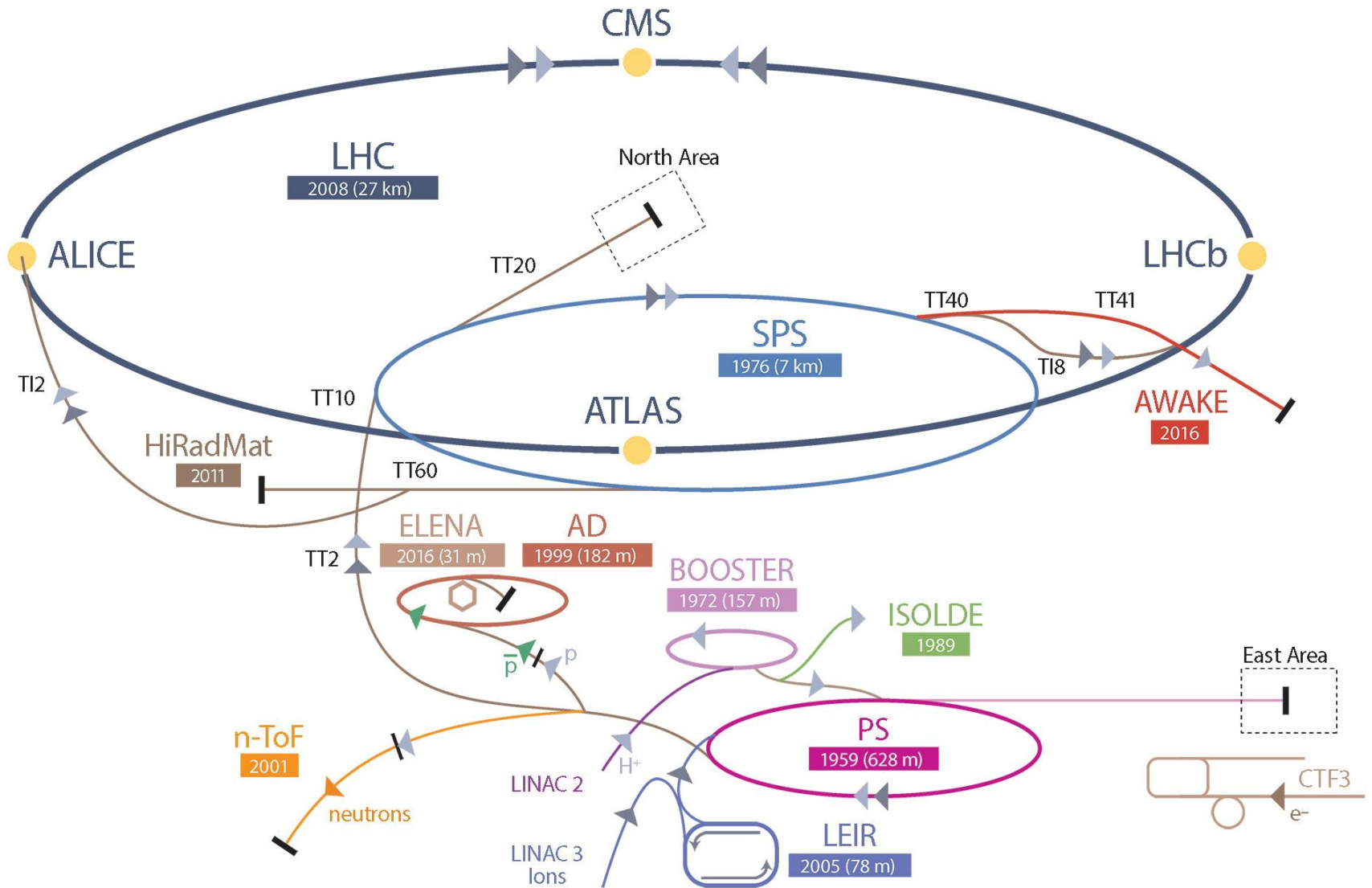
$$\Rightarrow \psi_1(s) = \exp\left(-i \frac{\Omega}{\beta c} s\right) \psi_1(0)$$

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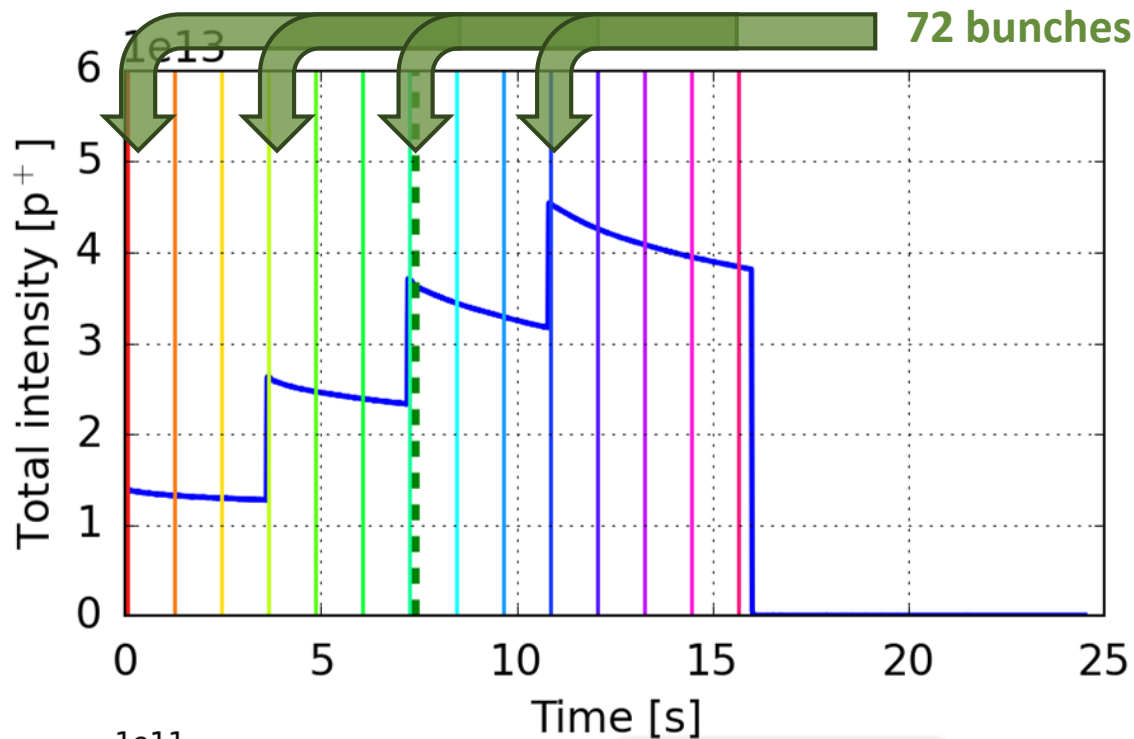
# Why worry about beam instabilities?

- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)
  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows dimensioning an active feedback system to prevent the instability

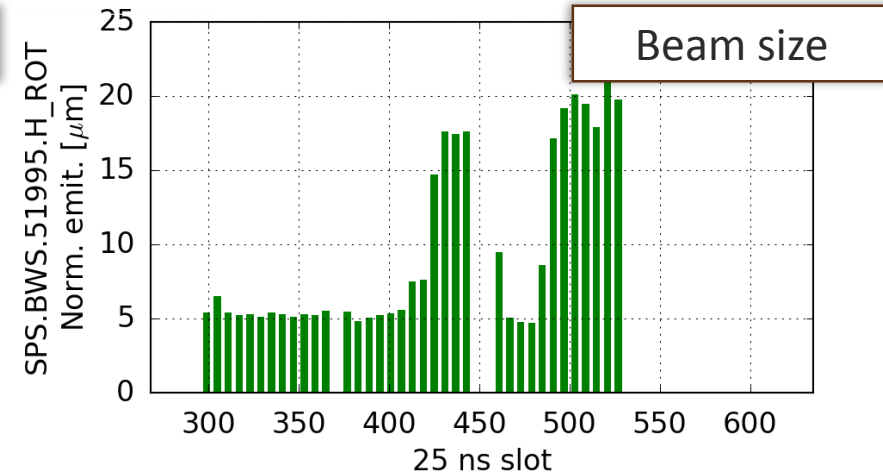
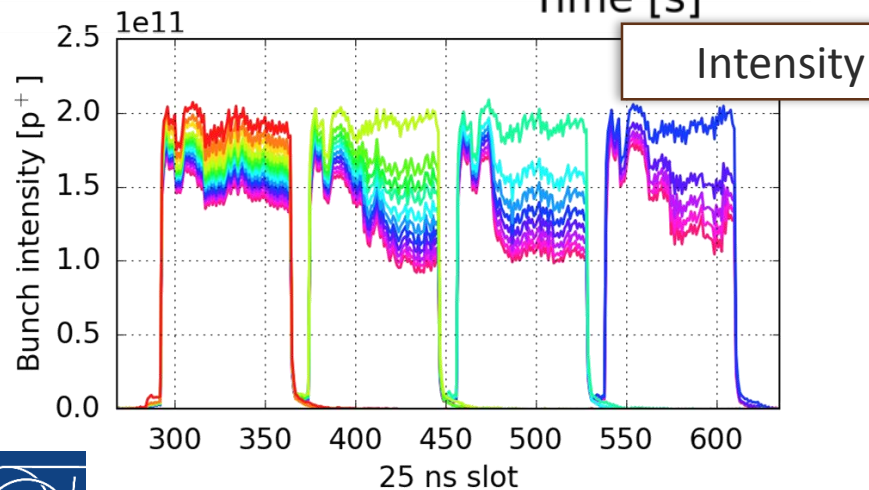
# The CERN accelerator complex



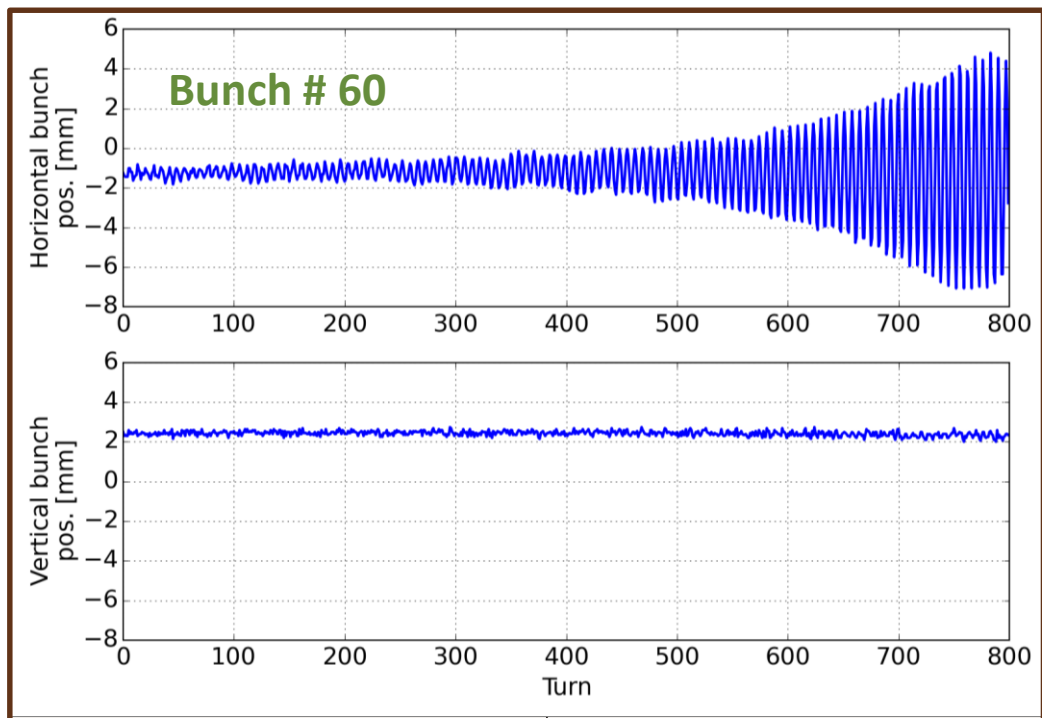
# Coupled bunch instability in the SPS



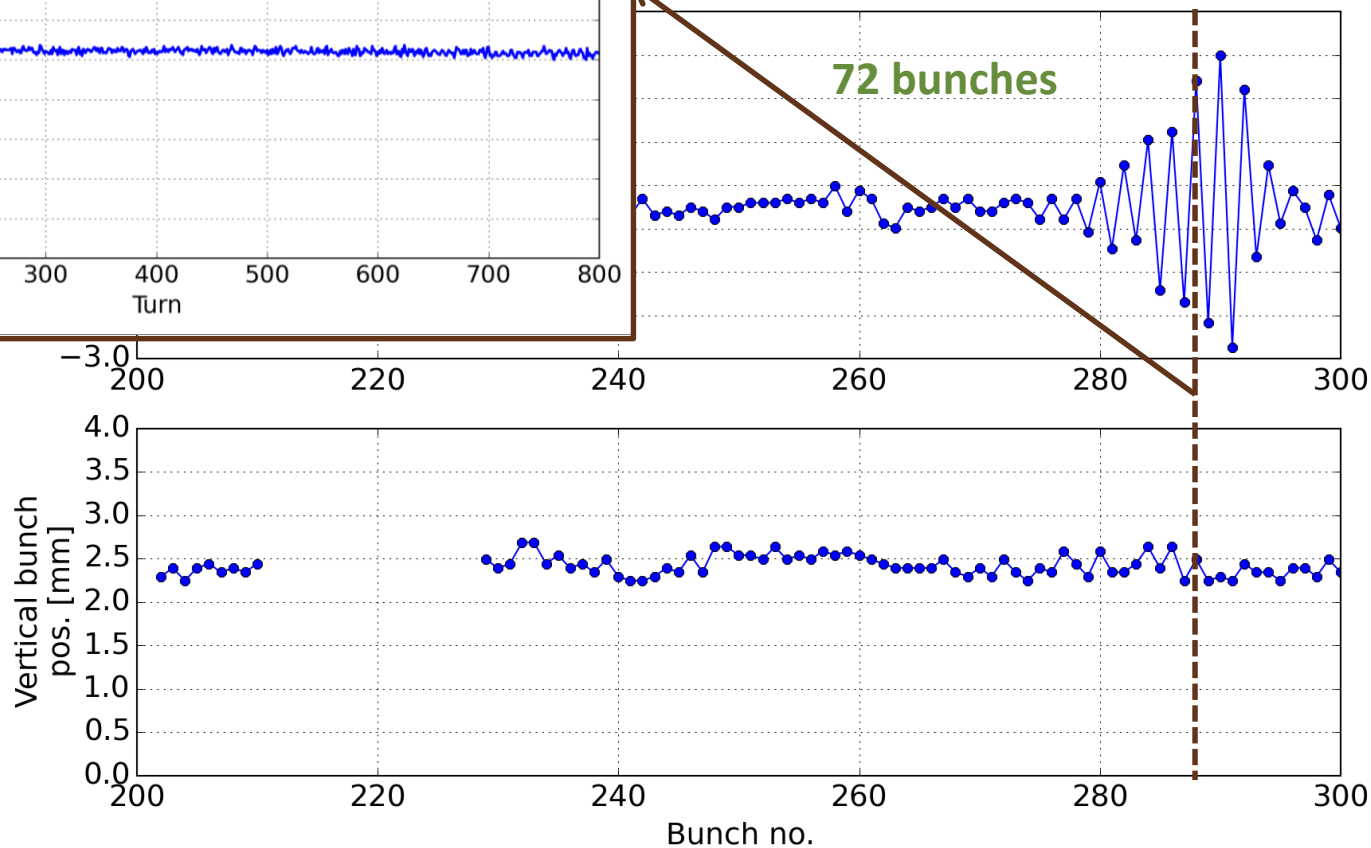
- Injection of 4 batches of 72 bunches trains into the SPS
- Later trains feature **strong losses (intensity)** and **large blow-up (emittance)** – this leads to a **strong loss of beam brightness**



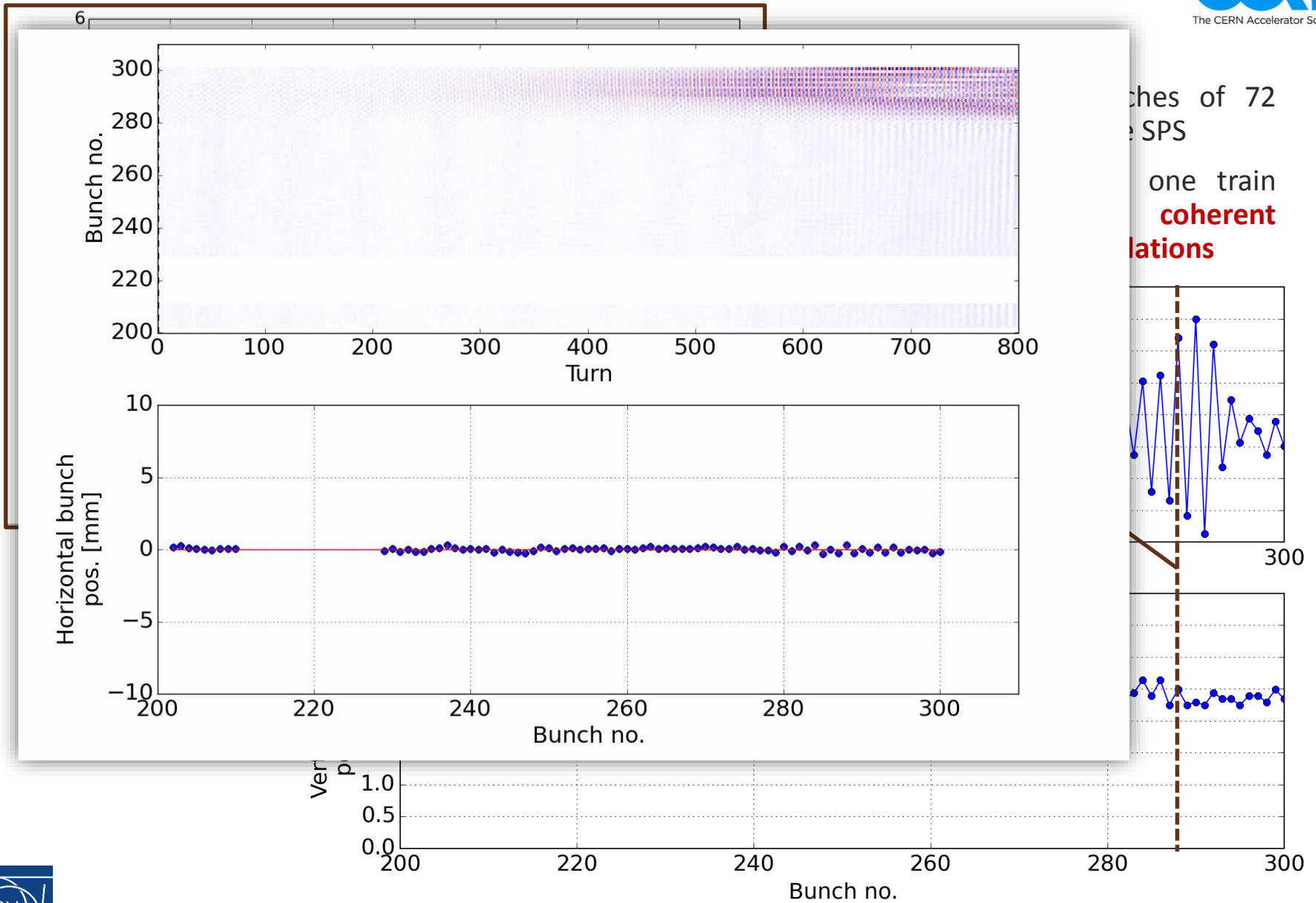
# Coupled bunch instability in the SPS



- Injection of 4 batches of 72 bunch trains into the SPS
- A closer look into one train exhibits **strong coherent coupled bunch oscillations**



# Coupled bunch instability in the SPS

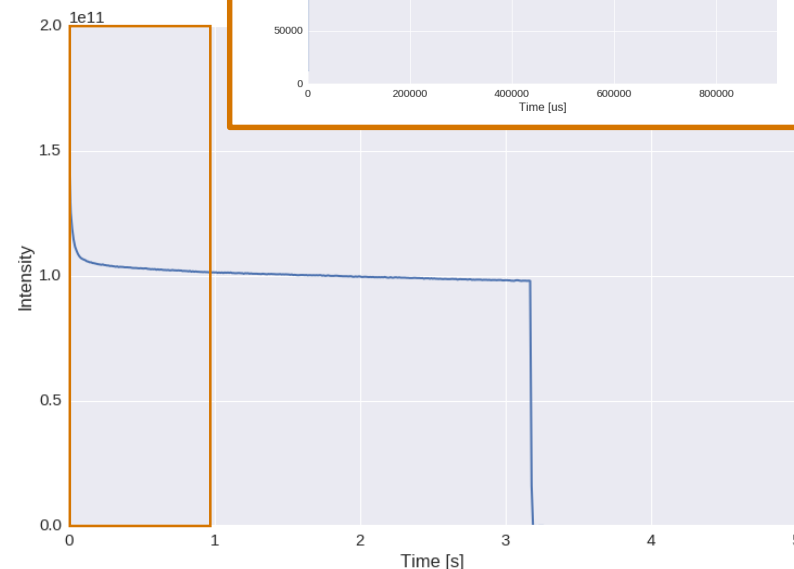
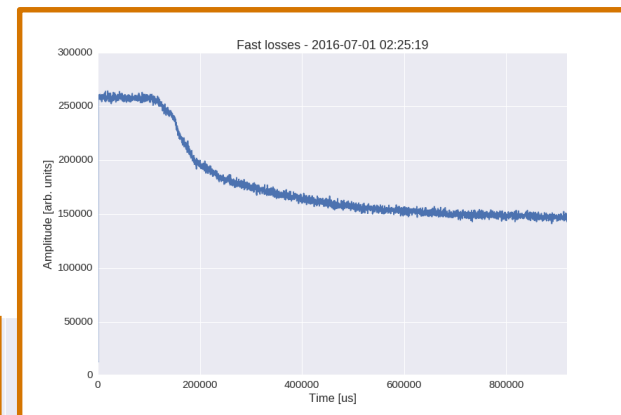
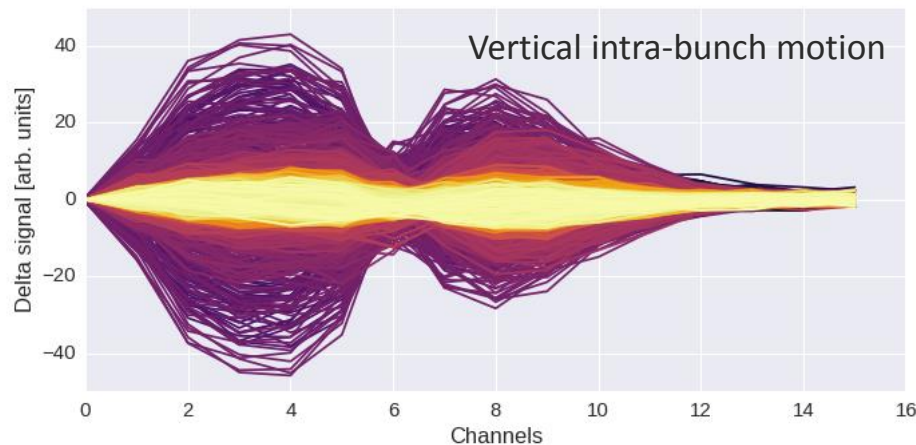
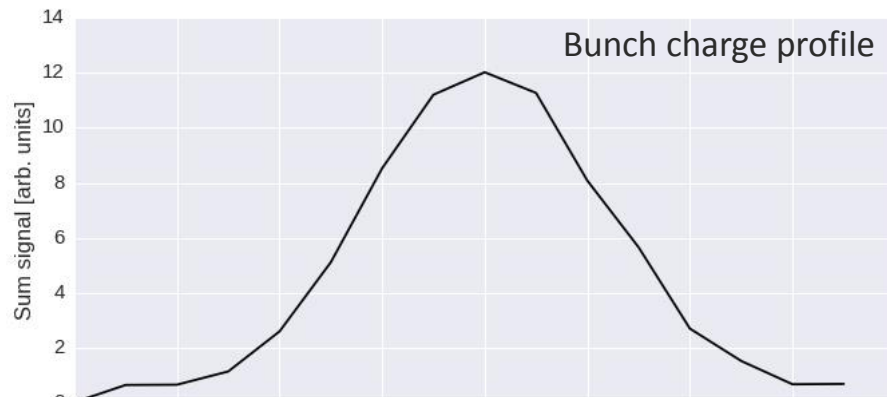




# Single bunch instability in the SPS

BOX data - SnapShot\_07-01-2016-0225

Open loop

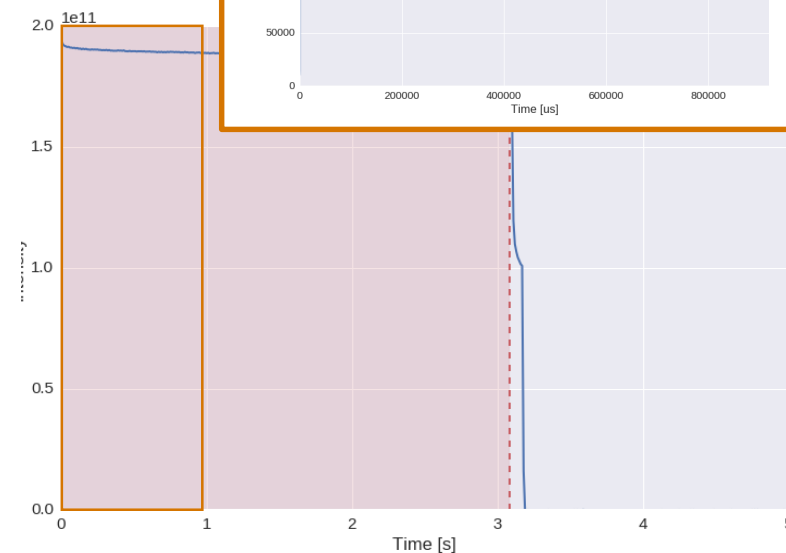
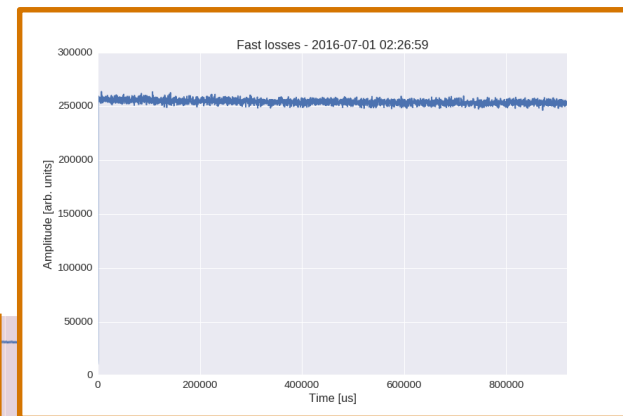
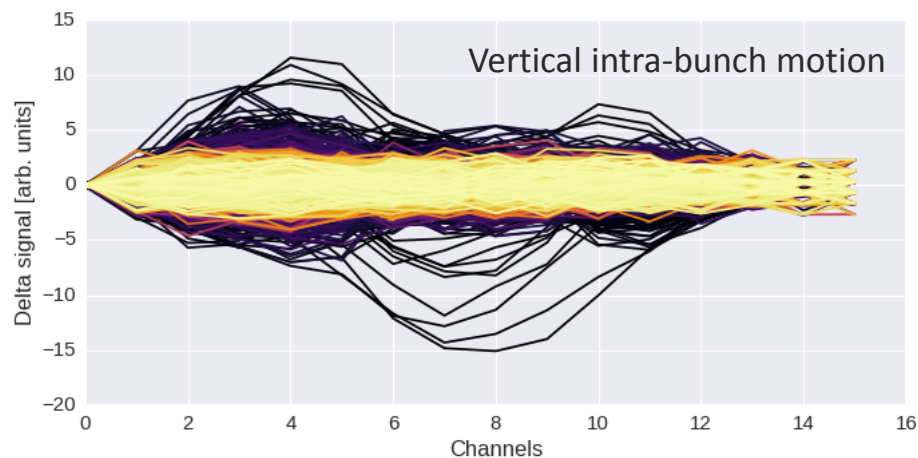
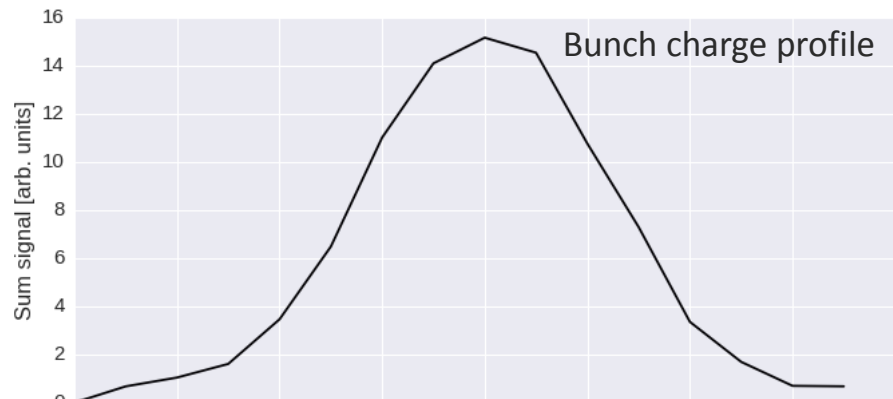


- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)**.

# Single bunch instability in the SPS

BOX data - SnapShot\_07-01-2016-0226

Closed loop

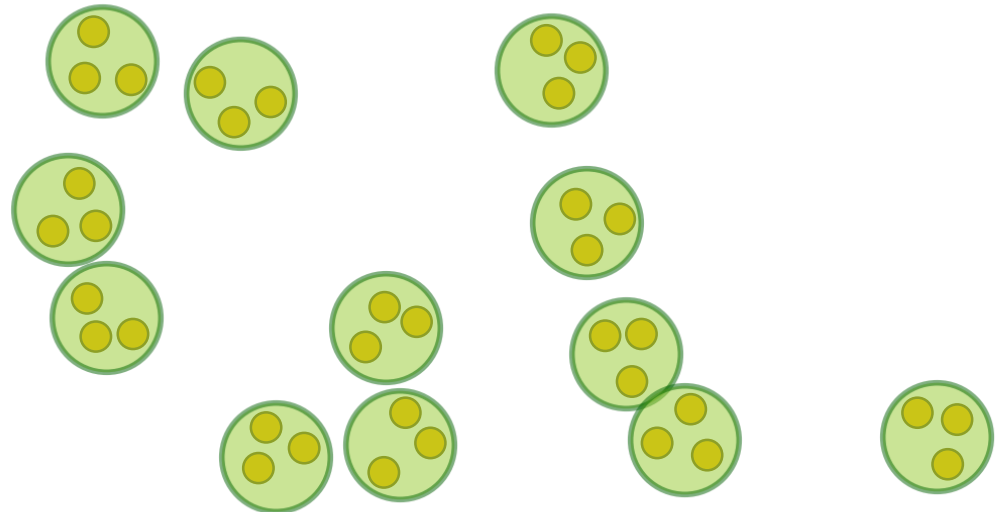
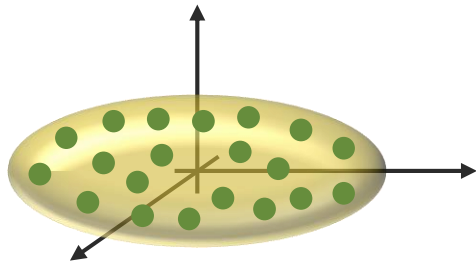


- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)** → can be mitigated by a **wideband feedback system**.

- We have seen the difference between **external forces** and **self-induced forces** which lead to **collective effects**.
- We have seen schematically how these collective effects can induce **coherent beam instabilities and some knobs to avoid them**.
- We have seen **examples of beam instabilities** and have understood how they can lead to serious **performance limitations**.
  
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# The particle description

- As seen earlier, and especially for the analytical treatment, we can represent a charged particle beam via a **particle distribution function**.
- In computer simulations, a charged particle beam is still represented as a multiparticle system. However, to be **compatible with computational resources**, we need to rely on **macroparticle models**.
- A **macroparticle** is a numerical **representation** of a **cluster of neighbouring physical particles**.
- Thus, instead of solving the system for the  $N$  ( $\sim 10^{11}$ ) physical particles one can significantly **reduce the number of degrees of freedom** to  $N_{MP}$  ( $\sim 10^6$ ). At the same time one must be aware that this **increases of the granularity** of the system which gives rise to numerical noise.



$$\Psi(x, x', y, y', z, \delta)$$

# Macroparticle representation of the beam

- Macroparticle models permit a **seamless mapping** of realistic systems into a **computational environment** – they are fairly easy to implement

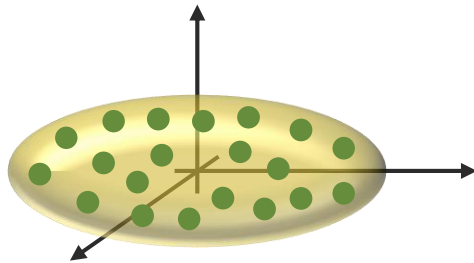
Beam:

$$\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \quad \begin{pmatrix} q_i \\ m_i \end{pmatrix}, \quad i = 1, \dots, N$$

Macroparticlenumber

$$\begin{pmatrix} y_i \\ y'_i \end{pmatrix} \quad \begin{pmatrix} z_i \\ \delta_i \end{pmatrix}$$

Canonically conjugate coordinates and momenta



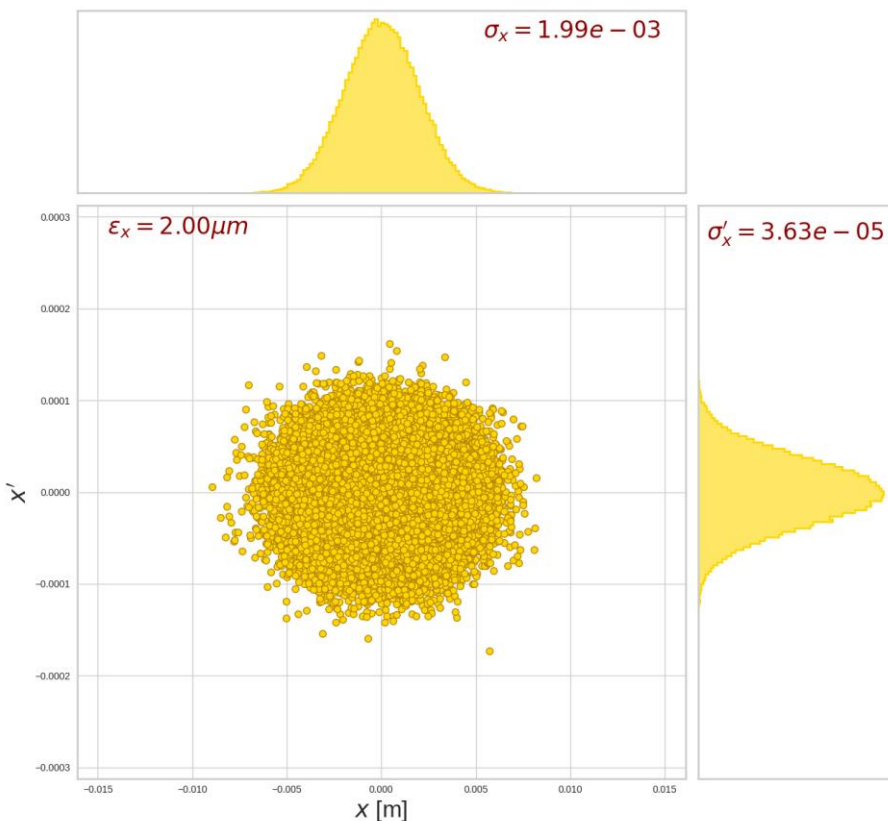
$$\Psi(x, x', y, y', z, \delta)$$

```
In [6]: df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
df
```

Out[6]:

	dp	x	xp	y	yp	z
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-06	0.353427
1	0.001978	0.000370	1.954404e-05	-0.000359	5.543904e-05	0.159670
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627340e-05	-0.251489
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1.870926e-05	-0.038597
4	0.000572	0.000990	5.493907e-05	0.000152	-1.951051e-05	0.492968
5	-0.000418	0.001088	4.778027e-05	0.003320	-7.716856e-06	0.415582
6	-0.000114	-0.000194	1.065400e-05	0.001798	-4.984276e-07	-0.349064
7	0.001100	-0.001257	-6.873217e-05	-0.002374	5.657645e-06	-0.023157
8	0.002706	0.005351	-1.867898e-07	-0.000765	3.012523e-05	-0.291095
9	0.003508	0.000499	1.865768e-05	-0.001032	-5.363820e-05	0.211726
10	-0.001711	-0.003168	4.372560e-05	-0.001933	-2.151020e-05	-0.145358
11	-0.002150	-0.000565	-1.853825e-05	-0.003895	-6.192450e-06	0.072499
12	0.002059	0.003453	-3.808703e-05	0.000118	3.179588e-05	-0.001816
13	0.002709	0.000241	-3.457535e-05	0.000474	5.057865e-05	-0.005464
14	-0.001593	0.000711	-1.667091e-05	-0.002523	-3.804168e-05	-0.089891
15	-0.000830	-0.000393	-7.473946e-05	-0.003690	5.712523e-06	0.000000
16	-0.001743	-0.003024	1.065400e-05	-0.001798	4.984276e-07	0.349064

# Macroparticle representation of the beam



- Initial conditions of the beam/particles

Profile	Size	Matching
Gaussian	Emittance	Optics
Parabolic		
Flat		
...		

- We use **random number generators** to obtain **random distributions of coordinates and momenta**
- Example transverse Gaussian beam in the SPS with normalized emittance of 2  $\mu\text{m}$  (0.35 eVs longitudinal)

$$\begin{aligned} \epsilon_{\perp} &= \beta\gamma\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \\ &= \beta\gamma\sigma_x\sigma_{x'} \\ \epsilon_{\parallel} &= 4\pi\sigma_z\sigma_{\delta}\frac{p_0}{e} \end{aligned}$$

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2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627000e-06	
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1.111111e-05	
4	0.002570	0.000000	5.400000e-05	0.000150	0.000000	

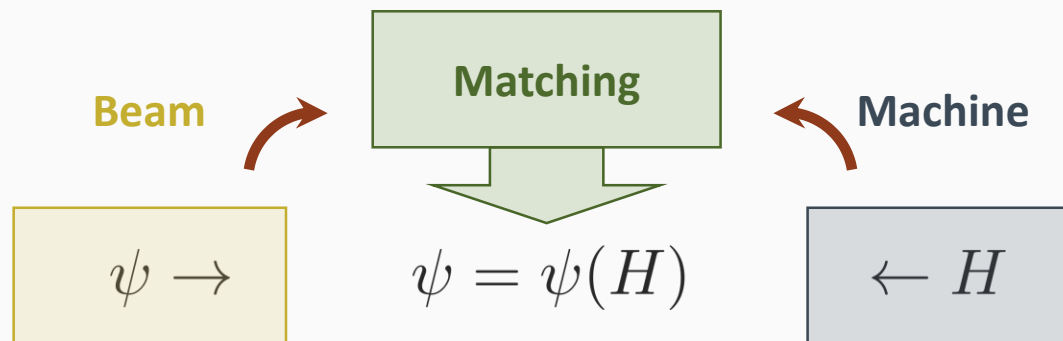
- We have learned about the **particle description** of a beam.
  - We have seen **macroparticles** and **macroparticle models**.
  - We have seen how **macroparticle models** are **mapped and represented in a computational environment**.
- 
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- As seen earlier, given a particle distribution function and a machine (described by a Hamiltonian  $H$ ) the stationary solution is given by:

$$\frac{\partial}{\partial s} \psi = [\mathbf{H}, \psi] = 0$$

and can be constructed via matching:

- In real life, an injected beam ought to be **matched to the machine** for best performance.
- Given a **particle distribution function** and a **machine optics** locally described by a Hamiltonian we ensure matching by targeting for:





We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(-\frac{H}{H_0}\right)$$

- Betatron motion

$$H = \frac{1}{2} x'^2 + \left(\frac{Q_x}{R}\right)^2 x^2$$
$$H_0 = \sigma_{x'}^2 = \left(\frac{Q_x}{R}\right)^2 \sigma_x^2 \implies \frac{\sigma_x}{\sigma_{x'}} = \frac{R}{Q_x} = \beta_x$$

- Synchrotron motion - linear

$$H(z, \delta) = -\frac{1}{2} \eta \beta c \delta^2 + \frac{eVh}{4\pi R^2 p_0} z^2$$
$$H_0 = \eta \beta c \sigma_\delta^2 = \frac{eVh}{2\pi R^2 p_0} \sigma_z^2 \implies \frac{\sigma_z}{\sigma_\delta} = R \eta \sqrt{\frac{2\pi \beta^2 E_0}{eV \eta h}} = \frac{R \eta}{Q_s} \sigma_\delta = \beta_z$$

# Matching examples

We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(-\frac{H}{H_0}\right)$$

- Betatron motion

In reality the synchrotron motion is described by the Hamiltonian:

$$H(z, \delta) = -\frac{1}{2}\eta\beta c\delta^2 + \frac{eV}{2\pi h p_0} \left( \cos\left(\frac{hz}{R}\right) - \cos\left(\frac{hz_c}{R}\right) + \frac{\Delta E}{eV} \left(\frac{hz}{R} - \frac{hz_c}{R}\right) \right)$$

- Synchrotron motion - linear

This leads to **nonlinear equations** and the matching procedure becomes more involved.

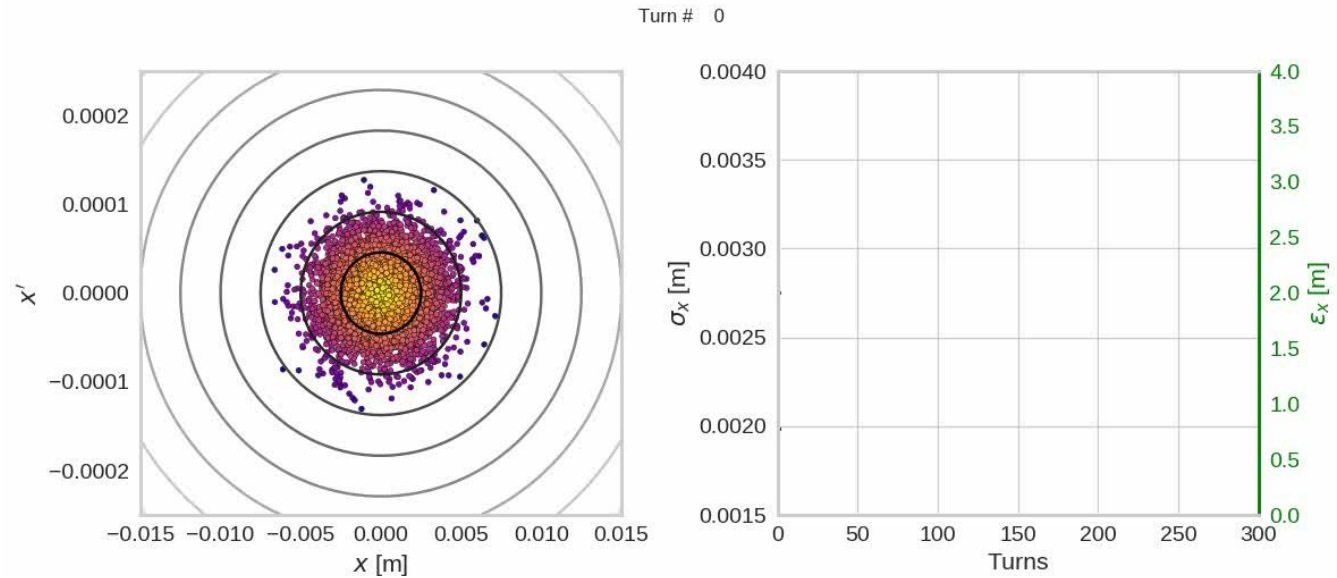
$$H_0 = \eta\beta c\sigma_\delta^2 = \frac{eVh}{2\pi R^2 p_0} \sigma_z^2 \Rightarrow \frac{\sigma_z}{\sigma_\delta} = R\eta \sqrt{\frac{2\pi\beta^2 E_0}{eV\eta h}} = \frac{R\eta}{Q_s} \sigma_\delta = \beta_z$$

# Matching illustration – matched beams

- Betatron motion  
– **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

Matched beams  
**maintain their beam moments** and their shape in phase space

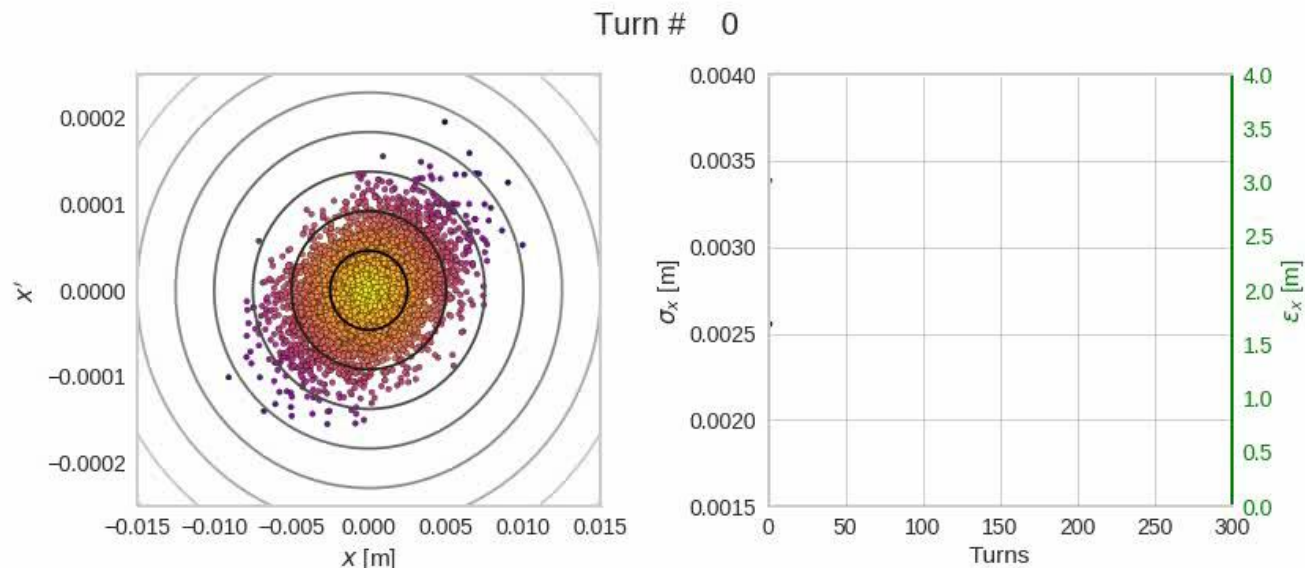


# Matching illustration – mismatched beams

- Betatron motion  
– **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

Mismatched beams show **oscillations in their beam moments** and may **change their shape due to filamentation**

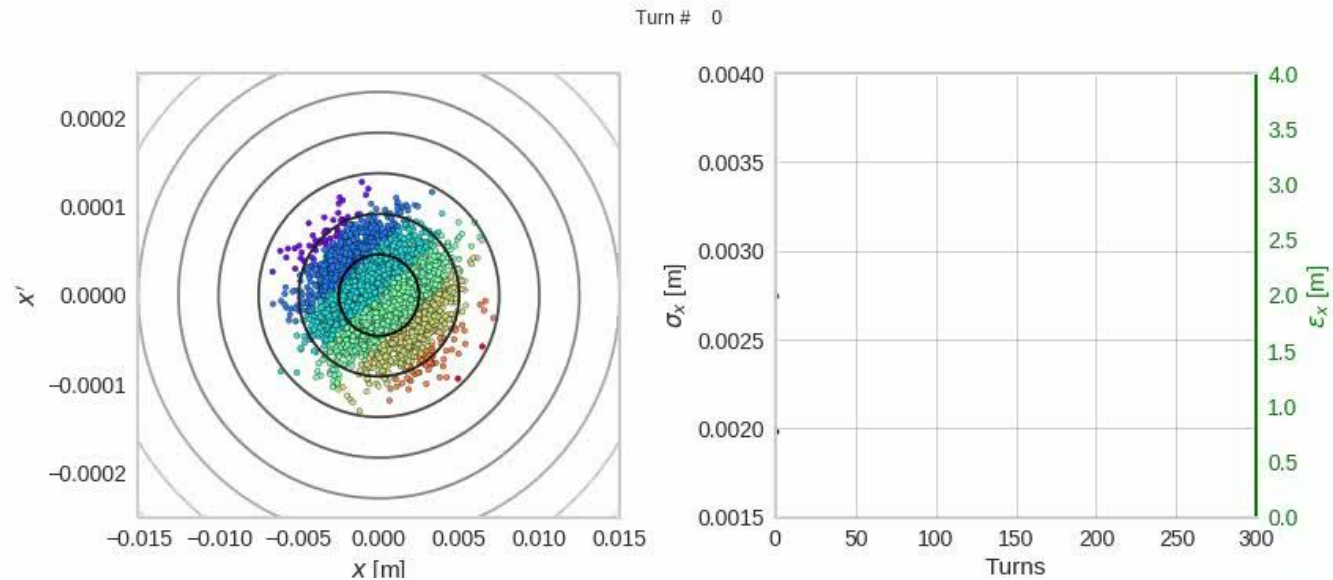


# Matching illustration – linear vs. nonlinear

- Betatron motion  
– **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

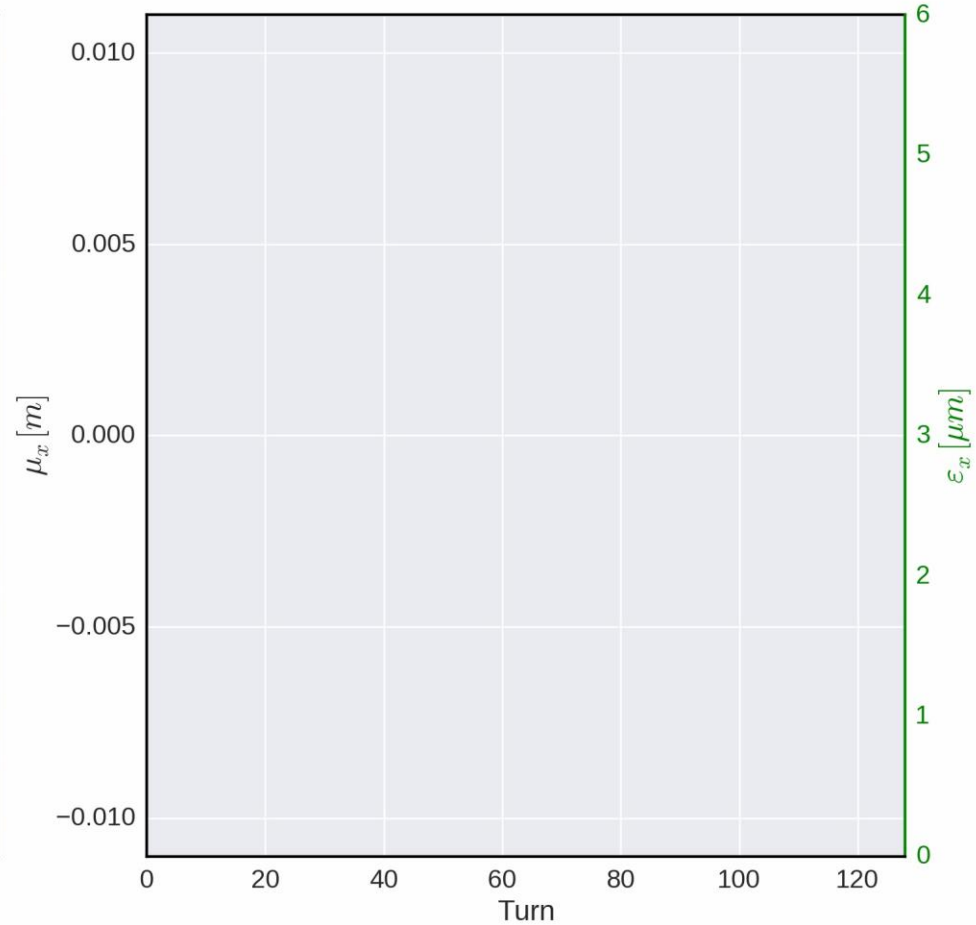
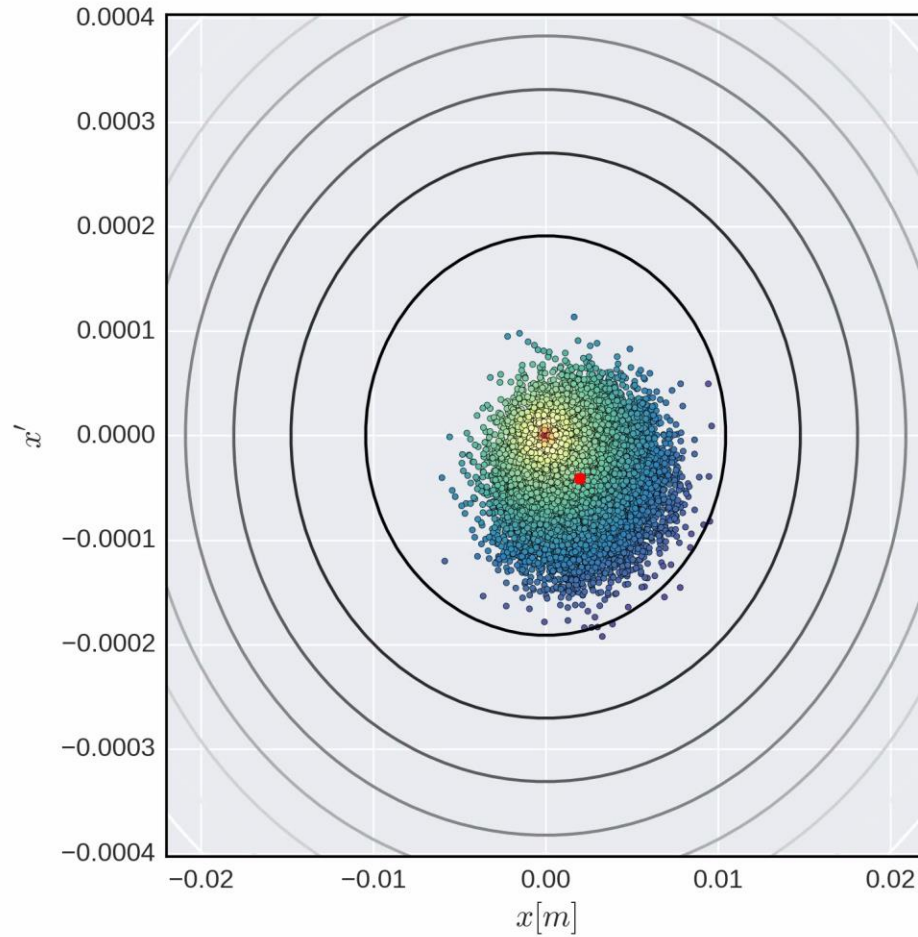
Nonlinearities lead to **detuning with amplitude**. This is visible as the **characteristic spiraling** of larger amplitude particles.



- We have learned about the **meaning of matching** a beam to the machine optics.
- We have seen how to **formally match a beam** to a given description of a machine.
- We have seen **examples of matched and mismatched beams** and have seen the difference between **linear and non-linear motion**.
- Part 1: Introduction – multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - Basic concepts
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    - Beam matching
    - Multiparticle effects – filamentation and decoherence
    - Wakefields as sources of collective effects

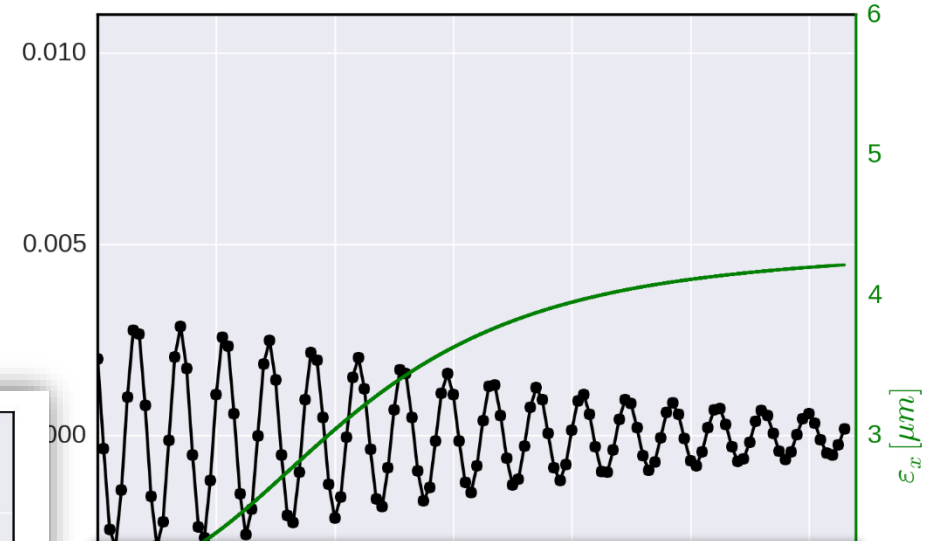
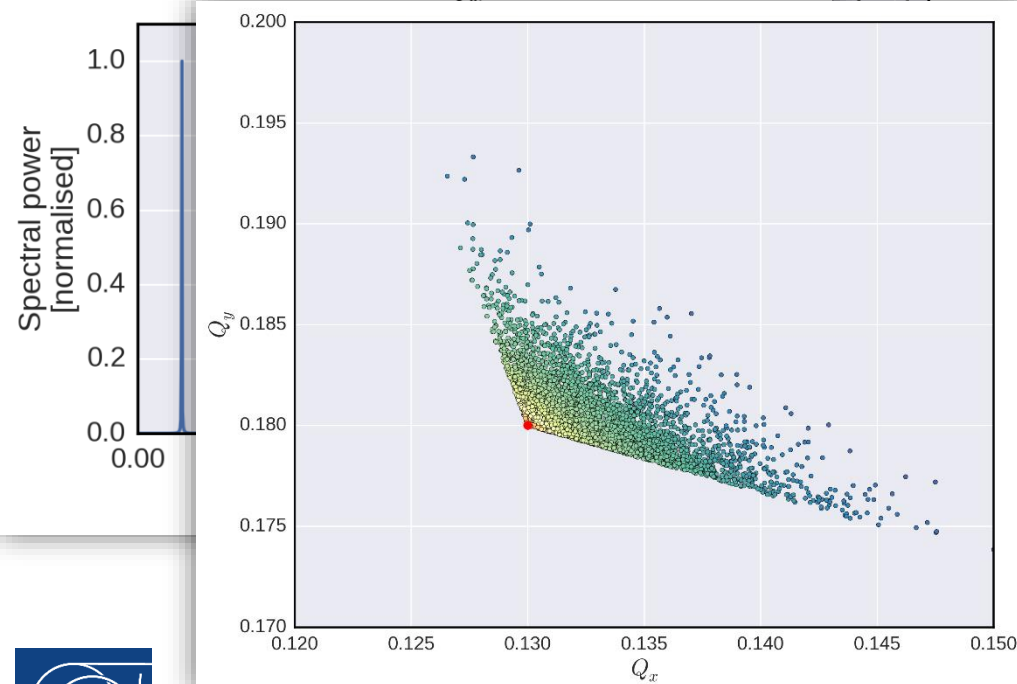
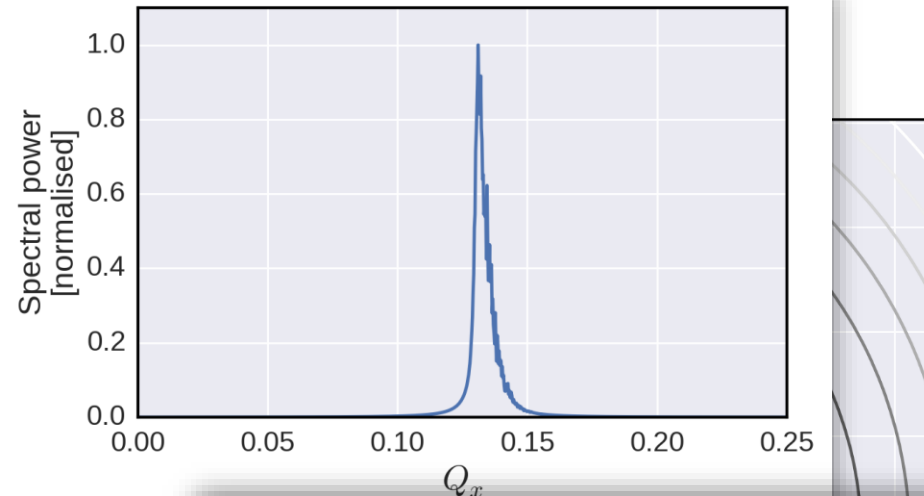
- We have learned or we may know from operational experience that there are a set of **crucial machine parameters to influence beam stability** – among them **chromaticity and amplitude detuning**
- Chromaticity
  - Controlled with sextupoles – provides **chromatic shift** of bunch spectrum wrt. impedance
  - Changes interaction of beam with impedance
  - Damping or excitation of **headtail modes**
- Amplitude detuning
  - Controlled with octupoles – provides (incoherent) **tune spread**
  - Leads to absorption of coherent power into the incoherent spectrum → **Landau damping**

# Example: filamentation as result of detuning



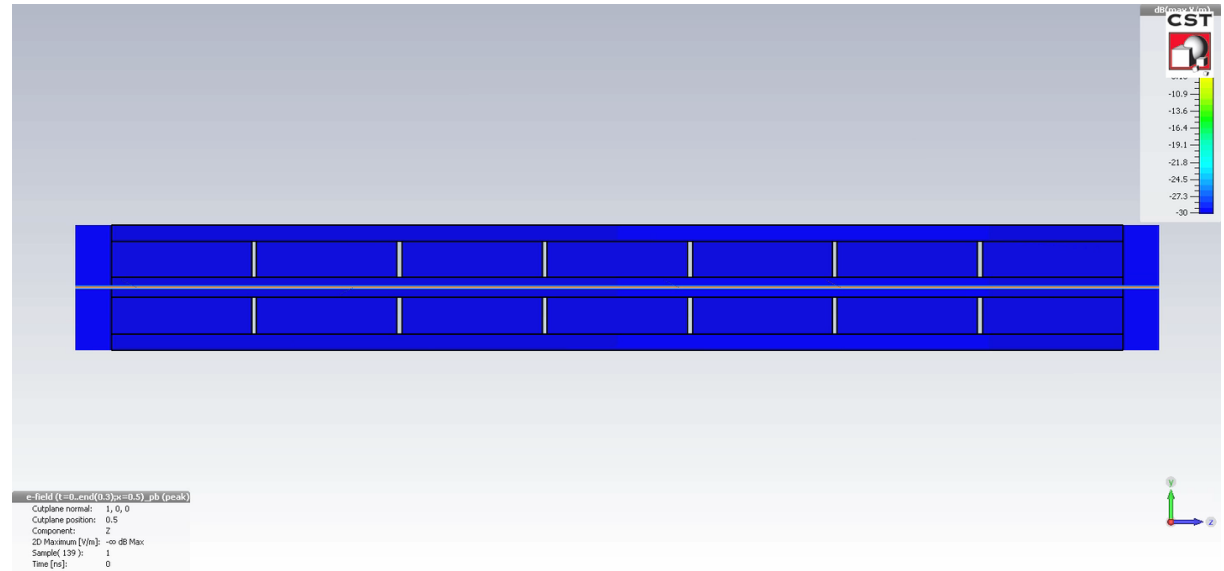
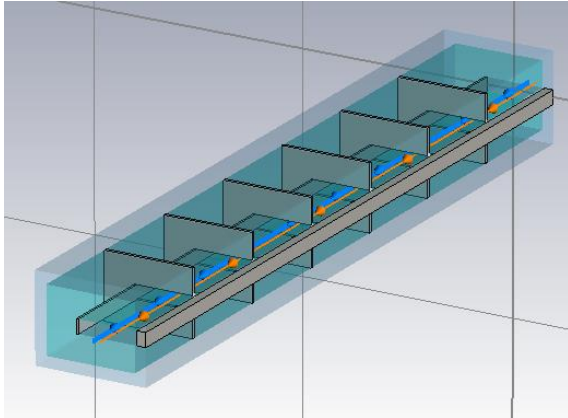


# Example: filamentation as result of detuning



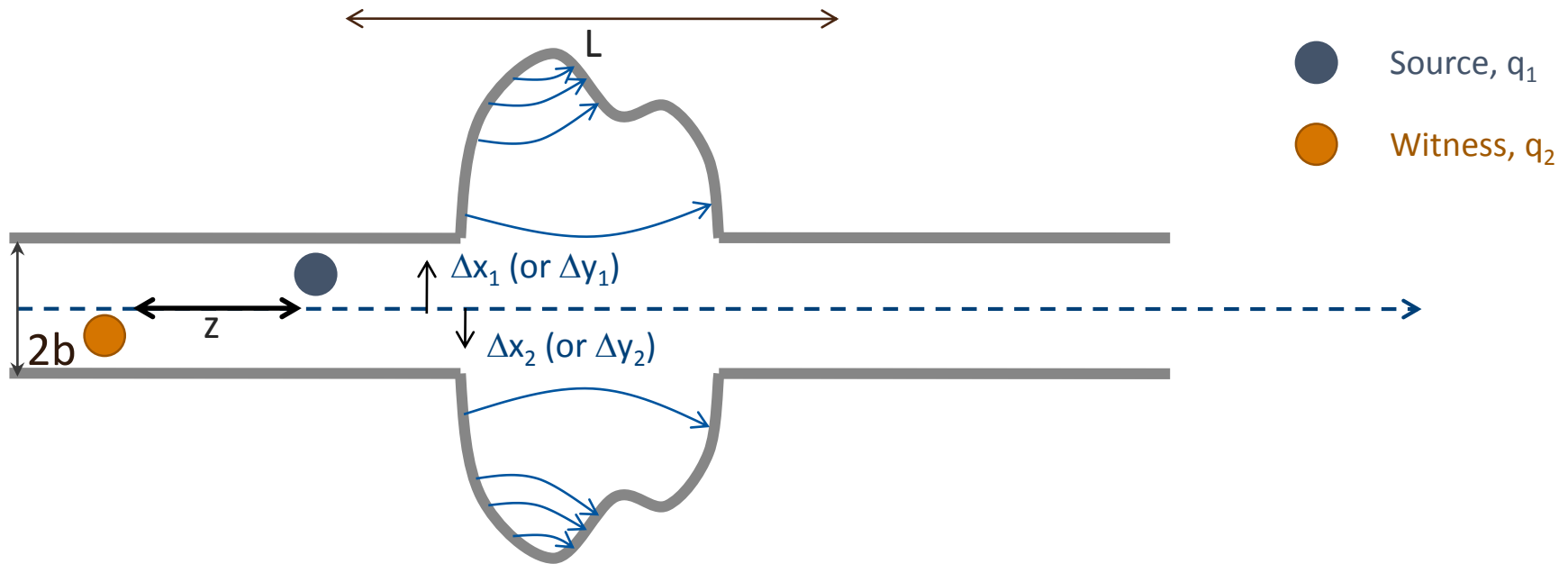
- Taking an **FFT of the centroid motion** (black curve) **reveals the tune** – interestingly there **is a spread**
- In the simulation we have access to the trajectory of **each individual macroparticle** – we can equally perform an **FFT of every macroparticle** and plot the horizontal vs. vertical tune to obtain the **tune footprint**

- Source for transverse nonlinearities are **chromaticity** and **detuning with amplitude** from octupoles, for example.
  - Transverse nonlinearities can lead to **decoherence** and **emittance blow-up**.
  - The effects seen so far are **characteristics for multiparticle systems** but are **not collective effects**.
- 
- Part 1: Introduction – multiparticle systems, macroparticle models and wake functions
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- The **wake function** is the **electromagnetic response** of an object to a charge pulse. It is an intrinsic property of any such object.
- The wake function **couples two charge distributions** as a function of the distance between them.
- The response depends on the boundary conditions and can occur e.g. due to **finite conductivity** (resistive wall) or more or less sudden **changes in the geometry** (e.g. resonator) of a structure.

# Wake functions in general



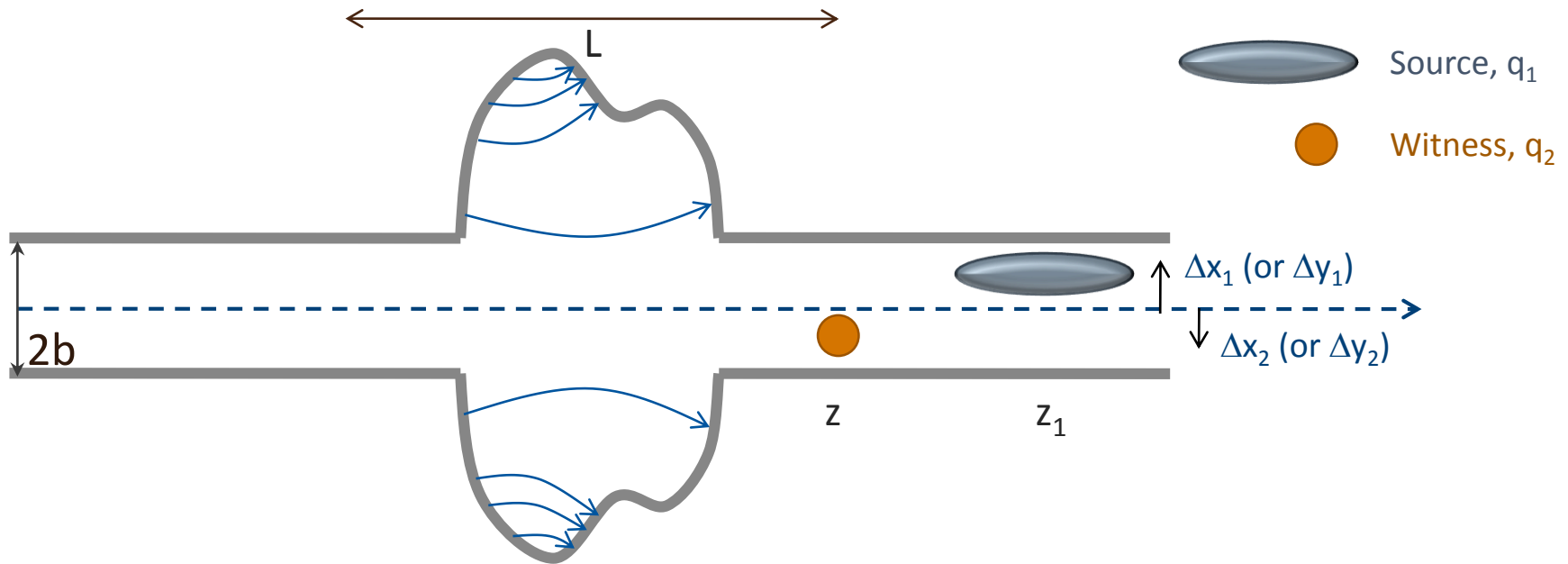
Definition as the **integrated force** associated to a change in energy:

- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s) ds = -q_1 q_2 w(x_1, x_2, z)$$

$w$  is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

# Wake potential for a distribution of particles



Definition as the **integrated force** associated to a change in energy:

- For an extended particle distribution this becomes

$$\Delta E_2(z) \propto \int \lambda_1(x_1, z_1) w(\mathbf{x}_1, \mathbf{x}_2, z - z_1) dx_1 dz_1$$

Forces become dependent on the **particle distribution function**

$$\Delta E_2(z) \propto \int \lambda_1(x_1, z_1) w(x_1, x_2, z - z_1) dx_1 dz_1$$

- We include the impact of wake field into the standard Hamiltonian for linear betatron (or synchrotron motion):

$$H = \frac{1}{2} x'^2 + \frac{1}{2} \left( \frac{Q_x}{R} \right)^2 x^2 + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1) w(x_1, x, z - z_1) dx_1 dz_1 dx$$

- The equations of motion become:

$$x'' + \left( \frac{Q_x}{R} \right)^2 x + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1) w(x_1, x, z - z_1) dx_1 dz_1 = 0$$

The presence of wake fields adds an **additional excitation** which depends on

1. The **moments of the beam distribution**
2. The **shape and the order** of the wake function

# How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances
- **Numerical approach**
  - Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the [ICFA mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators"](#), Erice, Sicily, 23-28 April, 2014
- **Bench measurements** based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations

- We have learned about the concept of **particles, distributions** and **macroparticles** as well as some **peculiarities of multiparticle dynamics** in accelerators, decoherence, filamentation.
  - We have learned about the basic **concept of wake fields** and how these can be characterized as a **collective effect** in that they depend on the particle distribution.
  - We now have a basic understanding of multiparticle systems and wakefields and are now ready to look at the **impact of these** in the longitudinal and transverse planes.
- 
- **Part 1: Introduction – multiparticle systems, macroparticle models and wake functions**
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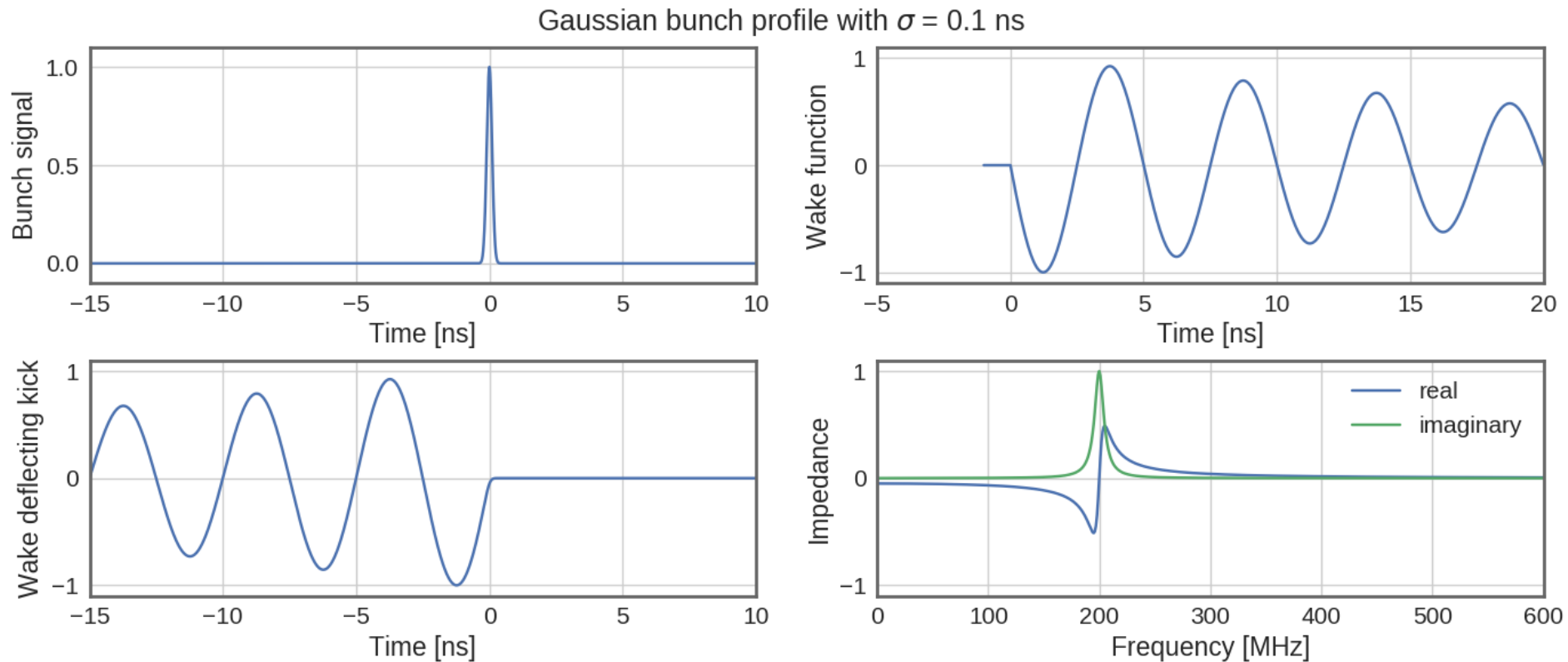


# End part 1



# Wake fields illustrative examples

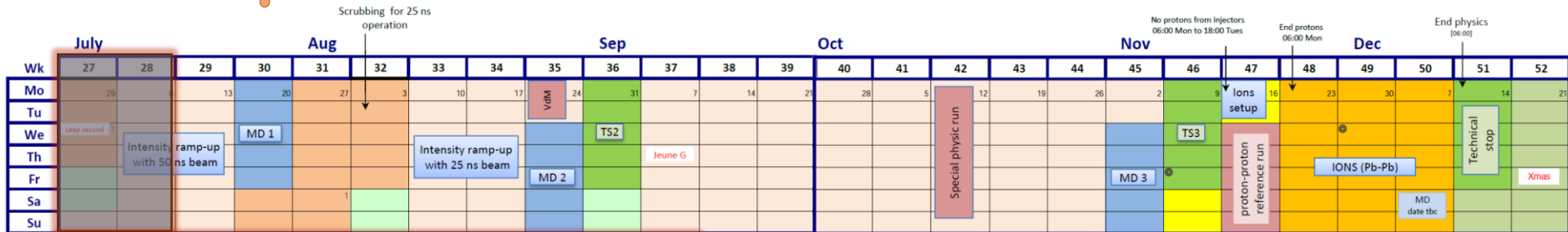
- Resonator wake:  $f_r = 200$  MHz,  $Q = 20$  – Gaussian bunch charge profile
- The plots show how the bunch moments and the wake function **convolve into an integrated deflecting kick** at the different positions along the bunch



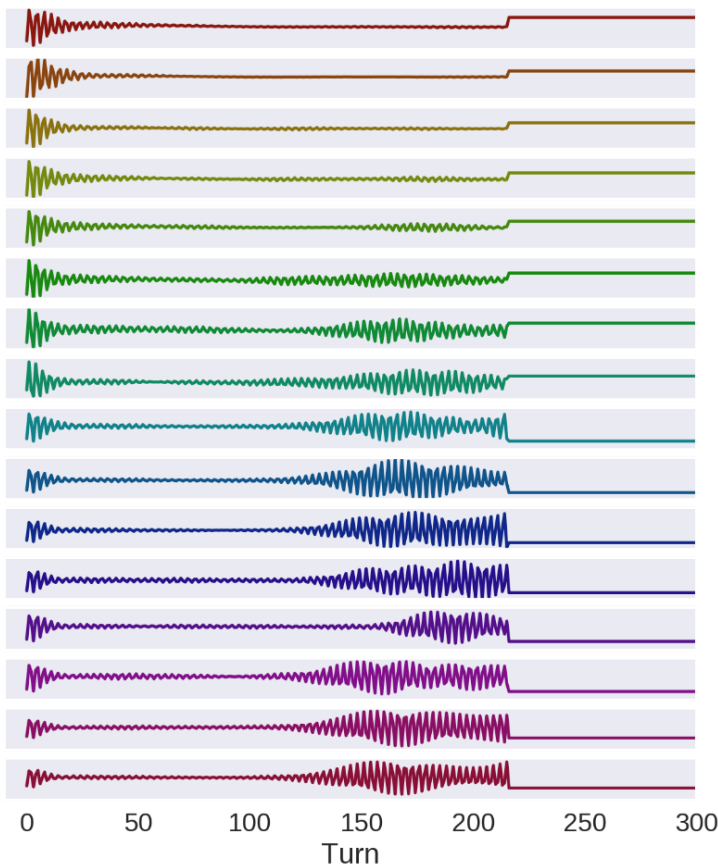
# Backup – instability examples

# E-cloud instabilities in the LHC

## Scrubbing run in 2015 – early stage



B2 - Vertical



Head of batch

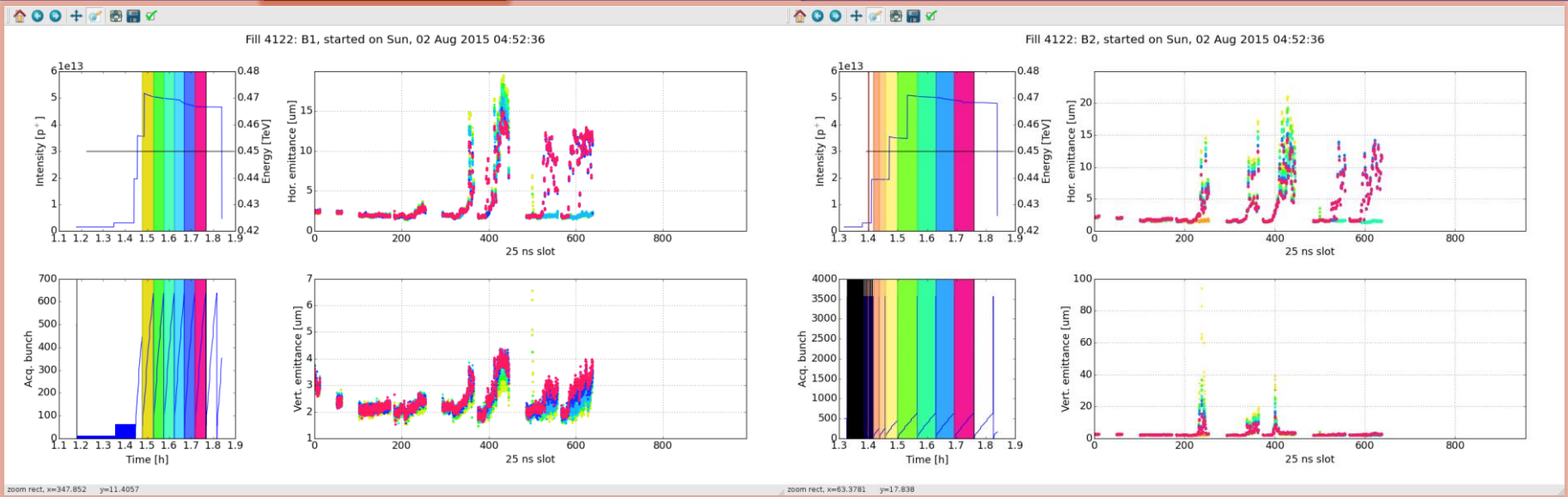
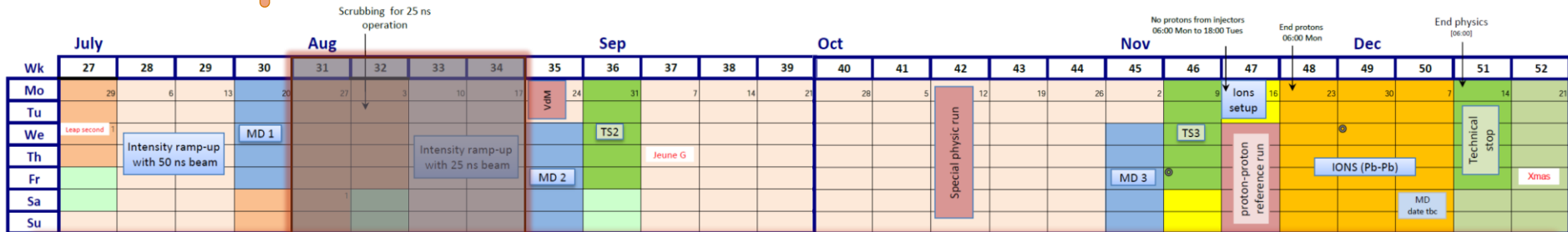
every 4<sup>th</sup> bunch just after injection

Tail of batch

- Injection of multiple bunch batches from the SPS into the LHC.
- Violent **instabilities during initial stages of scrubbing** – clear e-cloud signature
- Very hard to control in the beginning – **slow and staged ramp-up of intensity** (24 → 36 → 48 → 60 → 72 → 144 bpi)

# E-cloud instabilities in the LHC

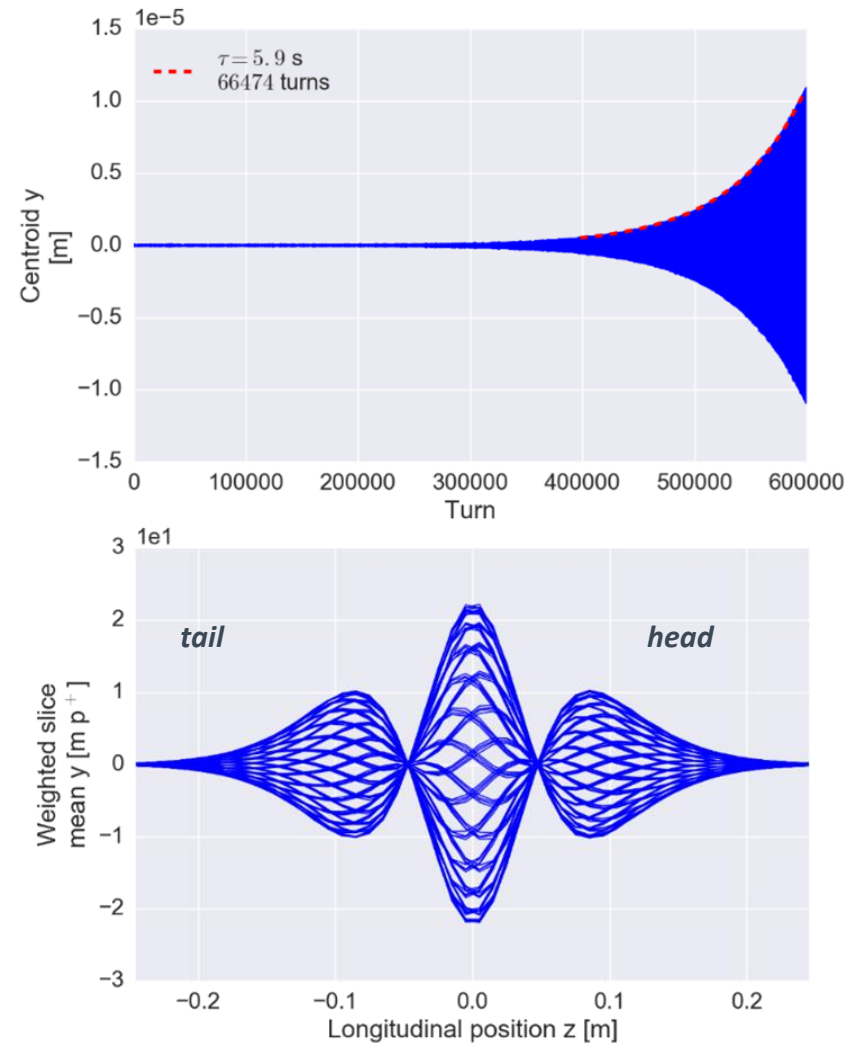
## Scrubbing run in 2015 – second stage



- At later stages dumps under control but still **emittance blow-up and serious beam quality degradation**.
- Beam and e-cloud induced **heating of kickers and collimators**.

# Headtail instabilities in the LHC

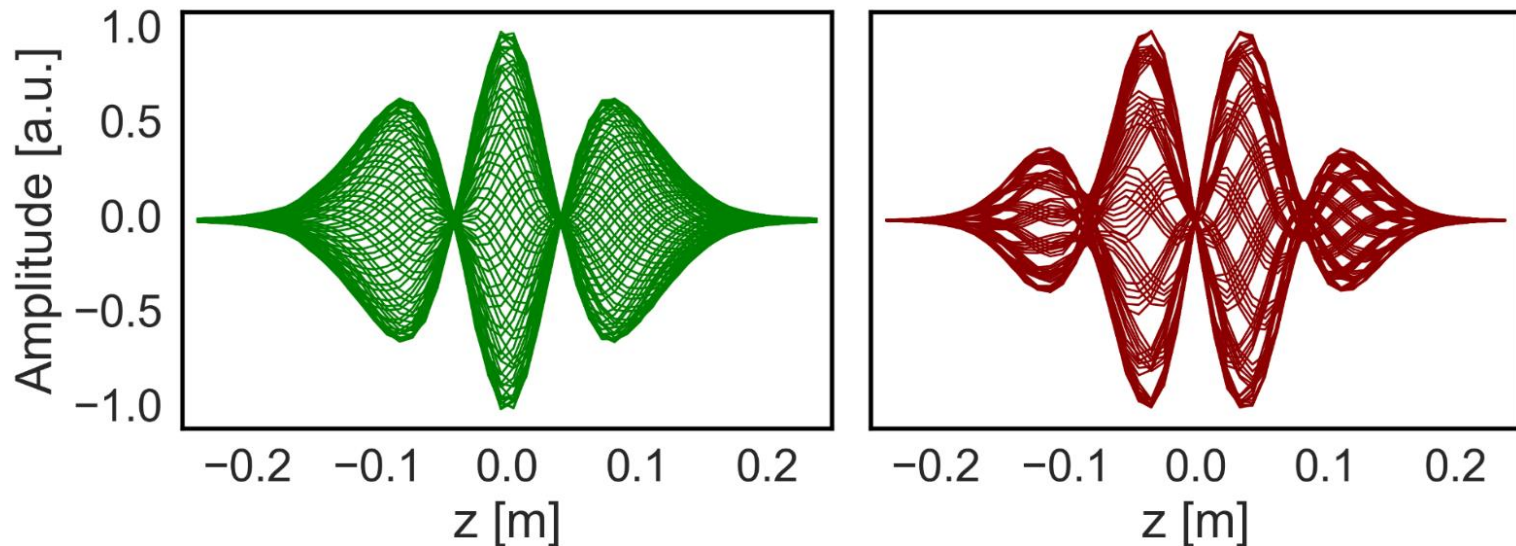
- The **impedance in the LHC** can give rise to coupled and single bunch instabilities which, when left untreated, can lead to **beam degradation and beam loss**.
- As an example, **headtail instabilities** are predicted from **macroparticle simulations** using the LHC impedance model.
- These simulations help to understand and to predict unstable modes which are observed in the real machine.



$m = 0$

$m = -1$

## Macroparticle simulations (PyHEADTAIL)



- These simulations help to understand and to **predict instabilities** which are **observed in the real machine**.