Timing and Synchronization

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Lecture Outline

• MOTIVATIONS
  ✓ Why accelerators need synchronization, and at what precision level

• DEFINITIONS AND BASICS
  ✓ Synchronization, Master Oscillator, Drift vs. Jitter
  ✓ Fourier and Laplace Transforms, Random processes, Phase noise in Oscillators
  ✓ Phase detectors, Phase Locked Loops

• SYNCRONIZATION ARCHITECTURE AND PERFORMANCES
  ✓ Phase lock of synchronization clients (RF systems, Lasers, Diagnostics, ...)
  ✓ Residual absolute and relative phase jitter
  ✓ Reference distribution – actively stabilized links

• BEAM ARRIVAL TIME FLUCTUATIONS
  ✓ Bunch arrival time measurement techniques
  ✓ Bunch arrival time downstream magnetic compressors
  ✓ Beam synchronization – general case

CONCLUSIONS AND REFERENCES
Every accelerator is built to produce some \textit{specific physical process}.

One \textit{necessary condition} for an efficient and stable machine operation is that \textit{some events have to happen at the same time} (simultaneously for an observer in the laboratory frame) or in a \textit{rigidly defined temporal sequence}, within a maximum allowed time error.

If the \textit{simultaneity} or the time separation \textit{of the events fluctuates} beyond the specifications, \textit{the performances of the machine are spoiled}, and the quantity and quality of the accelerator products are compromised.

Clearly, the tolerances on the time fluctuations are different for different kind of accelerators. The \textit{smaller the tolerances}, the \textit{tighter the level of synchronization required}. In the last decade a new generation of accelerator projects such as FEL radiation sources or plasma wave based boosters \textit{has pushed the level of the synchronization specifications down to the fs scale}. 
Bunches of the 2 colliding beams need to arrive at the Interaction Point (max vertical focalization) at the same time.

Waist length $\approx \beta_y \approx \sigma_z$ (hourglass effect)

**Synchronization requirement:**

$$\Delta t < \sigma_{t\,bunch} = \frac{1}{c} \cdot \sigma_{z\,bunch}$$

**CIRCULAR COLLIDERS:**

$\sigma_z \approx 1 \, cm \rightarrow \Delta t < 10 \, ps$

**LINEAR COLLIDER (ILC):**

$\sigma_z < 1 \, mm \rightarrow \Delta t < 1 \, ps$

RF Stability spec
Free Electro Laser machines had a crucial role in pushing the accelerator synchronization requirements and techniques to a new frontier in the last ≈15 years.

The simplest FEL regime, the SASE (Self-Amplified Spontaneous Emission), requires high-brightness bunches, being:

$$B \div \frac{I_{\text{bunch}}}{\epsilon_{\perp}^2}$$

Large peak currents $I_{\text{bunch}}$ are typically obtained by short laser pulses illuminating a photo-cathode embedded in an RF Gun accelerating structure, and furtherly increased with bunch compression techniques.

Small transverse emittances $\epsilon_{\perp}$ can be obtained with tight control of the global machine WP, including amplitude and phase of the RF fields, magnetic focusing, laser arrival time, ...

Global Synchronization requirements: < 500 fs rms
In a simple SASE configuration the **micro-bunching process**, which is the base of the FEL radiation production, starts from **noise**. Characteristics such as radiation intensity and envelope profile can vary considerably from shot to shot.

A better control of the radiation properties resulting in more **uniform** and **reproducible** shot to shot pulse characteristics can be achieved in the “**seeded**” FEL configuration.

To “trigger” and guide the avalanche process generating the exponentially-growing radiation intensity, the **high brightness bunch** is made to interact with a **VUV** short and intense **pulse** obtained by HHG (High Harmonic Generation) in gas driven by an IR pulse generated by a dedicated high power laser system (typically TiSa). The presence of the external radiation since the beginning of the micro-bunching process inside the magnetic undulators seeds and drives the FEL radiation growth in a steady, repeatable configuration. The **electron bunch** and the **VUV pulse**, both **very short**, must constantly **overlap** in **space** and **time** shot to shot.

**Synchronization requirements (e- bunch vs TiSa IR pulse):** < 100 fs rms
Pump-probe technique is widely requested and applied by user experimentalists.

Physical / chemical processes are \textit{initialized} by ultra-short \textit{laser pulses}, then the system status is \textit{probed} by \textit{FEL radiation}.

The dynamics of the process under study is captured and stored in a “snapshots” record.

Pump laser and FEL pulses need to be \textit{synchronized} at level of the \textit{time-resolution} required by the experiments (down to $\approx 10 \text{ fs}$).

The relative delay between pump and probe pulses needs to be finely and precisely scanned with proper time-resolution.

\textbf{Synchronization requirements (FEL vs Pump Laser pulses):} \hspace{0.5cm} $\approx 10 \text{ fs rms}$
Plasma acceleration is the new frontier in accelerator physics, to overcome the gradient limits of the RF technology in the way to compact, high energy machines.

Wakefield Laser-Plasma Acceleration (WLPA) is a technique using an extremely intense laser pulse on a gas jet to generate a plasma wave with large accelerating gradients (many GV/m).

To produce good quality beams external bunches have to be injected in the plasma wave. The “accelerating buckets” in the plasma wave are typically few 100 μm long.

The injected bunches have to be very short to limit the energy spread after acceleration, and ideally need to be injected constantly in the same position of the plasma wave to avoid shot-to-shot energy fluctuations.

This requires synchronization at the level of a small fraction of the plasma wave period.

**Synchronization requirements (external bunch vs laser pulse):**

< 10 fs rms
Synchronization of different facilities

SUMMARY

Circular colliders

Future Linear colliders

Seeded FELs

SASE FELs

Compton sources

FELs Pump-Probe

WLPA External injection

Upcoming ...

Low-level RF, beam FBKs, ...

10 ps

LLRF systems, beam FBKs, ...

1 ps

Seeding and PC lasers, LLRF, ...

100 fs

FEL (*), Pump laser, ...

10 fs

Injected beam (#), Power laser, ...

1 fs

Photocathode (PC) laser, LLRF, ...

Sub fs

(+ Interaction laser

(#) depends on all RF and laser systems of the injector

(*) depends on beam (LLRFs + PC laser) and laser seed (if any)
GLOSSARY
Every accelerator is built to produce some **specific physical processes** (shots of bullet particles, nuclear and sub-nuclear reactions, synchrotron radiation, FEL radiation, Compton photons, ...).

It turns out that a **necessary condition** for an efficient and reproducible event production is the **relative temporal alignment** of all the accelerator **sub-systems** impacting the beam longitudinal phase-space and time-of-arrival (such as RF fields, PC laser system, ...), and of the **beam bunches** with **any other system they have to interact with** during and after the acceleration (such as RF fields, seeding lasers, pump lasers, interaction lasers, ...).

The **synchronization system** is the complex including all the **hardware**, the **feedback processes** and the **control algorithms** required to keep **time-aligned** the **beam bunches** and **all the machine critical sub-systems** within the facility specifications.

50 m ÷ some km
Naive approach: can each sub-system be synchronized to a local high-stability clock to have a good global synchronization of the whole facility?

Best optical clocks $\rightarrow \Delta \omega / \omega \approx 10^{-18} \rightarrow \Delta T / T \approx 10^{-18} \rightarrow T \approx 10 \text{ fs} / 10^{-18} \approx 3 \text{ hours} !!!

It is impossible to preserve a tight phase relation over long time scales even with the state-of-the-art technology.

All sub-systems need to be continuously re-synchronized by a common master clock that has to be distributed to the all "clients" spread over the facility with a star network architecture.
Once the local oscillators have been locked to the reference, they can be shifted in time by means of delay lines of various types – translation stages with mirrors for lasers, trombone-lines or electrical phase shifters for RF signals. This allows setting, correcting, optimizing and changing the working point of the facility synchronization.

Delay lines can be placed either downstream the oscillators or on the reference signal on its path to the client oscillator. The function accomplished is exactly the same.

For simplicity, in most of the following sketches the presence of the delay line will be omitted.
The Master Oscillator of a facility based on particle accelerators is typically a good(*), low phase noise μ-wave generator acting as timing reference for the machine sub-systems. It is often indicated as the RMO (RF Master Oscillator).

The timing reference signal can be distributed straightforwardly as a pure sine-wave voltage through coaxial cables, or firstly encoded in the repetition rate of a pulsed (mode-locked) laser (or sometimes in the amplitude modulation of a CW laser), and then distributed through optical-fiber links.

Optical fibers provide less signal attenuation and larger bandwidths, so optical technology is definitely preferred for synchronization reference distribution, at least for large facilities.

(*) the role of the phase purity of the reference will be discussed later
Optical: mode-locked lasers

A **mode-locked laser** consists in an **optical cavity** hosting an active (amplifying) medium capable of sustaining a **large number of longitudinal modes** with frequencies $\nu_k = k\nu_0 = kc/L$ within the bandwidth of the active medium, being $L$ the cavity round trip length and $k$ integer. If the modes are forced to **oscillate in phase** and the medium emission BW is wide enough, a **very short pulse** ($\approx 100$ fs) travels forth and back in the cavity and a sample is coupled out through a leaking mirror.
The synchronization error of a client with respect to the reference is identified as *jitter* or *drift* depending on the *time scale* of the involved phenomena.

- **Jitter** = fast variations, caused by inherent residual lack of coherency between oscillators, even if they are locked at the best;
- **Drift** = slow variations, mainly caused by modifications of the environment conditions, such as temperature (primarily) but also humidity, materials and components aging, ...

The boundary between the 2 categories is somehow arbitrary. For instance, synchronization errors due to mechanical vibrations can be classified in either category:

- Acoustic waves → Jitter
- Infrasounds → Drift

For pulsed accelerators, where the beam is produced in the form of a sequence of bunch trains with a certain repetition rate (10 Hz ÷ 120 Hz typically), the *rep. rate value* itself can be taken as a reasonable definition of the *boundary* between *jitters* and *drifts*.

In this respect, *drifts* are phenomena significantly *slower* than *rep. rate* and will produced effects on the beam that can be *monitored* and *corrected* pulse-to-pulse.

On the contrary, *jitters* are *faster* than *rep. rate* and will result in a pulse-to-pulse *chaotic scatter* of the beam characteristics that has to be minimized but that *can not* be actively *corrected*. 

**GLOSSARY:**

**JITTER** vs. **DRIFT**
**Tasks of a Synchronization system:**

- Generate and transport the reference signal to any client local position with constant delay and minimal drifts;
- Lock the client (laser, RF, ...) fundamental frequency to the reference with minimal residual jitter;
- Monitor clients and beam, and apply delay corrections to compensate residual (out-of-loop) drifts.

**Triggers**

Digital signals still in the Timing business but the required precision is orders of magnitude less demanding.

**Not covered in this lecture** (but nevertheless an important aspect of machine operation).
BASICS

- Fourier and Laplace Transforms
- Random Processes
- Phase Noise in Oscillators
# Transforms summary

<table>
<thead>
<tr>
<th>Transforms</th>
<th>Fourier - $\mathcal{F}$</th>
<th>Laplace - $\mathcal{L}$</th>
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<tbody>
<tr>
<td>Definition</td>
<td>$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$</td>
<td>$X(s) = \int_{0}^{+\infty} x(t) e^{-st} dt$</td>
</tr>
<tr>
<td>Inverse transform</td>
<td>$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$</td>
<td>$x(t) = \frac{1}{2\pi i} \int_{Y-j\cdot\infty}^{Y+j\cdot\infty} X(s) e^{st} ds$</td>
</tr>
<tr>
<td>Transformability</td>
<td>$\int_{-\infty}^{+\infty}</td>
<td>x(t)</td>
</tr>
<tr>
<td>Linearity</td>
<td>$\mathcal{F} [a \cdot x(t) + b \cdot y(t)] = aX(\omega) + bY(\omega)$</td>
<td>$\mathcal{L} [a \cdot x(t) + b \cdot y(t)] = aX(s) + bY(s)$</td>
</tr>
<tr>
<td>Convolution product</td>
<td>$(x * y)(t) \overset{\text{def}}{=} \int_{-\infty}^{+\infty} x(t + \tau) \cdot y(\tau) d\tau$</td>
<td>$(x * y)(t) \overset{\text{def}}{=} \int_{0}^{t} x(t + \tau) \cdot y(\tau) d\tau$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{F} [(x * y)(t)] = X^*(\omega) \cdot Y(\omega)$</td>
<td>$\mathcal{L} [(x * y)(t)] = X^*(s) \cdot Y(s)$</td>
</tr>
<tr>
<td>Derivative</td>
<td>$\mathcal{F} \left[ \frac{dx}{dt} \right] = j\omega \cdot X(\omega)$</td>
<td>$\mathcal{L} \left[ \frac{dx}{dt} \right] = s \cdot X(s)$</td>
</tr>
<tr>
<td>Delay</td>
<td>$\mathcal{F} [x(t - \tau)] = X(\omega)e^{-j\omega \tau}$</td>
<td>$\mathcal{L} [x(t - \tau)] = X(s)e^{-st}$</td>
</tr>
</tbody>
</table>
Random process summary

Let’s consider a random variable $x(t)$ representing a physical observable quantity.

- Stationary process: statistical properties invariant for a $t'$ time shift $x(t) \rightarrow x(t + t')$

  - Stationarity:
    \[
    \overline{x_1} \approx \overline{x_2} \approx \overline{x_3} \approx \overline{x} \\
    \sigma_{x_1} \approx \sigma_{x_2} \approx \sigma_{x_3} \approx \sigma_x
    \]

- Ergodic process: statistical properties can be estimated by a single process realization

- Uncorrelation: if $x(t)$ and $y(t)$ are 2 random variables completely uncorrelated (statistically independent), then:

  \[
  \sigma^2_{x+y} = \sigma^2_x + \sigma^2_y \quad with \quad \sigma^2_x \equiv \overline{x^2} - \overline{x}^2
  \]
Power spectrum:

Since \( x_{\text{rms}} \neq 0 \), a real random variable \( x(t) \) is in general not directly Fourier transformable. However, if we observe \( x(t) \) only for a finite time \( \Delta T \) we may truncate the function outside the interval \([-\Delta T/2, \Delta T/2]\) as follows:

\[
x_{\Delta T}(t) = \begin{cases} 
  x(t) & -\Delta T/2 \leq t \leq \Delta T/2 \\
  0 & \text{elsewhere}
\end{cases}
\]

Let \( X_{\Delta T}(f) \) be the Fourier transform of the truncated function \( x_{\Delta T}(t) \). It might be demonstrated that the rms value of the random variable can be computed on the base of the Fourier transform \( X_{\Delta T}(f) \) according to:

\[
x_{\text{rms}}^2 = \int_{0}^{+\infty} S_x(f) \, df \quad \text{with} \quad S_x(f) \overset{\text{def}}{=} \lim_{\Delta T \to \infty} 2 \cdot \frac{|X_{\Delta T}(f)|^2}{\Delta T}
\]

The function \( S_x(f) \) is called “power spectrum” or “power spectral density” of the random variable \( x(t) \). The time duration of the variable observation \( \Delta T \) sets the minimum frequency \( f_{\text{min}} \approx 1/\Delta T \) containing meaningful information in the spectrum of \( x_{\Delta T}(t) \). 

Letting
Fourier and Laplace transforms are used to compute the response of *Linear Time Invariant (LTI)* systems:

**Time domain**

\[
\delta(t) \rightarrow \text{LTI system} \rightarrow h(t) \quad \text{Green's function}
\]

\[
v_{in}(t) \rightarrow \text{LTI system} \rightarrow v_{in}(t) \ast h(t)
\]

**Fourier transform**

\[
e^{j\omega t} \rightarrow \text{LTI system} \rightarrow H(j\omega)e^{j\omega t}
\]

\[
V_{in}(j\omega) \rightarrow \text{LTI system} \rightarrow V_{in}(j\omega) \cdot H(j\omega)
\]

**Laplace transform**

\[
e^{st} \rightarrow \text{LTI system} \rightarrow H(s)e^{st}
\]

\[
V_{in}(s) \rightarrow \text{LTI system} \rightarrow V_{in}(s) \cdot H(s)
\]
The most important task of a Synchronization system is to lock firmly the phase of each client to the reference oscillator in order to minimize the residual jitter. The clients are basically VCOs (Voltage Controlled Oscillators), i.e. local oscillators (electrical for RF systems, optical for laser systems) whose fundamental frequency can be changed by applying a voltage to a control port.

Before discussing the lock schematics and performances, it is worth introducing some basic concepts on phase noise in real oscillators.

In real oscillators the amplitude and phase will always fluctuate in time by a certain amount because of the unavoidable presence of noise. However, by common sense, a well behaving real oscillator has to satisfy the following conditions:

\[ |\alpha(t)| \ll 1; \quad \left| \frac{d\varphi}{dt} \right| \ll \omega_0 \]
A real oscillator signal can be also represented in **Cartesian Coordinates** \((\alpha, \varphi) \rightarrow (v_I, v_Q)\):

\[
V(t) = V_0 \cdot \cos(\omega_0 t) + v_I(t) \cdot \cos(\omega_0 t) - v_Q(t) \cdot \sin(\omega_0 t)
\]

*if* \(v_I(t), v_Q(t) \ll V_0\)

\[
\alpha(t) = v_I(t)/V_0, \quad \varphi(t) = v_Q(t)/V_0 \ll 1
\]

Cartesian representation only holds for small PM depth

Real oscillator outputs are **amplitude (AM)** and **phase (PM) modulated** carrier signals. In general it turns out that **close to the carrier** frequency the contribution of the **PM noise** to the signal spectrum **dominates** the contribution of the **AM noise**. For this reason the lecture will be focused on phase noise. However, amplitude noise in RF systems directly reflects in energy modulation of the bunches, that may cause bunch arrival time jitter when beam travels through dispersive and bended paths (i.e. when \(R_{56}\neq0\) as in magnetic chicanes).

Let’s consider a real oscillator and neglect the AM component:

\[
V(t) = V_0 \cdot \cos[\omega_0 t + \varphi(t)] = V_0 \cdot \cos[\omega_0(t + \tau(t))] \quad \text{with} \quad \tau(t) \equiv \varphi(t)/\omega_0
\]

The statistical properties of \(\varphi(t)\) and \(\tau(t)\) qualify the oscillator, primarily the values of the standard deviations \(\sigma_{\varphi}\) and \(\sigma_{\tau}\) (or equivalently \(\varphi_{rms}\) and \(\tau_{rms}\) since we may assume a zero average value). As for every noise phenomena they can be computed through the **phase noise power spectral density** \(S_\varphi(f)\) of the random variable \(\varphi(t)\).
Again, for practical reasons, we are only interested in observations of the random variable $\varphi(t)$ for a finite time $\Delta T$. So we may truncate the function outside the interval $[-\Delta T/2, \Delta T/2]$ to recover the function transformability.

Let $\Phi_{\Delta T}(f)$ be the Fourier transform of the truncated function $\varphi_{\Delta T}(t)$. We have:

$$
(\varphi_{rms}^2)_{\Delta T} = \int_{f_{\text{min}}}^{+\infty} S_{\varphi}(f) \, df \quad \text{with} \quad S_{\varphi}(f) \overset{\text{def}}{=} 2 \frac{|\Phi_{\Delta T}(f)|^2}{\Delta T}
$$

$S_{\varphi}(f)$ is the phase noise power spectral density, whose dimensions are $\text{rad}^2 / \text{Hz}$.

Again, the time duration of the variable observation $\Delta T$ sets the minimum frequency $f_{\text{min}} \approx 1/\Delta T$ containing meaningful information on the spectrum $\Phi_{\Delta T}(f)$ of the phase noise $\varphi_{\Delta T}(t)$.

**IMPORTANT:** we might still write

$$
\varphi_{rms}^2 = \lim_{\Delta T \to \infty} (\varphi_{rms}^2)_{\Delta T} = \int_{0}^{+\infty} \left( 2 \cdot \lim_{\Delta T \to \infty} \frac{|\Phi_{\Delta T}(f)|^2}{\Delta T} \right) df = \int_{0}^{+\infty} S_{\varphi}(f) \, df
$$

but we must be aware that $\varphi_{rms}$ in some case might diverge. This is physically possible since the power in the carrier does only depend on amplitude and not on phase. In these cases the rms value can only be specified for a given observation time $\Delta T$ or equivalently for a frequency range of integration $[f_1, f_2]$. 

$$
\varphi_{\Delta T}(t) = \begin{cases} 
\varphi(t) & -\Delta T/2 \leq t \leq \Delta T/2 \\
0 & \text{elsewhere}
\end{cases}
$$
BASICS:

Phase Noise in Oscillators

We have:

$$\varphi_{rms}^2|_{\Delta T} = 2 \cdot \int_{f_{min}}^{+\infty} L(f) \, df \quad \text{with} \quad L(f) = \begin{cases} \frac{|\Phi_{\Delta T}(f)|^2}{\Delta T} & f \geq 0 \\ 0 & f < 0 \end{cases}$$

The function $L(f)$ is defined as the “Single Sideband Power Spectral Density” and is called “script-L”

$$L(f) = \frac{\text{power in } 1 \text{ Hz phase modulation single sideband}}{\text{total signal power}} = \frac{1}{2} S_{\varphi}(f) \quad \text{IEEE standard 1139 – 1999}$$

Linear scale $\rightarrow L(f)$ units $\equiv Hz^{-1}$ or $rad^2/Hz$

Log scale $\rightarrow 10 \cdot \log[L(f)]$ units $\equiv dBc/Hz$

CONCLUSIONS:

✓ Phase (and time) jitters can be computed from the spectrum of $\varphi(t)$ through the $L(f)$ - or $S_{\varphi}(f)$ - function;

✓ Computed values depend on the integration range, i.e. on the duration $\Delta T$ of the observation. Criteria are needed for a proper choice (we will see ...).
### Close-in phase noise:

\[ S_\varphi(f) = \sum b_{-k} \frac{f^k}{f^k} \quad k = 0, 1, 2, 3, \ldots \]

<table>
<thead>
<tr>
<th>Type</th>
<th>Origin</th>
<th>( S_\varphi(f) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f^0 )</td>
<td>White</td>
<td>( F \cdot \frac{kT}{P_0} )</td>
</tr>
<tr>
<td>Shot</td>
<td>Current quantization</td>
<td>( 2q\bar{i}R/P_0 )</td>
</tr>
<tr>
<td>( f^{-1} )</td>
<td>Flicker</td>
<td>Flicking PM</td>
</tr>
<tr>
<td>( f^{-2} )</td>
<td>White FM</td>
<td>Thermal FM noise</td>
</tr>
<tr>
<td></td>
<td>Random walk</td>
<td>Brownian motion</td>
</tr>
<tr>
<td>( f^{-3} )</td>
<td>Flicker FM</td>
<td>Flicking FM</td>
</tr>
<tr>
<td>( f^{-4} )</td>
<td>Random walk FM</td>
<td>Brownian motion ( \rightarrow ) FM</td>
</tr>
<tr>
<td>( f^{-n} )</td>
<td>...</td>
<td>high orders ...</td>
</tr>
</tbody>
</table>

\[ F \equiv \frac{SNR_{in}}{SNR_{out}} \]

**Corner frequency**

\[ [b_{-k}] = \text{rad}^2 \text{Hz}^{k-1} \]
Time jitter can be computed according to:

\[ \sigma_t^2 = \frac{\sigma_\varphi^2}{\omega_c^2} = \frac{1}{\omega_c^2} \int_{f_{\text{min}}}^{+\infty} S_\varphi(f) \, df \]

same time jitter \( \rightarrow S_\varphi(f) \div \omega_c^2 \)

Phase noise spectral densities of different oscillators have to be compared at same carrier frequency \( \omega_c \) or scaled as \( \omega_c^{-2} \) before comparison.

---

**Commercial frequency synthesizer**

Low noise RMO

\[ f_c = 2856 \text{ MHz} \]

**Spurious**

\[ 60 \text{ fs} \]

\[ 10 \text{ Hz} \div 10 \text{ MHz} \]

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**OMO – Mode-locked laser – \( f = 3024 \text{ MHz} \)**

**Timing jitter = 3.34 fs**

SECTION III

BASICS

• Phase Detectors
• Phase Locked Loops
Phase detection on RF signals

The **Double Balanced Mixer** is the most diffused RF device for frequency translation (up/down conversion) and detection of the relative phase between 2 RF signals (LO and RF ports). The LO voltage is differentially applied on a diode bridge switching on/off alternatively the $D_1$-$D_2$ and $D_3$-$D_4$ pairs, so that the voltage at IF is:

$$V_{IF}(t) = V_{RF}(t) \cdot \text{sgn}[V_{LO}(t)]$$

$$V_{RF}(t) = V_{RF} \cdot \cos(\omega_{RF} t) ; \quad V_{LO}(t) = V_{LO} \cdot \cos(\omega_{LO} t)$$

$$V_{RF} \ll V_{LO}$$

$$V_{IF}(t) = V_{RF} \cos(\omega_{RF} t) \cdot \text{sgn}[\cos(\omega_{LO} t)] = V_{RF} \cos(\omega_{RF} t) \cdot \sum_{n=\text{odds}} \frac{4}{n\pi} \cos(n\omega_{LO} t) =$$

$$= \frac{2}{\pi} V_{RF} [\cos((\omega_{LO} - \omega_{RF}) t) + \cos((\omega_{LO} + \omega_{RF}) t) + \text{intermod products}]$$
Phase detection on RF signals

If \( f_{LO} = f_{RF} \) the IF signal has a DC component given by:

\[
V_{IF}^{DC} = \langle V_{IF}(t) \rangle = k_{CL} A_{RF} \cos \varphi
\]

\[
A_{RF} \cos(\omega t + \varphi)
\]

\[
A_{LO} \cos(\omega t)
\]

\[
V_{det} = V_{IF} = V(\varphi) + \text{high harm.}
\]

\[
A_{RF} \ll A_{LO} \implies V_{det}(\varphi) = k_{CL} A_{RF} \cos \varphi
\]

\[
\begin{align*}
V_I &= k_{CL} A_{RF} \cos(\varphi) + \text{high harmonics} \\
V_Q &= k_{CL} A_{RF} \sin(\varphi) + \text{high harmonics}
\end{align*}
\]

\[
\varphi = \arctan(V_Q/V_I) + \frac{\pi}{2} [1 - \text{sgn}(V_I)]
\]

\[
\frac{dV_{det}}{d\varphi} \bigg|_{\varphi=\pm \pi/2} = \pm k_{CL} A_{RF} \approx \frac{5}{10} \text{mV/Deg}
\]

- Passive
- Cheap, Robust
- Wideband
- Sensitivity proportional to level, AM \( \rightarrow \) PM not fully rejected
- Noise figure \( F \approx CL \)
- Good sensitivity but lower wrt optical devices
Direct conversion with photo detector (PD)
- Low phase noise
- Temperature drifts (0.4ps/C°)
- AM to PM conversion (0.5-4ps/mW)

Phase detection between RF and Laser – Sagnac Loop Interferometer or BOM-PD
(Balanced Optical Microwave Phase Detector)
Recently (∼ 10 years) special devices to perform direct measurements of the relative phase between an RF voltage and a train of short laser pulses have been developed
- balanced optical mixer to lock RF osc.
- insensitive against laser fluctuation
- Very low temperature drifts

Results: f=1.3GHz jitter & drift < 10 fs rms limited by detection!
**Balanced cross correlation** of very short optical pulses ($\sigma_t \approx 200\ f_s$) provides an **extremely sensitive** measurement of the **relative delay between** 2 pulses.

The two pulses have orthogonal polarization and generate a shorter wavelength pulse proportional to their time overlap in each branch by means of non-linear crystal.

In a second branch the two polarizations experience a differential delay $\Delta T = T_1 - T_2 \approx \sigma_t$. The amplitudes of the interaction radiation pulses are converted to voltages by photodiodes and their difference is taken as the detector output $V_0$.

If the initial time delay between the pulses is exactly $\Delta T/2$ then clearly $V_0 \approx 0$ (balance), while it grows rapidly as soon as initial delay deviates.

Detection sensitivity up to 10 mV/fs achievable with ultra-short pulses!!!
What is peculiar in PLLs for clients of a stabilization system of a Particle Accelerator facility?

- Both the reference and client oscillators can be either RF VCOs or laser cavities. Phase detectors are chosen consequently;
- Laser oscillators behave as VCOs by trimming the cavity length through a piezo controlled mirror.
  - Limited modulation bandwidth ($\approx$ few kHz typical);
  - Limited dynamic range ($\Delta f/f \approx 10^{-6}$), overcome by adding motorized translational stages to enlarge the mirror positioning range;
  - At frequencies beyond PLL bandwidth ($f > 1$ kHz) mode-locked lasers exhibit excellent low-phase noise spectrum.
PLLs are a very general subject in RF electronics, used to synchronize oscillators to a common reference or to extract the carrier from a modulated signal (FM tuning). In our context PLLs are used to phase-lock the clients of the synchronization system to the master clock (RMO or OMO). The building blocks are:

- A VCO, whose frequency range includes \((D/N) f_{\text{ref}}\);
- A phase detector, to compare the scaled VCO phase to the reference;
- A loop filter, which sets the lock bandwidth;
- A prescalers or synthesizer \((N/D)\) frequency multiplier, \(N\) and \(D\) integers) if different frequencies are required.

![PLL transfer function](image)

\[
\phi_{\text{out}}(s) = D \frac{H(s)}{N} \phi_{\text{ref}}(s) + \frac{1}{1 + H(s)} \phi_n(s)
\]

with \(H(s) = \frac{N}{D} \left( \frac{k_d k_m}{s} F(s) M(s) \right)\).
Loop filters provide PLL stability, tailoring the frequency response, and set loop gain and cut-off frequency.

The output phase spectrum is locked to the reference if $|H(j\omega)| \gg 1$, while it returns similar to the free run VCO if $|H(j\omega)| < 1$.

A flat-frequency response loop filter gives already a pure integrator loop transfer function thanks to a pole in the origin ($f=0$) provided by the dc frequency control of the VCO.

Loop filters properly designed can improve the PLL performance:

- By furtherly increasing the low-frequency gain and remove phase err. offsets due to systematic VCO frequency errors, by means of extra poles in the origin (integrators) compensated by zeroes properly placed;
- By enlarging the PLL BW through equalization of the frequency response of the VCO modulation port.

A very steep frequency response can be obtained (slope = 40 dB/decade) in stable conditions (see Nyquist plot).
Performances of Synchronization Systems

• Client Residual Jitter
• Stabilized Reference Distribution
Performances of Synchronization Systems: Residual Jitter of Clients

A client with a free-run phase noise $\varphi_{i_0}$ once being PLL locked to the reference with a loop gain $H_i(j2\pi f)$ will show a residual phase jitter $\varphi_i$ and a phase noise power spectrum $S_i(f)$ according to:

$$\varphi_i = \frac{H_i}{1 + H_i} \varphi_{ref} + \frac{1}{1 + H_i} \varphi_{i_0} \rightarrow S_i(f) = \frac{|H_i|^2}{|1 + H_i|^2} S_{ref}(f) + \frac{1}{|1 + H_i|^2} S_{i_0}(f)$$

**Incoherent noise contributions**

Client absolute residual time jitter

$$\sigma_{t_i}^2 = \frac{1}{\omega_{ref}^2} \int_{f_{min}}^{+\infty} \frac{|H_i|^2 S_{ref}(f) + S_{i_0}(f)}{|1 + H_i|^2} df$$
But we are finally interested in relative jitter between clients and reference $\varphi_{i-r} = \varphi_i - \varphi_{ref}$, and among different clients $\varphi_{i-j} = \varphi_i - \varphi_j$:

$$\varphi_{i-r} = \frac{\varphi_i - \varphi_{ref}}{1 + H_i} \rightarrow S_{i-r}(f) = \frac{S_i(f) + S_{ref}(f)}{|1 + H_i|^2}$$

Client residual relative time jitter

$$\sigma^2_{t_{i-r}} = \frac{1}{\omega^2_{ref}} \int_{f_{min}}^{+\infty} \frac{S_{ref}(f) + S_i(f)}{|1 + H_i|^2} df$$

$$\varphi_{i-j} = \frac{\varphi_i - \varphi_{ref}}{1 + H_i} - \frac{\varphi_j - \varphi_{ref}}{1 + H_j} \rightarrow S_{i-j}(f) = \frac{S_i(f)}{|1 + H_i|^2} + \frac{S_j(f)}{|1 + H_j|^2} + \frac{H_i - H_j}{(1 + H_i)(1 + H_j)}^2 S_{ref}(f)$$

$$\sigma^2_{t_{i-j}} = \frac{1}{\omega^2_{ref}} \int_{f_{min}}^{+\infty} S_{i-j}(f) df$$

Residual relative time jitter between clients i-j

If $H_i \neq H_j$ there is a direct contribution of the master clock phase noise $S_{ref}(f)$ to the relative jitter between clients i and j in the region between the cutoff frequencies of the 2 PLLs. That’s why a very low RMO phase noise is specified in a wide spectral region including the cut-off frequencies of all the client PLLs (0.1÷100 kHz typical).
Client jitters can be reduced by efficient PLLs locking to a local copy of the reference.

Reference distribution drifts need to be under control to preserve a good facility synchronization.

Depending on the facility size and specification the reference distribution can be:

**RF based, through coaxial cables**
- Passive (mainly) / actively stabilized
- Cheap
- Large attenuation at high frequencies
- Sensitive to thermal variations (copper linear expansion $\approx 1.7 \times 10^{-5}/^\circ C$)
- Low-loss 3/8" coaxial cables very stable for $\Delta T << 1^\circ C$ @ $T_0 \approx 24^\circ C$

**Optical based, through fiber links**
- Pulsed (mainly), also CW AM modulated
- High sensitivity error detection (cross correlation, interferometry, ...)
- Small attenuation, large BW
- Expensive
- Active stabilization always needed (thermal sensitivity of fibers)
- Dispersion compensation always needed for pulsed distribution
Around some optimal temperature \( T_{opt} \) cable physical elongation is compensated by dielectric constant variation. PPM relative delay variation is:

\[
\frac{\Delta \tau}{\tau} \bigg|_\text{PPM} \approx - \left( \frac{T - T_{opt}}{T_c} \right)^2
\]

For a 3/8" cable (FSJ2): \( T_{opt} \approx 24 \, ^\circ C \), \( T_c \approx 2 \, ^\circ C \). Good enough?

\( L \approx 1 \, km \rightarrow \tau \approx 5 \, \mu s \rightarrow \Delta \tau/\tau \approx 5 \, fs/5 \, \mu s \approx 10^{-3} \text{PPM} \) !!!

LONG DISTANCES → ACTIVE LINK STABILIZATION REQUIRED !!!

PERFORMANCES OF SYNCHRONIZATION SYSTEMS:

Drift of the reference distribution

RF distribution

\( f \sim 100 \, \text{MHz} \ldots \text{GHz} \)

Pulsed Optical distribution

\( \Delta f \sim 5 \, \text{THz} \)

**Sketches from H. Schlarb**
Active stabilized links are based on high resolution **round trip time measurements** and **path length correction** to stick at some stable reference value. Pulsed optical distribution is especially suitable, because of low signal attenuation over long links and path length monitoring through very sensitive pulse cross-correlators. However, **dispersion compensation of the link is crucial** to keep the optical pulses very short (≈ 100 fs).
Beam Synchronization

- Bunch Arrival Monitors
- Effects of Client Synchronization Errors on Bunch Arrival Time
The beam is\textit{ streaked} by a \textit{transverse RF cavity} on a \textit{screen}. The image is captured by a camera. Longitudinal charge distribution and centroid position can be measured.

- Works typically on single bunch. Bunch trains can be eventually resolved with fast gated cameras;
- Destructive (needs a screen ...)
- Measure bunch wrt to RF (relative measurement)
- with a spectrometer \(\rightarrow\) long. phase space imaging - \((z, \epsilon) \rightarrow (y, x)\)

\[
\tau_{res} = \frac{E/e}{\omega_{RF} V_\perp \sqrt{\beta_{\perp}^{defl}}} 
\]

\textbf{Achievable resolution down to } \approx 10 \text{ fs}
A reference laser pulse train (typically taken from the facility OMO) is connected to the optical input of a Mach-Zehnder interferometric modulator (EOM). The short laser pulses are amplitude-modulated by a bipolar signal taken from a button BPM placed along the beam path and synchronized near to the voltage zero-crossing. The bunch arrival time jitter and drift is converted in amplitude modulation of the laser pulses and measured.

- Works very well on bunch trains;
- Non-intercepting;
- Measure bunch wrt to a laser reference (OMO);
- Demonstrated high resolution

BAM 1 – 2 placed 60 m away along the beam path
An electro-optic crystal is placed near the beam trajectory. In correspondence to the beam passage the crystal is illuminated with a short reference laser pulse transversally enlarged and linearly polarized. The bunch electric field induces bi-rifrengence in the crystal, so that while propagating the laser gains elliptical polarization. A polarized output filter delivers a signal proportional to the polarization rotation, i.e. to the beam longitudinal charge distribution.

- Single shot, non-intercepting;
- Provides charge distribution and centroid position;
- Resolution $\approx 50 \, fs$ for the bunch duration, higher for centroid arrival time ($1 \, pixel \approx 10 \, fs$).

$\sigma_t \approx 20 \, fs_{rms}$ over 330 shots
Beam arrival time: Magnetic Compressor

Basics of magnetic bunch compression

Magnetic Chicane: Higher energy particles follow a shorter path. This allows the tail to catch the head of the bunch.

Typically operate at slightly less (or more) than full compression

Energy chirp $h$:

$$W_0 = W_{in} + qV_{RF} \cos(\varphi_0)$$

Final energy error of an accelerated particle starting with energy and phase errors $\Delta W_{in}$ and $\Delta \varphi$

$$\Delta W_0 = \Delta W_{in} - qV_{RF} \sin(\varphi_0) \Delta \varphi = \Delta W_{in} + h \frac{c}{\omega_{RF}} W_0 \Delta \varphi$$

with

$$h \overset{\text{def}}{=} \frac{\Delta W / W_0}{\Delta z} = \frac{\omega_{RF}}{c} \frac{\Delta W / W_0}{\Delta \varphi} = -\frac{qV_{RF} \sin(\varphi_0)}{W_{in} + qV_{RF} \cos(\varphi_0)} \frac{\omega_{RF}}{c}$$

chirp coefficient = relative energy deviation normalized to the particle z position
Non-isochronous transfer line:

The bunch compression process is completed by making the chirped beam travel along a non-isochronous transfer line. Particles with different energies travel along paths of different lengths according to:

$$\Delta L = R_{56} \left( \frac{\Delta W_0}{W_0} \right)$$

Path elongation normalized to the relative energy error

Overall, a particle entering the magnetic compressor with a time error $\Delta t_{in}$ and a relative energy error $\Delta W_{in}/W_{in}$ will leave it with time and relative energy errors $\Delta W_0/W_0$ and $\Delta t_o$ given by:

$$\Delta W_0/W_0 = h c \Delta t_{in} + \frac{W_{in}}{W_0} \Delta W_{in}/W_{in}$$

$$\Delta t_o = \Delta t_{in} + \frac{\Delta L}{c} = \Delta t_{in} + \frac{R_{56}}{c} \left( \frac{\Delta W_0}{W_0} \right) = (1 + h R_{56}) \Delta t_{in} + \frac{R_{56} W_{in}}{c W_0} \left( \frac{\Delta W_{in}}{W_{in}} \right)$$

In the end if the compressor is tuned to get $h \cdot R_{56} \approx -1$ it may easily noticed that the exit time of a particle is almost independent on the entering time. This mechanism describes the deformation (compression) of the longitudinal distribution of the particles in a bunch, but also the multi-shot dynamics of the bunch center of mass. The bunch arrival time downstream the compressor is weakly related to the upstream arrival time.
Compressor longitudinal transfer matrices:

Previous results can be summarized in a matrix notation, according to:

\[
\begin{pmatrix}
\Delta t \\
\Delta W/W
\end{pmatrix}_o = \begin{pmatrix}
1 & \frac{R_{56}}{c} \\
0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
hc & \frac{W_{in}}{W_0}
\end{pmatrix} \begin{pmatrix}
\Delta t \\
\Delta W/W
\end{pmatrix}_{in}
\]

\[
\begin{pmatrix}
\hat{\Delta t} \\
\hat{\Delta W/W}
\end{pmatrix}_{in} = \begin{pmatrix}
1 + hR_{56} & \frac{R_{56} W_{in}}{c W_0} \\
hc & \frac{W_{in}}{W_0}
\end{pmatrix} \begin{pmatrix}
\Delta t \\
\Delta W/W
\end{pmatrix}_{in}
\]

\[\hat{\text{B}} \quad \hat{\text{A}} \quad \hat{\text{C}} = \hat{\text{B}} \cdot \hat{\text{A}}\]
non-isochronous drift  
chirping acceleration  
total compressor stage
**Effects of PM and AM in the compressor RF on final bunch energy and arrival time:**

Let’s consider now the presence of phase ($\Delta \phi_o = -\omega_{RF} \Delta t_{RF}$) and amplitude ($\Delta V_{RF}/V_{RF}$) errors in the RF section of the compressor. The resulting energy error of the beam entering in the non-isochronous drift is:

\[
\Delta W_o = q \Delta V_{RF} \cos(\phi_o) - q V_{RF} \sin(\phi_o) \Delta \phi_o
\]

\[
\frac{\Delta W_o}{W_o} = -hc \Delta t_{RF} + \frac{W_o - W_{in}}{W_o} \frac{\Delta V_{RF}}{V_{RF}}
\]

The energy error will result in a time error downstream the drift through the transfer matrix $\hat{B}$.

\[
\begin{pmatrix}
\Delta t \\
\Delta W/W_o
\end{pmatrix} =
\begin{bmatrix}
1 & \frac{R_{56}}{c} \\
0 & 0
\end{bmatrix}
\begin{pmatrix}
\Delta t_{RF} \\
\Delta V_{RF}/V_{RF}
\end{pmatrix}
\]

\[
\hat{B} =
\begin{bmatrix}
-hc & \frac{R_{56} W_o - W_{in}}{c} \\
0 & \frac{W_o - W_{in}}{W_o}
\end{bmatrix}
\]

\[
\hat{R} = \hat{B} \cdot \hat{N}
\]

The energy error will result in a time error downstream the drift through the transfer matrix $\hat{B}$. RF AM&PM conversion to bunch time and energy.
**BC block diagram:**

To the first order the RF noise does not affect the bunch internal distribution, since RF amplitude and phase does not change significantly over a bunch time duration. It will more affect the bunch-to-bunch energy deviation and arrival time.

\[
\begin{align*}
\Delta t_0 &= (1 + hR_{56})\Delta t_{in} - hR_{56}\Delta t_{RF} + \frac{R_{56}}{c} \frac{W_{in}}{W_0} (\Delta W/W)_{in} + \frac{R_{56}}{c} \frac{W_0 - W_{in}}{W_0} (\Delta V_{RF}/V_{RF}) \\
\end{align*}
\]

The bunch arrival time downstream the compressor is strongly related to the phase of the chirping RF. It is also affected by initial energy errors and RF amplitude variations.
How beam arrival time is affected by synchronization errors of the sub-systems?

Perturbations of subsystem phasing $\Delta t_i$ will produce a change $\Delta t_b$ of the beam arrival time.

First-order approximation:

$$\Delta t_b = \sum_i a_i \Delta t_i = \sum_i \frac{\Delta t_i}{c_i} \quad \text{with} \quad \sum_i a_i = 1$$

Values of $a_i$ can be computed analytically, by simulations or even measured experimentally. They very much depends on the machine working point.
Beam synchronization

How beam arrival time is affected by synchronization errors of the sub-systems?

- **No compression:** Beam captured by the GUN and accelerated on-crest
  \[ a_{PC} \approx 0.7; \quad a_{RF_{GUN}} \approx 0.3; \quad \text{others } a_i \approx 0 \]

- **Magnetic compression:** Energy-time chirp imprinted by off-crest acceleration in the booster and exploited in magnetic chicane to compress the bunch
  \[ a_{RF_{boost}} \approx 1; \quad |a_{PC}| \ll 1; \quad \text{others } a_i \approx 0 \]
  Compression can be staged (few compressors acting at different energies). Bunch can be overcompressed (head and tail reversed, \( a_{PC} < 0 \)).

- **RF compression:** a non fully relativistic bunch \( (E_0 \approx \text{few MEV at Gun exit}) \) injected ahead the crest in an RF capture section slips back toward an equilibrium phase closer to the crest during acceleration, being also compressed in this process
  \[ a_{RF_{CS}} \approx 1; \quad |a_{PC}|, |a_{RF_{GUN}}| \ll 1; \quad \text{others } a_i \approx 0 \]
  The bunch gains also an Energy-time chirp. RF and magnetic compressions can be combined.

Particle distribution within the bunch and shot-to-shot centroid distribution behave similarly, but values of coefficients \( a_i \) might be different since space charge affects the intra-bunch longitudinal dynamics.
**Bunch Arrival Time Jitter**

If we consider uncorrelated residual jitters of $\Delta t_i$ (measured wrt the facility reference clock), the bunch arrival time jitter $\sigma_{tb}$ is given by:

$$\sigma_{tb}^2 = \sum_i a_i^2 \sigma_{ti}^2$$

while the jitter of the beam respect to a specific facility sub-system (such as the PC laser or the RF accelerating voltage of a certain group of cavities) $\sigma_{tb-j}$ is:

$$\sigma_{tb-j}^2 = (a_j - 1)^2 \sigma_{tj}^2 + \sum_{i \neq j} a_i^2 \sigma_{ti}^2$$

**EXAMPLE:** PC laser jitter $\sigma_{tPC} \approx 70 \text{ fs}$, RF jitter $\sigma_{tRF} \approx 30 \text{ fs}$

No Compression: $a_{PC} \approx 0.65$, $a_{RF_{GUN}} \approx 0.35$

Magnetic Compression: $a_{PC} \approx 0.2$, $a_{RF_{boost}} \approx 0.8
✓ Timing and Synchronization has growth considerably in the last ~ 15 years as a Particle Accelerators specific discipline

✓ It involves concepts and competences from various fields such as Electronics, RF, Laser, Optics, Control, Diagnostics, Beam dynamics, ...

✓ Understanding the real synchronization needs of a facility and proper specifications of the systems involved are crucial for successful and efficient operation (but also to avoid overspecification leading to extra-costs and unnecessary complexity ...)

✓ Synchronization diagnostics (precise arrival time monitors) is fundamental to understand beam behavior and to provide input data for beam-based feedback systems correcting synchronization residual errors

✓ Although stability down to the fs scale has been reached, many challenges still remain since requirements get tighter following the evolution of the accelerator technology. The battleground will move soon to the attosecond frontier ... (see A. Ferran Pousa et al 2017 J. Phys.: Conf. Ser. 874 012032)
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