
Superconducting Cavities

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DESY



Lecture I

1. Introduction and History
2. RF Parameters
3. Criteria for Cavity Design
4. Test Results of single-cell cavities

Lecture II

5. Multi-cell Structures and Weakly Coupled Structures
6. Tools for RF-design
7. LEC and Transient state
8. Performance test
9. Mechanical Design

Milestones that led to accelerators based on SRF

Superconductivity

1908: Heike Kamerlingh Onnes (Holland)
Liquefied Helium for the first time.

1911: Heike Kamerlingh Onnes
Discovered Superconductivity.

1928-34: Walther Meissner (Germany)
Discovered Superconductivity of Ta, V, Ti
and Nb.

RF Acceleration

1924: Gustaf Ising (Sweden)
The First Publication on RF Acceleration,
Arkiv för Matematik, Astronomi och Fysik,

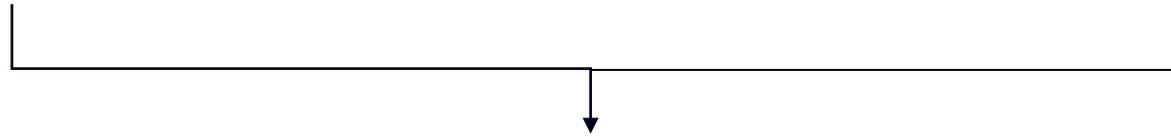
1928: Rolf Wideröe (Norway, Germany)
Built the first RF Accelerator,
Arch. für Elektrotechnik 21, vol.18.

1947: Luis Alvarez (USA)
Built first DTL (32 MeV protons).

1947: W. Hansen (USA)
Built first 6 MeV e-accelerator, Mark I
(TW- structure).

Superconductivity

RF Acceleration



1961: W. Fairbank (Stanford Univ.)

Presented the first proposal for a superconducting accelerator

1964: W. Fairbank, A. Schwettman, P. Wilson (Stanford Univ.)

First acceleration of electrons with sc lead cavity

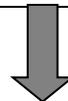
1970: J. Turneaure (Stanford Univ.)

$E_{\text{peak}} = 70 \text{ MV/m}$ and $Q \sim 10^{10}$ in 8.5 GHz cavity !

1968-1981: M. McAshan, A. Schwettman, T. Smith, J. Turneaure, P. Wilson
(Stanford Univ.)

Developed and Constructed the Superconducting Accelerator SCA

Since then, we have built many sc accelerators and we are constructing and making plans for many new facilities.



1. Introduction and History

Dismantled Facilities

1. TRISTAN (32/49m)*
2. LEP (288/490m)
3. HERA (16/19m)

Operating Facilities

1. SCA (4/28m)
2. S-DALINAC (10/10m)
3. CESR (4/1.2 m)
4. CEBAF (320/160m)
5. KEK B-Factory (8/2.4m)
6. Taiwan LS (1+1/0.3m)
7. Canadian LS (1+1/0.3m)
8. DIAMOND (3/0.9m)
9. SOLEIL (4/1.7m)
10. TTF II (56/58m)
11. SNS (81/65m)
12. JLab-FEL (24/14m)
13. LHC (16/6m)
14. ELBE (6/6m)

*(Number of cavities / total active length)

1. Introduction and History

Tomorrow Facilities

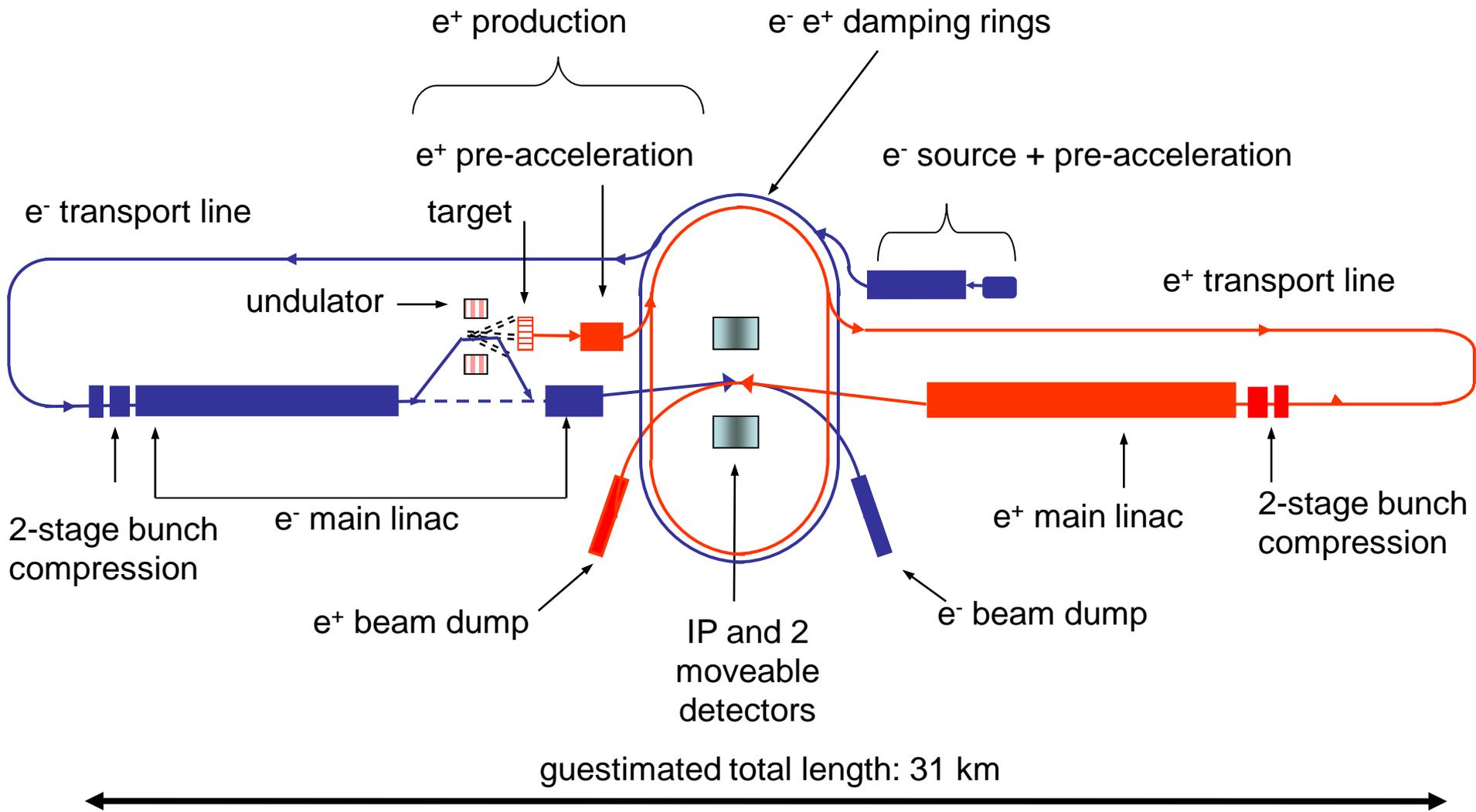
1. CEBAF-12GeV (400/216m)
2. SNS-upgrade (117/ 98m)
3. XFEL (648/674m)
4. ERL-Cornell (310/250m)
5. RHIC-cooling (1/1 m)
6. BEPC II (2/0.6m)
7.

Day after Tomorrow Facilities

1. FRIB (non-elliptical 47/~70m)
2. X-Ray MIT (option 176 / 184m)
3. Project X (elliptical 352 /~360m)
4. ERHIC.....
5. ELIC
6. ILC (~15764 /~16395m)

1. Introduction and History

ILC (~15764/~16395m)



1. Introduction and History

The “heart” of all mentioned facilities are (were) sc standing wave accelerating structures, here elliptical with $\beta \geq 0.61$.

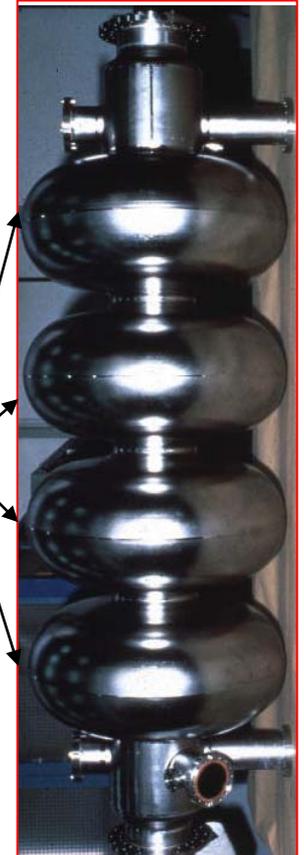
FERMI 3.9 GHz



TESLA/ILC 1.3 GHz



LEP 0.352 GHz



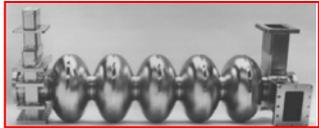
S-DALINAC 3 GHz



SNS $\beta=0.61, 0.81, 0.805$ GHz



CESR/CEBAF 1.5 GHz



HERA 0.5 GHz



HEPL 1.3 GHz



KEK-B 0.5 GHz



CESR 0.5 GHz



1. Introduction and History

We have also many “heavy” particles accelerators based on low- β sc cavities.
You had lecture on this topic by Dr. Maurizio Vretenar on Friday.
The low- β cavities are not discussed in this lecture.

Accelerator/Upgrade/Project

ISAC-II
SPIRAL-II
SARAF
IUAC
HIE-REX
EURISOL
SPES
LNL
ReA3
FRIB
IFMIF

QWR



HWR



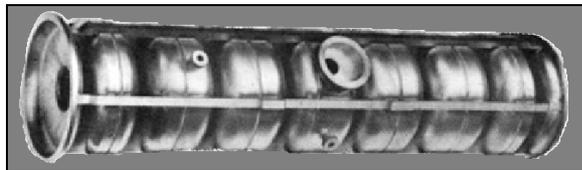
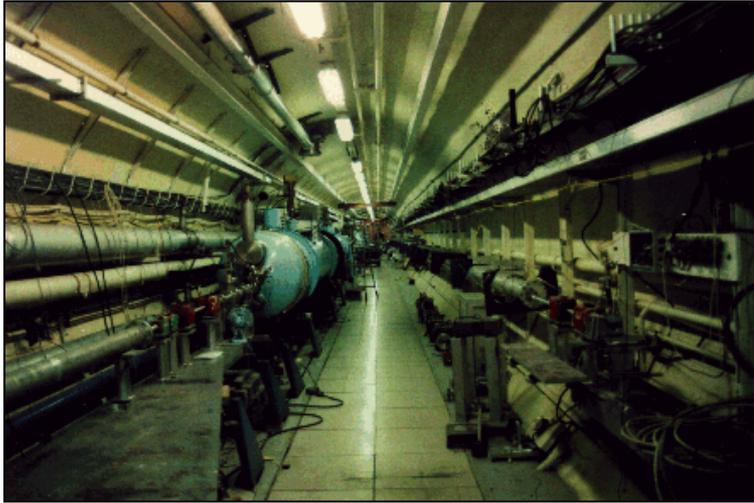
Spoke



1. Introduction and History

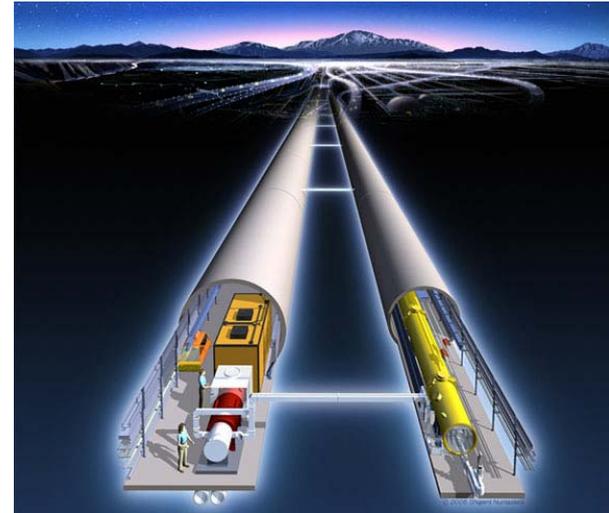
What was the progress in the 33 years and what do we need in the next 10 years?

~ 28 m long SCA at Stanford, 1977.



$E_{\text{acc}} \sim 2$ (2.5) MV/m in cw (10% DF).
4 Structures 5.65m + capture + pre-accelerator.

~ 21.6 km long ILC linac, 2018+



$E_{\text{acc}} > 34$ MV/m shown in
several 9-cells in the cw test.

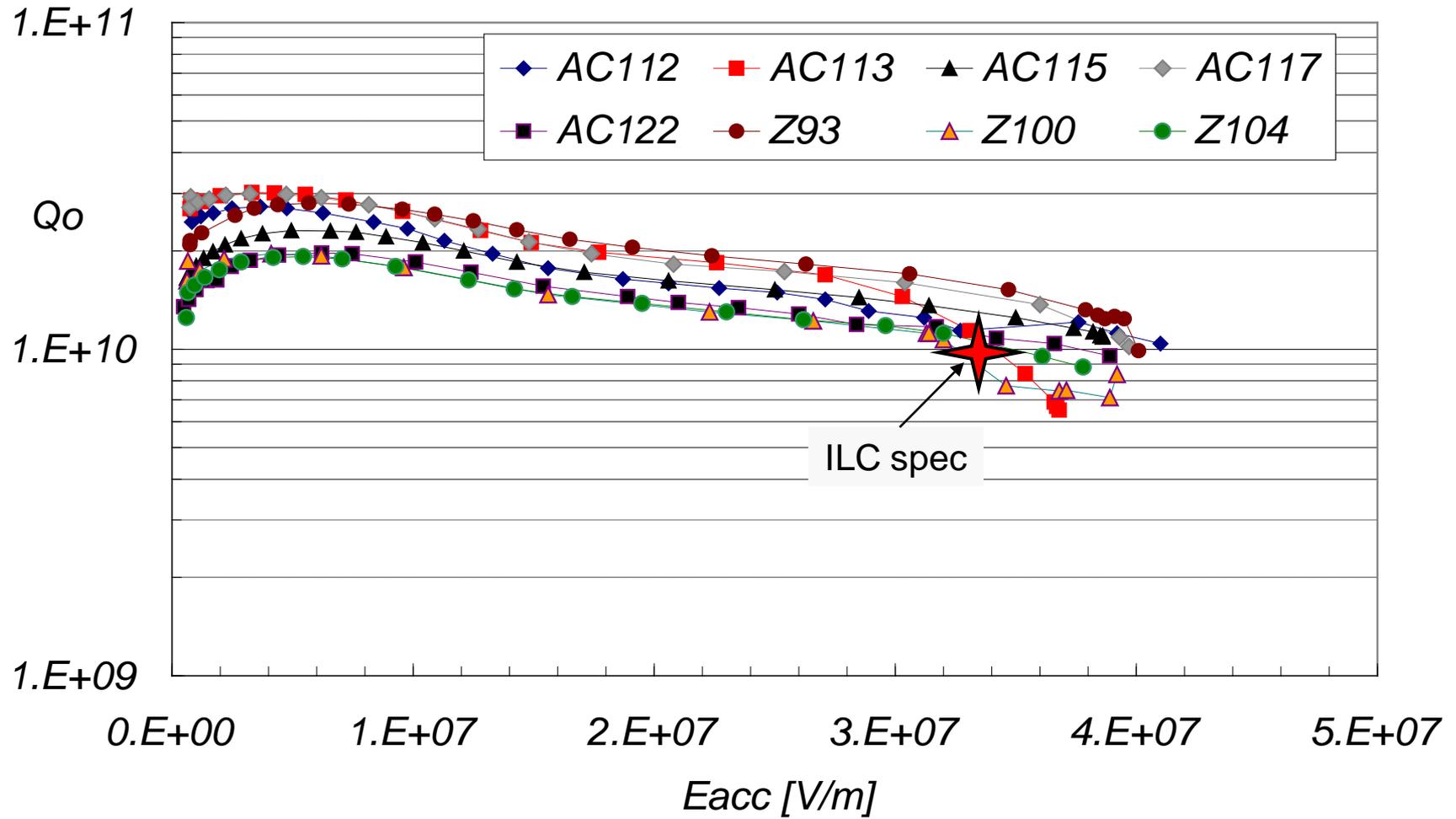


This gradient is required in all 15764 cavities.

1. Introduction and History

Results at DESY (status 2010):

cw test result at 2 K for 8 best electropolished 9-cell TESLA cavities.



2.1. Cavities and their Eigenmodes *(the simplest example)*

Cavity \equiv a volume, partially closed by the metal wall, capable to store the E-H energy

First assumption:

1. Stored E-H fields **are harmonic** in time.

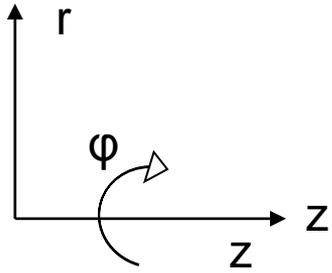
Maxwell equations for the harmonic,
lossless case with no free charge in the volume

$$\left\{ \begin{array}{l} \nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E} \\ \nabla \times \mathbf{E} = -i\omega\mu\mathbf{H} \\ \nabla \cdot \mathbf{E} = 0 \\ \nabla \cdot \mathbf{H} = 0 \end{array} \right.$$

2. RF Parameters

Second assumption (good approximation for the elliptical cavities):

2. The volume is **cylindrically symmetric**. We commonly use the (r, φ, z) coordinates.



z is conventional direction of the acceleration and it is symmetry axis

$$\left\{ \begin{array}{l} \nabla_{\mathbf{c}} \times \mathbf{H} = i\omega\epsilon\mathbf{E} \\ \nabla_{\mathbf{c}} \times \mathbf{E} = -i\omega\mu\mathbf{H} \\ \nabla_{\mathbf{c}} \cdot \mathbf{E} = 0 \\ \nabla_{\mathbf{c}} \cdot \mathbf{H} = 0 \end{array} \right.$$

$$\nabla_{\mathbf{c}} \times \mathbf{A} = \vec{i}_r \left(\frac{1}{r} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right) + \vec{i}_\varphi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \vec{i}_z \left(\frac{1}{r} \frac{\partial (rA_\varphi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \varphi} \right)$$

$$\nabla_{\mathbf{c}} \cdot \mathbf{A} = \frac{1}{r} \frac{\partial (rA_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

Third assumption

3. For the **acceleration** are suitable modes with **strong E** along the beam **trajectory**.

This ensures, by the proper phasing, maximal energy exchange between the cavity and beam.

TM_{0xx}-like monopole modes have “very strong” E_z component on the symmetry axis.

Fields of the monopole modes do not depend on φ .

$$\frac{\partial E}{\partial \varphi} = 0 \qquad \frac{\partial H}{\partial \varphi} = 0$$

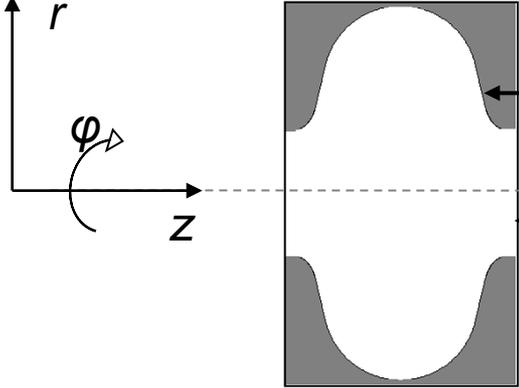
Non monopole modes (HOM) have component $E_z = 0$ on the symmetry axis.

Their fields dependent on φ .

2. RF Parameters

Maxwell equations + boundary conditions for E and H lead to the Helmholtz equation, which is an eigenvalue problem.

For $H(r,z)$ field of a monopole mode the equation is:



$$(\nabla_c^2 + \omega^2 \varepsilon \mu) H = 0$$

$$n \cdot H = 0 \quad \text{on metal wall}$$

$$\left\{ \begin{array}{l} H = 0 \\ n \cdot H = 0 \end{array} \right. \quad \text{optionally on non metal boundary}$$

$$\nabla_c^2 \mathbf{A} = \nabla_c (\nabla_c \cdot \mathbf{A}) - \nabla_c \times \nabla_c \times \mathbf{A}$$

There is infinity number of TM_{0xx} solutions (modes) to the Helmholtz equation.

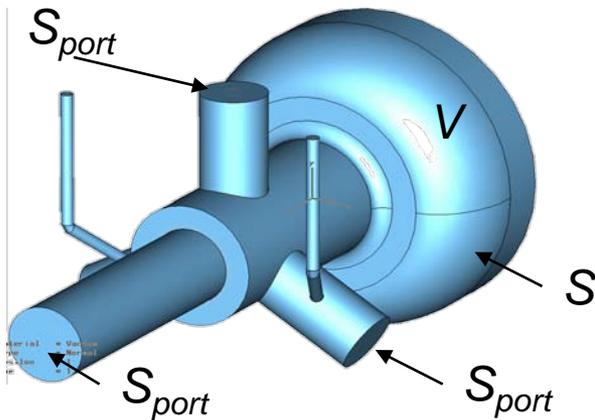
All modes are determine by:

$$H_n(r,z) = [0, H_{\varphi,n}(r,z), 0],$$

$$E_n(r,z) = [E_{r,n}(r,z), 0, E_{z,n}(r,z)]$$

and by frequency ω_n .

2.2 What are figures of merit for a cavity storing E-H energy?



$W_n \equiv$ stored energy of a mode $n : \{\omega_n, E_n, H_n\}$.

$$W_n \equiv 2\mu \int_V \frac{H_n^2}{4} dV = 2\varepsilon \int_V \frac{E_n^2}{4} dV$$

Quality Factors

The measure of the energy loss in metal wall and due to the radiation via open ports:

Intrinsic $Q \equiv Q_0$

$$Q_{0,n} \equiv \frac{\omega_n \cdot W_n}{P_n} = \frac{\omega_n \cdot W_n}{\frac{R_{s,n}}{2} \int_S H_n^2 ds}$$

External $Q \equiv Q_{ext}$

$$Q_{ext,n} \equiv \frac{\omega_n \cdot W_n}{P_{rad,n}} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int_{S_{port}} E_n \times H_n ds}$$

Geometric Factor

The measure of the energy loss in the metal wall for the surface resistance $R_{s,n}=1\Omega$

$$G_n \equiv Q_{0,n} \cdot R_{s,n} = \frac{\omega_n \cdot W_n \cdot R_{s,n}}{P_n} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int_S H_n^2 ds}$$

It is the ratio of stored energy to the integral of $(H_n)^2$ on metal wall S .

2.3 What are figures of merit for the beam-cavity interaction?

This interaction which is:

- Acceleration
- Deceleration (ERL)
- HOMs excitation

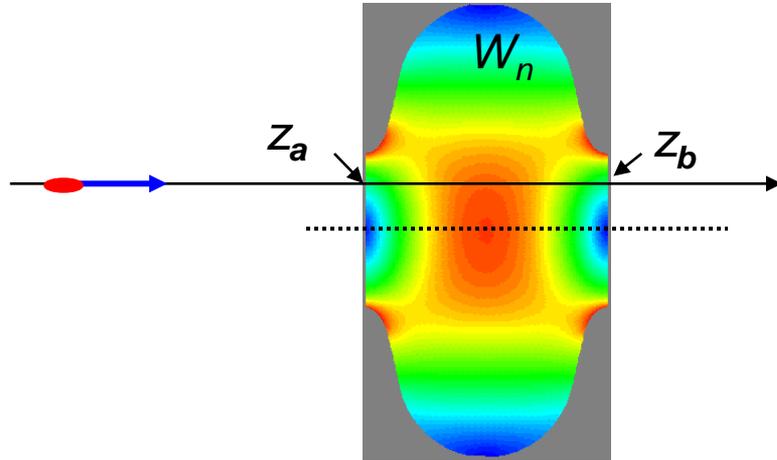
can be described in the Frequency Domain (FD) or/and in Time Domain (TD).

2. RF Parameters

$(R/Q)_n$; *beam impedance*

It is a “measure” of the energy exchange between point charge and mode n (FD).

Mode n : $\{\omega_n, E_n, H_n\}$.



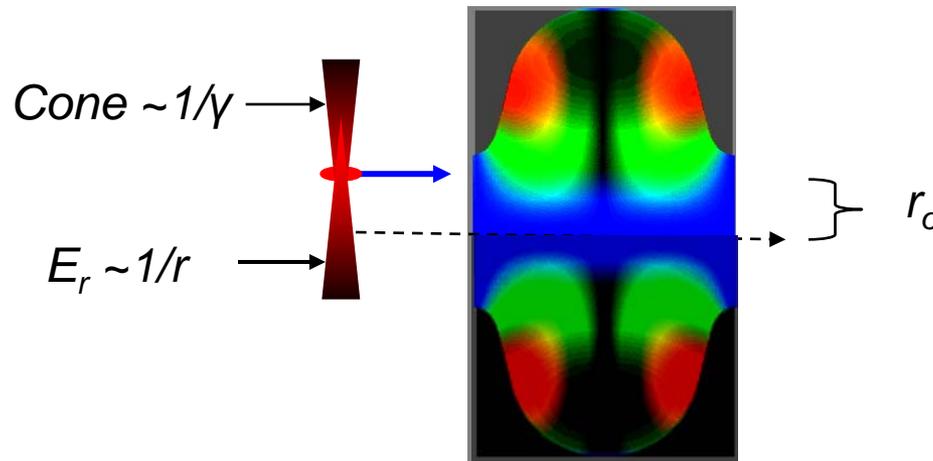
Trajectory of the point charge q, assumed here is a straight line.

$$V_n = \sqrt{\left(\int_{z_a}^{z_b} E_{n,z} \sin\left(\frac{\omega_n}{\beta c} (z - z_a)\right) dz \right)^2 + \left(\int_{z_a}^{z_b} E_{n,z} \cos\left(\frac{\omega_n}{\beta c} (z - z_a)\right) dz \right)^2}$$

$$(R/Q)_n \equiv \frac{V_n^2}{\omega_n W_n}$$

Longitudinal and Transverse Loss Factors (TD) (Excitation of cavity modes)

Ultra relativistic point charge q passes empty cavity



- Density of the inducted charge on the wall depends on the distance to beam trajectory
- The non uniform charge density on the metal wall causes the current flow on surface

2. RF Parameters

The amount of energy lost by charge q to the cavity is:

$$\Delta U_q = k_{\parallel} \cdot q^2 \quad \text{for monopole modes (max. on axis)}$$

$$\Delta U_q = k_{\perp} \cdot q^2 \quad \text{for non monopole modes (off axis)}$$

where k_{\parallel} and $k_{\perp}(r)$ are the loss factors for monopole and transverse modes respectively

The induced E-H field (wake) is a superposition of all cavity eigenmodes having the $E_n(r, \varphi, z)$ field along the particle trajectory.

Both description methods FD and TD are equivalent.

For an individual mode n and point-like charge:

$$k_{\parallel, n}^p = \frac{\omega_n \cdot (R/Q)_n}{4}$$

Note the linac convention for (R/Q) definition.

Similar for other loss factors.....

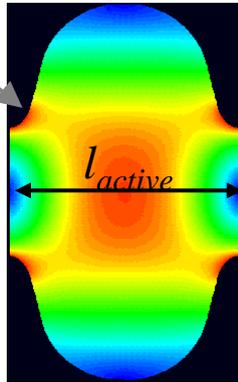
2.4. Some “practical” RF parameters of the accelerating mode

At stored energy W_{acc} the mean value of the accelerating gradient is:

$$E_{acc} = \frac{\sqrt{\omega_{acc} \cdot W_{acc} \cdot (R/Q)_{acc}}}{l_{active}}$$

E_{peak} on the wall

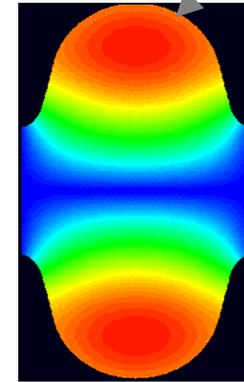
$$\frac{E_{peak}}{E_{acc}}$$



Contour plot of |E/

B_{peak} on the wall

$$\frac{B_{peak}}{E_{acc}}$$



Contour plot of |B/

The ratio shows sensitivity of the shape to the field electron emission phenomenon.

The ratio shows limit in E_{acc} due to the break-down of superconductivity (Nb ~190 mT).

$$G_{acc} \cdot (R/Q)_{acc}$$

For the accelerating mode we often use the product: $G_{acc} \cdot (R/Q)_{acc}$, as a “measure” of the power P dissipated in the wall at given accelerating voltage V_{acc} and given surface resistance R_s .

$$\frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G_{acc} \cdot (R/Q)_{acc}}$$

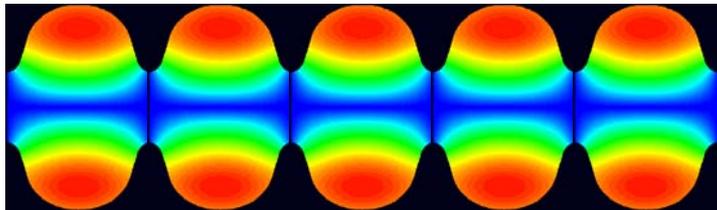
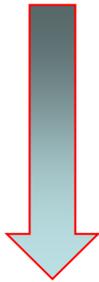
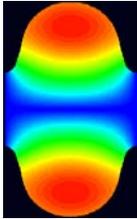
Big improvements are possible:

- Due to superconductivity
- Due to the surface quality.

This is due to the geometry of cells
Moderate improvement possible.

cell-to-cell coupling k_{cc}

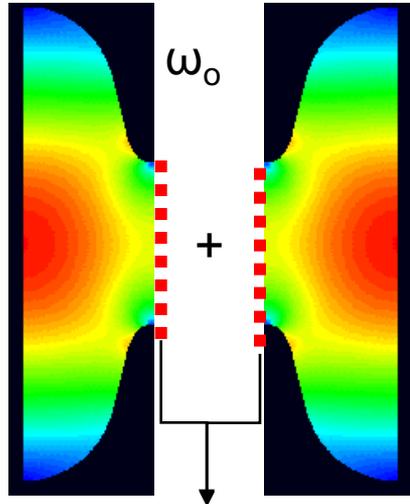
The k_{cc} is relevant for the accelerating mode passband of multi-cell structures



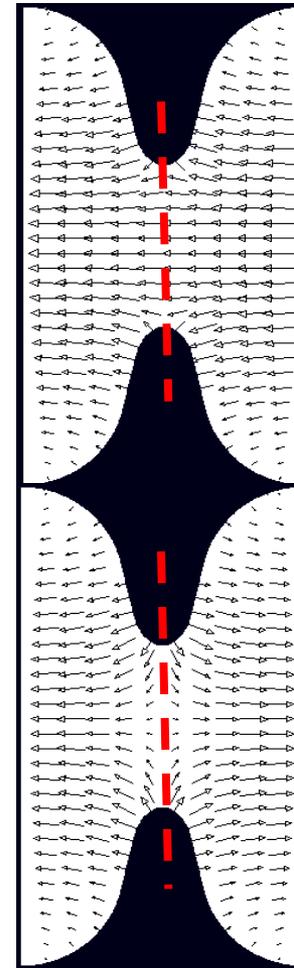
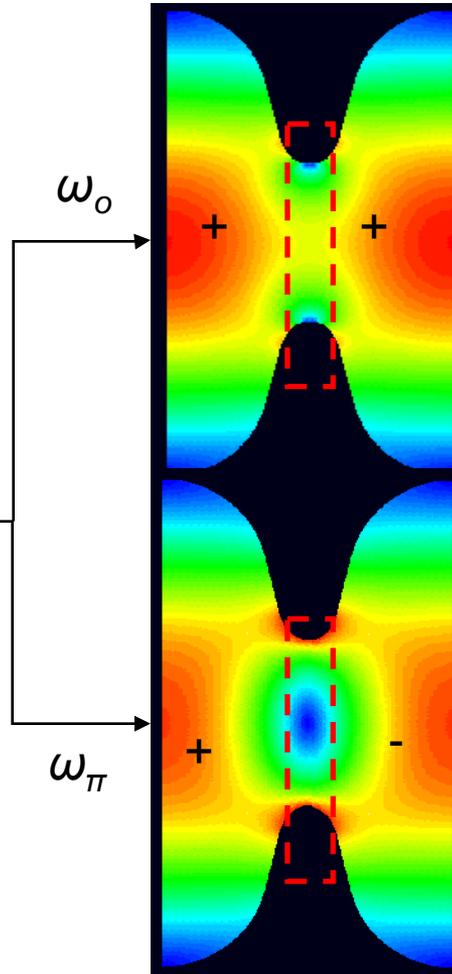
- Single-cell structures are attractive because:
- ◆ It is easier to manage HOM damping
 - ◆ There is no field flatness problem.
 - ◆ Input coupler transfers less power
 - ◆ They are easy for cleaning and preparation
 - ◆ **But it is expensive to base even a small linear accelerator on the single cells. We do it only for very high beam current machines.**
 - ◆ Multi-cell structures are less expensive/m and allow for higher real-estate gradient.

2. RF Parameters

Resonators closed by metal wall:



Symmetry planes for the H field

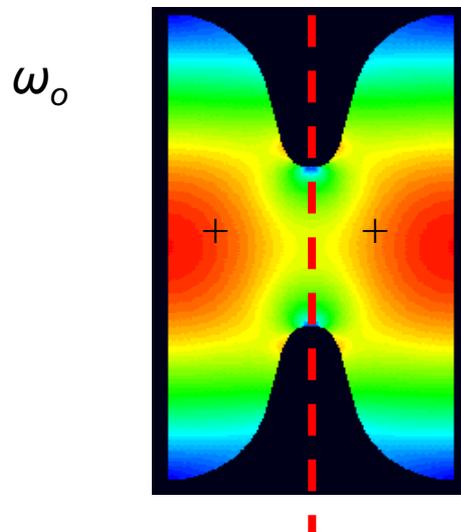


Symmetry plane for the H field

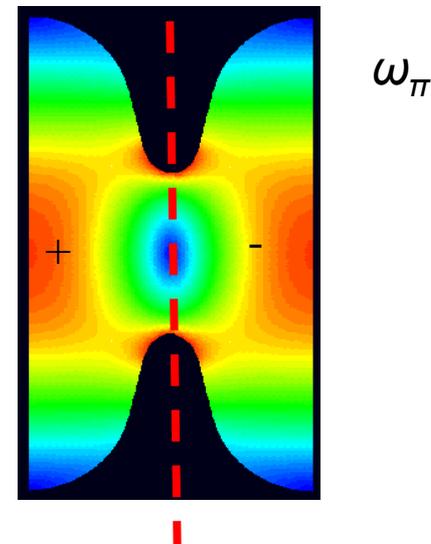
Symmetry plane for the E field,
which is an additional solution

2. RF Parameters

The energy flux across the coupling region, refilling energy loss is proportional to the transverse components: H_φ and E_r



Small E_r (due to the losses) + strong H_φ at the symmetry plane

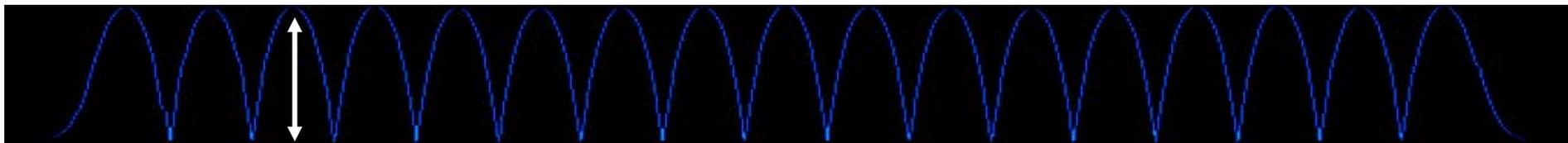
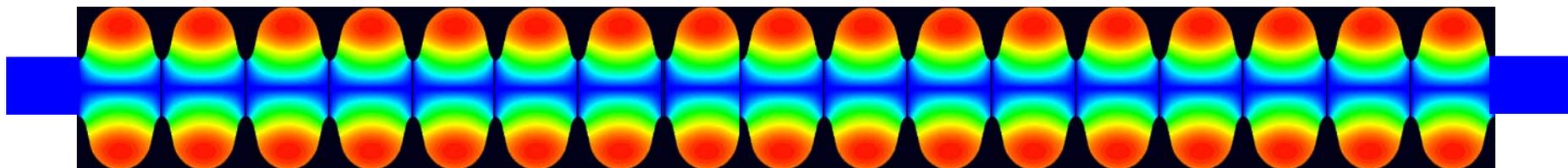


Small H_φ (due to the losses) + strong E_r at the symmetry plane

The normalized difference between these frequencies is a measure of the energy flow via the coupling region

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0} \cdot 2$$

2. RF Parameters



$$\frac{\Delta A_j}{A_j} = a_{ff} \frac{\Delta f_j}{f_j}$$

Field flatness factor a_{ff} for structure made of N cells and their coupling factor k_{cc}

$$a_{ff} = \frac{N^2}{k_{cc}}$$

The above formulae estimate the sensitivity of a multi-cell field profile to frequency errors of an individual cell for the accelerating mode (π -mode)

3. Criteria for Cavity Design

Criteria for inner cell design

We will discuss here design of inner cells because they “dominate” the RF properties of multi-cell superconducting accelerating structures.

RF parameters summary:

$$FM : (R/Q), G, E_{peak}/E_{acc}, B_{peak}/E_{acc}, k_{cc}$$

$$HOM : k_{\perp}, k_{\parallel}$$

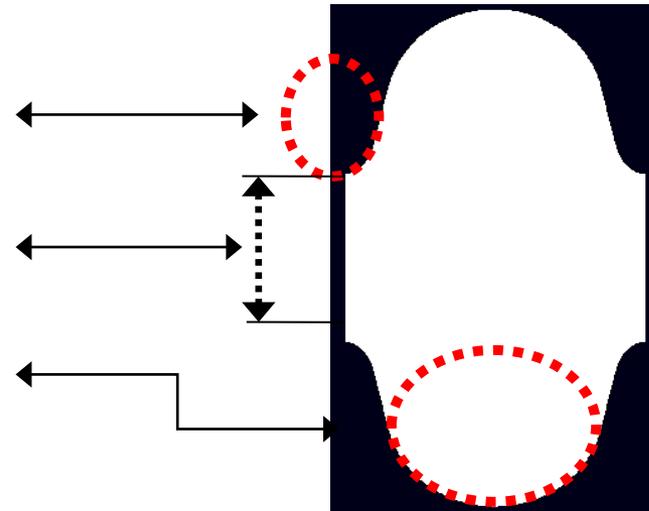
There are 7 parameters we want to optimize for an inner cell.

Geometry :

iris ellipsis : half-axis h_r, h_z

iris radius : r_i

equator ellipsis : half-axis h_r, h_z



There is some kind of conflict 7 parameters and only 5 variables to “tune”

3. Criteria for Cavity Design

<i>Criteria</i>	<i>RF-parameter</i>	<i>Improves when</i>	<i>Cavity examples</i>
<i>Operation at high gradient</i>	E_{peak}/E_{acc} B_{peak}/E_{acc} ↓	r_i ↓ <i>Iris & Equator shape</i>	<i>TESLA,</i> <i>HG CEBAF-12 GeV</i>
<i>Low cryogenic losses</i>	$(R/Q) \cdot G$ ↑	r_i ↓ <i>Equator shape</i>	<i>LL CEBAF-12 GeV</i>
<i>High $I_{beam} \leftrightarrow$</i> <i>Low HOM impedance</i>	k_{\perp}, k_{\parallel} ↓	r_i ↑	<i>B-Factory</i> <i>RHIC cooling</i>

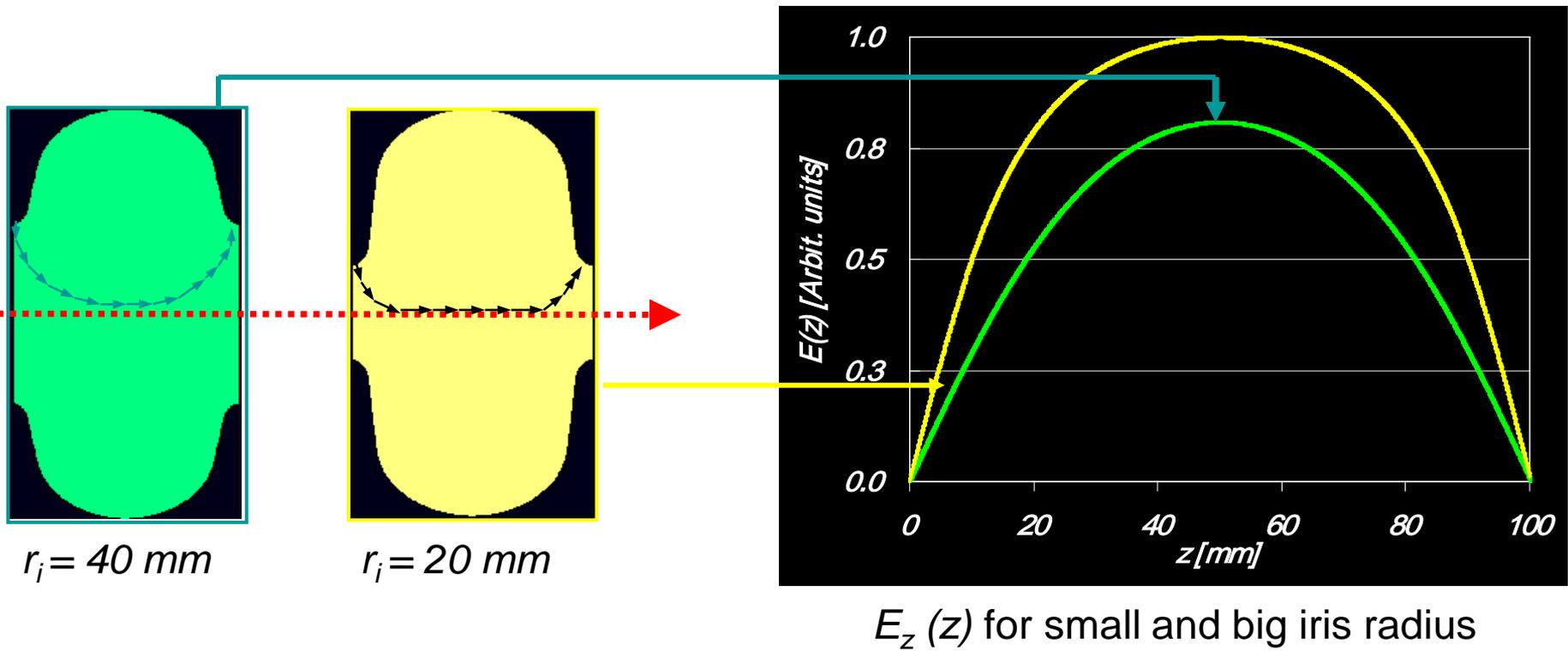
We see here that r_i is a very “powerful variable” to trim the RF-parameters of a cavity.

3. Criteria for Cavity Design

Why for a smaller aperture (r_i)

- (R/Q) is bigger
- E_{peak}/E_{acc} , B_{peak}/E_{acc} are lower ?

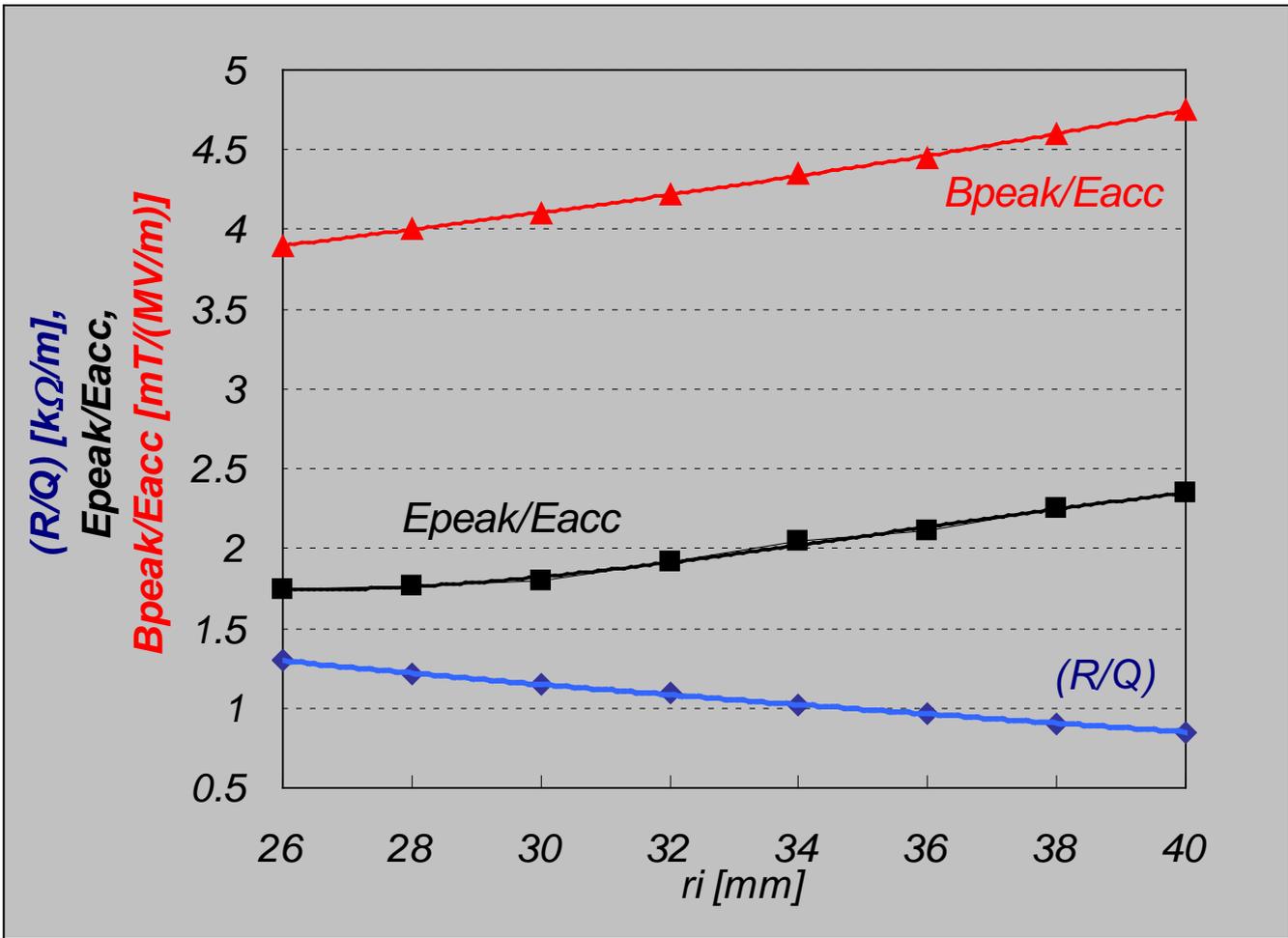
E_{acc} is higher at the same stored energy in the cell with smaller aperture.



3. Criteria for Cavity Design

Example:

$\{ (R/Q), E_{peak}/E_{acc}, B_{peak}/E_{acc} \}$ vs. r_i for cell at $f = 1.5$ GHz

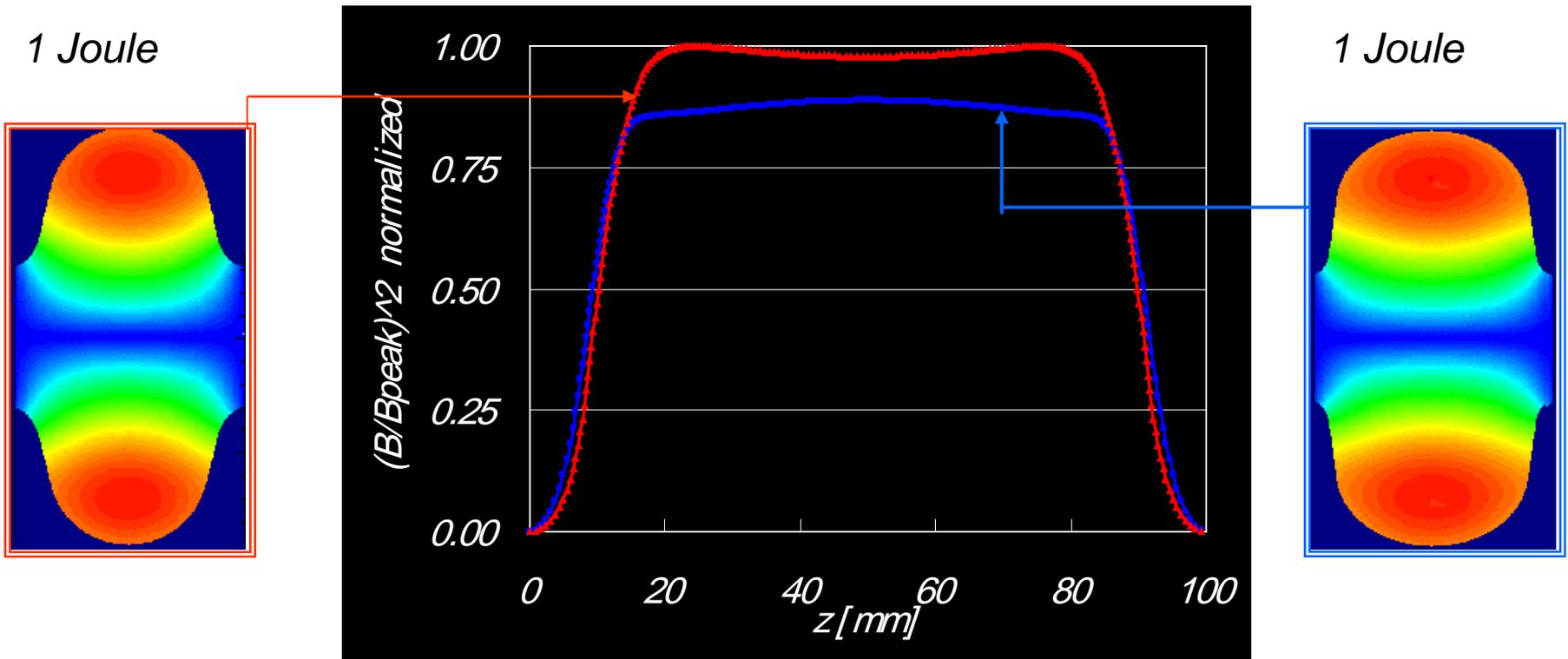


A. Mosnier, E. Haeibel, SRF Workshop 1991

3. Criteria for Cavity Design

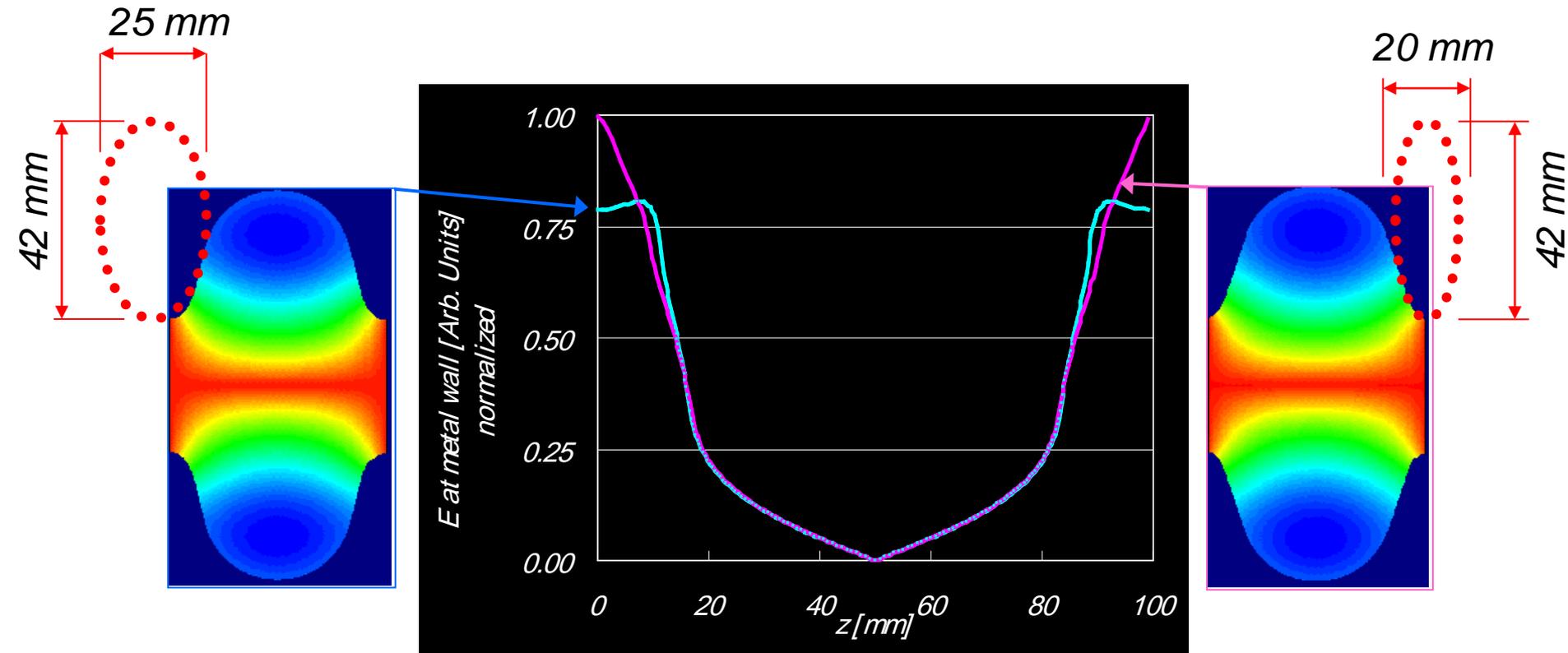
In addition to the iris radius r_i :

- B_{peak}/E_{acc} (and G) changes vs. the equator shape



3. Criteria for Cavity Design

Similarly : E_{peak}/E_{acc} changes vs. the iris shape



Both cells have the same: f , (R/Q) and r_i

3. Criteria for Cavity Design

We know that a smaller aperture r_i makes FM :

- (R/Q) higher
- B_{peak}/E_{acc} , E_{peak}/E_{acc} lower

} (+)

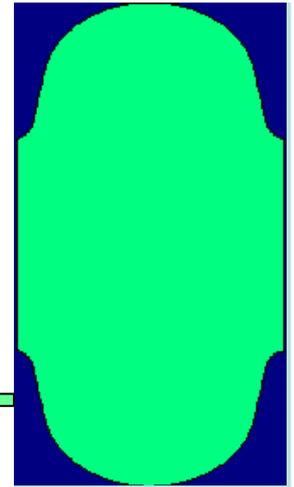
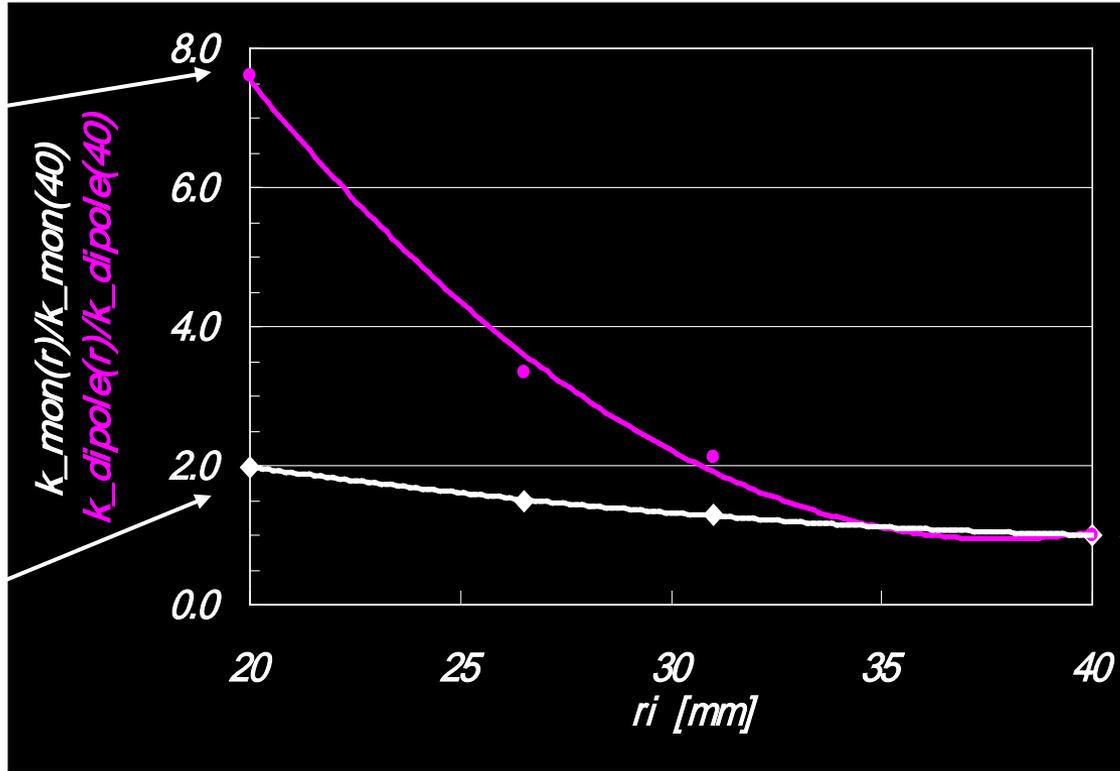
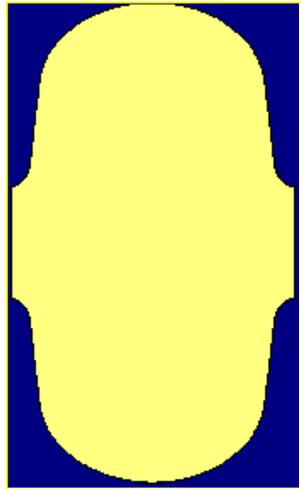
but unfortunately a smaller aperture r_i makes:

- HOMs impedances (k_{\perp} , k_{\parallel}) higher
- cell-to-cell coupling (k_{cc}) weaker

} (-)

3. Criteria for Cavity Design

HOMs loss factors (k_{\perp} , k_{\parallel})



$$(R/Q) = 152 \Omega$$

$$B_{peak} / E_{acc} = 3.5 \text{ mT}/(\text{MV}/\text{m})$$

$$E_{peak} / E_{acc} = 1.9$$

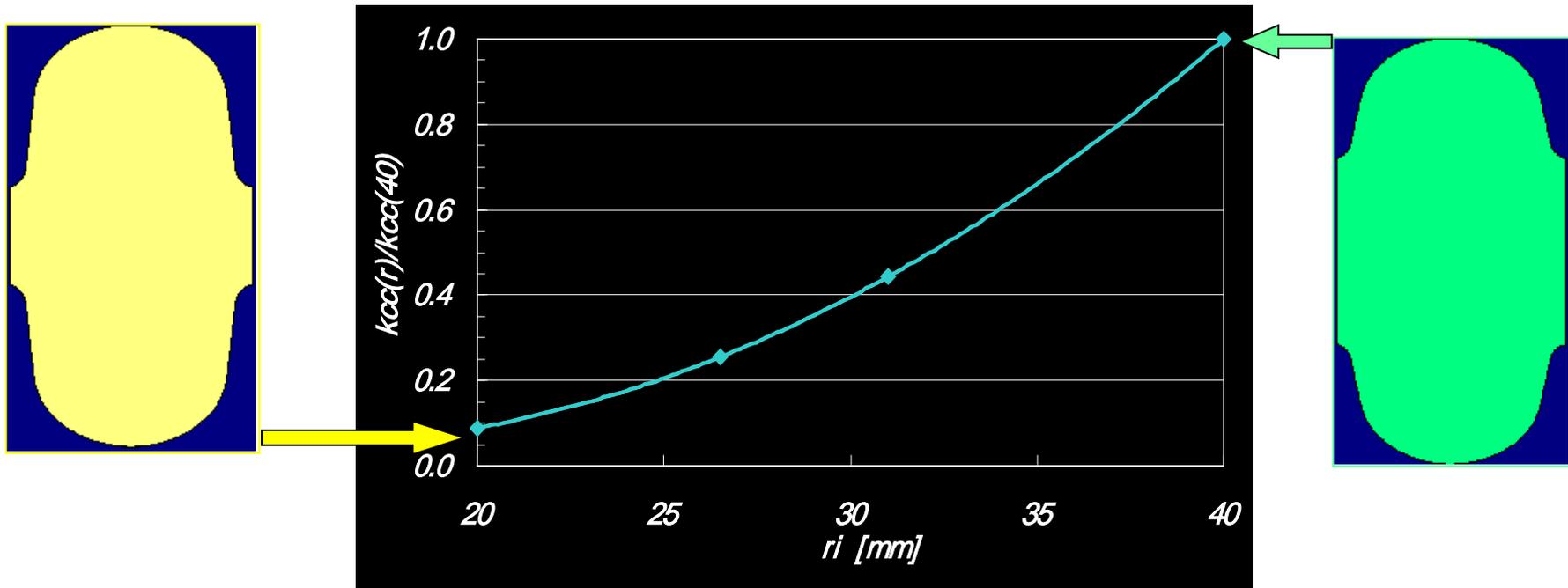
$$(R/Q) = 86 \Omega$$

$$B_{peak} / E_{acc} = 4.6 \text{ mT}/(\text{MV}/\text{m})$$

$$E_{peak} / E_{acc} = 3.2$$

3. Criteria for Cavity Design

Cell-to-cell coupling, k_{cc}



$$(R/Q) = 152 \Omega$$

$$B_{peak}/E_{acc} = 3.5 \text{ mT}/(\text{MV}/\text{m})$$

$$E_{peak}/E_{acc} = 1.9$$

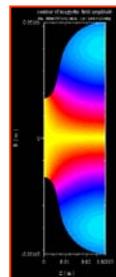
$$(R/Q) = 86 \Omega$$

$$B_{peak}/E_{acc} = 4.6 \text{ mT}/(\text{MV}/\text{m})$$

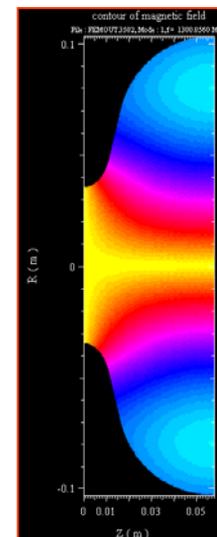
$$E_{peak}/E_{acc} = 3.2$$

3. Criteria for Cavity Design

Frequency of the accelerating mode frequency



x 2 =



f_{π}	[MHz]	2600
R/Q	[Ω]	57
$r/q=(R/Q)/l$	[Ω/m]	2000
G	[Ω]	271

f_{π}	[MHz]	1300
R/Q	[Ω]	57
$r/q=(R/Q)/l$	[Ω/m]	1000
G	[Ω]	271

$$r/q=(R/Q)/l \sim f$$

3. Criteria for Cavity Design

From the formula, we learned before, one obtains:

$$P_{dissipated} = \frac{R_s \cdot V_{acc}^2}{G_{acc} \cdot (r/q)_{acc} \cdot l_{active}}$$

Higher f_{π} would be a good choice to minimize dissipation in the metal wall when the length l_{active} and final energy V_{acc} are fixed.

Unfortunately this applies only to room temperature structures made of Cu, which

$$R_s \sim (f)^{1/2}.$$

For superconductors, like Nb:

$$R_s(f) = R_{res} + R_{BCS} = R_{res} + 0.0002 \cdot \frac{1}{T} \cdot \left(\frac{f[\text{GHz}]}{1.5}\right)^2 \cdot \exp\left(-\frac{17.67}{T}\right)$$

and R_s , which is $\sim (f)^2$ for higher f must be compensated with lower temperature T .

This is why ILC, XFEL, ERL,... (1.3GHz) will operate at 2 K (1.8 K), and HERA (0.5 GHz) and LEP (0.352 GHz) could operate at 4.2 K

3. Criteria for Cavity Design

Examples of inner cells

		<i>CEBAF Original Cornell $\beta=1$</i>	<i>CEBAF -12 Low Loss $\beta=1$</i>	<i>TESLA $\beta=1$</i>	<i>SNS $\beta=0.61$</i>	<i>SNS $\beta=0.81$</i>	<i>RHIC Cooler $\beta=1$</i>
f_{π}	[MHz]	1497.0	1497.0	1300.0	805.0	805.0	703.7
k_{cc}	[%]	3.29	1.49	1.9	1.52	1.52	2.94
E_{peak}/E_{acc}	-	2.56	2.17	1.98	2.66	2.14	1.98
B_{peak}/E_{acc}	[mT/(MV/m)]	4.56	3.74	4.15	5.44	4.58	5.78
R/Q	[Ω]	96.5	128.8	113.8	49.2	83.8	80.2
G	[Ω]	273.8	280	271	176	226	225
$R/Q * G$	[$\Omega * \Omega$]	26421	36064	30840	8659	18939	18045
k_{\perp} ($\sigma_z=1mm$)	[V/pC/cm ²]	0.22	0.53	0.23	0.13	0.11	0.02
k_{\parallel} ($\sigma_z=1mm$)	[V/pC]	1.36	1.71	1.46	1.25	1.27	0.85

Evolution of inner cells proposed for the ILC collider

TESLA optimized

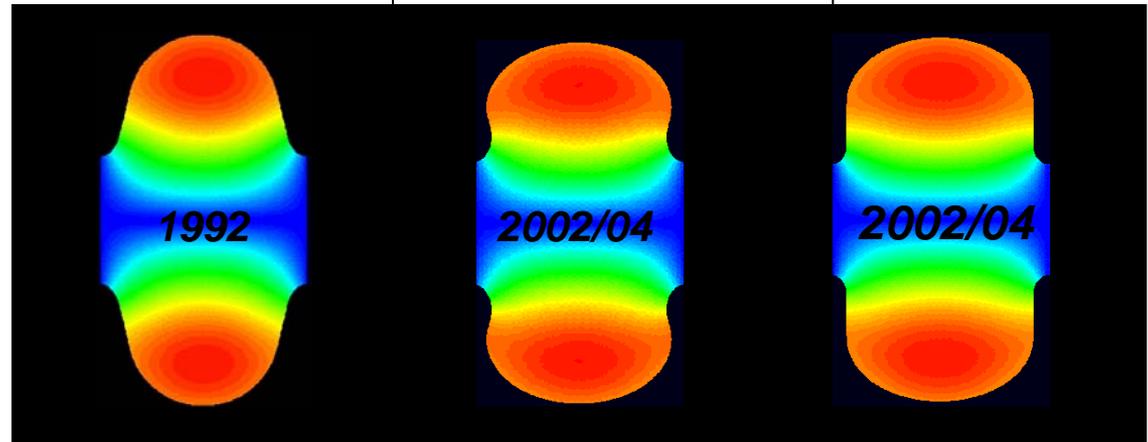
$$E_{peak}/E_{acc}$$

Re-entrant optimized

$$B_{peak}/E_{acc}$$

LL optimized

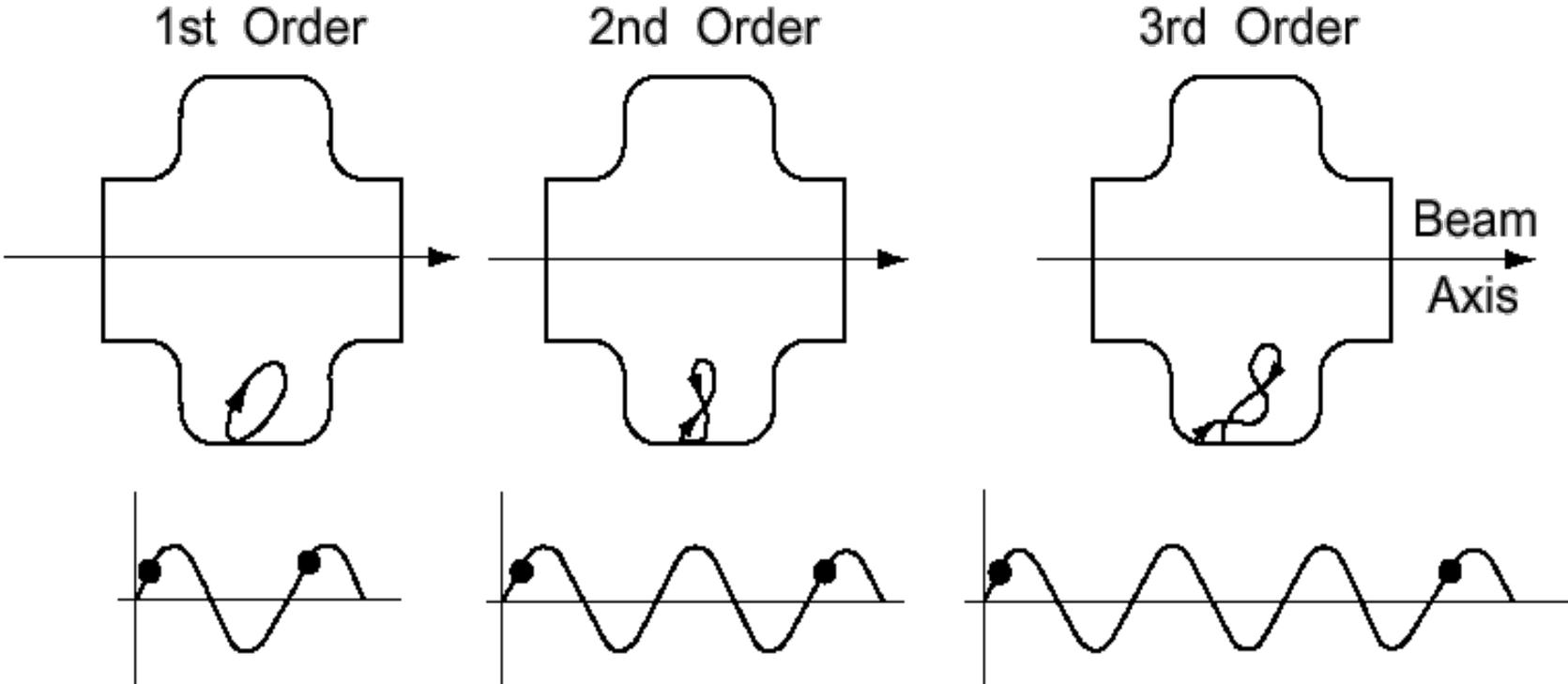
$$B_{peak}/E_{acc}$$



r_i	[mm]	35	30	30
k_{cc}	[%]	1.9	1.56	1.52
E_{peak}/E_{acc}	-	1.98	2.30	2.36
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15	3.57	3.61
R/Q	[Ω]	113.8	135	133.7
G	[Ω]	271	284.3	284
$R/Q * G$	[$\Omega * \Omega$]	30840	38380	37970
$k_{\perp} (\sigma_z=1mm)$	[V/pC/cm ²]	0.23	0.38	0.38
$k_{\parallel} (\sigma_z=1mm)$	[V/pC]	1.46	1.75	1.72

Multipacting

It is a phenomenon of resonant electron emission and multiplication.

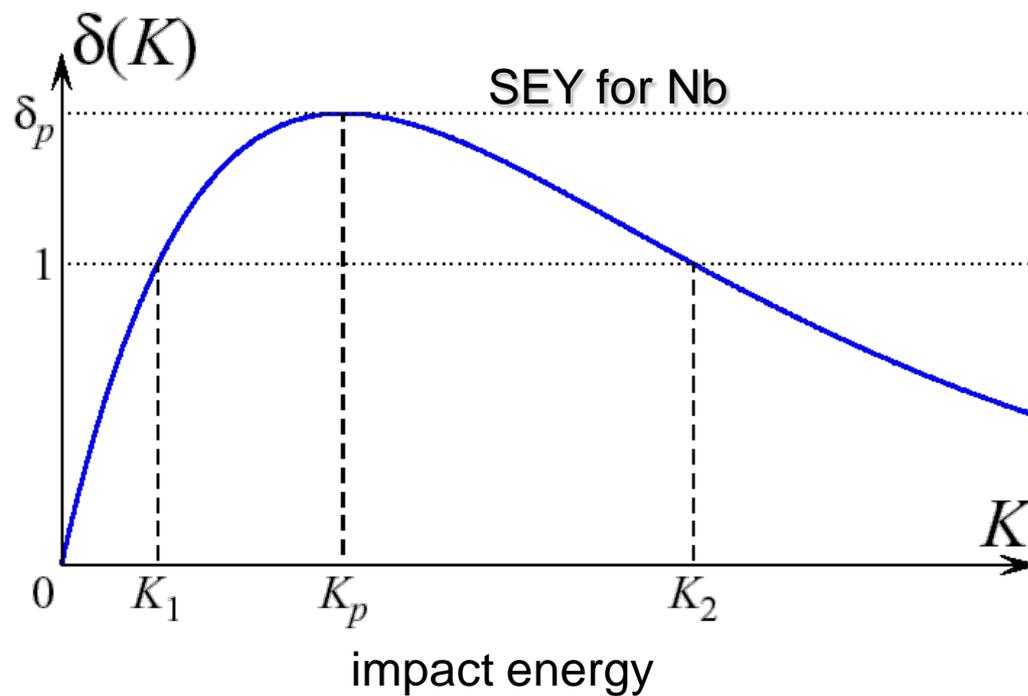


Curtsey W. Hartung

Impacting electron might create more than one secondary electron. This depends on the impact energy K and secondary emission yield $\delta(K)$.

3. Criteria for Cavity Design

SEY is function of the impact energy K and depends on the surface cleanliness.



Condition	K_1	K_2
high SEY	~ 27 eV	$\gtrsim 2000$ eV
typical SEY	~ 40 eV	~ 1000 eV
low SEY	~ 150 eV	~ 750 eV

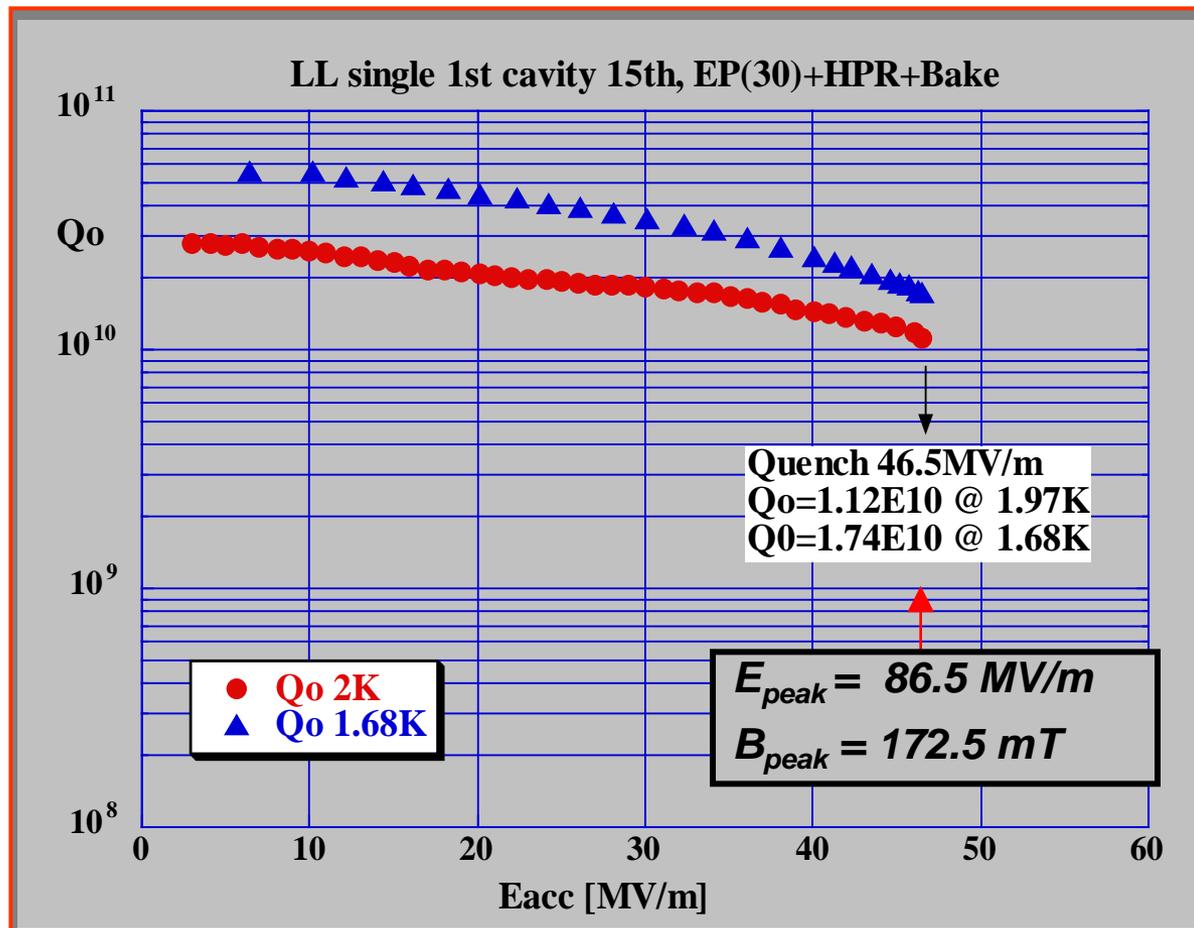
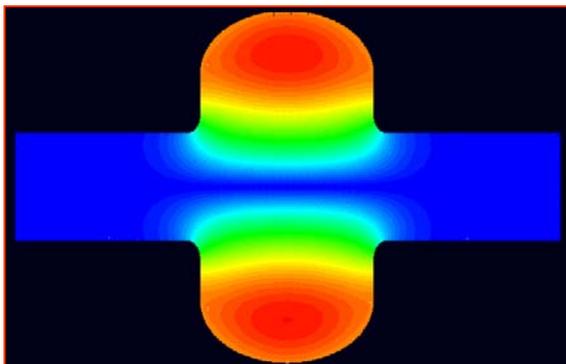
When happens, multipacting is barrier in rising the accelerating field in cavities and usually leads to quench.

In the design process we need to prove whether or not the shape of cell allows for multipacting.

4. Test Results of single-cell cavities

LL; KEK test September 2005

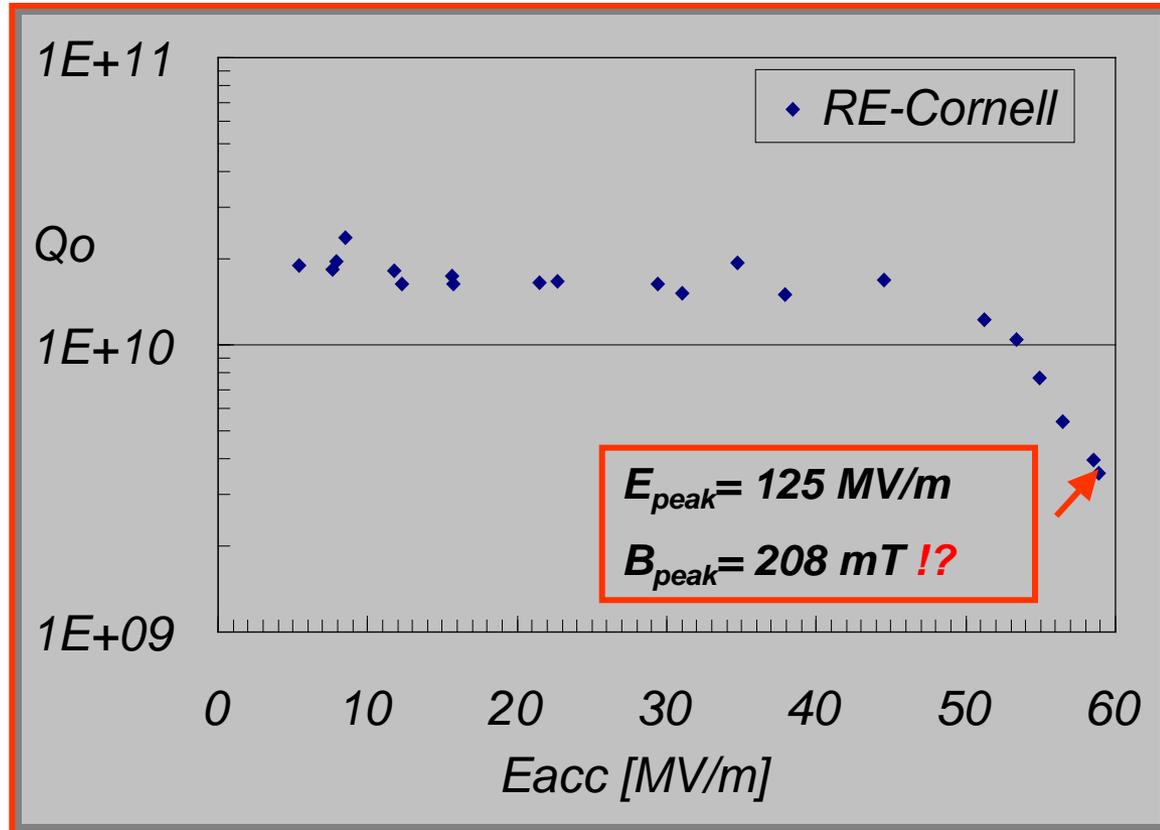
f	[MHz]	1286.6
E_{peak}/E_{acc}	-	1.86
B_{peak}/E_{acc}	[mT/(MV/m)]	3.71
R/Q	[Ω]	130.0
G	[Ω]	279
\varnothing_{iris}	[mm]	61



4. Test Results of single-cell cavities

RE; Cornell, test in March 2007 !!!!

f	[MHz]	1300.3
E_{peak}/E_{acc}	-	2.11
B_{peak}/E_{acc}	[mT/(MV/m)]	3.53
R/Q	[Ω]	126.0
G	[Ω]	283.3
\varnothing_{iris}	[mm]	60



Lecture I

1. Introduction and History
2. RF Parameters
3. Criteria for Cavity Design
4. Test Results of single-cell cavities

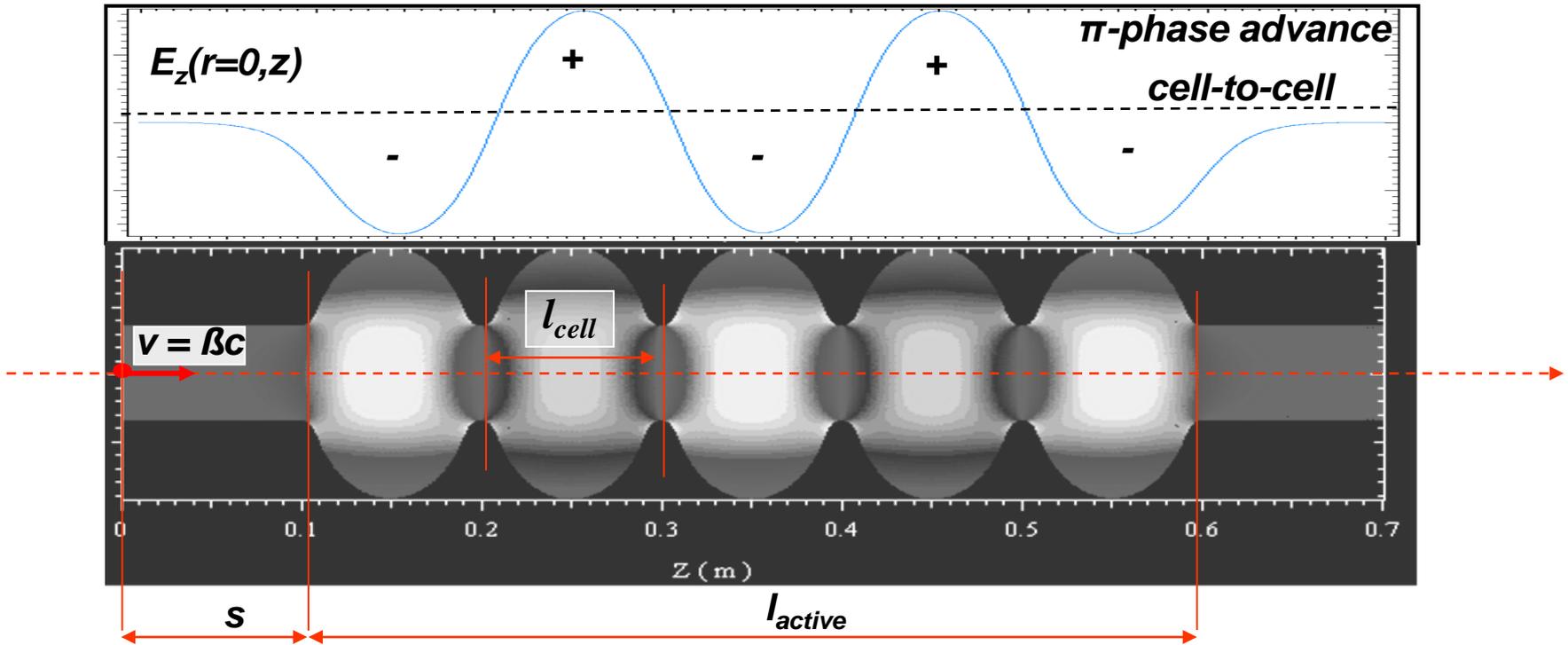
Lecture II

5. Multi-cell Structures and Weakly Coupled Structures
6. Tools for RF-design
7. LEC and Transient state
8. Performance test
9. Mechanical Design

We re-call **pros** and **cons** for a multi-cell structure:

- ◆ Cost of accelerators is lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics.....)
- ◆ Higher real-estate gradient (better fill factor)
- ◆ Field flatness vs. N
- ◆ HOM excitation and trapping vs. N
- ◆ Power capability of fundamental power couplers vs. N
- ◆ Surface cleaning procedures become more complicated
- ◆ The worst performing cell limits whole multi-cell structure

Accelerating mode in multi-cell structures



Synchronic acceleration and max of $(R/Q)_{acc}$ for a multi-cell structure when:

1. $l_{active} = Nl_{cell} = Nc\beta/(2f)$ and
2. the injection takes place at an optimum phase φ_{opt} , which ensures that particles arrive at the mid-plane of the first cell when E_{acc} reaches its maximum (minimum)

Field flatness in multi-cell structures

	<i>Original Cornell</i> $N = 5$	<i>High Gradient</i> $N = 7$	<i>Low Loss</i> $N = 7$	<i>TESLA</i> $N = 9$	<i>SNS</i> $\beta = 0.61$ $N = 6$	<i>SNS</i> $\beta = 0.81$ $N = 6$	<i>Low Loss</i> $N = 9$	<i>RHIC</i> $N = 5$
<i>year</i>	1982	2001	2002	1992	2000	2000	2003	2003
a_{ff}	1489	2592	3288	4091	3883	2924	5435	850



$$a_{ff} = \frac{N^2}{k_{cc} \cdot \beta}$$

Decades of experience with: heat and chemical treatment, handling and assembly allow to maintain good field profile, even in cavities with bigger N and weaker k_{cc} .

For many TESLA cavities: field flatness is better than 95 %

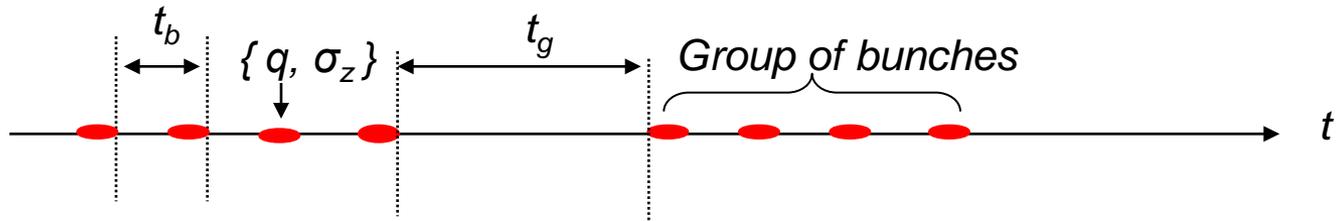
HOM excitation and trapping in multi-cell structures

HOM excitation causes:

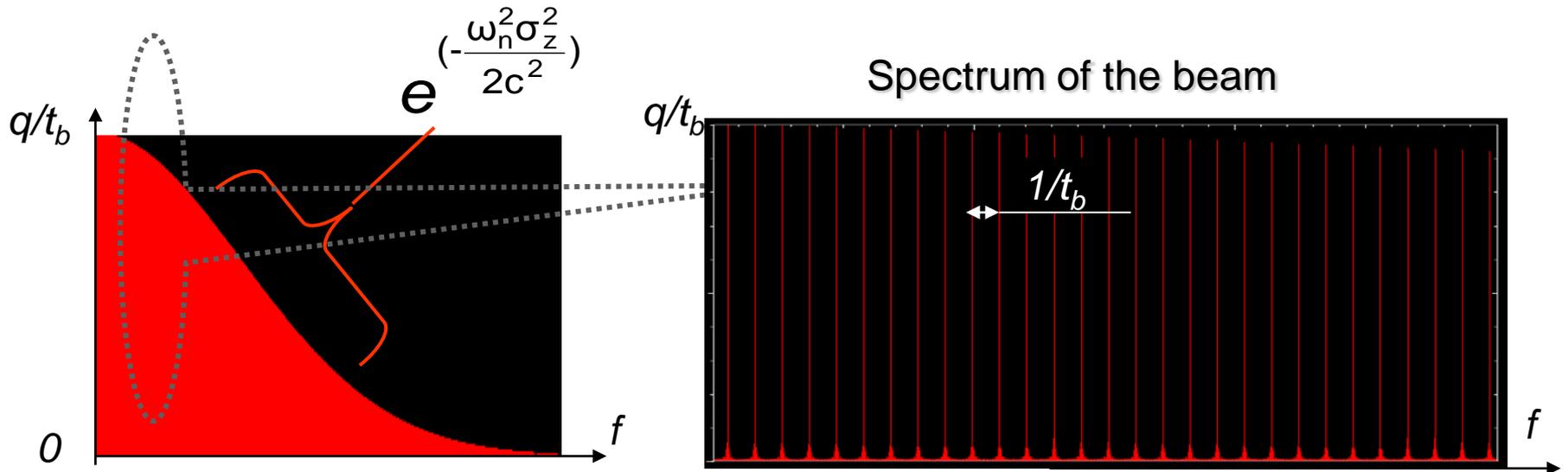
- Beam instabilities and/or dilution of emittance
- Bunch-to-bunch energy modulation
- Additional cryogenic loss

HOM excitation:

Time structure of the beam

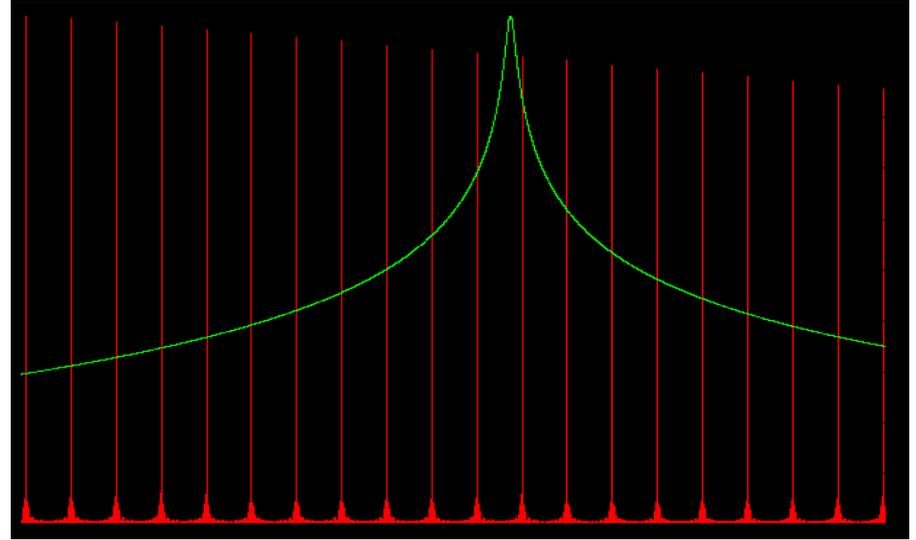
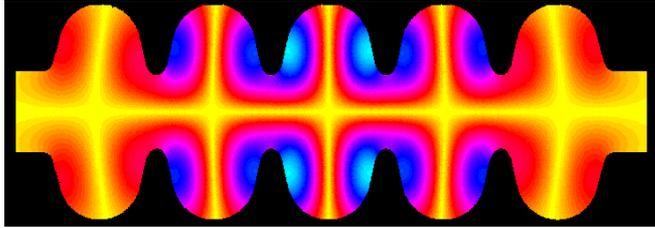


Spectrum of the beam



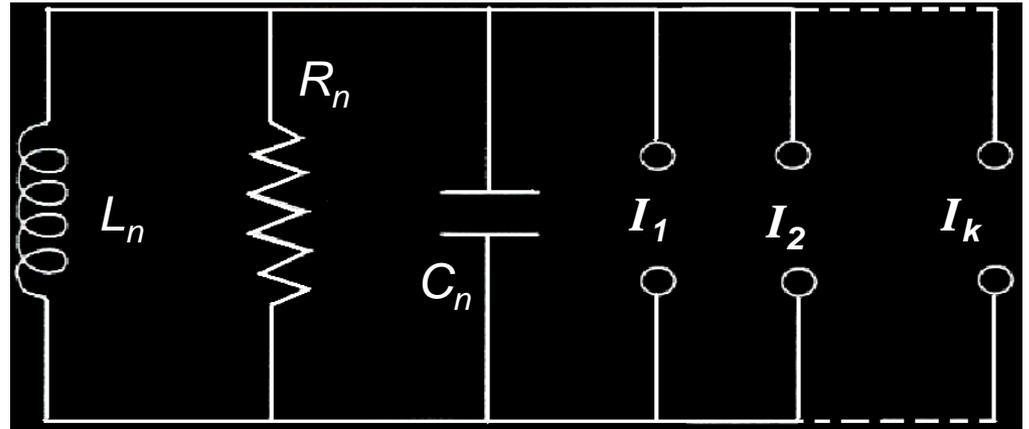
5. Multi-cell Structures and Weakly Coupled Structures

Mode No. n : $\{ \omega_n, (R/Q)_n, Q_{L,n} \}$



$$Z_n(\omega) = \frac{(R/Q)_n \cdot Q_{L,n}}{1 + jQ_{L,n} \left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega} \right)}$$

Impedance of the n -mode



Multi-source excitation

5. Multi-cell Structures and Weakly Coupled Structures

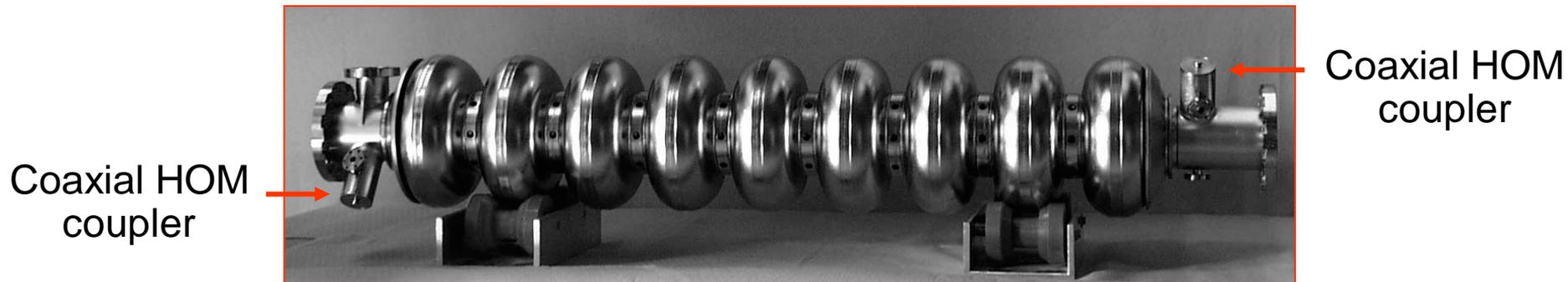
The power induced by “all” spectral lines (current sources) in mode No. n :

$$P_n = \frac{1}{2} \sum_k Z_n(\omega_k) \cdot I_k^2$$
$$Z_n(\omega) = \frac{(R/Q)_n \cdot Q_{L,n}}{1 + jQ_{L,n} \left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega} \right)}$$

where: $\frac{1}{Q_{L,n}} = \frac{1}{Q_{0,n}} + \frac{1}{Q_{ext,n}}$ ← Measure of the extracted power

The HOM couplers, devices extracting the energy from parasitic modes, are attached to cavities for mitigation of high and harmful E - M fields of HOM .

The experience is that, the HOM couplers can be attached to the beam tubes and must not be located at cells because this leads to the performance degradation.



5. Multi-cell Structures and Weakly Coupled Structures

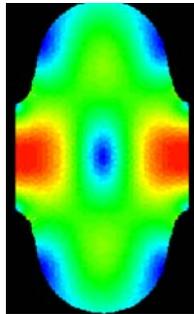


Waveguide
HOM ports

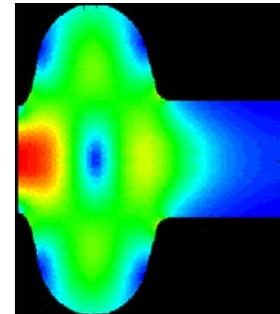
The HOM trapping is similar to the FM field profile unflatness mechanism:

- weak $k_{cc,HOM}$, cell-to-cell coupling for HOM
- difference in the HOM frequency between the end-cells and inner-cells

$f = 2385 \text{ MHz}$



That is why they
hardly resonate
together

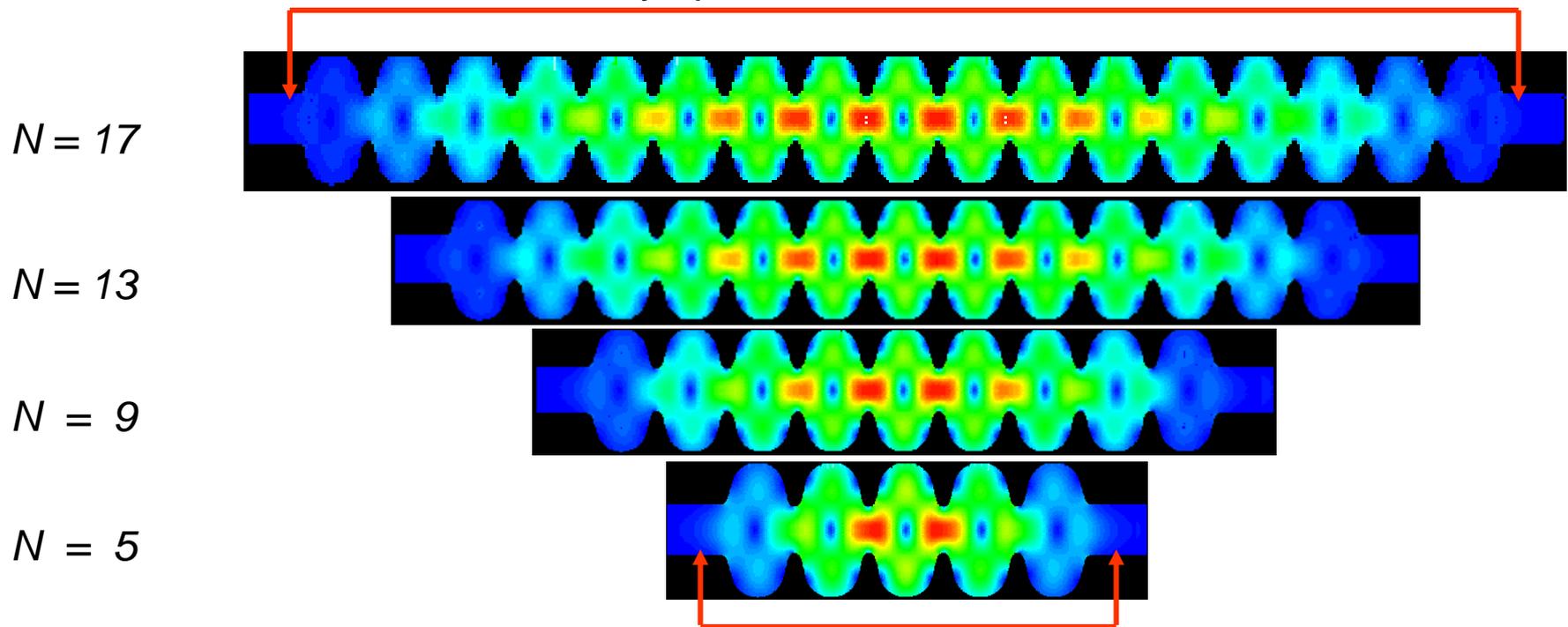


$f = 2415 \text{ MHz}$

5. Multi-cell Structures and Weakly Coupled Structures

Example of the trapping and how N influences strength of the $E-H$ fields at the HOM couplers locations

no $E-H$ fields at HOM couplers locations (trapping), which are always placed at the end beam tubes



$E-H$ fields at HOM couplers locations

Less cells in a structure helps always to reach low Q s of HOMs.

What else can help to avoid the trapping?

Adjustment of end-cells

The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes, HOM- and input couplers.

Their function is multifold and their geometry must fulfill three requirements:

- field flatness and frequency of the accelerating mode
- field strength of the accelerating mode must enable matching of Q_{ext} of FPC
- field strength of the dangerous HOMs must ensure required damping.

All three requirements make design of the end-cells more difficult than inner cells.

5. Multi-cell Structures and Weakly Coupled Structures

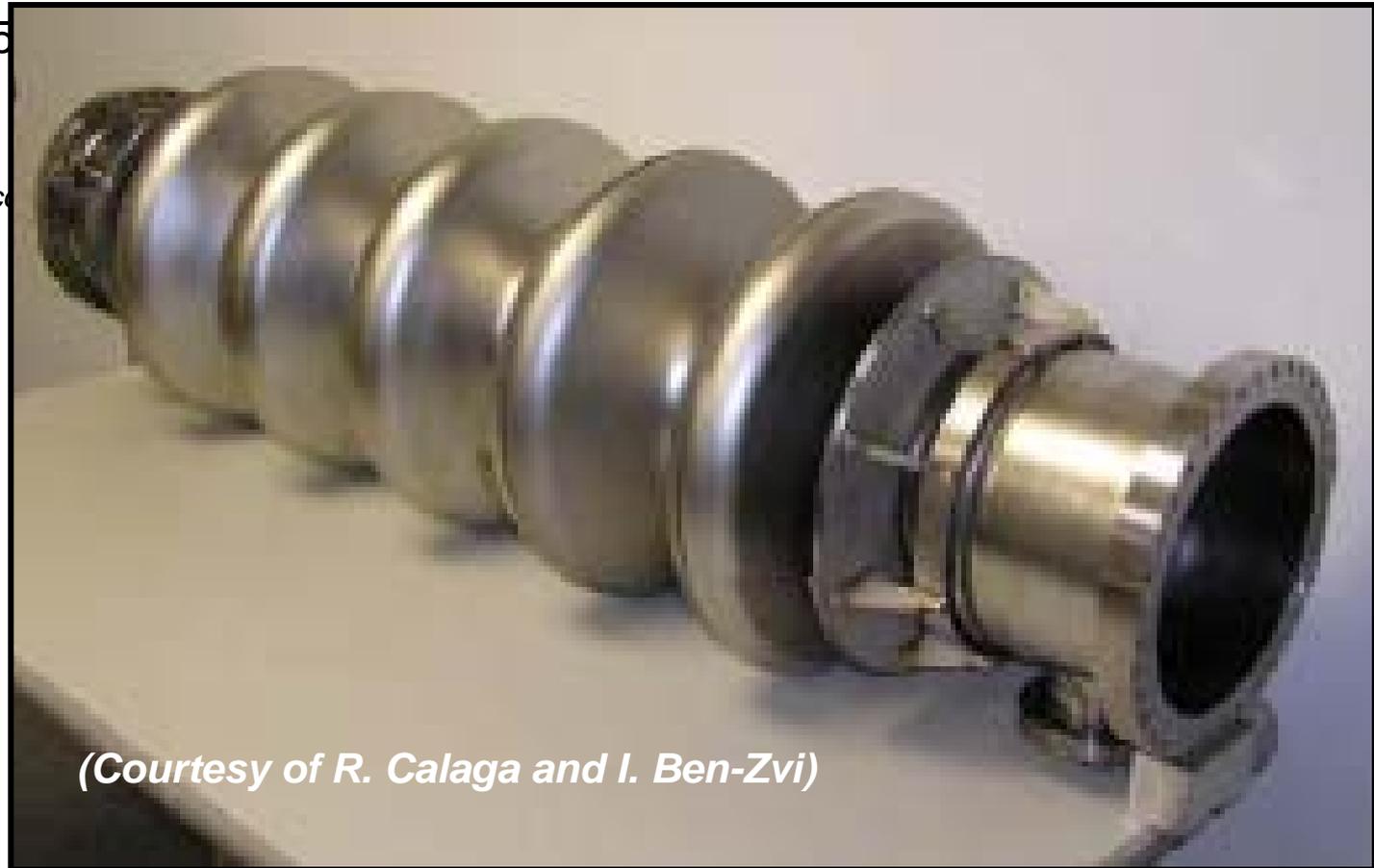
1. Open irises of the inner- and end-cells (bigger $k_{cc,HOM}$) and shaping them similarly.

Example: RHIC 5

Monopole mode k_c

$$f_{HOM} = 1394 \text{ MHz}$$

$$f_{HOM} = 1403 \text{ MHz}$$



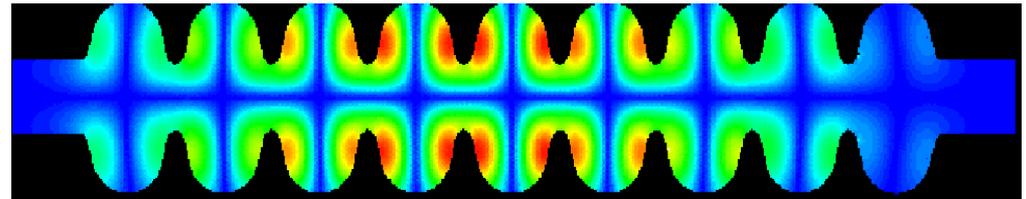
The method causes (R/Q) reduction of fundamental mode, which in this application is less relevant.

5. Multi-cell Structures and Weakly Coupled Structures

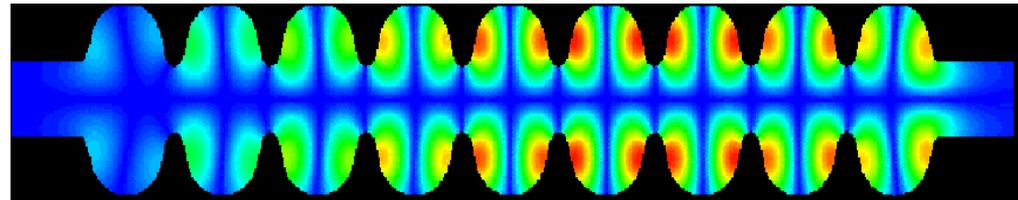
2. Tailoring the end-cells to equalize HOM frequencies of inner- and end-cells.

Example: TESLA 9-cell cavity, which has two different end-cells (asymmetric cavity)

The lowest mode in the passband $f_{HOM} = 2382$ MHz

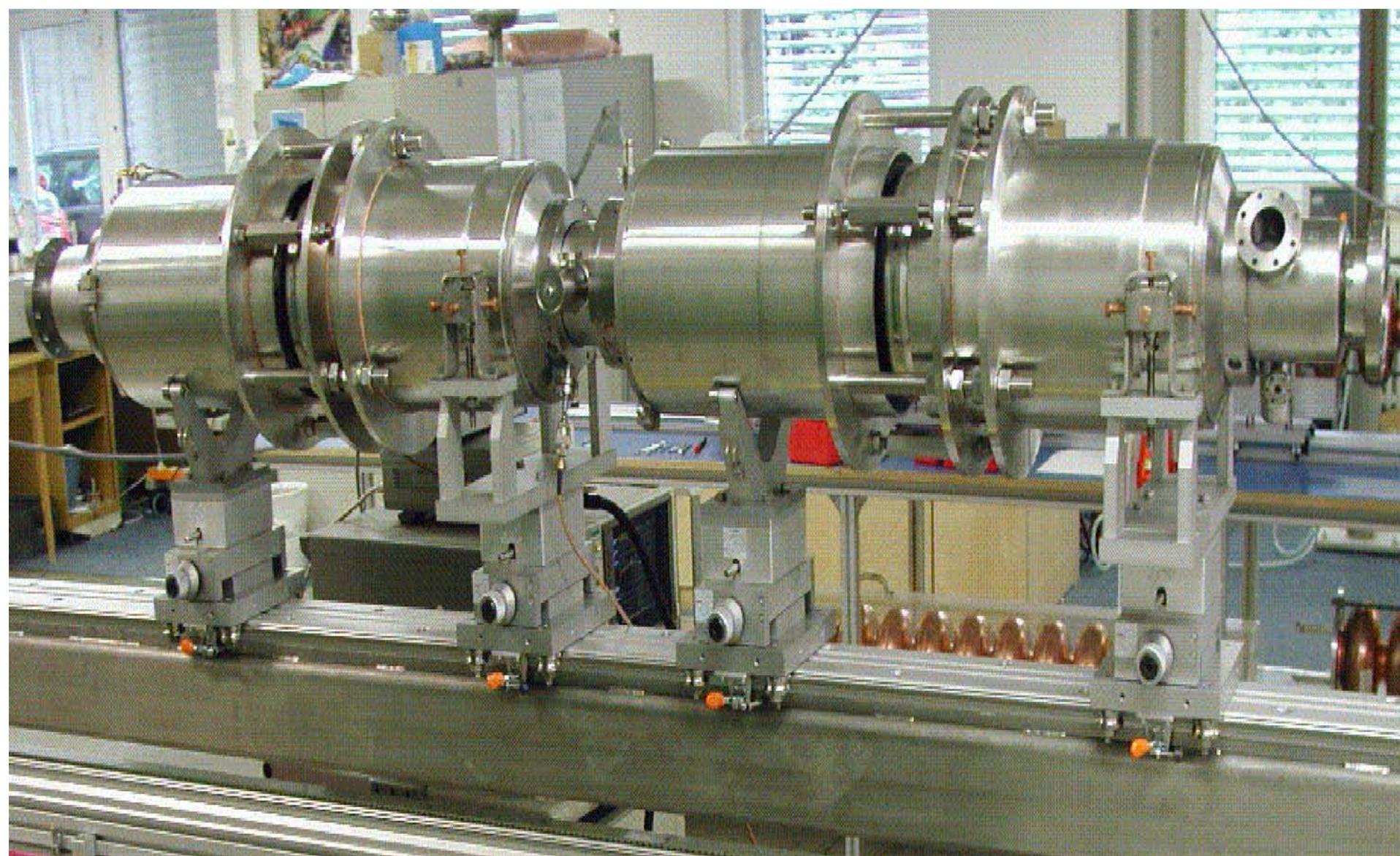


The highest mode in the passband $f_{HOM} = 2458$ MHz



The method works for few modes but keeps the (R/Q) of the fundamental mode high.

5. Multi-cell Structures and Weakly Coupled Structures



Power capability of the FPC for multi-cell structures

When I_{beam} and E_{acc} are specified and a superconducting multi-cell structure does not operate in the energy recovery mode:

$$P_{in} \sim N$$

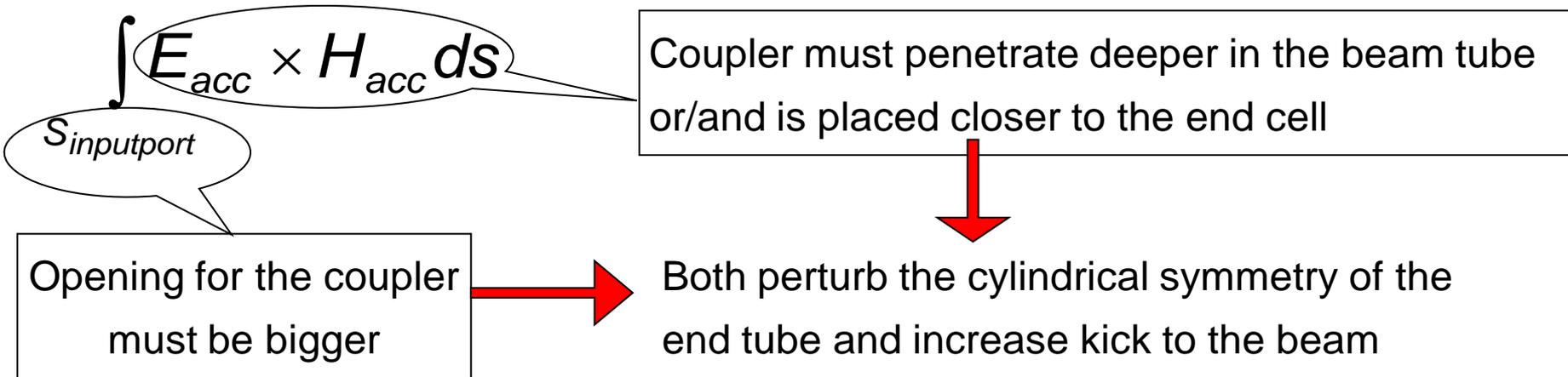
Q_{ext} of the FPC, which usually is \ll than intrinsic Q_0 , is:

$$Q_{ext} \approx \frac{E_{acc} \cdot \beta \cdot \lambda \cdot N}{I_{beam} \cdot (R/Q)_{cell} \cdot N} = \frac{E_{acc} \cdot \beta \cdot \lambda}{I_{beam} \cdot (R/Q)_{cell}} = \frac{\omega_{acc} \cdot W_{onecell} \cdot N}{\frac{1}{2} \int_{S_{inputport}} E_{acc} \times H_{acc} ds}$$

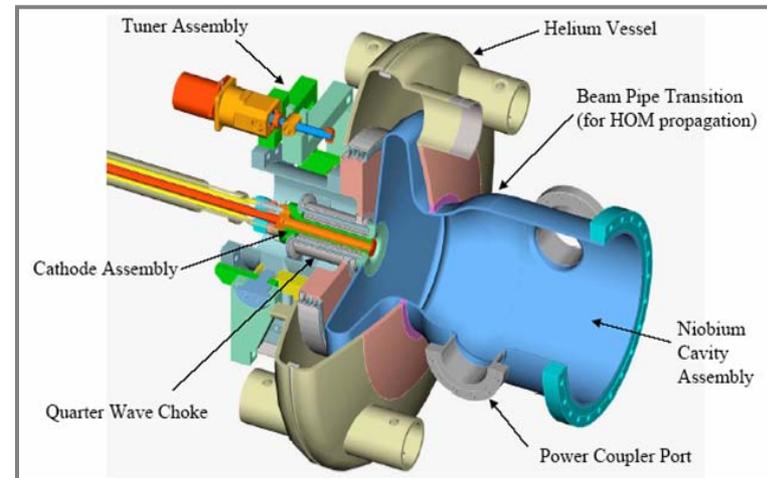
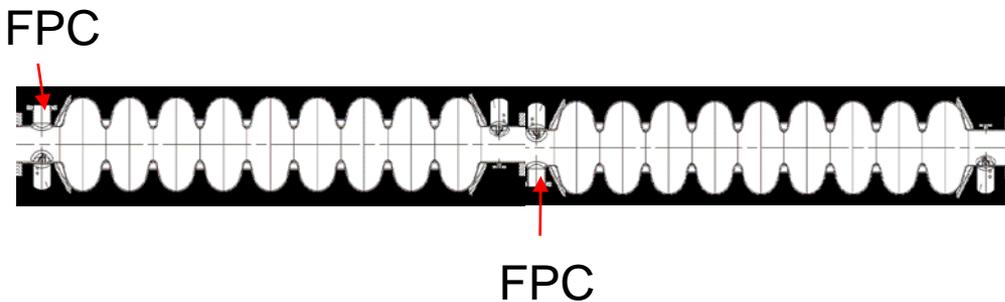
Independent of N

It must be $\sim N$ to keep the ratio constant

5. Multi-cell Structures and Weakly Coupled Structures



The remedies are: alternating positions of couplers or double couplers



Courtesy of Alan Todd (AES)

Surface cleaning procedures are more complicated

Few words on Nb and on the surface preparation procedures

All high gradient cavities are made of the pure metallic bulk Nb (II-type sc, $T_c = 9.2$ K):

- We use poly-crystal Nb from the very beginning.
- Recently, we made several cavities of large-grain Nb (with hope for single crystal)

Surface preparation has several steps with three major procedures:

- Chemical treatment: can be Buffered Chemical Polishing or Electro-Polishing
- Heat treatment
- High Pressure Water rinsing

Buffered Chemical Polishing (BCP)

Acids: HF (49%), HNO₃ (65%), H₃PO₄ (85%)

Mixture: 1:1:1 or 1:1:2 by volume

Electro-Polishing (EP)

Electrolyte:

1 part HF(49%), 9 parts H₂SO₄ (96%)

Al-cathode, Nb-anode, $J \sim 50$ mA/cm²

5. Multi-cell Structures and Weakly Coupled Structures

The sequence in the surface preparation is:

- **Heavy chemical etch (EP or BCP)**
 - Removal of damaged surface layer (100-150um) caused by fabrication and handling
- **Removal of surface contamination**
 - Ultrasonic cleaning of surface with detergent and DI water, or Alcohol rinse
- **Heat treatment (600-800C in vacuum furnace)**
 - Removes hydrogen from the bulk niobium to reduce the risk of Q-disease
- **RF tuning and mechanical inspection**
 - Field profile, calibration of test probes, check mechanical structure
- **Removal of surface contamination**
 - Ultrasonic cleaning of surface with detergent and DI water,
- **Light chemical etch (EP or BCP)**
 - Remove any risk from damage during handling and furnace contamination
- **High pressure rinse (UPW @100 Bar) + Class 10 drying of cavity**
 - Reduction of field emission sources, surface particulates
 - At least two passes over entire surface

Courtesy J. Mammosser

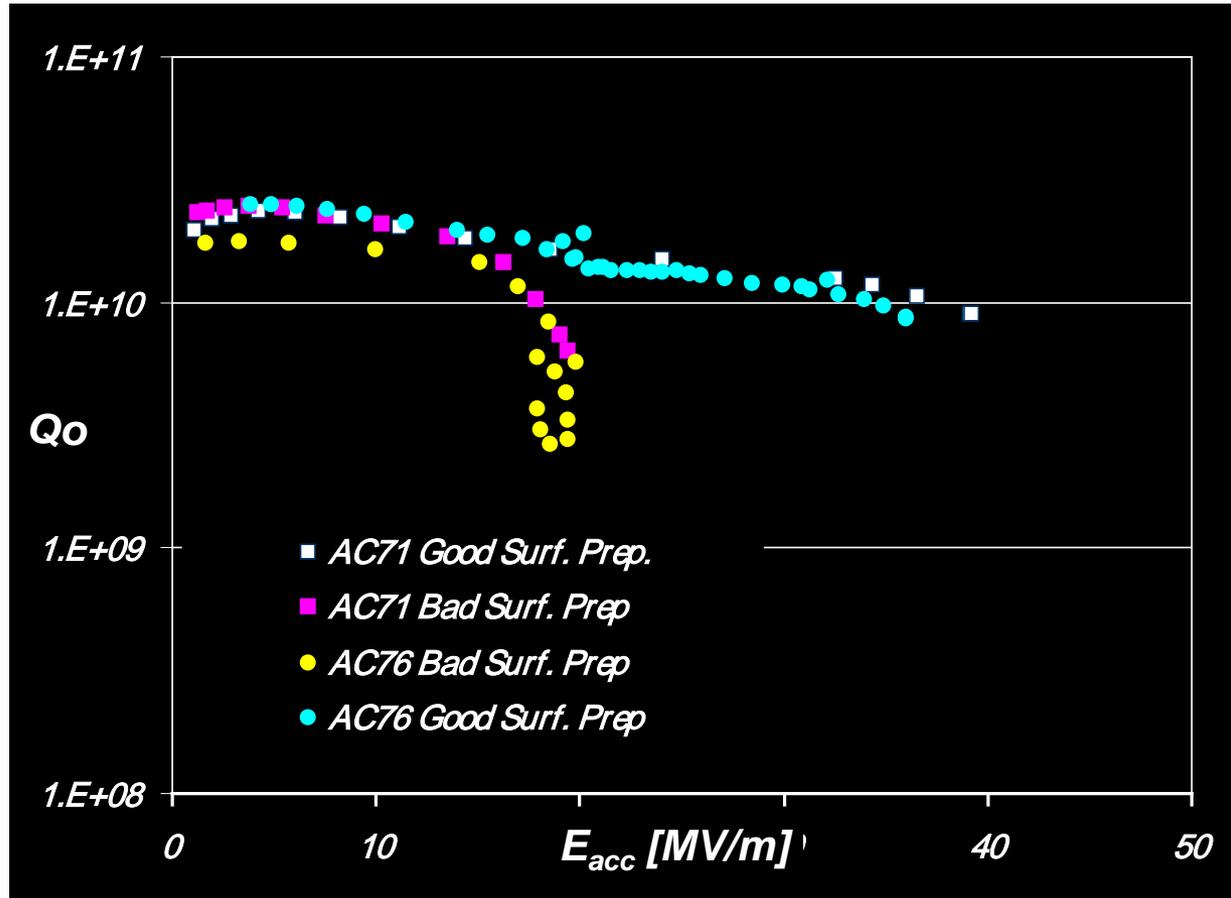
5. Multi-cell Structures and Weakly Coupled Structures

Example showing randomness in the performance due to the additional cleaning:

AC71 went from good to bad

AC76 went from bad to good

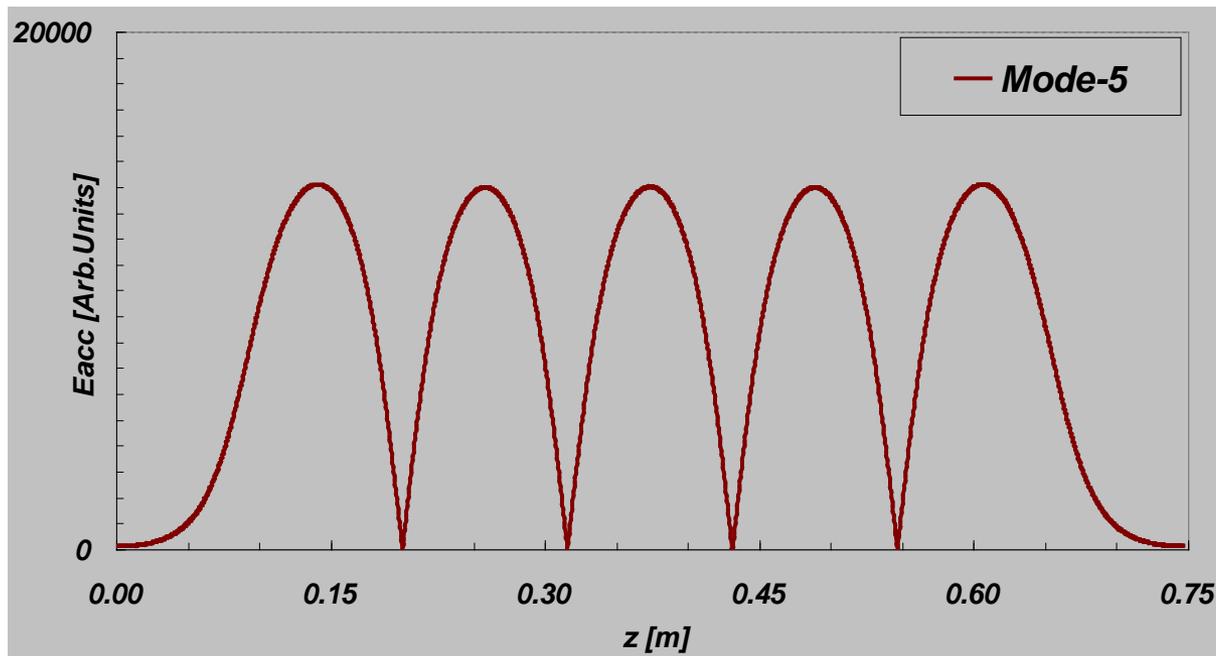
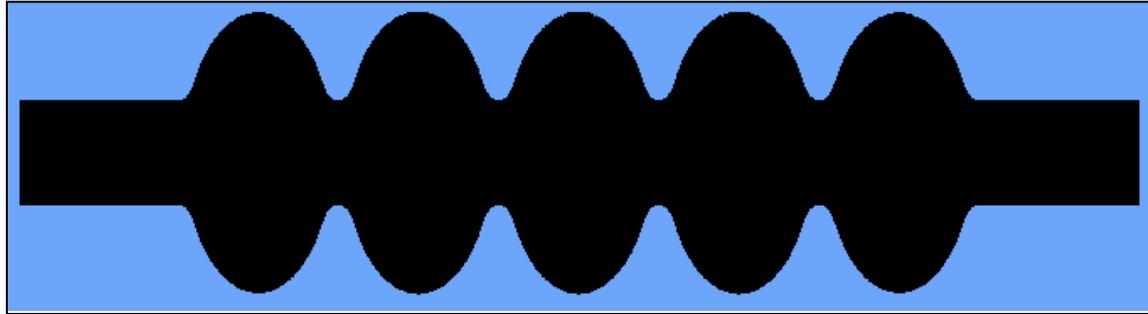
9-cell TESLA cavities



The best performance is still difficult to reach. The preparation must be repeated to reach the ultimate goal of 34 MV/m @ 10^{10} . It makes cavities very “expensive”.

The worst performing cell limits whole multi-cell structure

After the pre-tuning all cells have the same amplitude



List of multi-cell cavities $\beta=1$ optimized for various criteria

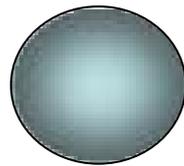
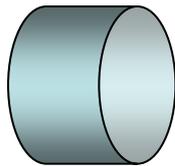
Criterion	Structure	Best parameter	Weakest parameter (point)	Comments
E_{acc}	HG: 1.5 GHz, N=7 TESLA: 1.3 GHz, N=9 ILC-LL: 1.3 GHz, N=9 ILC-RE 1.3 GHz, N=9	$E_{peak}/E_{acc}= 1.96$ $E_{peak}/E_{acc}= 1.98$ $B_{peak}/E_{acc}= 3.61$ $B_{peak}/E_{acc}= 3.57$	maximum -Eacc maximum -Eacc E_{peak}/E_{acc} E_{peak}/E_{acc}	Designed for $I_{beam} < 10 \text{ mA}$, Pulse operation
P_{loss}	LL: 1.5 GHz, N= 7	$B_{peak}/E_{acc}= 3.7$ (R/Q)·G	Not easy to clean, HOM damping	Designed for $I_{beam} < 1 \text{ mA}$ First LL-type cavity
Z_{HOM}	RHIC: 0.7 GHz, N= 5	Very low: k_{\perp} , k_{\parallel} $E_{peak}/E_{acc}= 1.98$	Cryogenic losses	First cavity for $I_{beam} \approx 2 \text{ A}$

6. Tools for RF-design

Design of an elliptical cavity is usually performed in two steps: “2D” and “3D” :

- “2D” is fast and allows to define geometry of a cylindrical symmetric body (inner and end-cells) of the cavity.
- “3D” is much more time consuming but necessary for modeling of full equipped cavity with FPC and HOM couplers.
Also coupling strength for FPC and damping of HOMs can be modeled only in 3D.

Solutions to 2D (or 3D) Helmholtz equation can be found analytically only for very few geometries (pillbox, spherical resonators or rectangular resonator)



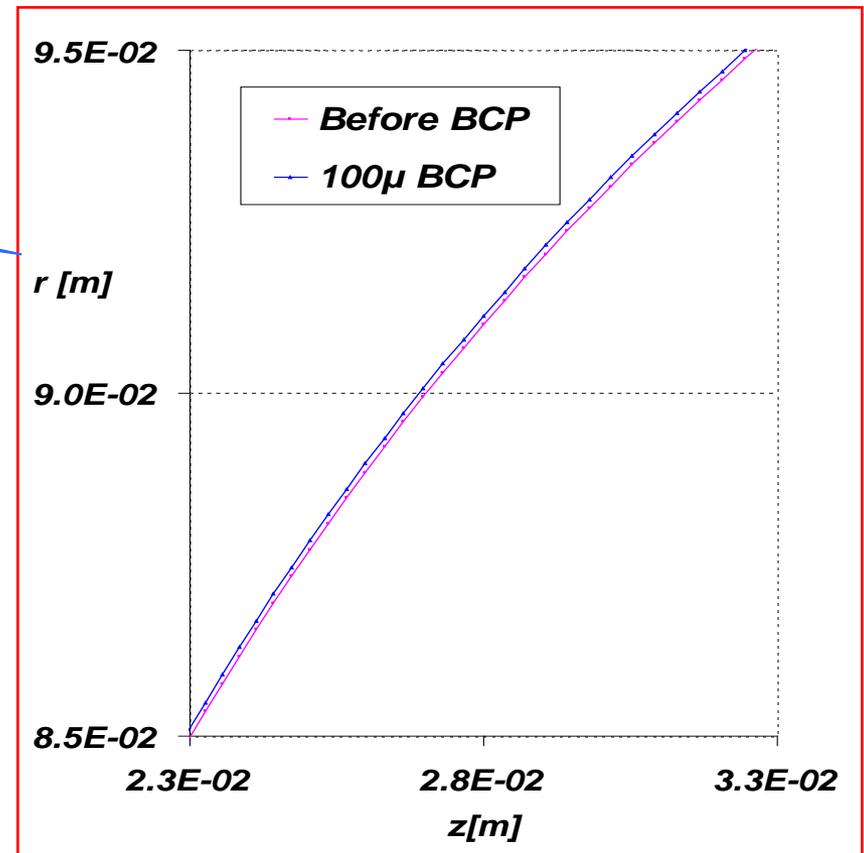
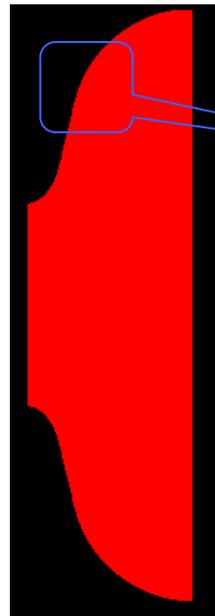
We need numerical methods: $(\nabla^2 + \omega^2 \epsilon \mu) \mathbf{A} = 0$

Approximating operator
(Finite Difference Methods)

Approximating function
(Finite Element Methods)

6. Tools for RF-design

Example: FEM-code modeling of the frequency change due to the chemical treatment (removed layer of 100 μm)



Zoomed difference in shape of the TESLA mid cup after 100 μm BCP

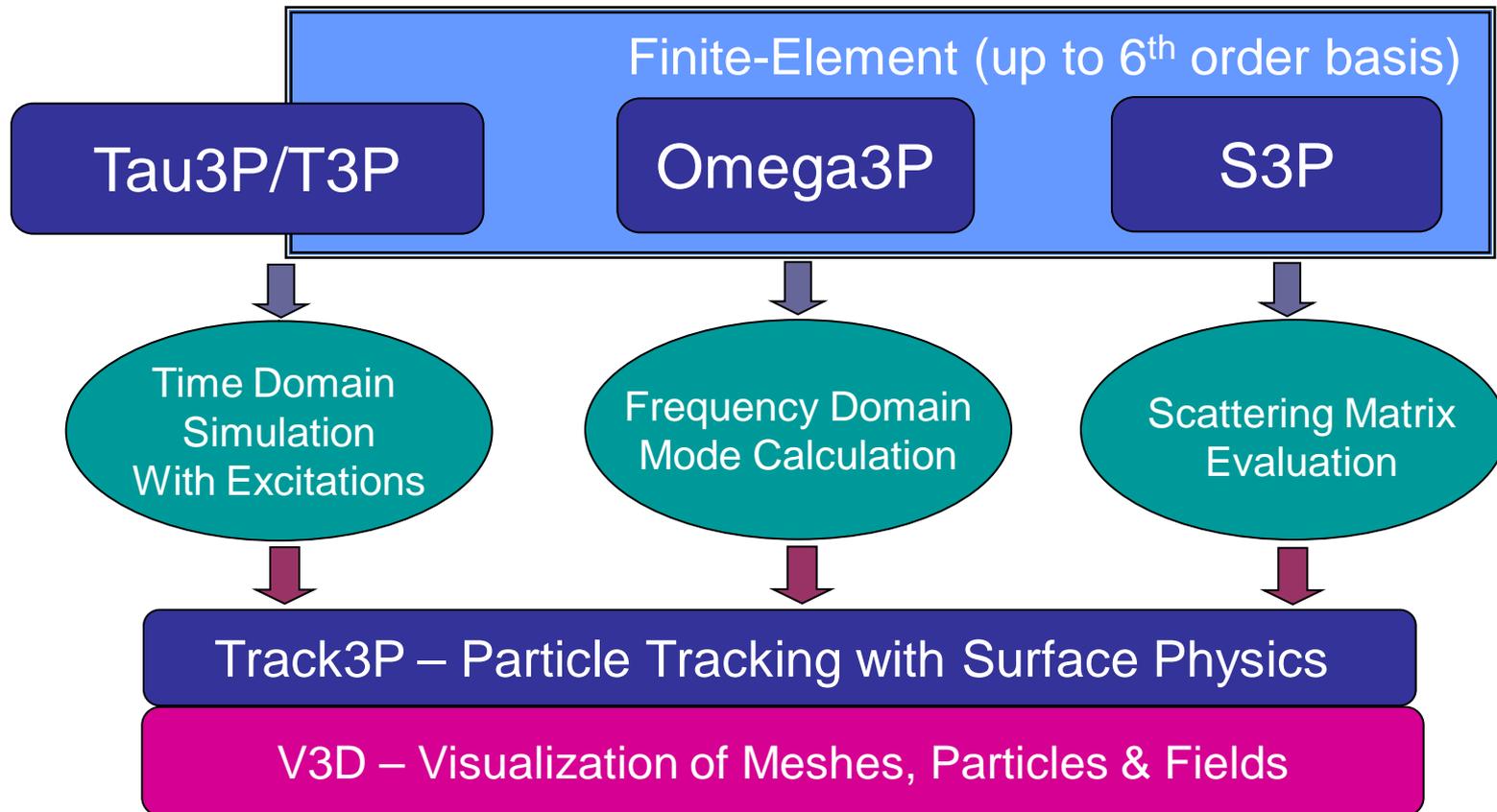
2D Modeling takes ~2min

$$\frac{\partial f}{100\mu\text{m}} = -10\text{kHz} / \mu\text{m}$$

which was measured for the uniform removal

Electromagnetic Code Development at SLAC by ACD

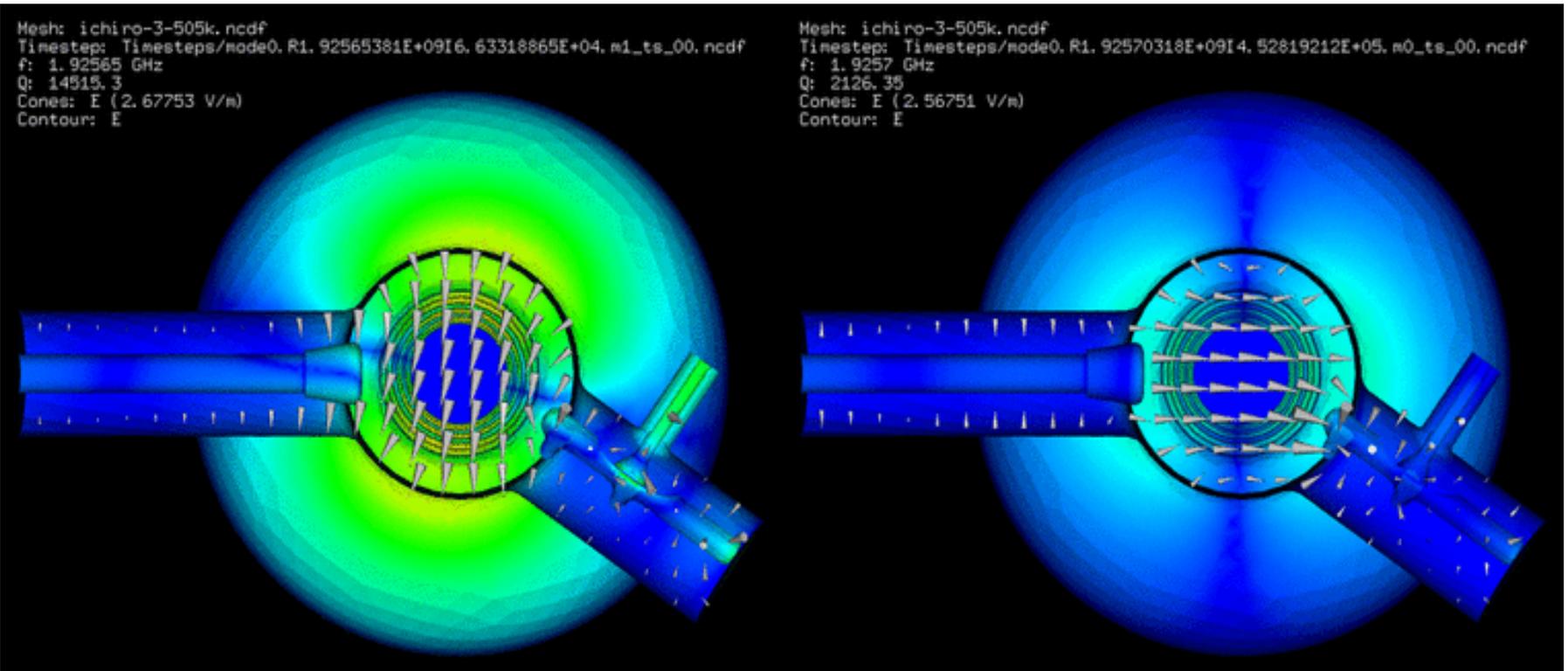
Solves Maxwell's equations with particles in TD & FD



(Courtesy of Kwok Ko and ACD Members)

6. Tools for RF-design

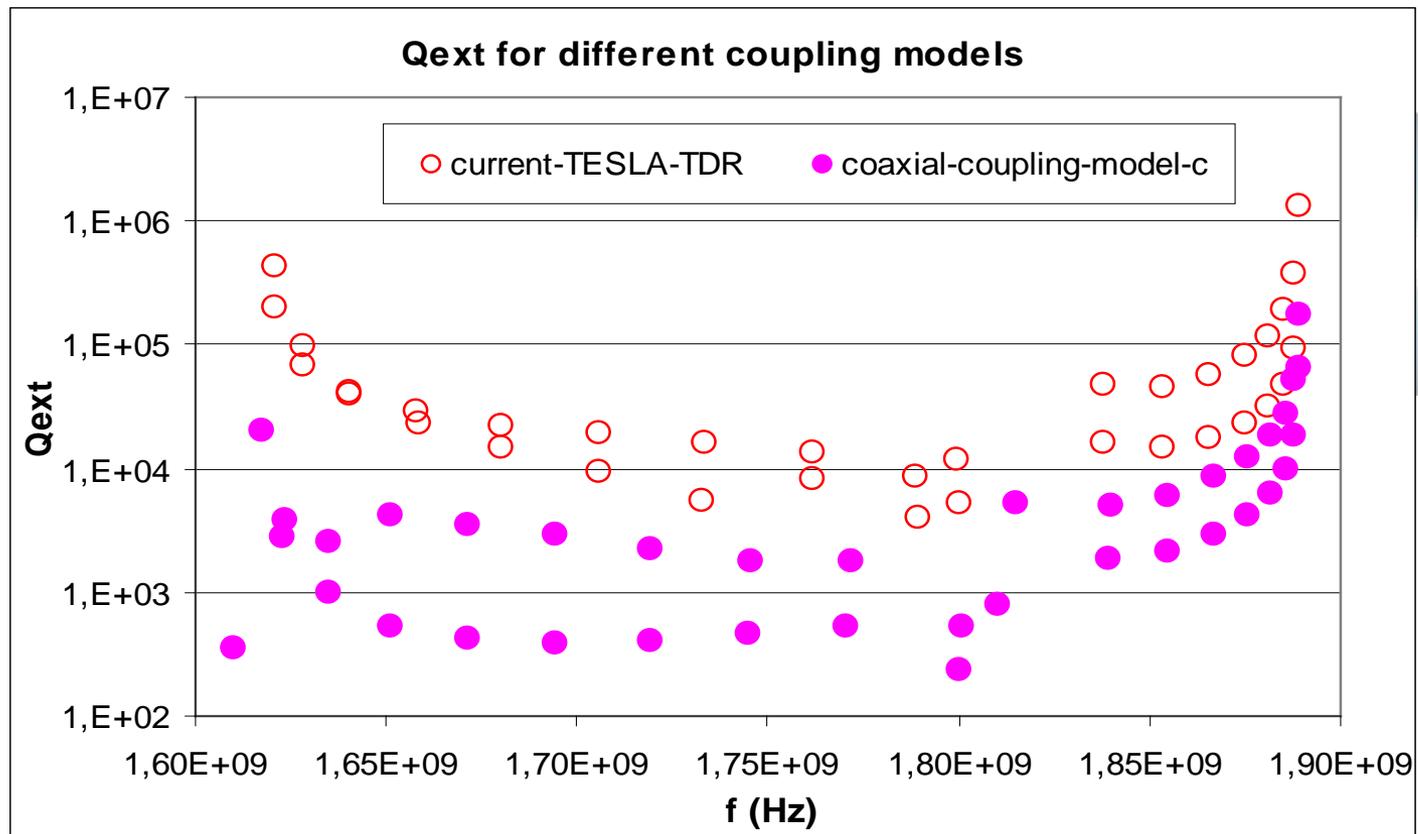
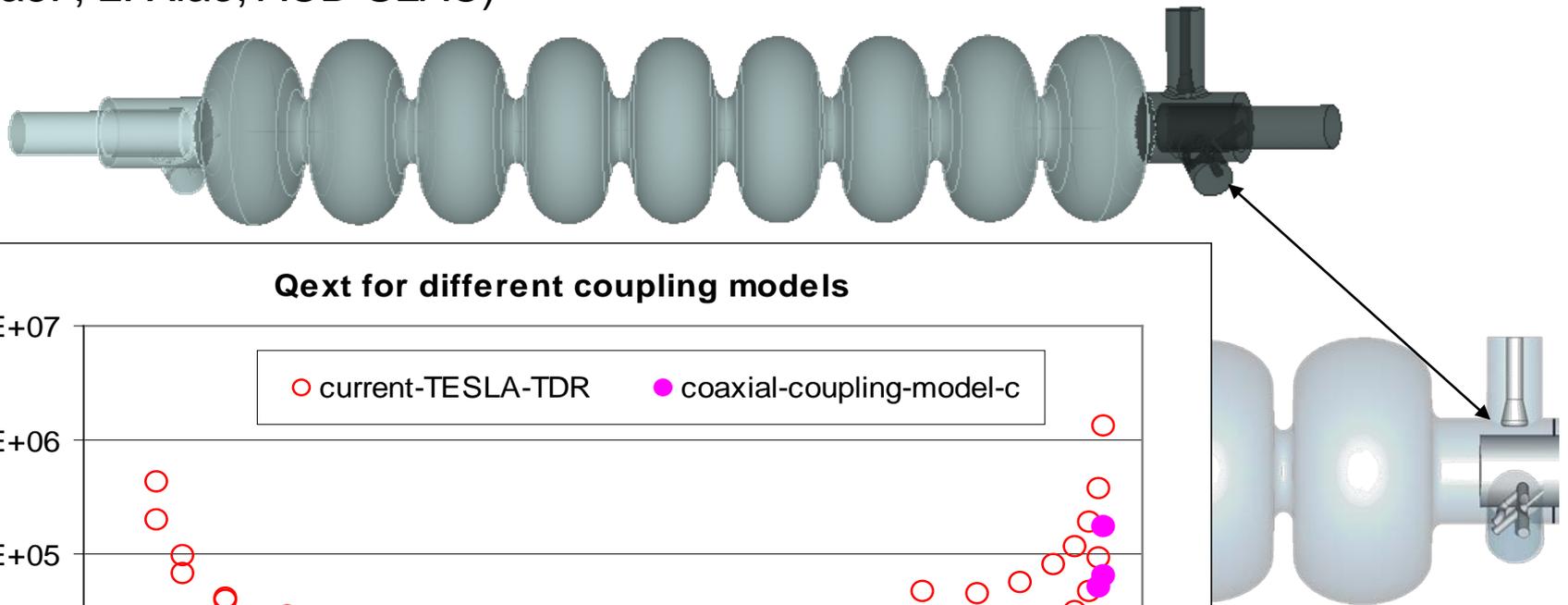
Example of 3D two dipoles overlapping modeling in the TESLA cavity with Omega3P



(Courtesy of Kwok Ko and ACD Members)

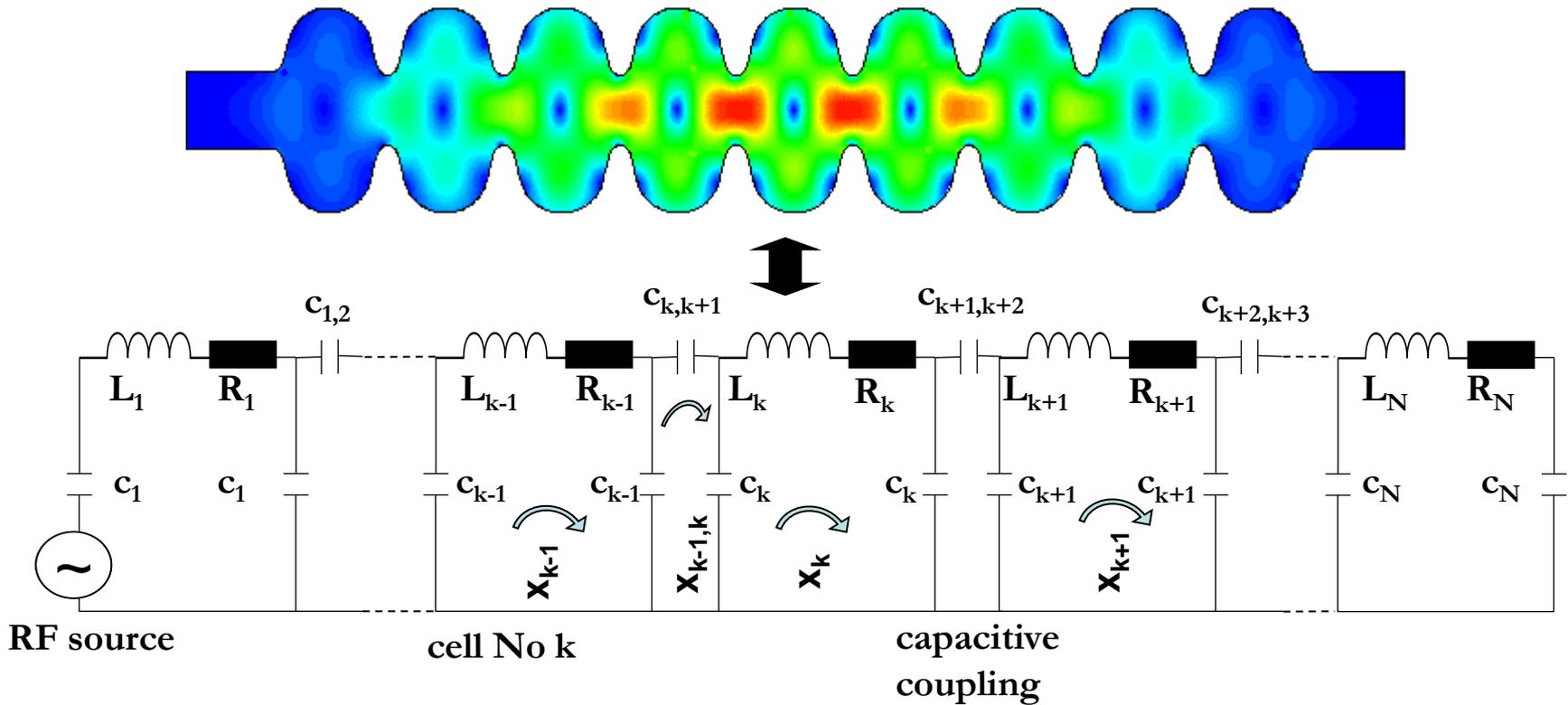
6. Tools for RF-design

Example of 3D dipole damping modeling for the TESLA cavity with the coaxial coupling (Omega3P, L. Xiao, ACD SLAC)



7. LEC and Transient State

In the design process we use 2D-codes (*SUPERFISH*, *SLANS*, *FEM..*) and 3D-codes (*MWS*, *HFSS*, *MAFIA* and *OMEGA-3P*) but still the Lumped Element replacement Circuit is helpful to investigate some RF properties.



$$\text{Where: } 2\pi f_{FM} = (L_k \cdot C_k)^{-0.5}; \quad (R/Q)_{FM} = (L_k / C_k)^{0.5}; \quad R = (R/Q)_{FM} \cdot Q_{L,FM};$$

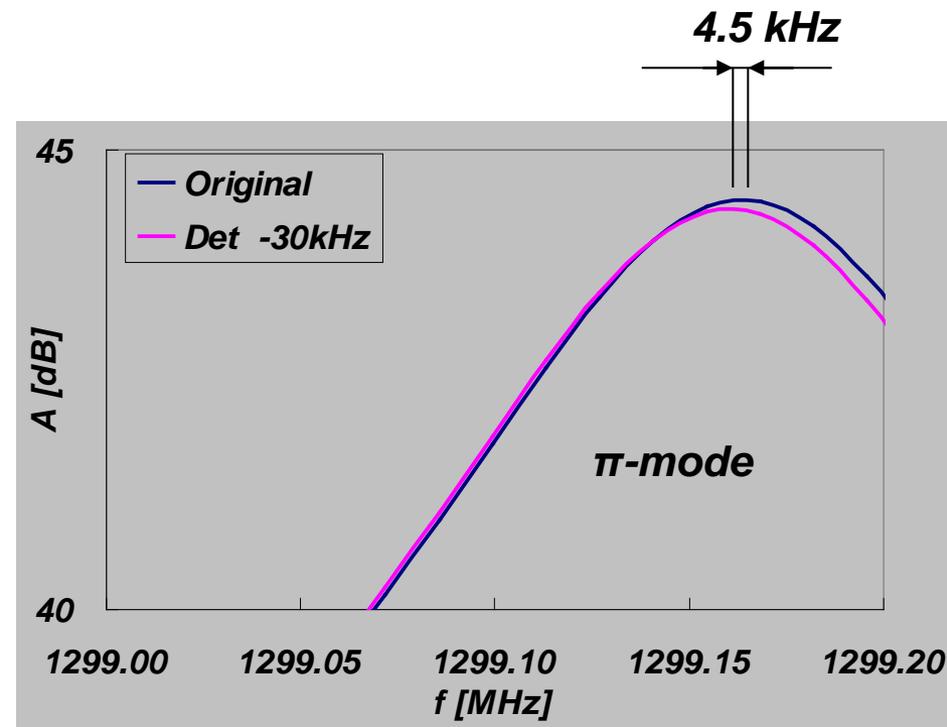
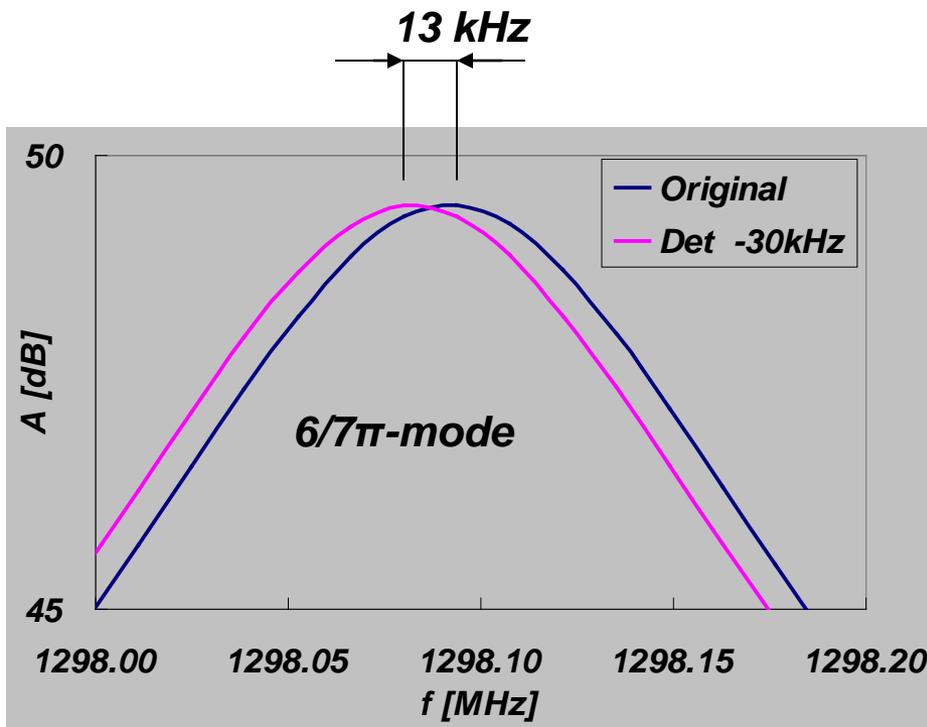
What can be done by means of the LEC:

- Cavity pre-tuning after the fabrication and bulk chemical treatment
- Investigation how the field profile in cells depends on their frequency errors ($\partial f/f < 10^{-4}$)
- Investigation how passband frequencies depend on cell frequency errors ($\partial f/f < 10^{-4}$)
- Modeling of the transient state (mode beating)
- Modeling of the voltage stability during acceleration

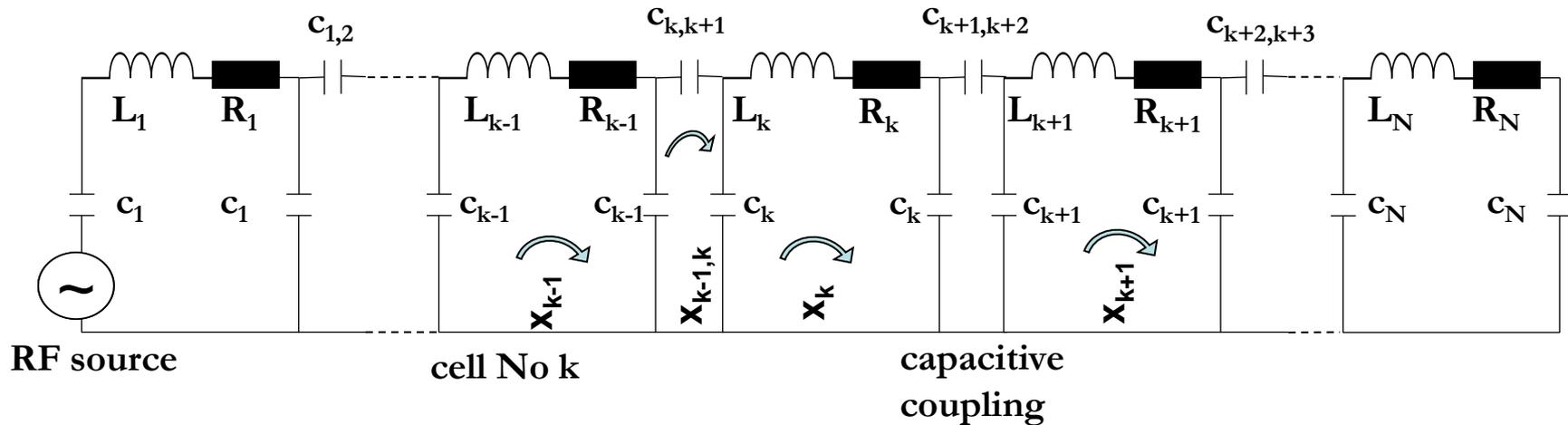
blue marked examples of the implementation are shown on next slides

Investigation of the FM passband frequencies sensitivity to cell frequency errors ($\partial f/f < 10^{-4}$)

Example: 7-cells, $k_{cc}=1.85\%$, 1st cell detuned by -30kHz (cell length changed $-11\mu\text{m}$!!, hard to model with 2D and 3D codes)



Transient State: Mode beating in the pulse operation



Solving the set of Kirchoff equations:

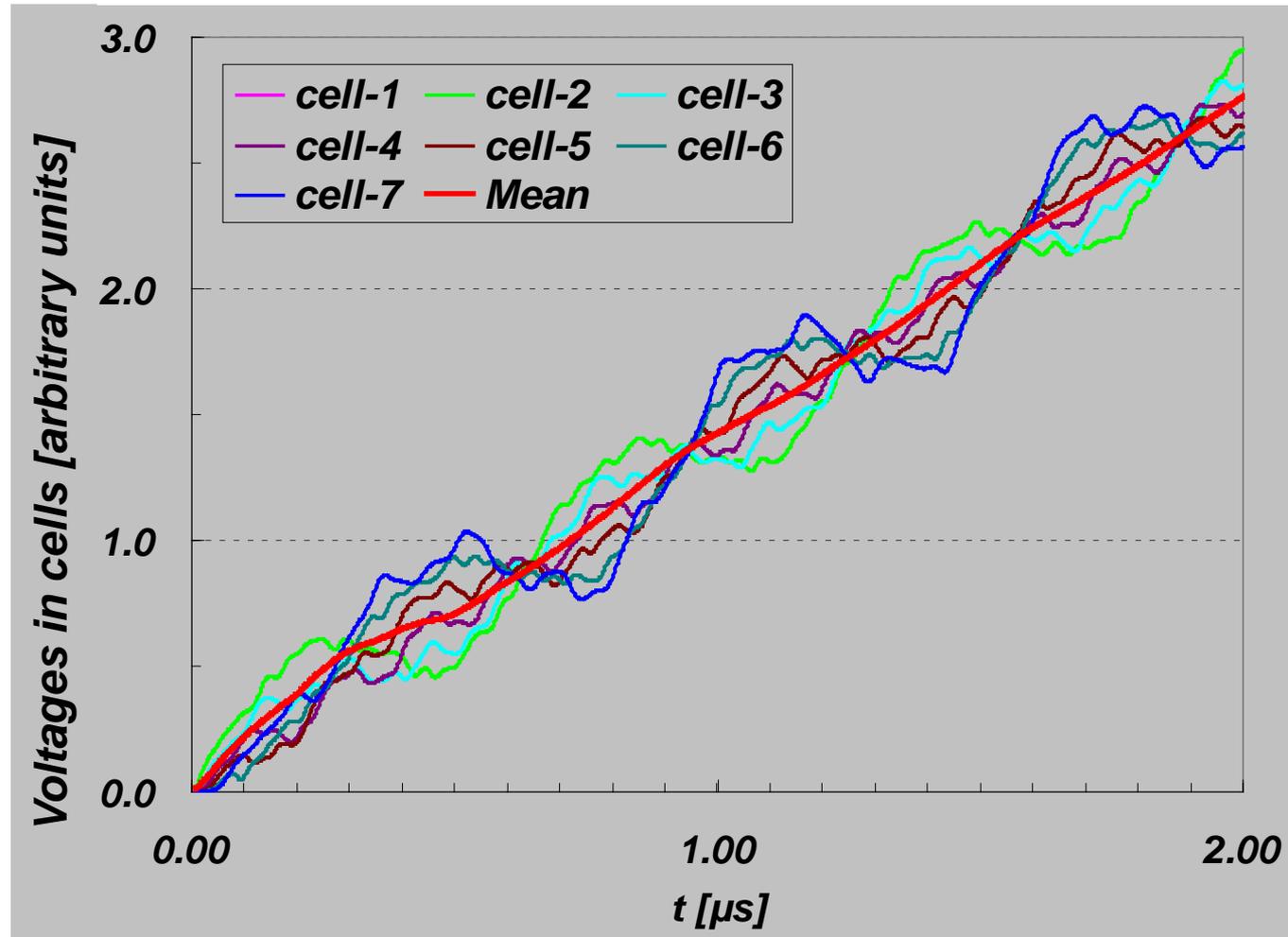
$$\begin{aligned}
 & R_1 \cdot x_1(t) + L_1 \cdot \dot{x}_1(t) + \frac{1}{c_1} \cdot \int_0^t x_1(\tau) d\tau - \frac{1}{c_1} \cdot \int_0^t x_{1,2}(\tau) d\tau = U_{-1}(t) \cdot e(t) \\
 & \dots\dots\dots \\
 & -\frac{1}{c_k} \cdot \int_0^t x_{k-1,k}(\tau) d\tau + R_k \cdot x_k(t) + L_k \cdot \dot{x}_k(t) + \frac{1}{c_k} \cdot \int_0^t x_k(\tau) d\tau - \frac{1}{c_k} \cdot \int_0^t x_{k,k+1}(\tau) d\tau = 0 \\
 & \dots\dots\dots \\
 & -\frac{1}{c_N} \cdot \int_0^t x_{N-1,N}(\tau) d\tau + R_N \cdot x_N(t) + L_N \cdot \dot{x}_N(t) + \frac{1}{c_N} \cdot \int_0^t x_N(\tau) d\tau = 0
 \end{aligned}$$

one can find voltages right after the RF-source is switched on and during the acceleration

7. LEC and Transient State

Modeling of the transient state (mode beating)

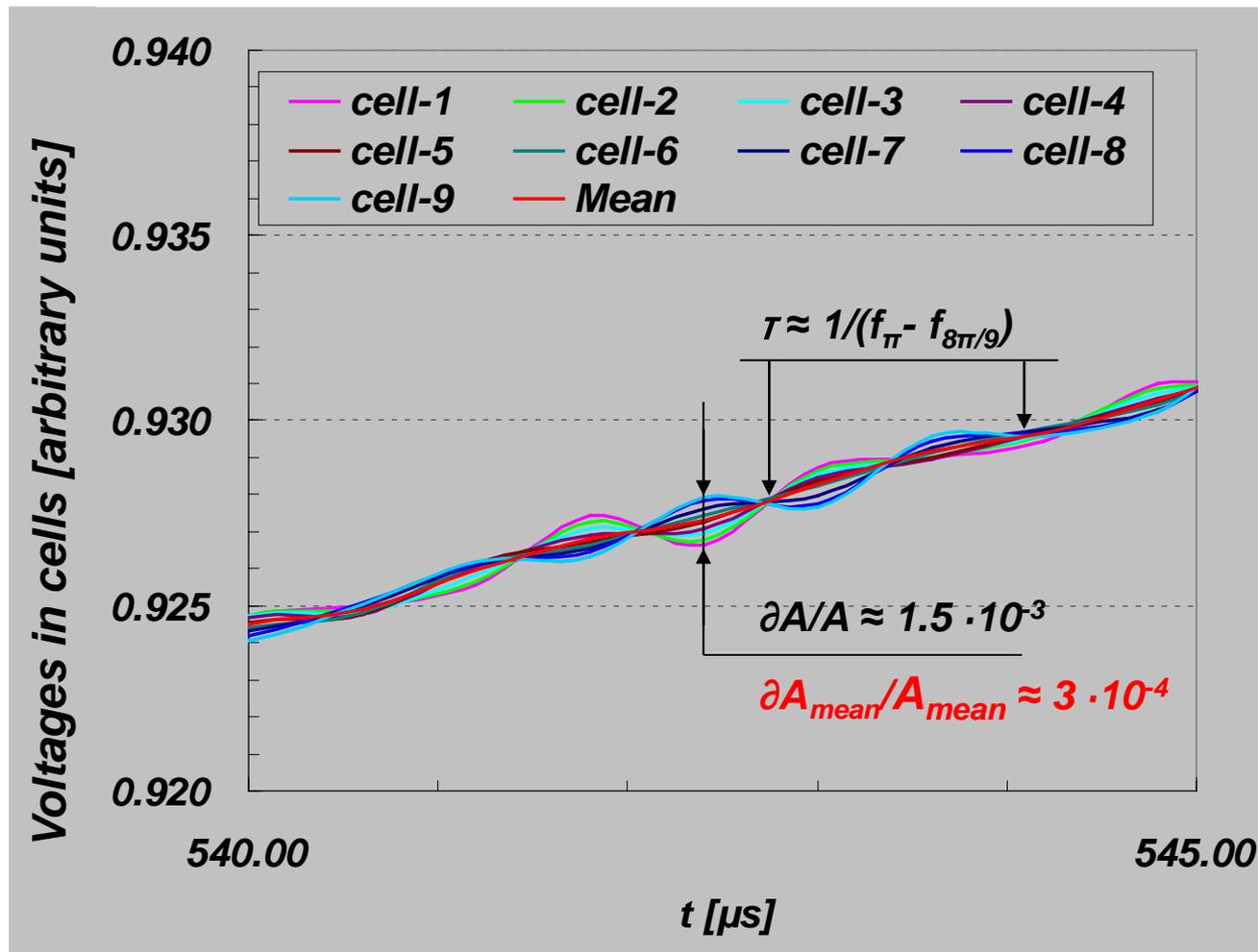
Example: 7-cells, $k_{cc}=1.85\%$, $Q_L=3.4 \cdot 10^6$



7. LEC and Transient State

Modeling of the transient state (mode beating at the beam arrival time)

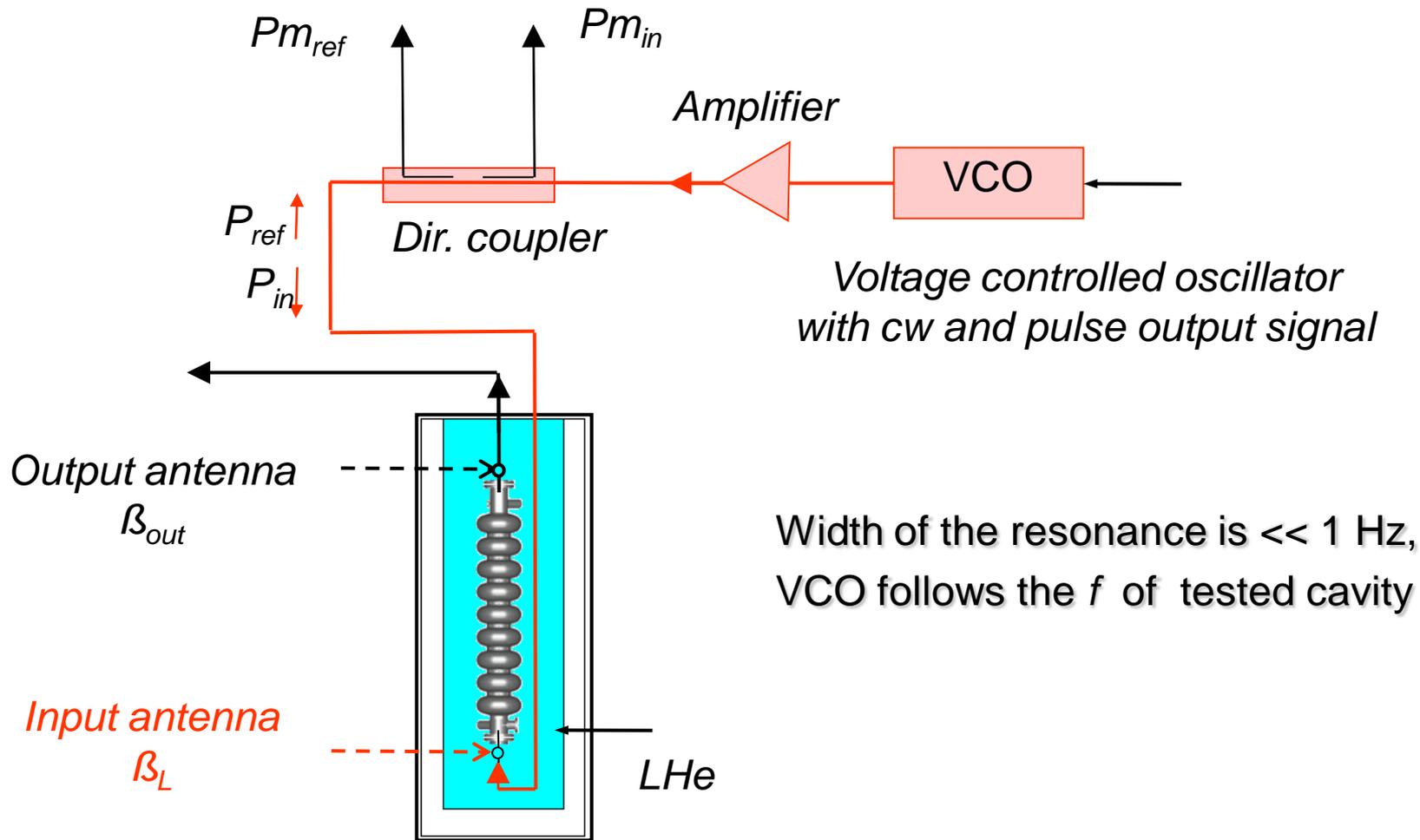
Example: 9-cell TESLA structure, $k_{cc}=1.85\%$, $Q_L=3.8 \cdot 10^6$



8. Performance test

“Vertical test” at $T < T_c$ (usually $\leq 2K$)

The goal is: Q_o vs. E_{acc} (E_{peak})



8. Performance test

At first, in these tests one measures the coupling strength β_L and β_{out} of the input and output antennae.

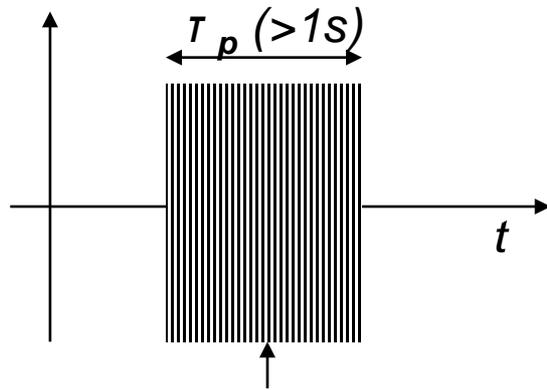
$$\beta_L = \frac{Q_0}{Q_{ext,input}}$$

$$\beta_{out} = \frac{Q_0}{Q_{ext,output}}$$

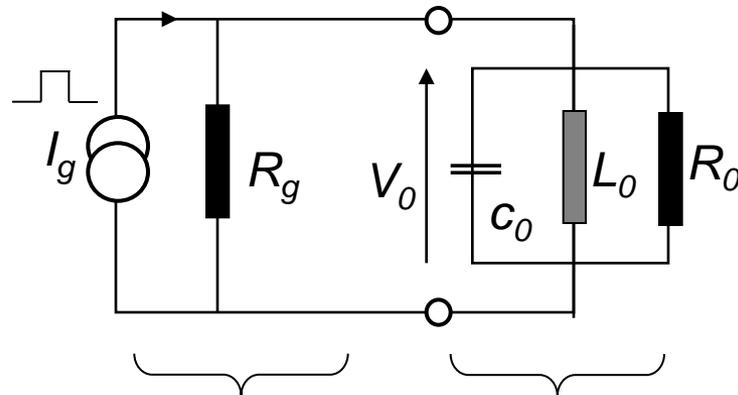
$$Q_{ext,input} \ll Q_{ext,output}$$

Step 1.

Cavity response (shape of the reflected wave amplitude) to the rectangular RF-pulse



$\sim 10^9$ RF oscillations at the resonant frequency



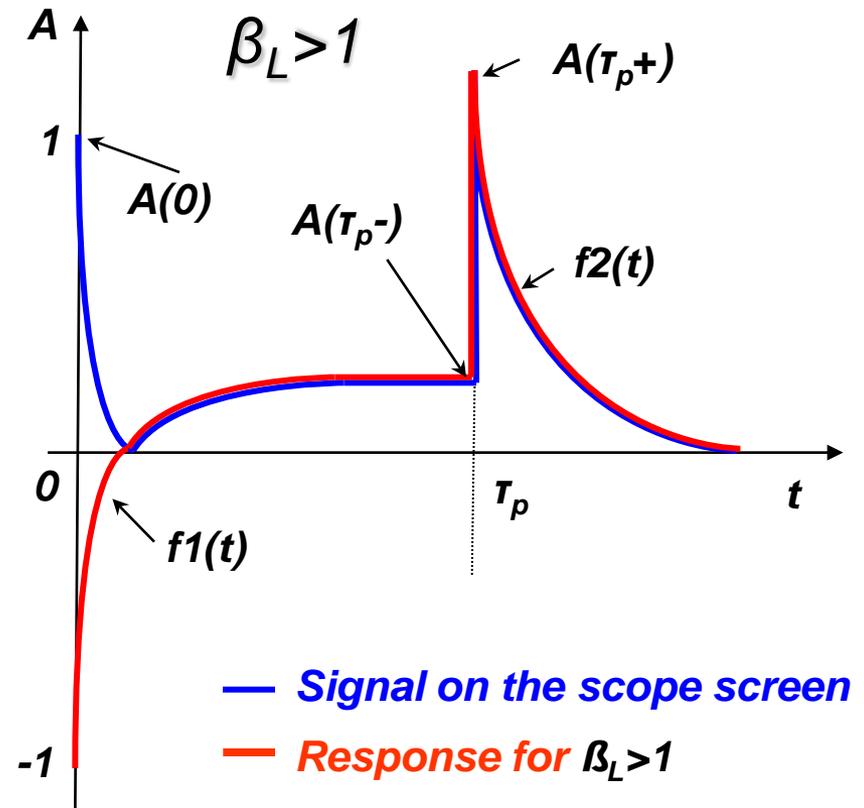
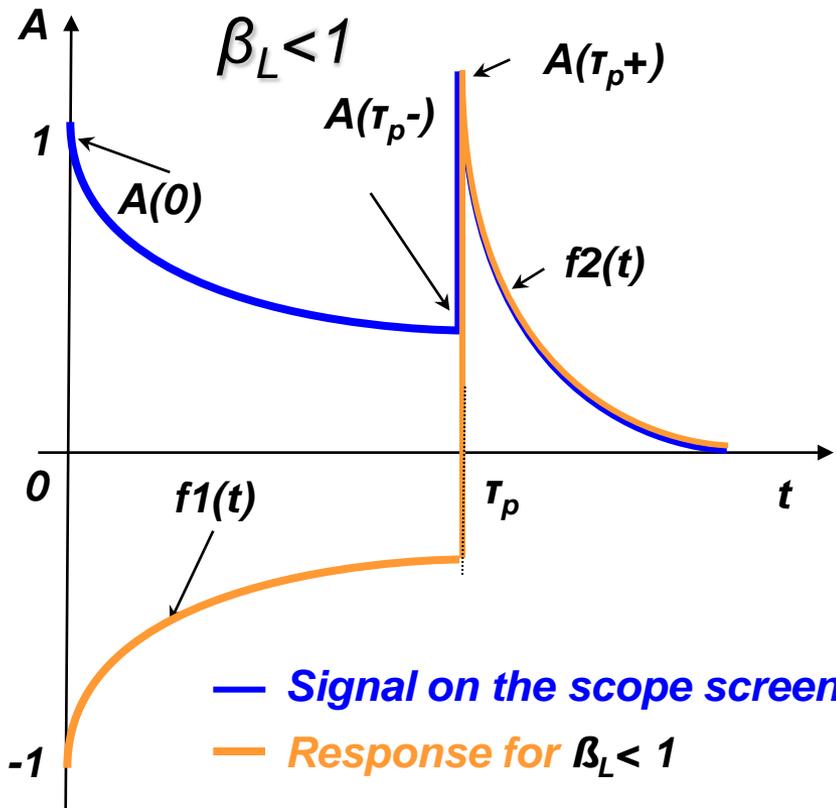
Replacement LEC
for the RF-source

Replacement LEC for
the tested cavity

8. Performance test

$$\begin{cases} f1(t) = -\frac{1-\beta_L}{1+\beta_L} - \frac{2\beta_L}{1+\beta_L} e^{-\frac{\omega_0 t}{2Q_L}} S(t) & \text{for } t \in \langle 0, \tau_p^- \rangle \\ f2(t) = f1(t) + f1(t-\tau_p) S(t-\tau_p) & \text{for } t \in \langle \tau_p, \infty \rangle \end{cases}$$

where $S(t)$ is the step function.



8. Performance test

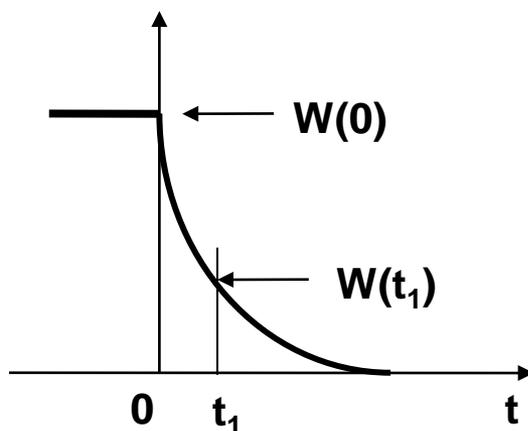
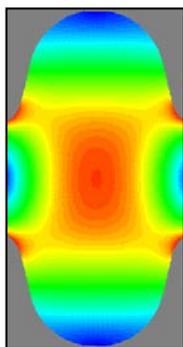
β_L can be computed with one of the there following formulas:

$$\beta_L = \frac{A(0) - A(\tau_p -)}{A(0) + A(\tau_p -)}$$

$$\beta_L = \frac{A(\tau_p +)}{2A(0) - A(\tau_p +)}$$

$$\beta_L = \frac{A(\tau_p +)}{2A(\tau_p -) + A(\tau_p +)}$$

Step 2. Energy decay right after the RF-pulse is switched off



$$W(t) = W(\tau_p +) e^{-\frac{\omega_0(t - \tau_p +)}{Q_L}}$$

and measuring the input and transmitted power (P_{in} and P_{tran}), one obtains:

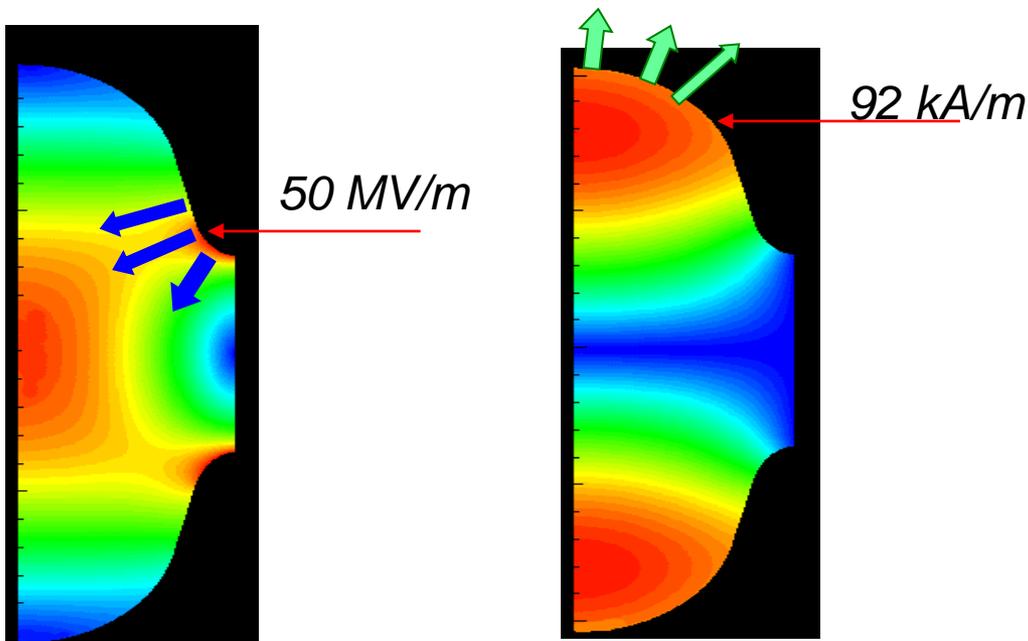
$$Q_0 = Q_L (1 + \beta_L) \left(1 + \frac{P_{tran}}{P_{in} - P_{tran}}\right)$$

9. Mechanical Design

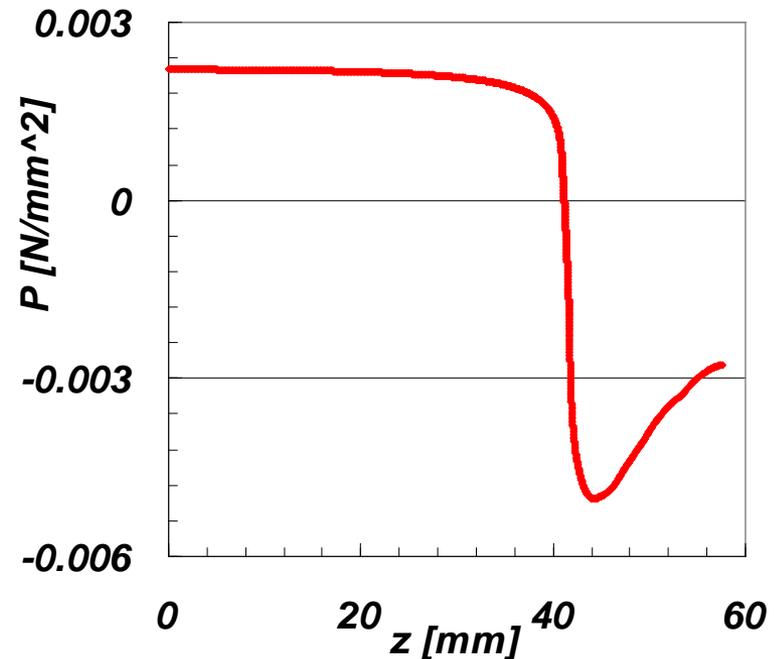
The mechanical design of a cavity follows its RF design:

- ◆ Lorentz Force Detuning
- ◆ Mechanical Resonances

Lorentz Force Detuning

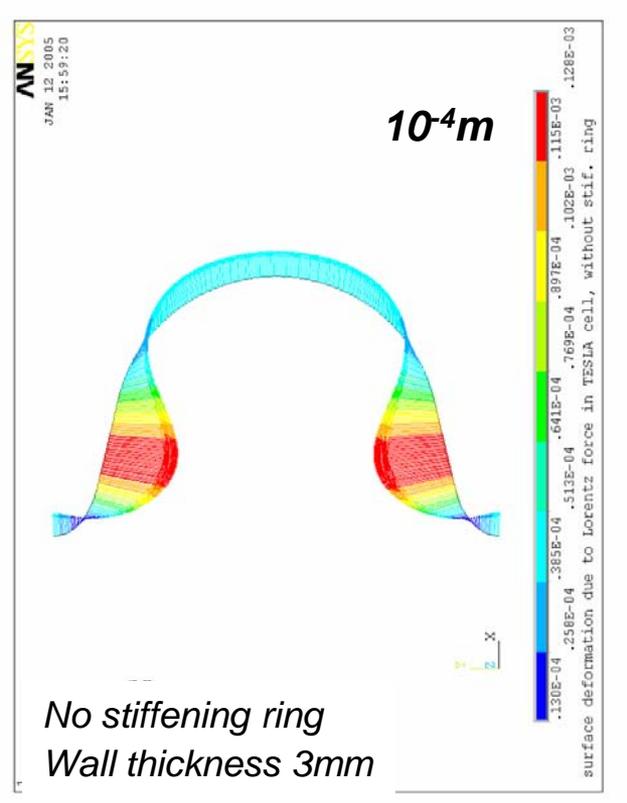
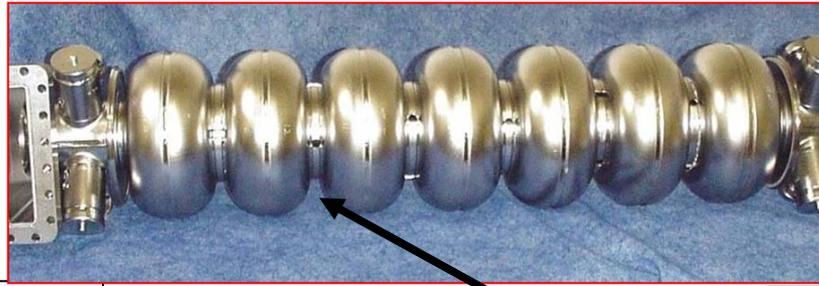


$$P = \frac{\mu_0 H_s^2 - \epsilon_0 E_s^2}{4}$$

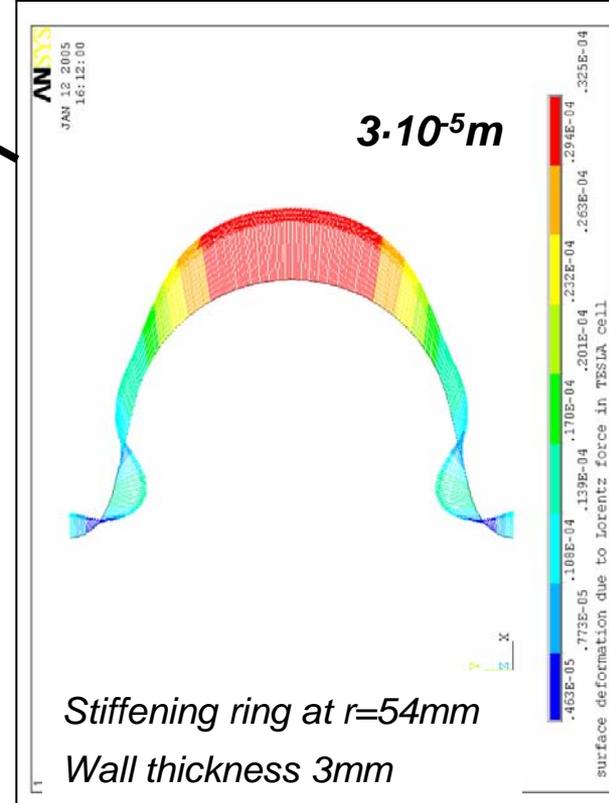


E and H at $E_{acc} = 25$ MV/m in TESLA inner-cup

9. Mechanical Design

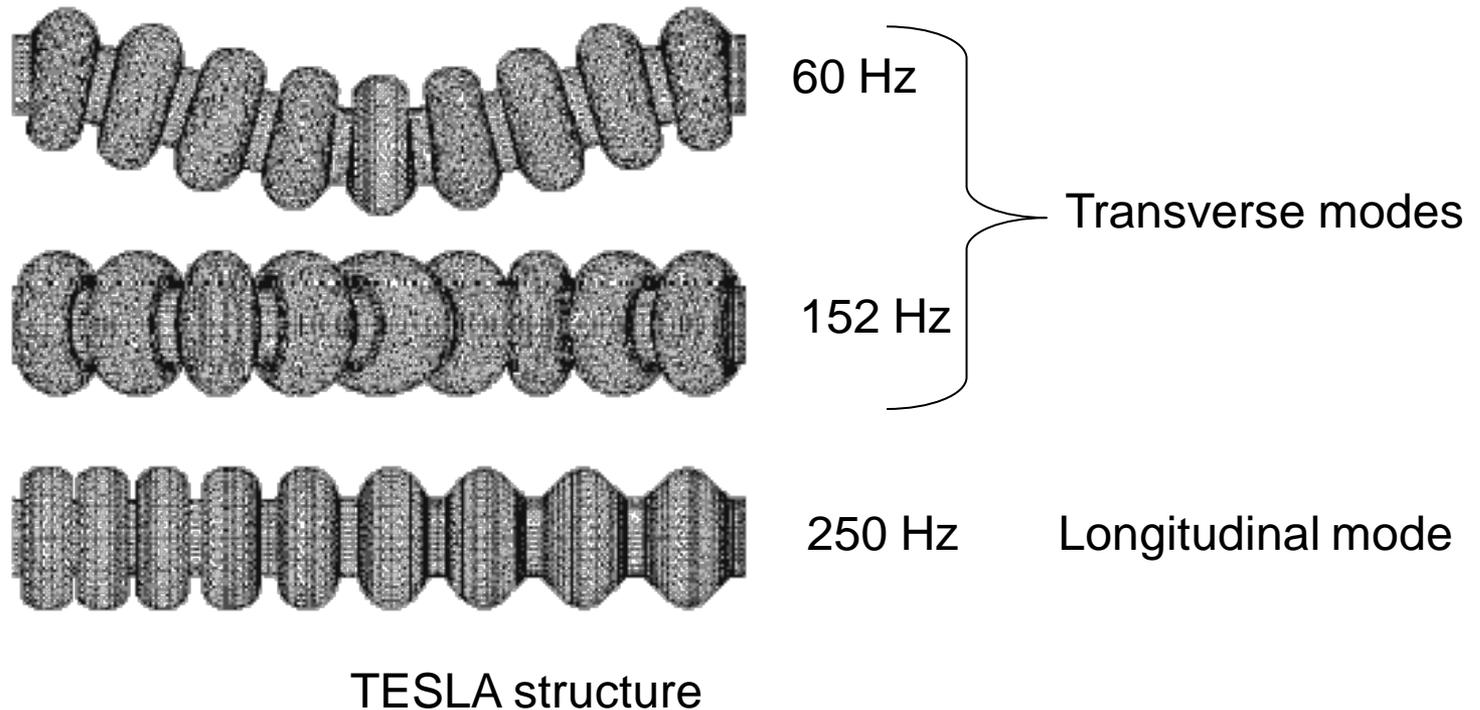


Surface deformation without and with stiffening ring (courtesy of I. Bonin, FERMI)



Essential for the operation of a pulsed accelerator $\Delta f = k_L (E_{acc})^2$ $k_L = -1 \text{ Hz}/(\text{MV}/\text{m})^2$

Mechanical Resonances of a multi-cell cavity



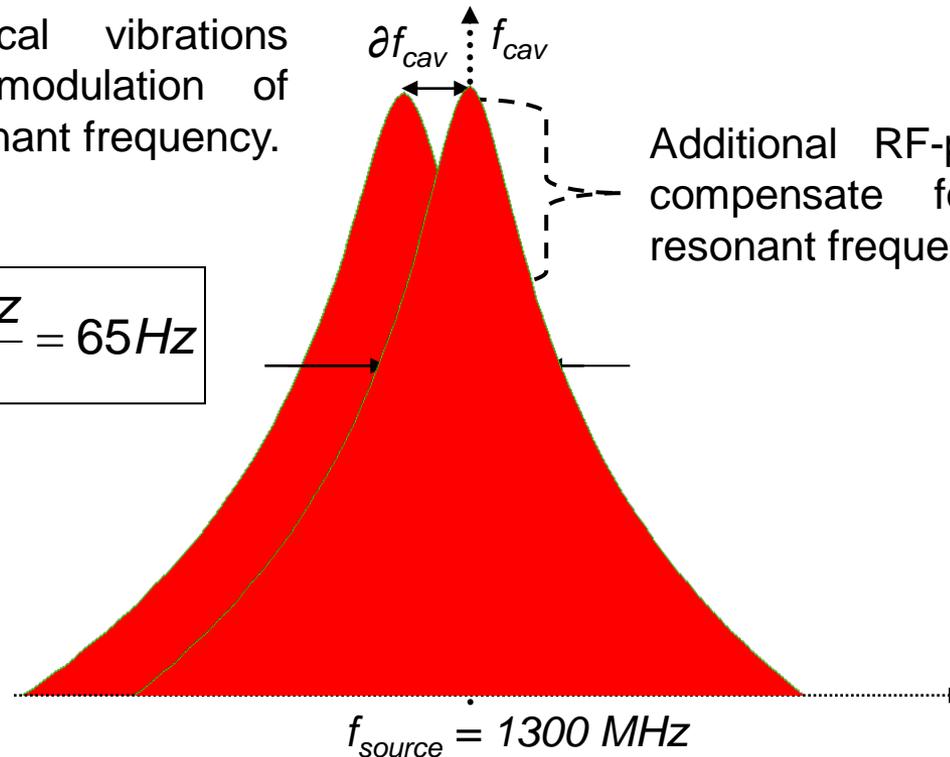
The mechanical resonances modulate frequency of the accelerating mode.
Sources of their excitation: vacuum pumps, ground vibrations...

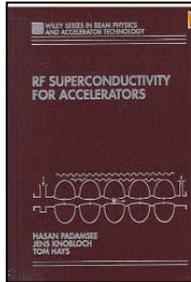
9. Mechanical Design

The mechanical resonances modulate frequency of the accelerating mode.
Sources of their excitation: vacuum pumps, ground vibrations...

Mechanical vibrations cause modulation of the resonant frequency.

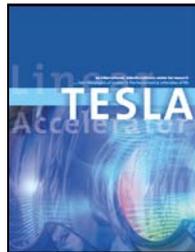
$$\Delta f_{3db} \equiv \frac{1.3 \cdot 10^9 \text{ Hz}}{2 \cdot 10^7} = 65 \text{ Hz}$$



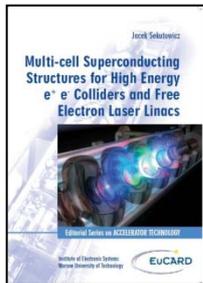


1. H. Padamsee, J. Knobloch, T. Hays, ***“RF-Superconductivity for Accelerators”***, Wiley Series in Beam Physics and Accelerator Technology, 1998.

2. Proceedings of all SRF Workshops; <http://accelconf.web.cern.ch/accelconf/>



3. TESLA TDR, DESY-Report 2003



4. J.S. ***“Multi-cell Superconducting Structures for High Energy e^+e^- colliders and Free electron Laser Linacs”***, CERN/FP7;
<http://eucard.web.cern.ch/eucard/>