Superconducting Cavities

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DESY
Topics

Lecture I
1. Introduction and History
2. RF Parameters
3. Criteria for Cavity Design
4. Test Results of single-cell cavities

Lecture II
5. Multi-cell Structures and Weakly Coupled Structures
6. Tools for RF-design
7. LEC and Transient state
8. Performance test
9. Mechanical Design
1. Introduction and History

Milestones that led to accelerators based on SRF

Superconductivity

RF Acceleration

1908: Heike Kamerlingh Onnes (Holland)
Liquefied Helium for the first time.

1911: Heike Kamerlingh Onnes
Discovered Superconductivity.

1928-34: Walther Meissner (Germany)
Discovered Superconductivity of Ta, V, Ti and Nb.

1924: Gustaf Ising (Sweden)
The First Publication on RF Acceleration,
Arkiv för Matematik, Astronomi och Fysik,

1928: Rolf Wideröe (Norway, Germany)
Built the first RF Accelerator,
Arch. für Elektrotechnik 21, vol.18.

1947: Luis Alvarez (USA)
Built first DTL (32 MeV protons).

1947: W. Hansen (USA)
Built first 6 MeV e-accelerator, Mark I
(TW- structure).
1. Introduction and History

Superconductivity

1961: W. Fairbank (Stanford Univ.)
Presented the first proposal for a superconducting accelerator

1964: W. Fairbank, A. Schwettman, P. Wilson (Stanford Univ.)
First acceleration of electrons with sc lead cavity

1970: J. Turneaure (Stanford Univ.)
$E_{\text{peak}} = 70 \text{MV/m}$ and $Q \approx 10^{10}$ in 8.5 GHz cavity!

Developed and Constructed the Superconducting Accelerator SCA

Since then, we have built many sc accelerators and we are constructing and making plans for many new facilities.
## 1. Introduction and History

### Dismantled Facilities
1. **TRISTAN** (32/49m)*
2. **LEP** (288/490m)
3. **HERA** (16/19m)

### Operating Facilities
1. **SCA** (4/28m)
2. **S-DALINAC** (10/10m)
3. **CESR** (4/1.2 m)
4. **CEBAF** (320/160m)
5. **KEK B-Factory** (8/2.4m)
6. **Taiwan LS** (1+1/0.3m)
7. **Canadian LS** (1+1/0.3m)
8. **DIAMOND** (3/0.9m)
9. **SOLEIL** (4/1.7m)
10. **TTF II** (56/58m)
11. **SNS** (81/65m)
12. **JLab-FEL** (24/14m)
13. **LHC** (16/6m)
14. **ELBE** (6/6m)

*(Number of cavities / total active length)
1. Introduction and History

### Tomorrow Facilities

<table>
<thead>
<tr>
<th>No.</th>
<th>Facility</th>
<th>Length/Width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>CEBAF-12GeV</td>
<td>400/216</td>
</tr>
<tr>
<td>2.</td>
<td>SNS-upgrade</td>
<td>117/98</td>
</tr>
<tr>
<td>3.</td>
<td>XFEL</td>
<td>648/674</td>
</tr>
<tr>
<td>4.</td>
<td>ERL-Cornell</td>
<td>310/250</td>
</tr>
<tr>
<td>5.</td>
<td>RHIC-cooling</td>
<td>1/1</td>
</tr>
<tr>
<td>6.</td>
<td>BEPC II</td>
<td>2/0.6</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
</tr>
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</table>

### Day after Tomorrow Facilities

<table>
<thead>
<tr>
<th>No.</th>
<th>Facility</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>FRIB</td>
<td>non-elliptical 47/~70m</td>
</tr>
<tr>
<td>2.</td>
<td>X-Ray MIT</td>
<td>option 176 / 184m</td>
</tr>
<tr>
<td>3.</td>
<td>Project X</td>
<td>elliptical 352 /~360m</td>
</tr>
<tr>
<td>4.</td>
<td>ERHIC</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>ELIC</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>ILC</td>
<td>(~15764 /~16395m)</td>
</tr>
</tbody>
</table>
1. Introduction and History

ILC (~15764/~16395m)

- e- main linac
- IP and 2 moveable detectors
- 2-stage bunch compression
- undulator
- e- transport line
- e- beam dump

- e+ production
- e+ pre-acceleration
- e+ transport line
- e+ beam dump
- e+ main linac
- e+ source + pre-acceleration
- e+ damping rings
- e- transport line
- e- beam dump

Guestimated total length: 31 km
1. Introduction and History

The “heart” of all mentioned facilities are (were) sc standing wave accelerating structures, here elliptical with $\beta \geq 0.61$.

FERMI 3.9 GHz

S-DALINAC 3 GHz

CESR/CEBAF 1.5 GHz

HEPL 1.3 GHz

TESLA/ILC 1.3 GHz

SNS $\beta = 0.61, 0.81, 0.805$ GHz

HERA 0.5 GHz

KEK-B 0.5 GHz

CESR 0.5 GHz

LEP 0.352 GHz
We have also many “heavy” particles accelerators based on low-β sc cavities. You had lecture on this topic by Dr. Maurizio Vretenar on Friday. The low-β cavities are not discussed in this lecture.

1. Introduction and History

Accelerator/Upgrade/Project

- ISAC-II
- SPIRAL-II
- SARAF
- IUAC
- HIE-REX
- EURISOL
- SPES
- LNL
- ReA3
- FRIB
- IFMIF
1. Introduction and History

What was the progress in the 33 years and what do we need in the next 10 years?

~ 28 m long SCA at Stanford, 1977.

~ 21.6 km long ILC linac, 2018+

E_{acc} \approx 2 \text{ (2.5) MV/m in cw (10\% DF).}

4 Structures 5.65m + capture + pre-accelerator.

E_{acc} > 34 \text{ MV/m shown in several 9-cells in the cw test.} \downarrow

This gradient is required in all 15764 cavities.
1. Introduction and History

Results at DESY (status 2010):

cw test result at 2 K for 8 best electropolished 9-cell TESLA cavities.
2. RF Parameters

2.1. Cavities and their Eigenmodes \textit{(the simplest example)}

Cavity \equiv a volume, partially closed by the metal wall, capable to store the E-H energy

First assumption:
1. Stored E-H fields are harmonic in time.

Maxwell equations for the harmonic, lossless case with no free charge in the volume

\[
\begin{aligned}
\nabla \times H &= i\omega \varepsilon E \\
\nabla \times E &= -i\omega \mu H \\
\n\nabla \cdot E &= 0 \\
\n\nabla \cdot H &= 0
\end{aligned}
\]
Second assumption (good approximation for the elliptical cavities):

2. The volume is **cylindrically symmetric**. We commonly use the \((r, \phi, z)\) coordinates.

\[
\begin{align*}
\nabla_c \times H &= i \omega \varepsilon E \\
\nabla_c \times E &= -i \omega \mu H \\
\nabla_c \cdot E &= 0 \\
\nabla_c \cdot H &= 0
\end{align*}
\]

\[
\begin{align*}
\nabla_c \times A &= \vec{i}_r \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_r}{\partial z} \right) + \vec{i}_\phi \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \vec{i}_z \left( \frac{1}{r} \frac{\partial (rA_\phi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right) \\
\nabla_c \cdot A &= \frac{1}{r} \frac{\partial (rA_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}
\end{align*}
\]
2. RF Parameters

Third assumption

3. For the **acceleration** are suitable modes with **strong** $E$ along the beam **trajectory**. This ensures, by the proper phasing, maximal energy exchange between the cavity and beam.

TM0xx-like monopole modes have “very strong” $E_z$ component on the symmetry axis. Fields of the monopole modes do not depend on $\phi$.

$$\frac{\partial E}{\partial \phi} = 0 \quad \frac{\partial H}{\partial \phi} = 0$$

Non monopole modes (HOM) have component $E_z = 0$ on the symmetry axis. Their fields dependent on $\phi$. 
2. RF Parameters

Maxwell equations + boundary conditions for E and H lead to the Helmholtz equation, which is an eigenvalue problem.

For $H(r,z)$ field of a monopole mode the equation is:

$$(\nabla_c^2 + \omega^2 \varepsilon \mu)H = 0$$

$n \cdot H = 0$ on metal wall

$H = 0$ optionally on non metal boundary

$n \cdot H = 0$

$\nabla^2 \mathbf{A} = \nabla_c (\nabla_c \cdot \mathbf{A}) - \nabla_c \times \nabla_c \times \mathbf{A}$

There is infinity number of TM0xx solutions (modes) to the Helmholtz equation. All modes are determine by:

$H_n(r,z)=[0, H_{\varphi,n}(r,z), 0],$

$E_n(r,z)=[E_{r,n}(r,z), 0, E_{z,n}(r,z)]$

and by frequency $\omega_n$. 
2.2 What are figures of merit for a cavity storing E-H energy?

\[ W_n \equiv \text{stored energy of a mode } n : \{ \omega_n, E_n, H_n \}. \]

\[ W_n = 2 \mu \int \frac{H_n^2}{V} dV = 2\varepsilon \int \frac{E_n^2}{V} dV \]

**Quality Factors**

The measure of the energy loss in metal wall and due to the radiation via open ports:

**Intrinsic** \( Q \equiv Q_0 \)

\[ Q_{0,n} = \frac{\omega_n \cdot W_n}{P_n} = \frac{\omega_n \cdot W_n}{R_{s,n} \int H_n^2 ds} \]

**External** \( Q \equiv Q_{ext} \)

\[ Q_{ext,n} = \frac{\omega_n \cdot W_n}{P_{rad,n}} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int E_n \times H_n ds} \]
2. RF Parameters

**Geometric Factor**

The measure of the energy loss in the metal wall for the surface resistance $R_{s,n}=1\Omega$

$$G_n \equiv Q_{0,n} \cdot R_{s,n} = \frac{\omega_n \cdot W_n \cdot R_{s,n}}{P_n} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int_S H_n^2 ds}$$

It is the ratio of stored energy to the integral of $(H_n)^2$ on metal wall $S$.

2.3 What are figures of merit for the beam-cavity interaction?

This interaction which is:

- Acceleration
- Deceleration (ERL)
- HOMs excitation

can be described in the Frequency Domain (FD) or/and in Time Domain (TD).
2. RF Parameters

\[(R/Q)_n ; \text{beam impedance}\]

It is a “measure” of the energy exchange between point charge and mode n (FD).

Mode n : \(\{\omega_n, E_n, H_n\}\).

Trajectory of the point charge q, assumed here is a straight line.

\[
V_n = \sqrt{\left(\int_{z_a}^{z_b} E_{n,z} \sin\left(\frac{\omega_n}{\beta c} (z - z_a)\right) dz\right)^2 + \left(\int_{z_a}^{z_b} E_{n,z} \cos\left(\frac{\omega_n}{\beta c} (z - z_a)\right) dz\right)^2}
\]

\[
(R/Q)_n \equiv \frac{V_n^2}{\omega_n W_n}
\]
2. RF Parameters

Longitudinal and Transverse Loss Factors (TD) (Excitation of cavity modes)

Ultra relativistic point charge $q$ passes empty cavity

- Cone $\sim 1/y$
- $E_r \sim 1/r$

a. Density of the inducted charge on the wall depends on the distance to beam trajectory

b. The non uniform charge density on the metal wall causes the current flow on surface
2. RF Parameters

The amount of energy lost by charge \( q \) to the cavity is:

\[
\Delta U_q = k_{\parallel} \cdot q^2 \quad \text{for monopole modes (max. on axis)}
\]
\[
\Delta U_q = k_{\perp} \cdot q^2 \quad \text{for non monopole modes (off axis)}
\]

where \( k_{\parallel} \) and \( k_{\perp}(r) \) are the loss factors for monopole and transverse modes respectively.

The induced E-H field (wake) is a superposition of all cavity eigenmodes having the \( E_n(r, \varphi, z) \) field along the particle trajectory.

Both description methods FD and TD are equivalent.

For an individual mode \( n \) and point-like charge:

\[
k_{\parallel,n}^p = \frac{\omega_n \cdot (R/Q)_n}{4}
\]

Note the linac convention for \( (R/Q) \) definition.

Similar for other loss factors…….
2. RF Parameters

2.4. Some “practical” RF parameters of the accelerating mode

At stored energy $W_{\text{acc}}$ the mean value of the accelerating gradient is:

$$E_{\text{acc}} = \sqrt{\frac{\omega_{\text{acc}} \cdot W_{\text{acc}} \cdot (R/Q)_{\text{acc}}}{I_{\text{active}}}}$$

The ratio shows sensitivity of the shape to the field electron emission phenomenon.

The ratio shows limit in $E_{\text{acc}}$ due to the break-down of superconductivity (Nb ~190 mT).
2. RF Parameters

For the accelerating mode we often use the product: $G_{acc} \cdot (R/Q)_{acc}$, as a “measure” of the power $P_dissipated$ dissipated in the wall at given accelerating voltage $V_{acc}$ and given surface resistance $R_s$.

$$\frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G_{acc} \cdot (R/Q)_{acc}}$$

Big improvements are possible:
- Due to superconductivity
- Due to the surface quality.

This is due to the geometry of cells
Moderate improvement possible.
The $k_{cc}$ is relevant for the accelerating mode passband of multi-cell structures.

Single-cell structures are attractive because:
- It is easier to manage HOM damping
- There is no field flatness problem.
- Input coupler transfers less power
- They are easy for cleaning and preparation
- But it is expensive to base even a small linear accelerator on the single cells. We do it only for very high beam current machines.

Multi-cell structures are less expensive/m and allow for higher real-estate gradient.
2. RF Parameters

Resonators closed by metal wall:

Symmetry planes for the H field

\( \omega_0 \)

\( \omega_{\pi} \)

Symmetry plane for the H field

Symmetry plane for the E field, which is an additional solution
2. RF Parameters

The energy flux across the coupling region, refilling energy loss is proportional to the transverse components: $H_\phi$ and $E_r$

Small $E_r$ (due to the losses) + strong $H_\phi$ at the symmetry plane

Small $H_\phi$ (due to the losses) + strong $E_r$ at the symmetry plane

The normalized difference between these frequencies is a measure of the energy flow via the coupling region

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0}$$
The above formulae estimate the sensitivity of a multi-cell field profile to frequency errors of an individual cell for the accelerating mode ($\pi$-mode).

Field flatness factor $a_{ff}$ for a structure made of $N$ cells and their coupling factor $k_{cc}$:

$$a_{ff} = \frac{N^2}{k_{cc}}$$

$$\frac{\Delta A_i}{A_i} = a_{ff} \frac{\Delta f_i}{f_i}$$
3. Criteria for Cavity Design

**Criteria for inner cell design**

We will discuss here design of inner cells because they “dominate” the RF properties of multi-cell superconducting accelerating structures.

**RF parameters summary:**

- $FM: (R/Q), G, E_{peak}/E_{acc}, B_{peak}/E_{acc}, k_{cc}$
- $HOM: k_{\perp}, k_{\parallel}$

There are 7 parameters we want to optimize for an inner cell.

**Geometry:**

- Iris ellipsis: half-axis $h_r, h_z$
- Iris radius: $r_i$
- Equator ellipsis: half-axis $h_r, h_z$

There is some kind of conflict 7 parameters and only 5 variables to “tune”
### 3. Criteria for Cavity Design

<table>
<thead>
<tr>
<th>Criteria</th>
<th>RF-parameter</th>
<th>Improves when</th>
<th>Cavity examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operation at high gradient</td>
<td>$E_{peak}/E_{acc}$, $B_{peak}/E_{acc}$</td>
<td>$r_i$, Iris &amp; Equator shape</td>
<td>TESLA, HG CEBAF-12 GeV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low cryogenic losses</td>
<td>$(R/Q) \cdot G$</td>
<td>$r_i$, Equator shape</td>
<td>LL CEBAF-12 GeV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High $I_{beam} \leftrightarrow$</td>
<td>$k_\perp, k_\parallel$</td>
<td>$r_i$</td>
<td>B-Factory RHIC cooling</td>
</tr>
<tr>
<td>Low HOM impedance</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We see here that $r_i$ is a very “powerful variable” to trim the RF-parameters of a cavity.
Why for a smaller aperture ($r_i$) 

- $(R/Q)$ is bigger 
- $E_{\text{peak}}/E_{\text{acc}}$, $B_{\text{peak}}/E_{\text{acc}}$ are lower?

$E_{\text{acc}}$ is higher at the same stored energy in the cell with smaller aperture.
3. Criteria for Cavity Design

Example:

\[ \{ (R/Q), \frac{E_{\text{peak}}}{E_{\text{acc}}}, \frac{B_{\text{peak}}}{E_{\text{acc}}} \} \text{ vs. } r_i \text{ for cell at } f = 1.5 \text{ GHz} \]

![Graph showing \( (R/Q) \), \( \frac{E_{\text{peak}}}{E_{\text{acc}}} \), \( \frac{B_{\text{peak}}}{E_{\text{acc}}} \) vs. \( r_i \) for cell at \( f = 1.5 \) GHz.](image)
3. Criteria for Cavity Design

In addition to the iris radius $r_i$:

- $\frac{B_{\text{peak}}}{E_{\text{acc}}}$ (and $G$) changes vs. the equator shape

![Graph showing changes in $(B/B_{\text{peak}})^2$ normalized vs. $z$ [mm]](image)
3. Criteria for Cavity Design

Similarly: \( \frac{E_{\text{peak}}}{E_{\text{acc}}} \) changes vs. the iris shape

Both cells have the same: \( f \), \( (R/Q) \) and \( r_i \)
3. Criteria for Cavity Design

We know that a smaller aperture $r_i$ makes FM:

- $(R/Q)$ higher
- $B_{\text{peak}}/E_{\text{acc}}, E_{\text{peak}}/E_{\text{acc}}$ lower

but unfortunately a smaller aperture $r_i$ makes:

- HOMs impedances ($k_\perp, k_\parallel$) higher
- cell-to-cell coupling ($k_{cc}$) weaker
3. Criteria for Cavity Design

HOMs loss factors \((k_\perp, k_\parallel)\)

\[
\begin{align*}
(R/Q) &= 152 \, \Omega \\
B_{\text{peak}} / E_{\text{acc}} &= 3.5 \, \text{mT}/(\text{MV/m}) \\
E_{\text{peak}} / E_{\text{acc}} &= 1.9 \\
(R/Q) &= 86 \, \Omega \\
B_{\text{peak}} / E_{\text{acc}} &= 4.6 \, \text{mT}/(\text{MV/m}) \\
E_{\text{peak}} / E_{\text{acc}} &= 3.2
\end{align*}
\]
3. Criteria for Cavity Design

Cell-to-cell coupling, $k_{cc}$

$\frac{R}{Q} = 152 \ \Omega$

$B_{peak}/E_{acc} = 3.5 \text{ mT/(MV/m)}$

$E_{peak}/E_{acc} = 1.9$

$\frac{R}{Q} = 86 \ \Omega$

$B_{peak}/E_{acc} = 4.6 \text{ mT/(MV/m)}$

$E_{peak}/E_{acc} = 3.2$
### 3. Criteria for Cavity Design

#### Frequency of the accelerating mode frequency

<table>
<thead>
<tr>
<th>$f_{\pi}$ [MHz]</th>
<th>2600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R/Q$ [Ω]</td>
<td>57</td>
</tr>
<tr>
<td>$r/q=(R/Q)/l$ [Ω/m]</td>
<td>2000</td>
</tr>
<tr>
<td>$G$ [Ω]</td>
<td>271</td>
</tr>
</tbody>
</table>

$\begin{array}{|c|c|c|}
\hline
f_{\pi} & [MHz] & 1300 \\
R/Q & [Ω] & 57 \\
r/q=(R/Q)/l & [Ω/m] & 1000 \\
G & [Ω] & 271 \\
\hline
\end{array}$

$x \times 2 =

r/q=(R/Q)/l \sim f$
From the formula, we learned before, one obtains:

\[ P_{\text{dissipated}} = \frac{R_s \cdot V_{\text{acc}}^2}{G_{\text{acc}} \cdot (r / q)_{\text{acc}} \cdot l_{\text{active}}} \]

Higher \( f_{\pi} \) would be a good choice to minimize dissipation in the metal wall when the length \( l_{\text{active}} \) and final energy \( V_{\text{acc}} \) are fixed.

Unfortunately this applies only to room temperature structures made of Cu, which \( R_s \sim (f)^{1/2} \).

For superconductors, like Nb:

\[ R_s(f) = R_{\text{res}} + R_{\text{BCS}} = R_{\text{res}} + 0.0002 \cdot \frac{1}{T} \cdot \frac{f[\text{GHz}]}{1.5} \cdot \exp\left(-\frac{17.67}{T}\right) \]

and \( R_s \), which is \( (f)^2 \) for higher \( f \) must be compensated with lower temperature \( T \).

This is why ILC, XFEL, ERL,... (1.3GHz) will operate at 2 K (1.8 K), and HERA (0.5 GHz) and LEP (0.352 GHz) could operate at 4.2 K.
### 3. Criteria for Cavity Design

#### Examples of inner cells

<table>
<thead>
<tr>
<th></th>
<th>CEBAF Original Cornell $\beta=1$</th>
<th>CEBAF -12 Low Loss $\beta=1$</th>
<th>TESLA $\beta=1$</th>
<th>SNS $\beta=0.61$</th>
<th>SNS $\beta=0.81$</th>
<th>RHIC Cooler $\beta=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\pi}$ [MHz]</td>
<td>1497.0</td>
<td>1497.0</td>
<td>1300.0</td>
<td>805.0</td>
<td>805.0</td>
<td>703.7</td>
</tr>
<tr>
<td>$k_{cc}$ [%]</td>
<td>3.29</td>
<td>1.49</td>
<td>1.9</td>
<td>1.52</td>
<td>1.52</td>
<td>2.94</td>
</tr>
<tr>
<td>$E_{\text{peak}}/E_{\text{acc}}$</td>
<td>-</td>
<td>2.56</td>
<td>2.17</td>
<td>1.98</td>
<td>2.66</td>
<td>2.14</td>
</tr>
<tr>
<td>$B_{\text{peak}}/E_{\text{acc}}$ [mT/(MV/m)]</td>
<td>4.56</td>
<td>3.74</td>
<td>4.15</td>
<td>5.44</td>
<td>4.58</td>
<td>5.78</td>
</tr>
<tr>
<td>$R/Q$ [Ω]</td>
<td>96.5</td>
<td>128.8</td>
<td>113.8</td>
<td>49.2</td>
<td>83.8</td>
<td>80.2</td>
</tr>
<tr>
<td>$G$ [Ω]</td>
<td>273.8</td>
<td>280</td>
<td>271</td>
<td>176</td>
<td>226</td>
<td>225</td>
</tr>
<tr>
<td>$R/Q<em>G$ [Ω</em>Ω]</td>
<td>26421</td>
<td>36064</td>
<td>30840</td>
<td>8659</td>
<td>18939</td>
<td>18045</td>
</tr>
<tr>
<td>$k_{\perp}$ ($\sigma_z=1\text{mm}$) [V/pC/cm²]</td>
<td>0.22</td>
<td>0.53</td>
<td>0.23</td>
<td>0.13</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>$k_{\parallel}$ ($\sigma_z=1\text{mm}$) [V/pC]</td>
<td>1.36</td>
<td>1.71</td>
<td>1.46</td>
<td>1.25</td>
<td>1.27</td>
<td>0.85</td>
</tr>
</tbody>
</table>
### Evolution of inner cells proposed for the ILC collider

<table>
<thead>
<tr>
<th></th>
<th>TESLA optimized</th>
<th>Re-entrant optimized</th>
<th>LL optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{peak}}/E_{\text{acc}}$</td>
<td>-</td>
<td>1.98</td>
<td>2.30</td>
</tr>
<tr>
<td>$B_{\text{peak}}/E_{\text{acc}}$ [mT/(MV/m)]</td>
<td>4.15</td>
<td>3.57</td>
<td>3.61</td>
</tr>
<tr>
<td>$R/Q$ [Ω]</td>
<td>113.8</td>
<td>135</td>
<td>133.7</td>
</tr>
<tr>
<td>$G$ [Ω]</td>
<td>271</td>
<td>284.3</td>
<td>284</td>
</tr>
<tr>
<td>$R/Q<em>G$ [Ω</em>Ω]</td>
<td>30840</td>
<td>38380</td>
<td>37970</td>
</tr>
<tr>
<td>$k_\perp (\sigma_z=1\text{mm})$ [V/pC/cm²]</td>
<td>0.23</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>$k_\parallel (\sigma_z=1\text{mm})$ [V/pC]</td>
<td>1.46</td>
<td>1.75</td>
<td>1.72</td>
</tr>
</tbody>
</table>

#### 3. Criteria for Cavity Design

- Evolution of inner cells proposed for the ILC collider
- Table of parameters for different optimizations:
  - TESLA optimized
  - Re-entrant optimized
  - LL optimized

- Criteria for cavity design:
3. Criteria for Cavity Design

Multipacting

It is a phenomenon of resonant electron emission and multiplication.

Impacting electron might create more than one secondary electron. This depends on the impact energy $K$ and secondary emission yield $\delta(K)$. 
3. Criteria for Cavity Design

SEY is function of the impact energy $K$ and depends on the surface cleanness.

$$\delta(K)$$

$\delta_p$ SEY for Nb

<table>
<thead>
<tr>
<th>Condition</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>high SEY</td>
<td>$\sim 27 \text{ eV}$</td>
<td>$\gtrsim 2000 \text{ eV}$</td>
</tr>
<tr>
<td>typical SEY</td>
<td>$\sim 40 \text{ eV}$</td>
<td>$\sim 1000 \text{ eV}$</td>
</tr>
<tr>
<td>low SEY</td>
<td>$\sim 150 \text{ eV}$</td>
<td>$\sim 750 \text{ eV}$</td>
</tr>
</tbody>
</table>

When happens, multipacting is barrier in rising the accelerating field in cavities and usually leads to quench.

In the design process we need to prove whether or not the shape of cell allows for multipacting.
## 4. Test Results of single-cell cavities

**LL; KEK test September 2005**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ (MHz)</td>
<td>1286.6</td>
</tr>
<tr>
<td>$E_{\text{peak}}/E_{\text{acc}}$</td>
<td>- 1.86</td>
</tr>
<tr>
<td>$B_{\text{peak}}/E_{\text{acc}}$ (mT/(MV/m))</td>
<td>3.71</td>
</tr>
<tr>
<td>$R/Q$ (Ω)</td>
<td>130.0</td>
</tr>
<tr>
<td>$G$ (Ω)</td>
<td>279</td>
</tr>
<tr>
<td>$\varnothing_{\text{iris}}$ (mm)</td>
<td>61</td>
</tr>
</tbody>
</table>

**Graph**

- $E_{\text{peak}} = 86.5 \text{ MV/m}$
- $B_{\text{peak}} = 172.5 \text{ mT}$
- $Q_0 = 1.12 \times 10^{10}$ @ 1.97K
- $Q_0 = 1.74 \times 10^{10}$ @ 1.68K
- Quench $46.5 \text{ MV/m}$

**Other parameters**

- $E_{\text{peak}}/E_{\text{acc}} = 1.86$
- $B_{\text{peak}}/E_{\text{acc}} (\text{mT}/(\text{MV/m})) = 3.71$
- $R/Q (\Omega) = 130.0$
- $G (\Omega) = 279$
- $\varnothing_{\text{iris}} (\text{mm}) = 61$
4. Test Results of single-cell cavities

RE; Cornell, test in March 2007 !!!!

<table>
<thead>
<tr>
<th>$f$</th>
<th>[MHz]</th>
<th>1300.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{peak}}/E_{\text{acc}}$</td>
<td>-</td>
<td>2.11</td>
</tr>
<tr>
<td>$B_{\text{peak}}/E_{\text{acc}}$</td>
<td>[mT/(MV/m)]</td>
<td>3.53</td>
</tr>
<tr>
<td>$R/Q$</td>
<td>[$\Omega$]</td>
<td>126.0</td>
</tr>
<tr>
<td>$G$</td>
<td>[$\Omega$]</td>
<td>283.3</td>
</tr>
<tr>
<td>$\phi_{\text{iris}}$</td>
<td>[mm]</td>
<td>60</td>
</tr>
</tbody>
</table>

$E_{\text{peak}} = 125 \text{ MV/m}$

$B_{\text{peak}} = 208 \text{ mT}$
Topics

Lecture I

1. Introduction and History
2. RF Parameters
3. Criteria for Cavity Design
4. Test Results of single-cell cavities

Lecture II

5. Multi-cell Structures and Weakly Coupled Structures
6. Tools for RF-design
7. LEC and Transient state
8. Performance test
9. Mechanical Design
We re-call **pros** and **cons** for a multi-cell structure:

- **Cost of accelerators is lower** (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics……..)

- **Higher real-estate gradient** (better fill factor)

- **Field flatness vs. N**

- **HOM excitation and trapping vs. N**

- **Power capability of fundamental power couplers vs. N**

- **Surface cleaning procedures become more complicated**

- **The worst performing cell limits whole multi-cell structure**
Accelerating mode in multi-cell structures

Synchrotron acceleration and max of \((R/Q)_{acc}\) for a multi-cell structure when:

1. \(l_{active} = Nl_{cell} = Nc\beta/(2f)\) and

2. the injection takes place at an optimum phase \(\phi_{opt}\), which ensures that particles arrive at the mid-plane of the first cell when \(E_{acc}\) reaches its maximum (minimum)
Decades of experience with: heat and chemical treatment, handling and assembly allow to maintain good field profile, even in cavities with bigger $N$ and weaker $k_{cc}$. For many TESLA cavities: field flatness is better than 95 %
HOM excitation and trapping in multi-cell structures

HOM excitation causes:

- Beam instabilities and/or dilution of emittance
- Bunch-to-bunch energy modulation
- Additional cryogenic loss

HOM excitation:

Time structure of the beam

\[ t_b \quad \{ q, \sigma_z \} \quad t_g \quad \text{Group of bunches} \]

Spectrum of the beam

\[ \frac{\omega_n^2 \sigma_z^2}{2c^2} \]

\[ q/t_b \]

\[ f \]

\[ 1/t_b \]
5. Multi-cell Structures and Weakly Coupled Structures

Mode No. $n : \{ \omega_n, (R/Q)_n, Q_{L,n} \}$

Impedance of the $n$-mode

\[
Z_n(\omega) = \frac{(R/Q)_n \cdot Q_{L,n}}{1 + jQ_{L,n}\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)}
\]
The power induced by “all” spectral lines (current sources) in mode No. $n$:

$$P_n = \frac{1}{2} \sum_k Z_n(\omega_k) \cdot I_k^2$$

where:

$$Z_n(\omega) = \frac{(R/Q)_n \cdot Q_{L,n}}{1 + jQ_{L,n}\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)}$$

$$\frac{1}{Q_{L,n}} = \frac{1}{Q_{0,n}} + \frac{1}{Q_{\text{ext},n}}$$

Measure of the extracted power

The HOM couplers, devices extracting the energy from parasitic modes, are attached to cavities for mitigation of high and harmful E-M fields of HOM.

The experience is that, the HOM couplers can be attached to the beam tubes and must not be located at cells because this leads to the performance degradation.
5. Multi-cell Structures and Weakly Coupled Structures

The HOM trapping is similar to the FM field profile unflatness mechanism:

- weak $k_{cc,HOM}$, cell-to-cell coupling for HOM
- difference in the HOM frequency between the end-cells and inner-cells

That is why they hardly resonate together:

$f = 2385 \text{ MHz}$

$f = 2415 \text{ MHz}$
5. Multi-cell Structures and Weakly Coupled Structures

Example of the trapping and how $N$ influences strength of the $E-H$ fields at the HOM couplers locations

- $N = 17$
- $N = 13$
- $N = 9$
- $N = 5$

Less cells in a structure helps always to reach low $Q$s of HOMs.
5. Multi-cell Structures and Weakly Coupled Structures

What else can help to avoid the trapping?

**Adjustment of end-cells**

The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes, HOM- and input couplers.

Their function is multifold and their geometry must fulfill three requirements:

- field flatness and frequency of the accelerating mode
- field strength of the accelerating mode must enable matching of $Q_{ext}$ of FPC
- field strength of the dangerous HOMs must ensure required damping.

All three requirements make design of the end-cells more difficult than inner cells.
1. Open irises of the inner- and end-cells (bigger $k_{cc,HOM}$) and shaping them similarly.

Example: RHIC 5-cell cavity for the electron cooling:

- $f_{HOM} = 1394$ MHz
- $f_{HOM} = 1407$ MHz
- $f_{HOM} = 1403$ MHz

Monopole mode $k_{cc}$

The method causes $(R/Q)$ reduction of fundamental mode, which in this application is less relevant.

(Courtesy of R. Calaga and I. Ben-Zvi)
2. Tailoring the end-cells to equalize HOM frequencies of inner- and end-cells.

Example: TESLA 9-cell cavity, which has two different end-cells (asymmetric cavity)

The lowest mode in the passband $f_{HOM} = 2382$ MHz

The highest mode in the passband $f_{HOM} = 2458$ MHz

The method works for few modes but keeps the $(R/Q)$ of the fundamental mode high.
Two main limitations in $N$:

- Field unflatness
- HOM trapping

can be overcome in weakly coupled structures. One FPC/$(2N)$ cells. Energy flows via very weak coupling $\sim 10^{-4}$.

Superstructure: Two (or more) $N$-cell structures are coupled by $\lambda/2$ long tubes. Each structure has its own cold tuner and HOM dampers.

Beam test of two 2x7-cell prototypes at DESY confirmed stable acceleration and very good damping of HOMs but the final BCP and cleaning were very difficult.

5. Multi-cell Structures and Weakly Coupled Structures
Power capability of the FPC for multi-cell structures

When $I_{beam}$ and $E_{acc}$ are specified and a superconducting multi-cell structure does not operate in the energy recovery mode:

$$P_{in} \sim N$$

$Q_{ext}$ of the FPC, which usually is $<<$ than intrinsic $Q_0$, is:

$$Q_{ext} \approx \frac{E_{acc} \cdot \beta \cdot \lambda \cdot N}{I_{beam} \cdot (R/Q)_{cell} \cdot N} = \frac{E_{acc} \cdot \beta \cdot \lambda}{I_{beam} \cdot (R/Q)_{cell}} = \frac{\omega_{acc} \cdot W_{onecell} \cdot N}{\frac{1}{2} \int_{S_{inputport}} E_{acc} \times H_{acc} \, ds}$$

Independent of $N$

It must be $\sim N$ to keep the ratio constant
5. Multi-cell Structures and Weakly Coupled Structures

\[ \int_{S_{\text{inputport}}} E_{\text{acc}} \times H_{\text{acc}} \, ds \]

Coupler must penetrate deeper in the beam tube or/and is placed closer to the end cell.

Opening for the coupler must be bigger. Both perturb the cylindrical symmetry of the end tube and increase kick to the beam.

The remedies are: alternating positions of couplers or double couplers.

Courtesy of Alan Todd (AES)
Surface cleaning procedures are more complicated

Few words on Nb and on the surface preparation procedures

All high gradient cavities are made of the pure metallic bulk Nb (II-type sc, $T_c = 9.2$ K):
- We use poly-crystal Nb from the very beginning.
- Recently, we made several cavities of large-grain Nb (with hope for single crystal)

Surface preparation has several steps with three major procedures:
- Chemical treatment: can be Buffered Chemical Polishing or Electro-Polishing
- Heat treatment
- High Pressure Water rinsing

Buffered Chemical Polishing (BCP)
Acids: HF (49%), HNO₃ (65%), H₃PO₄ (85%)
Mixture: 1:1:1 or 1:1:2 by volume

Electro-Polishing (EP)
Electrolyte:
1 part HF(49%), 9 parts H₂SO₄ (96%)
Al-cathode, Nb-anode, J~ 50 mA/cm²
5. Multi-cell Structures and Weakly Coupled Structures

The sequence in the surface preparation is:

- **Heavy chemical etch (EP or BCP)**
  - Removal of damaged surface layer (100-150um) caused by fabrication and handling

- **Removal of surface contamination**
  - Ultrasonic cleaning of surface with detergent and DI water, or Alcohol rinse

- **Heat treatment (600-800C in vacuum furnace)**
  - Removes hydrogen from the bulk niobium to reduce the risk of Q-disease

- **RF tuning and mechanical inspection**
  - Field profile, calibration of test probes, check mechanical structure

- **Removal of surface contamination**
  - Ultrasonic cleaning of surface with detergent and DI water,

- **Light chemical etch (EP or BCP)**
  - Remove any risk from damage during handling and furnace contamination

- **High pressure rinse (UPW @100 Bar) + Class 10 drying of cavity**
  - Reduction of field emission sources, surface particulates
  - At least two passes over entire surface
The best performance is still difficult to reach. The preparation must be repeated to reach the ultimate goal of 34 MV/m @ 10^{10}. It makes cavities very “expensive”. 

Example showing randomness in the performance due to the additional cleaning:
AC71 went from good to bad
AC76 went from bad to good
The worst performing cell limits whole multi-cell structure

After the pre-tuning all cells have the same amplitude
List of multi-cell cavities $\beta=1$ optimized for various criteria

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Structure</th>
<th>Best parameter</th>
<th>Weakest parameter (point)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{acc}}$</td>
<td>HG: 1.5 GHz, N=7, TESLA: 1.3 GHz, N=9, ILC-LL: 1.3 GHz, N=9, ILC-RE 1.3 GHz, N=9</td>
<td>$E_{\text{peak}}/E_{\text{acc}}=1.96$, $E_{\text{peak}}/E_{\text{acc}}=1.98$, $B_{\text{peak}}/E_{\text{acc}}=3.61$, $B_{\text{peak}}/E_{\text{acc}}=3.57$</td>
<td>maximum $-E_{\text{acc}}$, maximum $-E_{\text{acc}}$, $E_{\text{peak}}/E_{\text{acc}}$, $E_{\text{peak}}/E_{\text{acc}}$</td>
<td>Designed for $I_{\text{beam}} &lt; 10$ mA, Pulse operation</td>
</tr>
<tr>
<td>$P_{\text{loss}}$</td>
<td>LL: 1.5 GHz, N=7</td>
<td>$B_{\text{peak}}/E_{\text{acc}}=3.7$, $(R/Q)\cdot G$</td>
<td></td>
<td>Not easy to clean, HOM damping</td>
</tr>
<tr>
<td>$Z_{\text{HOM}}$</td>
<td>RHIC: 0.7 GHz, N=5</td>
<td>Very low: $k_{\perp}$, $k_{\parallel}$, $E_{\text{peak}}/E_{\text{acc}}=1.98$</td>
<td>Cryogenic losses</td>
<td>First cavity for $I_{\text{beam}} \approx 2$ A</td>
</tr>
</tbody>
</table>
6. Tools for RF-design

Design of an elliptical cavity is usually performed in two steps: “2D” and “3D”:

- “2D” is fast and allows to define geometry of a cylindrical symmetric body (inner and end-cells) of the cavity.

- “3D” is much more time consuming but necessary for modeling of full equipped cavity with FPC and HOM couplers. Also coupling strength for FPC and damping of HOMs can be modeled only in 3D.

Solutions to 2D (or 3D) Helmholtz equation can be found analytically only for very few geometries (pillbox, spherical resonators or rectangular resonator)

We need numerical methods:

\[
(\nabla^2 + \omega^2 \varepsilon \mu) A = 0
\]

Approximating operator (Finite Difference Methods) \hspace{1cm} \text{Approximating function (Finite Element Methods)}
The FEM is superior in mapping of curvilinear boundary. This is essential for modeling of:

- Multipacting
- Electron emission from the metal wall and generation of dark current
- Frequency change due to the chemical treatment (removed layer of ~150µm)

2D codes like SUPERFISH (linear boundary approx.), SLANS (parabolic boundary approx.) or FEM-code (3rd order boundary approx.) are commonly used.

Example from the FEM code (the code is used also in the multipacting package MultiPac by P. Yla-Oijala and D. Proch, Proceedings of SRF2001, KEK).

Smooth boundary ensures that emitted electrons follow the force of E-H fields which components are not perturbed by a “zigzag” boundary.

6. Tools for RF-design
6. Tools for RF-design

Example: FEM-code modeling of the frequency change due to the chemical treatment (removed layer of 100µm)

\[
\frac{\partial f}{100 \, \mu m} = -10 \, kHz / \mu m
\]

2D Modeling takes ~2min

which was measured for the uniform removal

Zoomed difference in shape of the TESLA mid cup after 100µm BCP
Electromagnetic Code Development at SLAC by ACD

Solves Maxwell’s equations with particles in TD & FD

- Tau3P/T3P
- Omega3P
- S3P

Finite-Element (up to 6th order basis)

- Time Domain Simulation With Excitations
- Frequency Domain Mode Calculation
- Scattering Matrix Evaluation

Track3P – Particle Tracking with Surface Physics

V3D – Visualization of Meshes, Particles & Fields

(Courtesy of Kwok Ko and ACD Members)
6. Tools for RF-design

Example of 3D two dipoles overlapping modeling in the TESLA cavity with Omega3P

(Courtesy of Kwok Ko and ACD Members)
Example of 3D dipole damping modeling for the TESLA cavity with the coaxial coupling (Omega3P, L. Xiao, ACD SLAC)
7. LEC and Transient State

In the design process we use 2D-codes (SUPERFISH, SLANS, FEM..) and 3D-codes (MWS, HFSS, MAFIA and OMEGA-3P) but still the Lumped Element replacement Circuit is helpful to investigate some RF properties.

Where:  
\[ 2\pi f_{FM} = (L_k \cdot c_k)^{-0.5} \];  
\[ (R/Q)_{FM} = (L_k/c_k)^{0.5} \];  
\[ R = (R/Q)_{FM} \cdot Q_{L,FM} \];
7. LEC and Transient State

What can be done by means of the LEC:

- Cavity pre-tuning after the fabrication and bulk chemical treatment
- Investigation how the field profile in cells depends on their frequency errors \( (\partial f/f < 10^{-4}) \)
- Investigation how passband frequencies depend on cell frequency errors \( (\partial f/f < 10^{-4}) \)
- Modeling of the transient state (mode beating)
- Modeling of the voltage stability during acceleration

blue marked examples of the implementation are shown on next slides
Investigation of the FM passband frequencies sensitivity to cell frequency errors \( (\partial f/f<10^{-4}) \)

Example: 7-cells, \( k_{cc}=1.85\% \), 1st cell detuned by \(-30kHz\) (cell length changed \(-11\mu m\), hard to model with 2D and 3D codes)
7. LEC and Transient State

Transient State: Mode beating in the pulse operation

Solving the set of Kirchoff equations:

\[ R_1 \cdot x_1(t) + L_1 \cdot \dot{x}_1(t) + \frac{1}{c_1} \int_0^t x_1(\tau) d\tau - \frac{1}{c_1} \int_0^t x_{1,2}(\tau) d\tau = U_{-1}(t) \cdot e(t) \]

\[
\vdots
\]

\[- \frac{1}{c_{k-1}} \int_0^t x_{k-1,k}(\tau) d\tau + R_k \cdot x_k(t) + L_k \cdot \dot{x}_k(t) + \frac{1}{c_k} \int_0^t x_k(\tau) d\tau - \frac{1}{c_k} \int_0^t x_{k,k+1}(\tau) d\tau = 0 \]

\[
\vdots
\]

\[- \frac{1}{c_N} \int_0^t x_{N-1,N}(\tau) d\tau + R_N \cdot x_N(t) + L_N \cdot \dot{x}_N(t) + \frac{1}{c_N} \int_0^t x_N(\tau) d\tau = 0 \]

one can find voltages right after the RF-source is switched on and during the acceleration
Modeling of the transient state (mode beating)

Example: 7-cells, $k_{cc}=1.85\%$, $Q_L=3.4\ 10^6$
Modeling of the transient state (mode beating at the beam arrival time)

Example: 9-cell TESLA structure, $k_{cc}=1.85\%$, $Q_L=3.8 \times 10^6$
8. Performance test

“Vertical test” at T<Tc (usually ≤ 2K)

The goal is: \(Q_0 \text{ vs. } E_{\text{acc}} (E_{\text{peak}})\)

Voltage controlled oscillator with cw and pulse output signal

Width of the resonance is << 1 Hz, VCO follows the \(f\) of tested cavity
8. Performance test

At first, in these tests one measures the coupling strength $\beta_L$ and $\beta_{\text{out}}$ of the input and output antennae.

$$\beta_L = \frac{Q_0}{Q_{\text{ext,input}}} \quad \beta_{\text{out}} = \frac{Q_0}{Q_{\text{ext,output}}} \quad Q_{\text{ext,input}} \ll Q_{\text{ext,output}}$$

Step 1.

Cavity response (shape of the reflected wave amplitude) to the rectangular RF-pulse

$\sim 10^9$ RF oscillations at the resonant frequency
8. Performance test

\[
\begin{cases}
  f_1(t) = -\frac{1 - \beta_L}{1 + \beta_L} - \frac{2\beta_L}{1 + \beta_L} e^{\frac{\omega_0 t}{2Q_L}} S(t) \\
  f_2(t) = f_1(t) + f_1(t - \tau_p) S(t - \tau_p)
\end{cases}
\]

for \( t \in <0, \tau_p - > \)

for \( t \in < \tau_p, \infty > \)

where \( S(t) \) is the step function.

**Signal on the scope screen**

- **Response for \( \beta_L < 1 \)**
- **Response for \( \beta_L > 1 \)**
8. Performance test

\( \beta_L \) can be computed with one of the there following formulas:

\[
\beta_L = \frac{A(0) - A(t_p^-)}{A(0) + A(t_p^-)} \quad \beta_L = \frac{A(t_p^+)}{2A(0) - A(t_p^+)} \quad \beta_L = \frac{A(t_p^+)}{2A(t_p^-) + A(t_p^+)}
\]

Step 2. Energy decay right after the RF-pulse is switched off

\[
W(t) = W(t_p^+) e^{-\frac{\omega_0 (t - t_p^+)}{Q_L}}
\]

and measuring the input and transmitted power \((P_{in} \text{ and } P_{tran})\), one obtains:

\[
Q_0 = Q_L (1 + \beta_L)(1 + \frac{P_{tran}}{P_{in} - P_{tran}})
\]
The mechanical design of a cavity follows its RF design:

- Lorentz Force Detuning
- Mechanical Resonances

Lorentz Force Detuning

\[ P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4} \]

\( E \) and \( H \) at \( E_{acc} = 25 \text{ MV/m} \) in TESLA inner-cup
9. Mechanical Design

Surface deformation without and with stiffening ring (courtesy of I. Bonin, FERMI)

Essential for the operation of a pulsed accelerator $\Delta f = k_L E_{acc}^2$

$k_L = -1 \text{ Hz/(MV/m)}^2$
9. Mechanical Design

Mechanical Resonances of a multi-cell cavity

TESLA structure

60 Hz

Transverse modes

152 Hz

Longitudinal mode

250 Hz

The mechanical resonances modulate frequency of the accelerating mode. Sources of their excitation: vacuum pumps, ground vibrations…
The mechanical resonances modulate frequency of the accelerating mode. Sources of their excitation: vacuum pumps, ground vibrations…

Mechanical vibrations cause modulation of the resonant frequency.

$$\Delta f_{3db} \equiv \frac{1.3 \cdot 10^9 \text{Hz}}{2 \cdot 10^7} = 65\text{Hz}$$

Additional RF-power needed to compensate for the shift of resonant frequency from 1.3GHz.

2. Proceedings of all SRF Workshops; http://accelconf.web.cern.ch/accelconf/

3. TESLA TDR, DESY-Report 2003