Cavity Basics

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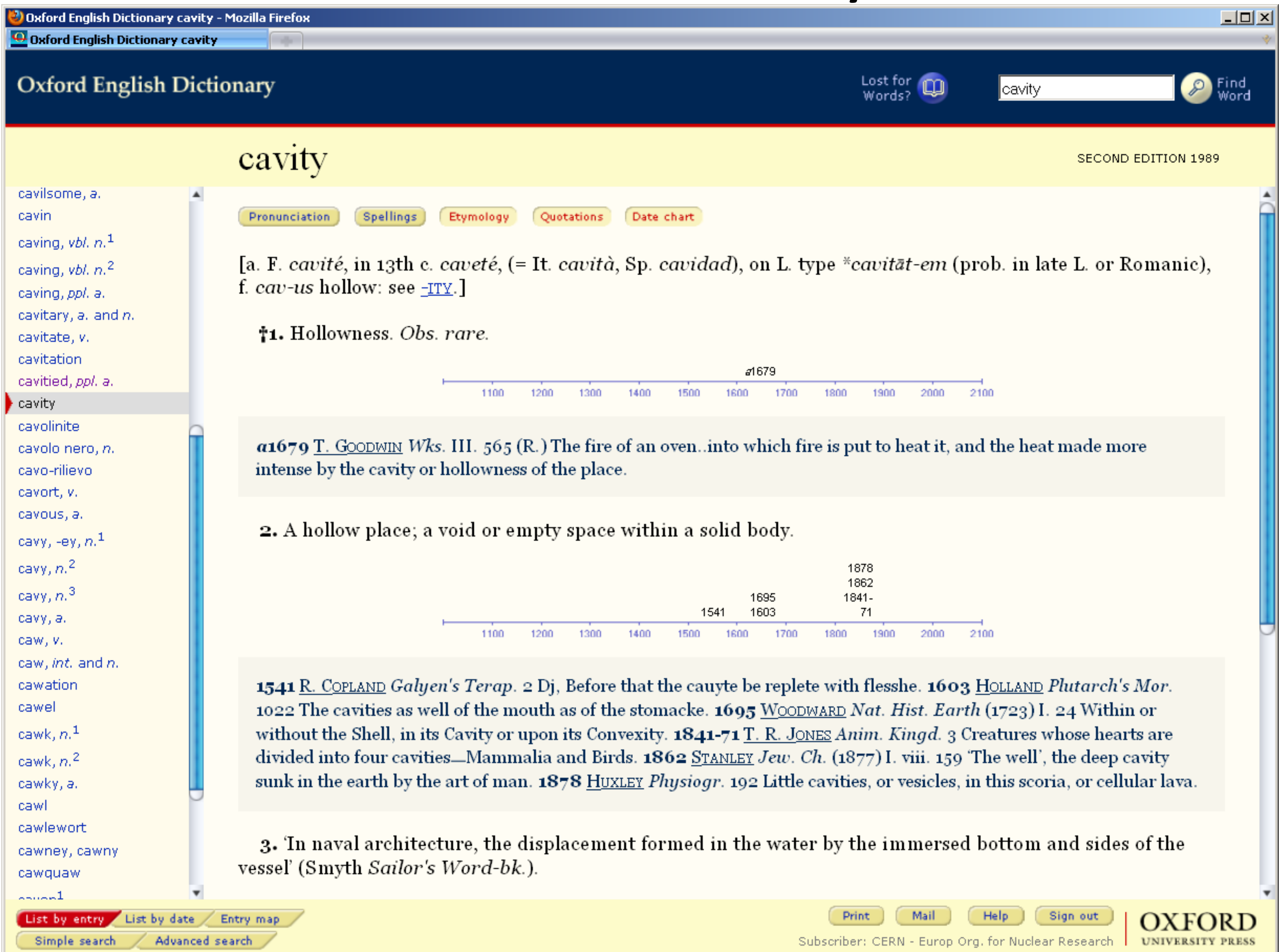
CERN BE-RF

What is a cavity?


Oxford English Dictionary cavity - Mozilla Firefox

Oxford English Dictionary

cavity

SECOND EDITION 1989

Pronunciation | Spellings | Etymology | Quotations | Date chart

[a. F. *cavité*, in 13th c. *caveté*, (= It. *cavità*, Sp. *cavidad*), on L. type **cavitāt-em* (prob. in late L. or Romanic), f. *cav-us* hollow: see -ITY.]

†1. Hollowness. *Obs. rare.*

a1679 T. GOODWIN *Wks.* III. 565 (R.) The fire of an oven..into which fire is put to heat it, and the heat made more intense by the cavity or hollowness of the place.

2. A hollow place; a void or empty space within a solid body.

1541 R. COPLAND *Galyen's Terap.* 2 Dj, Before that the cauyte be replete with flesshe. **1603** HOLLAND *Plutarch's Mor.* 1022 The cavities as well of the mouth as of the stomacke. **1695** WOODWARD *Nat. Hist. Earth* (1723) I. 24 Within or without the Shell, in its Cavity or upon its Convexity. **1841-71** T. R. JONES *Anim. Kingd.* 3 Creatures whose hearts are divided into four cavities—Mammalia and Birds. **1862** STANLEY *Jew. Ch.* (1877) I. viii. 159 'The well', the deep cavity sunk in the earth by the art of man. **1878** HUXLEY *Physiogr.* 192 Little cavities, or vesicles, in this scoria, or cellular lava.

3. 'In naval architecture, the displacement formed in the water by the immersed bottom and sides of the vessel' (Smyth *Sailor's Word-bk.*).

Lorentz force

A charged particle moving with velocity $\vec{v} = \frac{\vec{p}}{m\gamma}$ through an electromagnetic field experiences a force

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

The total energy of this particle is $W = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2} = \gamma mc^2$, the kinetic energy is $W_{kin} = mc^2\left(\gamma - 1\right)$

The role of acceleration is to increase the particle energy!

Change of W by differentiation:

$$W\mathrm{d}W = c^2\vec{p}\cdot\mathrm{d}\vec{p} = qc^2\vec{p}\cdot\left(\vec{E} + \vec{v}\times\vec{B}\right)\mathrm{d}t = qc^2\vec{p}\cdot\vec{E}\mathrm{d}t$$

$$\mathrm{d}W = q\vec{v}\cdot\vec{E}\mathrm{d}t$$

Note: **Only the electric field can change the particle energy!**

Maxwell's equations

The electromagnetic fields inside the "hollow place" obey these equations:

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \cdot \vec{E} = 0$$

With the curl of the 3rd, the time derivative of the 1st equation and the vector identity

$$\nabla \times \nabla \times \vec{E} \equiv \nabla\nabla \cdot \vec{E} - \Delta\vec{E}$$

this set of equations can be brought in the form

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0$$

which is the Laplace equation in 4 dimensions.

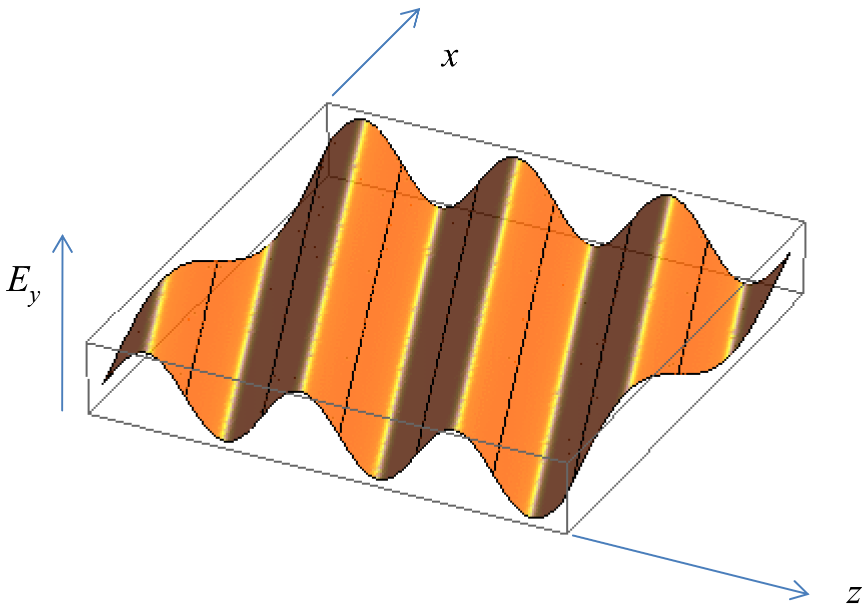
With the boundaries of the "solid body" around it (the cavity walls), there exist eigensolutions of the cavity at certain frequencies (eigenfrequencies).

Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$
$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$

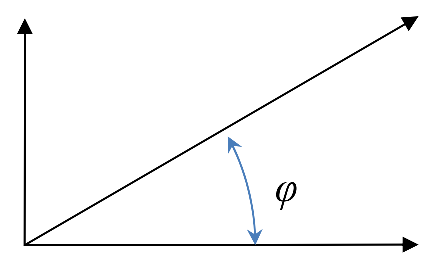
$$\vec{k}\cdot\vec{r} = \frac{\omega}{c}\left(\cos(\varphi)z + \sin(\varphi)x\right)$$

Wave vector $\vec{k}$:
the direction of $\vec{k}$ is the direction of propagation,
the length of $\vec{k}$ is the phase shift per unit length.
$\vec{k}$ behaves like a vector.



$$k_\perp = \frac{\omega_c}{c}$$

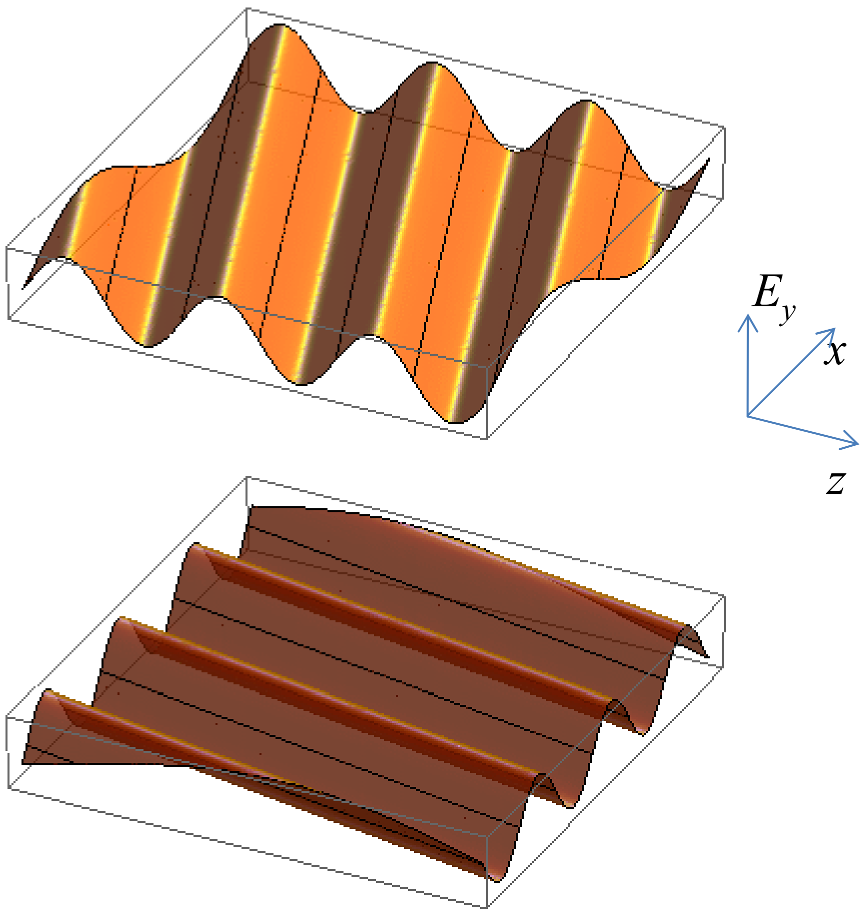
$$k = \frac{\omega}{c}$$



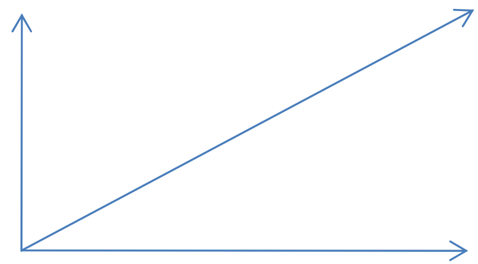
$$k_z = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

Wave length, phase velocity

The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\,\lambda_z$



$$k_\perp = \frac{\omega_c}{c}$$



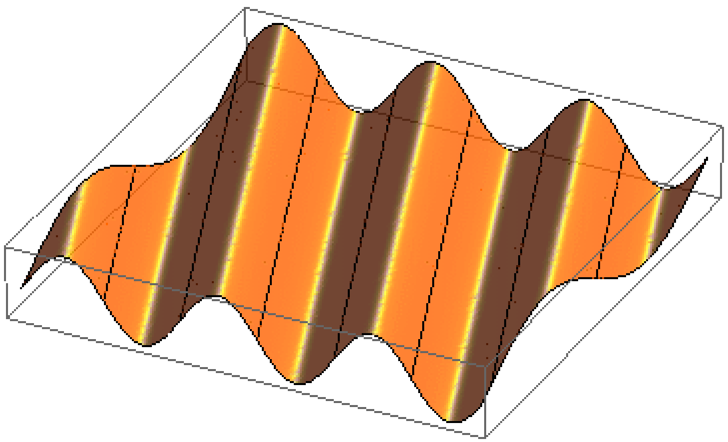
$$k = \frac{\omega}{c}$$

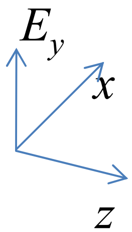
$$k_\perp = \frac{\omega_c}{c}$$

$$k = \frac{\omega}{c}$$

$$k_z = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

Superposition of 2 homogeneous plane waves





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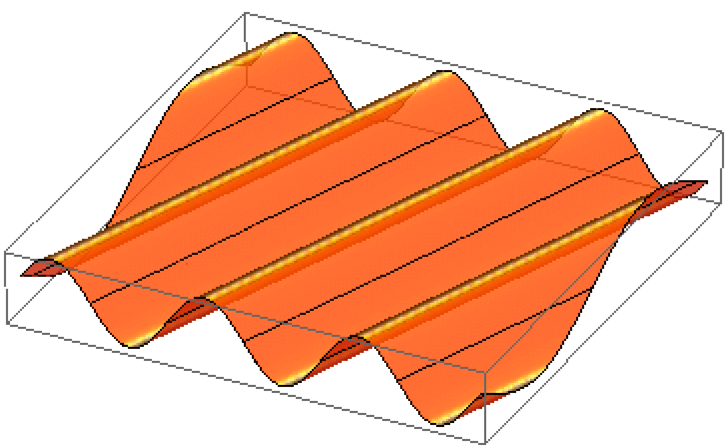

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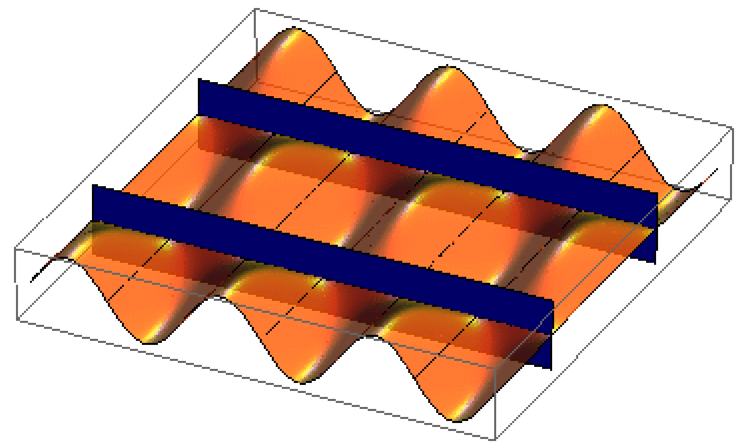


Metallic walls may be inserted where $E_y = 0$ without perturbing the fields.

Note the standing wave in *x*-direction!

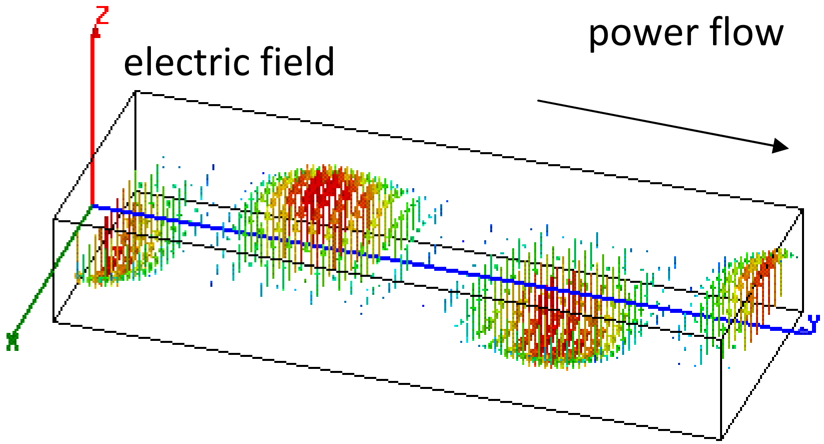
This way one gets a hollow rectangular waveguide

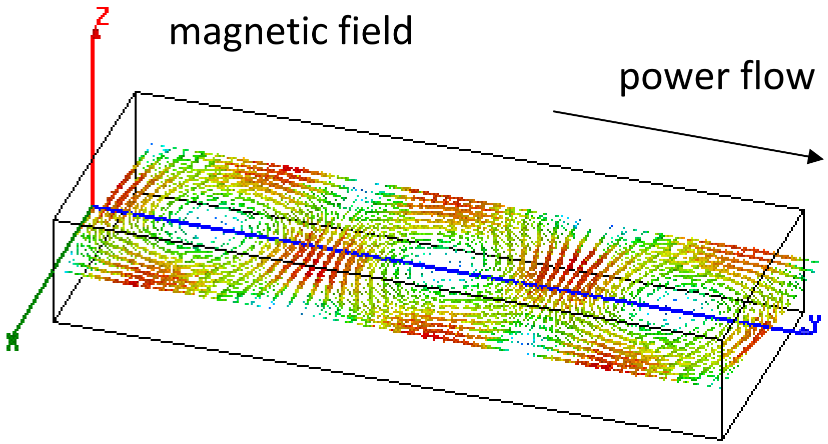
Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

Example: "S-band" : 2.6 GHz ... 3.95 GHz,
Waveguide type WR284 (2.84" wide),
dimensions: 72.14 mm x 34.04 mm.
Operted at f = 3 GHz.

power flow: $$\frac{1}{2}\mathrm{Re}\left\{ \iint_{\substack{\text{cross}\\ \text{section}}} \vec{E} \times \vec{H}^* \cdot \mathrm{d}\vec{A} \right\}$$

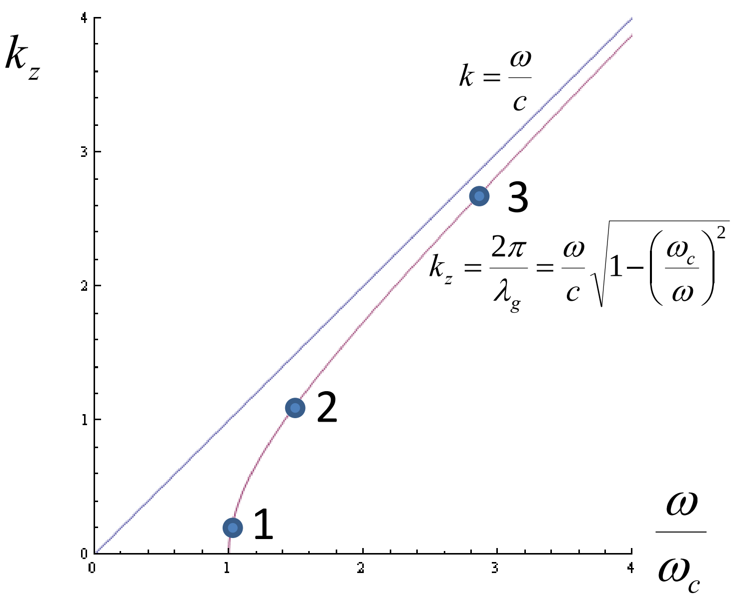




Waveguide dispersion

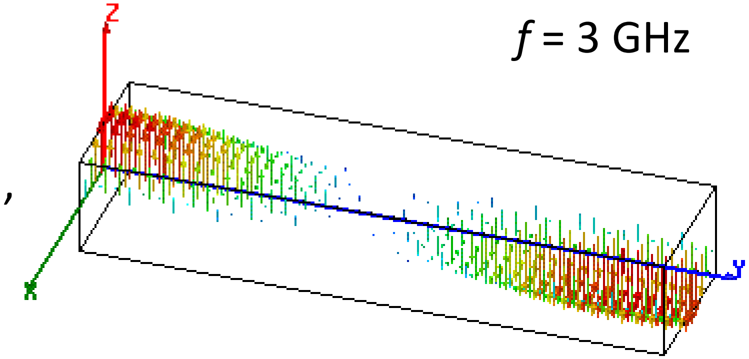
What happens with different waveguide dimensions (different width *a*)?

f = 3 GHz

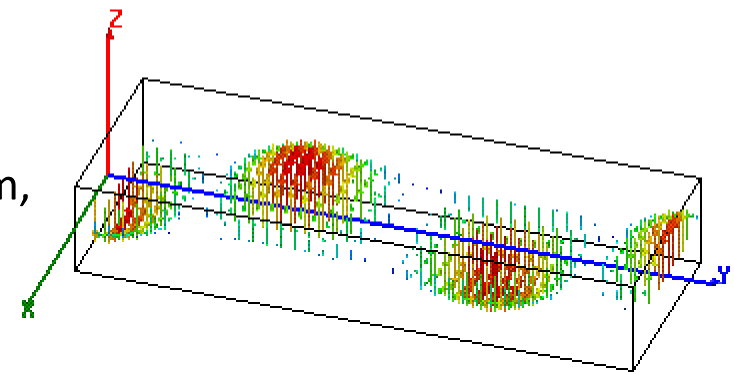


cutoff $f_c = \frac{c}{2a}$

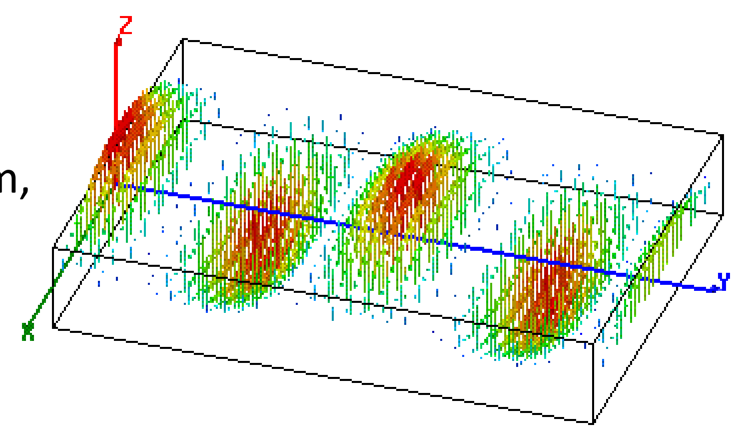
1:
a = 52 mm,
f/f_c = 1.04



2:
a = 72.14 mm,
f/f_c = 1.44

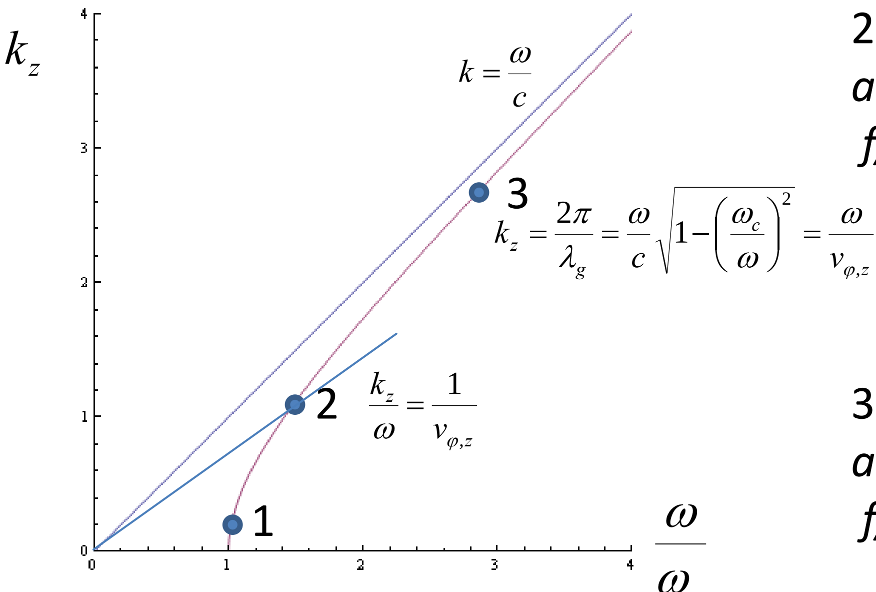


3:
a = 144.3 mm,
f/f_c = 2.88



Phase velocity

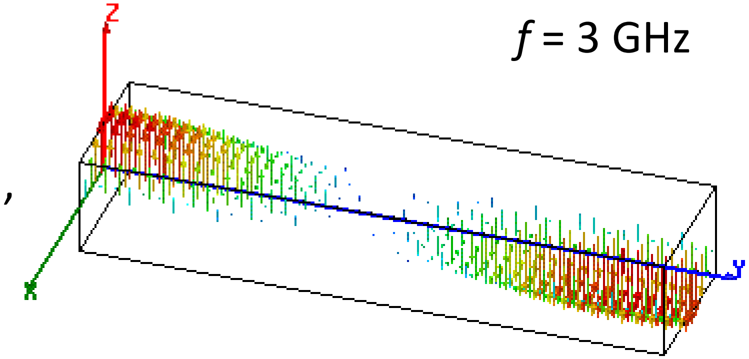
The phase velocity is the speed with which the crest or a zero-crossing travels in *z*-direction.
Note on the three animations on the right that, at constant *f*, it is $\propto \lambda_g$.
Note that at $f = f_c$, $v_{\varphi,z} = \infty$!
With $f \to \infty$, $v_{\varphi,z} \to c$!



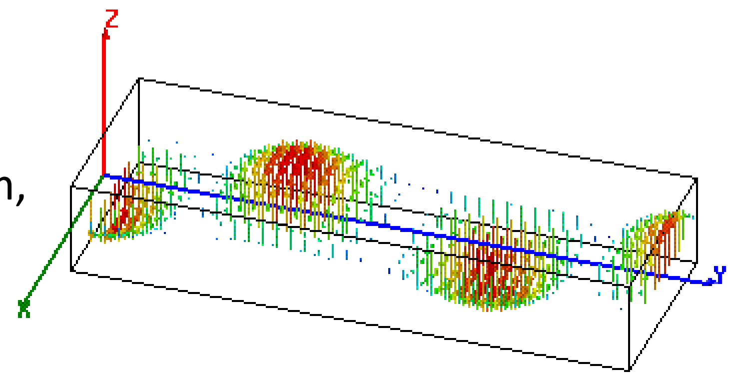
cutoff $f_c = \frac{c}{2a}$

f = 3 GHz

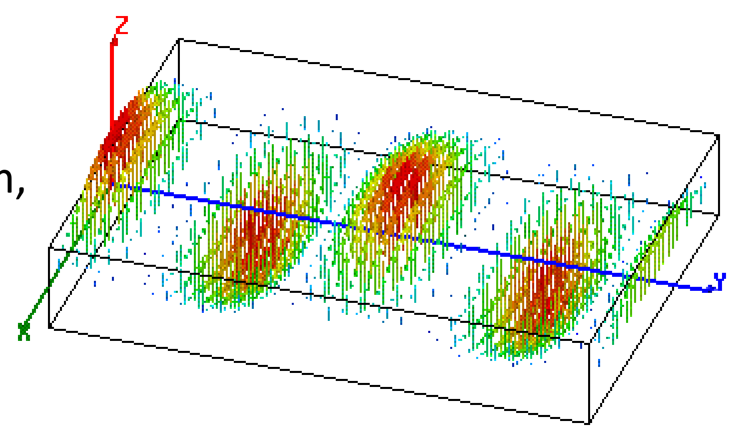
1:
a = 52 mm,
f/f_c = 1.04



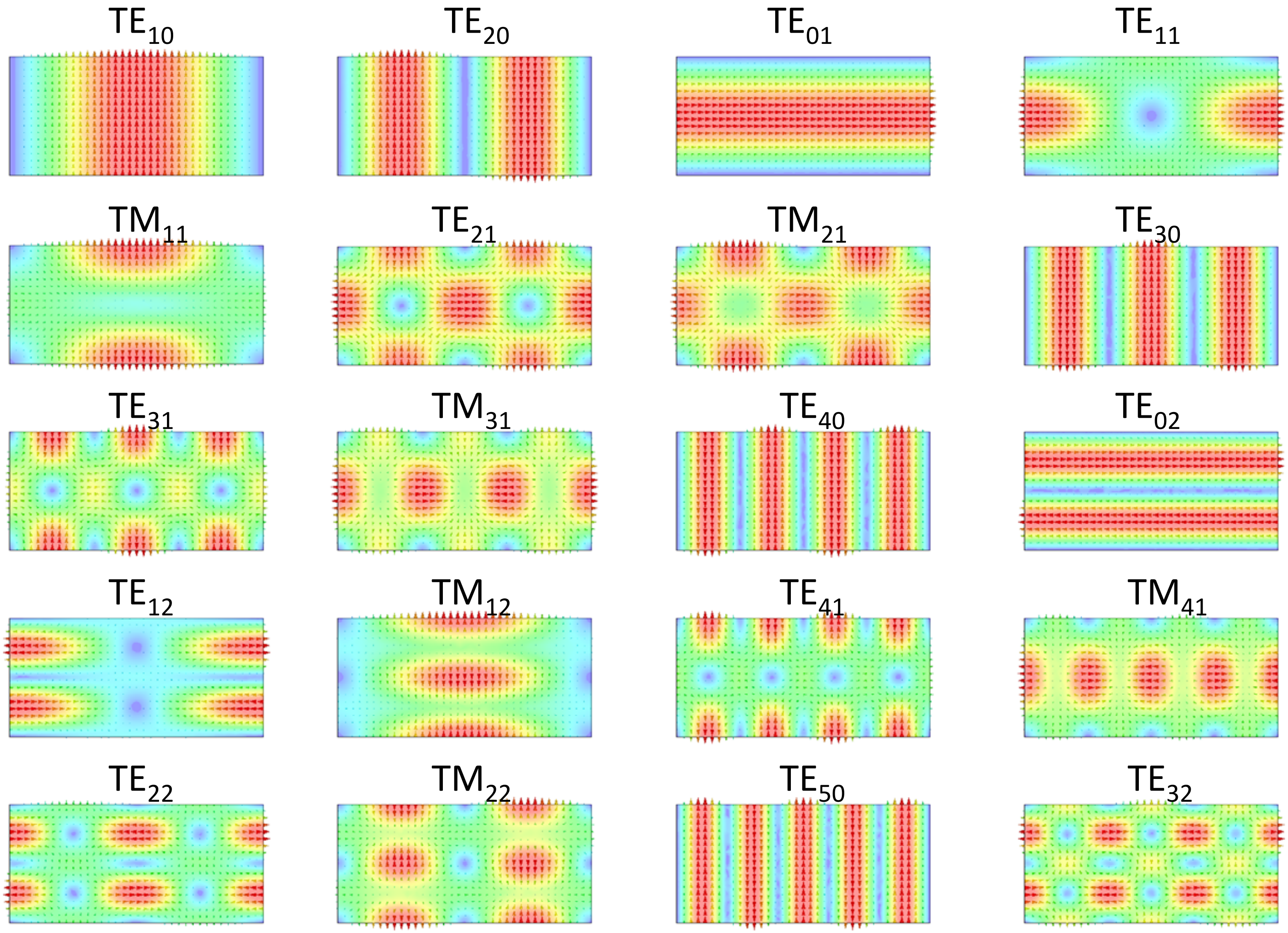
2:
a = 72.14 mm,
f/f_c = 1.44



3:
a = 144.3 mm,
f/f_c = 2.88



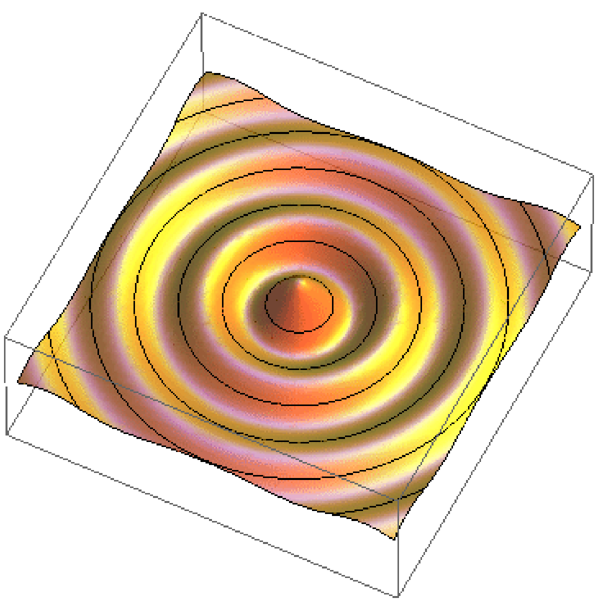
Rectangular waveguide modes



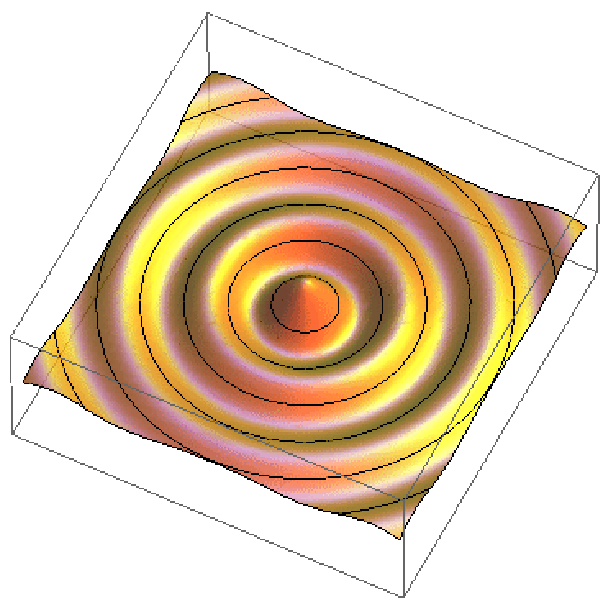
plotted: *E*-field

Radial waves

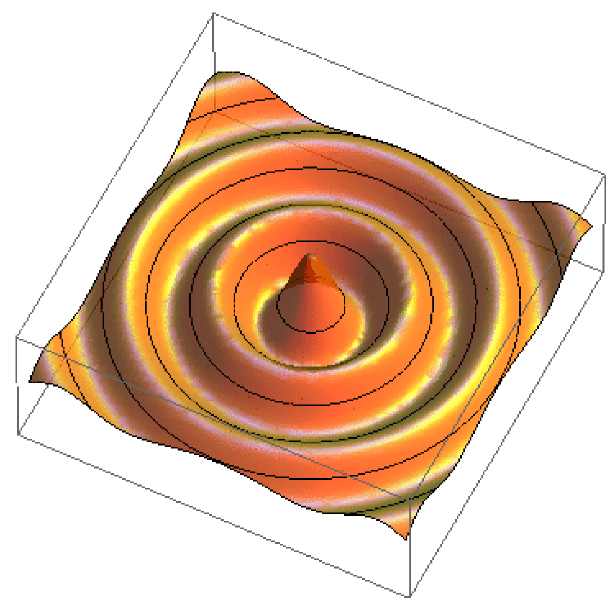
Also radial waves may be interpreted as superpositions of plane waves.
The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho)\cos(n\varphi)$$



$$E_z \propto H_n^{(1)}(k_\rho \rho)\cos(n\varphi)$$



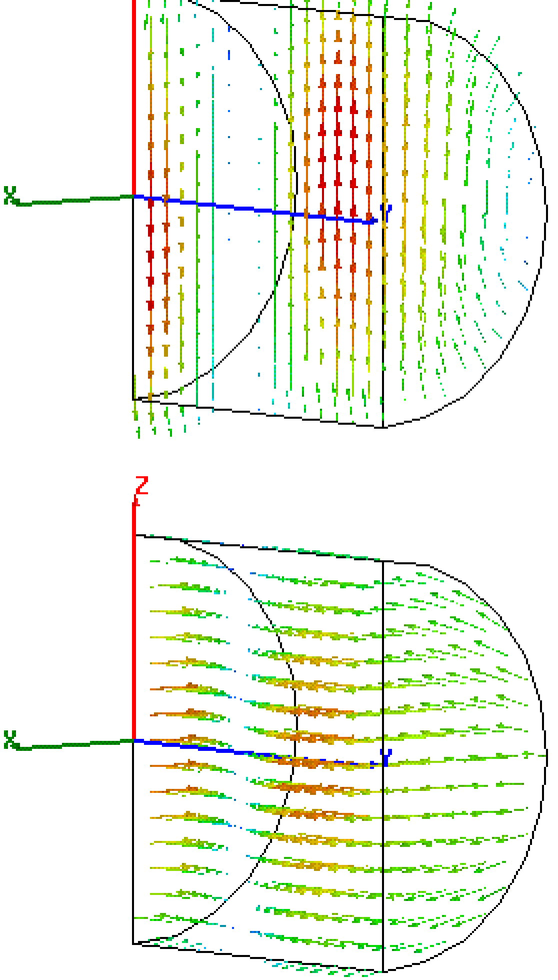
$$E_z \propto J_n(k_\rho \rho)\cos(n\varphi)$$

Round waveguide

$f/f_c = 1.44$

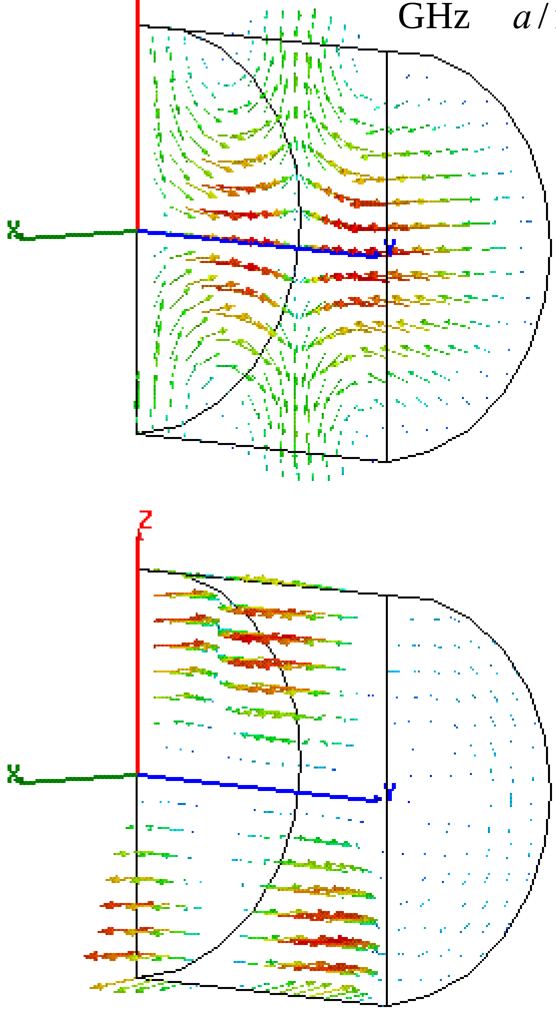
TE_{11} – fundamental

$$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}}$$



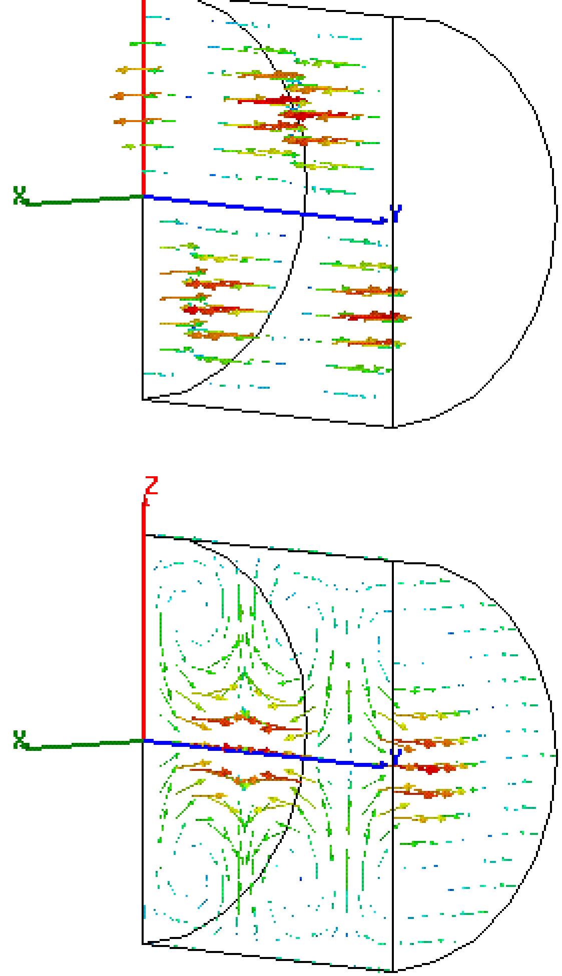
TM_{01} – axial field

$$\frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}}$$

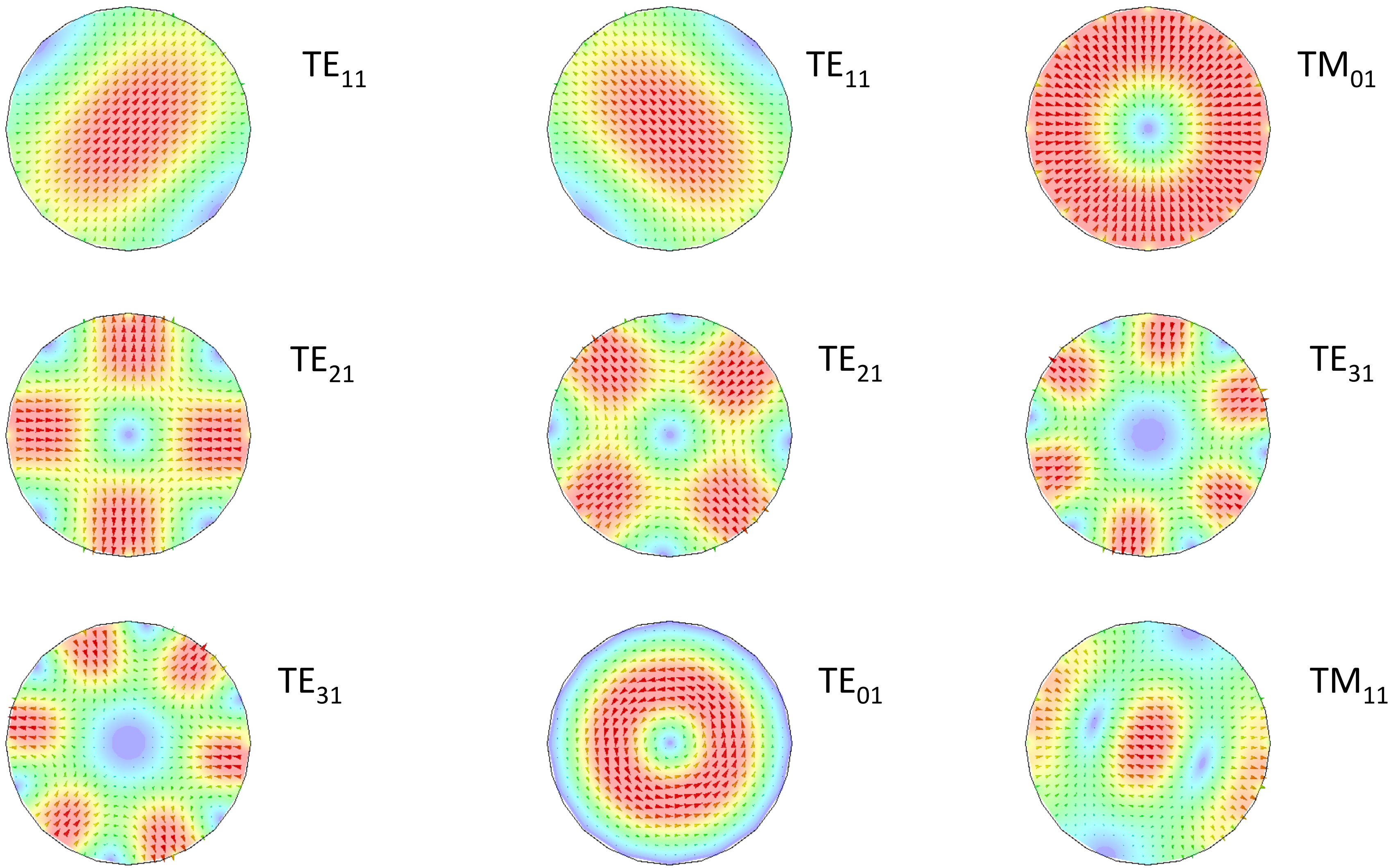


TE_{01} – low loss

$$\frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$$



Circular waveguide modes



plotted: *E*-field

General waveguide equations:

Transverse wave equation (membrane equation): $$\Delta T + \left(\frac{\omega_c}{c}\right)^2 T = 0$$

	TE (or H) modes	TM (or E) modes
boundary condition:	$\vec{n} \cdot \nabla T = 0$	$T = 0$

longitudinal wave equations (transmission line equations):

$$\frac{\mathrm{d}U(z)}{\mathrm{d}z} + \gamma Z_0 I(z) = 0$$

$$\frac{\mathrm{d}I(z)}{\mathrm{d}z} + \frac{\gamma}{Z_0} U(z) = 0$$

propagation constant:

$$\gamma = \mathrm{j}\frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

	TE (or H) modes	TM (or E) modes
characteristic impedance:	$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma}$	$Z_0 = \frac{\gamma}{\mathrm{j}\omega\varepsilon}$
ortho-normal eigenvectors:	$\vec{e} = \vec{u}_z \times \nabla T$	$\vec{e} = -\nabla T$

transverse fields:

$$\vec{E} = U(z)\vec{e}$$

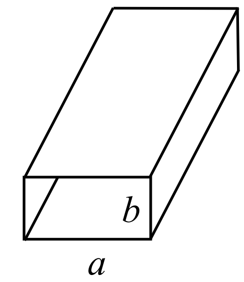
$$\vec{H} = I(z)\vec{u}_z \times \vec{e}$$

	TE (or H) modes	TM (or E) modes
longitudinal field:	$H_z = \left(\frac{\omega_c}{c}\right)^2 \frac{T U(z)}{\mathrm{j}\omega\mu}$	$E_z = \left(\frac{\omega_c}{c}\right)^2 \frac{T I(z)}{\mathrm{j}\omega\varepsilon}$

Rectangular waveguide: transverse eigenfunctions

TE (H) modes: $$T_{mn}^{(H)} = \frac{1}{\pi}\sqrt{\frac{ab\varepsilon_m\varepsilon_n}{(mb)^2+(na)^2}}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

TM (E) modes: $$T_{mn}^{(E)} = \frac{2}{\pi}\sqrt{\frac{ab}{(mb)^2+(na)^2}}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

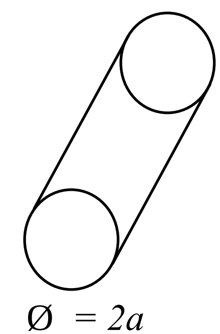


$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

Round waveguide: transverse eigenfunctions

TE (H) modes: $$T_{mn}^{(H)} = \sqrt{\frac{\varepsilon_m}{\pi\left(\chi_{mn}'^2-m^2\right)}}\frac{\mathrm{J}_m\left(\chi_{mn}'\frac{\rho}{a}\right)}{\mathrm{J}_m\left(\chi_{mn}'\right)}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$$

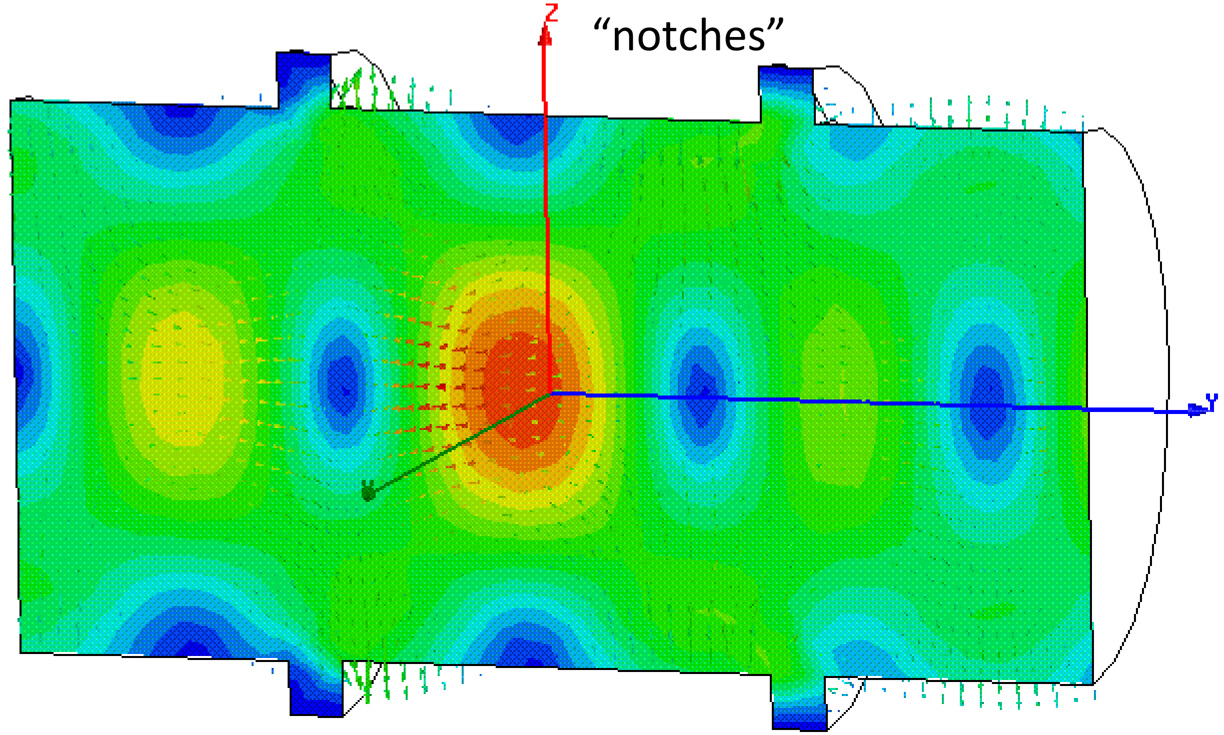
TM (E) modes: $$T_{mn}^{(E)} = \sqrt{\frac{\varepsilon_m}{\pi}}\frac{\mathrm{J}_m\left(\chi_{mn}\frac{\rho}{a}\right)}{\chi_{mn}\,\mathrm{J}_{m-1}\left(\chi_{mn}\right)}\begin{Bmatrix}\sin(m\varphi)\\ \cos(m\varphi)\end{Bmatrix}$$



$$\frac{\omega_c}{c} = \frac{\chi_{mn}}{a}$$

where $$\varepsilon_i = \begin{cases}1 & for \quad i=0\\ 2 & for \quad i\neq 0\end{cases}$$

Waveguide perturbed by notches

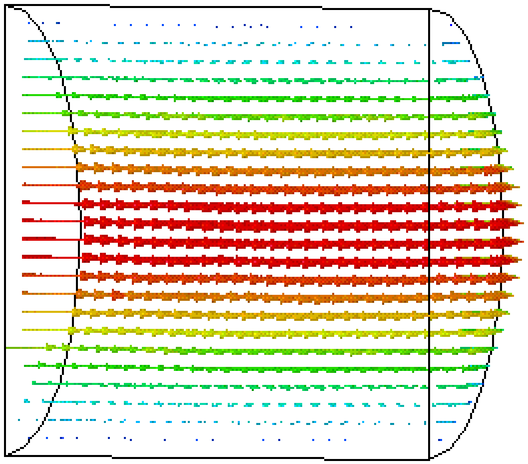


Signal flow chart

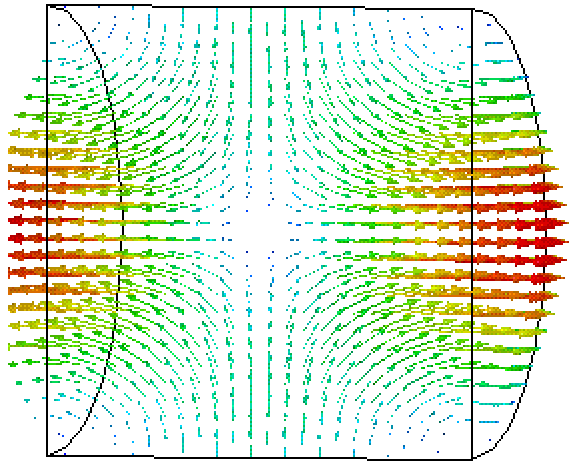
Reflections from notches lead to a superimposed standing wave pattern.
"Trapped mode"

Short-circuited waveguide

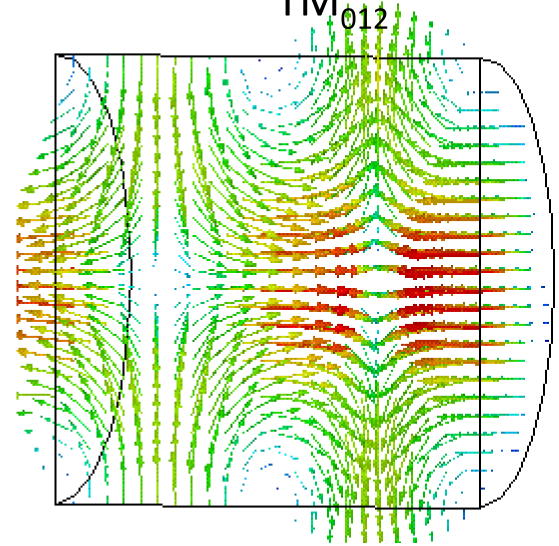
TM_{010} (no axial dependence)

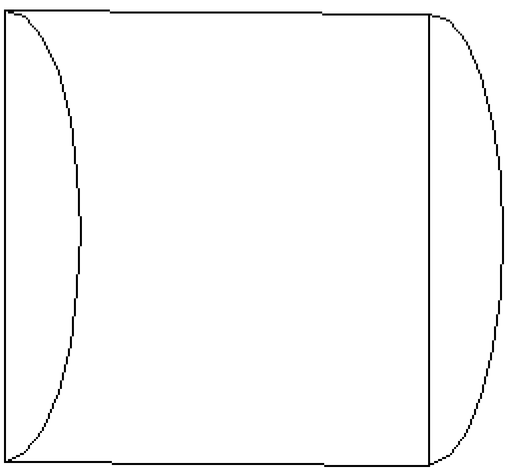


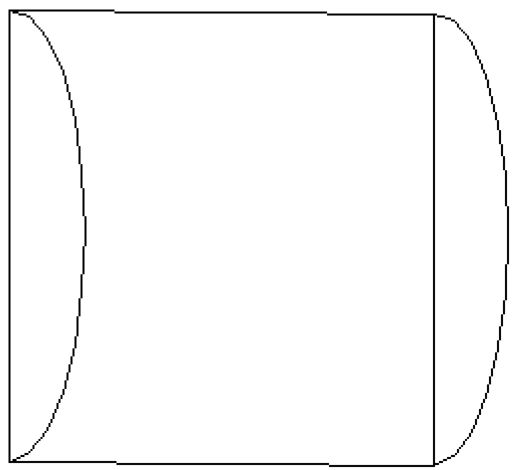
TM_{011}

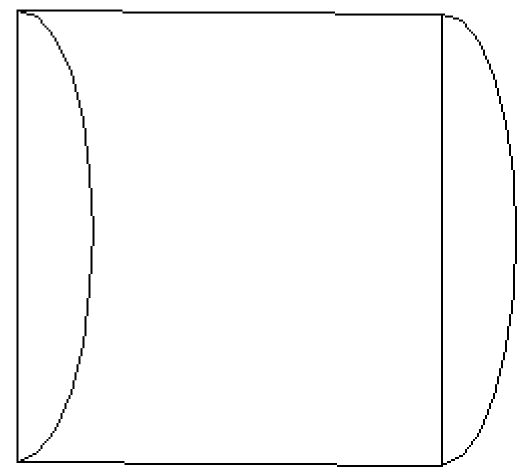


TM_{012}

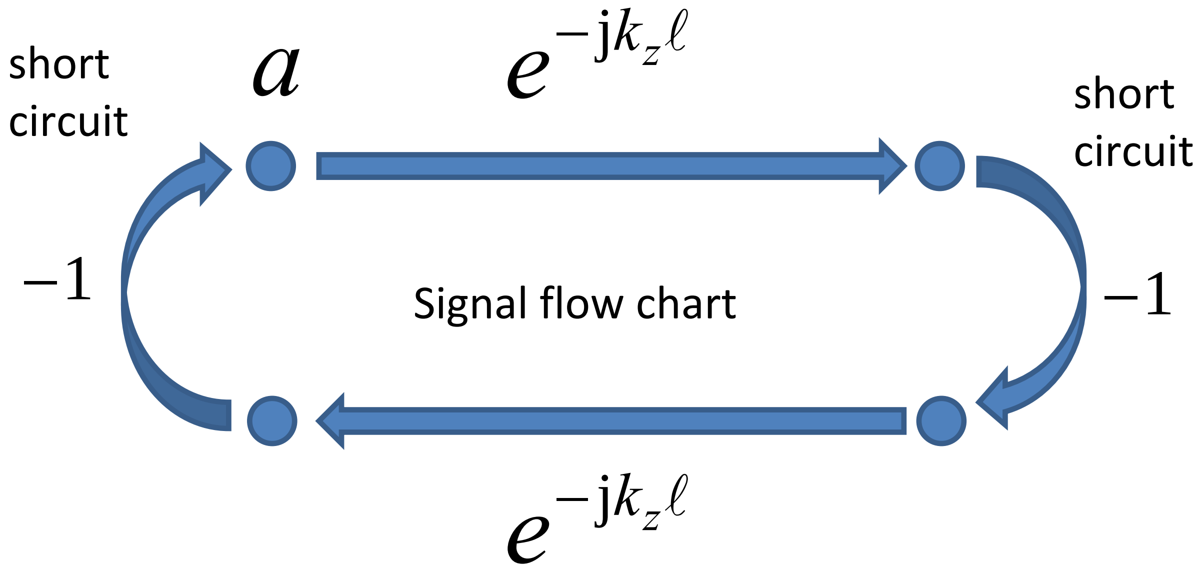








Single WG mode between two shorts



Eigenvalue equation for field amplitude *a*:

$$a = e^{-jk_z 2\ell} a$$

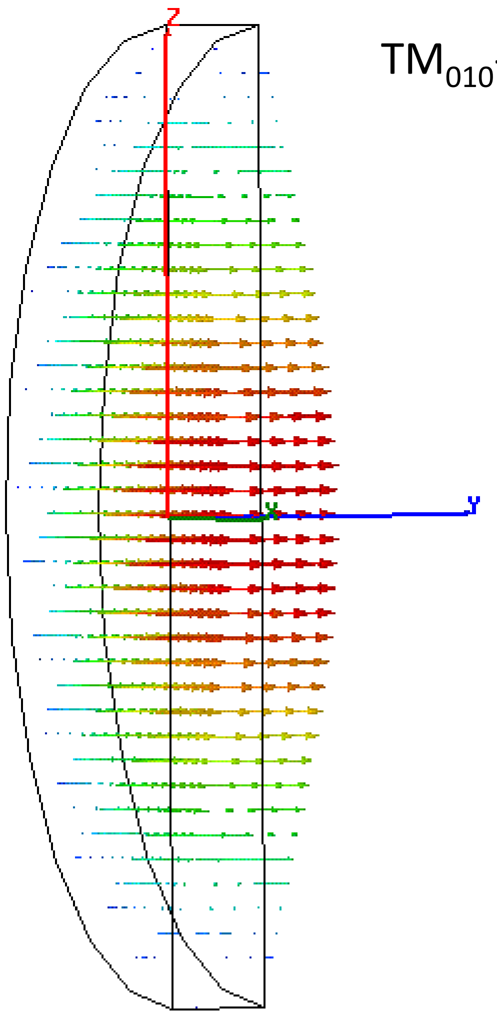
Non-vanishing solutions exist for $2k_z\ell = 2\pi m$:

With $k_z = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(c\frac{m}{2\ell}\right)^2$

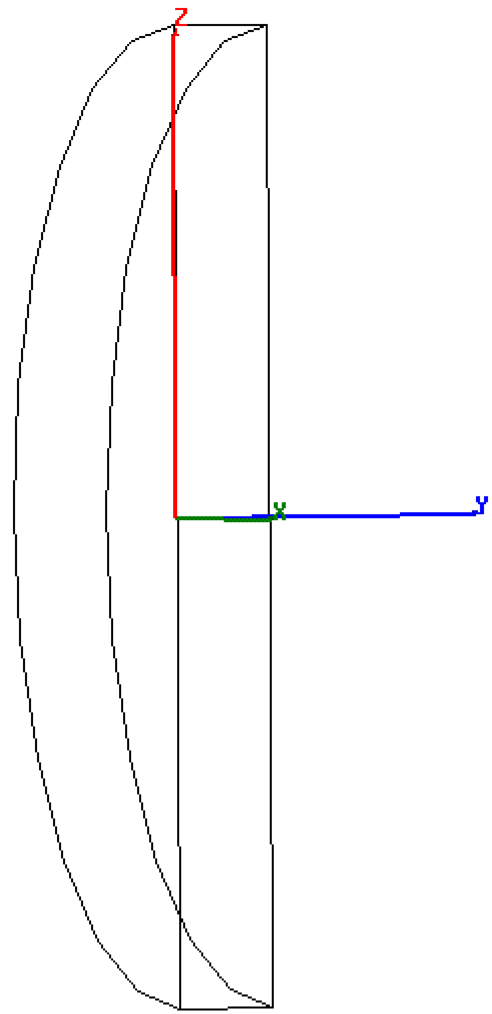
Simple pillbox

(only 1/2 shown)

TM_{010}-mode



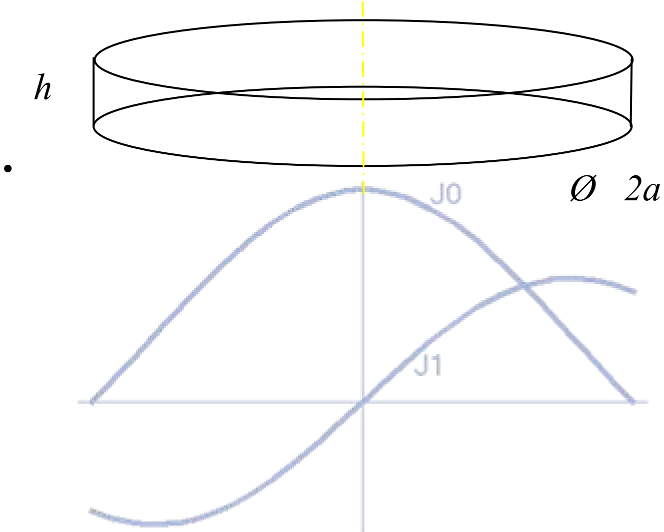
electric field (purely axial)



magnetic field (purely azimuthal)

Pillbox cavity field (w/o beam tube)

$$T(\rho,\varphi)=\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01}J_1\left(\frac{\chi_{01}}{a}\right)}\qquad \chi_{01}=2.40483...$$



The only non-vanishing field components :

$$E_z=\frac{1}{j\omega\varepsilon_0}\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi=\mu_0\sqrt{\frac{1}{\pi}}\frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

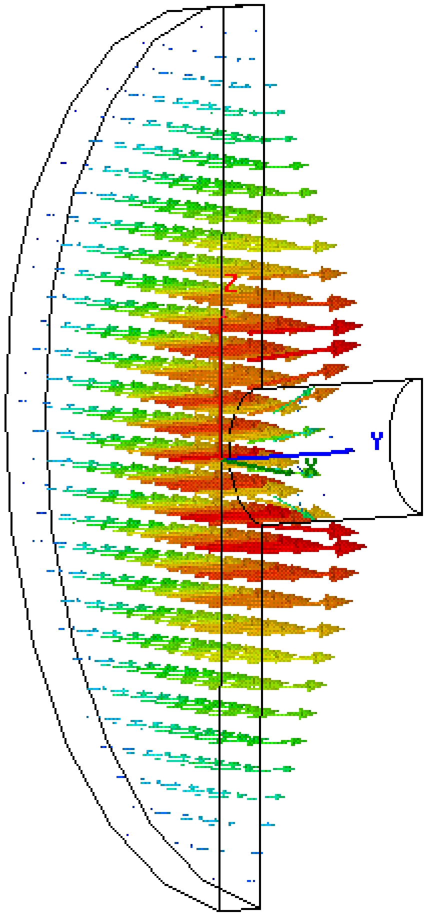
$$\left.\omega_0\right|_{pillbox}=\frac{\chi_{01}\,c}{a}\qquad \eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}=377\,\Omega$$

$$\left.Q\right|_{pillbox}=\frac{\sqrt{2a\,\eta\sigma\chi_{01}}}{2\left(1+\frac{a}{h}\right)}$$

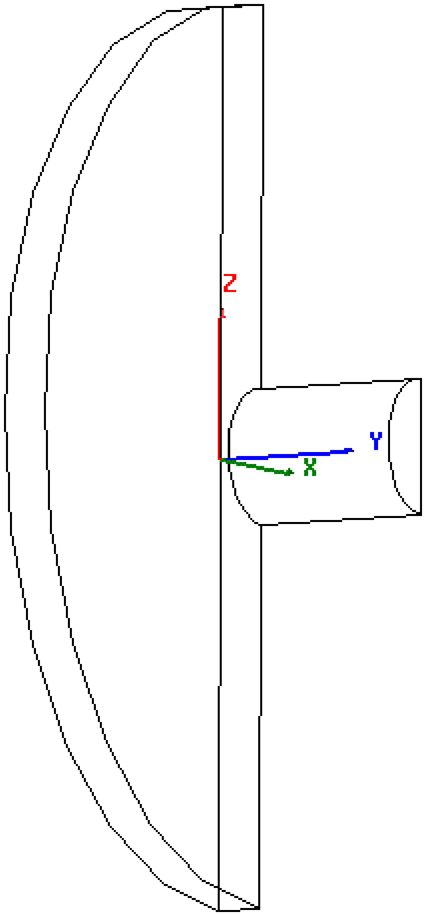
$$\left.\frac{R}{Q}\right|_{pillbox}=\frac{4\eta}{\chi_{01}{}^3\pi J_1^2(\chi_{01})}\frac{\sin^2\left(\frac{\chi_{01}}{2}\frac{h}{a}\right)}{h/a}$$

Pillbox with beam pipe TM_{010}-mode (only 1/4 shown)

One needs a hole for the beam pipe – circular waveguide below cutoff



electric field

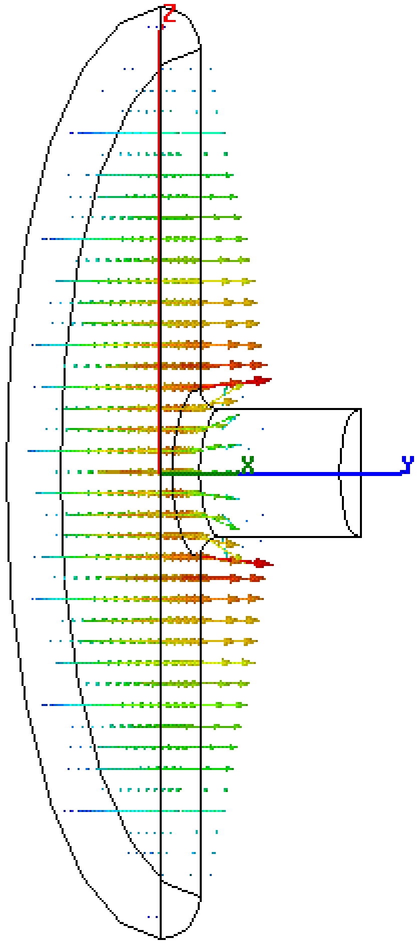


magnetic field

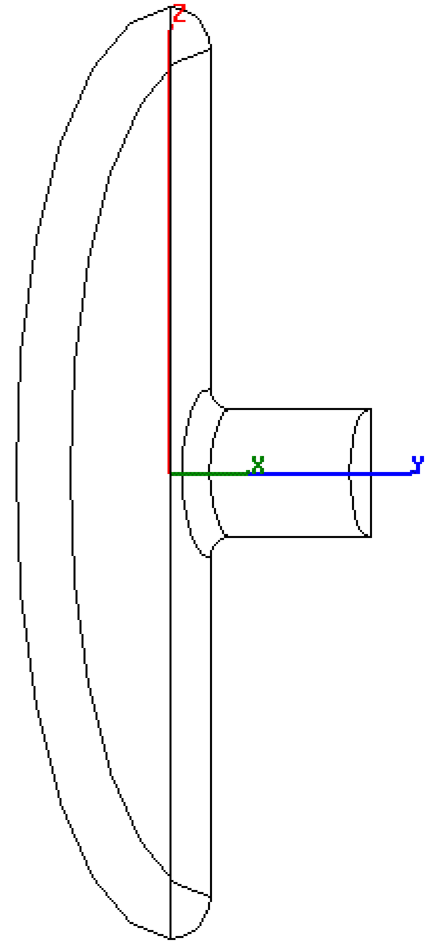
A more practical pillbox cavity

TM_{010}-mode (only 1/4 shown)

Round of sharp edges
(field enhancement!)



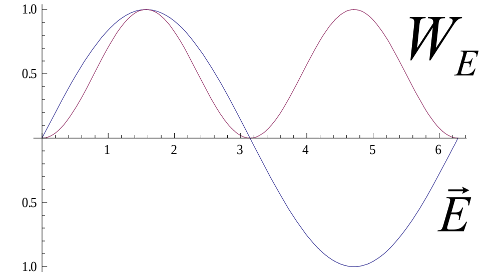
electric field



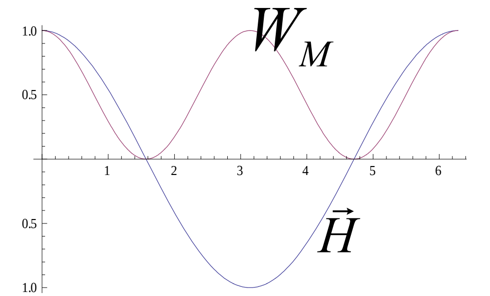
magnetic field

Stored energy

The energy stored in the electric field is $\iiint\limits_{cavity} \frac{\varepsilon}{2}\left|\vec{E}\right|^2 \mathrm{d}V$



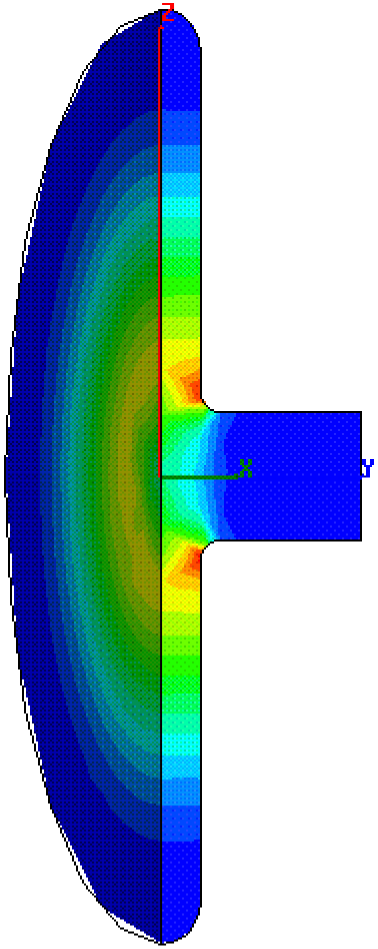
The energy stored in the magnetic field is $\iiint\limits_{cavity} \frac{\mu}{2}\left|\vec{H}\right|^2 \mathrm{d}V$



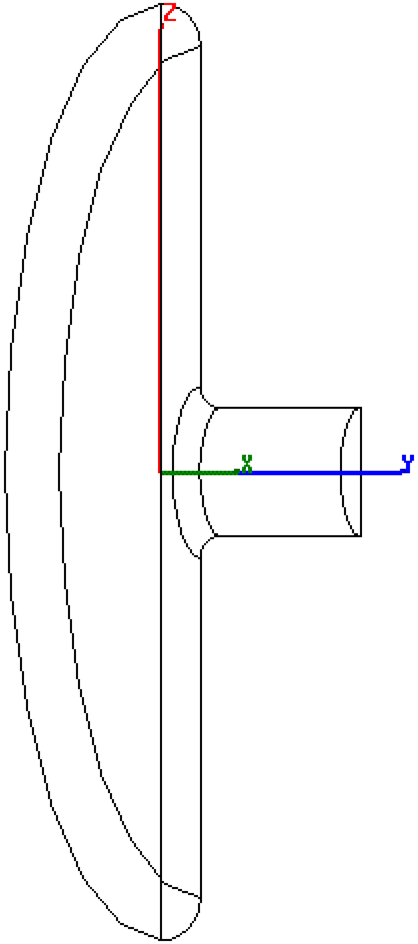
Since $\vec{E}$ and $\vec{H}$ are 90° out of phase, the stored energy continuously swaps from electric energy to magnetic energy. On average, electric and magnetic energy must be equal.
The (imaginary part of the) Poynting vector describes this energy flux.

In steady state, the total stored energy $W = \iiint\limits_{cavity} \left(\frac{\varepsilon}{2}\left|\vec{E}\right|^2 + \frac{\mu}{2}\left|\vec{H}\right|^2 \right) \mathrm{d}V$ is constant in time.

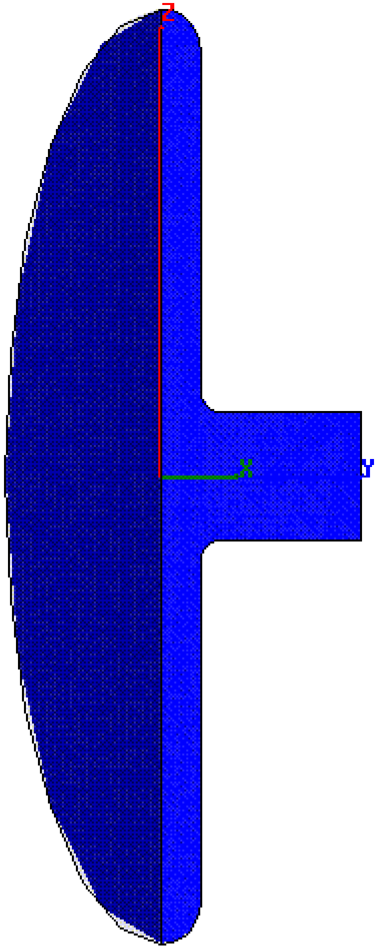
Stored energy & Poynting vector



electric field energy



Poynting vector



magnetic field energy

Losses & Q factor

The losses P_{loss} are proportional to the stored energy W.

The cavity quality factor Q is defined as the ratio $Q = \frac{\omega_0 W}{P_{loss}}$.

In a vacuum cavity, losses are dominated by the ohmic losses due to the finite conductivity of the cavity walls.
If the losses are small, one can calculate them with a **perturbation method**:

- The tangential magnetic field at the surface leads to a surface current.
- This current will see a wall resistance $R_A = \sqrt{\frac{\omega\mu}{2\sigma}}$
- {R_A is related to the skin depth δ by $\delta\sigma R_A = 1$. }
- The cavity losses are given by $P_{loss} = \iint_{wall} R_A |H_t|^2 \mathrm{d}A$
- If other loss mechanisms are present, losses must be added. Consequently, the inverses of the Q 's must be added!

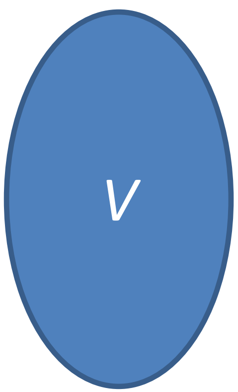
Small boundary perturbation – tuning

Another application of the perturbation method is to analyse the sensitivity to (small) surface geometry perturbations.

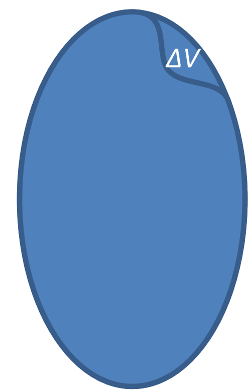
- This is relevant to understand the effect of fabrication tolerances.
- Intentional surface perturbation can be used to tune the cavity.

The basic idea of the perturbation theory is use a known solution (in this case the unperturbed cavity) and assume that the deviation from it is only small. We just used this to calculate the losses (assuming H_t would be that without losses).

The result of this calculation leads to a convenient expression for the detuning:



$$\frac{\omega - \omega_0}{\omega_0} = \frac{\iiint\limits_{\Delta V} \left(\mu |H_0|^2 - \varepsilon |E_0|^2 \right) \mathrm{d}V}{\iiint\limits_{V} \left(\mu |H_0|^2 + \varepsilon |E_0|^2 \right) \mathrm{d}V}$$



unperturbed ω_0

perturbed ω

"Slater-theorem"

Acceleration voltage & *R*-upon-*Q*

I define $V_{acc} = \int E_z e^{\mathrm{j}\frac{\omega}{\beta c}z} \mathrm{d}z$. The exponential factor accounts for the variation of the field while particles with velocity βc are traversing the gap (see next page).

With this definition, V_{acc} is generally complex – this becomes important with more than one gap. For the time being we are only interested in $\left|V_{acc}\right|$

Attention, different definitions are used!

The square of the acceleration voltage is proportional to the stored energy W .

The proportionality constant defines the quantity called *R*-upon-*Q*:

$$\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{2\,\omega_0 W}$$

Attention, also here different definitions are used!

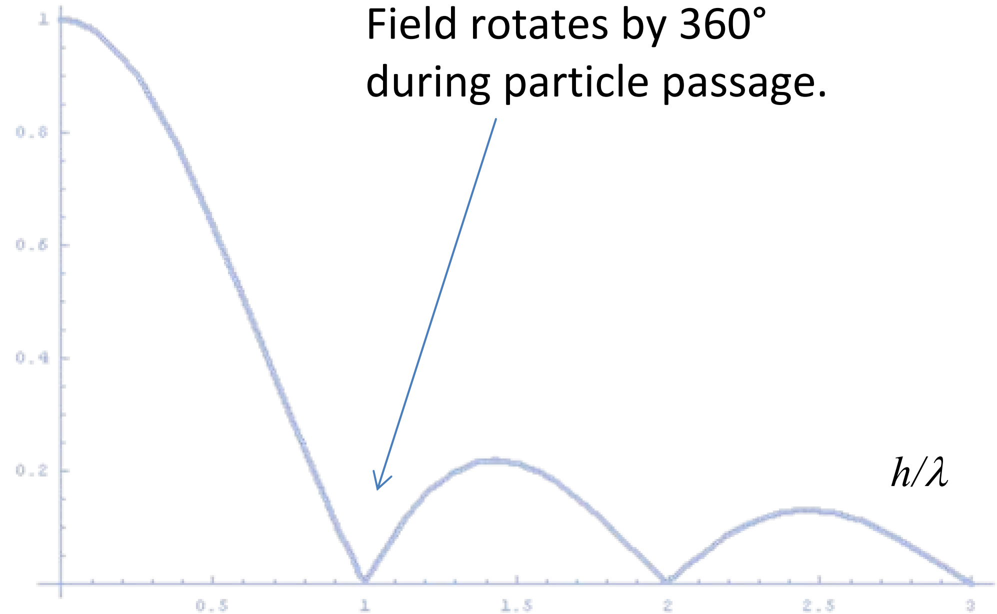
Transit time factor

The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see.

$$TT = \frac{|V_{acc}|}{\left|\int E_z \mathrm{d}z\right|} = \frac{\left|\int E_z e^{\mathrm{j}\frac{\omega}{\beta c}z} \mathrm{d}z\right|}{\left|\int E_z \mathrm{d}z\right|}$$

The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length) *h* is:

$$TT = \sin\left(\frac{\chi_{01} h}{2a}\right) \Big/ \left(\frac{\chi_{01} h}{2a}\right)$$



Shunt impedance

The square of the acceleration voltage is proportional to the power loss P_{loss} .

The proportionality constant defines the quantity “shunt impedance”

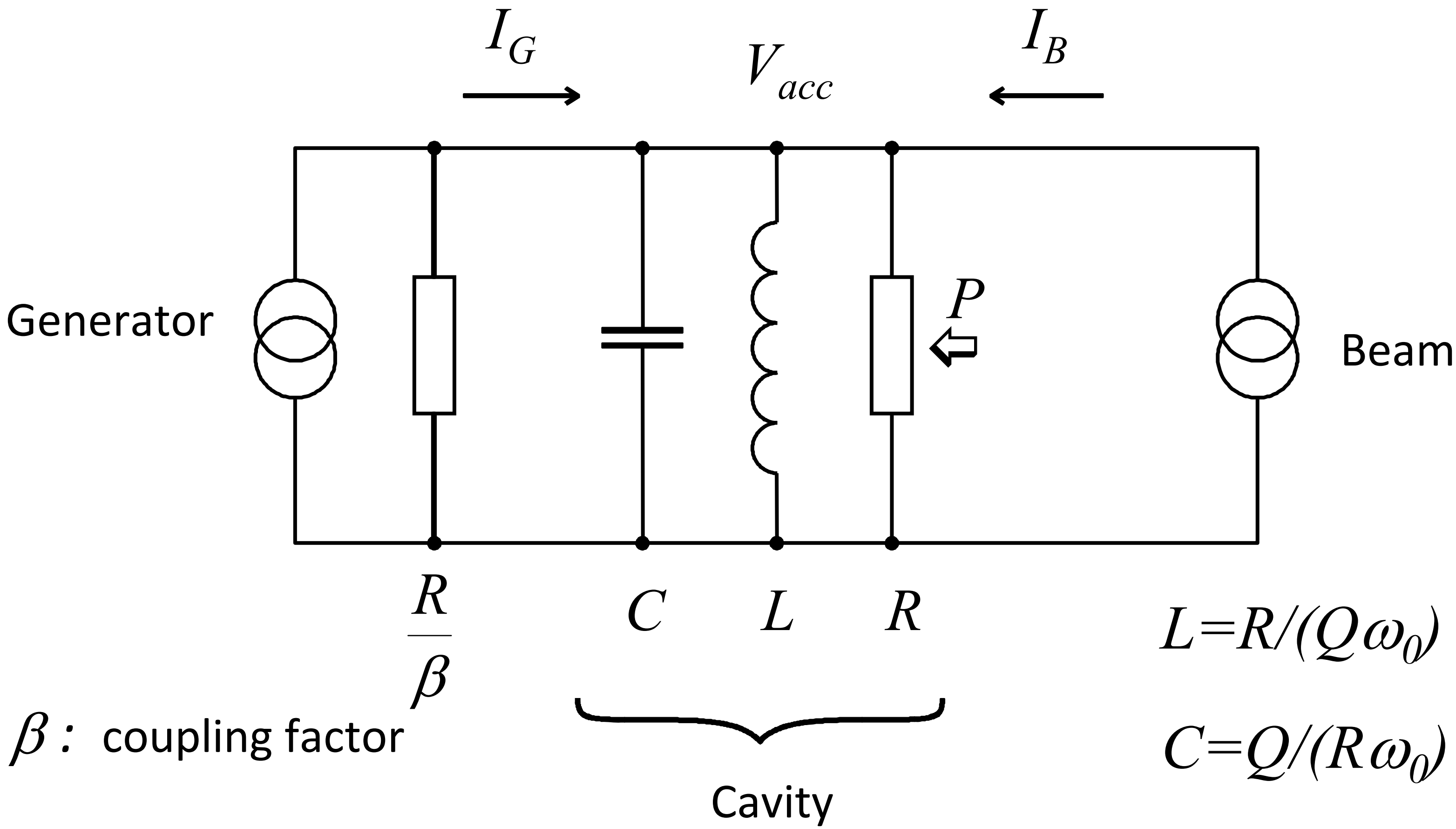
$$R = \frac{\left|V_{acc}\right|^2}{2\,P_{loss}}$$

Attention, also here different definitions are used!

Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.

Equivalent circuit

Simplification: single mode



β : coupling factor

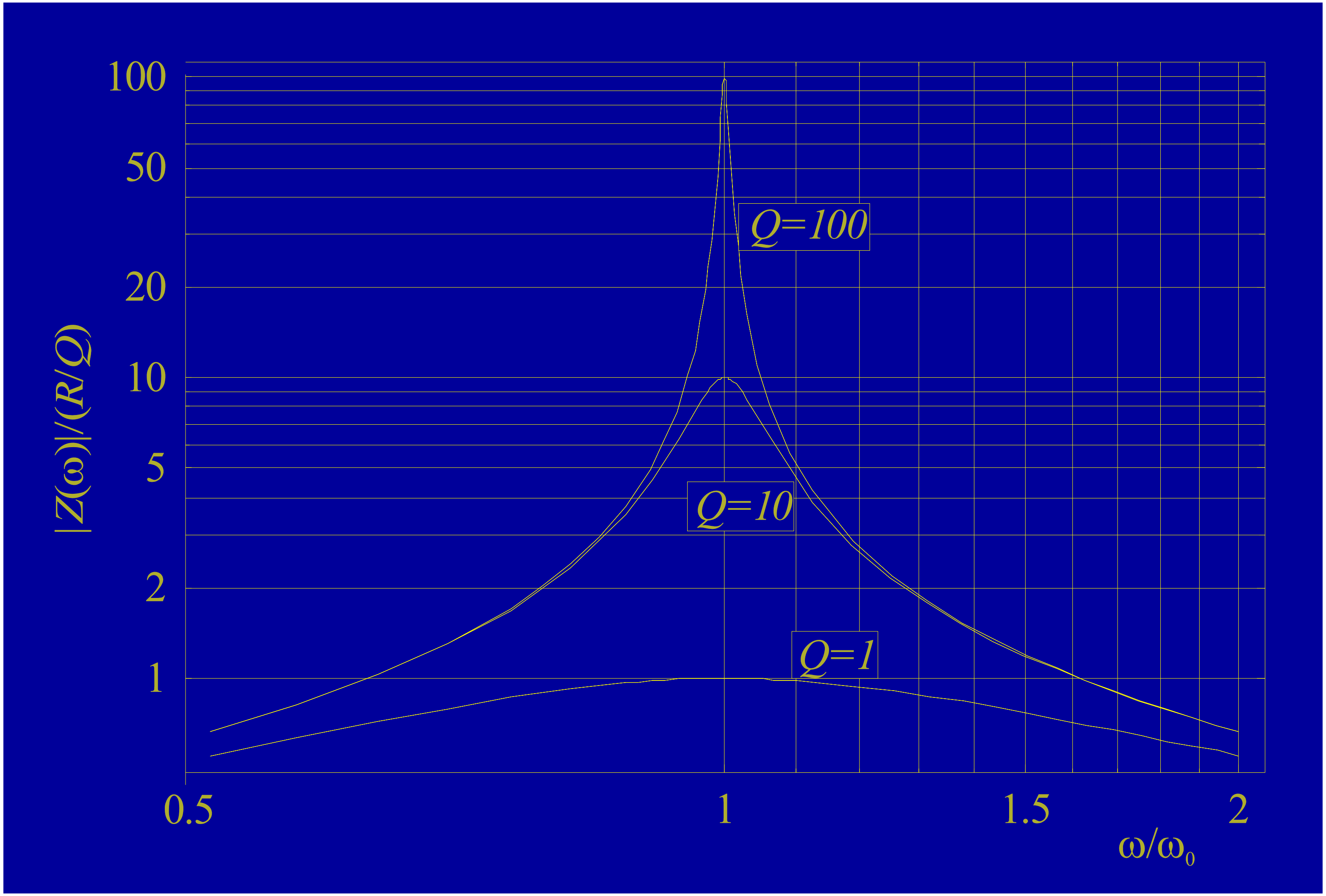
$L=R/(Q\omega_0)$

$C=Q/(R\omega_0)$

R: Shunt impedance

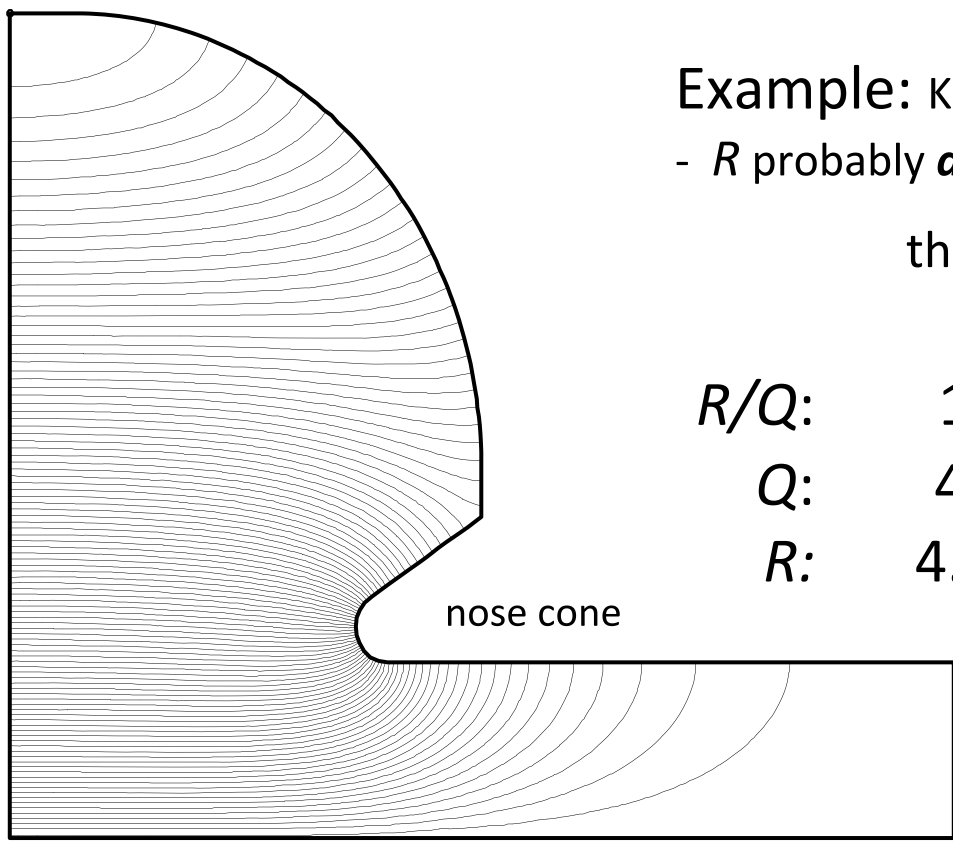
$\sqrt{L/C}$: *R*-upon-*Q*

Resonance



Reentrant cavity

Nose cones increase transit time factor, round outer shape minimizes losses.



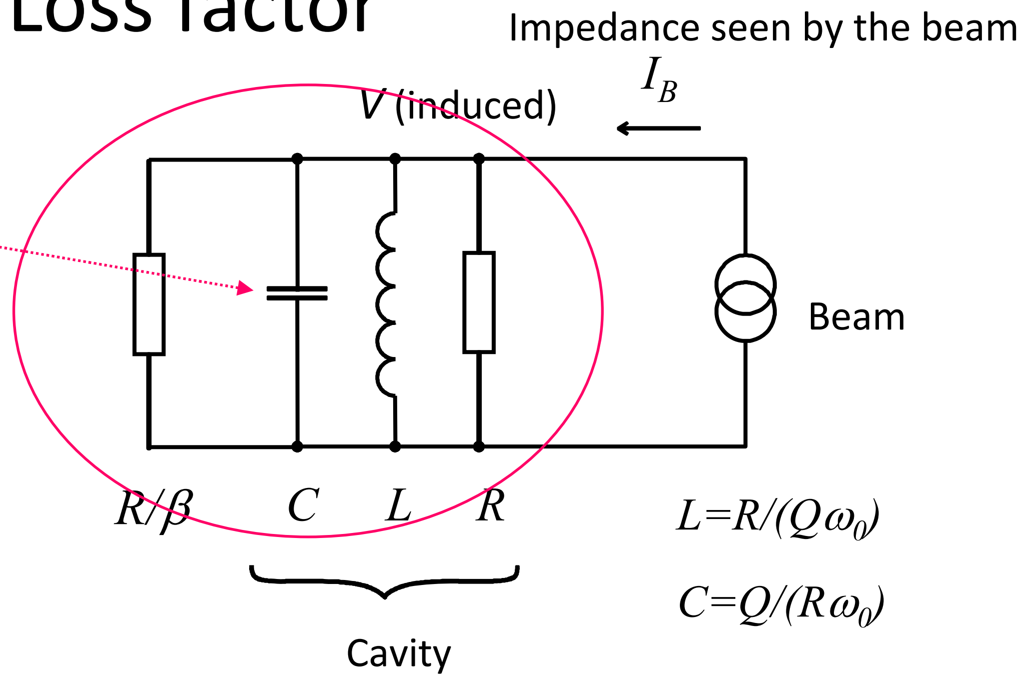
Example: KEK photon factory 500 MHz

- *R* probably ***as good as it gets*** -

	this cavity	optimized pillbox
R/Q:	111 Ω	107.5 Ω
Q:	44270	41630
R:	4.9 MΩ	4.47 MΩ

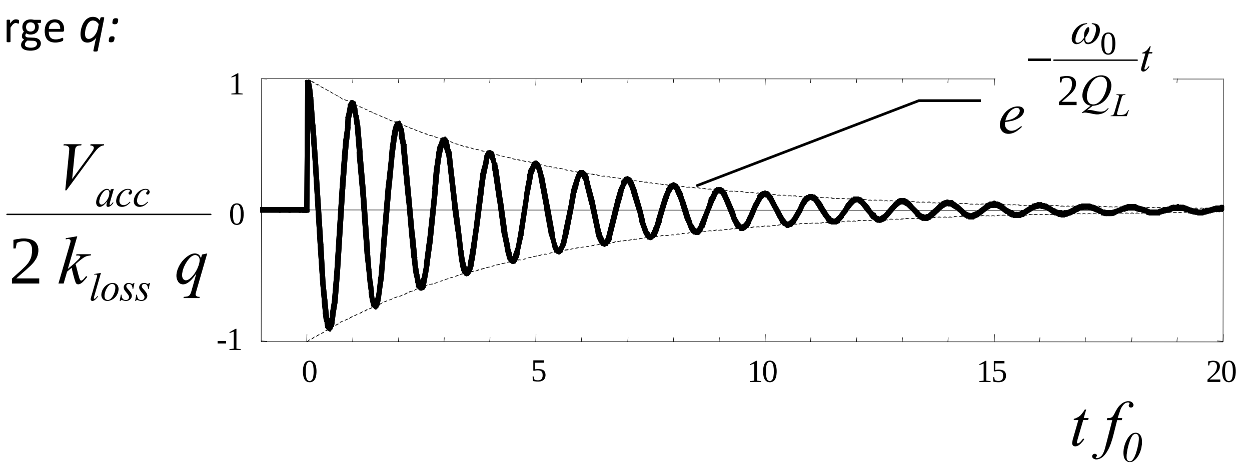
Loss factor

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{|V_{acc}|^2}{4W} = \frac{1}{2C}$$



Energy deposited by a single charge *q*: $k_{loss} q^2$

Voltage induced by a single charge *q:*



Summary: relations between V_{acc}, W, P_{loss}

_R_-upon-_Q_

$$\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{2\,\omega_0 W}$$

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\left|V_{acc}\right|^2}{4\,W}$$

gap voltage

V_{acc}

Shunt impedance

$$R = \frac{\left|V_{acc}\right|^2}{2\,P_{loss}}$$

Energy stored inside the cavity

W

$$Q = \frac{\omega_0 W}{P_{loss}}$$

Power lost in the cavity walls

P_{loss}

***Q* factor**

Beam loading – RF to beam efficiency

The beam current "loads" the generator, in the equivalent circuit this appears as a resistance in parallel to the shunt impedance.

If the generator is matched to the unloaded cavity, beam loading will cause the accelerating voltage to decrease.

The power absorbed by the beam is $-\frac{1}{2}\mathrm{Re}\left\{V_{acc}\; I_B^*\right\}$,

the power loss $P_{loss} = \frac{\left|V_{acc}\right|^2}{2\,R}$.

For high efficiency, beam loading should be high.

The RF to beam efficiency is $\eta = \frac{1}{1+\frac{V_{acc}}{R\left|I_B\right|}} = \frac{\left|I_B\right|}{\left|I_G\right|}$.

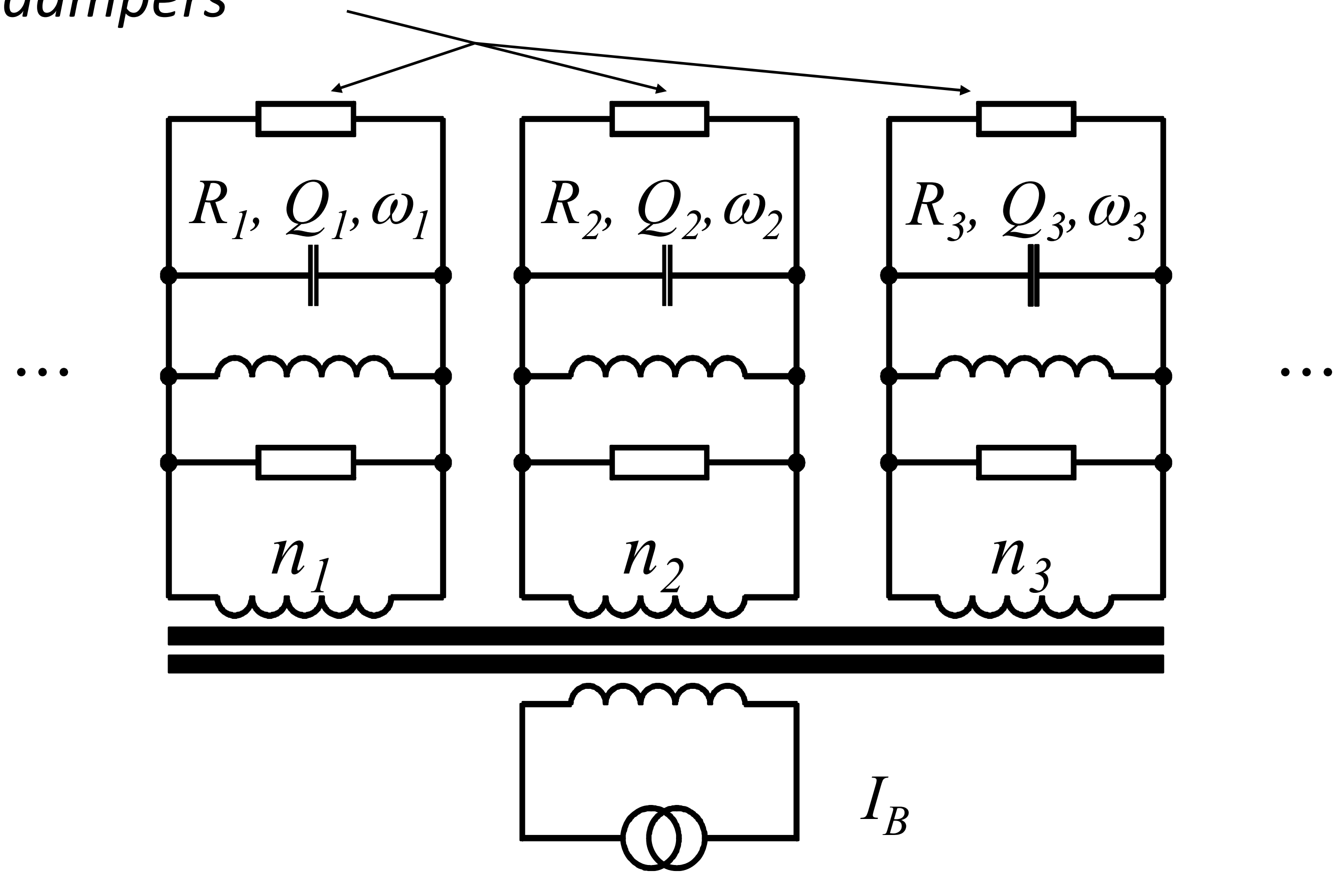
Characterizing cavities

- Resonance frequency $$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$
- Transit time factor
  field varies while particle is traversing the gap $$\frac{\left|\int E_z e^{\mathrm{j}\frac{\omega}{\beta c}z} \mathrm{d}z\right|}{\left|\int E_z \mathrm{d}z\right|}$$

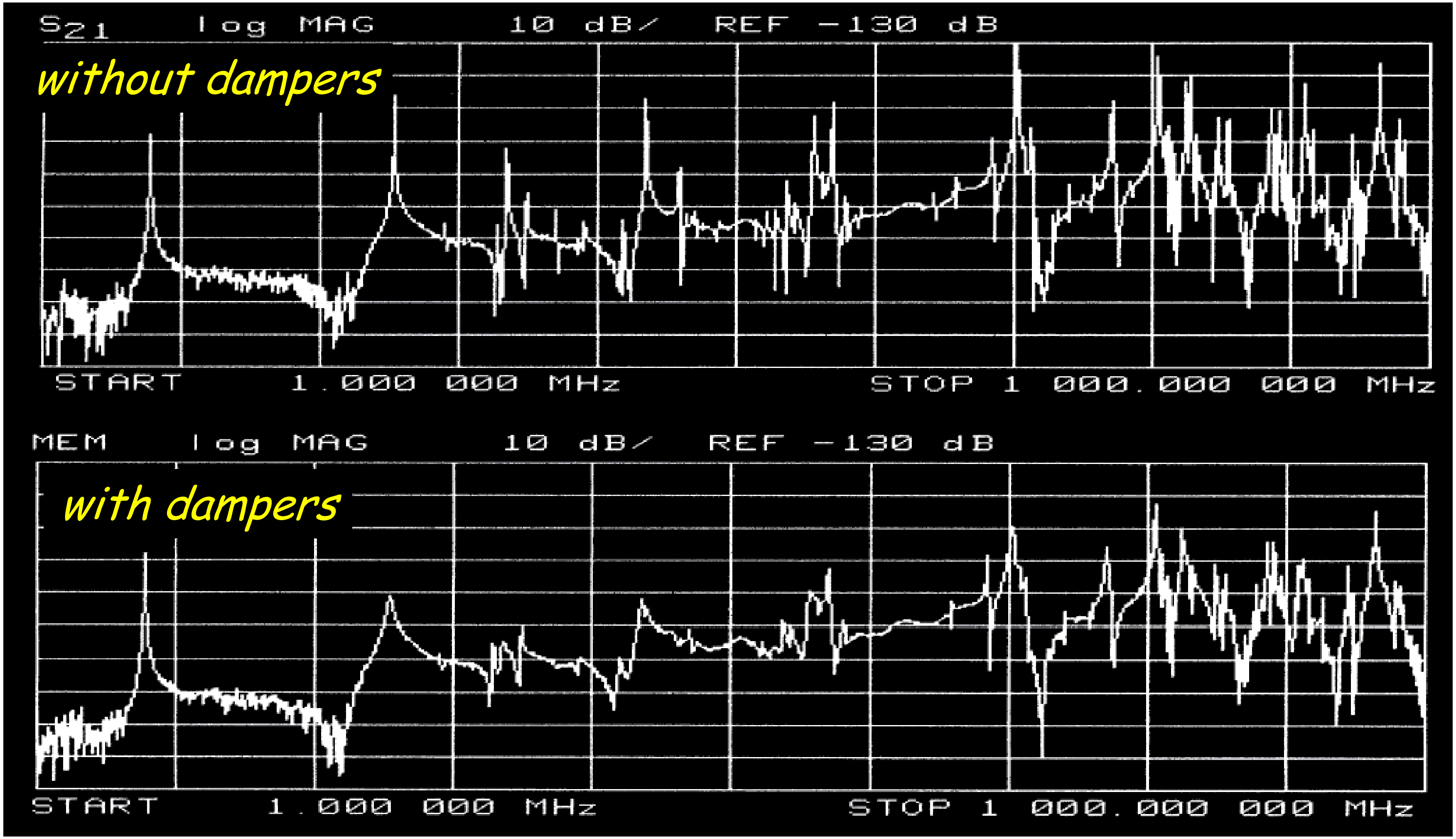
	Circuit definition	Linac definition
Shunt impedance gap voltage – power relation	$\left\|V_{acc}\right\|^2 = 2\,R\;P_{loss}$	$\left\|V_{acc}\right\|^2 = R\;P_{loss}$
Q factor	$\omega_0 W = Q P_{loss}$	
R/Q independent of losses – only geometry!	$\frac{R}{Q} = \frac{\left\|V_{acc}\right\|^2}{2\,\omega_0 W} = \sqrt{\frac{L}{C}}$	$\frac{R}{Q} = \frac{\left\|V_{acc}\right\|^2}{\omega_0 W}$
loss factor	$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{\left\|V_{acc}\right\|^2}{4\,W}$	$k_{loss} = \frac{\omega_0}{4}\frac{R}{Q} = \frac{\left\|V_{acc}\right\|^2}{4\,W}$

Higher order modes

external dampers

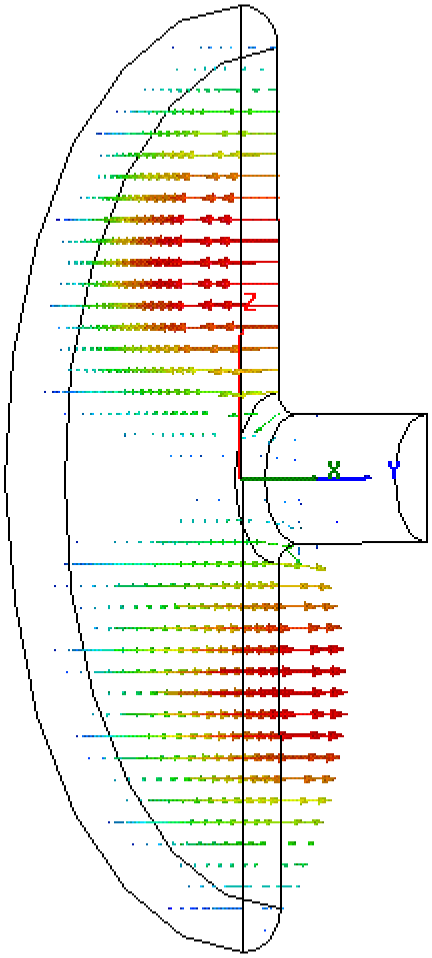


Higher order modes (measured spectrum)

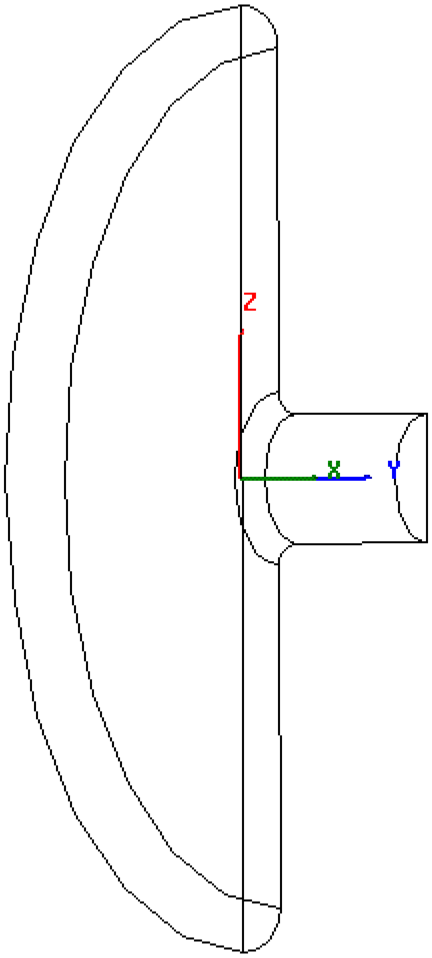


Pillbox: dipole mode

TM_{110}-mode (only 1/4 shown)

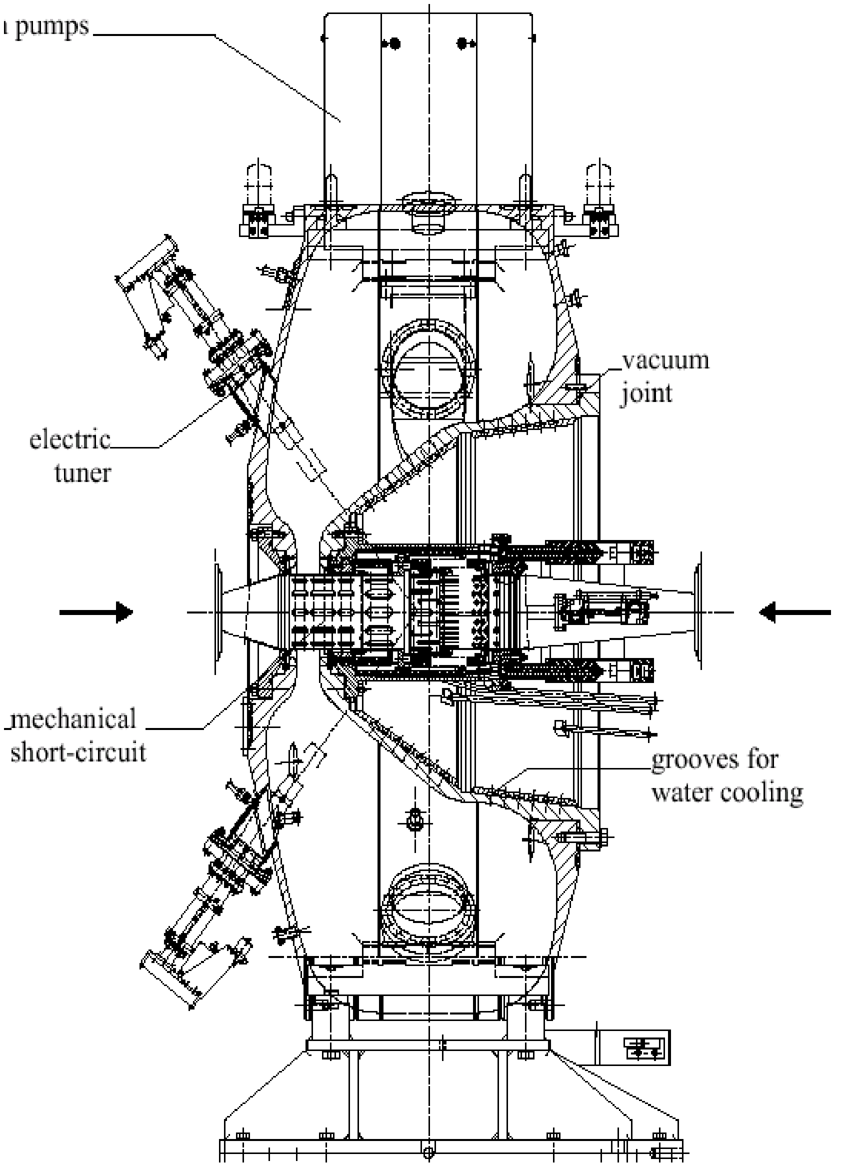


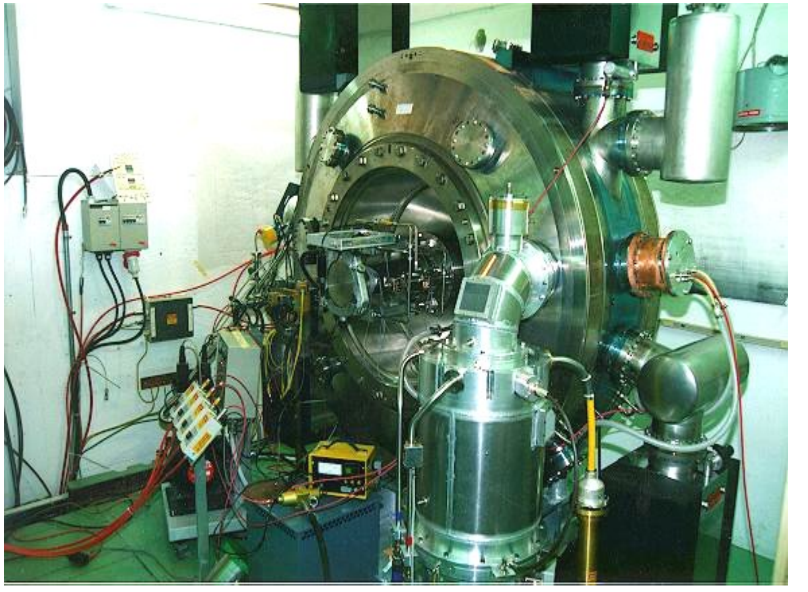
electric field



magnetic field

CERN/PS 80 MHz cavity (for LHC)



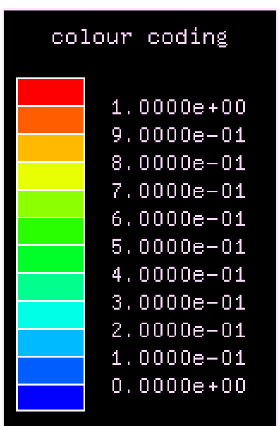


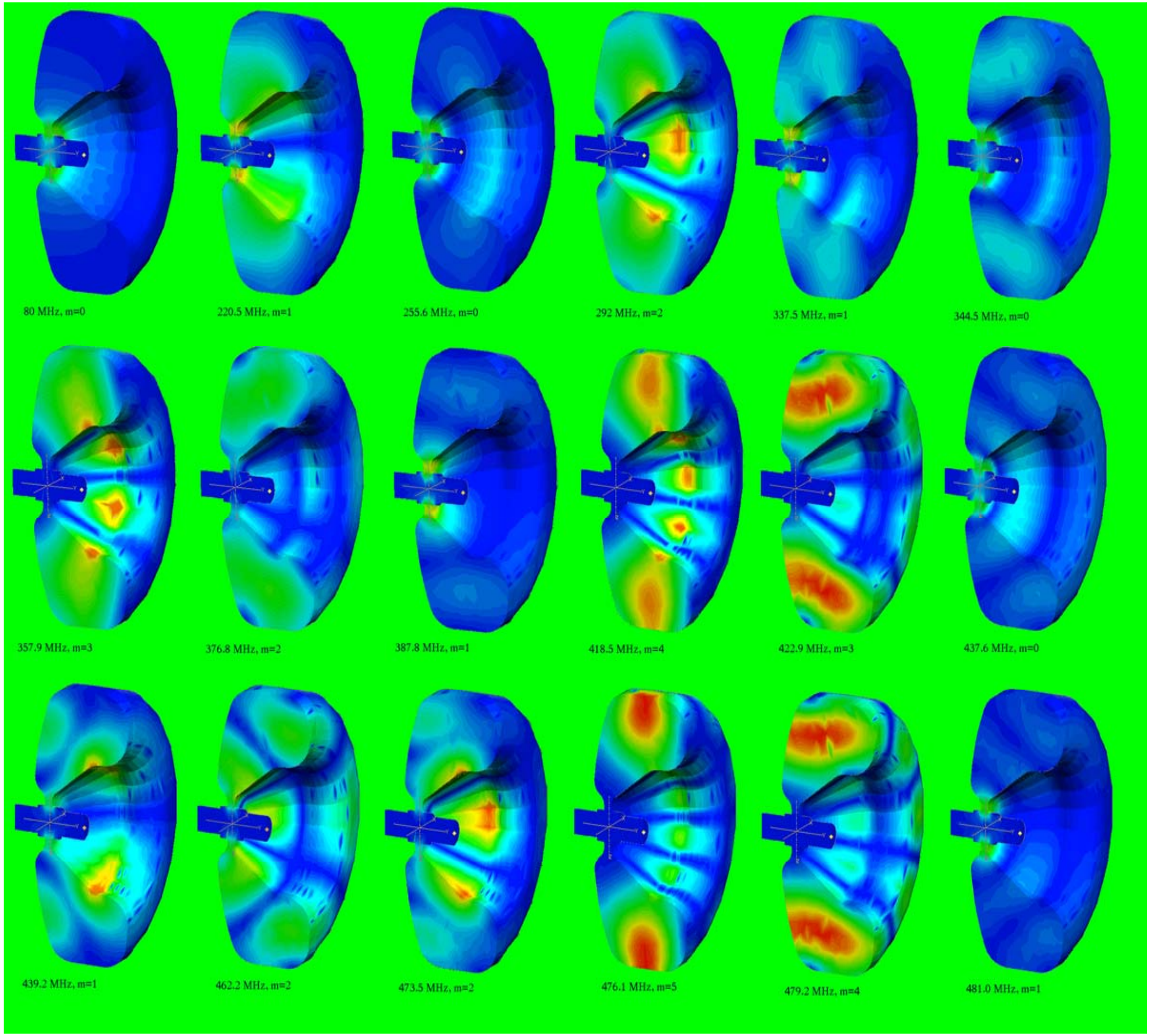
Higher order modes

Example shown: 80 MHz cavity PS for LHC.

Color-coded:

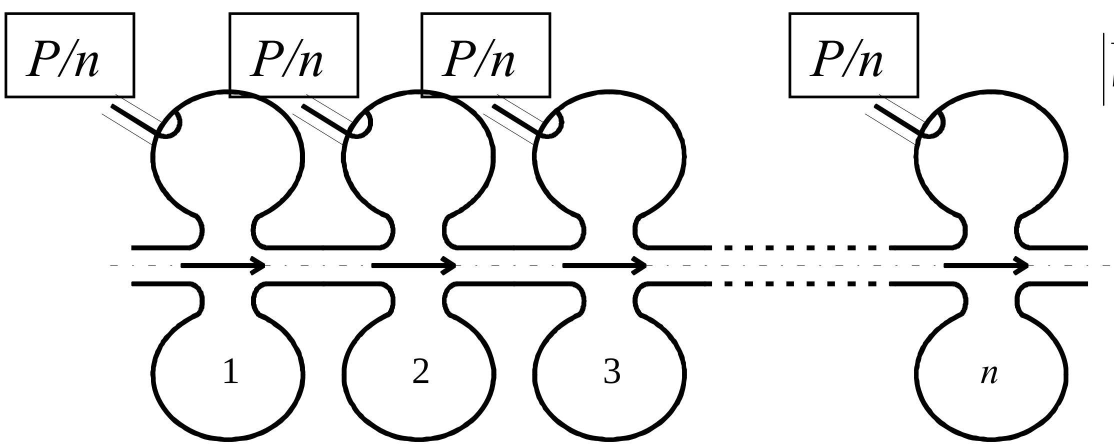
$$\left|\vec{E}\right|$$





What do you gain with many gaps?

- The *R/Q* of a single gap cavity is limited to some 100 Ω.
  Now consider to distribute the available power to *n* identical cavities: each will receive *P/n*, thus produce an accelerating voltage of $\sqrt{2R\,P/n}$.
  The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR.

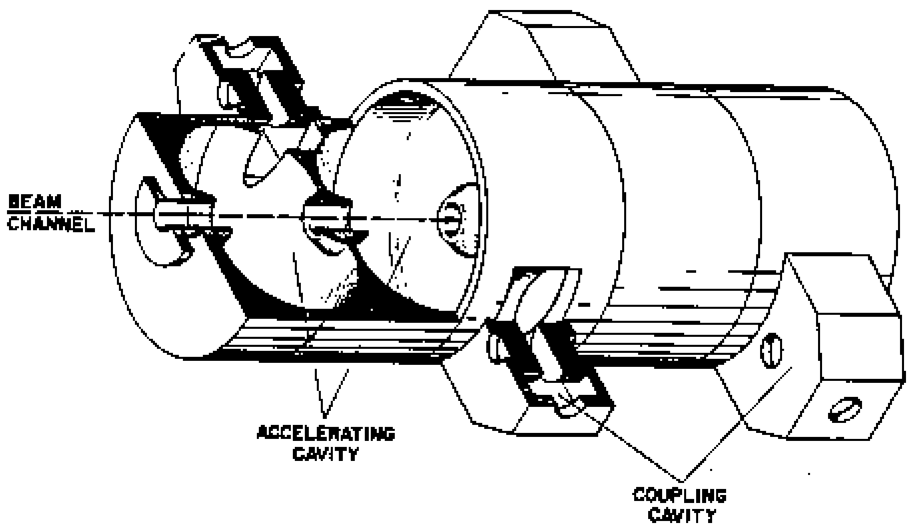


$$|V_{acc}| = n\sqrt{2R\frac{P}{n}} = \sqrt{2(nR)P}$$

Standing wave multicell cavity

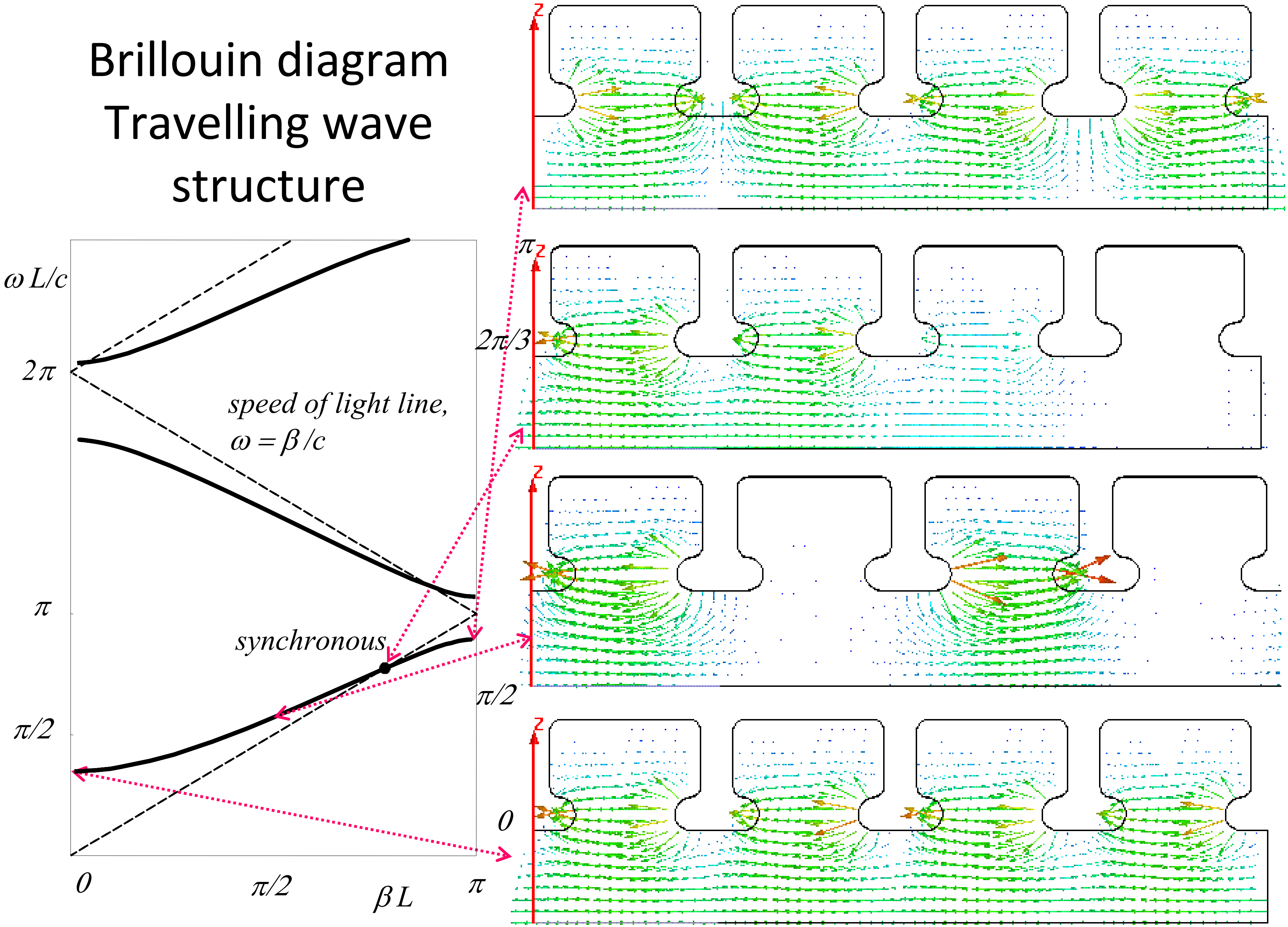
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).

- Coupled cavity accelerating structure (side coupled)



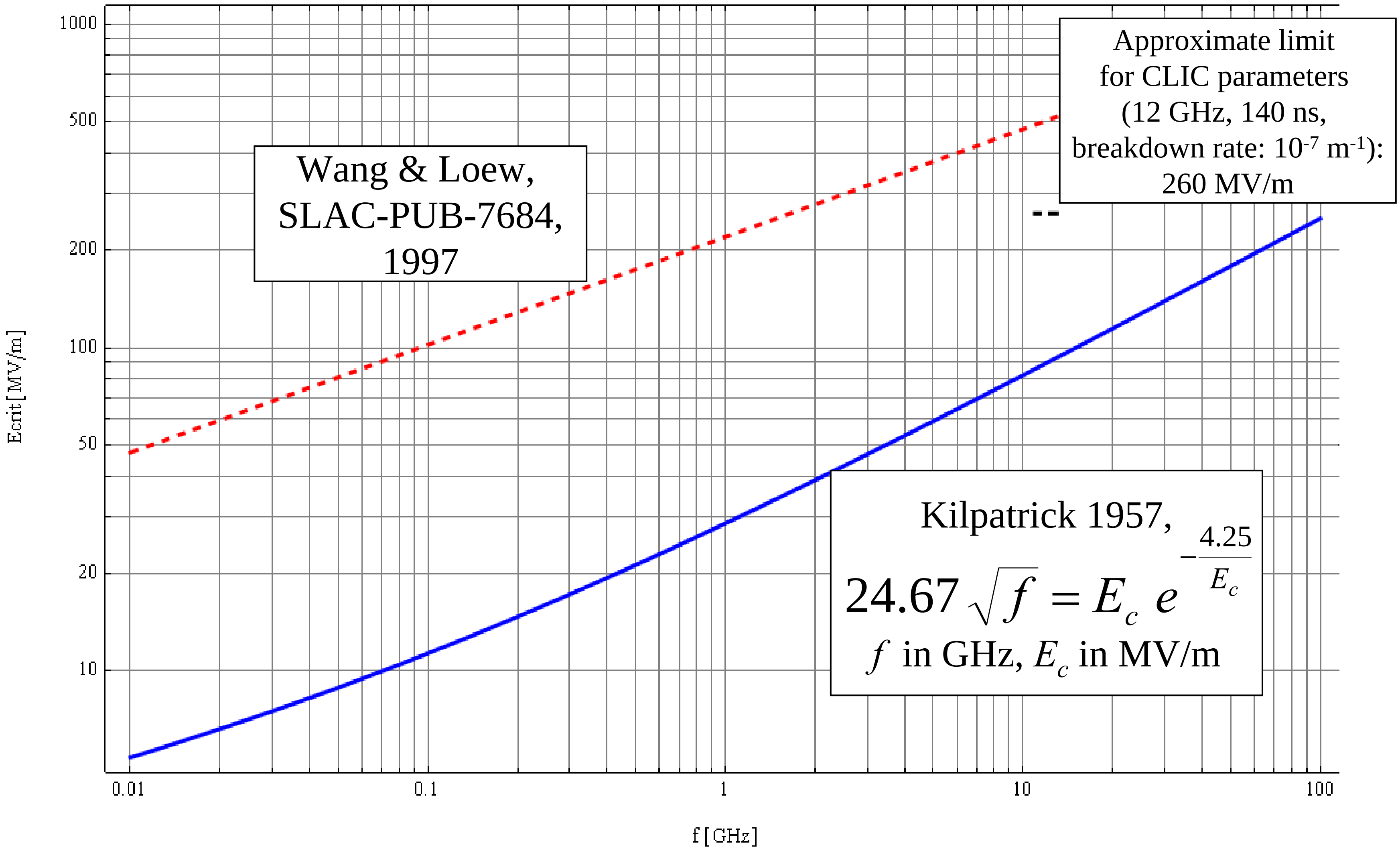
- The phase relation between gaps is important!

Brillouin diagram Travelling wave structure



What limits the acceleration

1: The surface electric field:



What limits the acceleration

2: The surface magnetic field

The surface magnetic field leads to a surface current.

For superconducting cavities, it must stay below a threshold value.

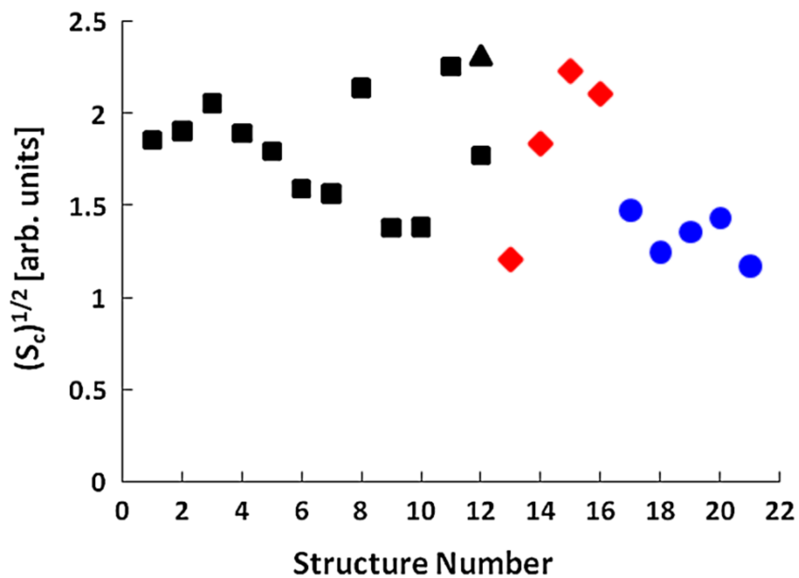
For normal conducting cavities, it will lead to local heating, which in turn can lead to mechanical stress and deformation; with pulsed RF, this can lead to fatigue stress issues. In the presence of an electric field, the heated surface can have an effect on the breakdown.

What limits the acceleration?

3: The (modified) Poynting vector (?)

Even though not yet fully understood, experimental evidence supports the model that a combination of electric and magnetic fields at the surface correlate well with the measured breakdown probability.

N	Name	f [GHz]	$\Delta\varphi$ [°]	v_g/c [%]	L [m]
1	DDS1 [3]	11.424	120	11.7–3	1.8
2	T53vg5R [3]	11.424	120	5.0–3.3	0.53
3	T53vg3MC [3]	11.424	120	3.3–1.6	0.53
4	H90vg3 [3]	11.424	150	3.1–1.9	0.9
5	H60vg3 [3]	11.424	150	3–1.2	0.6
6	H60vg3S18 [3,4]	11.424	150	3.3–1.2	0.6
7	H60vg3S17 [3,4]	11.424	150	3.6–1.0	0.6
8	H75vg4S18 [3]	11.424	150	4.0–1	0.75
9	H60vg4S17 [3,4]	11.424	150	4.5–1	0.6
10	HDX11 [5]	11.424	60	5.1	0.05
11	CLIC-X-band [6]	11.424	120	1.1	0.23
12	T18vg2.6 [7]	11.424	120	2.6–1.0	0.18
13	SW20a3.75 [3]	11.424	180	0	0.2
14	SW1a5.65t4.6 [8]	11.424	180	0	0.013
15	SW1a3.75t2.6 [8]	11.424	180	0	0.013
16	SW1a3.75t1.66 [8]	11.424	180	0	0.013
17	$2\pi/3$ [9]	29.985	120	4.7	0.1
18	$\pi/2$ [10]	29.985	90	7.4	0.1
19	HDS60 [11]	29.985	60	8.0–5.1	0.1
20	HDS60-Back [11]	29.985	60	5.1–8.0	0.1
21	PETS9mm [12]	29.985	120	39.8	0.4



$$S_c = \max\left(\mathrm{Re}\{S\} + \frac{1}{6}\mathrm{Im}\{S\}\right)$$

http://cdsweb.cern.ch/record/1216861

Cavity basics – Summary

- The EM fields inside a hollow cavity are superpositions of homogeneous plane waves.
- When operating near an eigenfrequency, one can profit from a resonance phenomenon (with high Q).
- R-upon-Q, Shunt impedance and Q factor were are useful parameters, which can also be understood in an equivalent circuit.
- The perturbation method allows to estimate losses and sensitivity to tolerances.
- Many gaps can increase the effective impedance.