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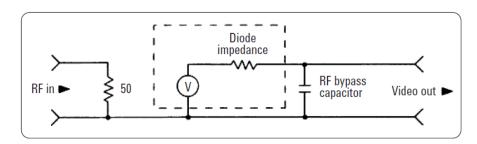
- The radio frequency (RF) diode
- Superheterodyne concept
- Spectrum analyzer
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- Noise basics
- Noise-figure measurement with the spectrum analyzer

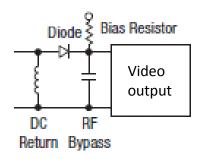
The RF diode (1)

- We are not discussing the generation of RF signals here, just the detection
- Basic tool: fast RF* diode
 (= Schottky diode)
- In general, Schottky diodes are fast but still have a voltage dependent junction capacity (metal – semiconductor junction)

A typical RF detector diode
Try to guess from the type of the
connector which side is the RF input
and which is the output

Equivalent circuit:





^{*}Please note, that in this lecture we will use RF for both the RF and micro wave (MW) range, since the borderline between RF and MW is not defined unambiguously

The RF diode (2)

- Characteristics of a diode:
- The current as a function of the voltage for a barrier diode can be described by the Richardson equation:

$$I = AA^{**}_{2} exp \left(-\frac{q \oplus B}{kT}\right) \left[exp\left(\frac{qv}{NkT}\right) - 1\right]$$

where

A = area (cm²)

A** = modified Richardson constant (amp/oK)²/cm²)

k = Boltzman's Constant

T = absolute temperature (°K)

φB = barrier heights in volts

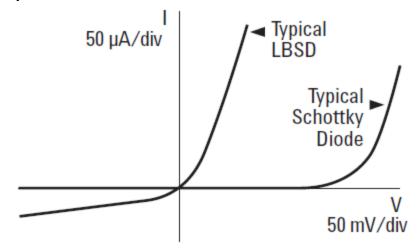
V = external voltage across the depletion layer

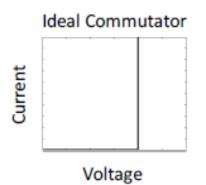
(positive for forward voltage) - V - IRS

R_S = series resistance

I = diode current in amps (positive forward current)

n = ideality factor





The RF diode is NOT an ideal commutator for small signals! We cannot apply big signals otherwise burnout

The RF diode (3)

• In a highly simplified manner, one can approximate this expression as:

$$I = I_S \left[exp \left(\frac{V_J}{0.028} \right) - 1 \right]$$
 V_J ... junction voltage

 and show as sketched in the following, that the RF rectification is linked to the second derivation (curvature) of the diode characteristics:

If the DC current is held constant by a current regulator or a large resistor, then the total junction current, including RF, is

$$I = I_0 = i \cos \omega t$$

and the I–V relationship can be written

$$V_J = 0.028 Ln \left(\frac{I_S + I_0 + i cos \omega t}{I_S} \right)$$

$$= 0.028 \operatorname{Ln}\left(\frac{l_0 + l_S}{l_S}\right) + 0.028 \operatorname{Ln}\left(\frac{i\cos\omega t}{l_0 + l_S}\right)$$

If the RF current, i, is small enough, the IN-term can be approximated in a Taylor series:

$$V_J \approx 0.028 \text{Ln} \left(\frac{l_0 + l_S}{l_S} \right) + 0.028 \left[\frac{i \cos \omega t}{l_0 + l_S} - \frac{i^2 \cos^2 \omega t}{2(l_0 + l_S)^2} + \dots \right]$$

=
$$V_{DC} + V_J \cos \omega t + higher frequency terms$$

If you use the fact that the average value of \cos^2 is 0.50, then the RF and DC voltages are given by the following equations:

$$V_J = \frac{0.028}{I_0 + I_S} \quad i = R_S i$$

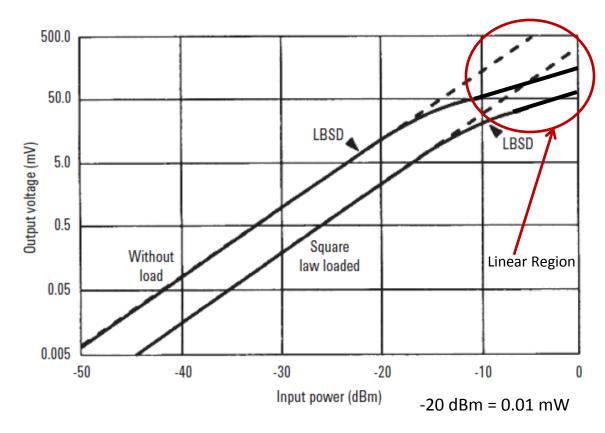
$$V_{DC} = 0.028/n \left(1 + \frac{I_0}{I_S}\right) - \frac{0.028^2}{4(I_0 + I_S)^2} = V_0 - \frac{V_J^2}{0.112}$$

The RF diode (4)

- This diagram depicts the so called square-law region where the output voltage (V_{Video}) is proportional to the input power
- Since the input power is proportional to the square of the input voltage (V_{RF}²) and the output signal is proportional to the input power, this region is called square-law region.



$$V_{Video} \sim V_{RF}^2$$

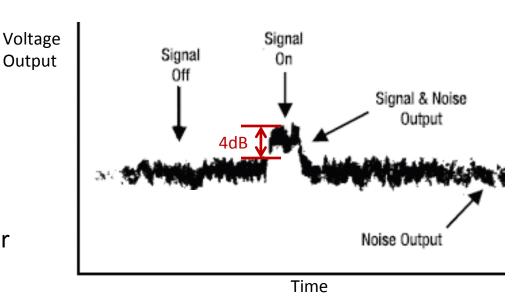


 The transition between the linear region and the square-law region is typically between -10 and -20 dBm RF power (see diagram)

The RF diode (5)

• Due to the square-law characteristic we arrive at the thermal noise region already for moderate power levels (-50 to -60 dBm) and hence the V_{Video} disappears in the thermal noise

This is described by the term
 tangential signal sensitivity (TSS)
 where the detected signal
 (Observation BW, usually 10 MHz)
 is 4 dB over the thermal noise floor



The RF mixer (1)

- For the detection of very small RF signals we prefer a device that has a linear response over the full range (from 0 dBm (= 1mW) down to thermal noise = $-174 \text{ dBm/Hz} = 4 \cdot 10^{-21} \text{ W/Hz}$)
- This is the RF mixer which is using 1, 2 or 4 diodes in different configurations (see next slide)
- Together with a so called LO (local oscillator) signal, the mixer works as a signal multiplier with a very high dynamic range since the output signal is always in the "linear range" provided, that the mixer is not in saturation with respect to the RF input signal (For the LO signal the mixer should always be in saturation!)
- The RF mixer is essentially a multiplier implementing the function

$$f_1(t) \cdot f_2(t)$$
 with $f_1(t) = RF$ signal and $f_2(t) = LO$ signal

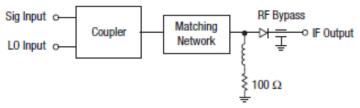
$$a_1 \cos(2\pi f_1 t + \varphi) \cdot a_2 \cos(2\pi f_2 t) = \frac{1}{2} a_1 a_2 [\cos((f_1 + f_2)t + \varphi) + \cos((f_1 - f_2)t + \varphi)]$$

 Thus we obtain a response at the IF (intermediate frequency) port that is at the sum and difference frequency of the LO and RF signals

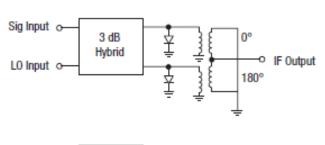
The RF mixer (2)

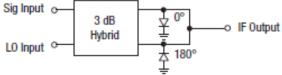
Examples of different mixer configurations

A. Single-Ended Mixer

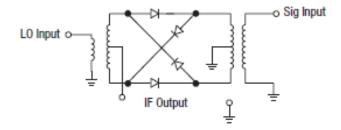


B. Balanced Mixers





C. Double-Balanced Mixer





A typical coaxial mixer (SMA connector)

The RF mixer (3)

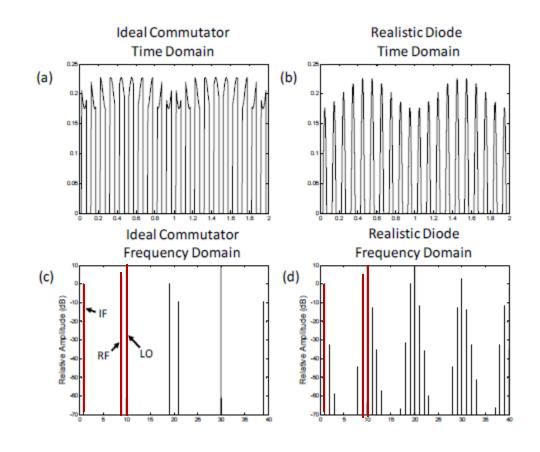
Response of a mixer in time and frequency domain:

Input signals here:

LO = 10 MHz

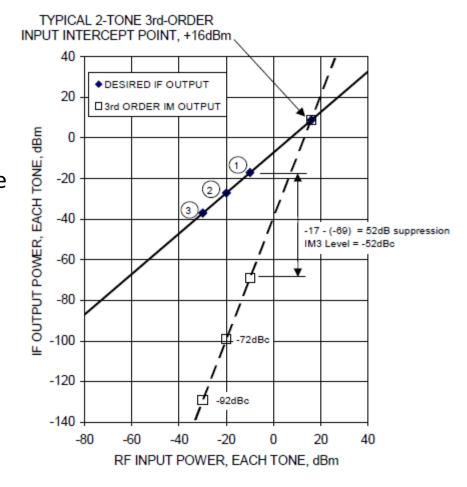
RF = 8 MHz

Mixing products at 2 and 18 MHz plus higher order terms at higher frequencies



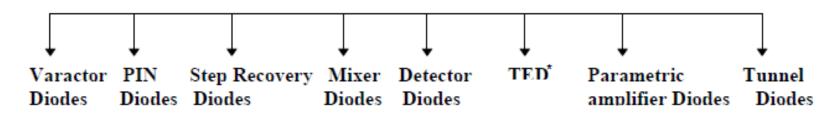
The RF mixer (4)

- Dynamic range and IP3 of an RF mixer
- The abbreviation IP3 stands for the third order intermodulation point where the two lines shown in the right diagram intersect. Two signals $(f_1, f_2 > f_1)$ which are closely spaced by Δf in frequency are simultaneously applied to the DUT. The intermodulation products appear at $+ \Delta f$ above f_2 and at Δf below f_1 .
- This intersection point is usually not measured directly, but extrapolated from measurement data at much smaller power levels in order to avoid overload and damage of the DUT



Solid state diodes used for RF applications

 There are many other diodes which are used for different applications in the RF domain



* TransferredElectron Devices

- Varactor diodes: for tuning application
- PIN diodes: for electronically variable RF attenuators
- Step Recovery diodes: for frequency multiplication and pulse sharpening
- Mixer diodes, detector diodes: usually Schottky diodes
- TED (GUNN, IMPATT, TRAPATT etc.): for oscillator
- Parametric amplifier Diodes: usually variable capacitors (vari caps)
- Tunnel diodes: rarely used these days, they have negative impedance and are usually used for very fast switching and certain low noise amplifiers

Measurement devices (1)

- There are many ways to observe RF signals. Here we give a brief overview of the four main tools we have at hand
- Oscilloscope: to observe signals in time domain
 - periodic signals
 - burst signal
 - application: direct observation of signal from a pick-up, shape of common
 230 V mains supply voltage, etc.
- Spectrum analyser: to observe signals in frequency domain
 - sweeps through a given frequency range point by point
 - application: observation of spectrum from the beam or of the spectrum emitted from an antenna, etc.

Measurement devices (2)

Dynamic signal analyser (FFT analyser)

- Acquires signal in time domain by fast sampling
- Further numerical treatment in digital signal processors (DSPs)
- Spectrum calculated using Fast Fourier Transform (FFT)
- Combines features of a scope and a spectrum analyser: signals can be looked at directly in time domain or in frequency domain
- Contrary to the SPA, also the spectrum of non-repetitive signals and transients can be observed
- Application: Observation of tune sidebands, transient behaviour of a phase locked loop, etc.

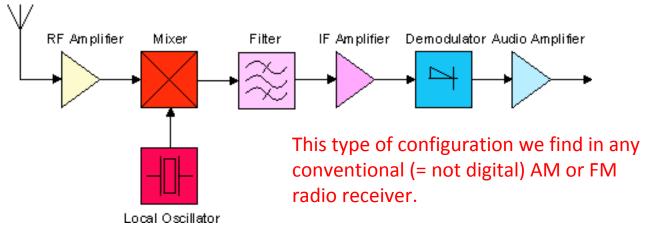
Network analyser

- Excites a network (circuit, antenna, amplifier or such) at a given CW frequency and measures response in magnitude and phase => determines S-parameters
- Covers a frequency range by measuring step-by-step at subsequent frequency points
- Application: characterization of passive and active components, time domain reflectometry by Fourier transforming reflection response, etc.

Superheterodyne Concept (1)

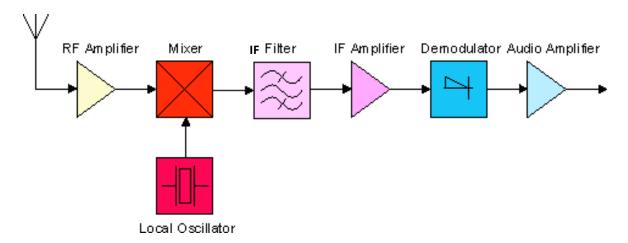
Design and its evolution

The diagram below shows the basic elements of a single conversion superheterodyne ("superhet" or just "super") receiver. The essential elements of a local oscillator and a mixer followed by a fixed-tuned filter and IF amplifier are common to all superhet circuits. [super $\varepsilon \tau \varepsilon \rho \omega \delta \upsilon \nu \alpha \mu \iota \sigma$] a mixture of latin and greek ... it means: another force becomes superimposed.



The advantage to this method is that most of the radio's signal path has to be sensitive to only a narrow range of frequencies. Only the front end (the part before the frequency converter stage) needs to be sensitive to a wide frequency range. For example, the front end might need to be sensitive to 1–30 MHz, while the rest of the radio might need to be sensitive only to 455 kHz, a typical IF.

Superheterodyne Concept (2)



RF Amplifier = wideband frontend amplification (RF = radio frequency)

The Mixer can be seen as an analog multiplier which multiplies the RF signal with the **LO** (local oscillator) signal.

The local oscillator has its name because it's an oscillator situated in the receiver locally and not far away as the radio transmitter to be received.

IF stands for intermediate frequency.

The demodulator can be an amplitude modulation (AM) demodulator (envelope detector) or a frequency modulation (FM) demodulator, implemented e.g. as a PLL (phase locked loop).

The tuning of a normal radio receiver is done by changing the frequency of the LO, not of the IF filter.

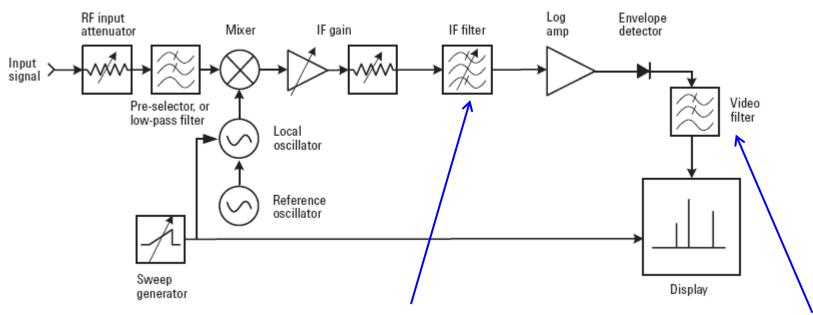
Note that also multiple (double, triple, quadruple) superhet stage concept exist and are used for very high quality receivers.

en.wikipedia.org

Spectrum Analyzer (1)

- Radio-frequency spectrum-analysers (SPA or SA) can be found in virtually every control-room of a modern particle accelerator.
- They are used for many aspects of beam diagnostics including Schottky signal acquisition and RF observation. We discuss first the application of classical super-heterodyne SPAs and later systems based on the acquisition of time domain traces and subsequent Fourier transform (FFT analysers).
- Such a super-heterodyne SPA is very similar (in principle) to any AM or FM radio receiver. The incoming RF-signal is moderately amplified (sometimes with adjustable gain) and then sent to the RF-port of a mixer.

Example for Application of the Superheterodyne Concept in a Spectrum Analyzer



The center frequency is fixed, but the bandwidth of the IF filter can be modified.

Agilent, 'Spectrum Analyzer Basics,' Application Note 150, page 10 f.

The video filter is a simple low-pass with variable bandwidth before the signal arrives to the vertical deflection plates of the cathode ray tube.

Spectrum Analyzer (2)

 The most important knobs on a spectrum analyzer are:

1. Vertical parameters:

- Reference Level
- 2. Sensitivity
- 3. Display format (Lin-log)
- 4. Video bandwidth (low pass for vertical deflection)
- 5. Vertical scale (1, 5, 10 dB per division)

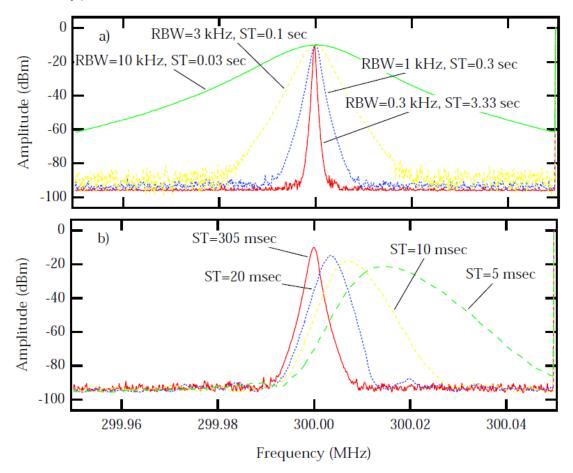
2. Horizontal parameters

- 1. Sweep time
- 2. Frequency span
- 3. Resolution bandwidth
- 4. Number of points

Caution: These Parameters should not be adjusted independent from each other The setting time of the resolution BW filter must be kept in mind.

Spectrum Analyzer (3)

 Impact of the variable resolution BW (upper half) and video BW (lower half respectively):



Spectrum Analyzer (4)

The optimum sweep time (ST) for a given RBW and frequency span can be found by equating the time spent within the RBW for a given frequency span with the risetime of the IF filter. The expression is

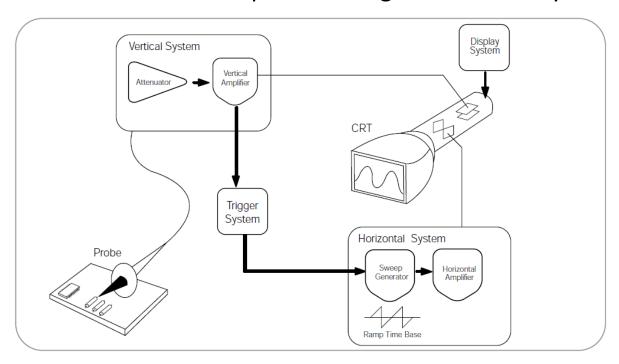
$$ST = k \frac{Span}{RBW^2}$$

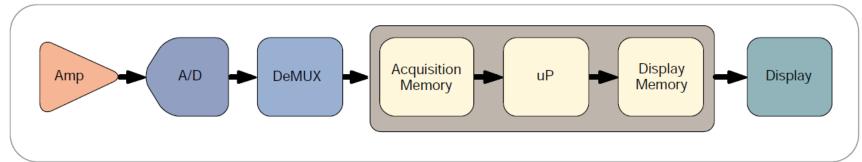
where k is a constant relating the filter risetime to the resolution bandwidth and depends in detail on the filter shape. For Gaussian filters it is about 2.5. Usually, the RBW and sweep time are automatically adjusted according to the frequency span such that a reasonable resolution is achieved. However, higher resolution can usually be achieved but at the expense of longer sweep times.

Oscilloscope (1)

The architecture of an oscilloscope has changed considerably over the last

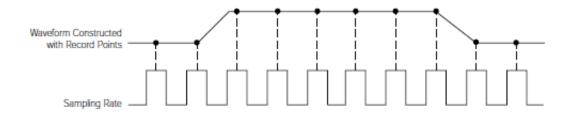
decades

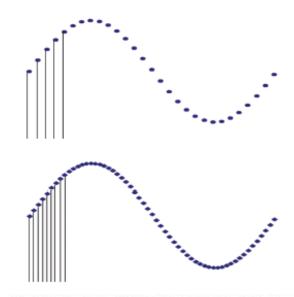


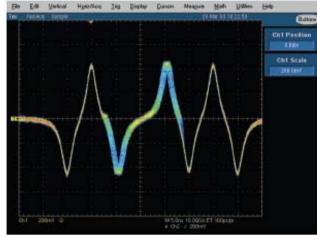


Oscilloscope (2)

- One of the many interesting feature of modern oscilloscopes is that they can change the sampling rate through the sweep in a programmed manner.
- This can be very helpful for detailed analysis in certain time windows
- Typical sampling rates are between a factor 2.5 and 4 of the maximum frequency (Nyquist requires a real time minimum sampling rate as twice f_{max})

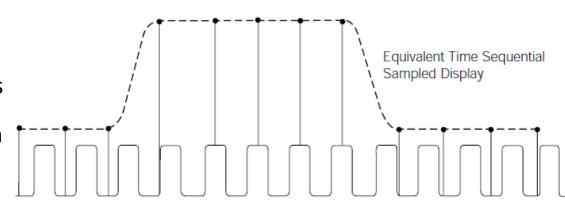




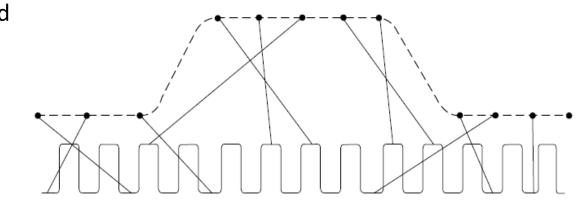


Oscilloscope (3)

 Sequential sampling requires a pre trigger (required to open the sampling gate) and permits a non real time bandwidth of more than 50GHz with modern scopes

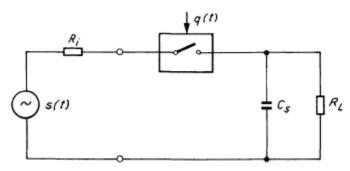


 Random sampling (rarely used these days) was developed about 40 years ago for the case that no pre trigger was availiable and relying on a strictly periodic signal to predict a pre trigger from the measured periodicity

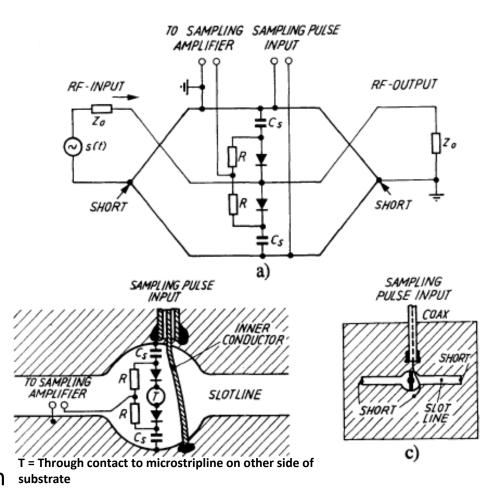


Oscilloscope (4)

How does sampling really work in detail?

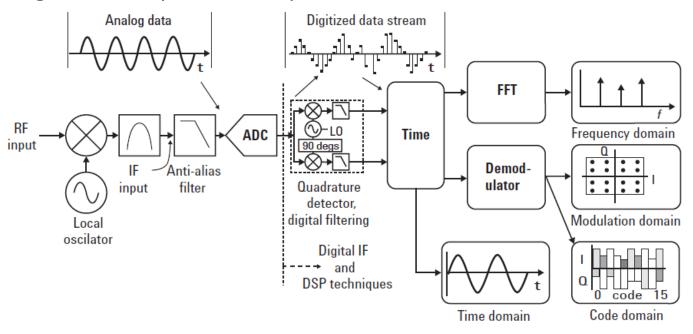


- We need a "sample and hold" circuit (shown above)
- A possible technological implementation is shown on the right side for a 20 GHz sampling head built around 1980.
- This kind of unit working with very short sampling spikes is also known as harmonic mixer



Vector spectrum analyzer

- The modern vector spectrum analyzer (VSA) is essentially a combination
 of a two channel digital oscilloscope and a spectrum analyzer FFT display
- The incoming signal gets down mixed, bandpass (BP) filtered and passes an ADC (generalized Nyquist for BP signals; $f_{\text{sample}} \ge 2BW$).
- The digitized time trace then is split into an I (in phase) and Q (quadrature, 90 degree offset) component with respect to the phase of some reference oscillator. Without this reference, the term vector is meaningless for a spectral component

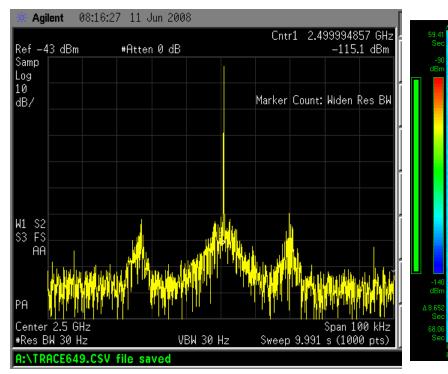


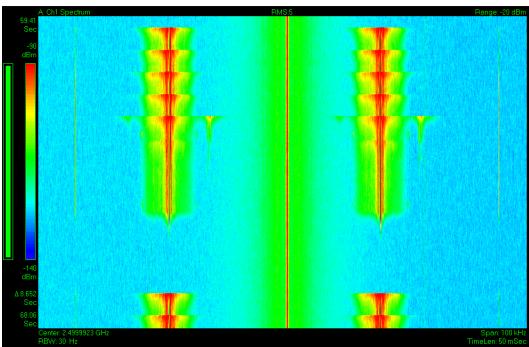
Vector spectrum analyzer

Example of vector spectrum analyzer display and performance:

Single shot FFT display similar to a very slow scan on a swept spectrum analyzer

Spectrogram display containing about 200 traces as shown on the left side in color coding. Time runs from top to bottom





Electron cloud measurements in the CERN SPS taken 2008

Decibel (1)

 The Decibel is the unit used to express relative differences in signal power. It is expressed as the base 10 logarithm of the ratio of the powers of two signals:

$$P [dB] = 10 \cdot log(P/P_0)$$

 Signal amplitude can also be expressed in dB. Since power is proportional to the square of a signal's amplitude, the voltage in dB is expressed as follows:

$$V [dB] = 20 \cdot log(V/V_0)$$

- P₀ and V₀ are the reference power and voltage, respectively.
- A given value in dB is the same for power ratios as for voltage ratios
- There are no "power dB" or "voltage dB" as dB values always express a ratio!!!

Decibel (2)

Conversely, the absolute power and voltage can be obtained from dB values by

$$P = P_0 \cdot 10^{\frac{P[dB]}{10}}, \quad V = V_0 \cdot 10^{\frac{V[dB]}{20}}$$

 Logarithms are useful as the unit of measurement because (1) signal power tends to span several orders of magnitude and (2) signal attenuation losses and gains can be expressed in terms of subtraction and addition.

Decibel (3)

- The following table helps to indicate the order of magnitude associated with dB:
- Power ratio = voltage ratio squared!
- S parameters are defined as ratios and sometimes expressed in dB, no explicit reference needed!

	power ratio	V, I, E or H ratio, S _{ij}
-20 dB	0.01	0.1
-10 dB	0.1	0.32
-3 dB	0.50	0.71
-1 dB	0.74	0.89
0 dB	1	1
1 dB	1.26	1.12
3 dB	2.00	1.41
10 dB	10	3.16
20 dB	100	10
n * 10 dB	10 ⁿ	10 ^{n/2}

Decibel (4)

- Frequently dB values are expressed using a special reference level and not SI units. Strictly speaking, the reference value should be included in parenthesis when giving a dB value, e.g. +3 dB (1W) indicates 3 dB at P₀ = 1 Watt, thus 2 W.
- For instance, dBm defines dB using a reference level of $P_0 = 1$ mW. Often a reference impedance of 50Ω is assumed.
- Thus, 0 dBm correspond to -30 dBW, where dBW indicates a reference level of $P_0=1W$.
- Other common units:
 - dBmV for the small voltages, $V_0 = 1 \text{ mV}$
 - dBµV/m for the electric field strength radiated from an antenna, E_0 = 1 $\,\mu\text{V/m}$

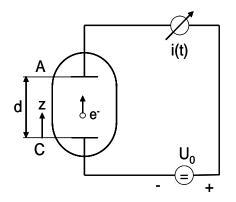
Definition of the Noise Figure

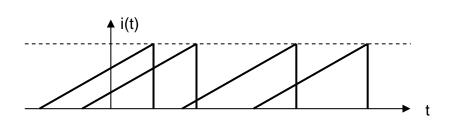
$$F = \frac{S_i N_i}{S_o N_o} = \frac{N_o}{GN_i} = \frac{N_o}{GkT_0B} = \frac{GkT_0B + N_R}{GkT_0B}$$

- F is the *Noise factor* of the receiver
- S_i is the available signal power at input
- $N_i = kT_0B$ is the available noise power at input
- T_0 is the absolute temperature of the source resistance
- N_o is the available noise power at the output , including amplified input noise
- G is the available receiver gain
- B is the effective noise bandwidth of the receiver
- If the noise factor is specified in a logarithmic unit, we use the term Noise Figure (NF).

$$\left| NF = 10 \lg \frac{S_i N_i}{S_o N_o} dB \right|$$

Shot noise in a vacuum diode (1)





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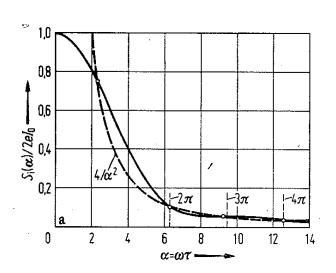
- Consider a vacuum diode where single electrons are passing through in a statistical manner (left figure) with the travel time τ
- Due to the dD/dt (D = ε E) we get a current linearly increasing vs time when the electron approaches the flat anode.
- We assume a diode in a saturated regime (space charge neglected) and obtain after some math for frequencies with a period >> τ for the spectral density S_i (ω) of the short circuit current the Schottky equation:

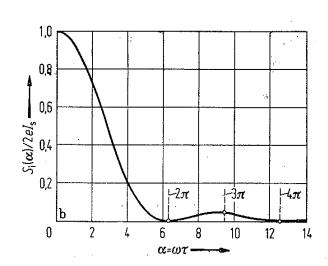
$$S_i(\omega) = 2I_0e$$

with e= 1.6e⁻¹⁹ As and the mean current $I_0 = e v_{mean}$

Shot noise in vacuum diodes (2)

- \clubsuit obviously the travel time τ plays a very important role for the frequency limit
- # The value for τ in typical vacuum diodes operated at a few 100 Volts is around a fraction of a ns. This translates to max frequencies of 1GHz





 $\tau = d\sqrt{\frac{2m_0}{e\,U_0}}$

Spectral current density of a planar ultra high vacuum diode in saturation (a) and solid state diode (b)

Thermal noise in resistors (1)

- In a similar way (saturated high vacuum diode ⇒ high vacuum diode in space charge region ⇒ biased solid state diode ⇒ unbiased solid state diode) one can arrive at the thermal noise properties of a resistor (often referred to as Johnson noise also W. Schottky made the first theoretical approach)
- * We obtain the general relation (valid also for very high frequencies f and/or low temperatures T of the open (unloaded) terminal voltage u) of some linear resistor R in thermo dynamical equilibrium for a frequency interval Δf as

$$\overline{u^2} = 4k_B TR \frac{hf/k_B T}{\exp(hf/k_B T) - 1} \Delta f$$

$$h = Planck's constant = 6.62 \cdot 10^{-34} Js$$

$$k_B = Boltzmann's constant = 1.38 \cdot 10^{-23} J/K$$

Thermal noise in resistors (2)

* From the general relation we can deduce the low frequency approximation which is still reasonably valid at ambient temperature up to about 500 GHz as

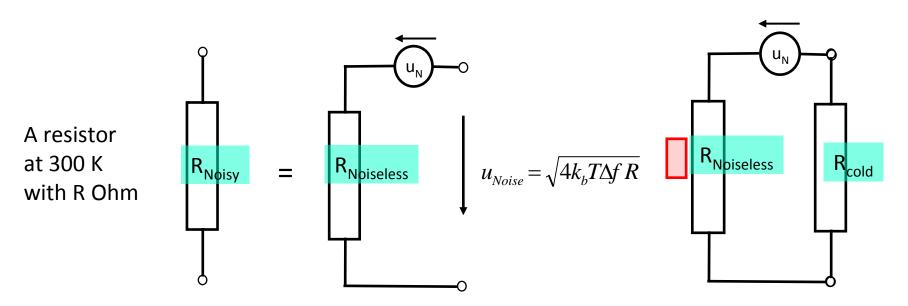
$$\overline{u^2} = 4k_b T \Delta f R$$

For the short circuit current we get accordingly

$$\overline{i^2} = 4k_b T \Delta f / R$$

Thermal noise in resistors (3)

Warm resistor



The open thermal noise voltage is reduced by a factor of two (matched load) if the warm (noisy) resistor is loaded by another resistor of same ohmic value but at 0 deg K. Then we get a net power flux density (per unit BW) of kT from the warm to the cold resistor. This power flux density is independent of R (matched load case);

more on resistor noise in: http://en.wikipedia.org/wiki/Noise and:http://www.ieee.li/pdf/viewgraphs_mohr_noise.pdf

Thermal noise in resistors (4)

Then we obtain for the power P delivered to this external load

$$P = k_b T \Delta f$$

Or for the power density p per unit bandwidth the very simple and useful relation

$$p = k_b T = -174 dBm / Hz$$
 @ $300K = 4 \cdot 10^{-21} Watt / Hz$

Note that this relation is also valid for networks of linear resistors at homogeneous temperature between any 2 terminals, but not for resistors which are not in thermo dynamical equilibrium like a biased diode or a transistor with supply voltage Such active elements can have noise temperatures well below their phys.temperature In particular a forward biased (solid state) diode may be used as a pseudocold load: By proper setting of the bias current the differential impedance can be set to 50 Ohm The noise temperature of this device is slightly above $T_0/2$. Alternatively the input stage of a low noise amplifier can be applied (example:1 dB NF= 70 deg K noise temp.)

R.H. Frater, D.R. Williams, An Active "Cold" Noise Source, IEEE Trans.on Microwave Theory and Techn., pp 344-347, April 1981

Noise figure measurements (1)

- The term "noise-figure" and "noise-factor" are used to describe the noise properties of amplifiers. F is defined as signal to noise (power) ratio and the input of the DUT versus signal to noise power ratio at the output. F is always >1 for linear networks i.e. the signal to noise ratio at the output of some 2-port or 4- pole is always more or less degraded. In other words, the DUT (which may be also an amplifier with a gain smaller than unity i.e. an attenuator) is always adding some of its own noise to the signal.
- F[dB] is called "noise figure"
- F[linear units of power ratio] sometimes noise factor
- F[dB] = 10 log F[linear units]

$$F[linearunit] = \frac{ENR[linearunit]}{Y[linearunit] - 1} = \frac{T_{ex}}{T_{o} \cdot (Y - 1)} \quad \text{with} \quad T_{ex} = T_{H} - T_{0}$$

- ENR stand for excess noise ratio delivered by the noise diode and tells us how much "warmer" than room temperature the noise diode appears. For an ENR of 16 dB this amounts roughly a factor of 40 in power or 40 times 300 K which is 12000 K.
- The quantity "Y" is the ratio of noise power densities measured on the SPA between the settings: noise source on and noise source off.

Noise figure measurements (2)

• As shown in the equations below, also the gain of the DUT can be found from the two readings on the SPA. Thus one can measure simultaneously gain and noise figure. $Y = \frac{measuredDUToutput\ power(density)withnoisesource = hot}{measuredDUToutput\ power(density)withnoisesource = hot}$

$$Y = \frac{measureaDUT output power(density)withnoisesource = not}{measuredDUT output power(density)withnoisesource = cold}$$

$$ENR[linear - unit] = \frac{\left(T_H - T_0\right)}{T_0}$$

$$ENR[dB] = 10 \cdot \log \left(\frac{\left(T_H - T_0 \right)}{T_0} \right)$$

$$G_{(DUT)}[lin] = \frac{N(SPA + DUT, Diodeon)[lin] - N(SPA + DUT, Diodeoff)[lin]}{N(SPA, Diodeon)[lin] - N(SPA, Diodeoff)[lin]}$$

N =noise power measured on the SPA for e.g. 1 MHz resolution bandwidth

$$F_{total}[linear-units] = F_1[linear-units] + \frac{F_2[linear-units]-1}{G_1[linear-units]} + \cdots$$

 The technique for noise figure measurement, which has been described above is commonly used for noise figure evaluation of amplifier in the RF and microwave range for frequencies higher than about 10 MHz.

Noise figure measurements (3)

Noise factor contributions of each stage in a chain follow this equation:

$$F_{cascade} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

Where the power gains and noise factors are the linear, not logarithmic, quantities. Note that the cascaded noise figure is based solely on the individual noise figure and gain at each stage.

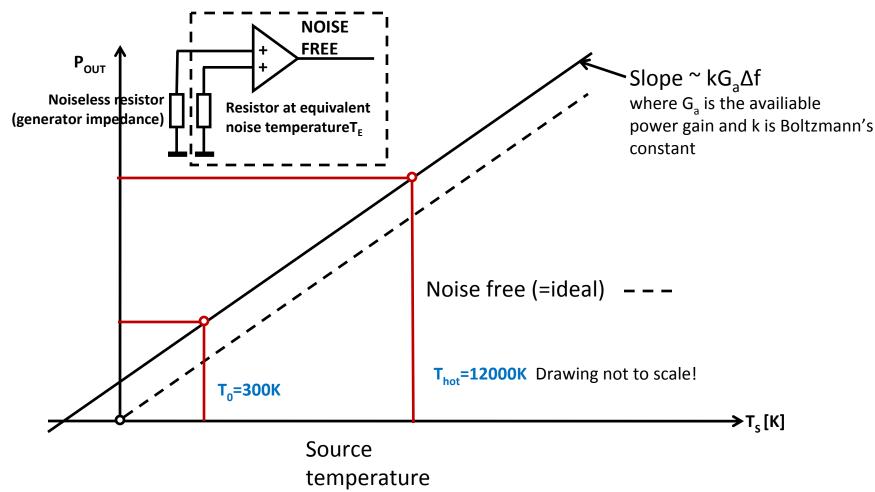
Example: Cascade of amplifiers

For a sufficiently high gain of the first amplifier, only the noise figure of this first stage is relevant for the overall performance of the complete chain.

Note that this formula also applies for attenuators where the power gain is smaller than unity

Noise figure measurements (4)

Visualisation of the Y-factor method:



Noise figure measurements (5)

- Practical application of the Y factor method:
- We obtain the gain of the DUT from the slope of the characteristic shown on the last slide and the noise temperature from the intersection with the negative horizontal axis
- Since we determine with the hot cold method two points on a straight line which defines unambiguously its position.
- In order to get clean measurements, we select the highest possible resolution BW (3 or 5 MHz) on the SPA as well as a very low Video BW (10 Hz) (smoothing of the trace) and a measurement time of about 1 second per frequency point
- At each measurement point, we have to turn on and off the noise diode, switching between 300 K and 12000K noise temperature
- Of course we would run this kind of measurements automatically, but for trouble shooting or if this automatic function is not available, we have to do it manually.