Low Level RF

1. Synchrotron Longitudinal dynamics
2. Beam based loops in synchrotrons
3. What will go wrong?
4. Power amplifier limits
5. Beam Loading
6. Longitudinal instabilities in synchrotrons
7. LLRF Cures
8. Design Example: Linac4

CAS RF

P. Baudrenghien CERN-BE-RF

Low Level RF

Part 1: Synchrotrons, Dynamics and Beam Based Loops

1. Synchrotron Longitudinal dynamics
2. Beam based loops in synchrotrons

CAS RF

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1. Synchrotrons RF

Definition:

Circular accelerator whose RF varies during acceleration to keep the particles on a centered orbit

1.1 Synchronous particle

- Definition: The Synchronous particle crosses the accelerating cavities at the same RF phase, turn after turn
- Equilibrium between momentum and dipole field
  - the radial component of the magnetic bending force must exactly compensate the centrifugal force (ρ being the magnet bending radius)

$$\frac{m.v^2}{\rho} = q.v.B$$

$$p = q.\rho.B$$

- To have coherent effect from turn to turn, the RF frequency must be locked to the revolution frequency (*h* is called the harmonic number)

$$f_{RF} = h.f_{rev}$$

$$f_{RF} = h.\frac{v}{2\pi R} = \frac{hc}{2\pi R}\beta$$

  - With $\beta = \frac{v}{c}$

- Using the relations between β (ratio of particle velocity to the velocity of light), *p* (momentum) and γ (ratio of particle total energy *E* to the rest energy E_0) we get (see appendix)

$$f_{RF} = \frac{hc}{2\pi R}\beta = \frac{hc}{2\pi R}\frac{1}{\sqrt{1+\left(\frac{E_0}{c.p}\right)^2}} = f_\infty\sqrt{1-\frac{1}{\gamma^2}}$$

- with the RF frequency at infinite energy $$f_\infty = \frac{hc}{2\pi R}$$

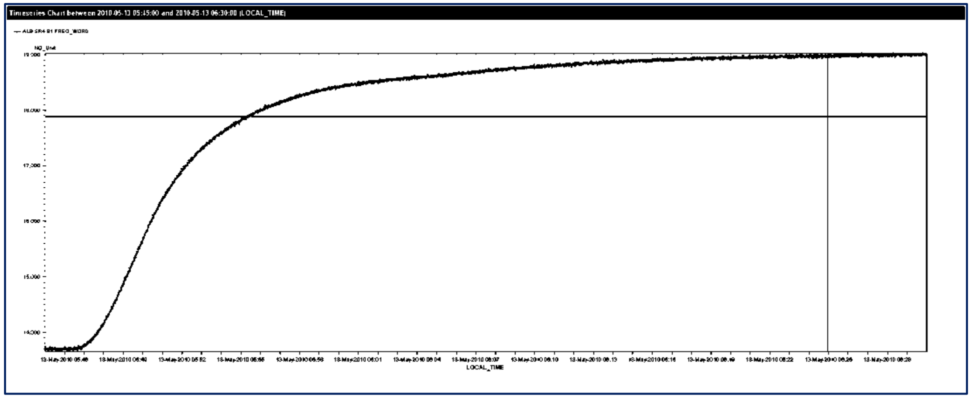
- Using the linear relation between the momentum and the dipole field, the above can be rewritten

$$\boxed{f_{RF} = f_\infty \frac{B}{\sqrt{B^2+\left(\frac{1}{c.\rho}\frac{E_0}{q}\right)^2}}}$$

- Frequency Ramp
  - Non-linear f_{RF} vs B relation
  - Frequency swing depends on the range of γ from injection to extraction
  - For highly relativistic machines (electrons) the RF frequency can be kept constant
  - Low energy proton or ion machines (high E_0/q) will have a large frequency swing
  - Heavy ions have a larger E_0/q ratio than protons because neutrons have no charge. (2.537 for the LHC Lead ions). If accelerated with the same magnetic ramp, the frequency swing will be larger
  - If the frequency swing is large it would best be controlled from a measurement of the dipole field

$$f_{RF} = f_\infty \frac{B}{\sqrt{B^2 + \left(\frac{1}{c\rho} \frac{E_0}{q} \right)^2}}$$

$$= f_\infty \sqrt{1 - \frac{1}{\gamma^2}}$$

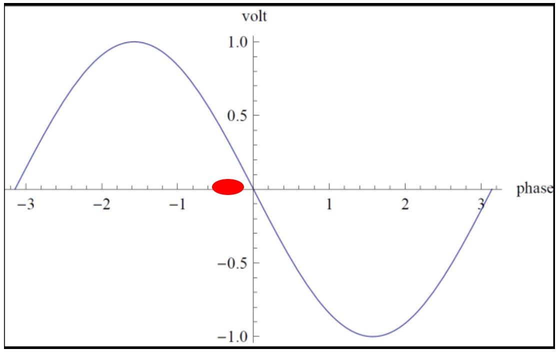


The 45 min long LHC frequency ramp from 450 GeV (400.788860 MHz) to 3.5 TeV (400.789 713 MHz)

Examples:

- e+e- (E_0=0.511 MeV) acceleration in the SPS as LEP injector, from 3 GeV to 22 GeV at constant frequency 200.395 MHz
- p (E_0=938.26 MeV) acceleration in the LHC from 450 GeV (400.788860 MHz) to 3.5 TeV (400.789713 MHz)
- Original p acceleration in the CPS (1959, h=20) from 50 MeV (2.9 MHz) to 25 GeV (9.54 MHz)
- Lead ions $^{208}Pb^{82+}$ acceleration in the SPS from 5.87 GeV/u (kinetic energy per nucleon) at 198.501 MHz to 160 GeV/u (200.393 MHz) for injection in the LHC

- Synchronous (or stable) phase
  - Let ϕ_s be the phase of the RF in the cavity when the particle crosses. The energy increase per turn is
    $$\Delta E_{turn} = q \cdot V \cdot \sin\phi_s$$
  - Now going to time derivative
    $$\frac{1}{f_{rev}}\frac{dE}{dt} = q \cdot V \cdot \sin\phi_s$$
  - or, from the relation between momentum and energy
    $$2\pi R\frac{dp}{dt} = q \cdot V \cdot \sin\phi_s$$
  - The LHS is defined by the machine momentum ramp. That, in turn, defines the product *V.sin* ϕ_s
    - In colliders *dp/dt=0* and the stable phase is zero
    - In fast ramping synchrotrons, ϕ_s can be as large as 30 degrees



Synchrotron above transition

1.2 Differential relations

- The previous section showed how the RF frequency must track the *B* field to keep the beam centered. This corresponds to imposing *R* *(R=0)* and *B*, and deriving *f*
- Of the four variables *(f,B,p,R)*, only two are independent. The relationship is non-linear but it can be linearized locally. This leads to four very useful differential relations:

$$p = p(R,B) \Rightarrow \frac{\Delta p}{p} = \gamma_t^{\,2} \frac{\Delta R}{R} + \frac{\Delta B}{B}$$

$$p = p(f,R) \Rightarrow \frac{\Delta p}{p} = \gamma^2 \frac{\Delta f}{f} + \gamma^2 \frac{\Delta R}{R}$$

$$B = B(f,p) \Rightarrow \frac{\Delta B}{B} = \gamma_t^{\,2} \frac{\Delta f}{f} + \frac{\gamma^2 - \gamma_t^{\,2}}{\gamma^2} \frac{\Delta p}{p}$$

$$B = B(f,R) \Rightarrow \frac{\Delta B}{B} = \gamma^2 \frac{\Delta f}{f} + \left(\gamma^2 - \gamma_t^{\,2}\right) \frac{\Delta R}{R}$$

- The transition energy γ_t will be presented later

- Matching the B field at injection: We measure radial displacement on first turn.

  Q: What is the correction to the B field to centre the beam?

  A: Use eq 1 with p constant
- Radial displacement of captured beam by trimming the RF (chromaticity measurement): We wish to displace the captured beam radially

  Q: How many Hz per mm ?

  A: Use eq 4 with B constant

$$p = p(R,B) \Rightarrow \frac{\Delta p}{p} = \gamma_t^{\,2}\frac{\Delta R}{R} + \frac{\Delta B}{B}$$

$$p = p(f,R) \Rightarrow \frac{\Delta p}{p} = \gamma^2 \frac{\Delta f}{f} + \gamma^2 \frac{\Delta R}{R}$$

$$B = B(f,p) \Rightarrow \frac{\Delta B}{B} = \gamma_t^{\,2}\frac{\Delta f}{f} + \frac{\gamma^2 - \gamma_t^{\,2}}{\gamma^2}\frac{\Delta p}{p}$$

$$B = B(f,R) \Rightarrow \frac{\Delta B}{B} = \gamma^2 \frac{\Delta f}{f} + \left(\gamma^2 - \gamma_t^{\,2}\right)\frac{\Delta R}{R}$$

1.3 Longitudinal Dynamics

- Let (p_s, ϕ_s) refer to the Synchronous particle. Consider another particle P with a slightly different (p, ϕ) . Given the small momentum difference P has also a different revolution frequency

$$\tilde{\phi} = \phi - \phi_s$$

$$\frac{d\tilde{\phi}}{dt} = -2\pi h \, \Delta f_{rev}$$

  we have a minus sign because ϕ is the RF phase when P crosses the cavity. This relation is kinematic only

- Dynamics: Crossing the cavity at a different RF phase, the momentum increase is different for P and for the Synchronous particle

$$2\pi R \frac{dp_s}{dt} = q \cdot V \cdot \sin\phi_s$$

$$2\pi R \frac{dp}{dt} = q \cdot V \cdot \sin\phi$$

$$2\pi R \frac{d\Delta p}{dt} = q \cdot V \cdot \sin\phi - q \cdot V \cdot \sin\phi_s$$

- The revolution frequency error can be related to the momentum error via the slippage factor η

$$\eta = \left[\frac{\frac{\Delta f_{rev}}{f_{rev}}}{\frac{\Delta p}{p}} \right]_{B=cst} = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}$$

- Differentiating the kinematic relation and using the above, we get

$$\frac{d^2 \tilde{\phi}}{dt^2} = -2\pi h \frac{d\Delta f_{rev}}{dt} = -\frac{2\pi \eta h f_{rev}}{p_s} \frac{d\Delta p}{dt}$$

- With

$$2\pi R \frac{d\Delta p}{dt} = q \cdot V \cdot \sin\phi - q \cdot V \cdot \sin\phi_s$$

we derive a second-order non-linear equation describing the synchrotron motion. Notice the non-linearity (sine)

$$\boxed{\frac{d^2 \tilde{\phi}}{dt^2} + \frac{\eta f_{RF}}{R p_s} q \cdot V \left(\sin\phi - \sin\phi_s \right) = 0}$$

- Small amplitude oscillations: We can linearise the previous equation and we get

$$\frac{d^2\tilde{\phi}}{dt^2}+\left[\frac{\eta f_{RF}\cos\phi_s}{Rp_s}q\cdot V\right]\tilde{\phi}=0$$

$$\boxed{\begin{gathered}\frac{d^2\tilde{\phi}}{dt^2}+\Omega_s^2\,\tilde{\phi}=0\\ \Omega_s=\sqrt{\frac{\eta f_{RF}\cos\phi_s}{Rp_s}q\cdot V}\end{gathered}}$$

- This synchrotron equation represents an undamped resonator with resonant frequency Ω_s called the synchrotron frequency
- Given a phase or momentum error as initial conditions, the particle will oscillate endlessly around the stable phase, exchanging longitudinal displacement with momentum offset
- The period of the synchrotron frequency is the characteristic time-response of the beam. We will call "adiabatic" the evolutions that are slow with respect to this period
- Periodic motion is possible only if $\eta.\cos(\phi_s) > 0$

$$\boxed{\eta=\frac{1}{\gamma^2}-\frac{1}{\gamma_t^2}}$$

$$\boxed{\begin{gathered}\gamma\le\gamma_t\Rightarrow\eta\ge0\Rightarrow\cos\phi_s\ge0\Rightarrow\phi_s\in\left[0,\frac{\pi}{2}\right]\\ \gamma\ge\gamma_t\Rightarrow\eta\le0\Rightarrow\cos\phi_s\le0\Rightarrow\phi_s\in\left[\frac{\pi}{2},\pi\right]\end{gathered}}$$

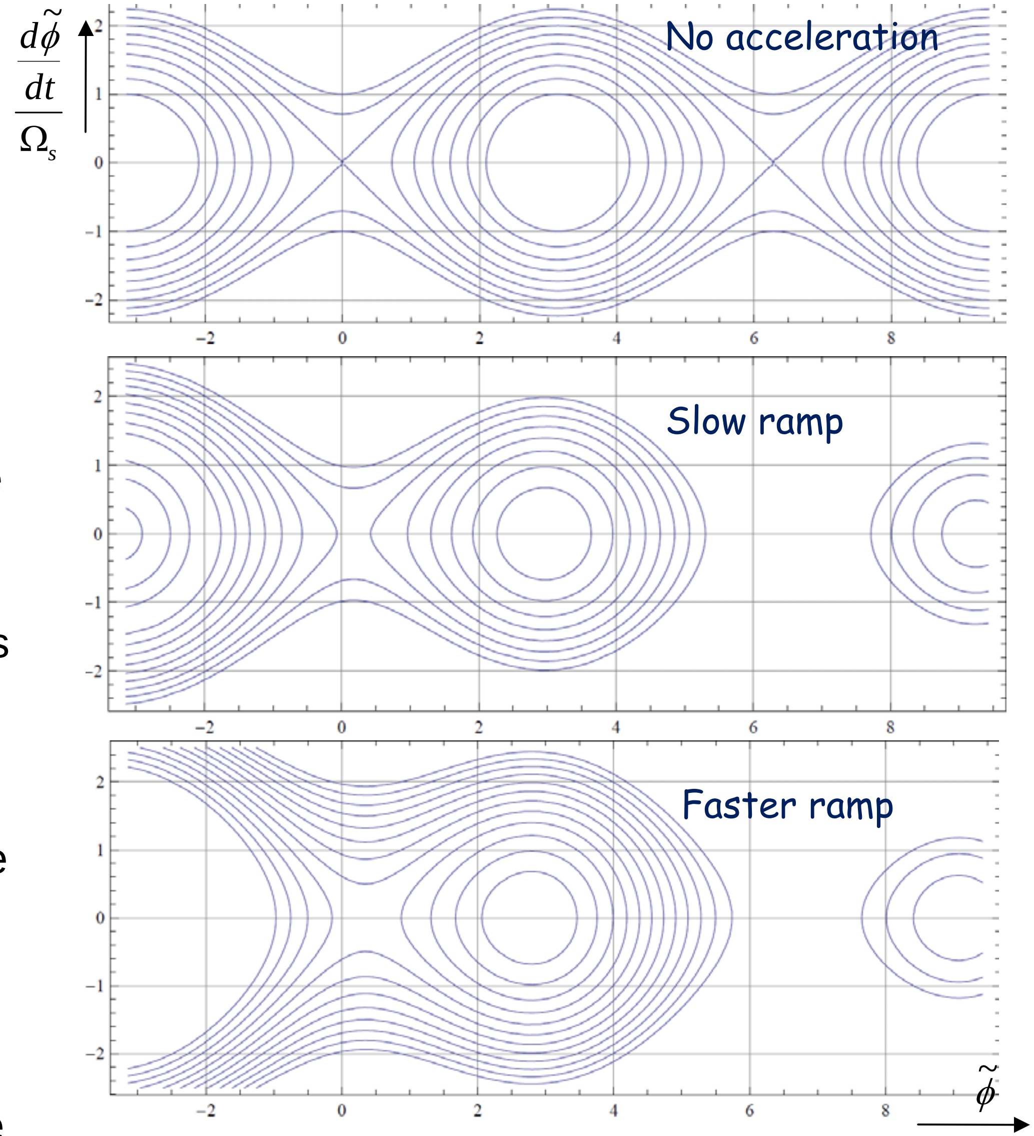
Acceleration below transition

Acceleration above transition

- After integration, the equation of synchrotron motion becomes

$$\frac{1}{2}\left(\frac{\frac{d\tilde{\phi}}{dt}}{\Omega_s}\right)^2 - \frac{\left(\cos\tilde{\phi} + \tilde{\phi}\sin\phi_s\right)}{\cos\phi_s} = C$$

- For small deviations from the stable phase the trajectories are ~ circular in phase space
- For larger deviations the trajectories are deformed, but still closed (stable)
- Above some excursion the trajectories are not closed any more and these particles are not controlled by the RF
- The limiting closed trajectory is called the separatrix. The enclosed surface in phase space is called the bucket area



Trajectories in (ϕ, $d\phi/dt/\Omega s$) phase space for synchronous phase 180 degrees (top), 170 and 160 degrees

- The previous phase space plots are in normalized (ϕ, $1/\Omega s\,(d\phi/dt)$) units. The trajectories are similar if the horizontal axis is in time, and the vertical axis in Δp or ΔE (momentum or energy deviation)
- The bucket area A is usually expressed in physical Energy.Time unit (eVs)

Constant

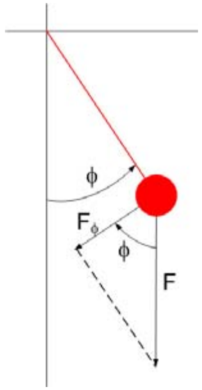
Energy dependent

RF parameters

$$\boxed{A = \left(\frac{16\sqrt{h\,q}}{(2\pi)^{3/2}}\right)\left(\frac{\beta\sqrt{E}}{f_{RF}\sqrt{|\eta|}}\right)\sqrt{V}\,\alpha(\phi_s)}$$

- The function $\alpha(\phi_s)$ is a non-linear function describing the rapid reduction of bucket area with the stable phase. It is equal to 1 for 0 or 180 degrees and drops to 0.3 for 30 or 150 degrees
- The particles will occupy an area inside the bucket. We call this area the bunch longitudinal emittance.
- The RF voltage must be dimensioned to allow for capture and acceleration without loss. The bucket area must always be significantly larger than the bunch emittance. The ratio is called the filling factor.

Analogy with the pendulum

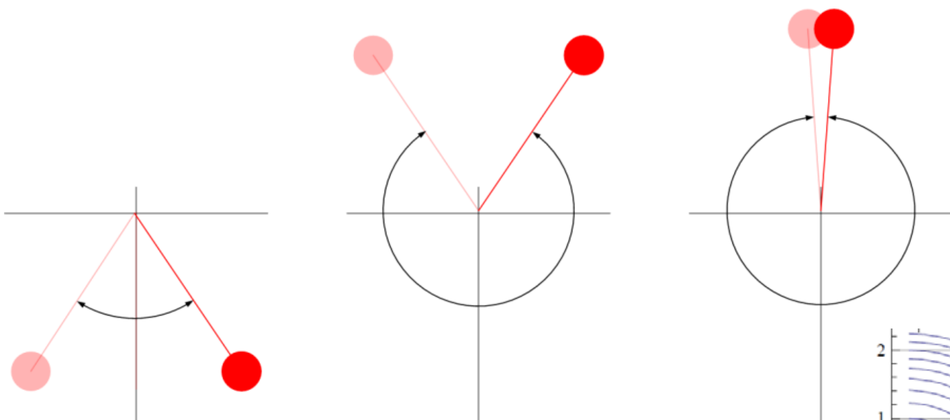


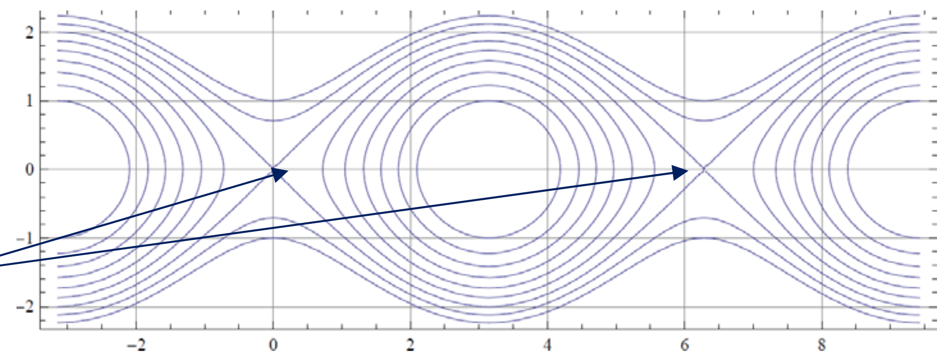
$$mj_\phi = F_\phi$$

$$-mR\frac{d^2\phi}{dt^2} = mg\sin\phi$$

$$\frac{d^2\phi}{dt^2} + \frac{g}{R}\sin\phi = 0$$

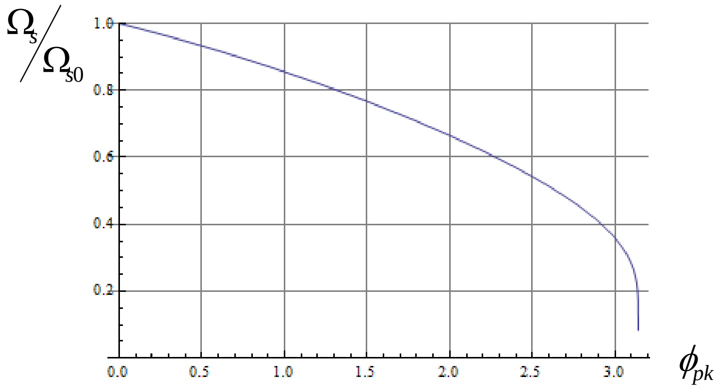
- Same as synchrotron oscillation for $\phi_s = 0$
- The period of oscillation does not depend on the amplitude for small deviations
- For larger deviations the period increases
- The period becomes infinite is the oscillation starts at the unstable +-π angles. This is the exact equivalent of the separatrix

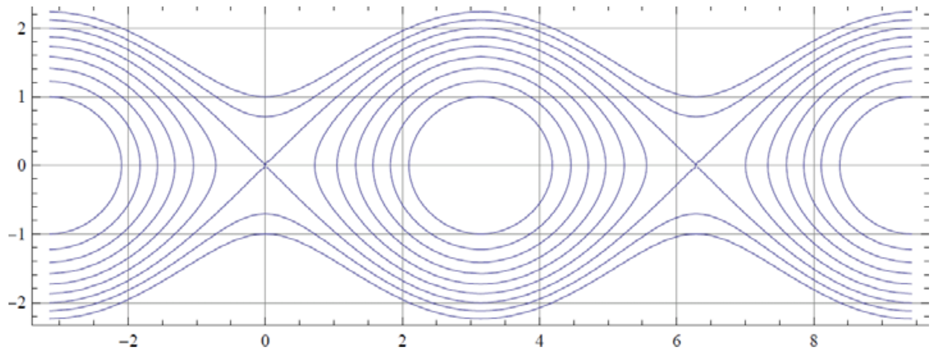




When the amplitude of the oscillation gets large, the pendulum spends more time at the extremes of deviations.

Variation of Synchrotron Frequency vs Peak deviation of trajectory





- For $\phi_s = 0$ we have a formula for the frequency of synchrotron oscillation vs peak phase ϕ_{pk} between 0 and π

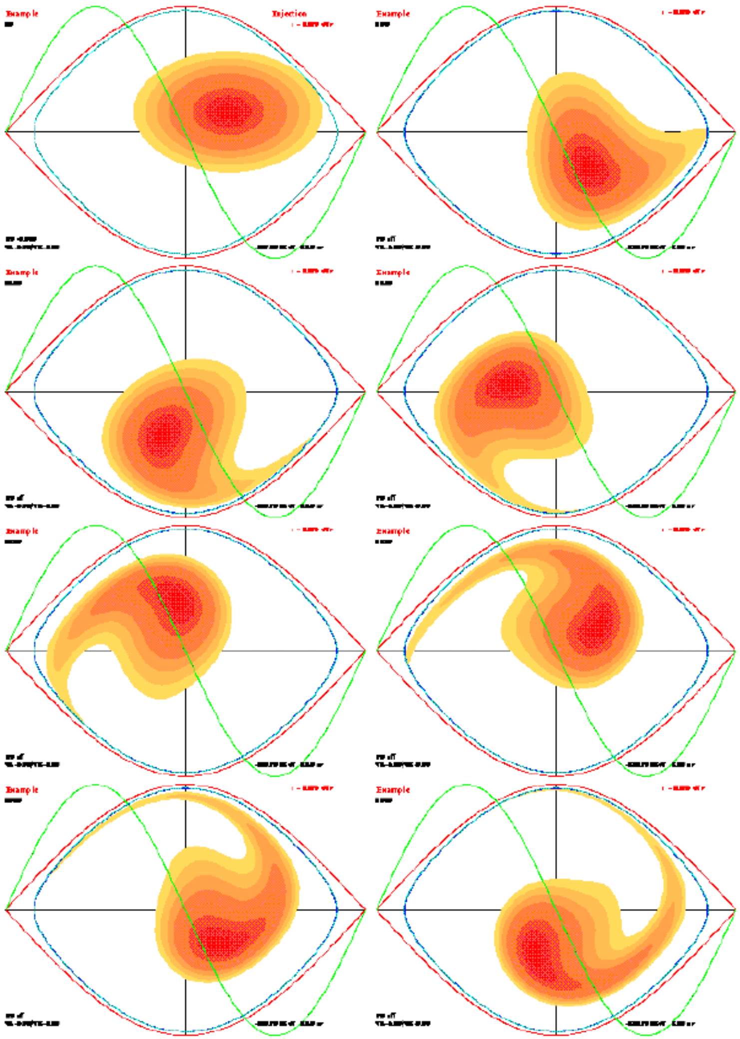
$$\boxed{\Omega_s(\phi_{pk}) = \frac{\pi\,\Omega_{s0}}{2\displaystyle\int_0^{\pi/2} \frac{du}{\sqrt{1-\sin^2\left(\frac{\phi_{pk}}{2}\right)\sin^2 u}}}}$$

- Given its length, the bunch will have a spread in the Synchrotron Tunes of the various particles. The longer the bunch, the larger the Tune Spread.
- In Hadron machines this Tune Spread will provide a stabilizing mechanism against coherent instabilities, called Landau Damping.

Harmonic systems: Adding an harmonic system (2x or 4x RF) we can shape the Synchrotron Tune vs. Peak deviation curve.
We may wish to increase the spread to increase Landau damping (200/800 MHz systems in the SPS).
Or we may wish to reduce the spread, that is make the potential more linear, to reduce the filamentation at injection and give time for a longitudinal damper to centre the bunch in phase space (example in Lecture 2).
Both are possible by adjusting the relative amplitude and phase of the fundamental and harmonic

Filamentation

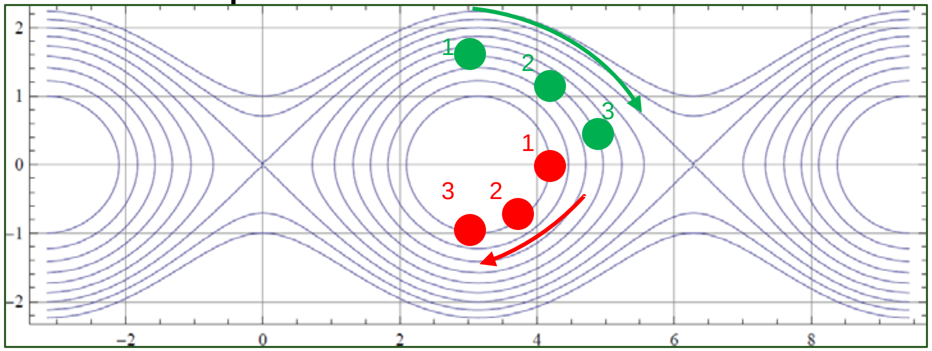
- Because the synchrotron equation is non-linear, the period of the various closed orbits are not all equal
- The synchrotron period is a correct approximation for small deviations from the synchronous particle
- For large oscillations the motion is slower
- On the separatrix the period is infinite. This is just like the pendulum with a +-180 degrees excursion
- If the bunch is injected off-centered, parts of the bunch will lag behind the core, resulting in filamentation in phase space
- After complete filamentation, the emittance will be much larger, filling the entire space within the blue trace in the simulation



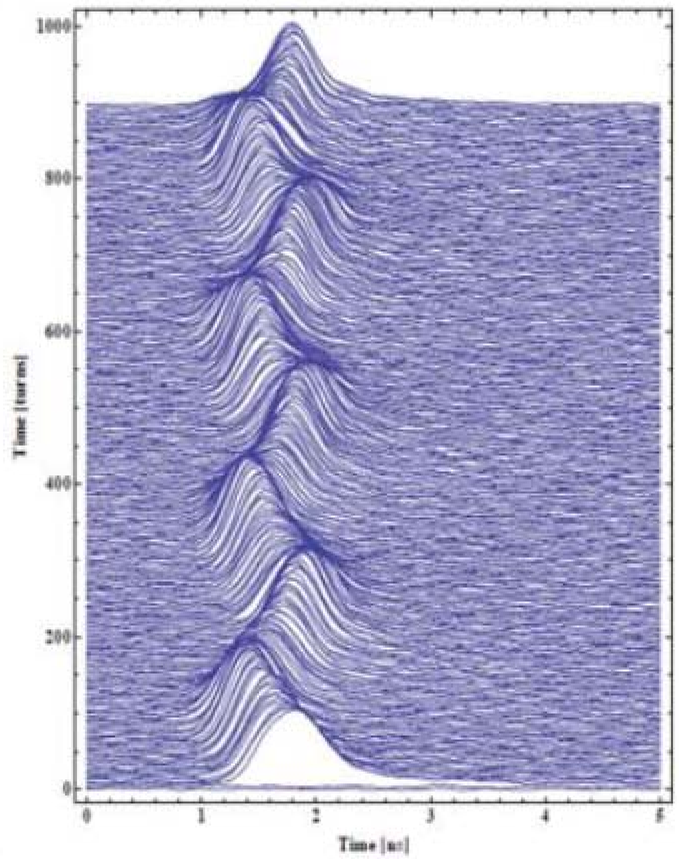
Simulation of the filamentation at injection in the LHC bucket. The bunch is injected with a small phase/momentum error. The separatrix is in red. After filamentation the bunch will fill the area inside the blue contour
Courtesy of J. Tuckmantel

Application: Fine-adjust the capture

- If the bunch is injected RF ON, with phase or energy error, it will first undergo a dipole oscillation that can be measured on a Pick Up

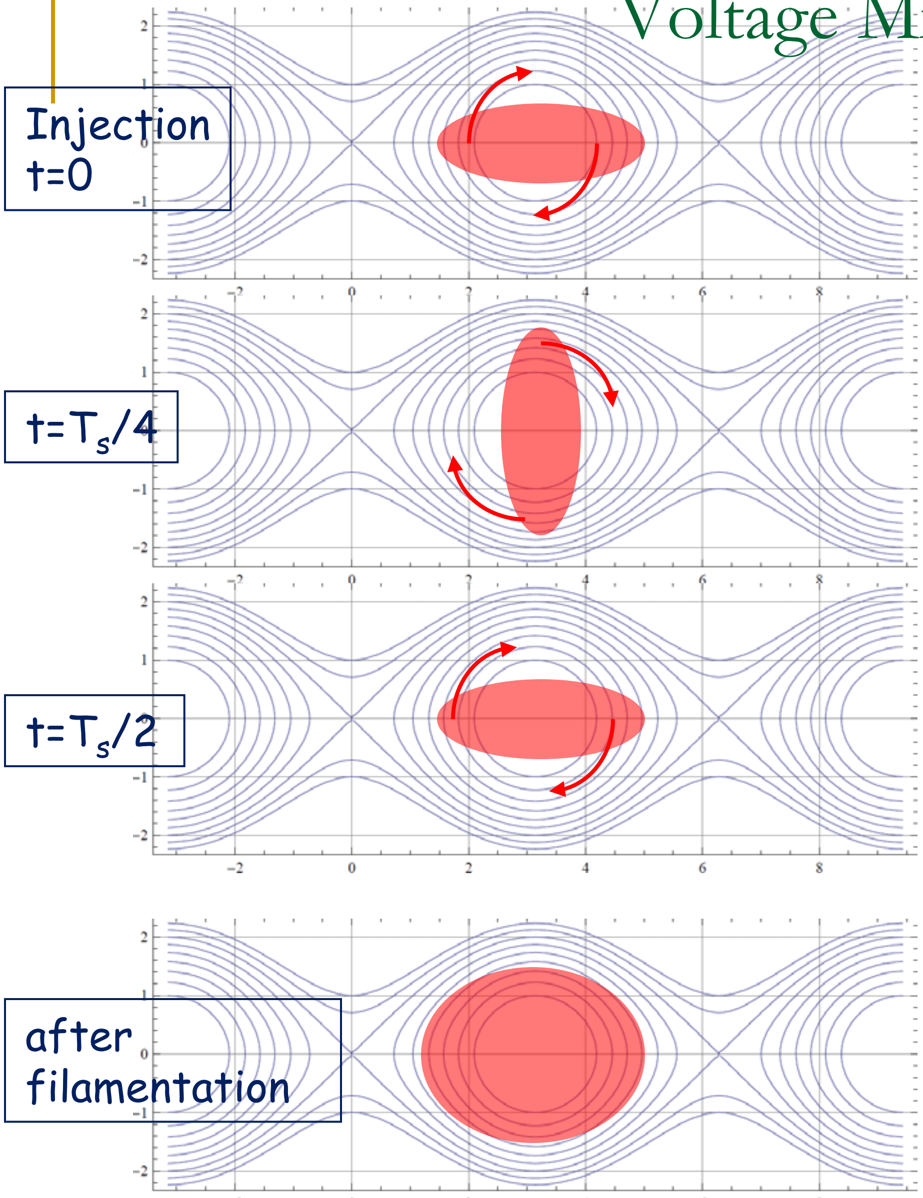


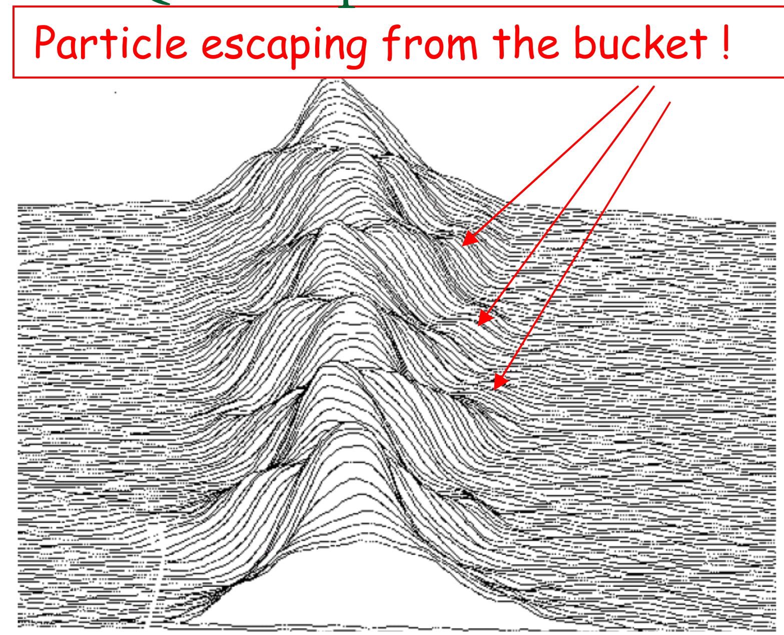
- For phase error only, the phase oscillation is a cosine wave as it starts with the maximal amplitude (red dots 1,2,3,… above)
- For energy error only, the oscillation is a sine wave (green dots 1,2,3,…)



Dipole oscillation at injection. The phase is a sine wave, indicating an energy error

Voltage Mismatch. Quadrupole oscillations



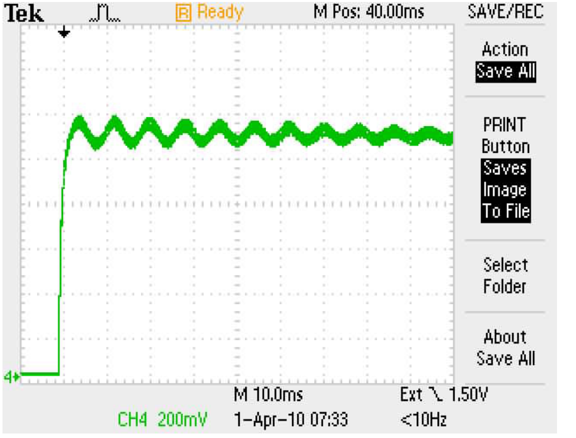


Quadrupole oscillation at injection (plus some dipole and loss) indicating a voltage mismatch. Voltage too high.

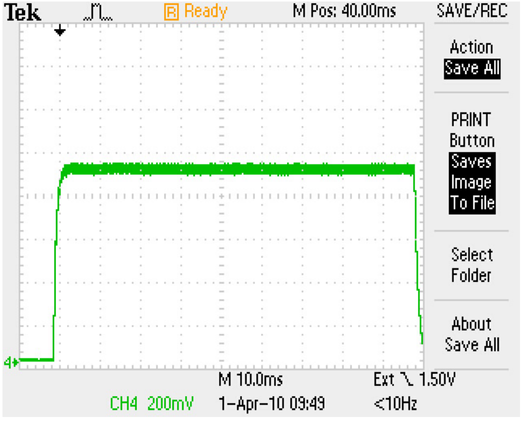
- If the bunch emittance does not match the RF voltage (phase space trajectories), the bunch length/peak are modulated at 2 Ω_s
- Filamentation will end-up filling an area contained within a closed trajectory -> emittance blow-up

Voltage matching looking at PU pk

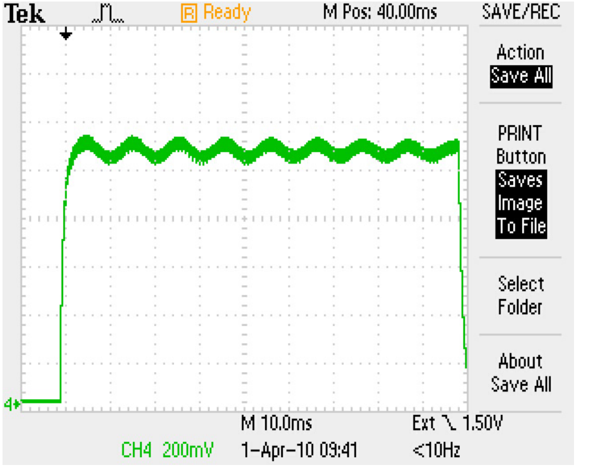
- Capture of LHC fat pilot 1E10
- 0.23 eVs, 1.2 ns long bunch



8 MV



2.5 MV GOOD!



5 MV

Radiation damping

- When a relativistic particle is accelerated it radiates energy at a rate proportional to the square of the accelerating force
- In a circular accelerator the main accelerating force is the bending of the trajectory. For a particle moving at constant speed on a circular orbit of radius ρ, the power radiated is

$$P_\gamma \propto \frac{\beta^4 \gamma^4}{\rho^2}$$

- The radiated energy must be compensated by the RF voltage. At 104.5 GeV per beam, LEP required 3.66 GV RF
- As the radiated power increases with energy, the mechanism will have a damping effect on the synchrotron oscillation: When the energy is larger than the synchronous particle it will radiate more and thereby loose part of the excess. GOOD

- The damping rate α_ε is proportional to the change of radiated loss with energy
- One can derive the simple expression

$$\alpha_\varepsilon \approx \frac{\langle P_\gamma \rangle}{E_s}$$

where the numerator is the power radiated by the synchronous particle and the denominator is the energy of the synchronous particle

- With radiation damping the synchrotron equation becomes

$$\boxed{\frac{d^2\tilde{\phi}}{dt^2} + 2\alpha_\varepsilon \frac{d\phi}{dt} + \frac{\eta f_{RF}}{Rp_s} q \cdot V\left(\sin\phi - \sin\phi_s\right) = 0}$$

- Radiation damping is significant for circular electron accelerators and colliders only because the radiated power scales as γ^4 and hadrons are not relativistic enough yet. In the LHC the radiation damping time is ~24 hours at 7 TeV. Not much damping! The 7 TeV p in the LHC have γ~7000 while the 100 GeV e- in LEP had γ~200000

Adiabatic evolution during ramping

- So far we have considered the synchronous particle parameters (p_s, ϕ_s, V) as constant and have moved them out of the derivatives
- In an accelerator the momentum increases during the ramp. The voltage is matched to the injector at capture, then increased during the ramp to keep a sufficient bucket area
- These parameters will be modified slowly compared to the synchrotron period
- Boltzman-Ehrenfest adiabatic theorem: "*If (p,q) are canonically conjugate variables of an oscillatory system with slowly changing parameters, then the action integral, evaluated over one period of oscillation, is constant*"

$$I = \oint p\,dq = C$$

- By applying this theorem to a closed trajectory in the longitudinal phase space we can get the following relations describing the evolution of the maximum time (Δt) and energy (ΔE) deviations in E-t phase space, for adiabatic ramping (changes of E) and adiabatic voltage (V) variations

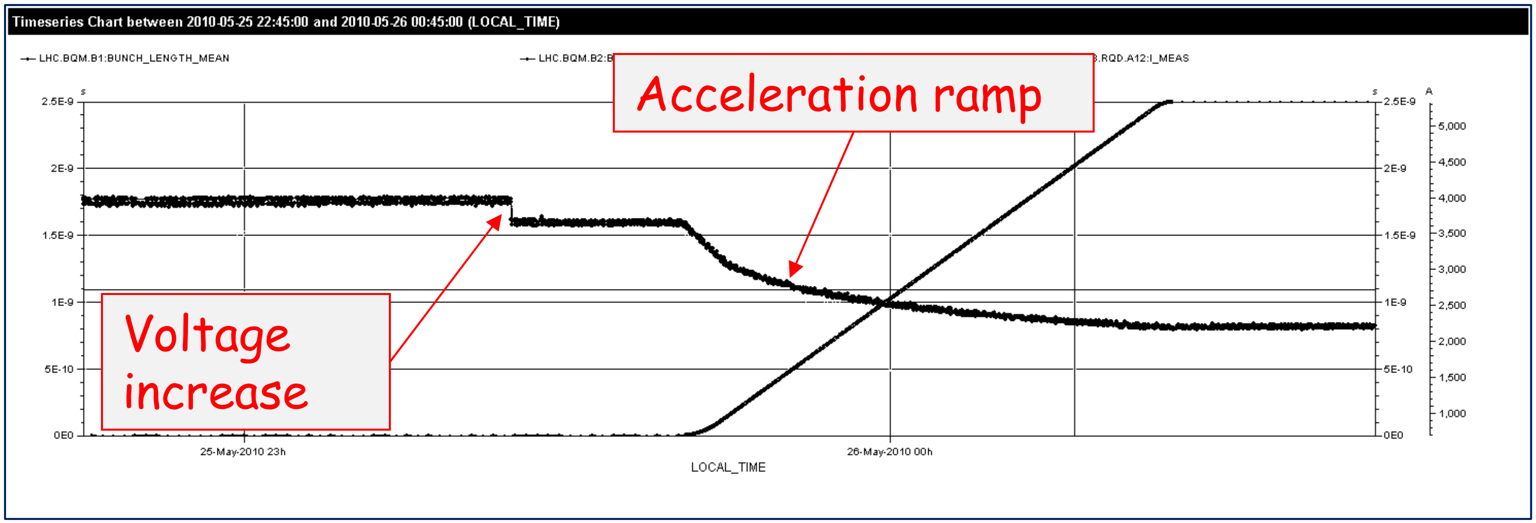
$$\Delta t \propto \frac{1}{f_{RF}} \sqrt[4]{\frac{|\eta|}{\gamma V \cos\phi_s}}$$

$$\Delta E \propto f_{RF} \sqrt[4]{\frac{\gamma V \cos\phi_s}{|\eta|}}$$

To be recalled:

Apart from the singularity at transition ($\eta=0$), bunch length shrinks and energy spread increases with voltage increase and slow ramping (constant stable phase), if adiabatic. The effect is moderate (fourth root)

- Applying these formulas to the outer trajectory of the bunch in E-t phase space we conclude that the longitudinal emittance (in eVs) remains constant during adiabatic evolution



Bunch length evolution in the LHC: capture at 450 GeV with 3.5 MV, voltage increase to 5 MV before start ramp then rise to 8 MV in first part of the ramp (up to 3.5 TeV).
Bunch length (4σ) B1: 1.82 ns -> 1.61 ns -> 0.83 ns / B2: 1.75 ns -> 1.58 ns -> 0.77 ns
Measured by the LHC Beam Quality Measurement (BQM) system, G. Papotti

Annex

- Useful relations between E (total energy), E_0 (rest energy), p (momentum), v (speed), γ and β

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{E}{E_0}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$E^2 = E_0^2 + c^2 p^2$$

$$\frac{dE}{dp} = \frac{p c^2}{E} = v$$

$$\beta = \frac{1}{\sqrt{1 + \left(\frac{E_0}{cp}\right)^2}}$$

- Electron and proton rest energy

$$E_{0p} = 938.26\ MeV$$

$$E_{0e} = 0.511\ MeV$$

2. Beam based loops for Synchrotrons

Loops that use signals from beam Pick-ups, either longitudinal (beam phase) or transverse (beam position) and that act on all bunches

2.1 Beam Phase Loop

- Motivation:
  - Transients will be triggered by energy/voltage mismatch at capture
  - RF noise will excite the synchrotron oscillation of each particle individually.
- In electron machines the synchrotron light provides a natural damping mechanism and will be sufficient in most cases except for the injection transient
- In proton and ion machines the injection oscillations will last, resulting in emittance blow-up due to filamentation. The bunch lengthening caused by the RF noise may lead to beam loss when particles reach the separatrix (major concern in colliders where beams are kept colliding for 10 hours).

Analysis

- Let us first consider the synchrotron oscillation in presence of a small $\delta\omega_{RF}$ modulation of the RF frequency. The kinematic relation for the RF phase at cavity crossing time becomes

$$\frac{d\tilde{\phi}}{dt} = -2\pi h \Delta f_{rev} + \delta\omega_{RF}$$

  The first term is the effect of the momentum error and the second term is the cavity RF frequency modulation.

- Following the derivation of the previous section, the linearised synchrotron oscillation equation becomes

$$\boxed{\frac{d^2\tilde{\phi}}{dt^2} + \Omega_s^2\,\tilde{\phi} = \frac{d\,\delta\omega_{RF}}{dt}}$$

Phase Loop

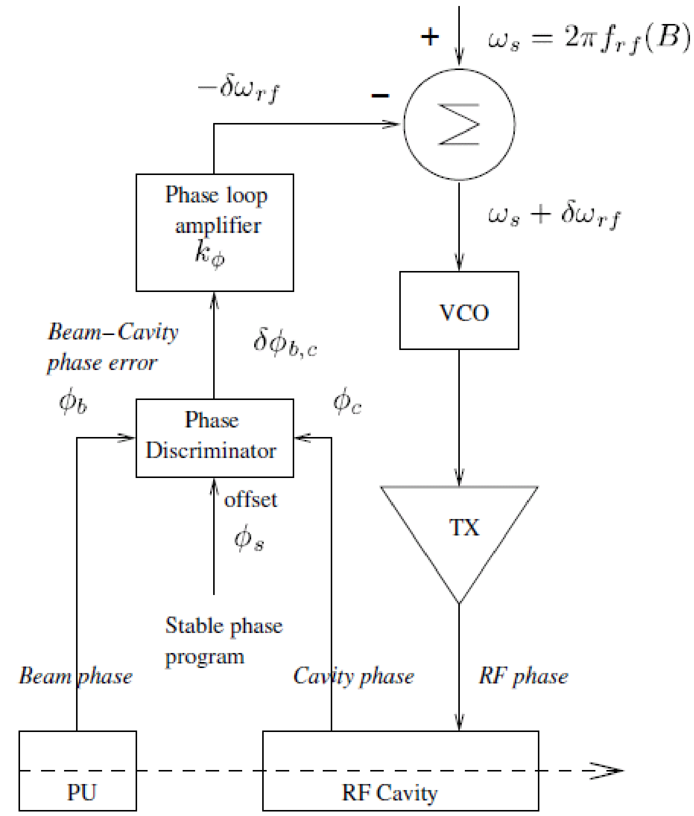
- We measure the phase error between Cavity Sum and the Beam (Longitudinal PU) averaged over all bunches
- This error is used to correct the RF frequency via the Phase Loop amplifier. The simplest regulation is proportional only.
- We have

$$\delta\omega_{RF} = -k_\varphi \left\langle \tilde{\phi} \right\rangle$$

and the differential equation becomes

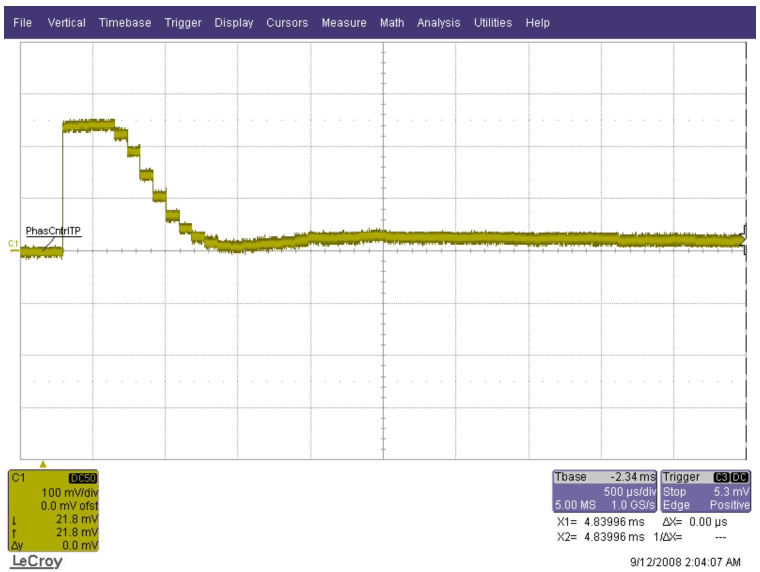
$$\boxed{\frac{d^2\left\langle \tilde{\phi} \right\rangle}{dt^2} + k_\varphi \frac{d\left\langle \tilde{\phi} \right\rangle}{dt} + \Omega_s^2 \left\langle \tilde{\phi} \right\rangle = 0}$$

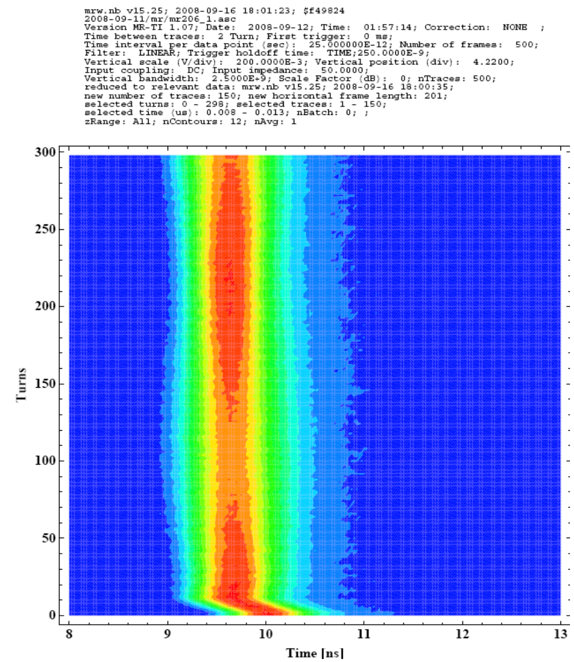
the synchrotron frequency is not changed but we have introduced the desired damping term



The phase loop effect is similar to radiation damping but…it acts on the average phase error only

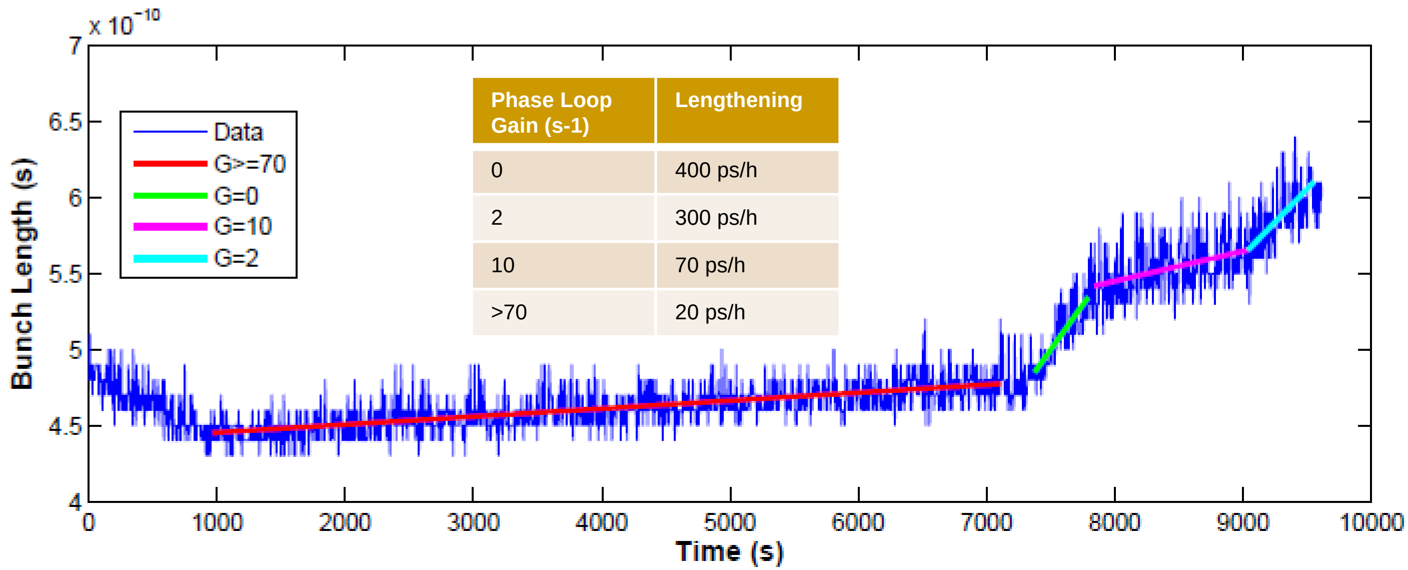
- Damping of injection transient:
  - The phase loop must be fast compared to the synchrotron period to avoid filamentation
  - It damps the average phase oscillation of the bunch. This is called dipole mode as the bunch moves back and forth in the bucket without shape change
  - It is very effective on injection phase and energy errors





Sept 12, 2008. First capture of LHC ring 2 beam with Phase Loop ON. Left: Phase error. Rigth: Mountain range dislay. Notice the rapid damping of the injection phase error and the lasting quadrupole oscillations caused by voltage mismatch (see next slides). Courtesy of T. Bohl

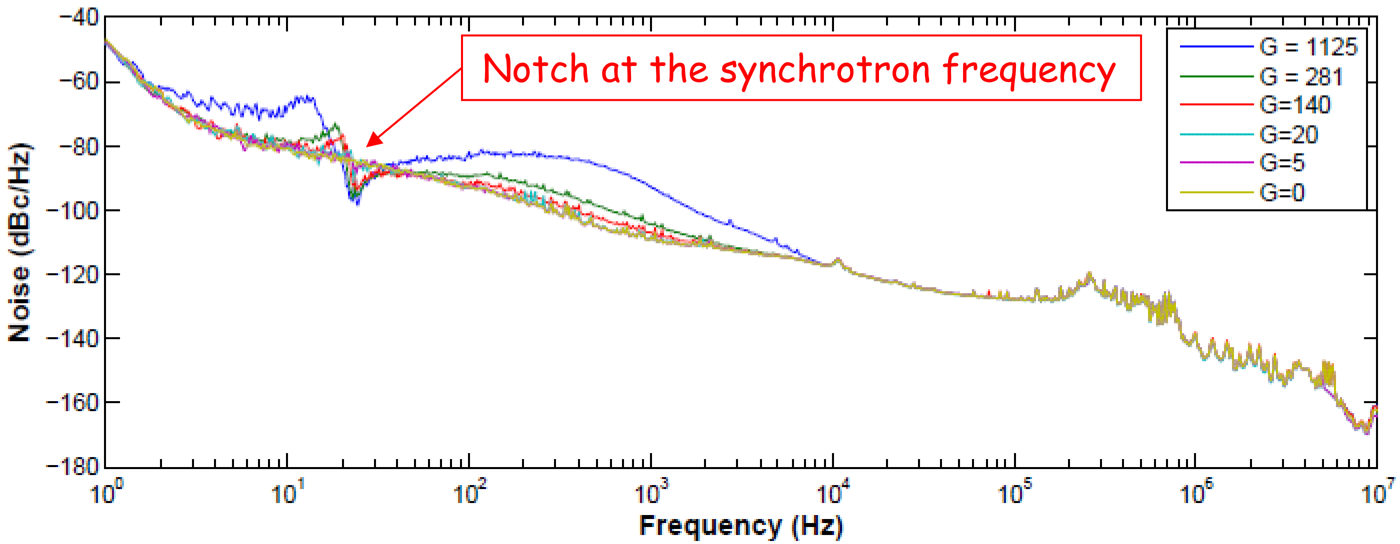
- Preventing emittance blow-up:
  - In colliders phase noise is much more damaging than amplitude noise because the beam sits at stable phase 0 or 180 degree
  - Beams must be kept colliding for > 10 hours. So effects of noise is critical
  - The Phase Loop was essential in the SPS proton-antiproton collider and it is essential in the LHC



Phase Loop Gain (s-1)	Lengthening
0	400 ps/h
2	300 ps/h
10	70 ps/h
>70	20 ps/h

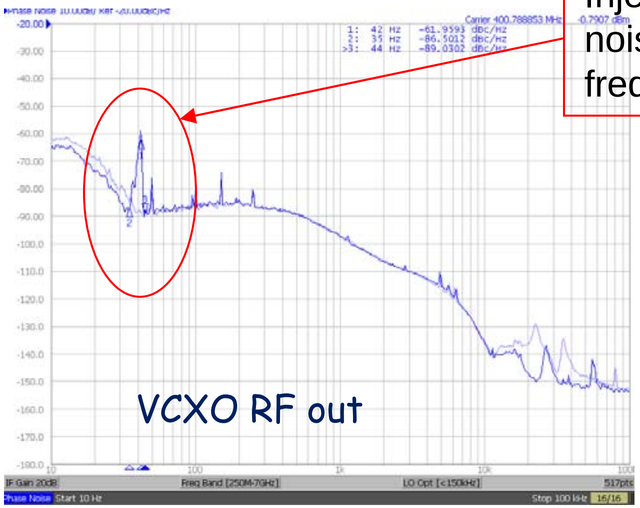
LHC single bunch at 3.5 TeV 1E10 p. Evolution of bunch length with time as a function of the Phase Loop gain. Fixed 8 MV RF (T. Mastorides, P. Baudrenghien, May 2010). LHC BQM system.

- But how can this work?
  - The Phase loop measures the bunch average only and there is no coherent dipole oscillation in bunch lengthening…
  - The explanation is that the phase loop reduces the phase noise in the cavity sum signal in the synchrotron band. In this band, the beam gives a coherent response that is measured by the loop and damped via the modulation of the VCXO
  - Outside the synchrotron band the phase loop does nothing…except inject noise in the cavity, as there is no response from the beam
  - Note that it acts on the first synchrotron band only

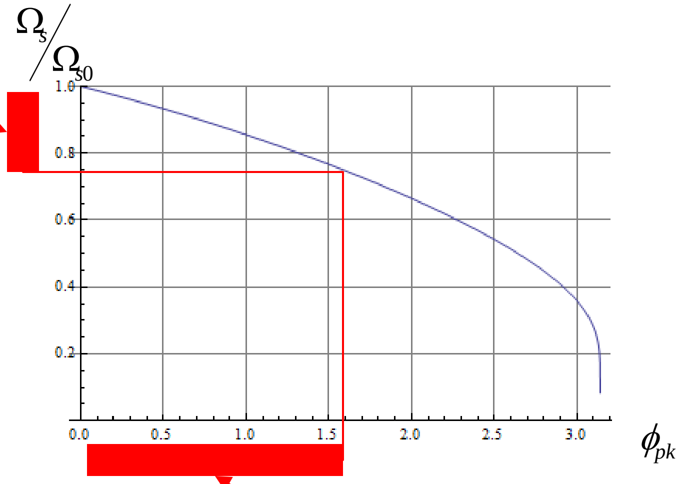


PSD of the phase noise in dBc/Hz in a LHC cavity with circulating 3.5 TeV bunch, for various phase loop gains (in s^{-1}), 1.66 MV. The synchrotron frequency is ~ 24 Hz

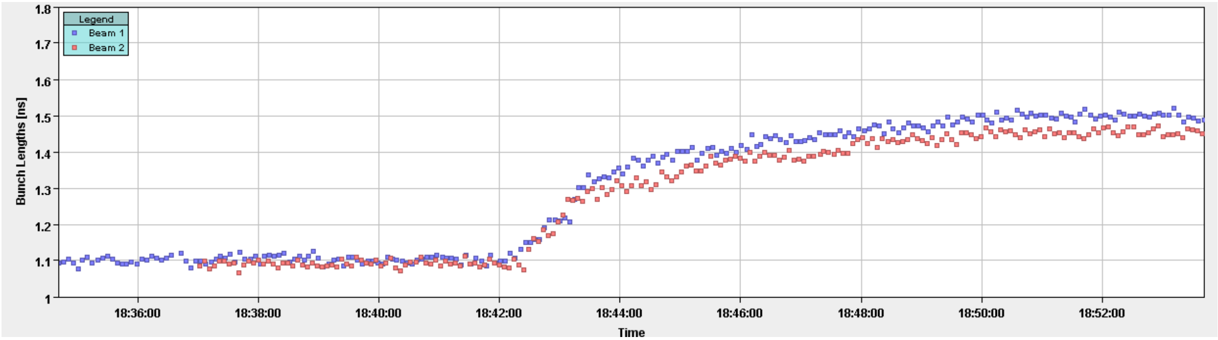
Reciprocity: Phase noise with narrowband spectrum at the Synchrotron Frequency of the core of the bunch, can be injected for controlled emittance blow-up



Inject phase noise in a narrow frequency band…



…blows-up the core of the bunch only



LHC single bunch at 450 GeV 2E10 p. Evolution of bunch length during emittance blow-up with a 0.6 deg rms phase noise covering the 36-42Hz band (42 Hz = Ω_{s0}). The bunch length grows quickly from 1.1 to 1.4 ns then settles (June 7, 2010. E. Chapochnikova, J. Tuckmantel, G. Papotti, A. Butterworth, M.E. Angoletta, P. Baudrenghien)

… and now a bit of discipline

- Motivation: In the phase loop, the beam is the Master. The RF will do its best to please it. If there is an energy error at injection, the RF will change its frequency. If there is noise in the cavity in the delicate synchrotron band the RF will be modulated to minimize this noise. That will preserve emittance but… it is not a stand-alone solution because:
  - If there are several injections, the RF must be restored to an injection frequency after transient to prepare for the next injection
  - When we start ramping, the RF must track the B field to keep the beam centered
  - If we transfer to another machine, the RF must be synchronized to the buckets of the receiving machine
  - There is no mechanism to keep the beam centred
- Solution: we will add a slower loop that will "discipline" the beam. It will be gentle enough so that it does not perturb the all-important phase loop. Gentle means adiabatic = "slow compared to the synchrotron period".
- Several options. Two big classics:
  - Radial Loop: We slowly adjust the RF to keep beam centered as measured in one or several PUs
  - Synchro Loop with frequency program reference: We keep the RF softly locked onto a Synthesizer whose frequency tracks the B field to keep the beam centered

2.3 Radial Loop

- We measure the radial position error average
- This error is used to correct the RF frequency via the Radial Loop amplifier. The simplest regulation is proportional only.
- We have

$$\delta\omega_{RF} = -k_\varphi \left\langle \tilde{\phi} \right\rangle - k_R \,\delta R$$

- At constant B, the radial position error is proportional to a momentum error

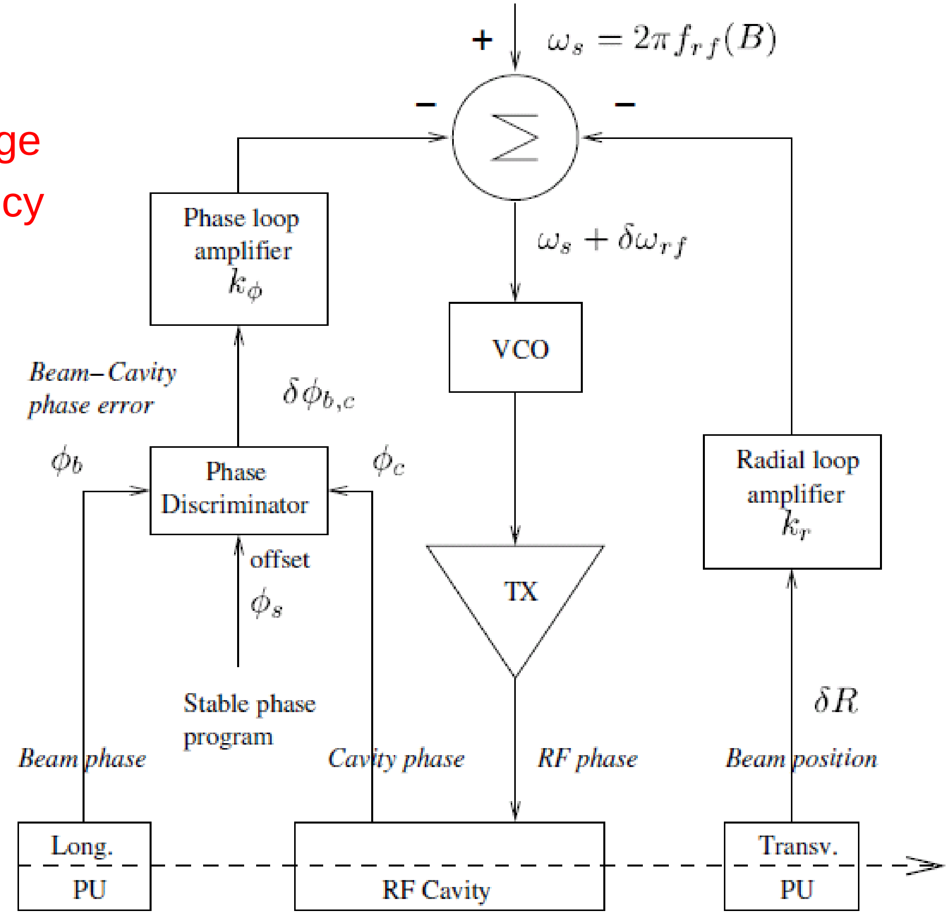
$$\frac{\delta R}{R} = \frac{1}{\gamma_t^{\,2}} \frac{\langle \Delta p \rangle}{p}$$

- We have

$$\delta\omega_{RF} = -k_\varphi \left\langle \tilde{\phi} \right\rangle - k_R \frac{R}{\gamma_t^2 \, p_s} \langle \Delta p \rangle$$

and the differential equation becomes

$$\boxed{\frac{d^2 \left\langle \tilde{\phi} \right\rangle}{dt^2} + k_\varphi \frac{d \left\langle \tilde{\phi} \right\rangle}{dt} + \left(\Omega_s^2 + k_R \frac{qV\cos\phi_s}{2\pi p_s \gamma_t^2} \right) \left\langle \tilde{\phi} \right\rangle = 0}$$



A classic combination for proton and ion synchrotrons: Phase loop and Radial loop

$$\boxed{\frac{d^2\langle\widetilde{\phi}\rangle}{dt^2}+k_\varphi\frac{d\langle\widetilde{\phi}\rangle}{dt}+\left(\Omega_s^2+k_R\frac{qV\cos\phi_s}{2\pi p_s\gamma_t^2}\right)\langle\widetilde{\phi}\rangle=0}$$

- The radial loop does not provide damping. It only increases the frequency of oscillation
- The sign of the gain must be changed at transition because *cos* ϕ_s changes sign there. It is the preferred loop for crossing transition
- Recall that, at constant B we have

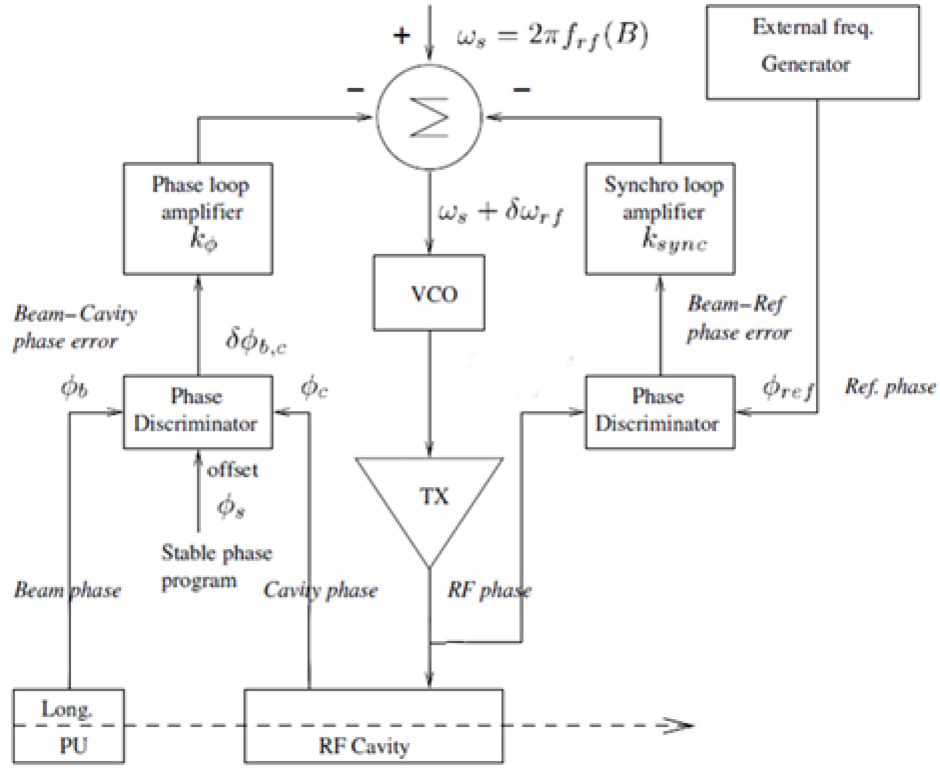
$$\frac{\Delta f}{f}=\left(\frac{\gamma_t^{\ 2}}{\gamma^2}-1\right)\frac{\Delta R}{R}$$

  the radial loop amplifier must have positive gain below transition and negative gain above
- It reduces the effect of frequency errors on the radial position
- Its gain must be small enough that its effects on the beam remain adiabatic
- Limits: It couples the two planes using transverse measurements to estimate momentum and correct the frequency. This causes problems:
  - Betatron oscillations interpreted as momentum error. Can be minimized by using two pick-ups at 180 degrees in betatron phase
  - Typically looks at one or few PUs only -> centers the beam in one location only instead of the average orbit
  - Position measurements are more noisy and more sensitive to intensity than frequency measurements

2.4 Synchro Loop

- We measure the phase of the RF (or beam) and compare it to a reference generator
- This error is used to correct the RF frequency via the Synchro Loop amplifier
- The reference generator will be set at the injection frequency during filling, then follow the frequency program during ramping and finally be locked to the receiving machine if needed for transfer
- The overall system is described by a third order differential equation. Its analysis is easier if we use Laplace Transforms
- First the beam transfer function (undamped resonator at Ω_s)

$$\tilde{\phi}(s) = \frac{s}{s^2+\Omega_s^2}\,\delta\omega_{RF}(s) = B_\phi(s)$$



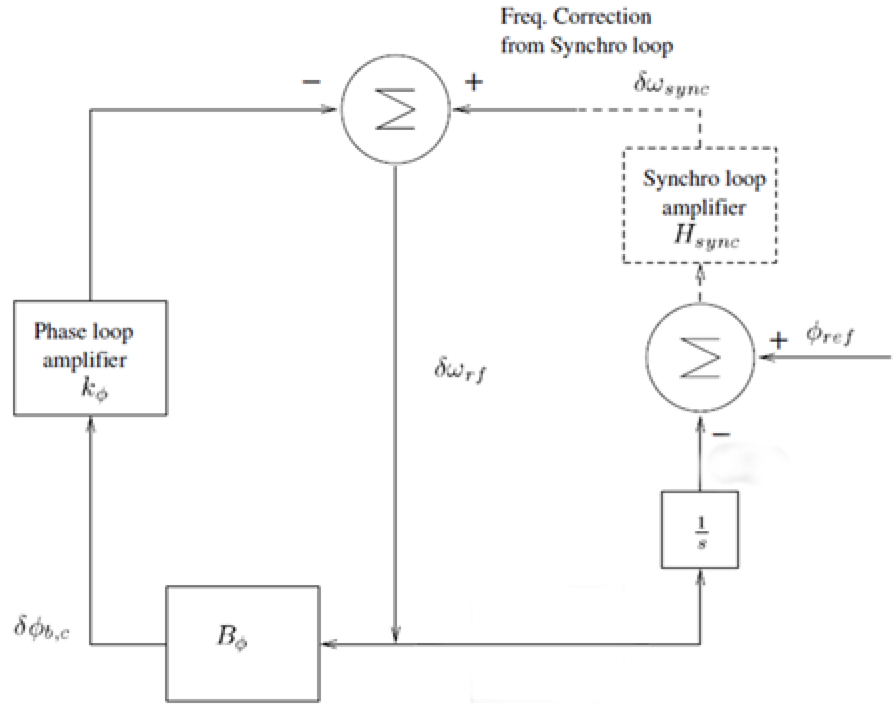
Another classic combination for proton and ion synchrotrons: Phase loop and Synchro loop

- The open-loop transfer function from $\delta\omega_{RF}$ to $\delta\phi_{RF}$ (the phase modulation applied to the RF) with phase loop closed is

$$H_{ol}(s)=\frac{\delta\varphi_{RF}(s)}{\delta\omega_{RF}(s)}=\frac{1}{1+k_\phi B_\phi(s)}\frac{1}{s}=\frac{s^2+\Omega_s^2}{s\left(s^2+k_\phi s+\Omega_s^2\right)}$$

- Without correction this response gives too low a phase margin. It can be corrected by the Synchro Loop amplifier. We use a phase advance network as a corrector that increases the phase margin

$$H_{sync}(s)=k_{sync}\frac{1+a\,\tau s}{1+\tau s}$$

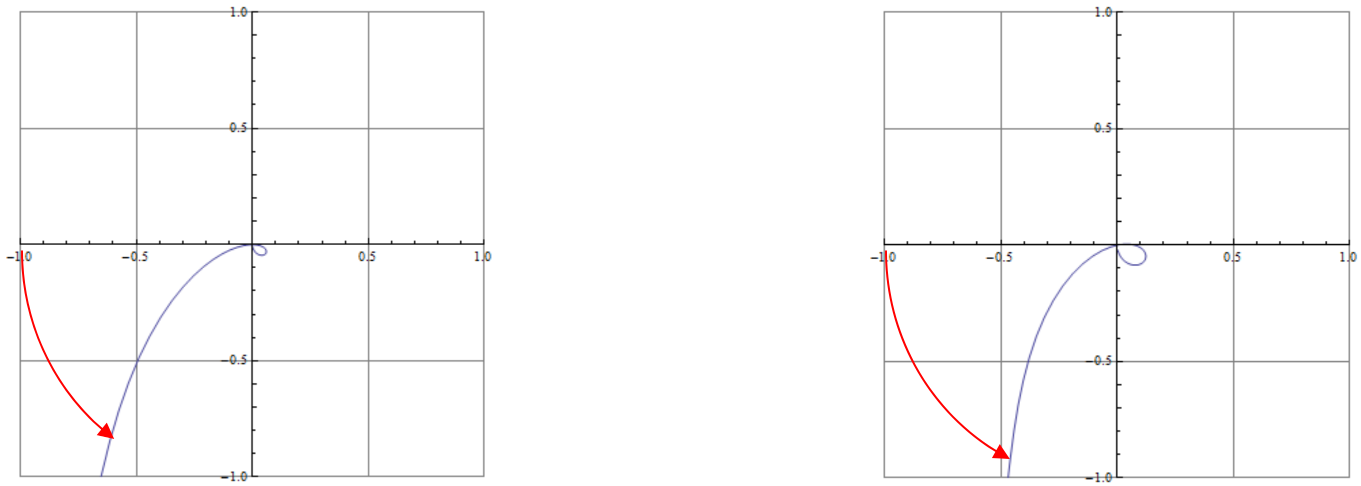


Block diagram of the Phase-Synchro loop in Laplace transforms

Correction using a Phase Advance Network

$$H_{sync}(s) = k_{sync} \frac{1 + a\tau s}{1 + \tau s}$$

- Optimize the parameters *a* and τ using classic Controls Theory (Nyquist Plots)



Nyquist plots of $H_{ol}(j\omega)$ without Phase Advance Network (left) and with correction optimized. Phase margin increased from ~45 to >60 degrees

- If the synchrotron frequency varies much during the acceleration ramp, the parameters of the corrector must track

- The Synchro Loop must respond in a time that is larger than the synchrotron period to remain adiabatic and to avoid exciting the beam with noise
- The Loop can be used throughout the acceleration cycle: set at the injection frequency during filling, then ramped following either a measurement of the B field (SPS p for LHC) or a function (LHC)
- If the accelerator is an injector, the loop can remain in use while rephasing takes place locking the reference generator on the receiving machine RF (SPS p for LHC)
- If it is a collider, using a single reference for both rings from fillings to physics, makes the beam cross in the correct position from injection on (LHC)
- If needed, a slow measurement of the orbit displacement using all ring PUs can feed back on the reference frequency (LHC real-time orbit correction)
- Limitation: It is not practical if transition is crossed during the acceleration. We have

$$\frac{\Delta R}{R} = \frac{\gamma^2}{\gamma_t^{\,2} - \gamma^2} \frac{\Delta f}{f}$$

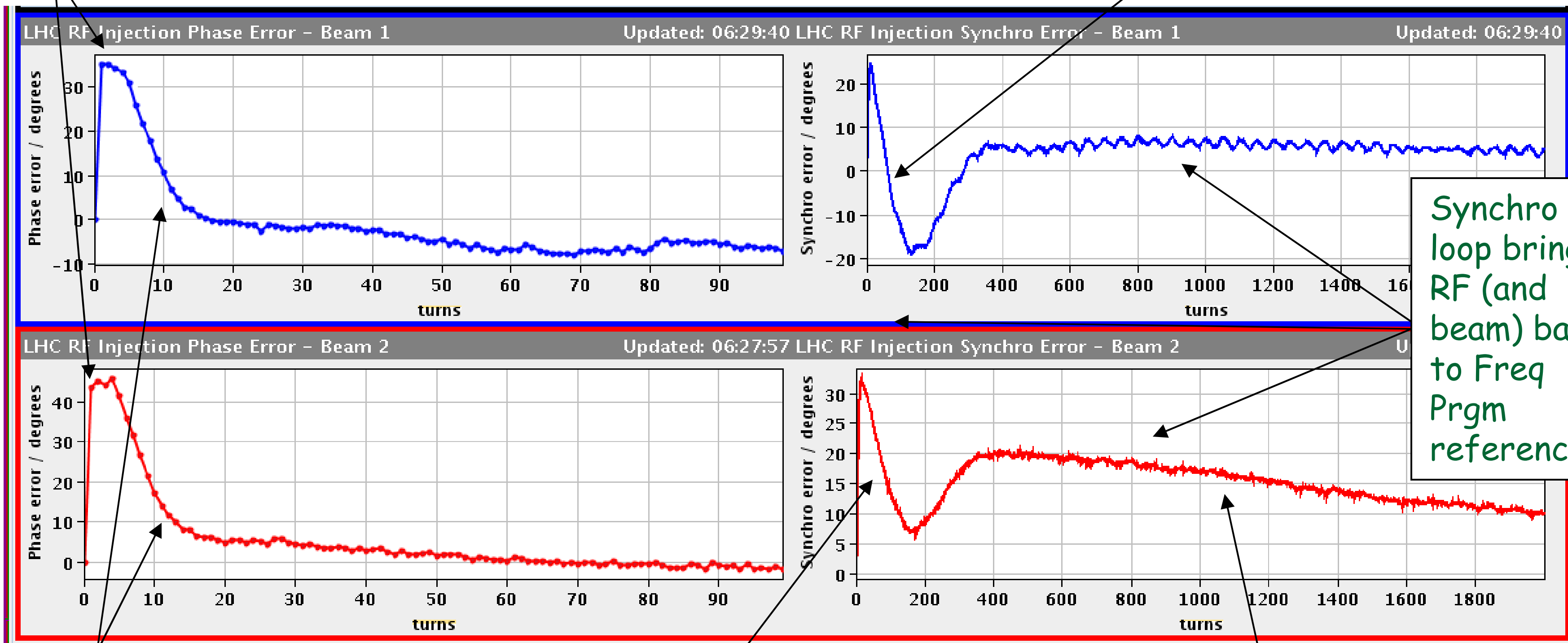
  At transition a very small frequency error will cause a large radial displacement. The Radial Loop is preferred is the acceleration cycle crosses transition
- Limitation: The range of the phase discriminator is only 360 degrees. And the synchro loop is much slower than the phase loop. At injection, with large offsets (energy error or stable phase offset), the phase discri may reach its limits before the loop is locked. It will later lock with an error of one or several RF periods. Not acceptable if multiple injections.

LHC Capture Transients

Inj Phase Error 35 deg/45 deg

Phase loop is fast: "jumps" the RF on the beam at injection
Synchro loop is slow. No reaction in first 100 turns. Slope gives frequency (energy) error at injection

-30 deg in 60 turns -> -15 Hz @ 400 MHz
$\Delta p/p \sim 10^{-4}$



Synchro loop brings RF (and beam) back to Freq Prgm reference

Cavity field "jumps" on the beam in ~ 10 turns

Phase Loop Error: Beam PU-Cav Sum

Synchro Loop Error: VCXO-Freq Prgm

-15 deg in 80 turns -> -6 Hz @ 400 MHz

Very slow (seconds) time constant. Boosts DC gain to minimize thermal drifts