Luminosity Diagnostics

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• Concept of luminosity
• Luminosity measurements
• Beam overlap scans
Process Cross-section

Consider the process where a particle encounters a target composed of a certain material - which could also be another beam.

There is a certain probability that the encounter produces a certain final state composed of various particles (Higgses and SUSYs partners are very popular!).

The likelihood of the process is expressed by a CROSS-SECTION $\sigma$. "Image" : $\sigma$ represents the 'area' over which the process occurs.

- The cross-section $\sigma$ is expressed in units of m$^2$.
- The standard unit in nuclear & high energy physics is the barn,
  
  $1$ barn $= 10^{-24}$ cm$^2$.

- Typical cross-sections for Higgs and exotic particle production are in the pico- to femtobarn range ($10^{-36}$ to $10^{-39}$ cm$^2$).
**Luminosity Definition**

In the case of two colliding beams, the number of collisions per unit time interval (event rate) that produce the final state of interest is proportional to the cross-section, but it also depends on how well the beams collide and how much beam there is.

The **luminosity** $L$ quantifies the performance ('brillance') of the collider and relates the cross-section and the event rate at time $t$:

$$ R(t) = \frac{dN(t)}{dt} = \dot{N}(t) = L(t) \sigma $$

Integrating over time one obtains the **integrated luminosity** $\int L$ and the total number of events $N$:

$$ N = (\int L(t)dt) \sigma = \int L \sigma $$

- Units of $L$: m$^{-2}$ s$^{-1}$.
- Typical luminosities: $10^{27}$ to $10^{34}$ cm$^{-2}$ s$^{-1}$.
- $\int L$ is frequently expressed in pb$^{-1}$ ($10^{36}$ cm$^{-2}$) or fm$^{-1}$ ($10^{39}$ cm$^{-2}$):
  
  With $\int L = 1$ pb$^{-1}$, a process with $\sigma = 1$ pb yields... 1 event!
# Luminosity Table

<table>
<thead>
<tr>
<th>Machine</th>
<th>Beam type</th>
<th>Beam energy (GeV)</th>
<th>Luminosity (cm$^{-2}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP I</td>
<td>e+ e-</td>
<td>45</td>
<td>$3 \times 10^{30}$</td>
</tr>
<tr>
<td>LEP II</td>
<td>e+ e-</td>
<td>90-104</td>
<td>$&gt;10^{31}$</td>
</tr>
<tr>
<td>PEP-II</td>
<td>e+ e-</td>
<td>$9 \times 3.1$</td>
<td>$&gt;10^{34}$</td>
</tr>
<tr>
<td>KEKB</td>
<td>e+ e-</td>
<td>$8 \times 3.5$</td>
<td>$2 \times 10^{34}$</td>
</tr>
<tr>
<td>SppS</td>
<td>p anti-p</td>
<td>270</td>
<td>$6 \times 10^{30}$</td>
</tr>
<tr>
<td>TEVATRON</td>
<td>p anti-p</td>
<td>980</td>
<td>$2 \times 10^{32}$</td>
</tr>
<tr>
<td>RHIC</td>
<td>Au Au</td>
<td>100 (/nucleon)</td>
<td>$10^{27}$</td>
</tr>
<tr>
<td>LHC</td>
<td>p p</td>
<td>7000</td>
<td>$10^{34}$</td>
</tr>
<tr>
<td>LHC</td>
<td>Pb Pb</td>
<td>2760 (/nucleon)</td>
<td>$10^{27}$</td>
</tr>
</tbody>
</table>
How is the luminosity related to accelerator parameters?

Intuitively the luminosity depends on:

- The number of particles that collide.
- The density of the particles, i.e. the beam sizes ~ particle distributions $\rho$.
- The collision frequency, i.e. ring size & number of bunches.
Luminosity Expression

General expression of the luminosity for two bunched beams (labelled 1 and 2):

\[ L = f k N_1 N_2 \int \rho_1(x, y, s, t) \rho_2(x, y, s, t) dx dy ds dt \]

Collision frequency
Beam overlap
Bunch populations

For bunches with gaussian profiles, the distributions \( \rho \) can be decomposed into:

\[ \rho(x, y, s, t) = \rho_x(x) \rho_y(y) \rho_s(s - vt) \]

with

\[ \rho_u(u) = \frac{1}{\sigma_u \sqrt{2\pi}} \exp\left\{ -\frac{(u - u_0)^2}{2\sigma_u^2} \right\} \]

N = no. particles per bunch
f = revolution frequency
k = number of bunches
v = speed (vector)
x, y, s = coordinates
\( x_0, y_0, s_0 \) = beam offsets
\( \sigma \) = rms size
c = speed of light
\( \rho \) = density distribution of the particles, normalized to 1
'Gaussian Beams' Luminosity

The general expression for two beams with Gaussian profiles (labelled 1 and 2) colliding without crossing angle:

\[
L = \frac{kN_1 N_2 f}{2\pi \sqrt{(\sigma_{x,1}^2 + \sigma_{x,2}^2)(\sigma_{y,1}^2 + \sigma_{y,2}^2)}} \exp\left\{-\frac{(x_1 - x_2)^2}{2(\sigma_{x,1}^2 + \sigma_{x,2}^2)} - \frac{(y_1 - y_2)^2}{2(\sigma_{y,1}^2 + \sigma_{y,2}^2)}\right\}
\]

- \(k\) = number of bunches
- \(f\) = revolution frequency
- \(N\) = no. particles per bunch
- \(\sigma_{x,y}\) = beam sizes at collision point (hor./vert.)
- \(x,y\) = transverse coordinates of the beam

For beams of equal sizes this reduces to:

\[
L = \frac{kN_1 N_2 f}{4\pi \sigma_x \sigma_y} \exp\left\{-\frac{(x_1 - x_2)^2}{4\sigma_x^2} - \frac{(y_1 - y_2)^2}{4\sigma_y^2}\right\}
\]
Peak Luminosity

From the expression for beams of equal sizes

\[ L = \frac{kN_1 N_2 f}{4\pi \sigma_x \sigma_y} \exp \left\{ -\frac{(x_{0,1} - x_{0,2})^2}{4\sigma_x^2} - \frac{(y_{0,1} - y_{0,2})^2}{4\sigma_y^2} \right\} \]

One obtains the standard expression for the luminosity when the beams collide head-on \((x_1-x_2 = y_1-y_2 = 0)\):

\[ L = \frac{kN_1 N_2 f}{4\pi \sigma_x \sigma_y} \]

- The luminosity follows a gaussian profile for non-zero collision offsets.
- For a separation of \(1\sigma\) in one plane, for example \(x_{0,1}-x_{0,2} = \sigma_x\), the reduction is \(\exp(-1/4) \sim 0.78\).

Example for the LHC:
\[ k = 2808 \quad f = 11.25 \text{ kHz} \]
\[ N = 1.15 \times 10^{11} \text{ protons} \quad \sigma_x, \sigma_y = 16 \mu\text{m} \]
\[ >> L = 1.2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \]
Crossing Angles

If the beams cross at an angle, the luminosity is reduced because the particles no longer traverse the entire length of the counter-rotating bunch:

\[
L = \frac{kN_1N_2f}{4\pi\sigma_x\sigma_y} \left( \frac{1}{1 + \left( \frac{\sigma_s \tan \phi}{\sigma_x} \right)^2} \right)
\]

L degrades as the bunch length 'projection' \((\sigma_s \tan(\phi/2))\) becomes more important with respect to the transverse beam size.

In all previous equations, it is assumed that the beam sizes are constant over the collision region. This is usually a good assumption, but for very strong focusing, the beam size may vary significantly over the length of the collision region. In that case numerical integration of the equation may become necessary.
Luminosity Measurements
The most evident way to determine the luminosity is to compute it from machine parameters:

- $k$, $f$: perfectly well known.
- $N$: can be well known, to the level of ~ 1%.
- $\sigma$: more tricky! Since in general we do not have diagnostics at the collision point, it is necessary to determine the emittance and the betatron function at the IP. Difficult to obtain with much better than 5-10% accuracy.
- $\phi$: measured with beam position monitors around the collision point.

>> In general one does not know the luminosity to better than 5-10% from the machine parameters!
'Van de Meer' Scan

- ISR: coasting (un-bunched) beams colliding with a crossing angle $\phi$ (hor. Plane).
- For a coasting beam $L$ does not depend on the horizontal offset!
- $L$ only depends on the vertical offset $y_0$ and may be expressed as (Van de Meer):

$$L(y_0) = \frac{I_1 I_2 h_{\text{eff}}(y_0)}{ce^2 \tan(\phi/2)}$$

$$h_{\text{eff}}(y_0) = \frac{\hat{N}(y_0)}{\int \hat{N}(y_0)dy_0} = \frac{1}{\sqrt{4\pi\sigma_y}}$$

@ optimum

- Just need a process that provides a signal proportional to $L$!
- Need excellent calibration of the scale ($y_0$).
- Beam sizes must be stable – not suited for strong beam-beam ($e^+e^-$).
- In case of bunched beams $\rightarrow$ scan in H & V.

Area measures the beam size!
Luminosity Measurement from $\sigma$

To measure the luminosity, invert the definition

$$L(t) = \frac{\dot{N}(t)}{\sigma}$$

Now find a process that satisfies the following criteria:

- The process is theoretically very well established: $\sigma$ can be computed with the desired accuracy!
- The rate $dN/dt$ is high (low statistical error) and can be measured with desired accuracy!
- Background processes contributing to $dN/dt$ can be identified and/or their contribution can be subtracted.

It is unfortunately simpler to write this down than to perform the measurement!
Bhabha Scattering

For e+e- colliders, the Bhabha scattering process is normally used for luminosity measurements (γ = photon):

\[ e^+ e^- \rightarrow e^+ e^- (\gamma) \]

- Large cross-section.
- Almost purely electromagnetic (QED) interaction.
- Can be computed to high accuracy. LEP : < 0.05%.

At small angle from the beam the cross-section for a detector covering the angular range (wrt beam axis) from \( \theta_{\text{min}} \) to \( \theta_{\text{max}} \) is:

\[
\sigma = k \left( \frac{1}{\theta_{\text{min}}^2} - \frac{1}{\theta_{\text{max}}^2} \right)
\]

For high rates, aim for the smallest possible \( \theta_{\text{min}} \).
Bhabha Scattering: LEP Example

- At LEP each experiment had a luminosity monitor covering the angular region of 30 to 50-100 mrad. They achieved measurements with accuracies of ~0.1%.
- The LEP machine had its own detectors, based on a 'sandwich' of Tungsten plates and Silicon strip detectors:
  - Installed at smaller angles (2-5 mrad) for faster update rates, but also more background!
  - Poor(er) accuracy, calibration wrt to the monitors of the experiments.
  - The machine monitors were used for luminosity optimization (see later).
LEP Machine Lumi Detectors

- 'Sandwich' of Tungsten plates (radiation length ~3.5 mm) and Silicon detectors.
- The Tungsten plates are used to induce and contain the electromagnetic particle shower.
- The Si detectors consisted either of strips (S) for shower position determination or of a full plane (F) for energy measurements.
• A Silicon track detector (SLUM) for precise impact/angle measurements.
• A scintillating crystal (BGO) electro-magnetic calorimeter to measure the energy of the particle, optimized for electrons and photons.
The BGO detector can be 'opened' to protect the sensitive BGO calorimeter from background particles during injection and ramping.

The detector was only moved to its data taking position when the beams were colliding and 'stable'.
The Proton Case

For the case of (anti-)proton beams, it is more difficult to find processes where the theoretical cross-section is well known!

For the LHC (also TEVATRON) the processes that are potentially used:

- **Lepton pair production**: 
  
  \[ pp \rightarrow X + l^+l^- + Y \]

- **W/Z boson production**: 
  
  \[ pp \rightarrow X + Z \rightarrow X + l^+l^- \]
  
  \[ pp \rightarrow X + W \rightarrow X + l\nu \]

  \[ l = e, \mu \]
  
  \[ X,Y = p, \text{hadron (jet)} \]

>> theoretical errors ~ few %
The Proton Case at Small Angle

An alternative for protons is to measure the cross-section at very small angles:

• Measure the rates (R) for elastic and inelastic pp collisions (R_{el} and R_{inel}) \[1\]

• Measure the dependence of the elastic rate on the momentum transfer \( t \) (\( \sim \) scattering angle \( \theta \)) and extrapolate to \( t \) (angle) = 0 \[2\]

\[
\sigma = \frac{16\pi}{1 + \rho^2} \frac{(dR_{el} / dt)_{t=0}}{R_{el} + R_{inel}}
\]

This requires

• Special detectors that can move very close to the beam (\( \sim 1-4 \text{ mm} \)) - so called 'Roman Pots' to reach angles of a few \( \mu \text{rad} \).

• Special optics with low divergence (and large size) beams at the collision points.

\[\text{dR/dt} \sim \text{dR/d\theta}\]

\( \gg \text{ may reach } \sim 1\% \)
Silicon tracking detectors are installed here. They can come as close as 1-4 mm to the beam. At the LHC the stored beam energy is 360 MJ!
Luminosity Optimization

Very frequently the primary goal of the machine operation crews is to maximize the luminosity by achieving the best possible overlap of the beams:

>> scan the beams across each other and observe $L$

In that case the absolute luminosity is not relevant, a relative measurement is sufficient (also true for Van de Meer Scan!):

• Any process that yields a (high) rate is OK for the luminosity measurement.
• For $e^+e^-$ Bhabha scattering is the optimum, for protons (ions) one can monitor anything that comes out at small angle. But watch out for systematic errors due to overlapping events (protons at high luminosity).
LHC Machine Luminosity Monitors

• Machine monitors only provide relative measurements – calibration is obtained from the LHC experiments!
• The monitors are installed in an absorber for neutral particles ("TAN").
• They measure the flux of neutral particles $\propto L$.
• High radiation environment – as little access as possible !!:
  ➢ $10^8$ Gy / year.
  ➢ Heat load > 100 W.
  ➢ >$10^{16}$ neutrons and charged particle per cm$^{-2}$ and per year.
LHC High Luminosity Machine Detectors

- $N_{\text{GAP}} = 6$
- $x_{\text{GAP}} = 1\text{mm}$

- Ar+N[4%] chamber.
- Supposedly radiation hard.
- Design optimized for high speed, but bunch by bunch measurement not fully achieved...

Overlap of consecutive collisions...

![Graph showing overlap of consecutive collisions](image-url)
LHC Low Luminosity Machine Detectors

- Fast semiconductor devices.
- $5 \cdot 10^4$ charges (300 $\mu$m thickness).
- Radiation hard to at least $10^{16}$ n/cm$^2$.
- $\times$ Important rise in dark current at $10^{17}$ n/cm$^2$.

>> Used for the LHC low luminosity collision points

CdTe Detectors (2 cm diameter)
Example of luminosity optimization scans at LEP for the 4 collision points.

The scans are made in each point individually.

In this example one can see the luminosity for the 3 bunches in a train (a,b,c). The optimum is not the same for each bunch due to parasitic beam-beam deflections.
Beam-beam Deflections

• When the beams collide, each particle senses the fields of the counter-rotating beam: the 'beam-beam' force.

• The beams can also be deflected collectively by the opposing beam:

- The deflection angles may be interpolated from each side of the collision point using beam position monitors.
- The total deflection angle \((\theta_1 + \theta_2)\) vanishes when the offset is zero, and varies anti-symmetrically wrt to the optimumum:

>> Beam-beam deflection scan!
Luminosity Optimization by Beam-beam Deflection

- Luminosity optimization by beam-beam deflection has been used extensively at SLC and at LEP2.
- The analytical expression for the deflection angle is complex:
  - Linear near centre.
  - Saturation at distance of few $\sigma$.
  - Decrease as $1$/distance at large separations.
- The beam parameters (size and luminosity) at the collision point can be reconstructed from the scan.
- At KEKB beam-beam deflections are even used as input for collision point feedback.