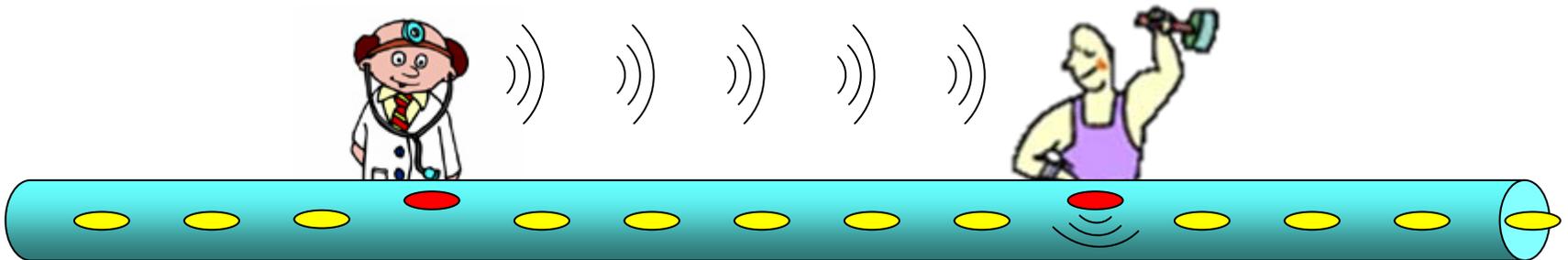


# Multi-bunch Feedback Systems

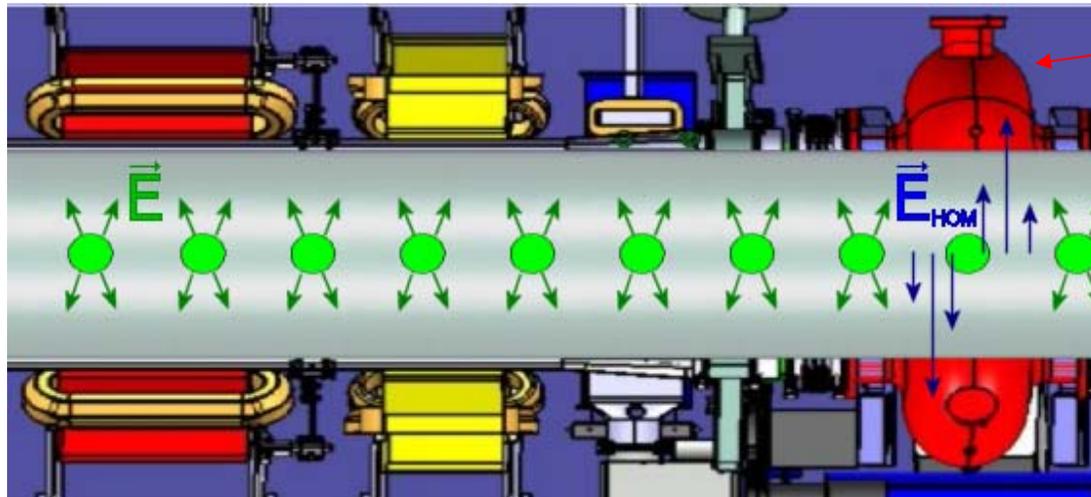
Marco Lonza  
Sincrotrone Trieste - Elettra



- Coupled-bunch instabilities
- Basics of feedback systems
- Feedback system components
- Digital signal processing
- Integrated diagnostic tools
- Conclusions

# Coupled-bunch instabilities

- ▶ Beam in a storage ring made of bunches of charged particles
- ▶ Transverse (betatron) and longitudinal (synchrotron) oscillations normally damped by natural damping
- ▶ Interaction of the electromagnetic field with metallic surroundings ("wake fields")
- ▶ Wake fields act back on the beam and produces growth of oscillations
- ▶ If the growth rate is stronger than the natural damping the oscillation gets unstable
- ▶ Since wake fields are proportional to the bunch charge, the onset of instabilities and their amplitude are normally current dependent



**Example:** interaction with an RF cavity can excite its Higher Order Modes (HOM)

Bunch

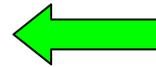
"Zoomed" beam pipe

# Objective of storage ring based particle accelerators

High brightness in synchrotron light sources  
 High luminosity in high energy physics experiments



High currents  
 Many bunches



Storage of intense particle beams



The interaction of these beams with the surrounding metallic structures gives rise to collective effects called "coupled-bunch instabilities"



Large amplitude instabilities can cause beam loss

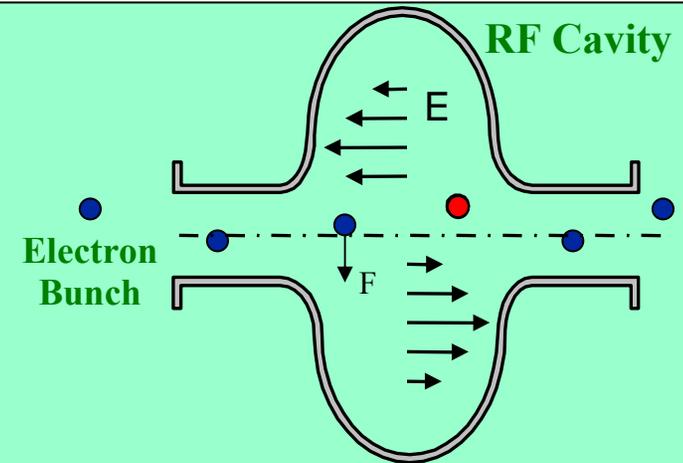
- Limitation of the stored current to low values

If the growth of instability saturates, the beam may stay in the ring

- Large instabilities degrade the beam quality: brightness or luminosity

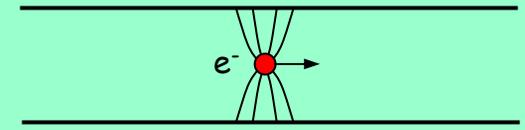
## Cavity High Order Modes (HOM)

High Q spurious resonances of the accelerating cavity excited by the bunched beam act back on the beam itself  
Each bunch affects the following bunches through the wake fields excited in the cavity  
The cavity HOM can couple with a beam oscillation mode having the same frequency and give rise to instability



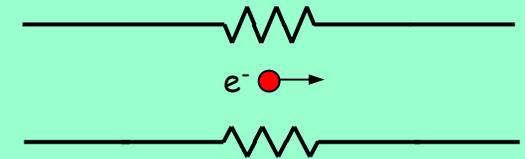
## Resistive wall impedance

Interaction of the beam with the vacuum chamber (skin effect)  
Particularly strong in low-gap chambers and in-vacuum insertion devices (undulators and wigglers)



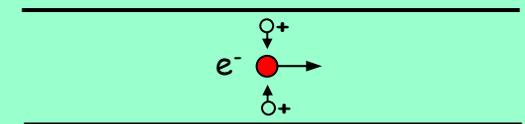
## Interaction of the beam with other objects

Discontinuities in the vacuum chamber, small cavity-like structures, ...  
Ex. BPMs, vacuum pumps, bellows, ...



## Ion instabilities

Gas molecules ionized by collision with the electron beam  
Positive ions remains trapped in the negative electric potential  
Produce electron-ion coherent oscillations

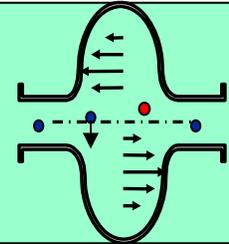


## Cavity High Order Modes (HOM)

Thorough design of the RF cavity

Mode dampers with antennas and resistive loads

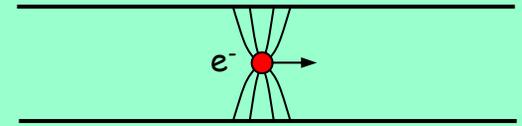
Tuning of HOMs frequencies through plungers or changing the cavity temperature



## Resistive wall impedance

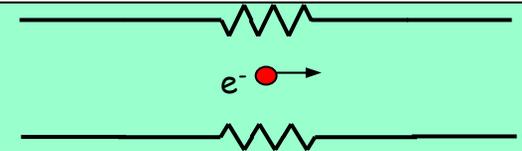
Usage of low resistivity materials for the vacuum pipe

Optimization of vacuum chamber geometry



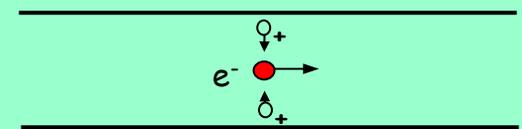
## Interaction of the beam with other objects

Proper design of the vacuum chamber and of the various installed objects



## Ion instabilities

Ion cleaning with a gap in the bunch train



## Landau damping by increasing the tune spread

Higher harmonic RF cavity (bunch lengthening)

Modulation of the RF

Octupole magnets (transverse)

**Active Feedbacks**

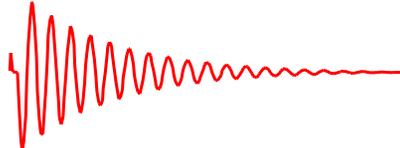
" $x$ " is the oscillation coordinate (transverse or longitudinal displacement)

Natural damping

Betatron/Synchrotron frequency:  
*tune ( $\nu$ )  $\times$  revolution frequency ( $\omega_0$ )*

$$\ddot{x}(t) + 2D \dot{x}(t) + \omega^2 x(t) = 0$$

If  $\omega \gg D$ , an approximated solution of the differential equation is a damped sinusoidal oscillation:

$$x(t) = e^{-\frac{t}{\tau_D}} \sin(\omega t + \varphi)$$


where  $\tau_D = 1/D$  is the "damping time constant" ( $D$  is called "damping rate")

Excited oscillations (ex. by quantum excitation) are damped by natural damping (ex. due to synchrotron radiation damping). The **oscillation** of individual particles is **uncorrelated** and shows up as an emittance growth

Coupling with other bunches through the interaction with surrounding metallic structures add a "driving force" term  $F(t)$  to the equation of motion:

$$\ddot{x}(t) + 2D \dot{x}(t) + \omega^2 x(t) = F(t)$$

Under given conditions the oscillation of individual particles becomes correlated and the centroid of the bunch oscillates giving rise to **coherent bunch (coupled bunch) oscillations**

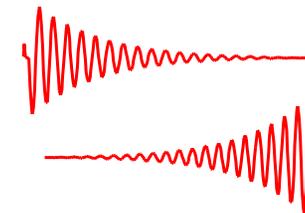
Each bunch oscillates according to the equation of motion:

$$\ddot{x}(t) + 2(D-G) \dot{x}(t) + \omega^2 x(t) = 0$$

where  $\tau_G = 1/G$  is the "growth time constant" ( $G$  is called "growth rate")

If  $D > G$  the oscillation amplitude decays exponentially

If  $D < G$  the oscillation amplitude grows exponentially



as:  $x(t) = e^{-\frac{t}{\tau}} \sin(\omega t + \varphi)$       where  $\frac{1}{\tau} = \frac{1}{\tau_D} - \frac{1}{\tau_G}$

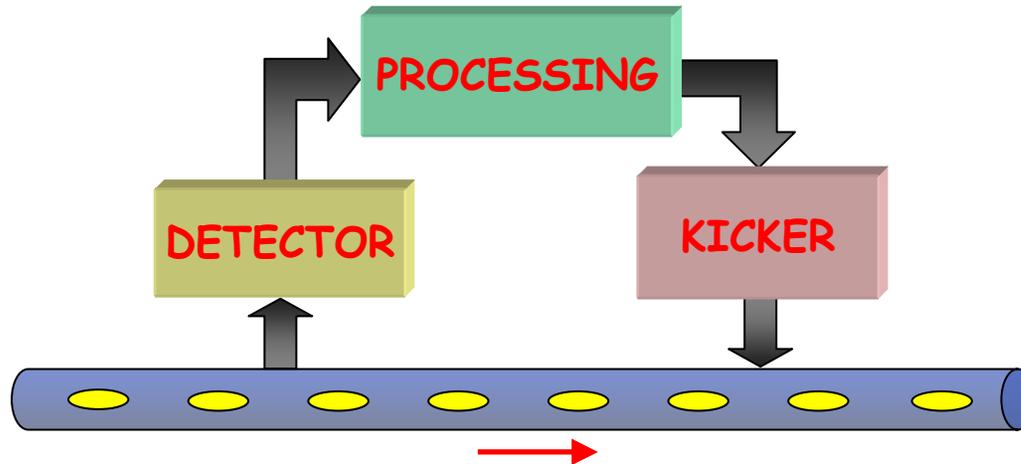
Since  $G$  is proportional to the beam current, if the latter is lower than a given current threshold the beam remains stable, if higher a coupled bunch instability is excited

# Feedback Damping Action

The feedback action adds a damping term  $D_{fb}$  to the equation of motion

$$\ddot{x}(t) + 2(D - G + D_{fb}) \dot{x}(t) + \omega^2 x(t) = 0 \quad \text{Such that } D - G + D_{fb} > 0$$

A multi-bunch feedback detects an instability by means of one or more Beam Position Monitors (BPM) and acts back on the beam by applying electromagnetic 'kicks' to the bunches

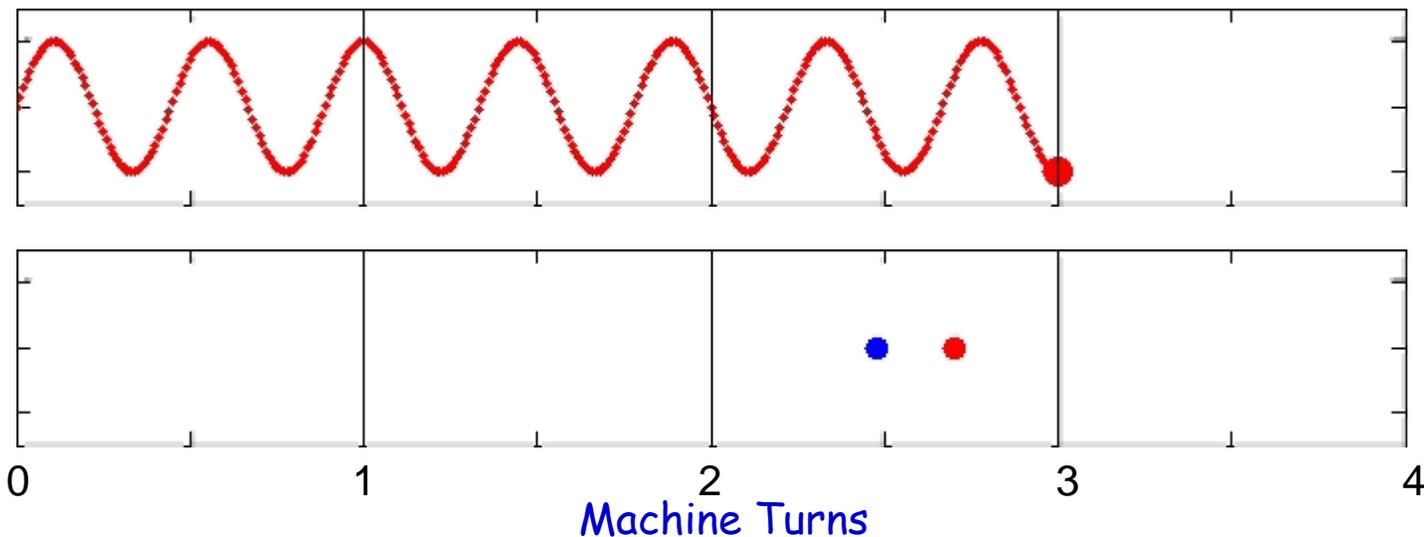


In order to introduce damping, the feedback must provide a kick proportional to the derivative of the bunch oscillation

Since the oscillation is sinusoidal, the kick signal for each bunch can be generated by shifting by  $\pi/2$  the oscillation signal of the same bunch when it passes through the kicker

# Multi-bunch modes

Typically, betatron tune frequencies (horizontal and vertical) are higher than the revolution frequency, while the synchrotron tune frequency (longitudinal) is lower than the revolution frequency



Ex.

Vertical

Tune = 2.25

Longitudinal

Tune = 0.5

Although each bunch oscillates at the tune frequency, there can be different modes of oscillation, called **multi-bunch modes** depending on how each bunch oscillates with respect to the other bunches

# Multi-bunch modes

Let us consider  $M$  bunches equally spaced around the ring

Each multi-bunch mode is characterized by a bunch-to-bunch phase difference of:

$$\Delta\Phi = m \frac{2\pi}{M} \quad m = \text{multi-bunch mode number } (0, 1, \dots, M-1)$$

Each multi-bunch mode is associated to a characteristic set of frequencies:

$$\omega = pM\omega_0 \pm (m+\nu)\omega_0$$

Where:

$p$  is an integer number  $-\infty < p < \infty$

$\omega_0$  is the **revolution frequency**

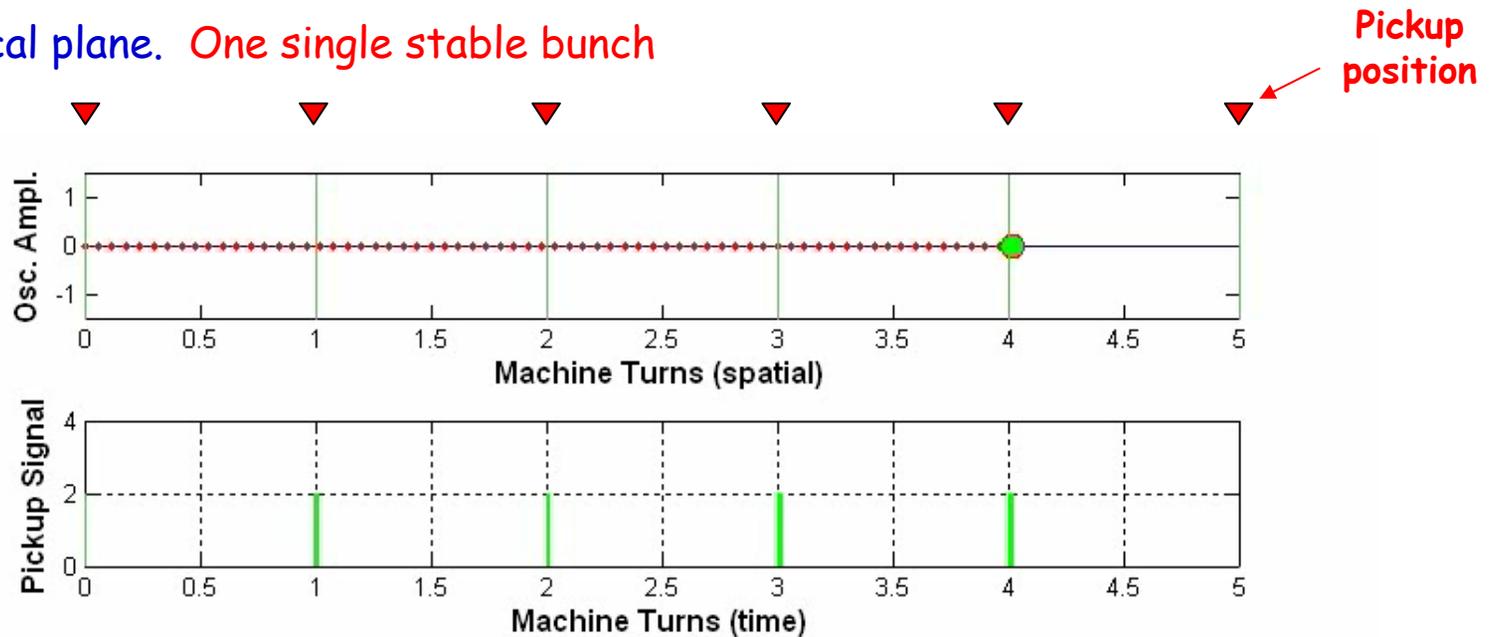
$M\omega_0 = \omega_{rf}$  is the RF frequency (bunch repetition frequency)

$\nu$  is the **tune**

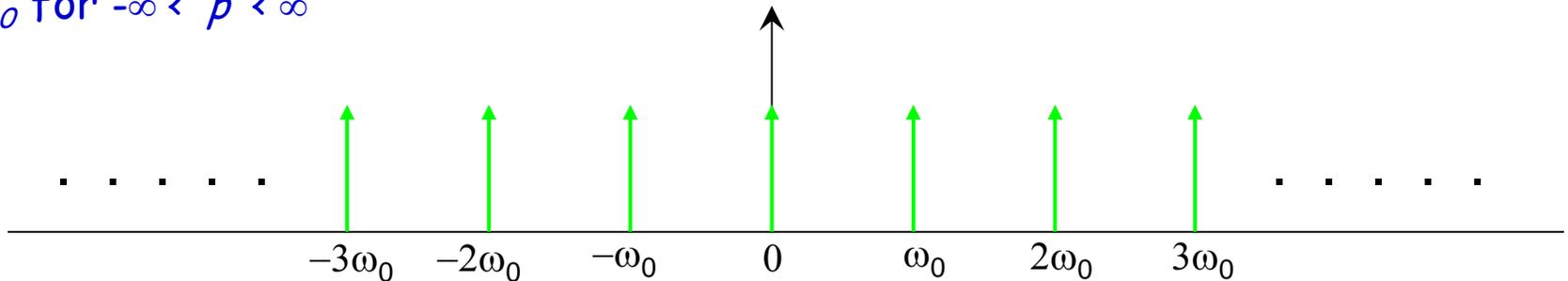
Two sidebands at  $\pm(m+\nu)\omega_0$  for each multiple of the RF frequency

# Multi-bunch modes: example1

Vertical plane. One single stable bunch

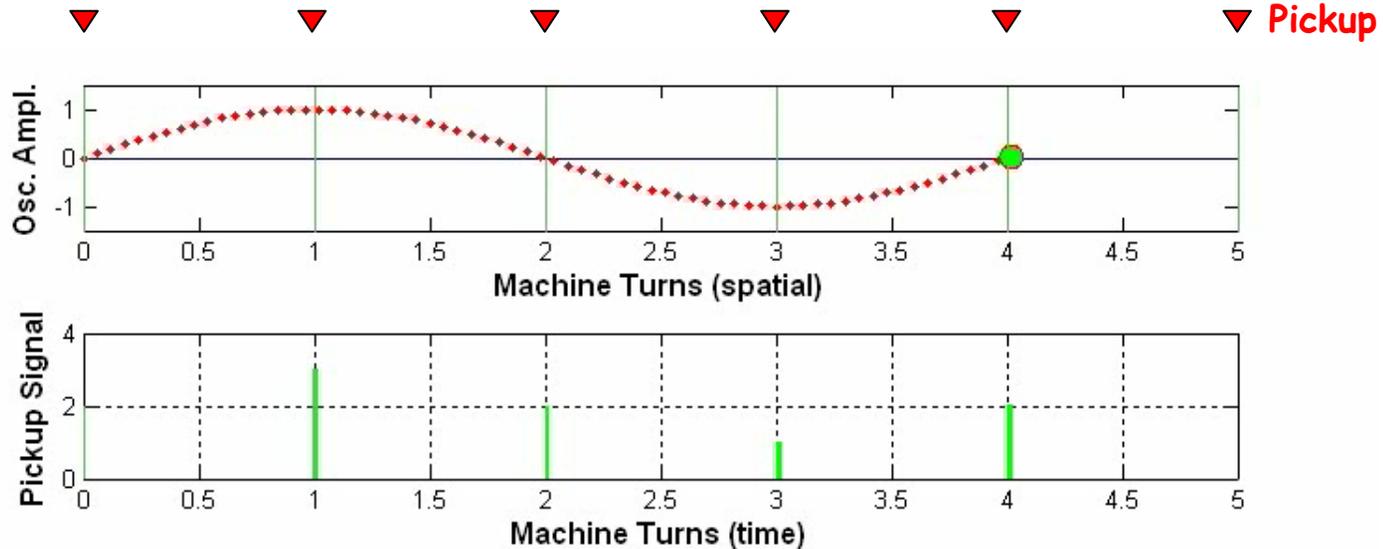


Every time the bunch passes through the pickup ( $\nabla$ ) placed at coordinate 0, a pulse with constant amplitude is generated. If we think it as a Dirac impulse, the spectrum of the pickup signal is a repetition of frequency lines at multiple of the revolution frequency:  $p\omega_0$  for  $-\infty < p < \infty$

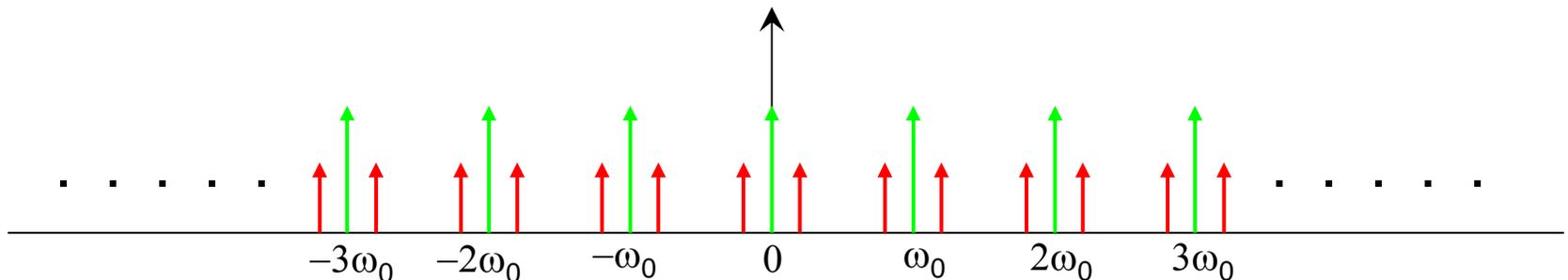


# Multi-bunch modes: example2

One single unstable bunch oscillating at the tune frequency  $\nu\omega_0$ : for simplicity we consider a vertical tune  $\nu < 1$ , ex.  $\nu = 0.25$ .  $M = 1 \rightarrow$  only mode #0 exists

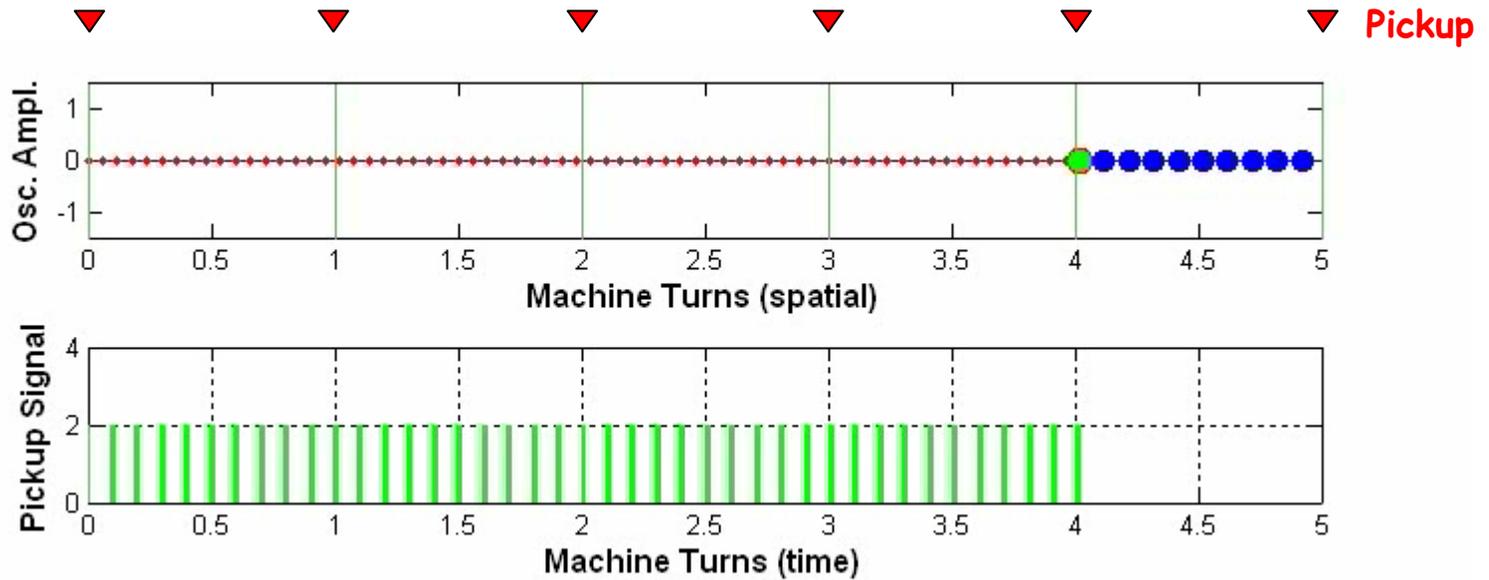


The pickup signal is a sequence of pulses modulated in amplitude with frequency  $\nu\omega_0$   
 Two sidebands at  $\pm\nu\omega_0$  appear at each of the revolution harmonics

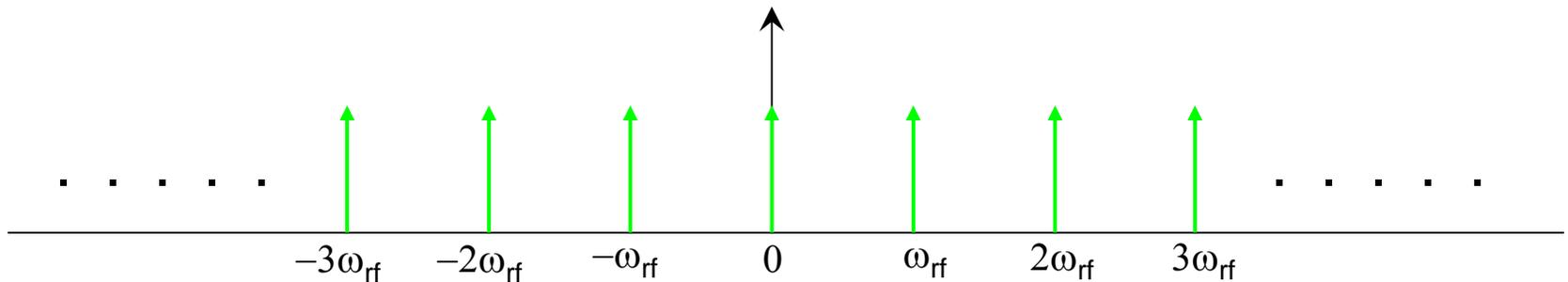


# Multi-bunch modes: example3

Ten identical equally-spaced stable bunches filling all the ring buckets ( $M = 10$ )



The spectrum is a repetition of frequency lines at multiples of the bunch repetition frequency:

$$\omega_{rf} = 10 \omega_0 \text{ (RF frequency)}$$


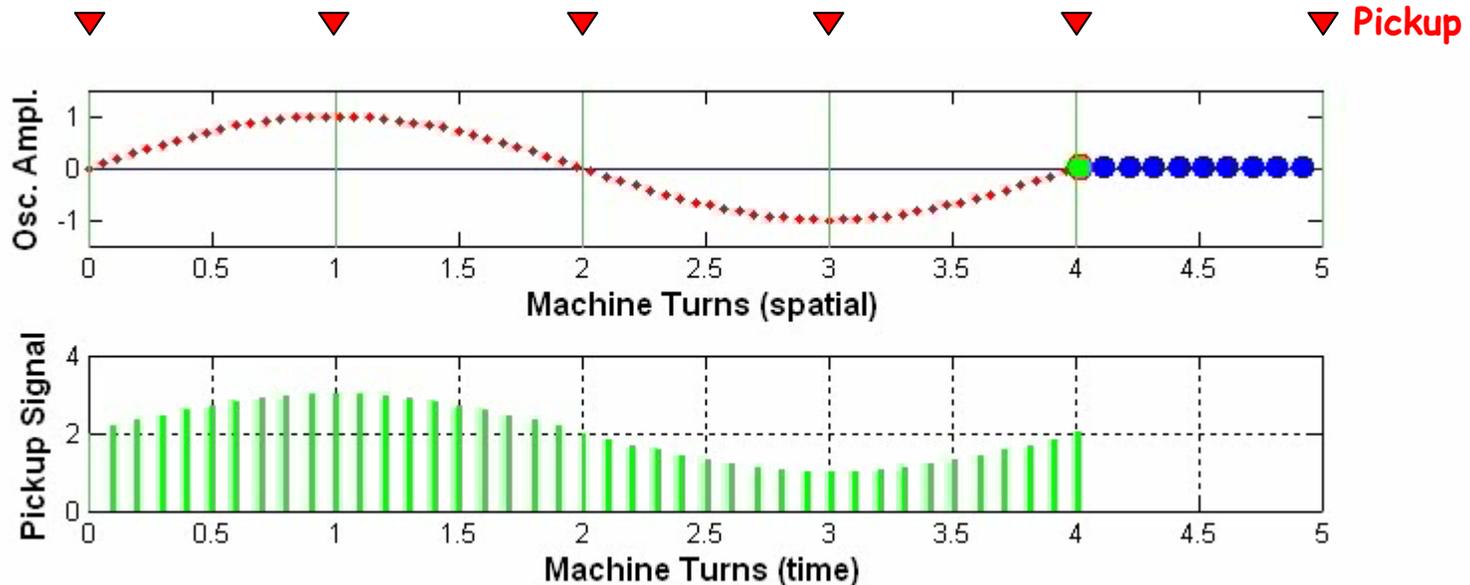
# Multi-bunch modes: example4

Ten identical equally-spaced unstable bunches oscillating at the tune frequency  $\nu\omega_0$  ( $\nu = 0.25$ )

$M = 10 \rightarrow$  there are 10 possible modes of oscillation

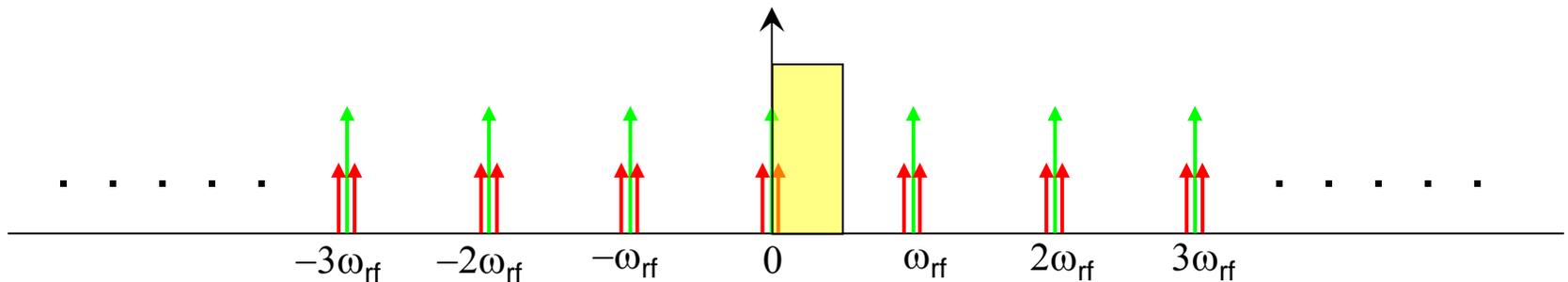
$$\Delta\Phi = m \frac{2\pi}{M} \quad m = 0, 1, \dots, M-1$$

Ex.: mode #0 ( $m = 0$ )  $\Delta\Phi=0$  all bunches oscillate with the same phase

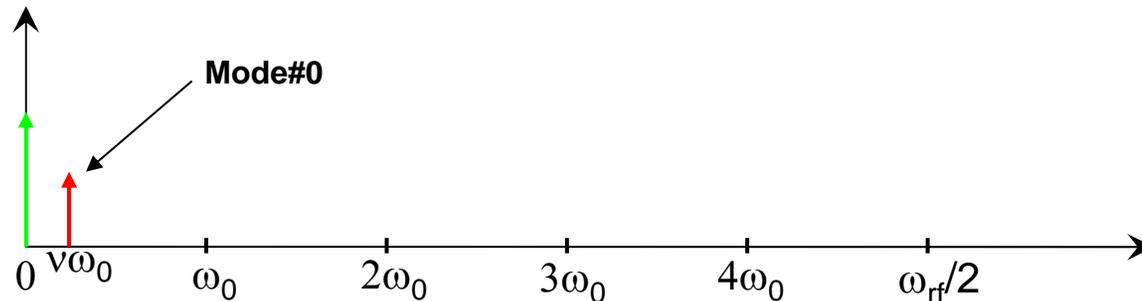


# Multi-bunch modes: example4

The spectrum is a repetition of frequency lines at multiples of the bunch repetition frequency with sidebands at  $\pm v\omega_0$ :  $\omega = p\omega_{rf} \pm v\omega_0 \quad -\infty < p < \infty \quad (v = 0.25)$

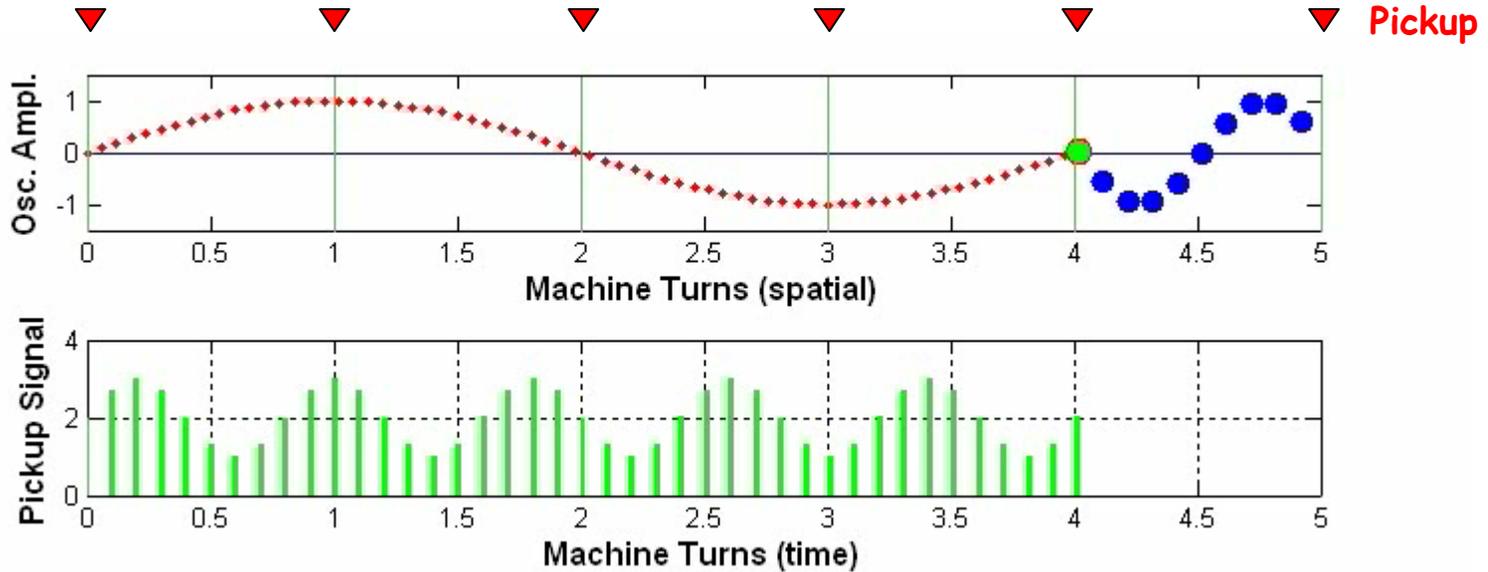


Since the spectrum is periodic and each mode appears twice (upper and lower side band) in a  $\omega_{rf}$  frequency span, we can limit the spectrum analysis to a  $0-\omega_{rf}/2$  frequency range

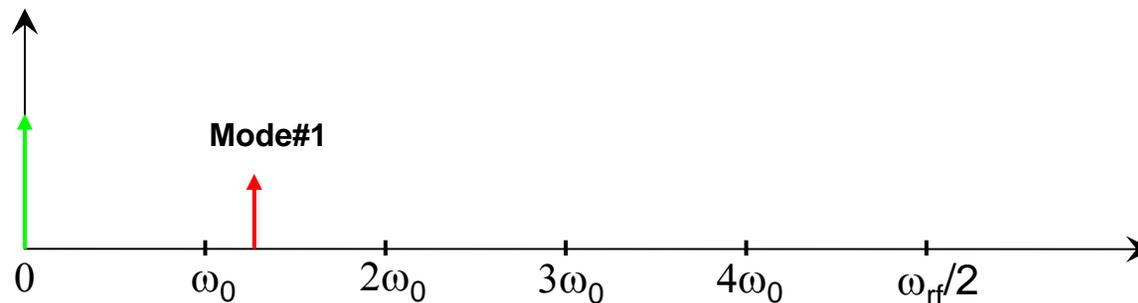


# Multi-bunch modes: example5

Ex.: mode #1 ( $m = 1$ )  $\Delta\Phi = 2\pi/10$  ( $\nu = 0.25$ )

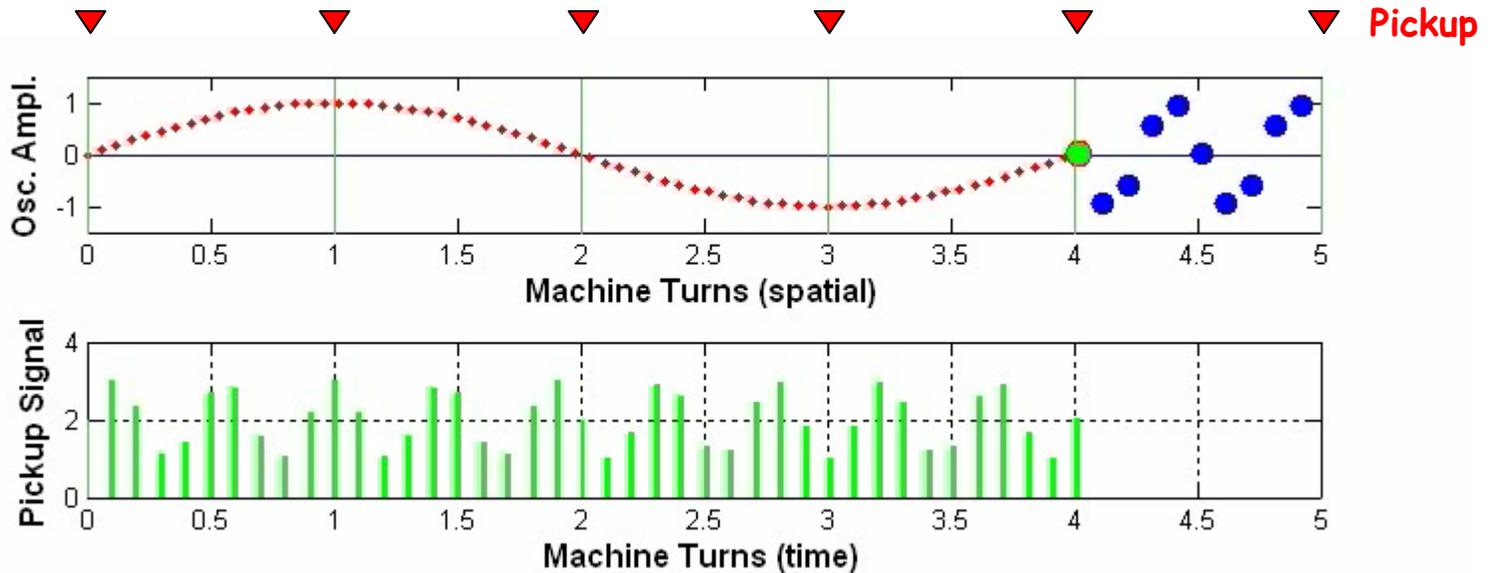


$$\omega = p\omega_{rf} \pm (\nu+1)\omega_0 \quad -\infty < p < \infty$$

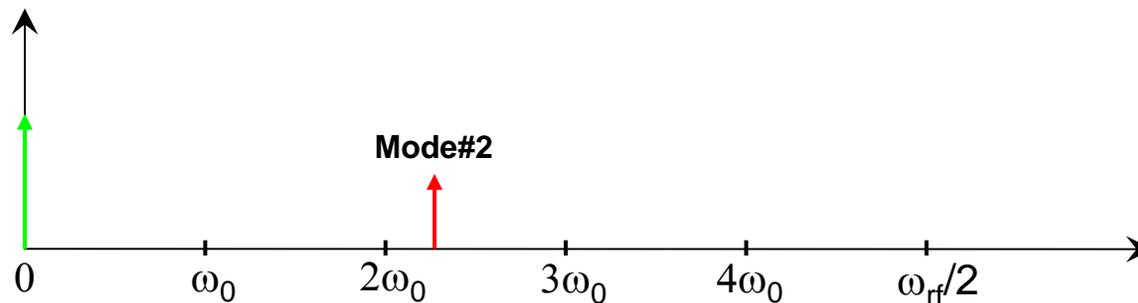


# Multi-bunch modes: example6

Ex.: mode #2 ( $m = 2$ )  $\Delta\Phi = 4\pi/10$  ( $\nu = 0.25$ )

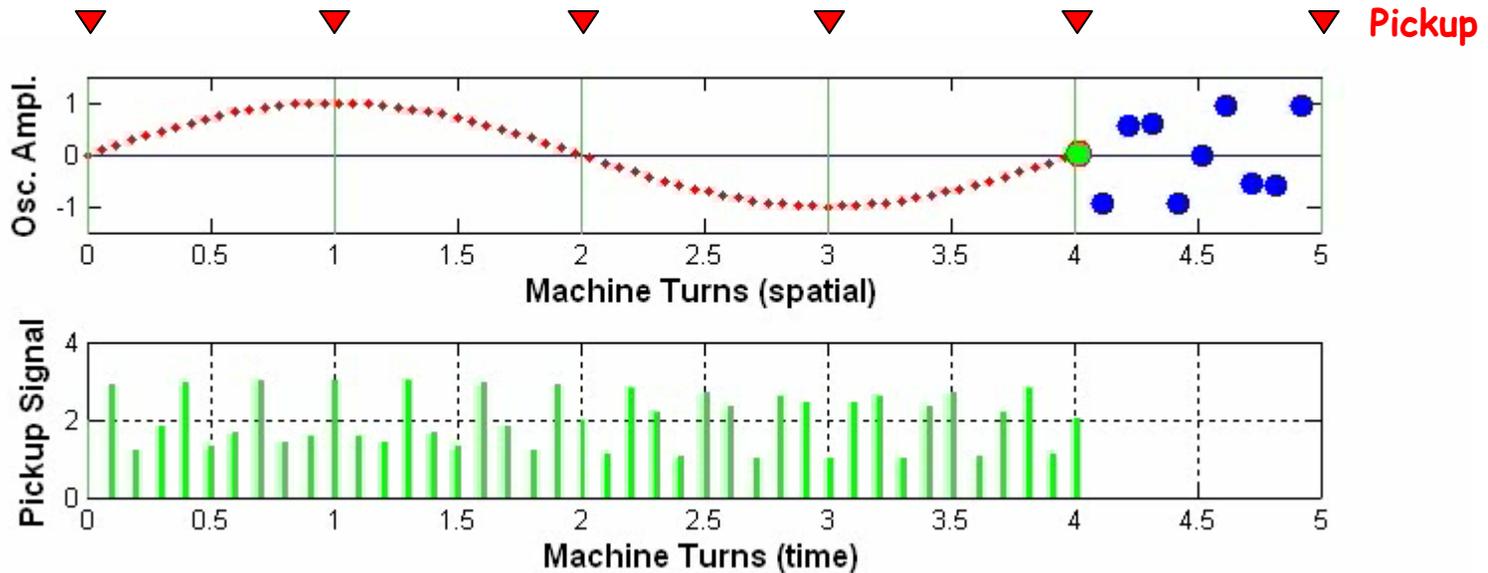


$$\omega = p\omega_{rf} \pm (\nu+2)\omega_0 \quad -\infty < p < \infty$$

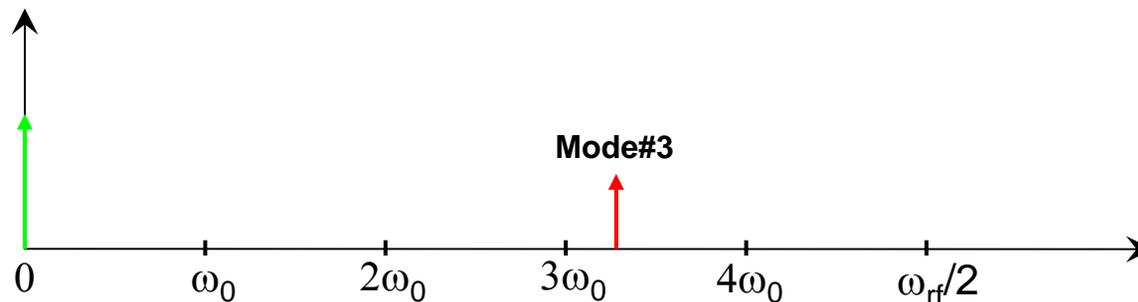


# Multi-bunch modes: example7

Ex.: mode #3 ( $m = 3$ )  $\Delta\Phi = 6\pi/10$  ( $\nu = 0.25$ )

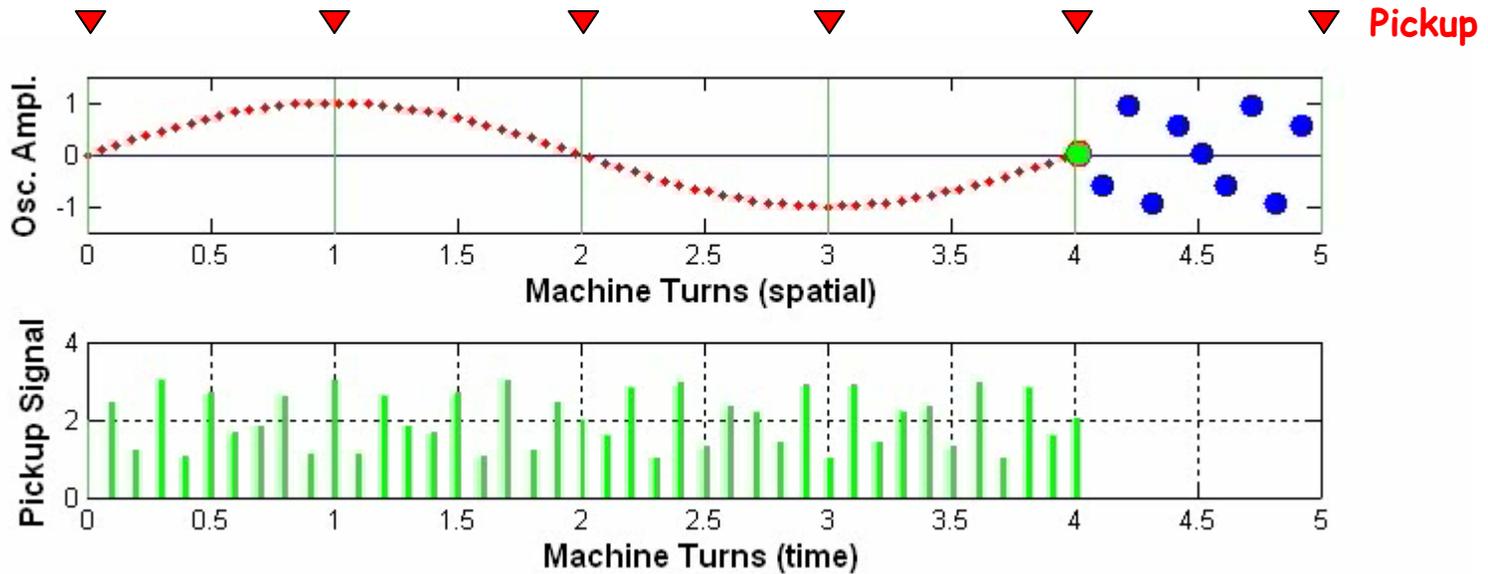


$$\omega = p\omega_{rf} \pm (\nu+3)\omega_0 \quad -\infty < p < \infty$$

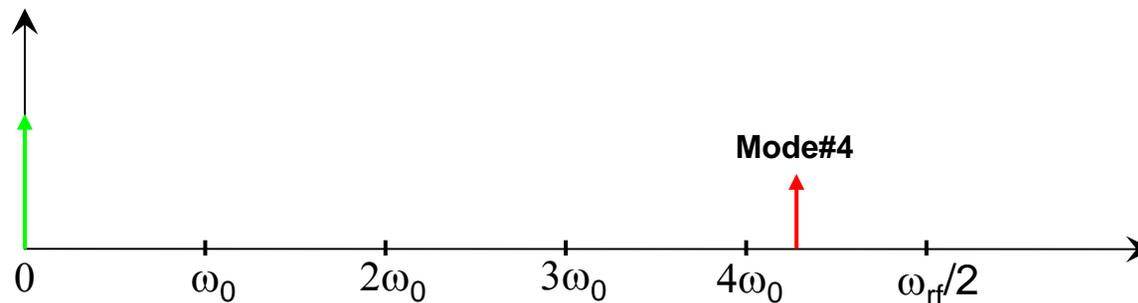


# Multi-bunch modes: example8

Ex.: mode #4 ( $m = 4$ )  $\Delta\Phi = 8\pi/10$  ( $\nu = 0.25$ )

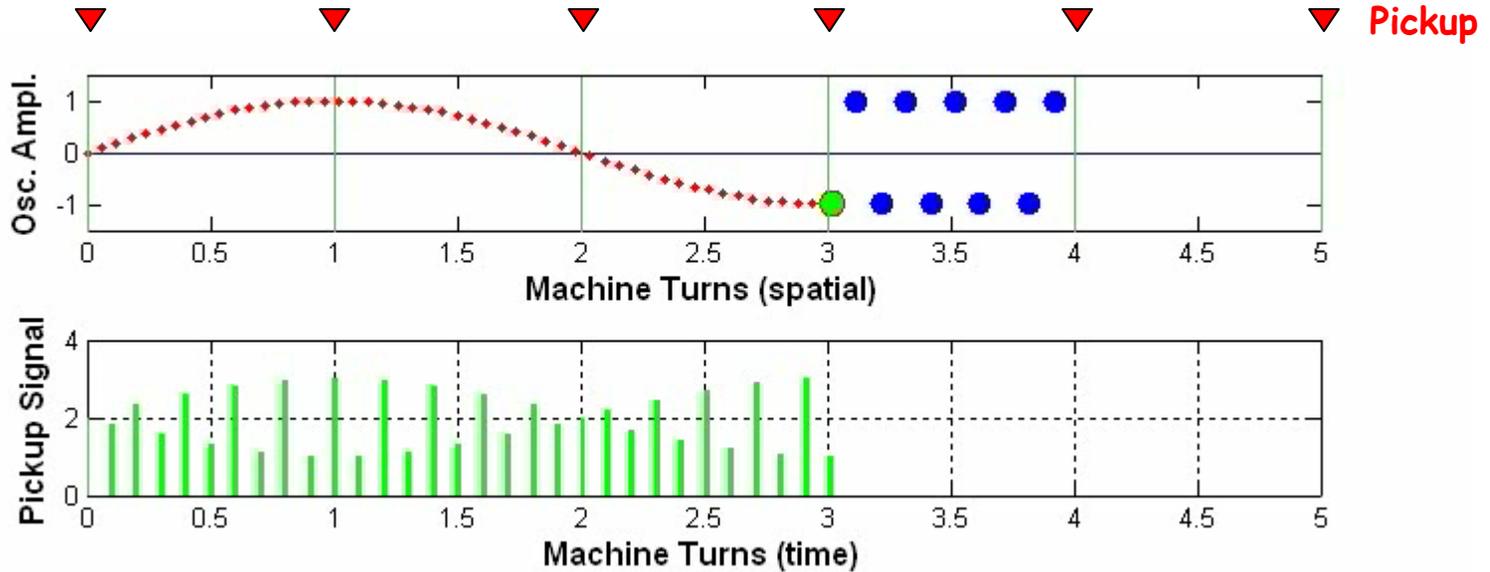


$$\omega = p\omega_{rf} \pm (\nu+4)\omega_0 \quad -\infty < p < \infty$$

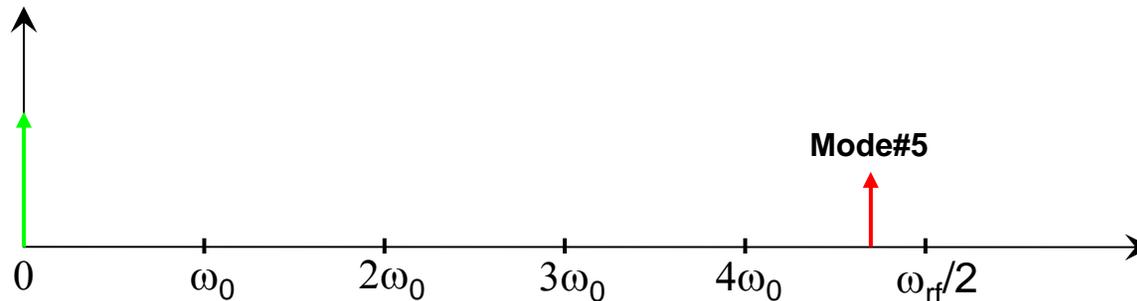


# Multi-bunch modes: example9

Ex.: mode #5 ( $m = 5$ )  $\Delta\Phi = \pi$  ( $\nu = 0.25$ )

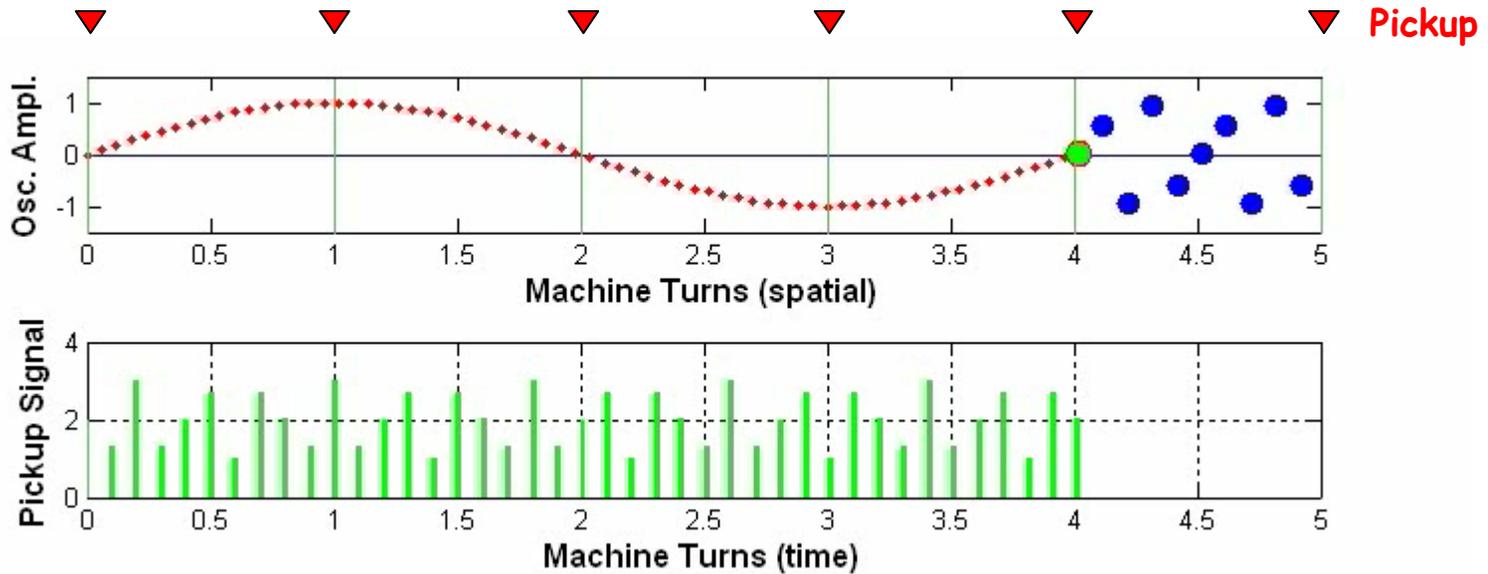


$$\omega = p\omega_{rf} \pm (\nu+5)\omega_0 \quad -\infty < p < \infty$$

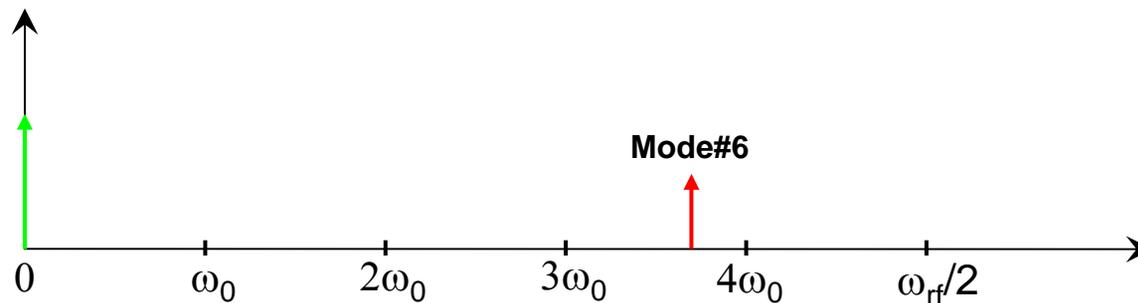


# Multi-bunch modes: example10

Ex.: mode #6 ( $m = 6$ )  $\Delta\Phi = 12\pi/10$  ( $\nu = 0.25$ )

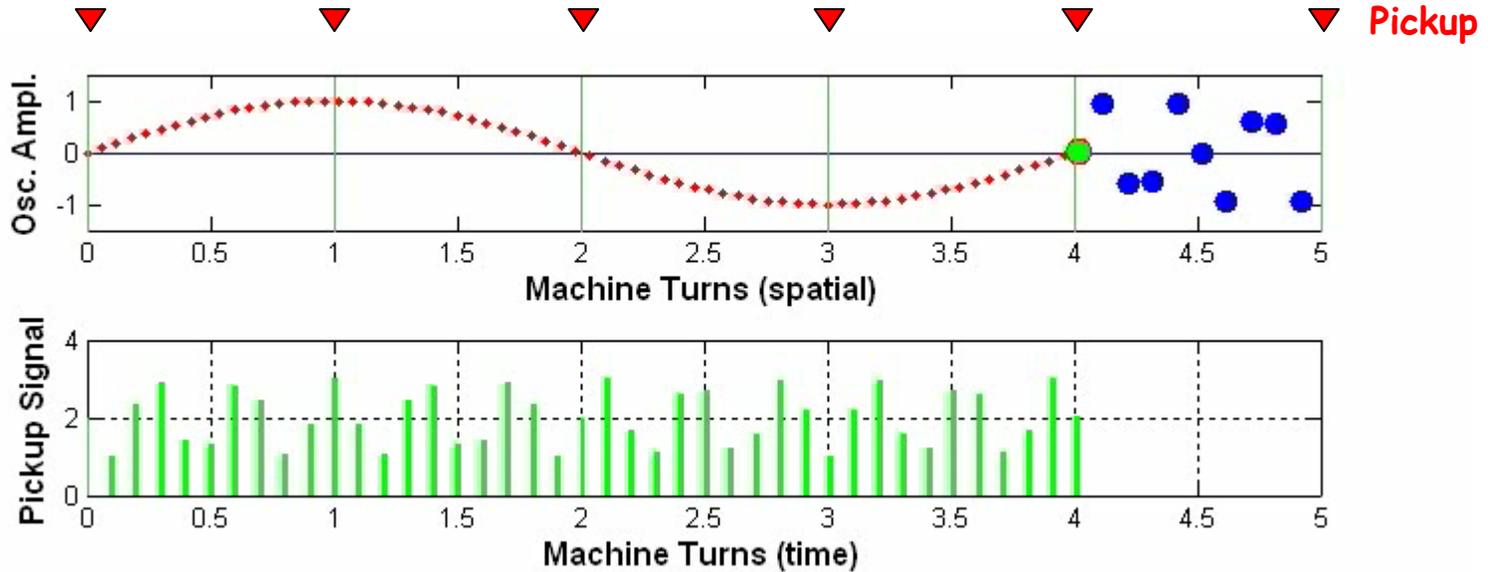


$$\omega = p\omega_{rf} \pm (\nu+6)\omega_0 \quad -\infty < p < \infty$$

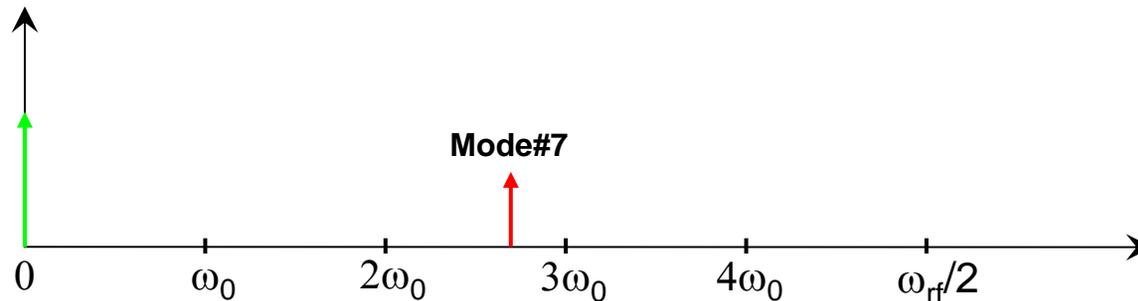


# Multi-bunch modes: example11

Ex.: mode #7 ( $m = 7$ )  $\Delta\Phi = 14\pi/10$  ( $\nu = 0.25$ )

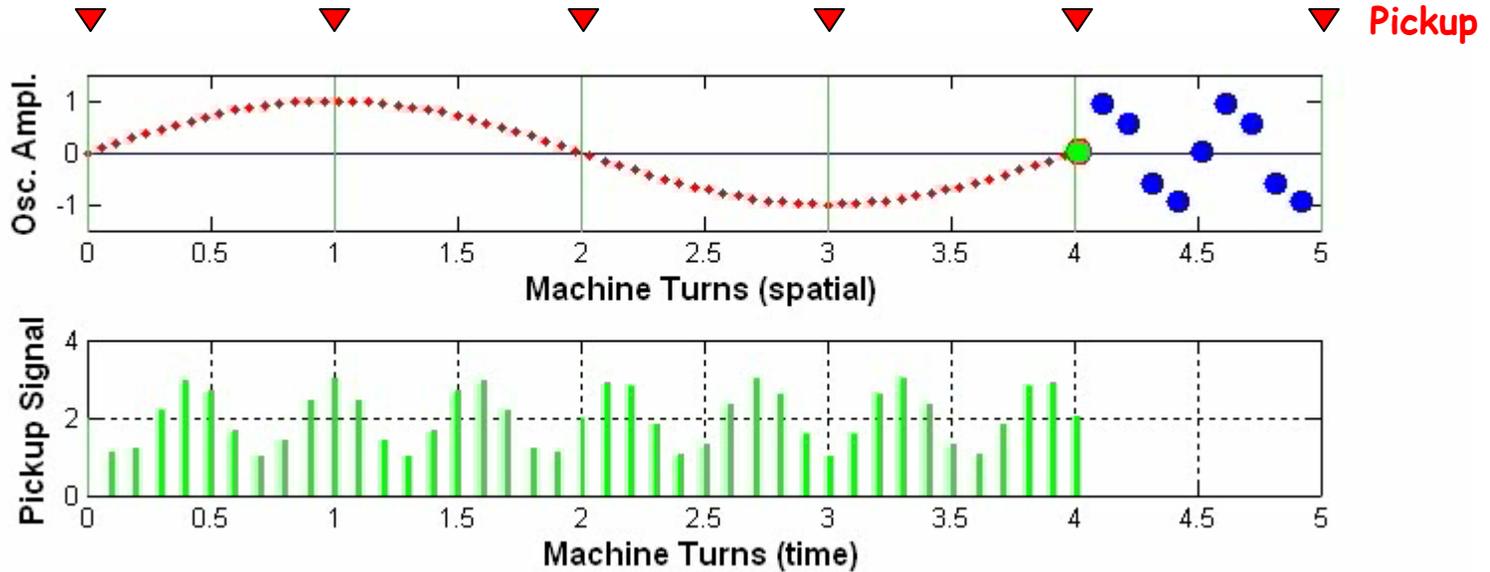


$$\omega = p\omega_{rf} \pm (\nu+7)\omega_0 \quad -\infty < p < \infty$$

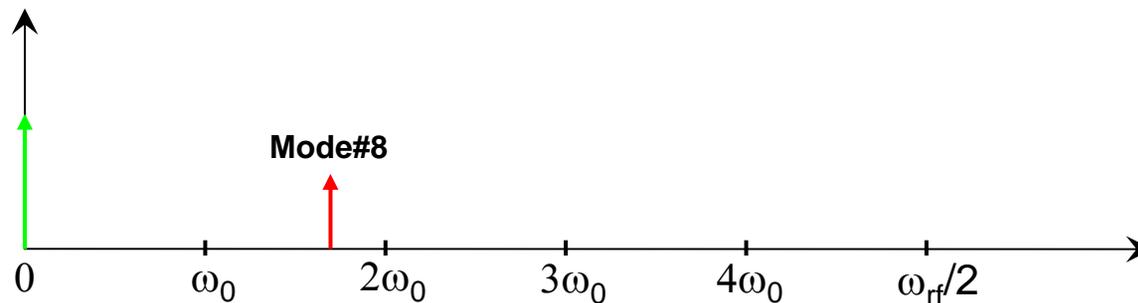


# Multi-bunch modes: example12

Ex.: mode #8 ( $m = 8$ )  $\Delta\Phi = 16\pi/10$  ( $\nu = 0.25$ )

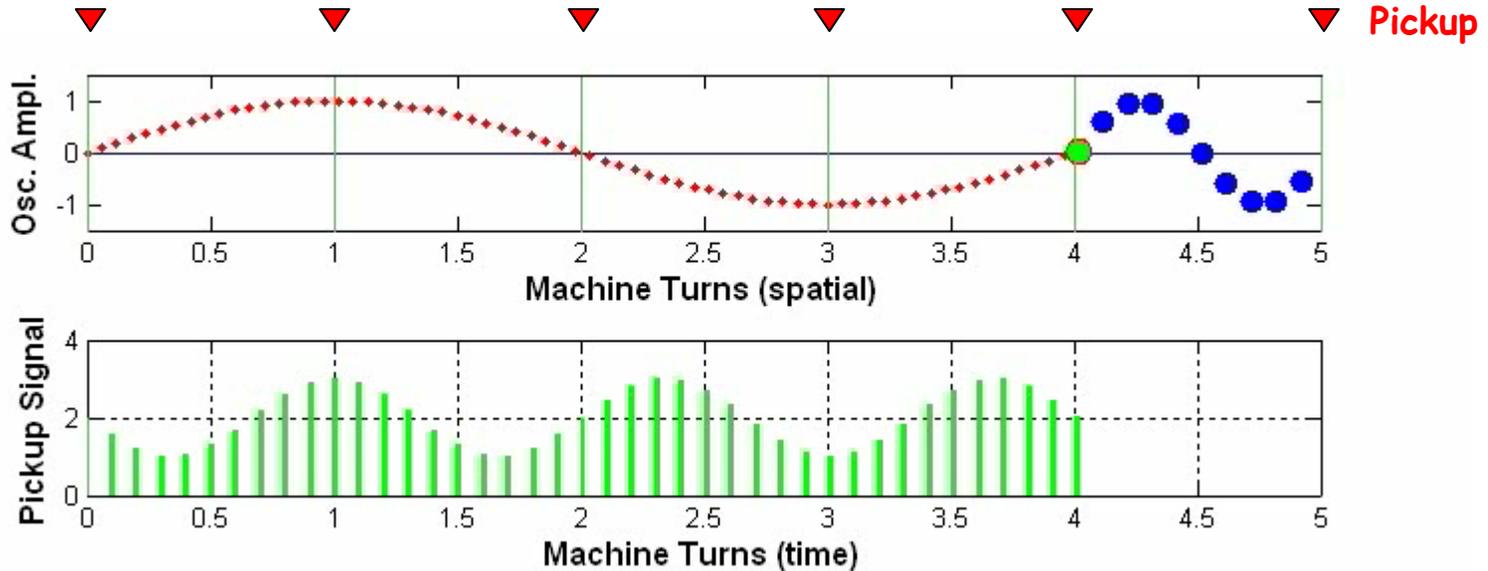


$$\omega = p\omega_{rf} \pm (\nu+8)\omega_0 \quad -\infty < p < \infty$$

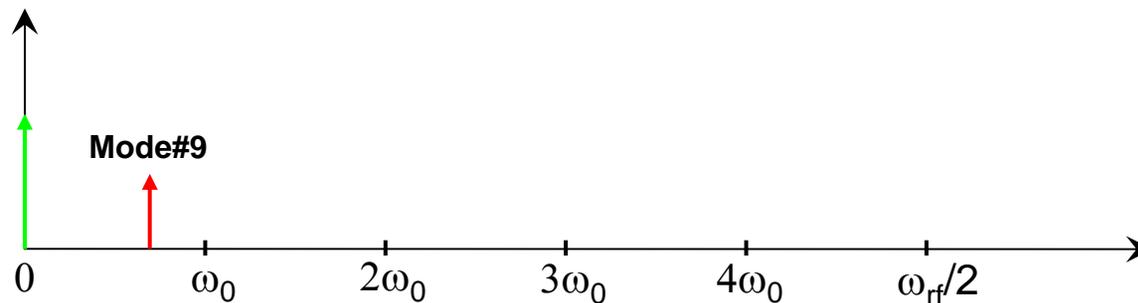


# Multi-bunch modes: example13

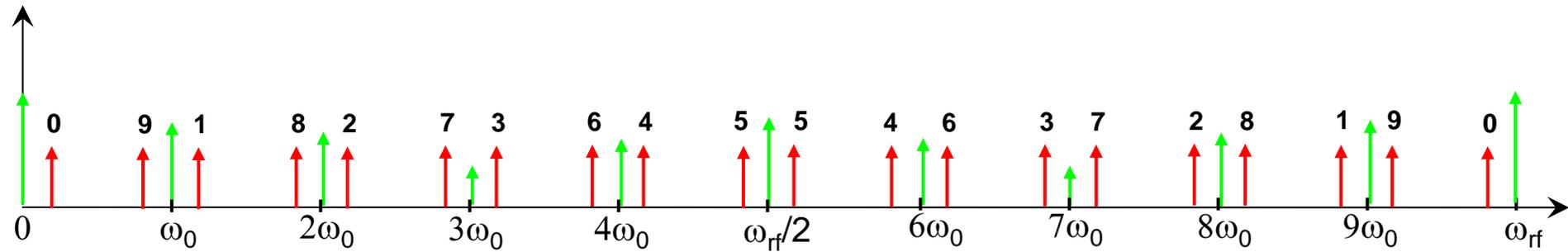
Ex.: mode #9 ( $m = 9$ )  $\Delta\Phi = 18\pi/10$  ( $\nu = 0.25$ )



$$\omega = p\omega_{rf} \pm (\nu+9)\omega_0 \quad -\infty < p < \infty$$

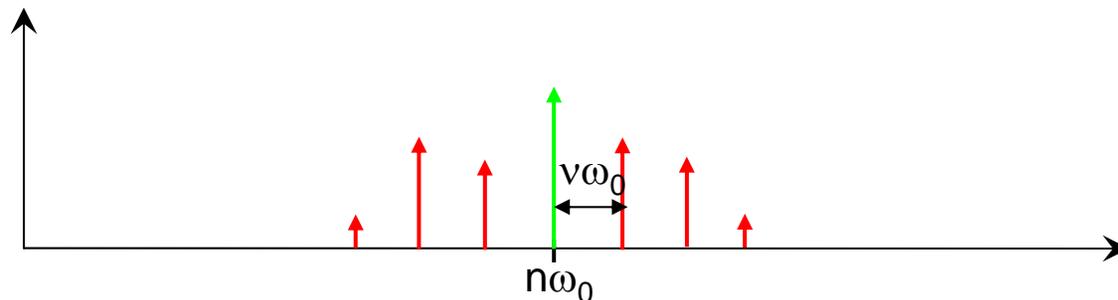


# Multi-bunch modes: uneven filling and longitudinal modes



If the bunches have **not the same charge**, i.e. the buckets are not equally filled (uneven filling), the spectrum has frequency **components** also **at the revolution harmonics** (multiples of  $\omega_0$ ). The amplitude of each revolution harmonic depends on the filling pattern of one machine turn

In case of **longitudinal modes**, we have a **phase modulation** of the stable beam signal. Components at  $\pm v\omega_0, \pm 2v\omega_0, \pm 3v\omega_0, \dots$  can appear aside the revolution harmonics. Their amplitude depends on the depth of the phase modulation (Bessel series expansion)

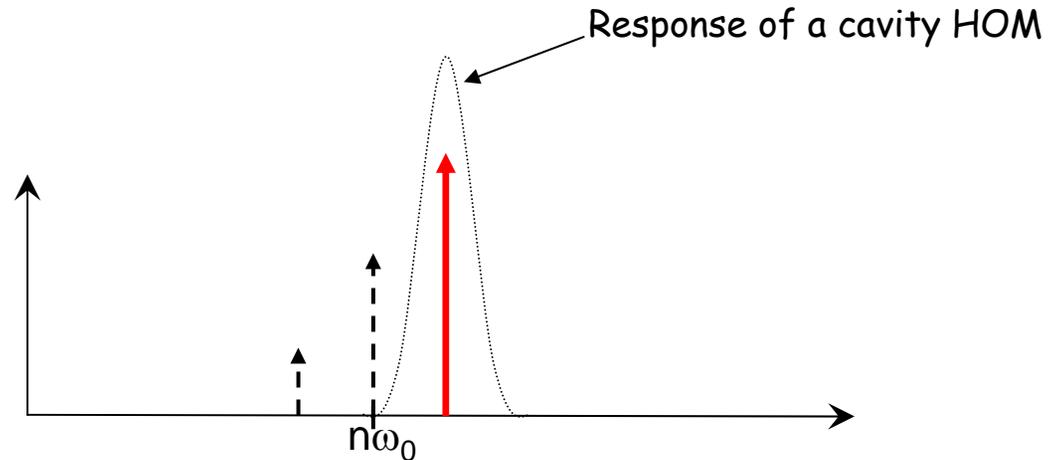


# Multi-bunch modes: coupled-bunch instability

One multi-bunch mode can become unstable if one of its sidebands overlaps, for example, with the frequency response of a cavity high order mode (HOM). The HOM couples with the sideband giving rise to a **coupled-bunch instability**, with consequent increase of the sideband amplitude



Synchrotron Radiation Monitor showing the transverse beam shape



## Effects of coupled-bunch instabilities:

- ☹ increase of the transverse beam dimensions
- ☹ increase of the effective emittance
- ☹ beam loss and max current limitation
- ☺ increase of lifetime due to decreased Touschek scattering (dilution of particles)

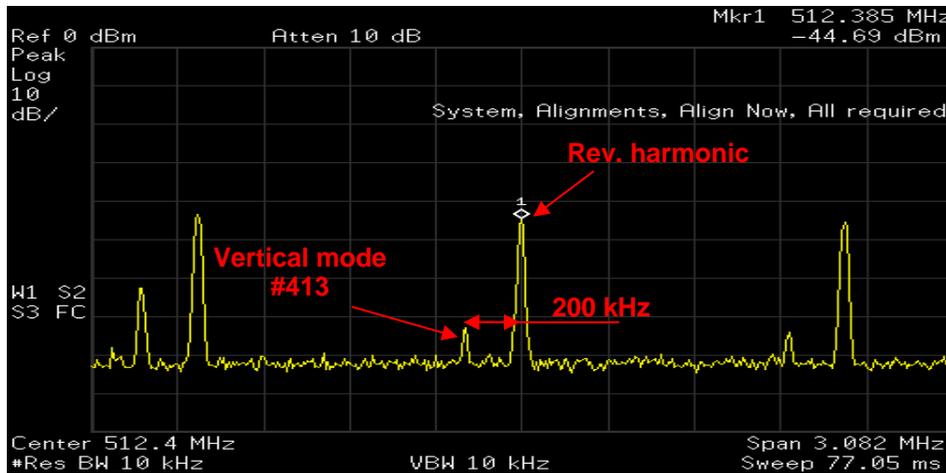
# Real example of multi-bunch modes

ELETTRA Synchrotron:  $f_{rf}=499.654$  MHz, bunch spacing  $\approx 2$  ns, 432 bunches,  $f_0 = 1.15$  MHz

$V_{hor} = 12.30$  (fractional tune frequency = 345 kHz),  $V_{vert} = 8.17$  (fractional tune frequency = 200 kHz)

$V_{long} = 0.0076$  (8.8 kHz)

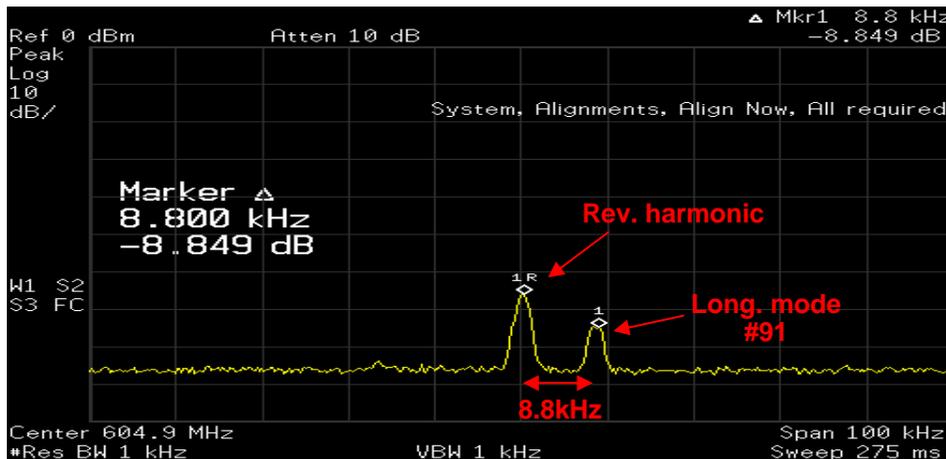
$$\omega = pM\omega_0 \pm (m+v)\omega_0$$



Spectral line at 512.185 MHz

Lower sideband of  $2f_{rf}$ , 200 kHz apart from the 443<sup>rd</sup> revolution harmonic

→ vertical mode #413

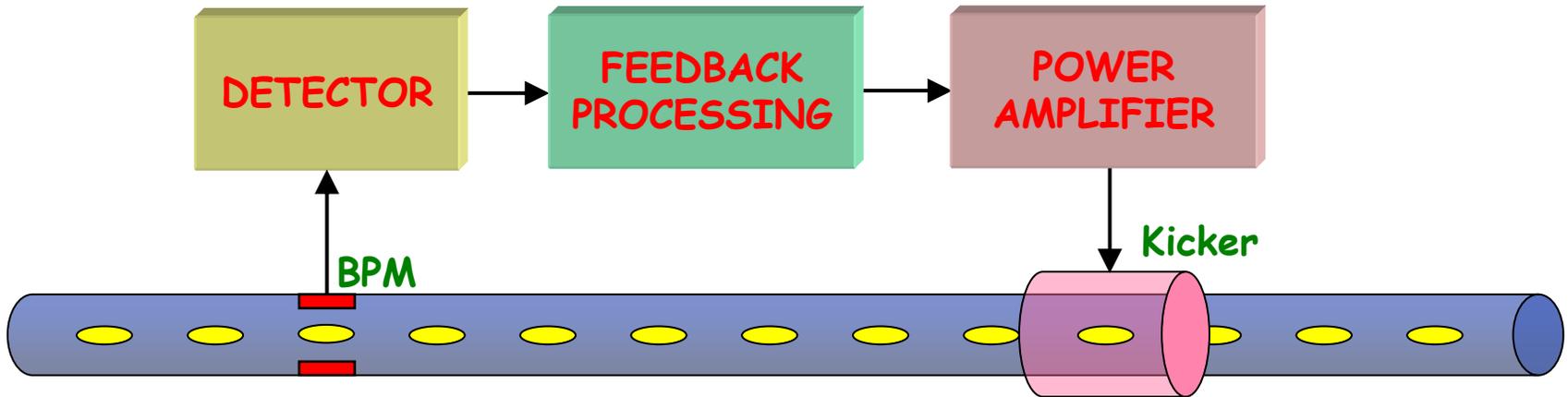


Spectral line at 604.914 MHz

Upper sideband of  $f_{rf}$ , 8.8 kHz apart from the 523<sup>rd</sup> revolution harmonic

→ longitudinal mode #91

A multi-bunch feedback system detects the instability using one or more Beam Position Monitors (BPM) and acts back on the beam to damp the oscillation through an electromagnetic actuator called **kicker**



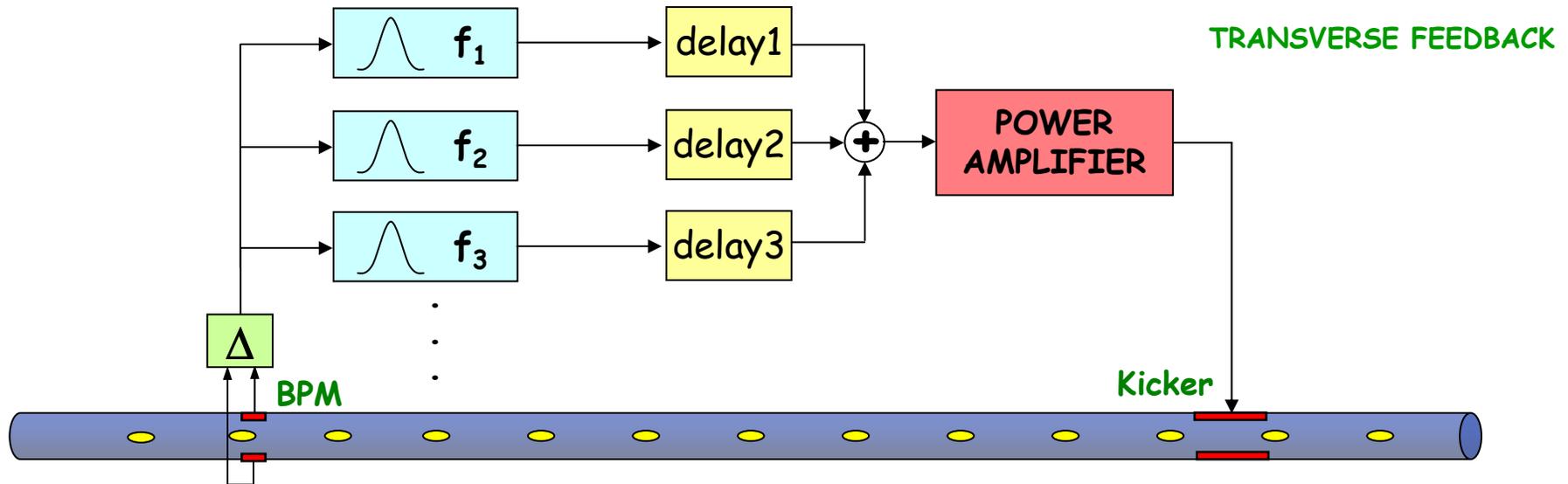
BPM and detector measure the beam oscillations

The feedback processing unit generates the correction signal

The RF power amplifier amplifies the signal

The kicker generates the electromagnetic field

A **mode-by-mode** (frequency domain) feedback acts separately on each unstable mode



An analog electronics generates the position error signal from the BPM buttons

A number of processing channels working in parallel each dedicated to one of the controlled modes

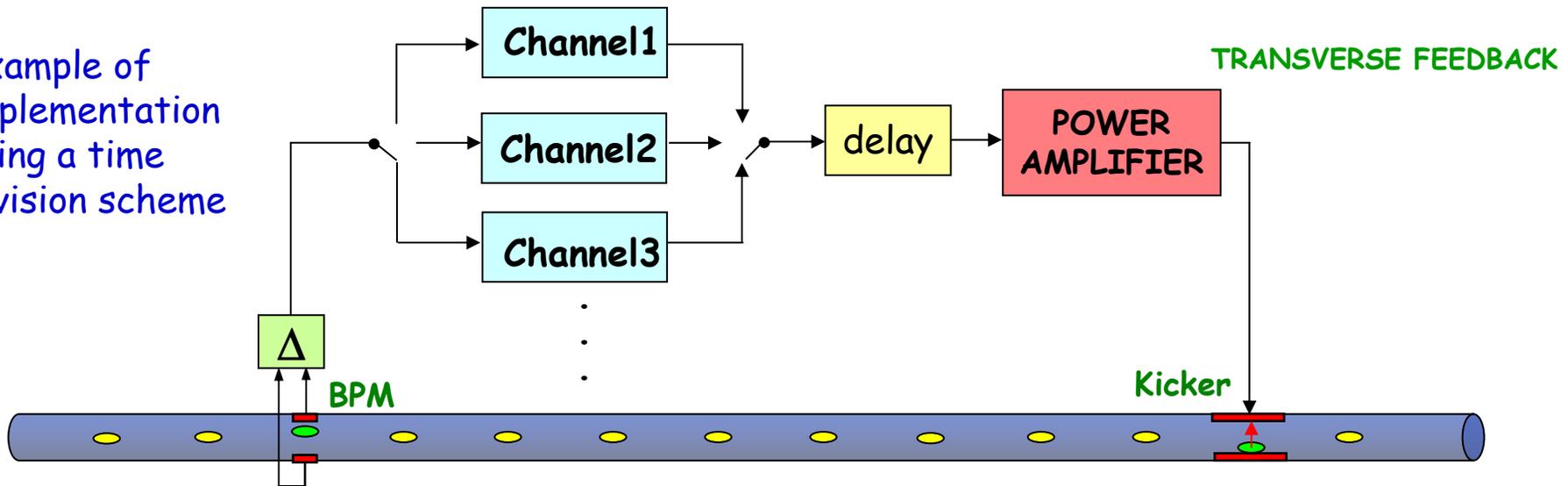
The signals are band-pass filtered, phase shifted by an adjustable delay line to produce a negative feedback and recombined

# Bunch-by-bunch feedback

A **bunch-by-bunch** (time domain) feedback individually steers each bunch by applying small electromagnetic kicks every time the bunch passes through the kicker: the result is a damped oscillation lasting several turns

The correction signal for a given bunch is generated based on the motion of the same bunch

Example of implementation using a time division scheme



Every bunch is measured and corrected at every machine turn but, due to the delay of the feedback chain, the correction kick corresponding to a given measurement is applied to the bunch **one or more turns later**

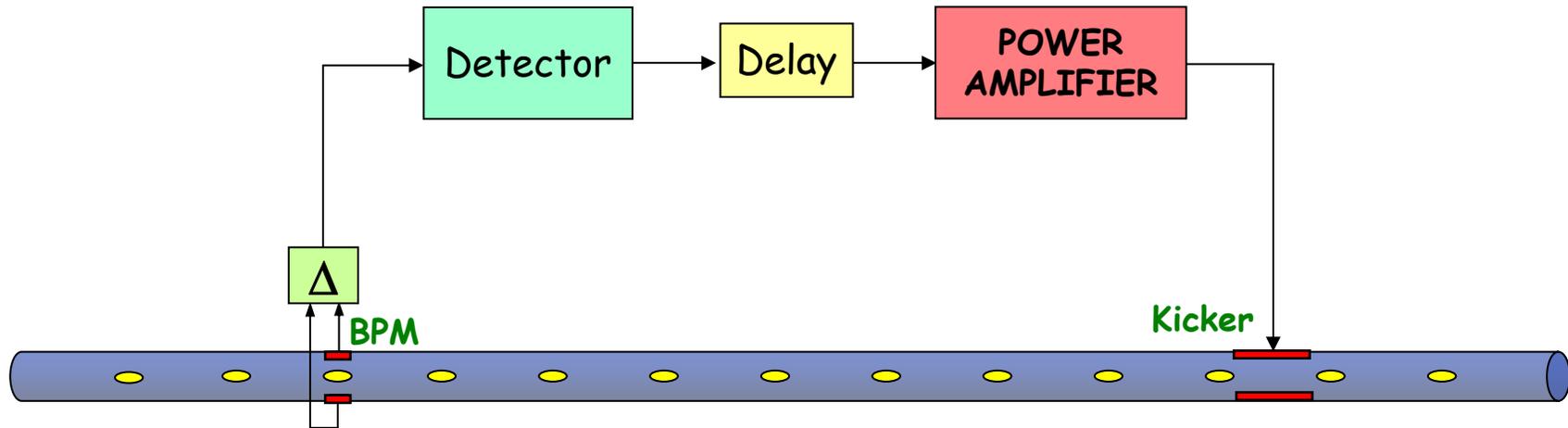
**Damping the oscillation of each bunch is equivalent to damping all multi-bunch modes**

# Analog bunch-by-bunch feedback: one-BPM feedback

## Transverse feedback

The correction signal applied to a given bunch must be proportional to the derivative of the bunch oscillation at the kicker, thus it must be a sampled sinusoid shifted  $\pi/2$  with respect to the oscillation of the bunch when it passes through the kicker

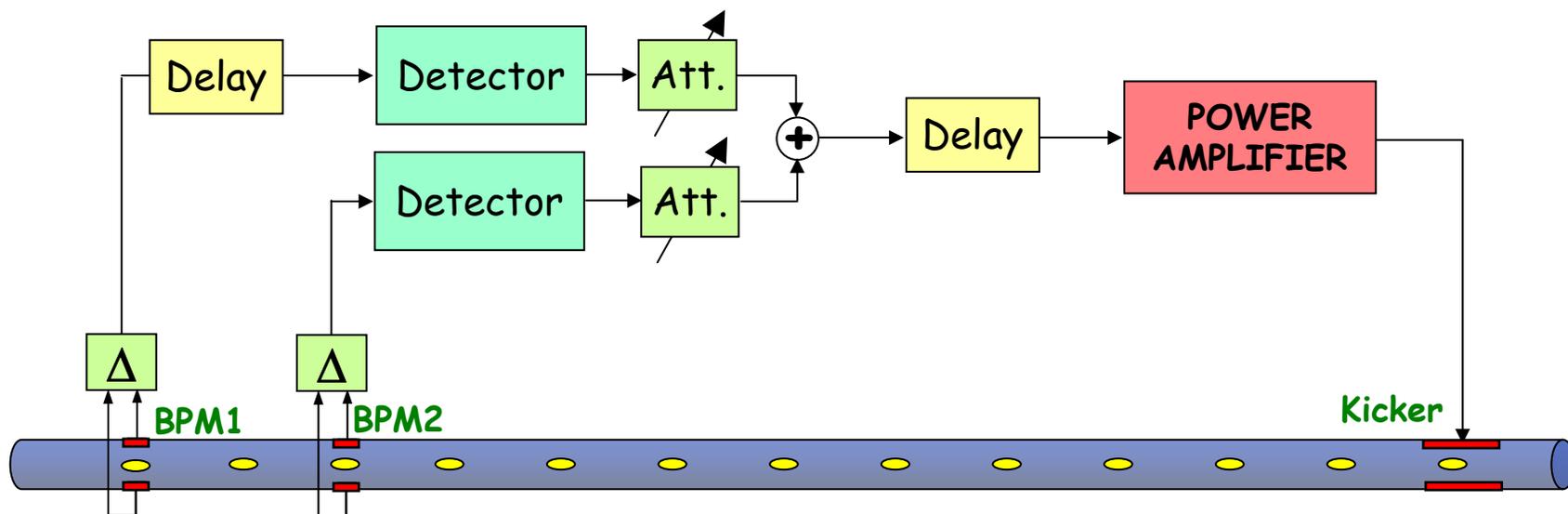
The signal from a BPM with the appropriate betatron phase advance with respect to the kicker can be used to generate the correction signal



The detector down converts the high frequency (typically a multiple of the bunch frequency  $f_{rf}$ ) BPM signal into base-band (range  $0 - f_{rf}/2$ )

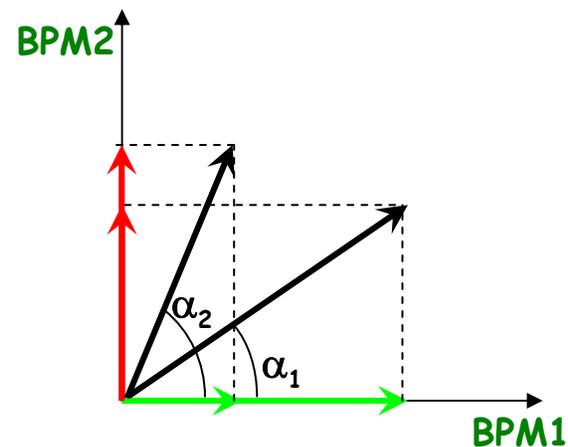
The delay line assures that the signal of a given bunch passing through the feedback chain arrives at the kicker when, after one machine turn, the same bunch passes through it

## Transverse feedback case



The **two BPMs** can be placed in any ring position with respect to the kicker providing that they are **separated by  $\pi/2$  in betatron phase**

Their signals are combined with variable attenuators in order to provide the required phase of the resulting signal

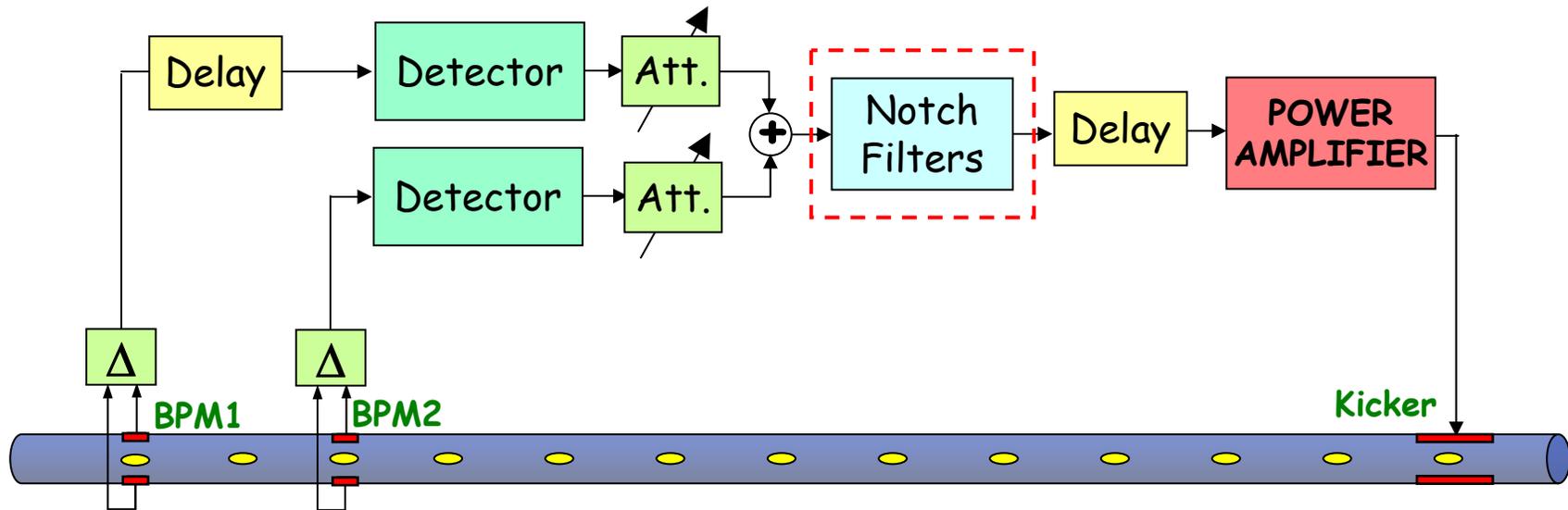


# Analog feedback: revolution harmonics suppression

## Transverse feedback case

The revolution harmonics (frequency components at multiples of  $\omega_0$ ) are useless components that have to be eliminated in order not to saturate the RF amplifier

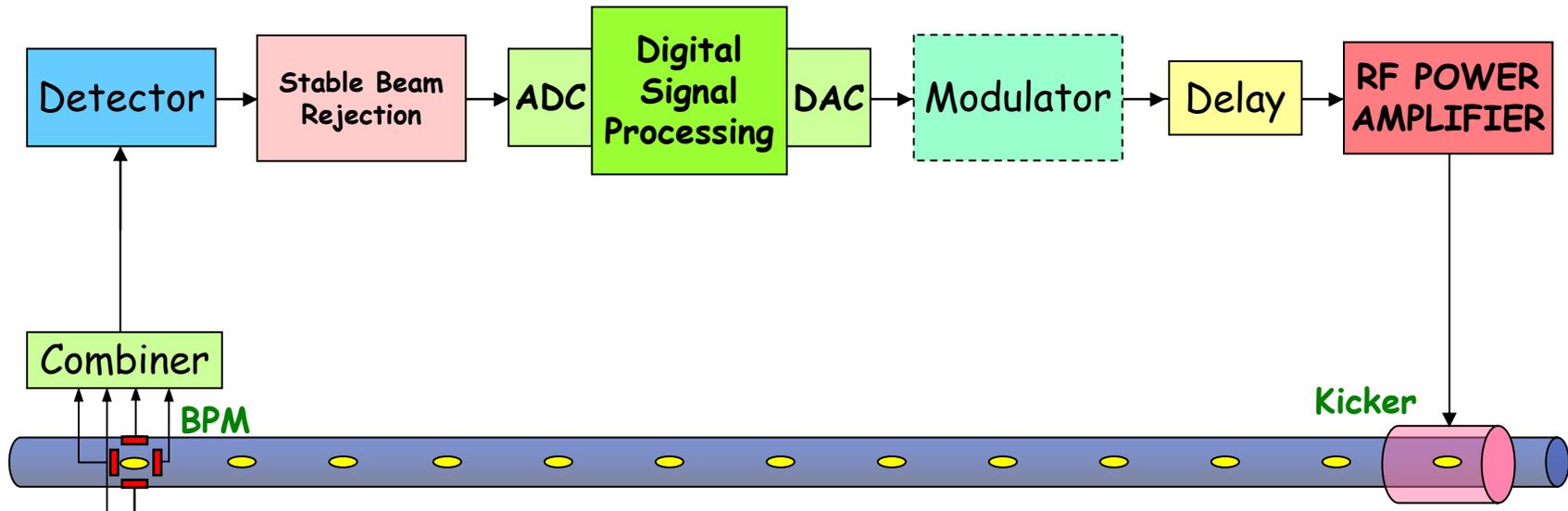
This operation is also called "stable beam rejection"



Similar feedback architectures have been used to build the transverse multi-bunch feedback system of a number of light sources: ex. ALS, BessyII, PLS, ANKA, ...

# Digital bunch-by-bunch feedback

*Transverse and longitudinal case*



The **combiner** generates the X, Y or  $\Sigma$  signal from the BPM button signals

The **detector** (RF front-end) demodulates the position signal to base-band

"Stable beam components" are suppressed by the **stable beam rejection module**

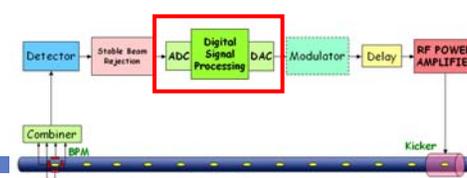
The resulting signal is digitized, processed and re-converted to analog by the **digital processor**

The **modulator** translates the correction signal to the kicker working frequency (long. only)

The **delay line** adjusts the timing of the signal to match the bunch arrival time

The **RF power amplifier** supplies the power to the **kicker**

# Digital vs. analog feedbacks



## ADVANTAGES OF DIGITAL FEEDBACKS



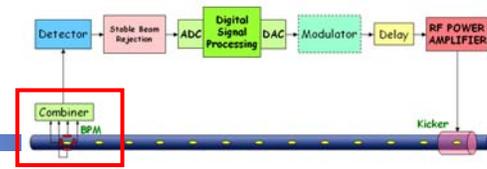
- ↘ **reproducibility**: when the signal is digitized it is not subject to temperature/environment changes or aging
- ↘ **programmability**: the implementation of processing functionalities is usually made using DSPs or FPGAs, which are programmable via software/firmware
- ↘ **performance**: digital controllers feature superior processing capabilities with the possibility to implement sophisticated control algorithms not feasible in analog
- ↘ **additional features**: possibility to combine basic control algorithms and additional useful features like signal conditioning, saturation control, down sampling, etc.
- ↘ **implementation of diagnostic tools**, used for both feedback commissioning and machine physics studies
- ↘ **easier and more efficient integration** of the feedback in the accelerator control system for data acquisition, feedback setup and tuning, automated operations, etc.

## DISADVANTAGE OF DIGITAL FEEDBACKS



- ↘ **High delay** due to ADC, digital processing and DAC

# BPM and Combiner

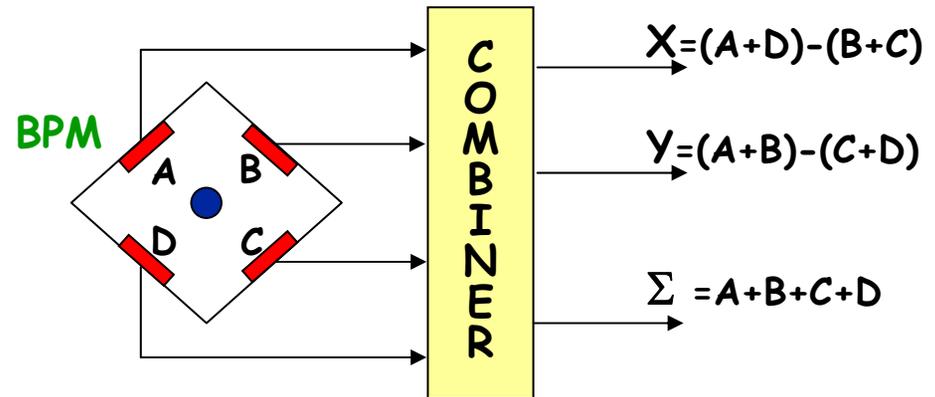
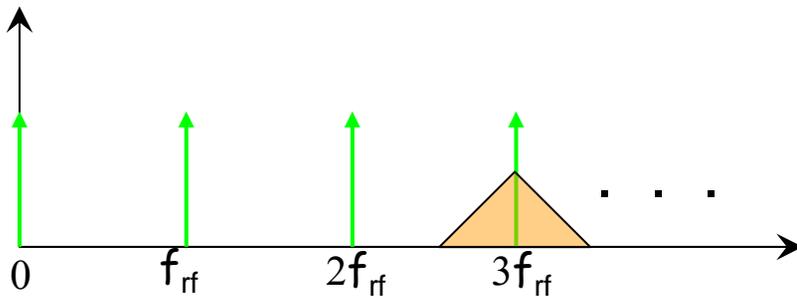


The four signals from a standard four-button BPM can be opportunely combined to obtain the wide-band  $X$ ,  $Y$  and  $\Sigma$  signals used respectively by the horizontal, vertical and longitudinal feedbacks

Any  $f_{rf}/2$  portion of the beam spectrum contains the information of all potential multi-bunch modes and can be used to detect instabilities and measure their amplitude

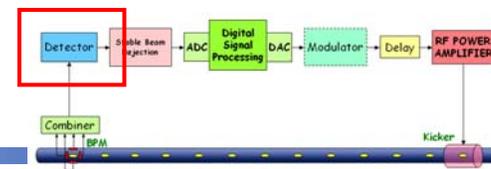
Usually BPM and combiner work around a multiple of  $f_{rf}$ , where the amplitude of the overall frequency response of BPM and cables is maximum

Moreover, a higher  $f_{rf}$  harmonic is preferred for the longitudinal feedback because of the better sensitivity of the phase detection system

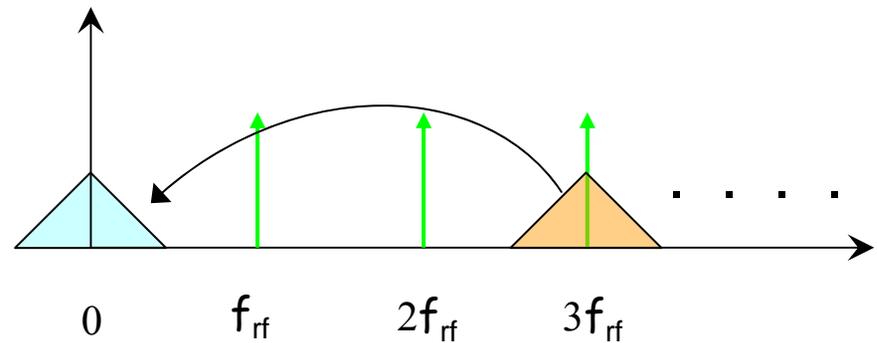


The SUM ( $\Sigma$ ) signal contains only information of the phase (longitudinal position) of the bunches, since the sum of the four button signals has almost constant amplitude

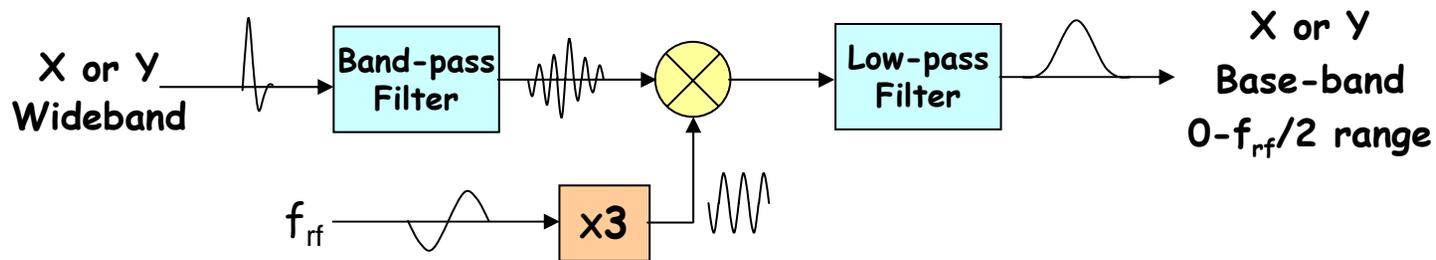
# Detector: transverse feedback



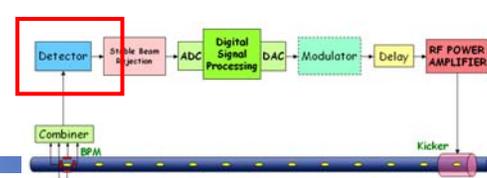
The **detector** (or **RF front-end**) translates the wide-band signal to base-band ( $0-f_{rf}/2$  range): the operation is an **amplitude demodulation**



**Heterodyne technique:** the "local oscillator" signal is derived from the RF by multiplying its frequency by an integer number corresponding to the chosen harmonic of  $f_{rf}$

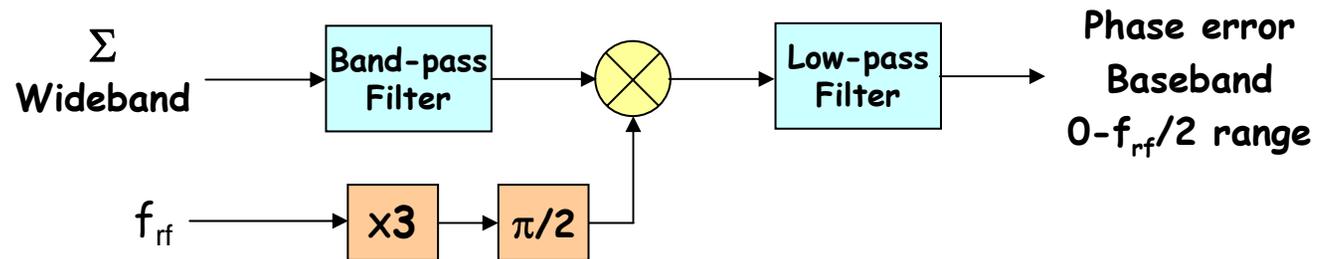


# Detector: longitudinal feedback



The detector generates the base-band longitudinal position (phase error) signal ( $0-f_{rf}/2$  range) by processing the wide-band signal: the operation is a **phase demodulation**

The phase demodulation can be obtained with the same heterodyne technique but using a local oscillator signal in quadrature (shifted  $\pi/2$ ) with respect to the bunches



**Amplitude demodulation:**

$$A(t) \sin(3\omega_{rf} t) \cdot \sin(3\omega_{rf} t) \propto A(t) (\cos(0) - \cos(6\omega_{rf} t)) \approx A(t)$$

~~Low-pass filter~~

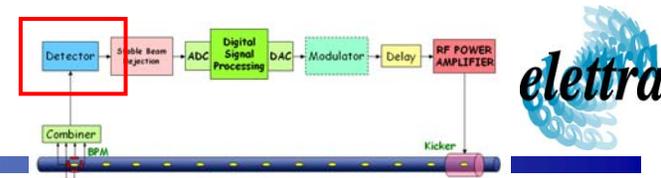
**Phase demodulation:**

$$\sin(3\omega_{rf} t + \varphi(t)) \cdot \cos(3\omega_{rf} t) \propto \sin(6\omega_{rf} t + \varphi(t)) + \sin(\varphi(t)) \approx \varphi(t)$$

~~Low-pass filter~~

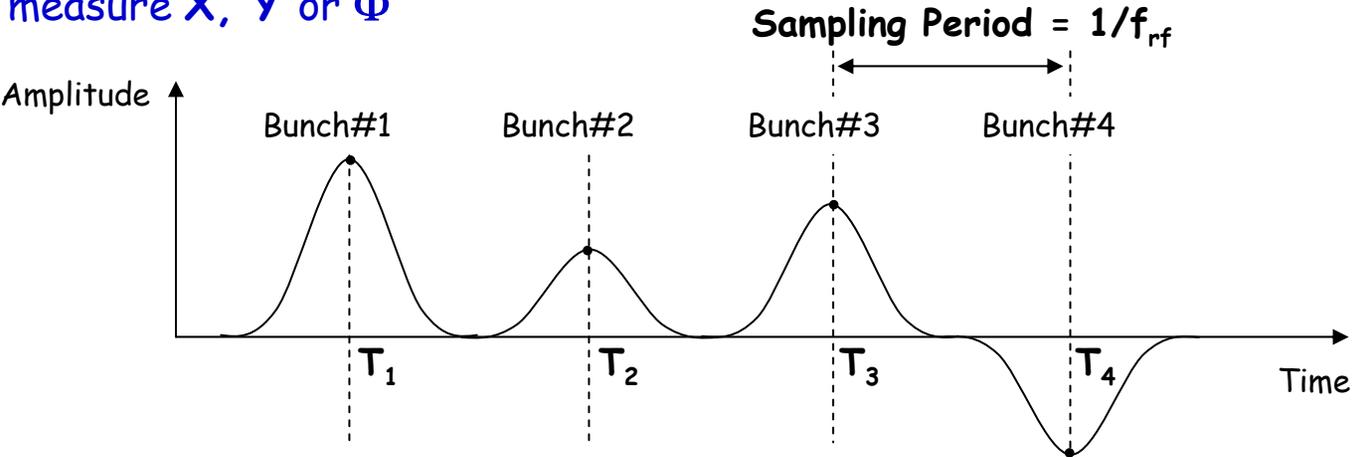
For small signals

# Detector: time domain considerations



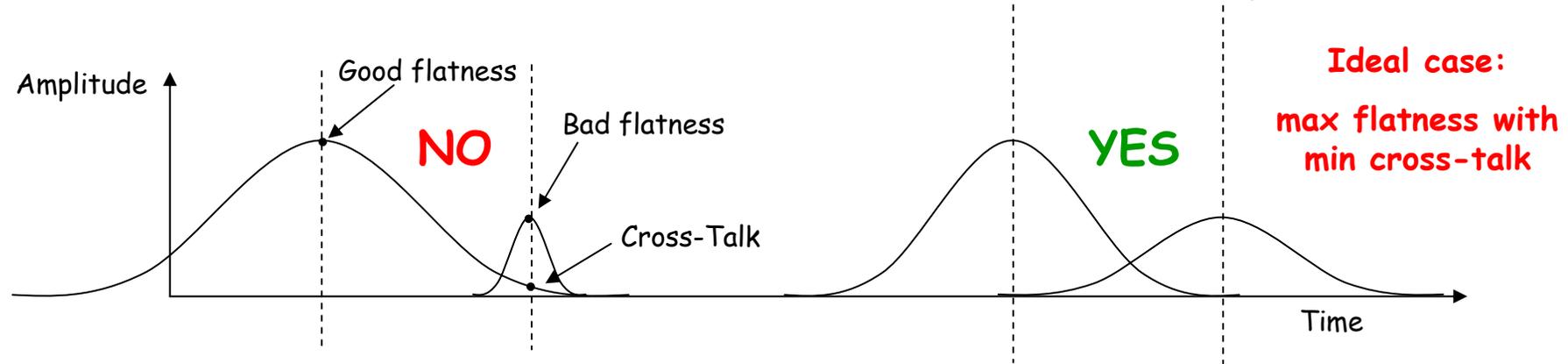
The base-band signal can be seen as a sequence of "pulses" each with amplitude proportional to the position error ( $X$ ,  $Y$  or  $\Phi$ ) and to the charge of the corresponding bunch

By sampling this signal with an A/D converter synchronous to the bunch frequency, one can measure  $X$ ,  $Y$  or  $\Phi$

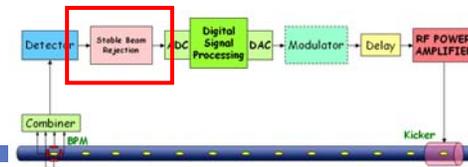


The multi-bunch-mode number  $M/2$  is the one with higher frequency ( $\approx f_{rf}/2$ ): the pulses have almost the same amplitude but alternating signs

The design of band-pass and low-pass filters is a compromise between **maximum flatness** of the top of the pulses and **cross-talk** between bunches due to overlap with adjacent pulses



# Rejection of stable beam signal



The turn-by-turn pulses of each bunch can have a constant offset (stable beam signal) due to:

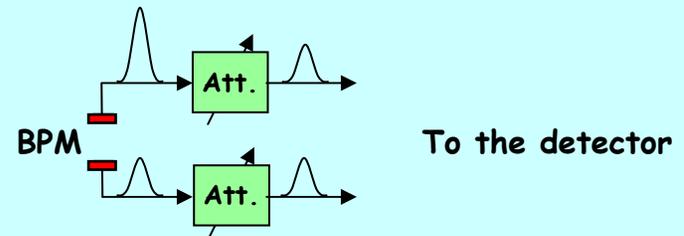
- ↘ **transverse case:** off-centre beam or unbalanced BPM electrodes or cables
- ↘ **longitudinal case:** beam loading, i.e. different synchronous phase for each bunch

In the frequency domain, the stable beam signal carries non-zero revolution harmonics

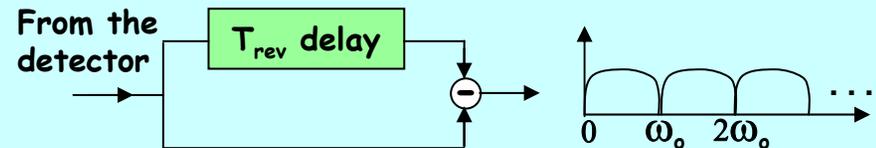
These components have to be suppressed because don't contain information about multi-bunch modes and can saturate ADC, DAC and amplifier

*Examples of used techniques:*

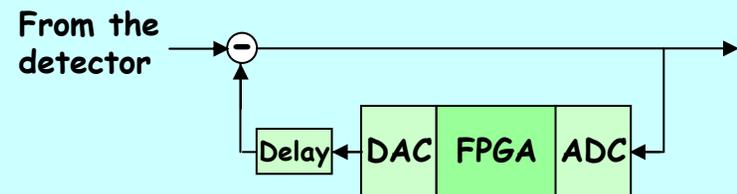
**Balancing of BPM buttons:** variable attenuators on the electrodes to equalize the amplitude of the signals (transverse feedback)



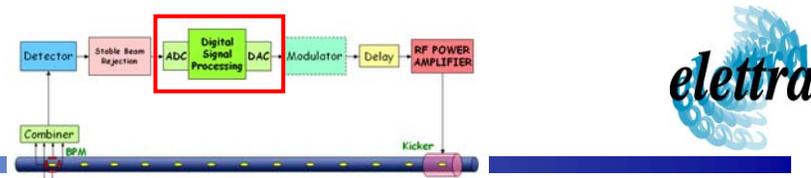
**Comb filter** using delay lines and combiners: the frequency response is a series of notches at multiple of  $\omega_0$ , DC included



**Digital DC rejection:** the signal is sampled at  $f_{rf}$ , the turn-by-turn signal is integrated for each bunch, recombined with the other bunches, converted to analog and subtracted from the original signal



# Digital processor

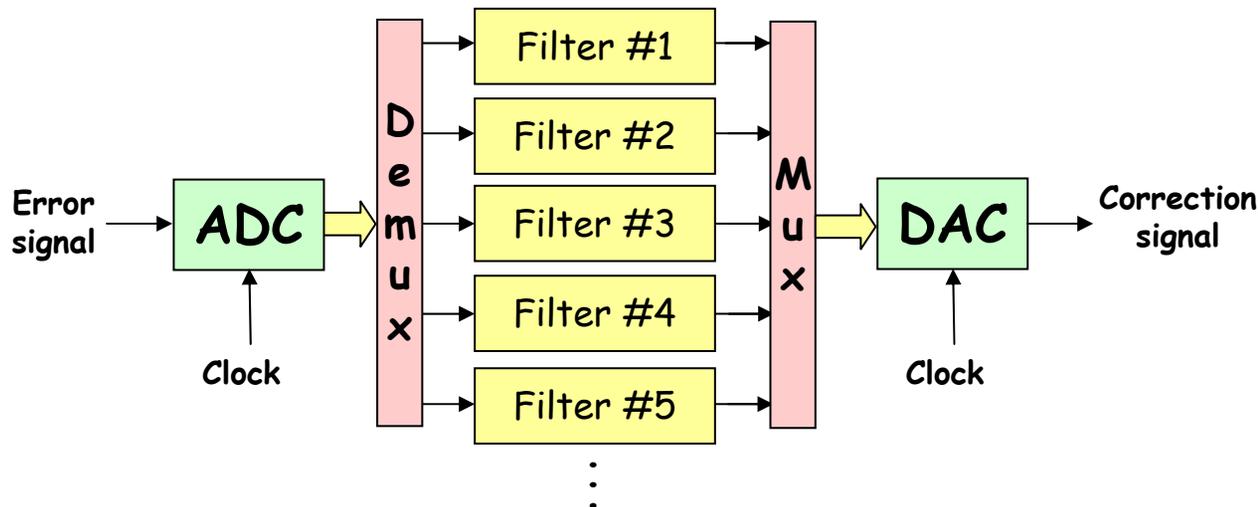


The **A/D converter** samples and digitizes the signal at the bunch repetition frequency: each sample corresponds to the position ( $X$ ,  $Y$  or  $\Phi$ ) of a given bunch. Precise synchronization of the sampling clock with the bunch signal must be provided

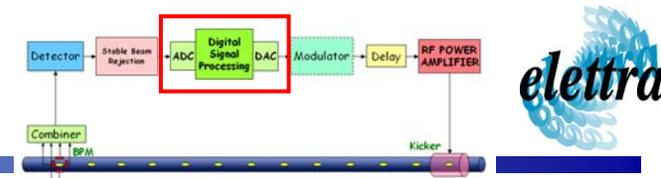
The **digital samples** are then **de-multiplexed** into  $M$  channels ( $M$  is the number of bunches): in each channel the turn-by-turn samples of a given bunch are processed by a dedicated **digital filter** to calculate the correction samples

The basic processing consists in **DC component suppression** (if not completely done by the external stable beam rejection) and **phase shift** at the betatron/synchrotron frequency

After processing, the correction sample streams are **recombined** and eventually converted to analog by the **D/A converter**



# Digital processor implementation



**ADC:** existing multi-bunch feedback systems usually employ 8-bit ADCs at up to 500 Msample/s; some implementations use a number of ADCs with higher resolution (ex. 14 bits) and lower rate working in parallel. ADCs with enhanced resolution have some advantages:

- lower quantization noise (crucial for low-emittance machines)
- higher dynamic range (external stable beam rejection not necessary)

**DAC:** usually employed DACs convert samples at up to 500 Msample/s and 14-bit resolution

**Digital Processing:** the feedback processing can be performed by discrete digital electronics (obsolete technology), DSPs or FPGAs

	Pros	Cons
DSP	<ul style="list-style-type: none"> <li>➤ Easy programming</li> <li>➤ Flexible</li> </ul>	<ul style="list-style-type: none"> <li>➤ Difficult HW integration</li> <li>➤ Latency</li> <li>➤ Sequential program execution</li> <li>➤ A number of DSPs are necessary</li> </ul>
FPGA	<ul style="list-style-type: none"> <li>➤ Fast (only one FPGA is necessary)</li> <li>➤ Parallel processing</li> <li>➤ Low latency</li> </ul>	<ul style="list-style-type: none"> <li>➤ Trickier programming</li> <li>➤ Less flexible</li> </ul>

# Examples of digital processors

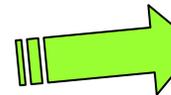
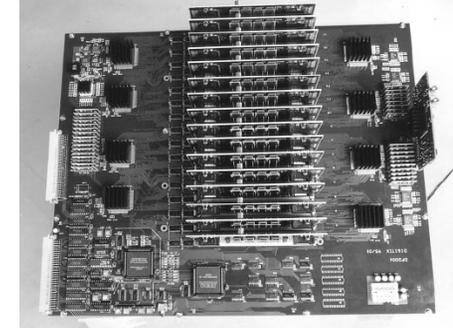
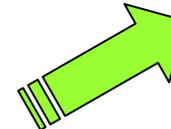
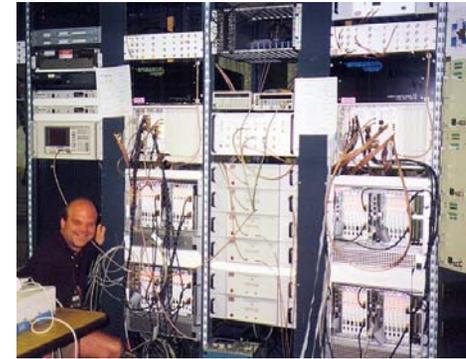
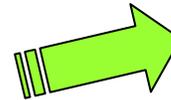
➤ **PETRA** transverse and longitudinal feedbacks: one ADC, a digital processing electronics made of discrete components (adders, multipliers, shift registers, ...) implementing a FIR filter, and a DAC

➤ **ALS/PEP-II/DAΦNE** longitudinal feedback (also adopted at **SPEAR**, **Bessy II** and **PLS**): A/D and D/A conversions performed by VXI boards, feedback processing made by DSP boards hosted in a number of VME crates

➤ **PEP-II** transverse feedback: the digital part, made of two ADCs, a FPGA and a DAC, features a digital delay and integrated diagnostics tools, while the rest of the signal processing is made analogically

➤ **KEKB** transverse and longitudinal feedbacks: the digital processing unit, made of discrete digital electronics and banks of memories, performs a two tap FIR filter featuring stable beam rejection, phase shift and delay

➤ **Elettra/SLS** transverse and longitudinal feedbacks: the digital processing unit is made of a VME crate equipped with one ADC, one DAC and six commercial DSP boards (Elettra only) with four microprocessors each



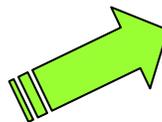
# Examples of digital processors

↘ **CESR** transverse and longitudinal feedbacks: they employ VME digital processing boards equipped with ADC, DAC, FIFOs and PLDs



↘ **HERA-p** longitudinal feedback: it is made of a processing chain with two ADCs (for I and Q components), a FPGA and two DACs

↘ **SPring-8** transverse feedback (also adopted at **TLS**, **KEK Photon Factory** and **Soleil**): fast analog de-multiplexer that distributes analog samples to a number of slower ADC FPGA channels. The correction samples are converted to analog by one DAC

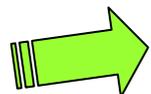


↘ **ESRF** transverse/longitudinal and **Diamond** transverse feedbacks: commercial product 'Libera Bunch by Bunch' (by Instrumentation Technologies), which features four ADCs sampling the same analog signal opportunely delayed, one FPGA and one DAC

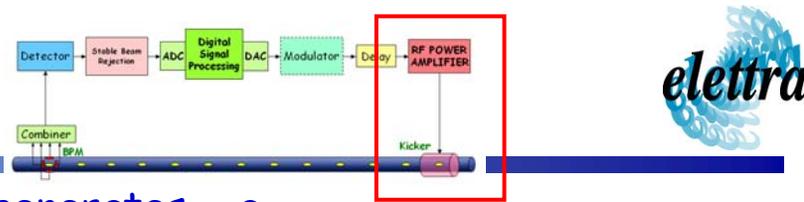


↘ **HLS** transverse feedback: the digital processor consists of two ADCs, one FPGA and two DACs

↘ **DAΦNE** transverse and **KEK-Photon-Factory** longitudinal feedbacks: commercial product called 'iGp' (by Dimtel), featuring an ADC-FPGA-DAC chain



# Amplifier and kicker



The **kicker** is the **feedback actuator**. It generates a transverse/longitudinal electromagnetic field that steers the bunches with small kicks as they pass through the kicker. The overall effect is damping of the betatron/synchrotron oscillations

The **amplifier** must provide the necessary RF power to the kicker by amplifying the signal from the DAC (or from the modulator in the case of longitudinal feedbacks)

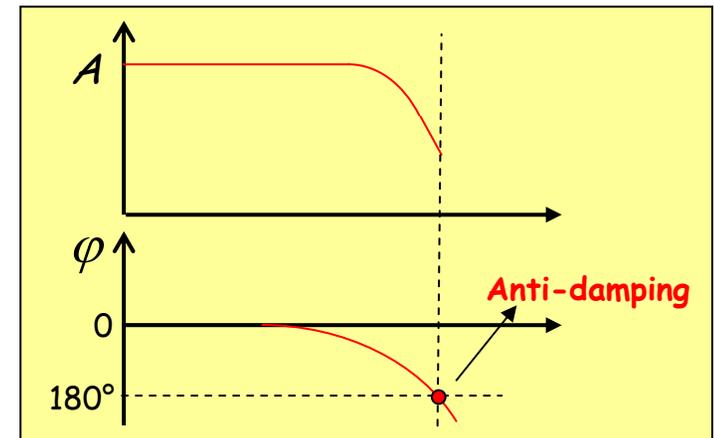
A **bandwidth** of at least  $f_{rf}/2$  is necessary: from  $\sim DC$  (all kicks of the same sign) to  $\sim f_{rf}/2$  (kicks of alternating signs)

*The bandwidth of amplifier-kicker must be sufficient to correct each bunch with the appropriate kick without affecting the neighbour bunches. The amplifier-kicker design has to maximize the kick strength while minimizing the cross-talk between corrections given to adjacent bunches*

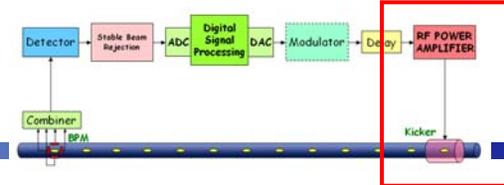
**Important issue:** the group delay of the amplifier must be as constant as possible, i.e. the phase response must be linear, otherwise the feedback efficiency is reduced for some modes and the feedback can even become positive

**Shunt impedance**, ratio between the squared voltage seen by the bunch and twice the power at the kicker input:

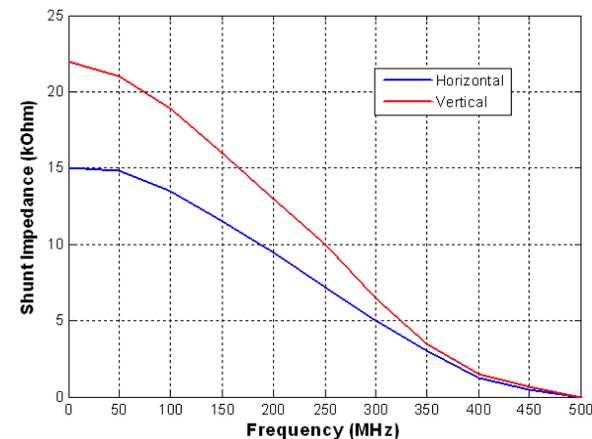
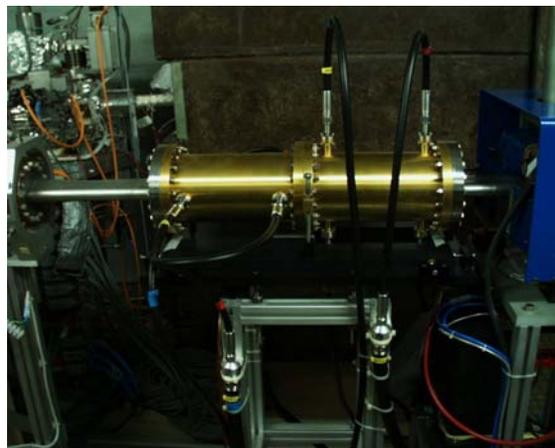
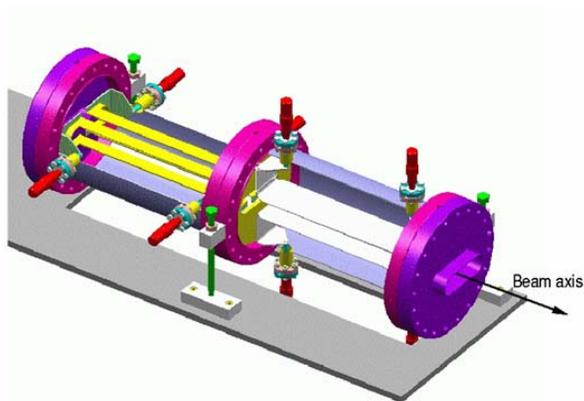
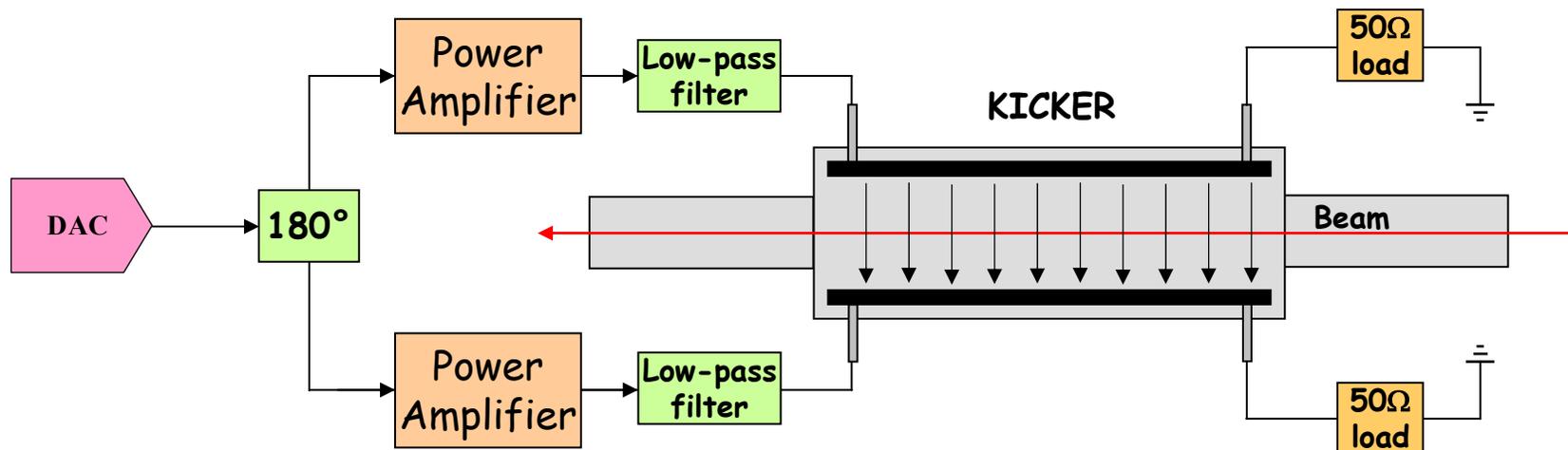
$$R = \frac{V^2}{2P_{IN}}$$



# Kicker and Amplifier: transverse FB



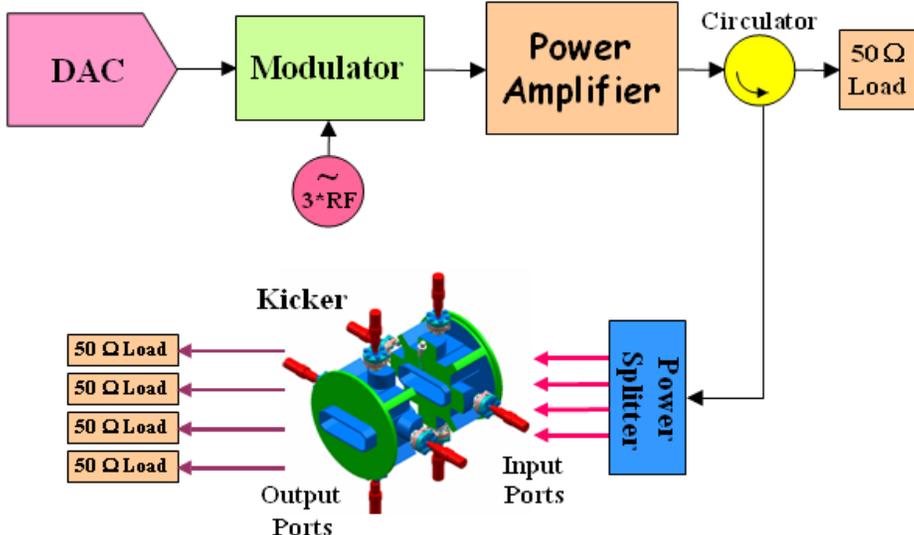
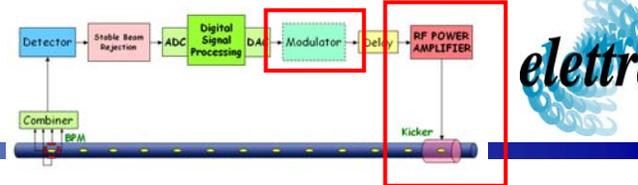
For the transverse kicker a **stripline** geometry is usually employed  
 Amplifier and kicker work in the  $\sim DC - \sim f_{rf}/2$  frequency range



The ELETTRA/SLS transverse kicker (by Micha Dehler-PSI)

Shunt impedance of the ELETTRA/SLS transverse kickers

# Kicker and Amplifier: longitudinal FB



A "cavity like" kicker is usually preferred  
Higher shunt impedance and smaller size

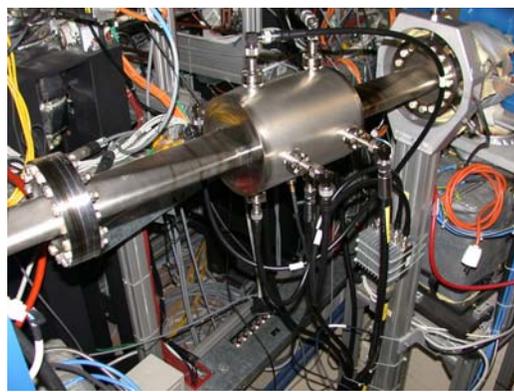
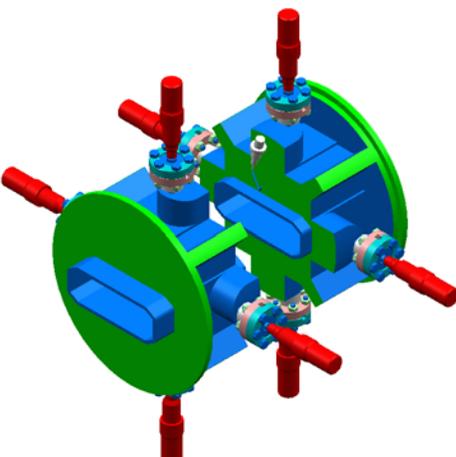
The operating frequency range is typically  $f_{rf}/2$  wide and placed on one side of a multiple of  $f_{rf}$ :

ex. from  $3f_{rf}$  to  $3f_{rf} + f_{rf}/2$

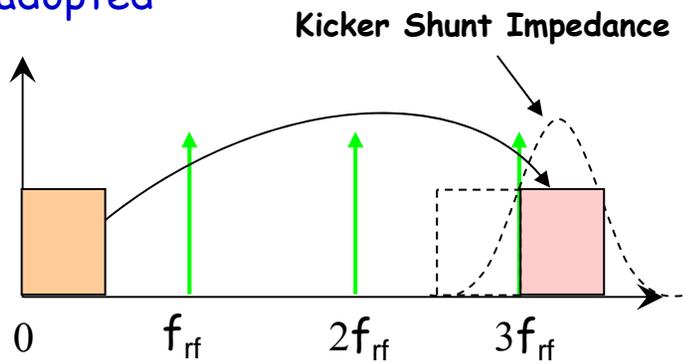
A "pass-band" instead of "base-band" device

The base-band signal from the DAC must be modulated, i.e. translated in frequency

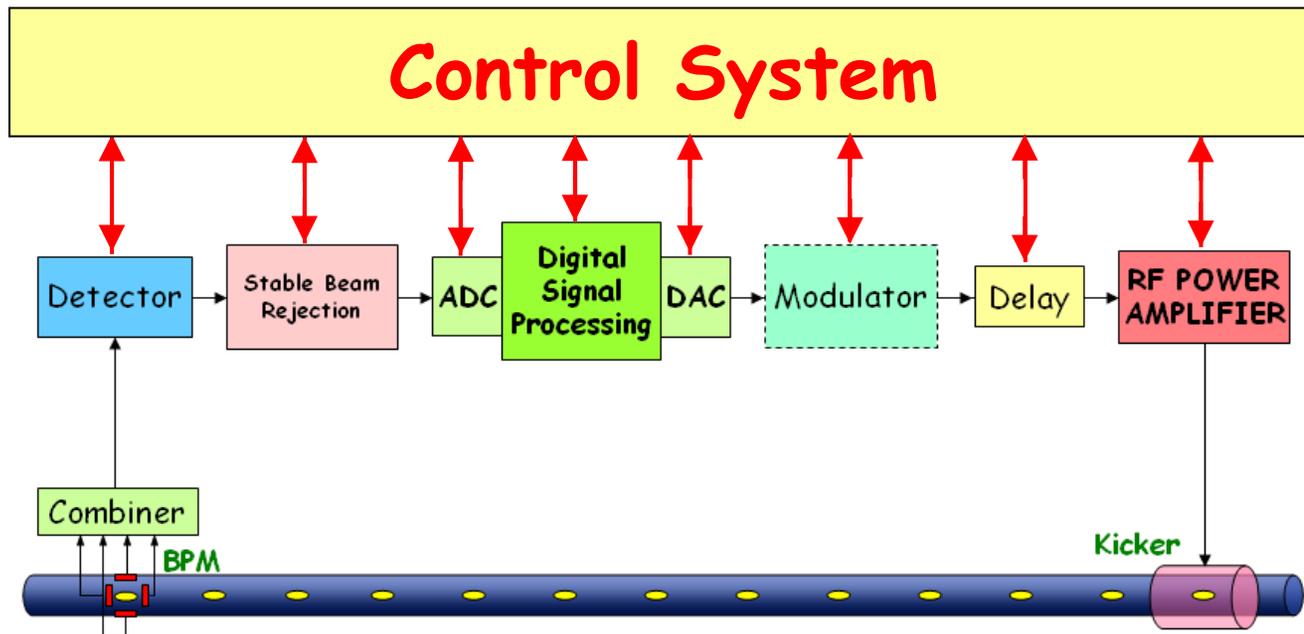
A SSB (Single Side Band) amplitude modulation or similar techniques (ex. QPSK) can be adopted



The ELETTRA/SLS longitudinal kicker (by Micha Dehler-PSI)



- It is desirable that each component of the feedback system that needs to be configured and adjusted has a **control system interface**
- Any **operation** must be possible from **remote** to facilitate the system commissioning and the optimization of its performance
- An effective data acquisition channel has to provide **fast transfer** of large amounts of data for analysis of the feedback performance and beam dynamics studies
- It is preferable to have a direct connection to a numerical computing environment and/or a script language (ex. **Matlab, Octave, Scilab, Python, IGOR Pro, IDL, ...**) for quick development of measurement procedures using scripts as well as for data analysis and visualization



# RF power requirements: transverse feedback

The transverse motion of a bunch of particles not subject to damping or excitation can be described as a pseudo-harmonic oscillation with amplitude proportional to the square root of the  $\beta$ -function

$$x(s) = a \sqrt{\beta(s)} \cos \varphi(s), \quad \text{where} \quad \varphi(s) = \int_0^s \frac{d\bar{s}}{\beta(\bar{s})}$$

The derivative of the position, i.e. the angle of the trajectory is:

$$x' = -\frac{a}{\sqrt{\beta}} \sin \varphi + \frac{a\beta'}{2\sqrt{\beta}} \cos \varphi, \quad \text{with} \quad \varphi' = \frac{1}{\beta}$$

By introducing  $\alpha = -\frac{\beta'}{2}$  we can write:  $x' = \frac{a}{\sqrt{\beta}} \sqrt{1 + \alpha^2} \sin(\varphi + \arctan \alpha)$

At the coordinate  $s_k$ , the electromagnetic field of the kicker deflects the particle bunch which varies its angle by  $k$ : as a consequence the bunch starts another oscillation

$x_1 = a_1 \sqrt{\beta} \cos \varphi_1$  which must satisfy the following constraints:

$$\begin{cases} x(s_k) = x_1(s_k) \\ x'(s_k) = x_1'(s_k) + k \end{cases}$$

By introducing  $A = a\sqrt{\beta}$ ,  $A_1 = a_1\sqrt{\beta}$  the two-equation two-unknown-variables system becomes:

$$\begin{cases} A \cos \varphi = A_1 \cos \varphi_1 \\ A \frac{\sqrt{1 + \alpha^2}}{\beta} \sin(\varphi + \arctg(\alpha)) = A_1 \frac{\sqrt{1 + \alpha^2}}{\beta} \sin(\varphi_1 + \arctg(\alpha)) + k \end{cases}$$

The solution of the system gives amplitude and phase of the new oscillation:

$$\begin{cases} A_1 = \sqrt{(A \sin \varphi - k\beta)^2 + A^2 \cos^2 \varphi} \\ \varphi_1 = \arccos\left(\frac{A}{A_1} \cos \varphi\right) \end{cases}$$

# RF power requirements: transverse feedback

From  $A_1 = \sqrt{(A \sin \varphi - k\beta)^2 + A^2 \cos^2 \varphi}$  if the kick is small ( $k \ll \frac{A}{\beta}$ ) then  $\frac{\Delta A}{A} = \frac{A - A_1}{A} \cong \frac{\beta}{A} k \sin \varphi$

In the linear feedback case, i.e. when the turn-by-turn kick signal is a sampled sinusoid proportional to the bunch oscillation amplitude, in order to maximize the damping rate the kick signal must be in-phase with  $\sin \varphi$ , that is in quadrature with the bunch oscillation

$$k = g \frac{A}{\beta} \sin \varphi \quad \text{with } 0 < g < 1$$

The optimal gain  $g_{opt}$  is determined by the maximum kick value  $k_{max}$  that the kicker is able to generate. The feedback gain must be set so that  $k_{max}$  is generated when the oscillation amplitude  $A$  at the kicker location is maximum:

$$g_{opt} = \frac{k_{max}}{A_{max}} \beta \quad \text{Therefore} \quad k = \frac{k_{max}}{A_{max}} A \sin \varphi$$

For small kicks  $\frac{\Delta A}{A} \cong \frac{k_{max}}{A_{max}} \beta \sin^2 \varphi$

the relative amplitude decrease is monotonic and its average is:  $\left\langle \frac{\Delta A}{A} \right\rangle \cong \frac{\beta k_{max}}{2 A_{max}}$

The average relative decrease is therefore constant, which means that, in average, the amplitude decrease is exponential with time constant  $\tau$  (damping time) given by:

$$\frac{1}{\tau} = \left\langle \frac{\Delta A}{A} \right\rangle \frac{1}{T_0} = \frac{\beta k_{max}}{2 A_{max} T_0} \quad \text{where } T_0 \text{ is the revolution period.}$$

By referring to the oscillation at the BPM location:

$$\frac{1}{\tau} = \frac{k_{max}}{2 T_0 A_{Bmax}} \sqrt{\beta_k \beta_B}$$

$A_{Bmax}$  is the max oscillation amplitude at the BPM

# RF power requirements: transverse feedback

For relativistic particles, the change of the transverse momentum  $p$  of the bunch passing through the kicker can be expressed by:

$$\Delta p = \frac{e}{c} V_{\perp} \quad \text{where} \quad V_{\perp} = \int_0^L (\bar{E} + c \times \bar{B})_{\perp} dz \quad \text{is the kick voltage and} \quad p = \frac{E_B}{c}$$

$e$  = electron charge,  $c$  = light speed,  $\bar{E}, \bar{B}$  = fields in the kicker,  $L$  = length of the kicker,  $E_B$  = beam energy

$$V_{\perp} \text{ can be derived from the definition of kicker shunt impedance: } R_k = \frac{V_{\perp}^2}{2P_k}$$

The max deflection angle in the kicker is given by:

$$k_{\max} = \frac{\Delta p}{p} = e \frac{V_{\perp}}{E_B} = \left( \frac{e}{E_B} \right) \sqrt{2P_k R_k}$$

From the previous equations we can obtain the power required to damp the bunch oscillation with time constant  $\tau$ :

$$P_k = \frac{2}{R_k \beta_k} \left( \frac{E_B}{e} \right)^2 \left( \frac{T_0}{\tau} \right)^2 \left( \frac{A_{B\max}}{\sqrt{\beta_B}} \right)^2$$

# RF power requirements

- $\tau$  = feedback damping time
- $\omega_0$  = revolution frequency
- $\omega_S$  = synchrotron frequency
- $\alpha$  = momentum compaction factor
- $f_{rf}$  = RF frequency
- $R_k$  = kicker shunt impedance
- $E_B$  = beam energy
- $\varphi_{\max}$  = maximum oscillation amplitude

$$P_k = \frac{2}{R_k} \left( \frac{\omega_S E_B \varphi_{\max}}{\omega_0 \alpha f_{RF} \tau} \right)^2 \quad P_K = \frac{2}{R_K \beta_K} \left( \frac{E_B}{e} \right)^2 \left( \frac{T_0}{\tau} \right)^2 \left( \frac{A_{B\max}}{\sqrt{\beta_B}} \right)^2$$

Longitudinal
Transverse

Required damping time

Max oscillation amplitude

The required RF power depends on:

- the strength of the instability
- the maximum oscillation amplitude

If we switch the feedback on when the oscillation is small, the required power is lower

**Example:**

Elettra Transverse feedback

$R_k = 15 \text{ k}\Omega$  (average value)

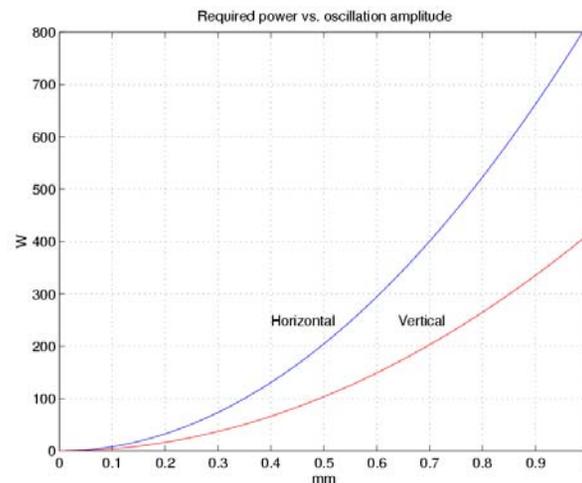
$E_B/e = 2 \text{ GeV}$

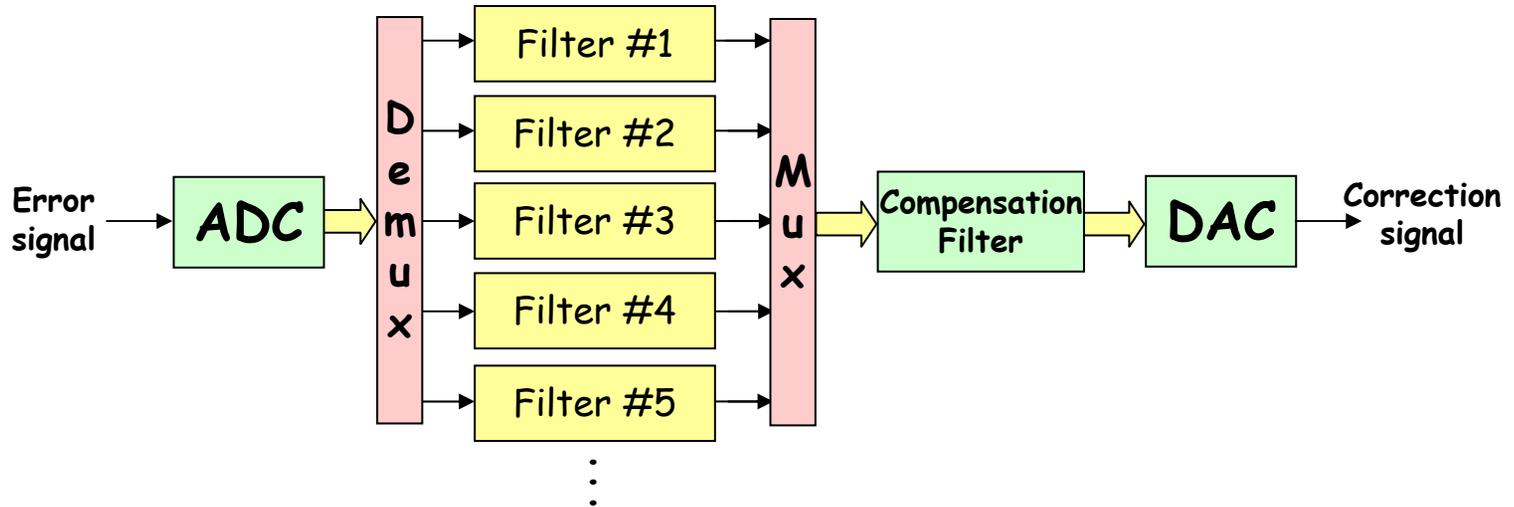
$T_0 = 864 \text{ ns}$

$\tau = 120 \text{ }\mu\text{s}$

$\beta_{B,H,V} = 5.2, 8.9 \text{ m}$

$\beta_{K,H,V} = 6.5, 7.5 \text{ m}$





$M$  channel/filters each dedicated to one bunch:  $M$  is the number of bunches

To damp the bunch oscillations the **turn-by-turn kick signal** must be the **derivative** of the bunch position at the kicker: for a given oscillation frequency a  $\pi/2$  **phase shifted** signal must be generated

In determining the real phase shift to perform in each channel, the phase advance between BPM and kicker must be taken into account as well as any additional delay due to the feedback latency (multiple of one machine revolution period)

The digital processing must also **reject** any residual **constant offset** (stable beam component) from the bunch signal to avoid DAC saturation

Digital filters can be implemented with **FIR** (Finite Impulse Response) or **IIR** (Infinite Impulse Response) structures. Various techniques are used in the design: ex. frequency domain design and model based design

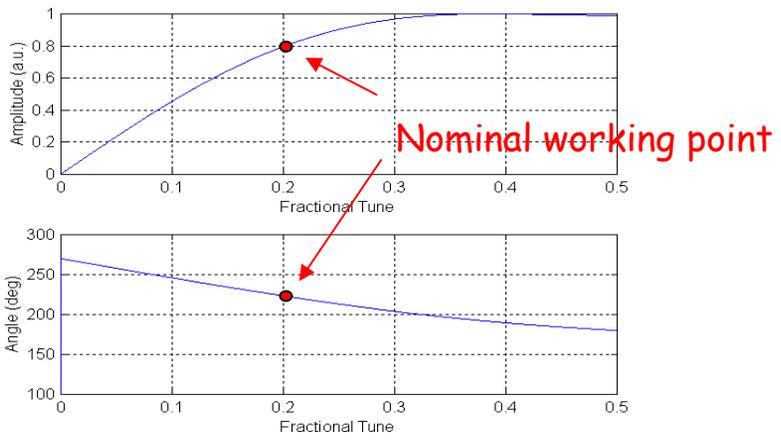
A filter on the full-rate data stream can compensate for amplifier/kicker not-ideal behaviour

# Digital filter design: 3-tap FIR filter

The minimum requirements are:

1. DC rejection (coefficients sum = 0)
2. Given amplitude response at the tune frequency
3. Given phase response at the tune frequency

A 3-tap FIR filter can fulfil these requirements: the filter coefficients can be calculated analytically



Example:

- Tune  $\omega / 2\pi = 0.2$
- Amplitude response at tune  $|H(\omega)| = 0.8$
- Phase response at tune  $\alpha = 222^\circ$

$$H(z) = -0.63 + 0.49 z^{-1} + 0.14 z^{-2}$$

## Z transform of the FIR filter response

In order to have zero amplitude at DC, we must put a "zero" in  $z=1$ . Another zero in  $z=c$  is added to fulfill the phase requirements.

$c$  can be calculated analytically:

$$H(z) = k(1 - z^{-1})(1 - cz^{-1})$$

$$H(z) = k(1 - (1+c)z^{-1} + cz^{-2}) \quad z = e^{j\omega}$$

$$H(\omega) = k(1 - (1+c)e^{-j\omega} + ce^{-2j\omega})$$

$$e^{-j\omega} = \cos \omega - j \sin \omega, \quad \alpha = \text{ang}(H(\omega))$$

$$\text{tg}(\alpha) = \frac{c(\sin(\omega) - \sin(2\omega)) + \sin(\omega)}{c(\cos(2\omega) - \cos(\omega)) + 1 - \cos(\omega)}$$

$$c = \frac{\text{tg}(\alpha)(1 - \cos(\omega)) - \sin(\omega)}{(\sin(\omega) - \sin(2\omega)) - \text{tg}(\alpha)(\cos(2\omega) - \cos(\omega))}$$

$k$  is determined given the required amplitude response at tune  $|H(\omega)|$ :

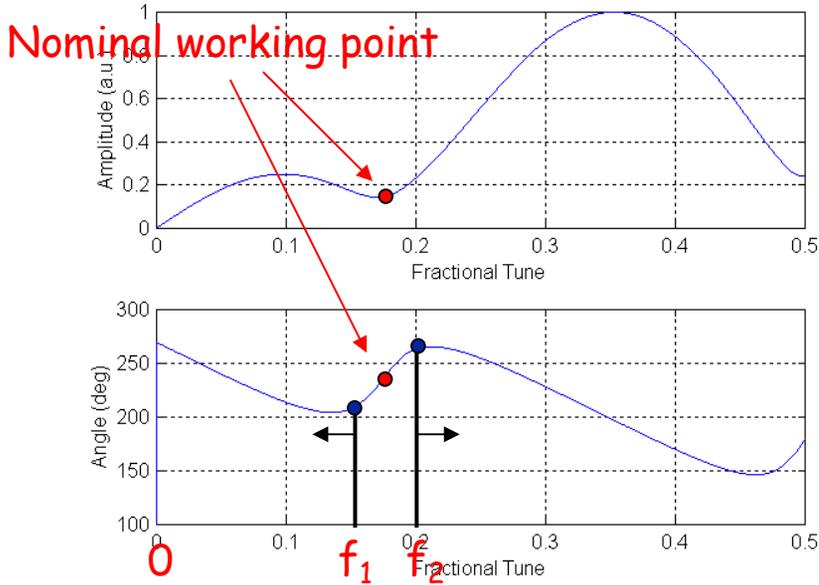
$$k = \frac{|H(\omega)|}{\sqrt{(1 - (1+c)\cos(\omega) + c\cos(2\omega))^2 + ((1+c)\sin(\omega) - c\sin(2\omega))^2}}$$

# Digital filter design: 5-tap FIR filter

With more degrees of freedom additional features can be added to a FIR filter

Ex.: *transverse feedback*. The **tune frequency** of the accelerator can significantly **change** during machine operations. The filter response must guarantee the same feedback efficiency in a given frequency range by performing **automatic compensation** of phase changes.

In this example the feedback delay is four machine turns. When the tune frequency increases, the phase of the filter must increase as well, i.e. the **phase response** must have a **positive slope** around the working point.



The filter design can be made using the Matlab function *invfreqz()*

This function calculates the filter coefficients that best fit the required frequency response using the **least squares method**

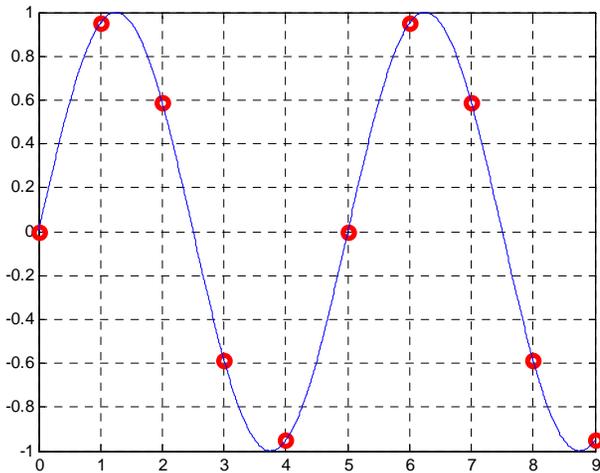
The desired response is specified by defining amplitude and phase at three different frequencies:  $0$ ,  $f_1$  and  $f_2$

# Digital filter design: selective FIR filter

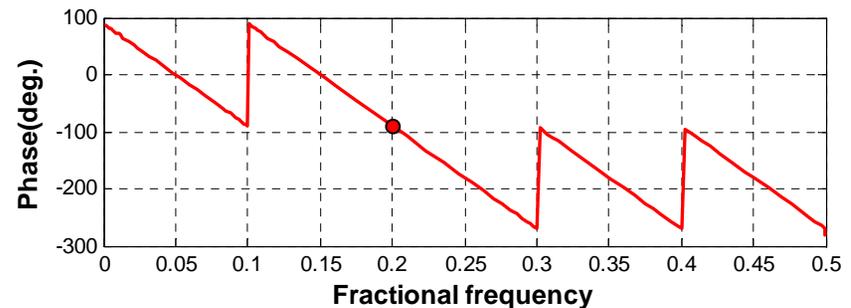
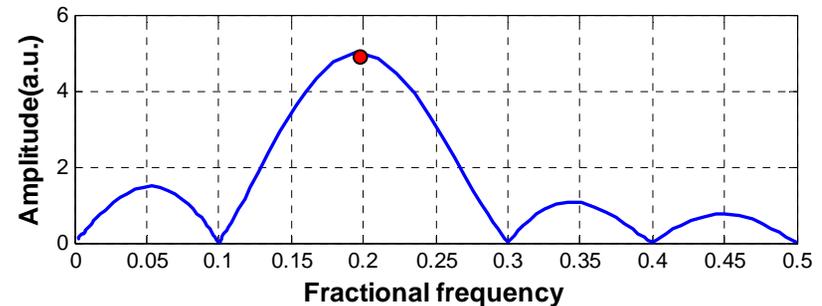
A filter often employed in longitudinal feedback systems is a selective FIR filter which impulse response (the filter coefficients) is a **sampled sinusoid** with frequency equal to the synchrotron tune

The filter amplitude response has a maximum at the tune frequency and linear phase

The more filter coefficients we use the more selective is the filter



Samples of the filter impulse response  
(= filter coefficients)



Amplitude and phase response of the filter

More sophisticated techniques using longer FIR or IIR filters enable a variety of additional features exploiting the potentiality of digital signal processing:

- ↘ enlarge the working frequency range with no degradation of the amplitude response
- ↘ enhance filter selectivity to better reject unwanted frequency components (noise)
- ↘ minimize the amplitude response at frequencies that must not be fed back
- ↘ stabilize different tune frequencies simultaneously by designing a filter with two separate working points (for example when horizontal and vertical as well as dipole and quadrupole instabilities have to be addressed by the same feedback system)
- ↘ improve the robustness of the feedback under parametric changes of accelerator or feedback components (ex. optimal control, robust control, etc.)

# Down sampling (longitudinal feedback)

The synchrotron frequency is usually much lower than the revolution frequency: one complete synchrotron oscillation is accomplished in many machine turns

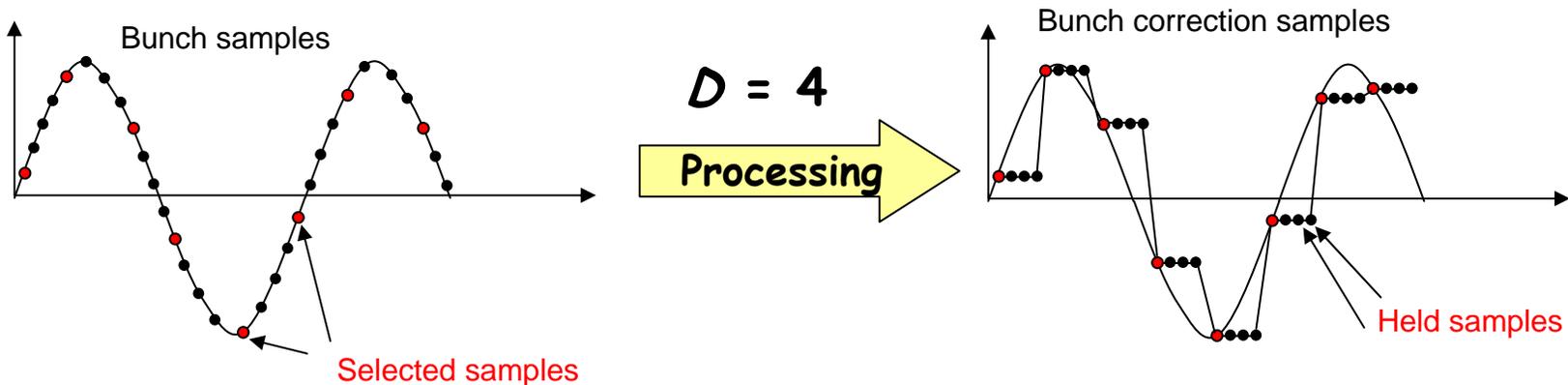
In order to be able to properly filter the bunch signal **down sampling** is usually carried out

One out of  $D$  samples is used:  $D$  is the **down sampling factor**

The processing is performed over the down sampled digital signal and the filter design is done in the down sampled frequency domain (the original one enlarged by  $D$ )

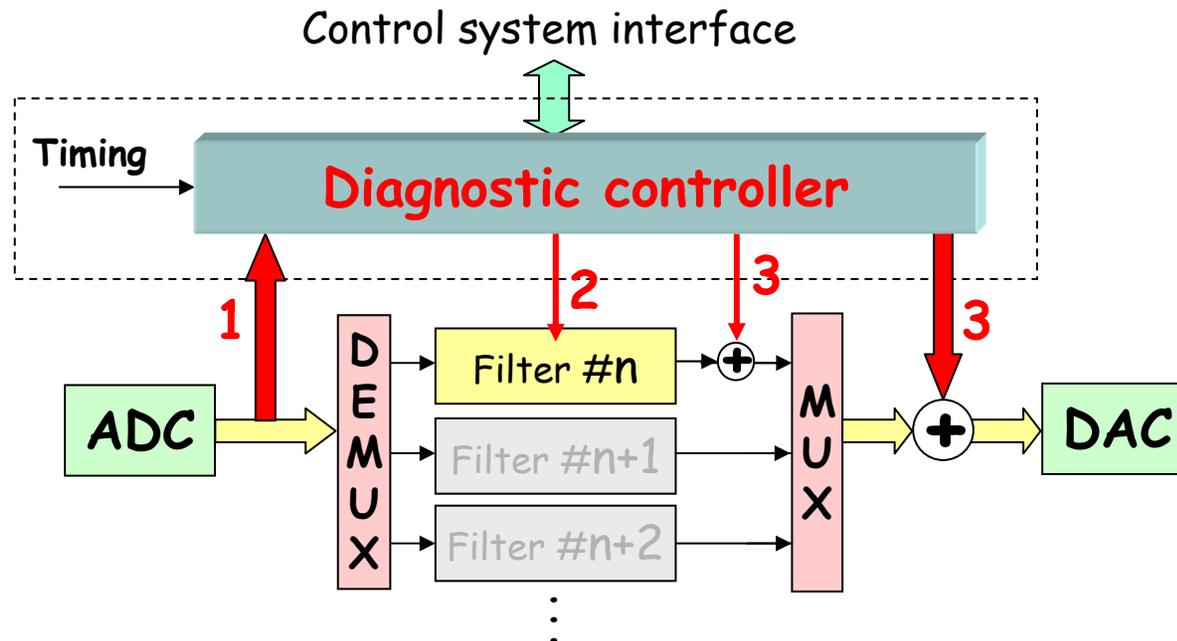
The turn-by-turn correction signal is reconstructed by a **hold buffer** that keeps each calculated correction value for  $D$  turns

The **reduced data rate** allows for more time available to perform filter calculations and more complex filters can therefore be implemented



A feedback system can implement a number of diagnostic tools useful for commissioning and optimization of the feedback system as well as for machine physics studies:

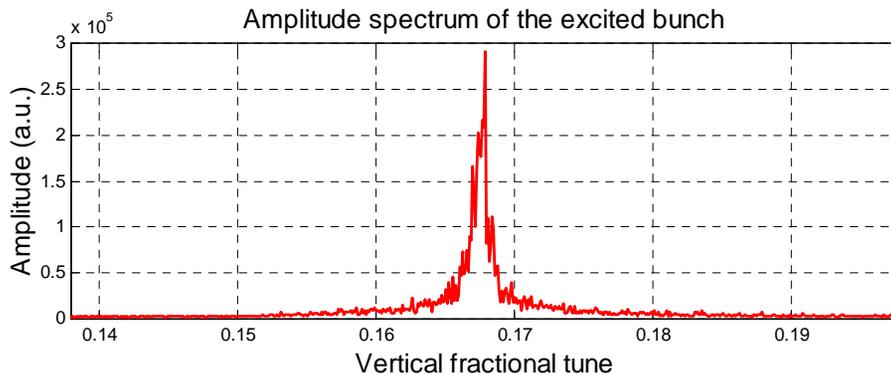
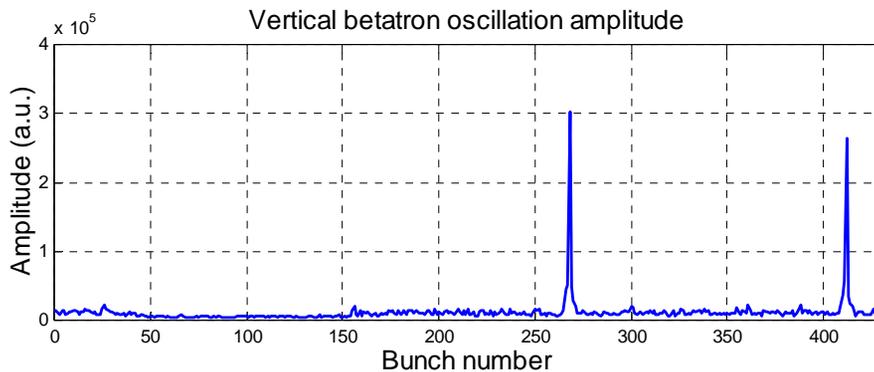
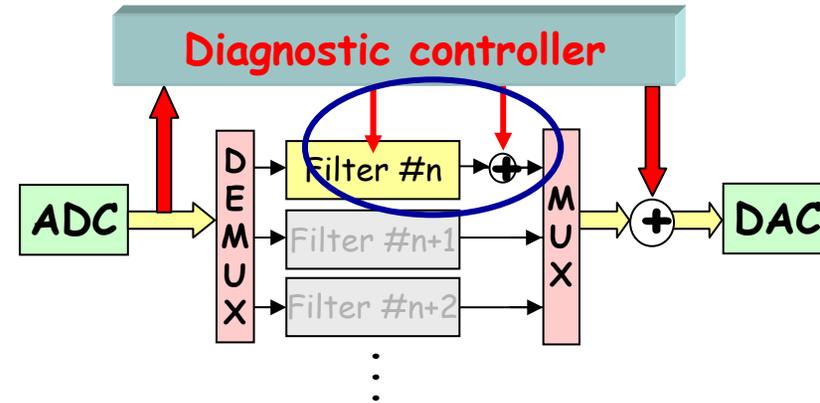
1. **ADC data recording:** acquisition and recording, in parallel with the feedback operation, of a large number of samples for off-line data analysis
2. **Modification of filter parameters on the fly** with the required timing and even individually for each bunch: switching ON/OFF the feedback, generation of grow/damp transients, optimization of feedback performance, ...
3. **Injection of externally generated digital samples:** for the excitation of single/multi bunches



# Diagnostic tools: excitation of individual bunches

The feedback loop is switched off for one or more selected bunches and the excitation is injected in place of the correction signal. Excitations can be:

- white (or pink) noise
- sinusoids

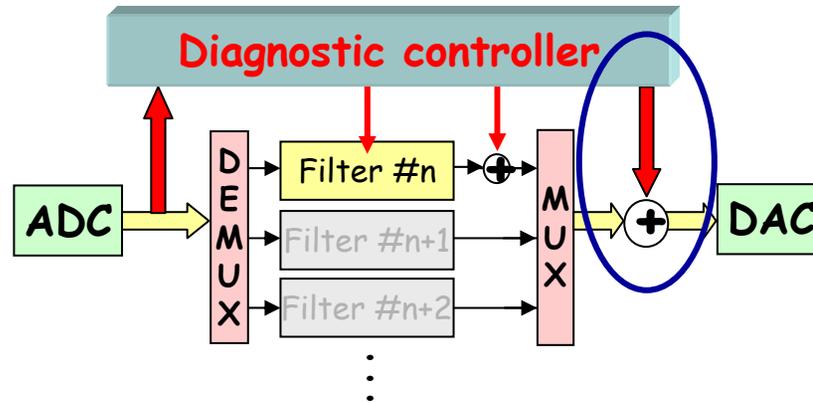


In this example two bunches are vertically **excited** with **pink noise** in a range of frequencies centered around the tune, while the feedback is applied on the other bunches. The spectrum of one excited bunch reveals a **peak at the tune frequency**

This technique is used to **measure the betatron tune** with almost no deterioration of the beam quality

# Diagnostic tools: multi-bunch excitation

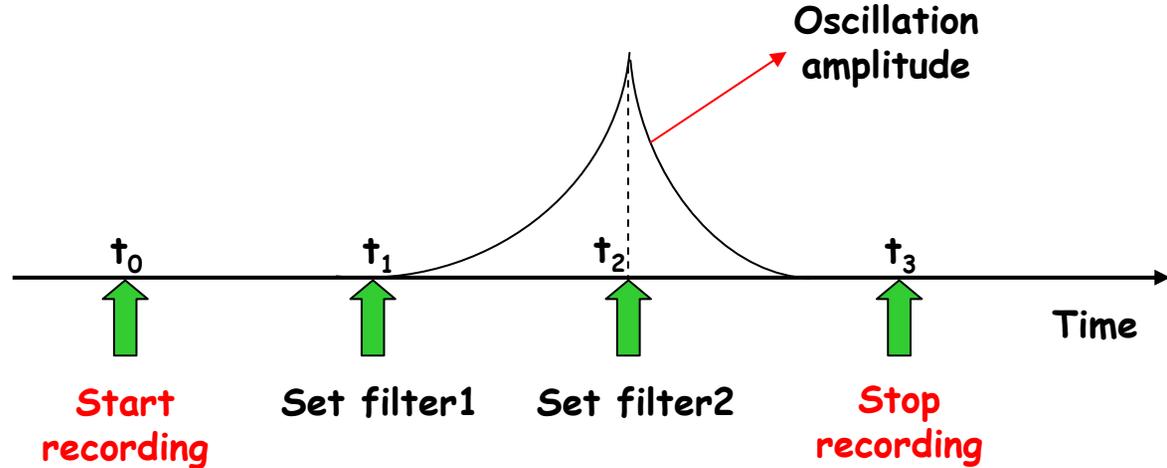
Interesting measurements can be performed by adding pre-defined signals in the output of the digital processor



1. By injecting a **sinusoid** at a given frequency, the corresponding beam **multi-bunch mode can be excited** to test the performance of the feedback in damping that mode
2. By injecting an appropriate signal and recording the ADC data with filter coefficients set to zero, the **beam transfer function** can be calculated
3. By injecting an appropriate signal and recording the ADC data with filter coefficients set to the nominal values, the **closed loop transfer function** can be determined

# Diagnostic tools: transient generation

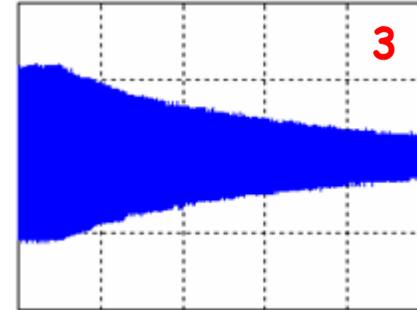
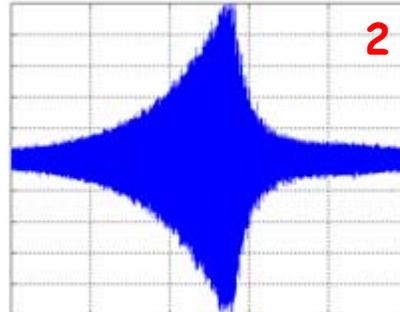
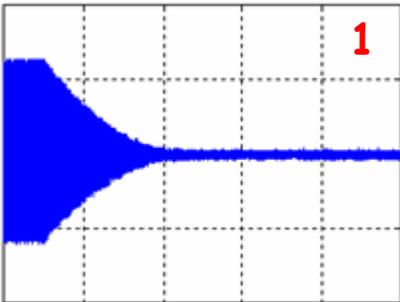
A powerful diagnostic application is the **generation of transients**. Transients can be generated by **changing the filter coefficients** accordingly to a predefined timing and by concurrently recording the oscillations of the bunches



Different types of transients can be generated, **damping times and growth rates** can be calculated by exponential fitting of the transients:

1. **Constant multi-bunch oscillation** → **FB on**: damping transient
2. **FB on** → **FB off** → **FB on**: grow/damp transient
3. **Stable beam** → **positive FB on (anti-damping)** → **FB off**: natural damping transient

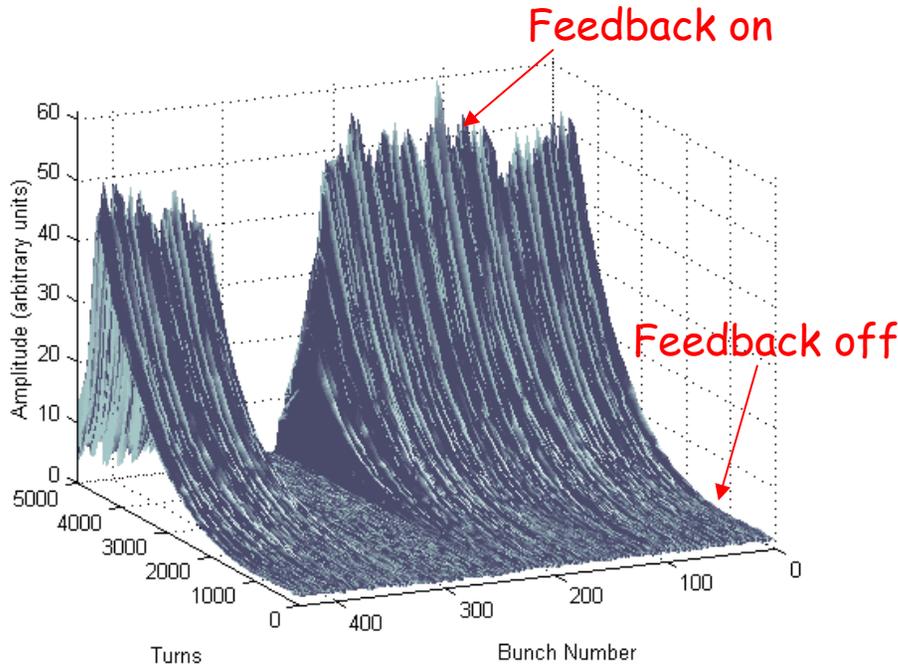
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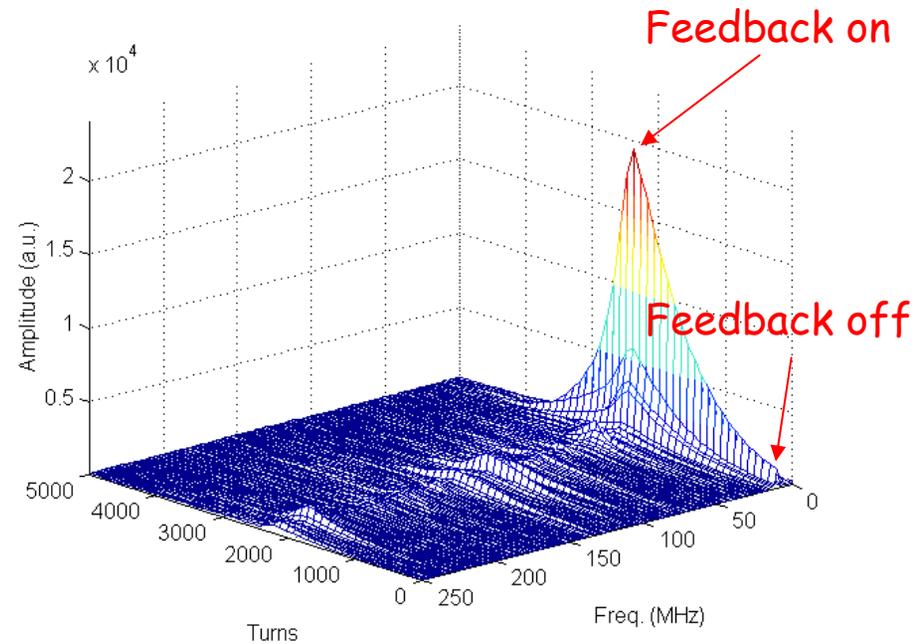
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# Grow/damp transients: 3-D graphs

Grow/damp transients can be analyzed by means of 3-D graphs



Evolution of the bunches oscillation amplitude during a grow-damp transient



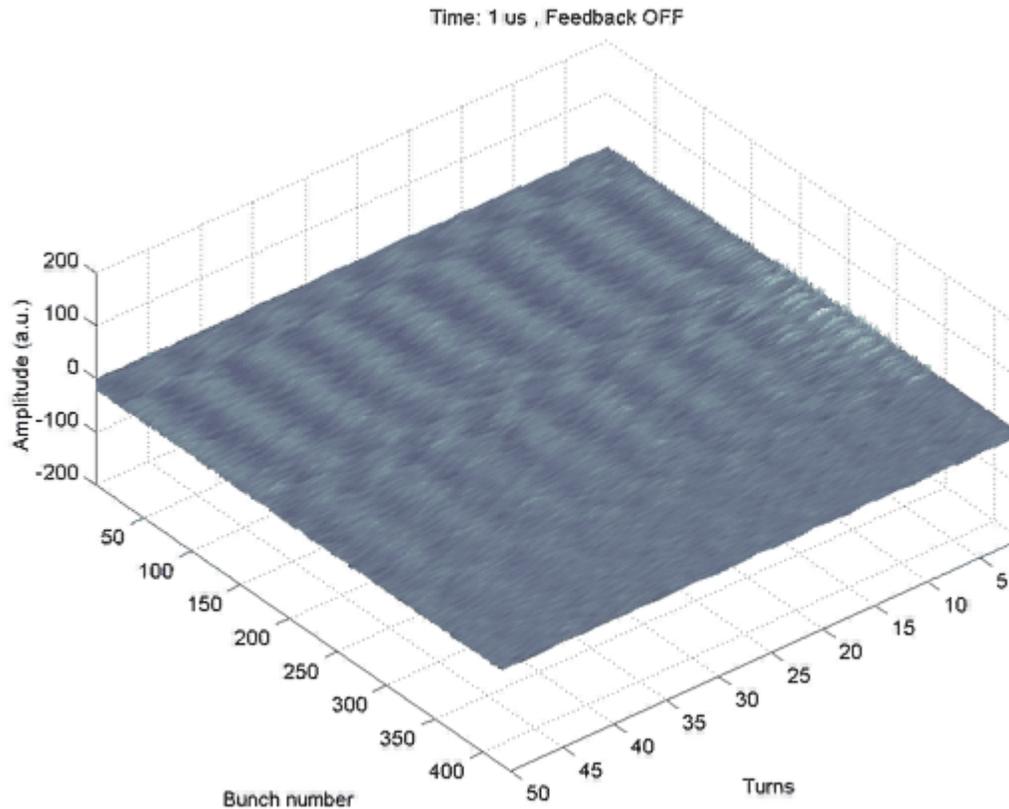
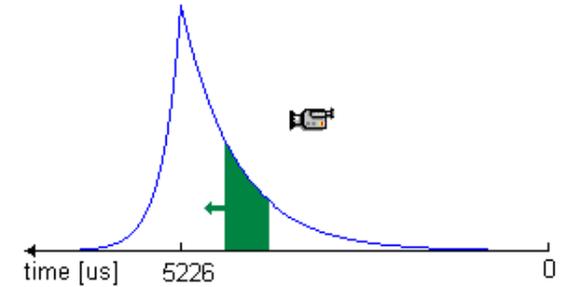
Evolution of coupled-bunch unstable modes during a grow-damp transient

# Grow/damp transients: real movie

'Movie' sequence:

1. Feedback OFF
2. Feedback ON after 5.2 ms

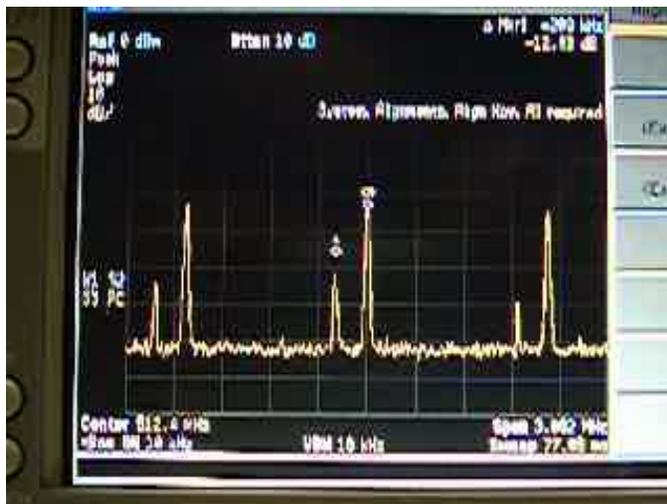
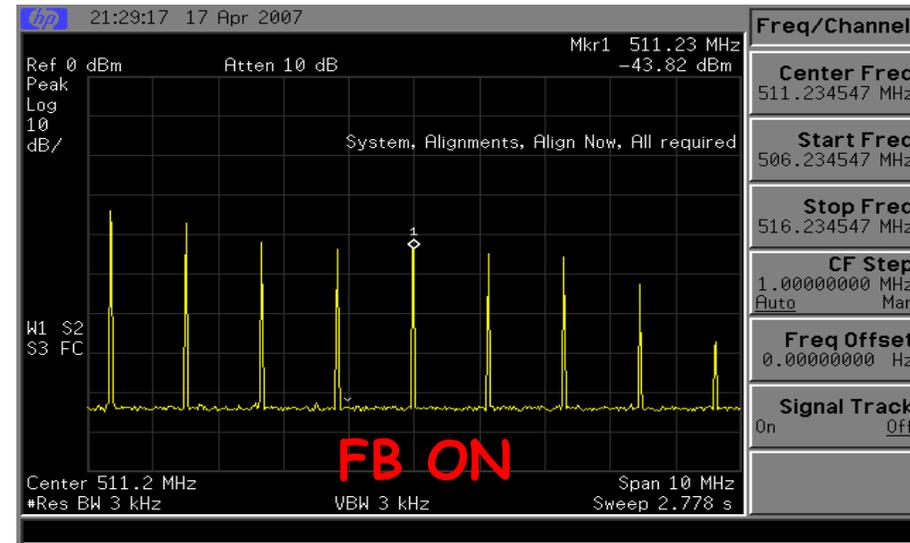
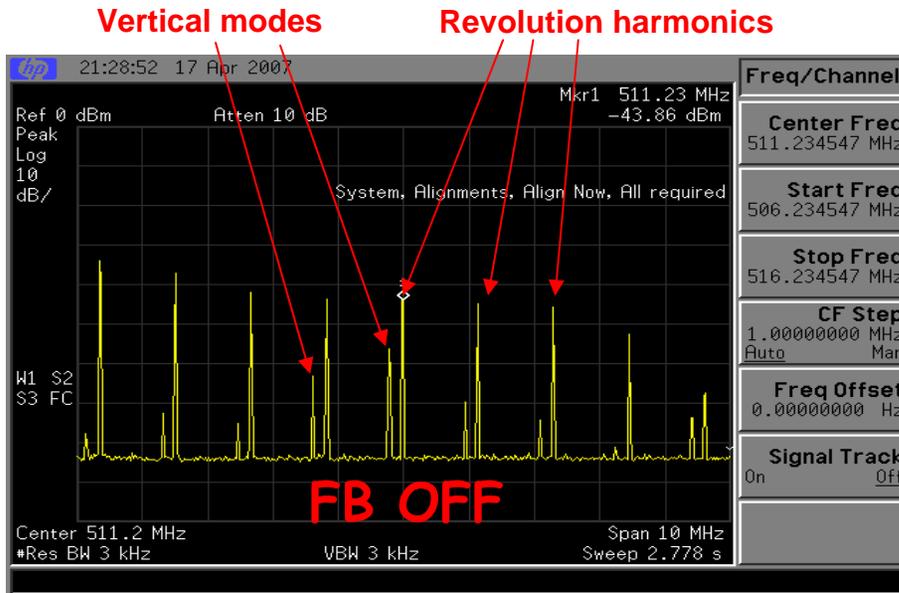
'Camera' view slice is 50 turns long (about  $43 \mu\text{s}$ )



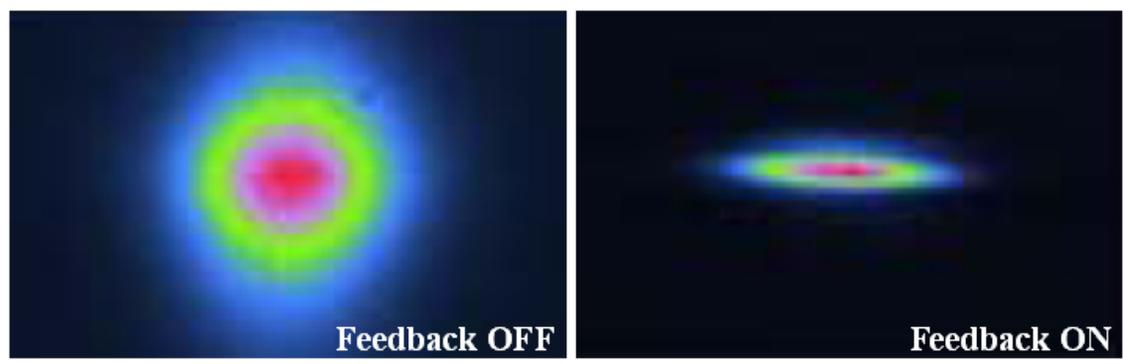
**Diagnostic tools** are helpful to tune feedback systems as well as to study coupled-bunch modes and beam dynamics. Here are some examples of measurements and analysis:

- **Feedback damping times**: can be used to characterize and optimize feedback performance
- **Resistive and reactive response**: a feedback not perfectly tuned has a reactive behavior (induces a tune shift when switched on) that has to be minimized
- **Modal analysis**: coupled-bunch mode complex eigenvalues, i.e. growth rates (real part) and oscillation frequency (imaginary part)
- **Accelerator impedance**: analysis of complex eigenvalues and bunch synchronous phases can be used to evaluate the machine impedance
- **Stable modes** : coupled-bunch modes below the instability threshold can be studied to predict their behavior at higher currents
- **Bunch train studies**: analysis of different bunches in the train give information on the sources of coupled-bunch instabilities
- **Phase space analysis**: phase evolution of unstable coupled-bunch modes for beam dynamics studies

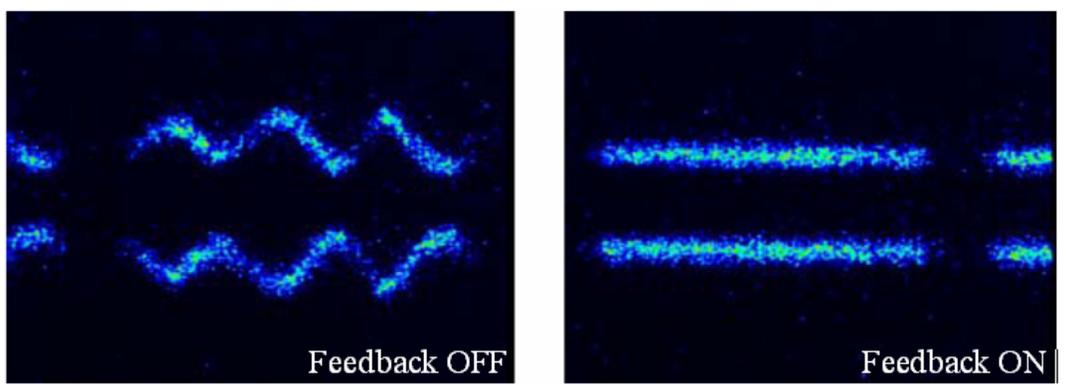
# Effects of a feedback: beam spectrum



Spectrum analyzer connected to a stripline pickup: observation of vertical instabilities. The sidebands corresponding to vertical coupled-bunch modes disappear as soon as the transverse feedback is activated



Synchrotron Radiation Monitor images taken at TLS

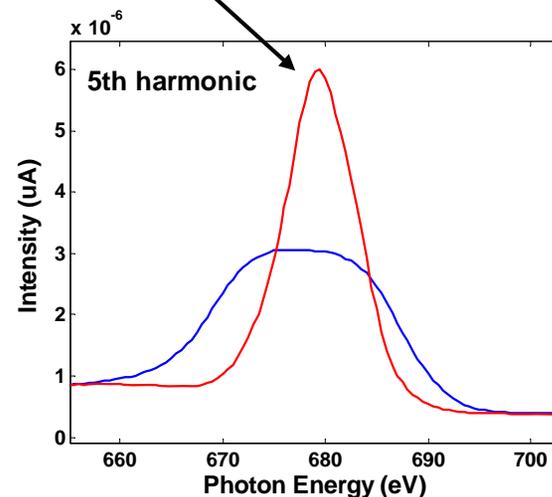
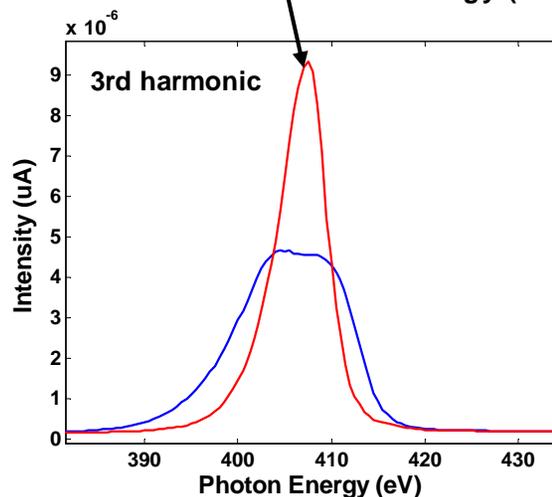
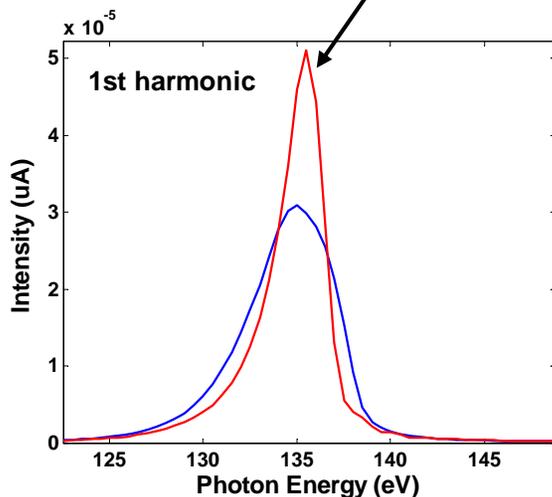
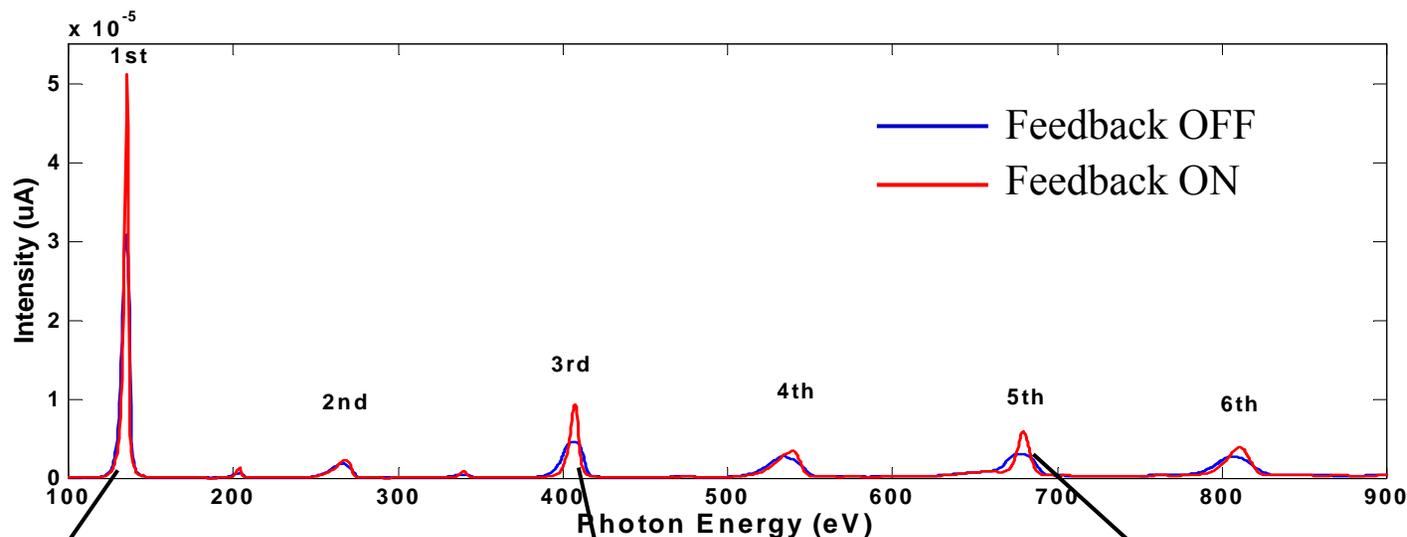


Images of one machine turn taken with a streak camera in 'dual scan mode' at TLS. The horizontal and vertical time spans are 500 and 1.4 ns respectively

# Effects of a feedback: photon beam spectra

Effects on the synchrotron light: spectrum of photons produced by an undulator  
The spectrum is noticeably improved when vertical instabilities are damped by the feedback

SuperESCA  
beamline at  
Elettra



- Feedback systems are indispensable tools to cure multi-bunch instabilities in storage rings
- Technology advances in digital electronics allow implementing digital feedback systems using programmable devices
- Digital signal processing theory widely used to design and implement filters as well as to analyze data acquired by the feedback
- Feedback systems not only for closed loop control but also as powerful diagnostic tools for:
  - optimization of feedback performance
  - beam dynamics studies
- Many potentialities of digital feedback systems still to be discovered and exploited



- Herman Winick, "Synchrotron Radiation Sources", World Scientific
- Many papers about coupled-bunch instabilities and multi-bunch feedback systems (PETRA, KEK, SPring-8, DaΦne, ALS, PEP-II, SPEAR, ESRF, Elettra, SLS, CESR, HERA, HLS, DESY, PLS, BessyII, SRRC, ...)
- Special mention for the articles of the SLAC team (J.Fox, D.Teytelman, S.Prabhakar, etc.) about development of feedback systems and studies of coupled-bunch instabilities