Schottky Signals

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Outline

1. **Introduction**
   - Some history and the motivation
   - Shot noise in a vacuum diode
   - Thermal noise (Johnson noise) in a resistor
   - The "field slice" of fast and slow beams
2. **Coasting beam**
   - Longitudinal signal
   - Transverse signal
3. **Bunched beam**
   - Longitudinal signal
   - Transverse signal
4. **Discussion of pick-up structures**
5. **Signal treatment**
6. **Measurement examples**

From thermo-ionic tubes to accelerators

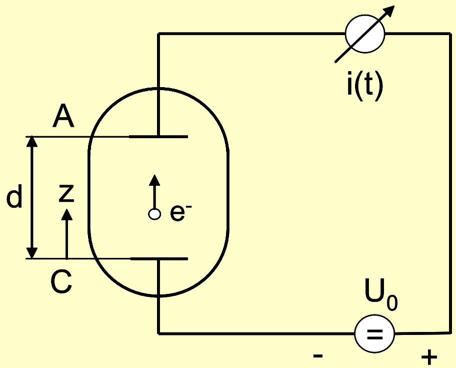
- 1918 : W.Schottky described spontaneous current fluctuations from DC electron beams; "Über spontane Stromschwankungen in verschiedenen Elektrizitätsleitern" Ann. Phys.57 (1918) 541-567
- 1968 : Invention of stochastic cooling (S. van der Meer)
- 1972 : Observation of proton Schottky noise in the ISR (H.G.Hereward, W. Schnell, L.Thorndahl, K. Hübner, J. Borer, P. Braham)
- 1972 : Theory of emittance cooling (S. van der Meer)
- 1975 : Pbar accumulation, schemes for the ISR (P. Strolin, L.Thorndahl)
- 1976 : Experimental proof of proton cooling (L.Thorndahl, G. Carron)

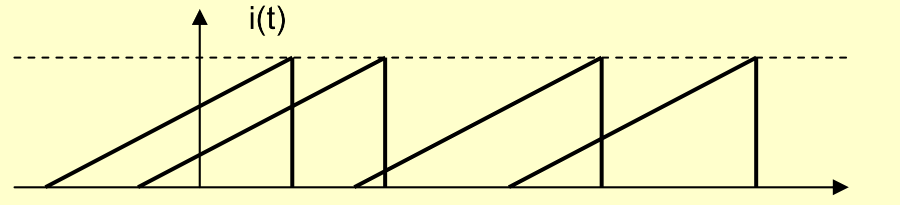
Why do we need it ?

- **Classic instruments**
  - EM instruments : most of them are blind when the beam is unbunched
  - H/V profiles instruments: slightly perturb the beam or even kill it

- **Diagnostics with Schottky Pick-ups**
  - Non perturbing method
  - Statistical-based : information is extracted from rms noise
  - Applications : Stochastic cooling and diagnostics
  - Provide a large set of fundamental informations
    - ✓ revolution frequency
    - ✓ momentum spread
    - ✓ incoherent tune
    - ✓ chromaticity
    - ✓ number of particles
    - ✓ rms emittance

Shot noise in a vacuum diode (1)





- Consider a vacuum diode where single electrons are passing through in a statistical manner (left figure) with the travel time τ
- Due to the dD/dt ($D = \varepsilon E$) we get a current linearly increasing vs time when the electron approaches the flat anode.
- We assume a diode in a saturated regime (space charge neglected) and obtain after some math for frequencies with a period $\gg\tau$ for the spectral density $S_i(\omega)$ of the short circuit current the Schottky equation:

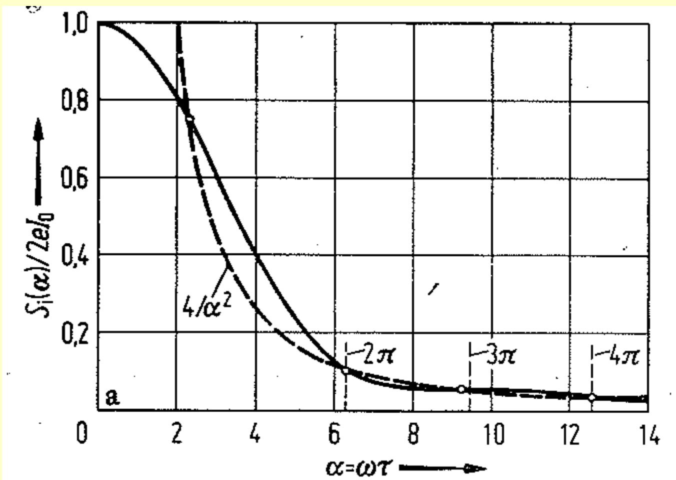
$$S_i(\omega) = 2 I_0 e$$

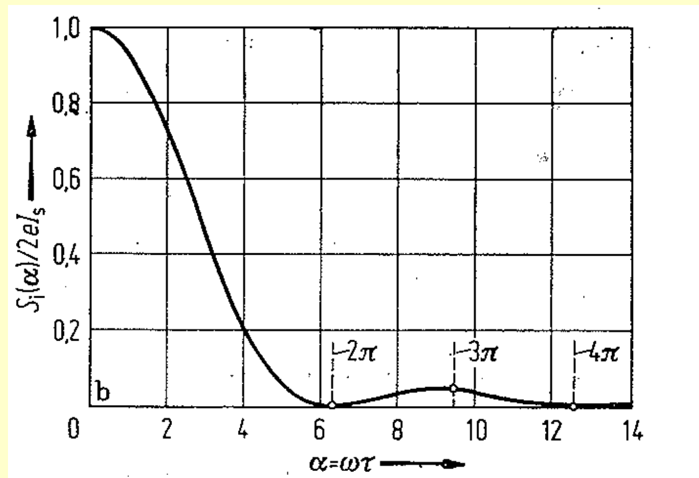
with e= 1.6e^{-19} As and the mean current I_0 =e v_{mean}

Shot noise in vacuum diodes (2)

- obviously the travel time τ plays a very important role for the frequency limit
- The value for τ in typical vacuum diodes operated at a few 100 Volts is around a fraction of a ns. This translates to max frequencies of 1GHz

$$\tau = d\sqrt{\frac{2m_0}{eU_0}}$$





Spectral current density of a planar ultra high vacuum diode in saturation (a) and solid state diode (b)

From:Zinke/Brunswig: Lehrbuch der Hochfrequenztechnik, zweiter Band ,Page 116

Thermal noise in resistors (1)

- In a similar way (saturated high vacuum diode ⇒ high vacuum diode in space charge region ⇒biased solid state diode ⇒unbiased solid state diode) one can arrive at the thermal noise properties of a resistor
- We obtain the general relation (valid also for very high frequencies f and/or low temperatures T of the open (unloaded) terminal voltage u) of some linear resistor R in thermo dynamical equilibrium for a frequency interval Δf as

$$\overline{u^2} = 4k_B TR \frac{hf / k_B T}{\exp(hf / k_B T) - 1} \Delta f$$

h = Planck's constant=6.62· 10^{-34}Js

k_B =Boltzmann's constant=1.38 ·10^{-23} J/K

Thermal noise in resistors (2)

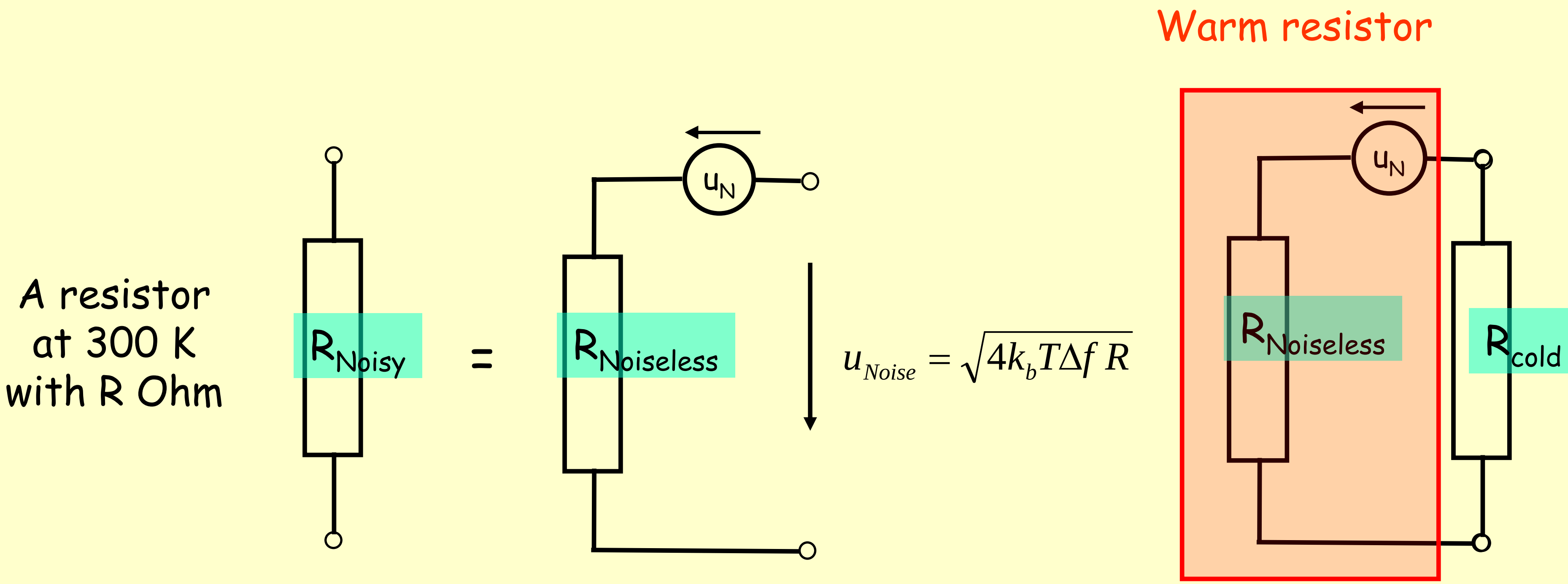
- From the general relation we can deduce the low frequency approximation which is still reasonably valid at ambient temperature up to about 500 GHz as

$$\overline{u^2} = 4k_b T\Delta f\, R$$

For the short circuit current we get accordingly

$$\overline{i^2} = 4k_b T\Delta f \;/\; R$$

Thermal noise in resistors (3)



$$u_{Noise} = \sqrt{4k_b T \Delta f\, R}$$

The open termial noise voltage is reduced by a factor of two (matched load) if the warm (noisy) resistor is loaded by another resistor of same ohmic value but at 0 deg K. Then we get a net power flux density (per unit BW) of kT from the warm to the cold resistor.This power flux density is independent of R (matched load case);

more on resistor noise in: http://en.wikipedia.org/wiki/Noise

and:http://www.ieee.li/pdf/viewgraphs_mohr_noise.pdf

Thermal noise in resistors (4)

Then we obtain for the power P delivered to this external load

$$P = k_b T \Delta f$$

Or for the power density p per unit bandwidth the very simple and useful relation

$$p = k_b T = -174 dBm/Hz @ 300K = 4\,10^{-21} Watt/Hz$$

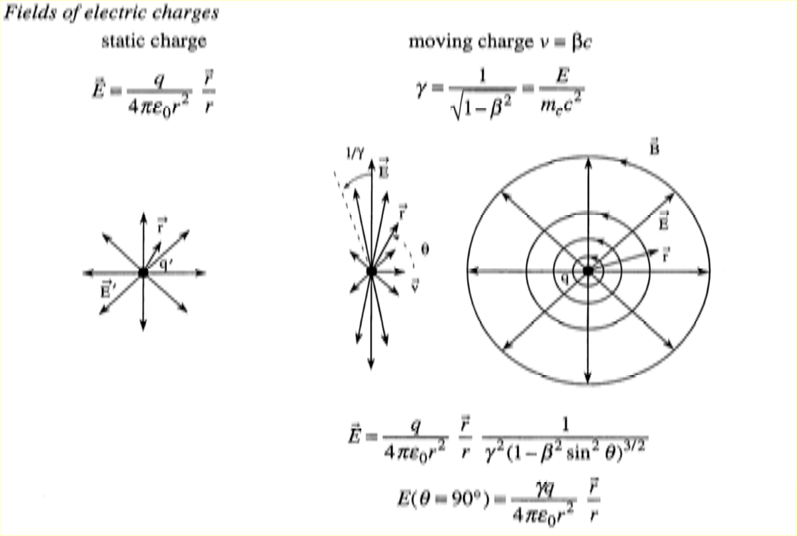
Note that this relation is also valid for networks of linear resistors at homogeneous temperature between any 2 terminals, but not for resistors which are not in thermo dynamical equilibrium like a biased diode or a transistor with supply voltage

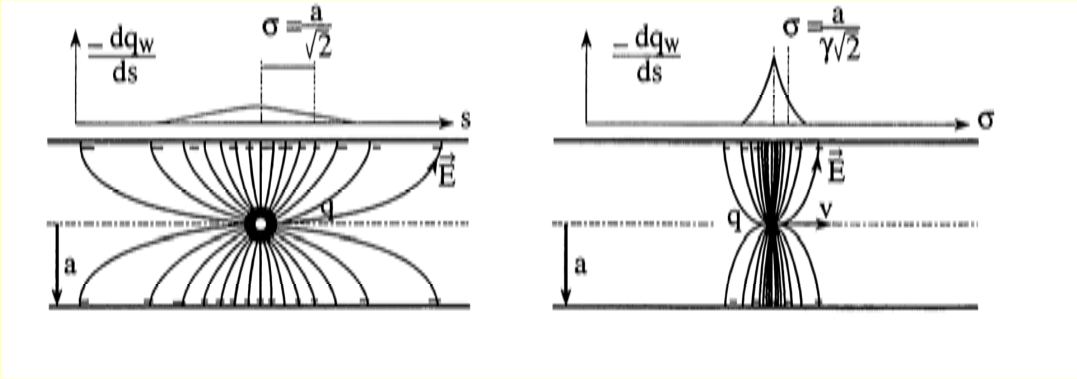
Such active elements can have noise temperatures well below their phys.temperature
In particular a forward biased (solid state) diode may be used as a pseudocold load:
By proper setting of the bias current the differential impedance can be set to 50 Ohm
The noise temperature of this device is slightly above $T_0/2$. Alternatively the input stage of a low noise amplifier can be applied (example:1 dB NF= 70 deg K noise temp.)

R.H. Frater, D.R. Williams, An Active „Cold" Noise Source, IEEE Trans.on Microwave Theory and Techn., pp 344-347, April 1981

The "field slice" of fast and slow beams

- Similar to what has been shown for the vacuum diode we experience a modification of the particle distribution spectrum (low pass characteristic) when we use Schottky signal monitors which are based on the interaction with the wall current (this is the case for the vast majority of applications)

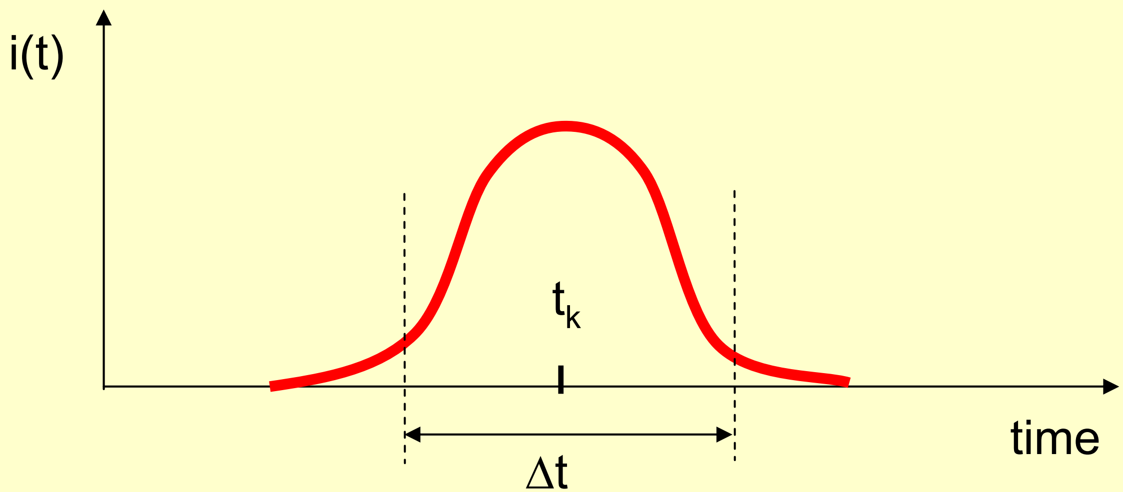




From: A.Hofmann: Electromagnetic field for beam observation CERN-PE-ED 001-92

Single-particle current (1)

- a single particle rotating in a storage ring
- constant frequency $\omega_0 = 2\pi f_0 = 2\pi / T$
- Signal induced on a pick-up at passage time t_k



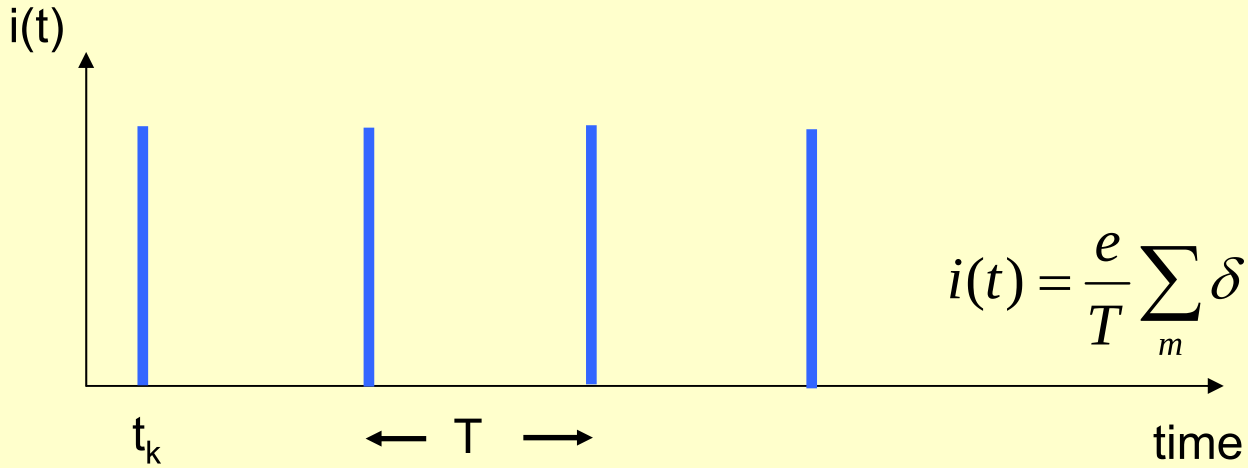
Single-particle current (2)

- Approximation by a Dirac distribution.
- Periodic signal over many revolutions $$i_k(t) = \frac{e}{T}\sum_m \delta(t - t_k - mT)$$
- Applying the Fourier expansion to $i_k(t)$:

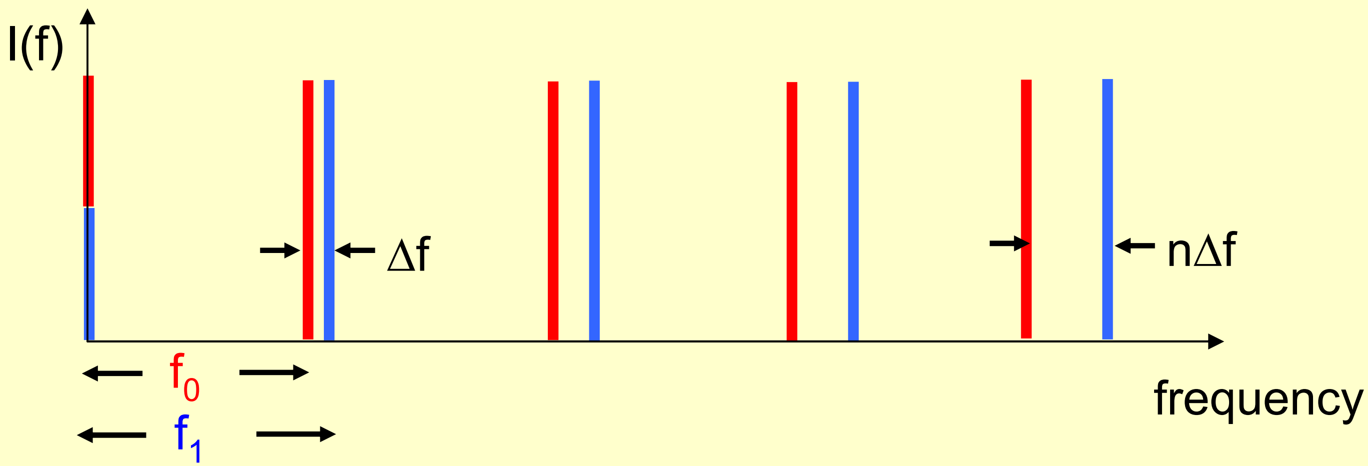
$$i_k(t) = i_0 + 2i_0 \sum_{n=1}^{\infty} a_n \cdot \cos n\omega_0 t + b_n \cdot \sin n\omega_0 t$$

$$\text{with} \quad \begin{cases} i_0 = ef_0 & \text{DC part of the beam (single particle)} \\ a_n = \cos n\varphi_k & \text{and} \quad \mathrm{b}_n = \sin n\varphi_k \end{cases}$$

Time and frequency domains



$$i(t)=\frac{e}{T}\sum_{m}\delta(t-t_k-mT)$$



$$\Delta f = f_0\,\eta\,\frac{\Delta p}{p}$$

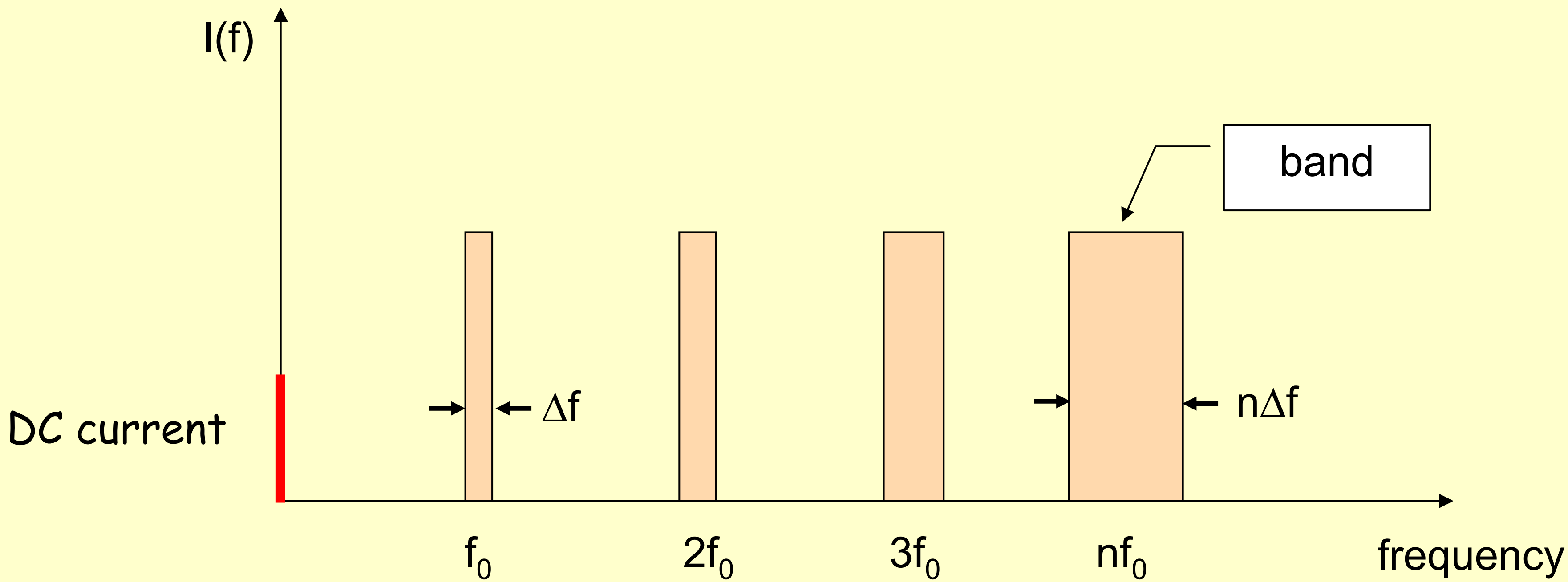
A second particle of revolution frequency $f_1 = f_0 + \Delta f$ is added

Coasting beam : frequency Domain

N particles with a distribution of revolution frequencies $f_0 \pm \Delta f /2$
for n=1 (fundamental)

One expects a spectrum with bands around each **harmonic** nf_0

The band height is arbitrary at this stage

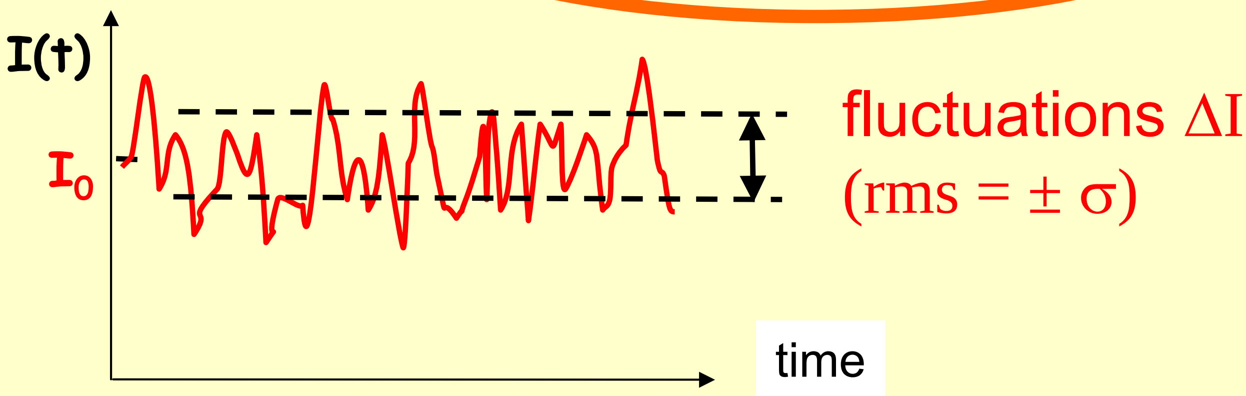


Current fluctuations

- Assume a group of ***N*** particles having the ***same*** revolution frequency
- with a random distribution of initial phases $\varphi_k = \omega_0 t_k$

$$I(t) = I_0 + 2 i_0 \sum_{n=1}^{\infty} A_n \cdot \cos n\omega_0 t + B_n \cdot \sin n\omega_0 t$$

$$\begin{cases} I_0 = N \times i_0 = N \times (e f_0) \\ A_n = \sum_{k=1}^{N} \cos n\varphi_k \\ B_n = \sum_{k=1}^{N} \sin n\varphi_k \end{cases}$$

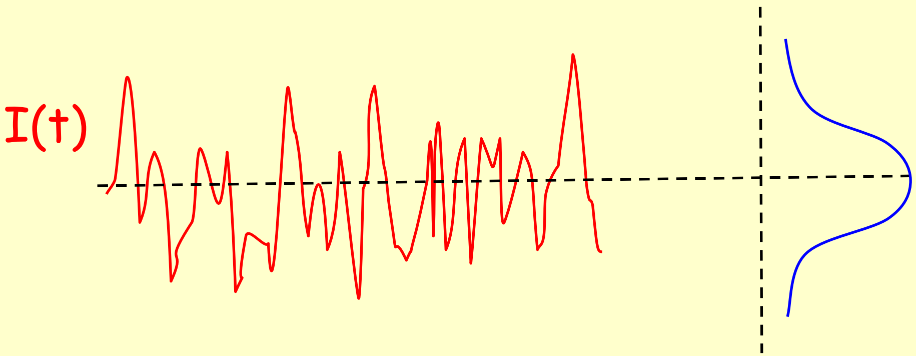


- In the total current ***I(t)***, the nth harmonic contains the contribution of all ***N*** particles

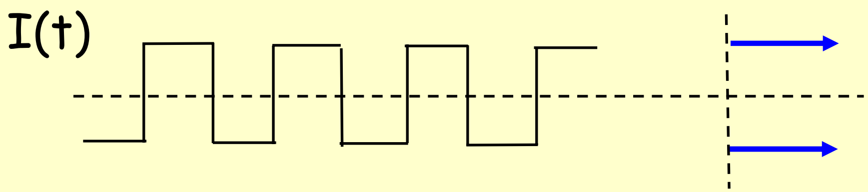
- $< \Delta I >_T = 0$. We are interested in the <u>mean squared fluctuations</u>

Amplitude density distribution and "color" of noise

- One talks often about white Gaussian noise; what does this really mean?
- The "color" of the noise refers to its spectral distribution over the frequency range of interest in analogy to the color of light;
  - White light has nearly constant spectral density
  - Red light is low pass filtered
  - Blue light is high pass filtered



The amplitude density distribution can be visualized as the intensity of the green trace seen on an old style scope when the horizontal deflection is turned off; try to draw the amplitude density distribution for a sinewave !



Mean squared fluctuations (1)

- Calculate the instantaneous squared fluctuation of all spectral lines

$$(\Delta I)^2$$

- The mean over the revolution period is given by

$$\left\langle \Delta I^2 \right\rangle_T = \frac{1}{T}\int_0^T (\Delta I)^2 \cdot dt = (2i_0)^2 \times \sum_{n=1}^{\infty} \frac{A_n^2}{2} + \frac{B_n^2}{2}$$

$$\left\langle \Delta I^2 \right\rangle_T = \sum_{n=1}^{\infty} \left\langle I_n \right\rangle^2 = \sum_{n=1}^{\infty} \frac{(2i_0)^2}{2} \left(\left[\sum_{k=1}^{N} \cos n\varphi_k \right]^2 + \left[\sum_{k=1}^{N} \sin n\varphi_k \right]^2 \right)$$

- Assuming a random distribution of the initial phases, the sums over cross terms cancel and one gets for harmonic n :

$$\left\langle I_n \right\rangle^2 = \frac{(2i_0)^2}{2} \sum_{k=1}^{N} \cos^2 n\varphi_k + \sin^2 n\varphi_k = 2e^2 f_0^2 N = 4e^2 f_0^2 \frac{N}{2}$$

Mean squared fluctuations (2)

$$\langle I_n \rangle^2 = 2e^2 f_0^2 N = 2eI_0 f_0 \quad [\mathrm{A}^2]\,\mathrm{with}\ \mathrm{I}_0 = ef_0 N$$

- "probable power contribution" to the nth "Schottky band" for a group of N mono-energetic particles for each particle

- Proportional to N

- No harmonic number dependency & constant for any spectral line

- Current fluctuations : $\Delta I = I_{rms} \sum_{n=1}^{\infty} \cos(n\omega_0 t - \varphi_n) \qquad \text{with } I_{rms} = 2ef_0 \sqrt{\frac{N}{2}}$

- For ions : substitute "e" by "Ze" and $\langle I_n \rangle^2 = 2(Ze)^2 f_0^2 N$

Spectral density of the noise

- Assume a distribution of revolution frequencies f_r between $f_0 \pm \frac{\Delta f}{2}$
- For a <u>subgroup</u> of particles having a very narrow range df_r : $N \rightarrow \left(\frac{dN}{df_r}\right) df_r$

$$d\langle I_n \rangle^2 = 2e^2 f_r^2 \left(\frac{dN}{df_r}\right) df_r$$

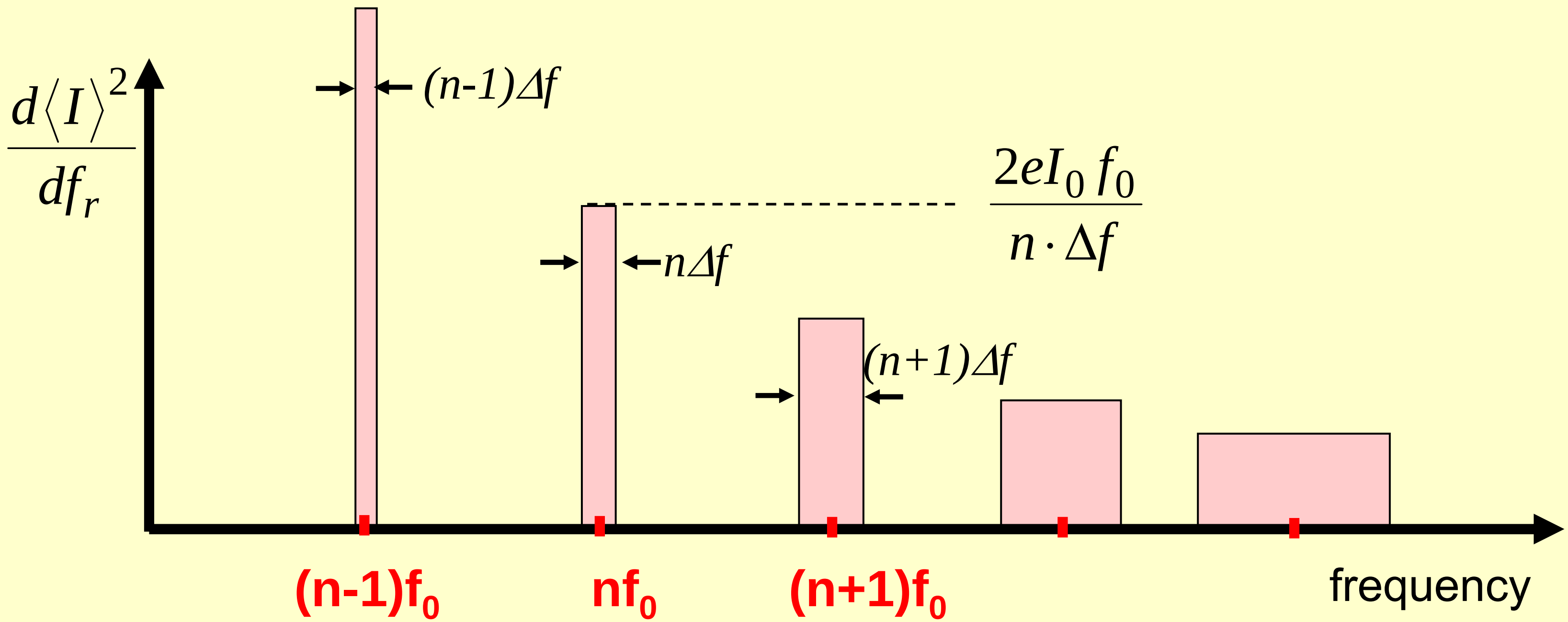
$$\Rightarrow \frac{d\langle I_n \rangle^2}{df_r} = 2e^2 f_r^2 \left(\frac{dN}{df_r}\right)$$ **spectral density of the noise in nth band**

- $\left(\frac{d\langle I_n \rangle^2}{df_r}\right)$ in units of $[A^2 / Hz]$
- Integrate over a band $(f_0 \pm \frac{\Delta f}{2})$: one gets the total noise per band

$$\langle I_n \rangle^2 = 2e^2 f_0^2 N = 2eI_0 f_0$$

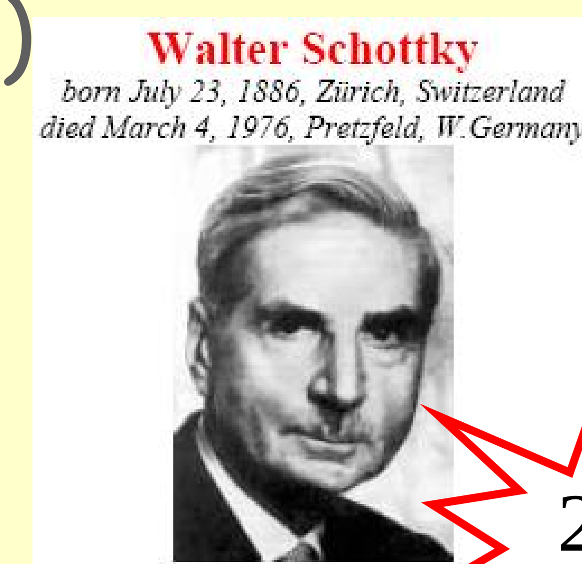
Schottky bands (1)

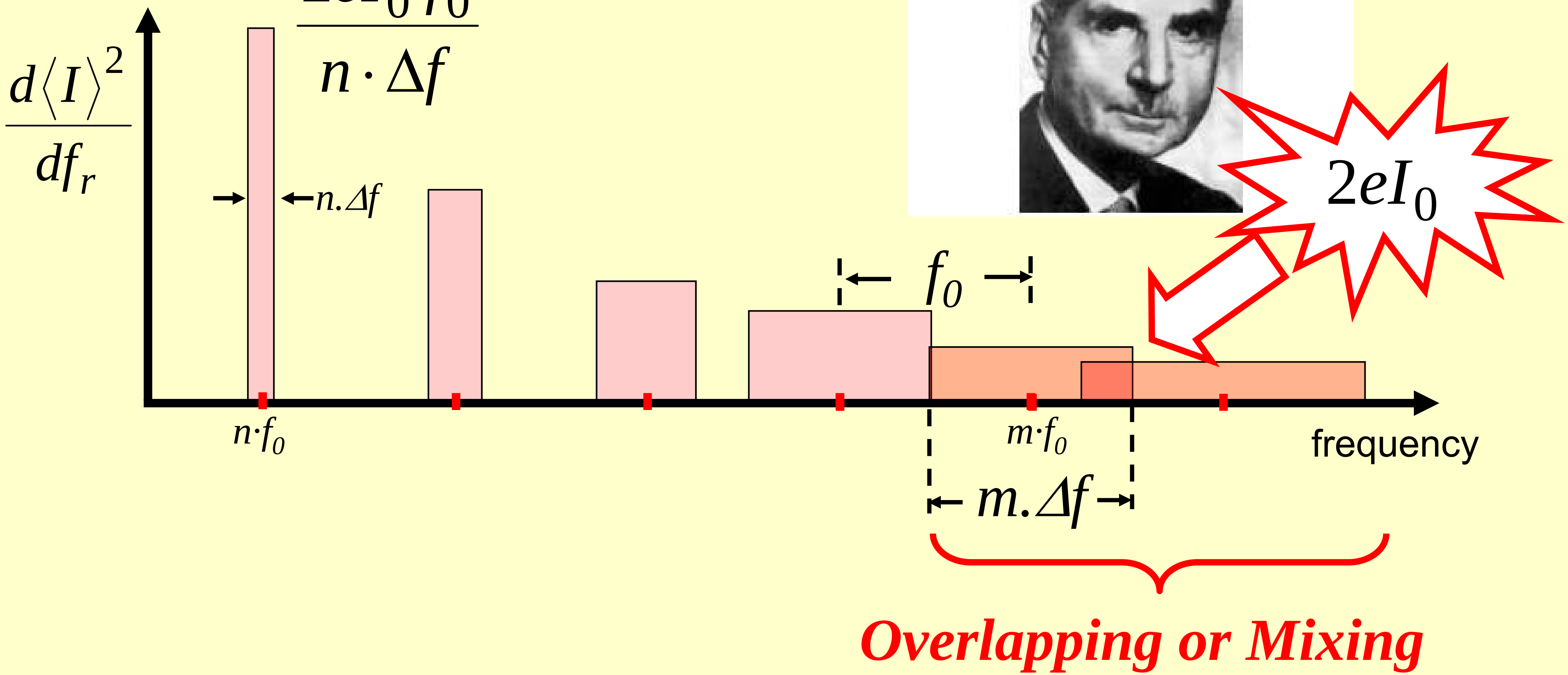
- In a diagram $\left(\frac{d\langle I\rangle^2}{df_r}\right)$ versus f the area of each Schottky band is constant
- Since the n^th band has a width $(n \times \Delta f)$, the spectral density decreases with $1/f$



Each longitudinal Schottky band has the same „area" = integrated power

Schottky bands (2)



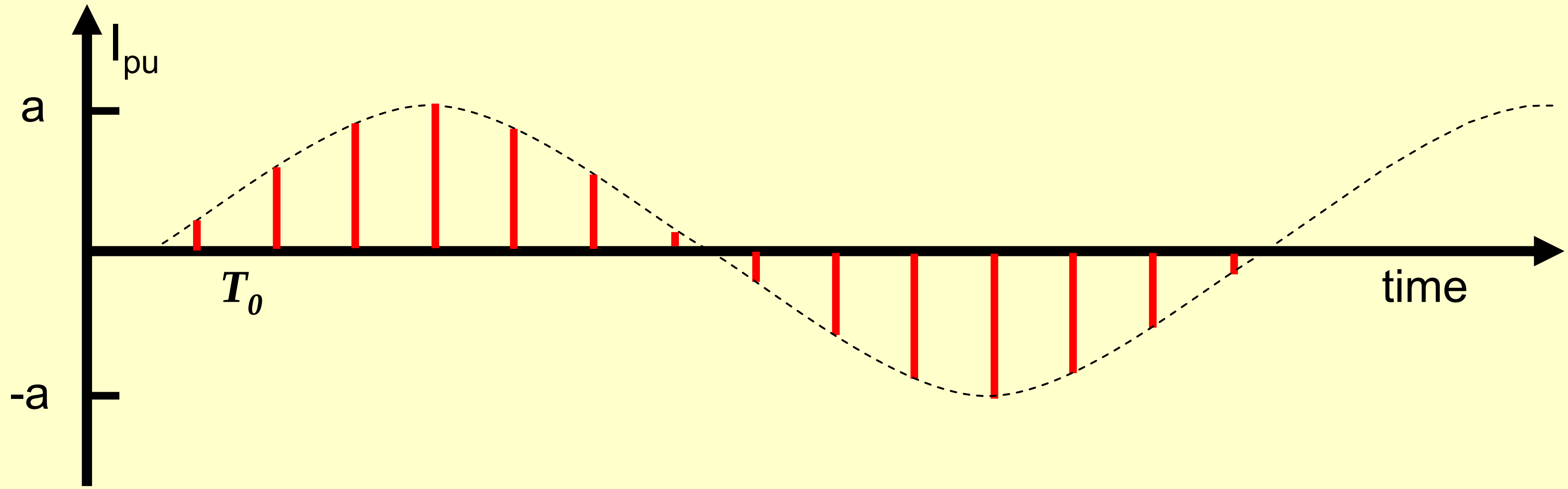


With a Spectrum Analyser

Longitudinal Schottky bands from a coasting beam give

- mean revolution frequency
- frequency distribution of particles
- momentum spread
- number of particles

Transverse signal



A single particle passing through a position sensitive pick-up generates a periodic series of delta functions :

$$i_{pu}(t) = \frac{e}{T} \sum_{n}^{\infty} \underbrace{\delta(t - nT + \varphi_k)} \times \underbrace{a_k \cos(q\omega t + \phi_k)}$$

Amplitude modulation of the longitudinal signal by the betatron oscillations

Transverse signal : Dipolar momentum

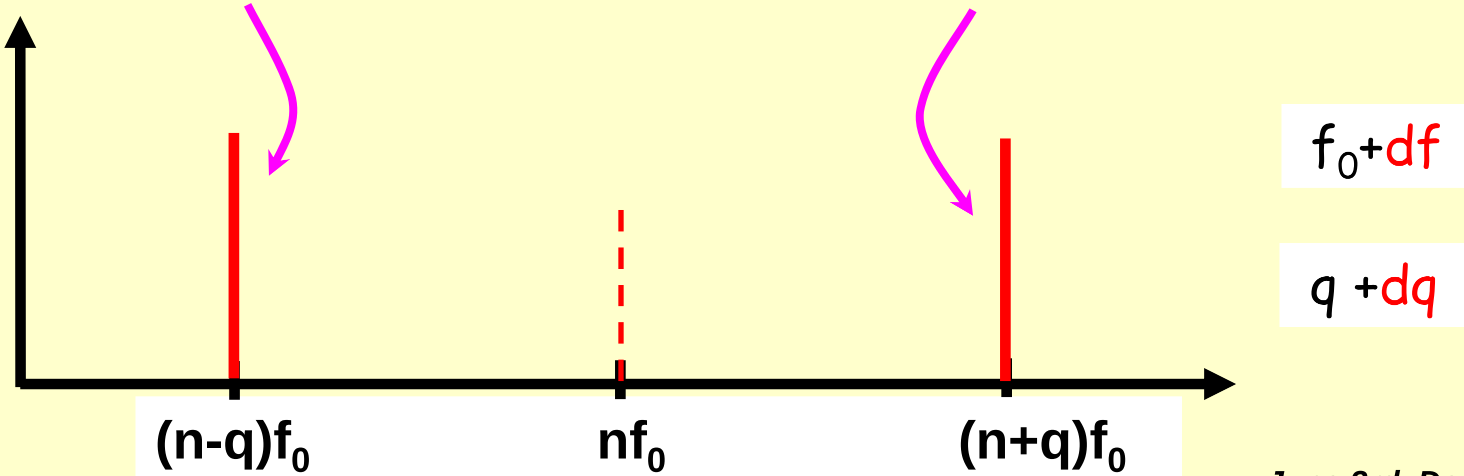
- Single particle
- The difference signal from a position sensitive PU gives

$$\Delta i_{PU} = S_\Delta \times a_k(t) \times i_k(t)$$

$$= S_\Delta \times a_k \cos(q\omega_0 t + \phi_k) \times \left[i_0 + 2i_0 \sum_{n=1}^{\infty} \cos(n\omega_0 t + n\varphi_k) \right]$$

- harmonic n =

$$d_n = S_\Delta \cdot a_k \cdot i_0 \cdot \left[\cos\left((n-q)\omega_0 t + n\varphi_k - \phi_k\right) + \cos\left((n+q)\omega_0 t + n\varphi_k + \phi_k\right) \right]$$



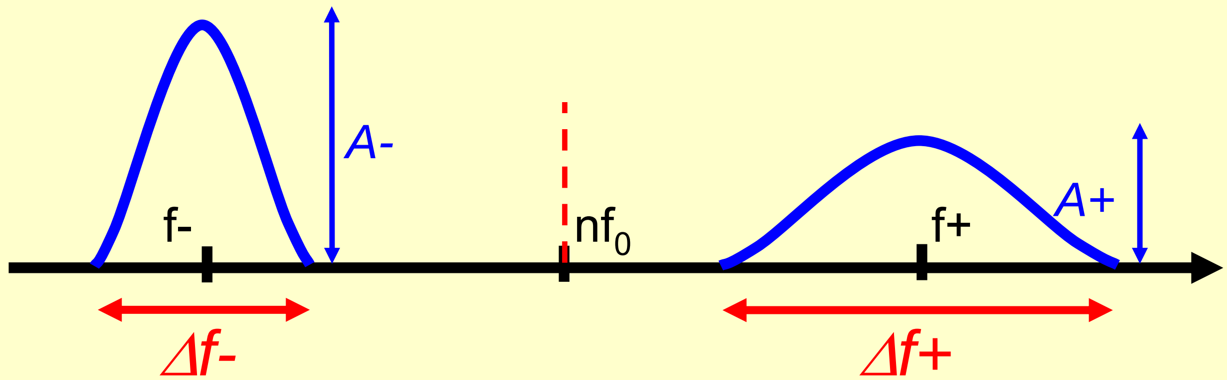
Transverse signal with N particles

- N charges having random phases
- The total power in one transverse sideband is :

$$\left\langle \Delta I_{PU}^2 \right\rangle = S_\Delta^2 a_{rms}^2 i_0^2 \frac{N}{2} = S_\Delta^2 a_{rms}^2 e^2 f_0^2 \frac{N}{2}$$

- a_{rms}^2 (squared rms beam size) is proportional to **the transverse emittance**
- Not harmonic-dependent
- All bands have the same integrated power
- Frequency width of the side bands $\Delta f_\pm = (n \pm q) \times df \pm f_0 dq$

A spectrum analyser gives…



- the fractional part of the tune $$q = \frac{1}{2} + \frac{f_+ - f_-}{2 f_0}$$
- the tune spread $$\frac{dq}{Q} = \frac{\Delta f_+ - \Delta f_-}{2 f_0 Q}$$
- the momentum spread $$\frac{dp}{p} = \frac{1}{\eta} \times \frac{\Delta f_+ + \Delta f_-}{2 n f_0}$$
- the chromaticity $$\xi = \left(\frac{dq}{Q}\right) / \left(\frac{dp}{p}\right)$$
- the emittance $$\varepsilon \propto A_- \Delta f_- + A_+ \Delta f_+$$

Bunched beams (1)

- When particles are oscillating in an RF bucket the revolution period $T=T_0$ is no longer constant but modulated with the synchrotron frequency f_S and we get for the time difference τ with respect to the synchronous particle (single particle case)

$$\tau(t) = A_s \sin(2\pi f_s t + \psi)$$

where A_S stands for the amplitude of the synchrotron oscillation and ψ is some initial phase. Introducing this time dependency into the equation for the single particle movement without RF bucket we obtain:

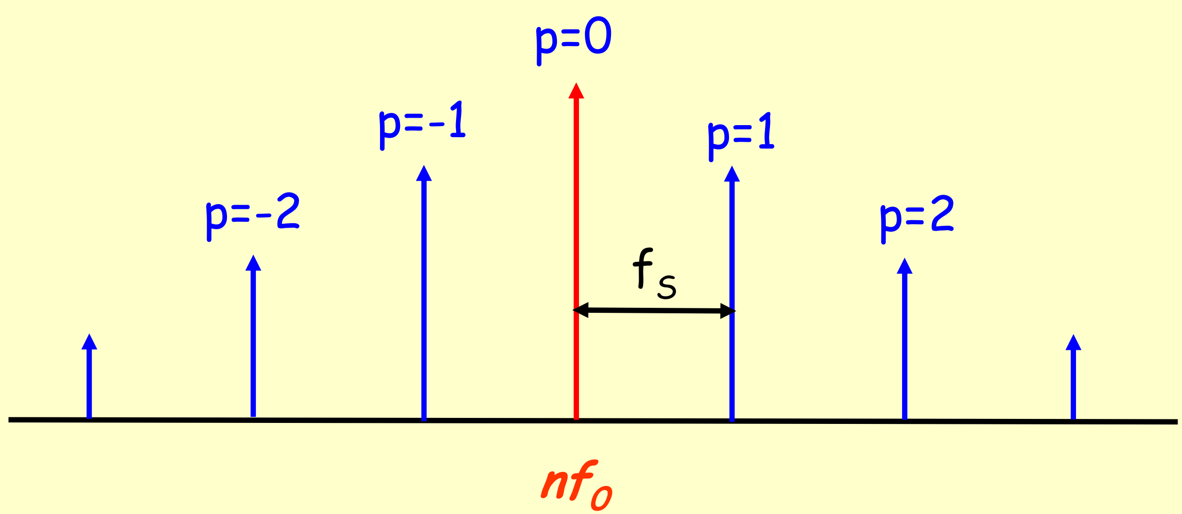
$$i(t) = e\ f_o + 2\,e\,f_0 \sum_{n=0}^{\infty} \cos\left\{2\pi n\,f_0\left[t + A_s \sin(2\pi f_s t + \psi)\right]\right\}$$

- In other words: each spectral line for a single particle splits up into an infinite number of modulation lines by this synchrotron oscillation related phase modulation. The mutual spacing between adjacent lines of this modulation spectrum is equal to f_S

Bunched beams (2)

For each of those modulation lines with index p the amplitude of the current, I_P can be expressed as

$$I_p = \sum_{p=-\infty}^{n} J_P(2\pi n f_0 A_s)\cos(2\pi n f_0 + 2\pi n f_s + p\psi)$$



Here J_P stands for the Bessel function of order p

Bunched beams (3)

In a similar way one can derive an expression for the time dipole moment d(t) of a single particle travelling on an RF bucket of some circular machine

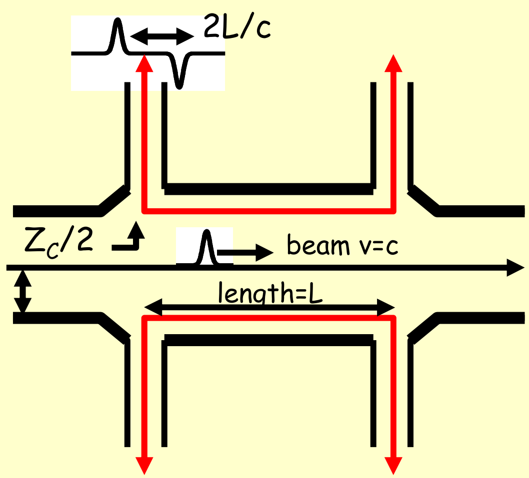
$$d(t) = \underbrace{a\cos(q2\pi f_0 t)}_{\text{Amplitude modulation proportional to a and at the fractional tune q}}\; ef_0 \underbrace{\cos\left[2nf_0 t + \tau(t)\sin(2\pi f_0 t + \psi)\right]}_{\text{phase modulation with the the synchrotron frequency } f_s}$$

Finally we obtain a rather lengthy expression for the amplitude of a particular line

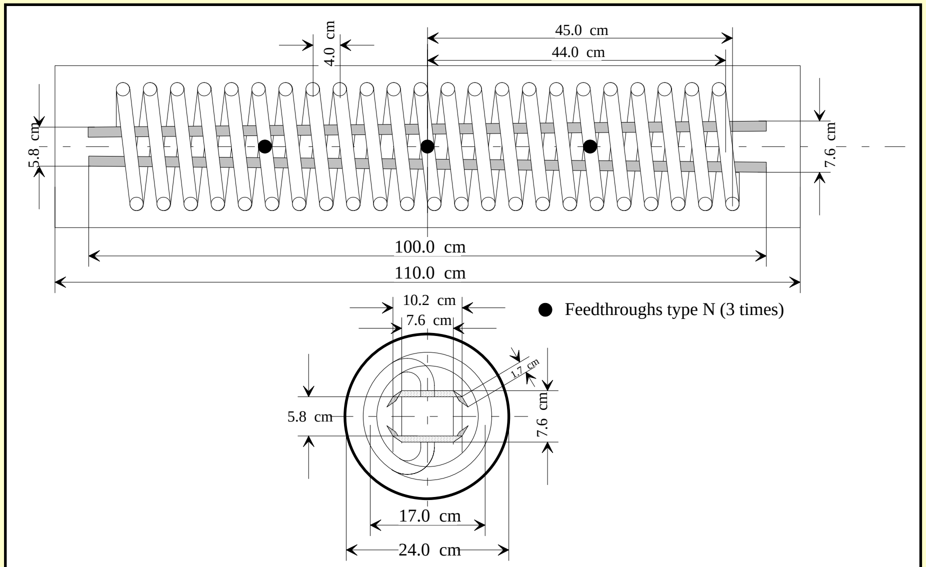
$$d_n = ef_0\, a \sum_{p=-\infty}^{p=+\infty} J_p\left[(n \pm q)2\pi f_0 \tau\right] \cos\left\{ \left[(n \pm q)2\pi f_0 + p2\pi f_S\right] t + p\psi + \Phi \right\}$$

Discussion of pick-up structures (1)

- The well known stripline pickup: (often referred to as λ/4 pick-up)
  - simple construction
  - may be operated resonant or non-resonant
  - can be used simultaneously as longitudinal and also as transverse pick-up
  - can be used for highly relativistic and also for slow beams
  - can be used with low impedance termination (upstream end) or at low frequencies also as capacitive pickup with high impedance termination
  - can be installed in dedicated sections or inside magnets



Basic stripline PU structure



AD Resonant Transverse LF Schottky PU
Low beta version

From: M.E. Angoletta Chamonix WS 2007; Structure was Designed and build By F. Pedersen

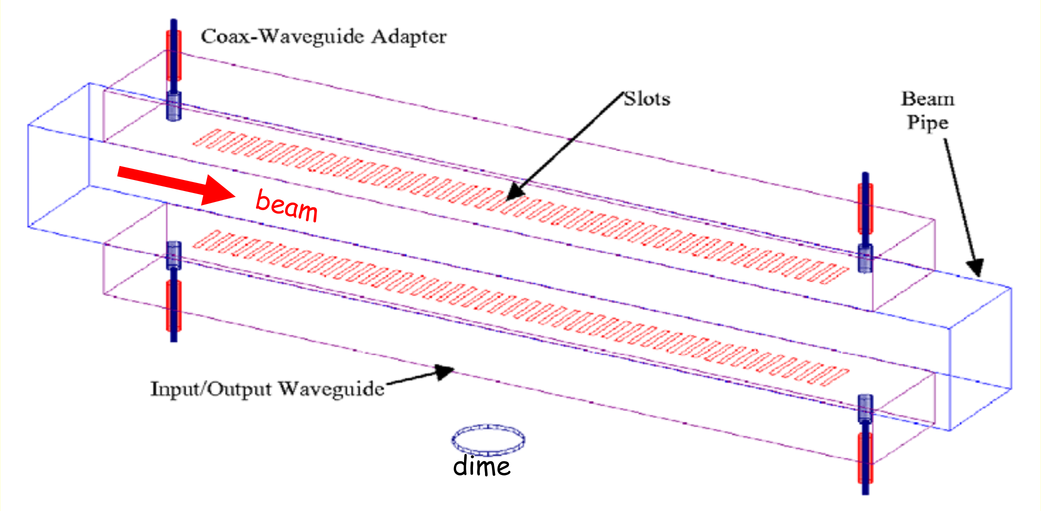
Discussion of pick-up structures (2)

- Shoe box type structures (low sensitivity unless resonant, but good transverse linearity; rather for low frequencies)
- Printed loop and printed slot couplers (e.g. FNAL and GSI)
- Ferrite ring structures (up to 500 Mhz; CERN „old“AA) with variable geometry during operation (shutter type)
- Wall current monitor type devices (low sensitivity !); mainly for coherent signals
- Travelling wave structures with multiple elements such as:
  - Cascaded stripline (superelectrode concept; e.g. CERN-AD and LEIR) or also knows as „n directional couplers in series“
  - Travelling wave cavities (e.g. the 200 MHz CERN SPS travelling wave cavities were used at one of their higher order modes (HOMs) at 460 Mhz during the p-pbar project in the SPS as Schottky PUs)
  - Faltin type slot couplers (a coax line with a slotted iris);
  - Slotted waveguide structures (FNAL, BNL and LHC)
  - Cerenkov type dielectric PUs (CERN- AA) around 5 GHz

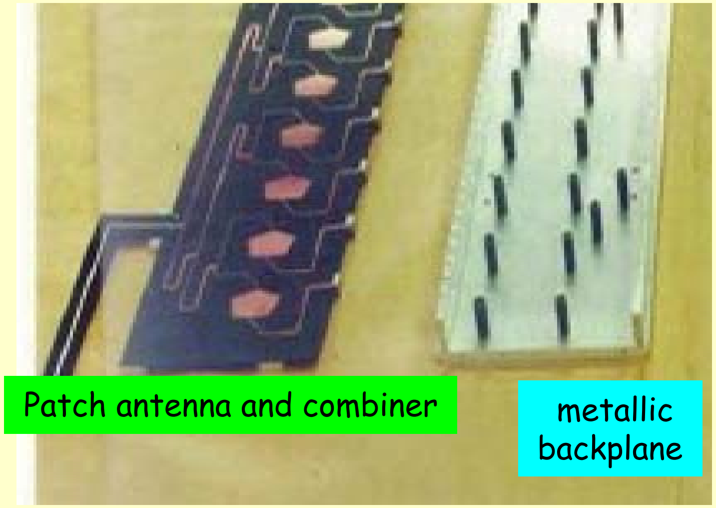
Discussion of pick-up structures (3)

- Resonant cavities above 1 GHz; operated as transverse and long. PU and kicker; such cavities may require mechnical displacement during operation in order to reduce large coherent signals from the longitudinal mode if operated as transverse PU
- Very low noise ferrite filled cavities at a few MHz ; a cavity at room temperature which has a noise temperature of a few deg K due to feedback with an ultra low noise amplifiers which is also at room temperature; [F. Pedersen, used in the CERN AD]; high longitudinal sensitivity for faint p-bars shots even at low v/c values
- Capacitive couplers (button like) in a trap; they are used to cool the few particles (e.g. pbars) in the trap by dissipating the induced signal (dE/dt) in some resistor; single particles can be seen this way
- And there are proposals for the future:
    - Undulator type PUs for optical stochastic cooling
    - Coherent electron cooling (the electron beam is the PU for the hadron Schottky signal)
- NOTE: In certain stochastic cooling systems each particles induces just a single microwave photon (10E-5 eV) per passage in the pickup; try to do a „back of an envelope" calculation to check this case

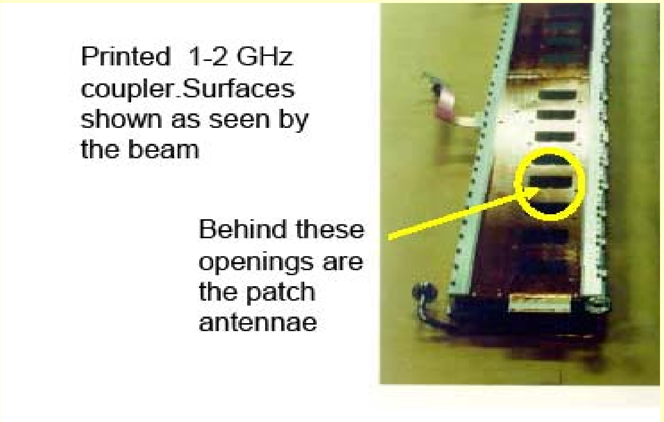
Discussion of pick-up structures (4)



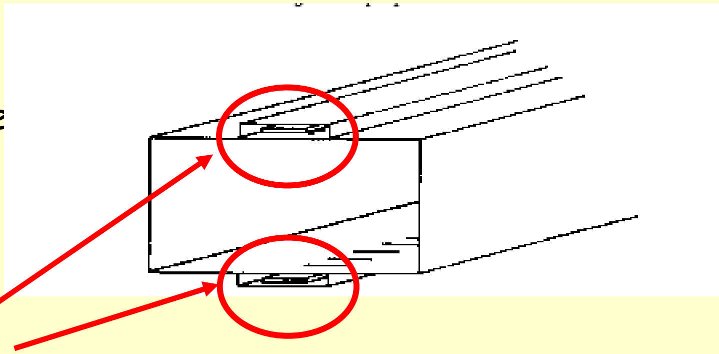
Slotted waveguide coupler (McGinnis, FNAL)
D. McGinnis, "Slotted Waveguide Slow-Wave Stochastic Cooling Arrays", PAC 1999, 1999, New York



Printed loop coupler (FNAL)

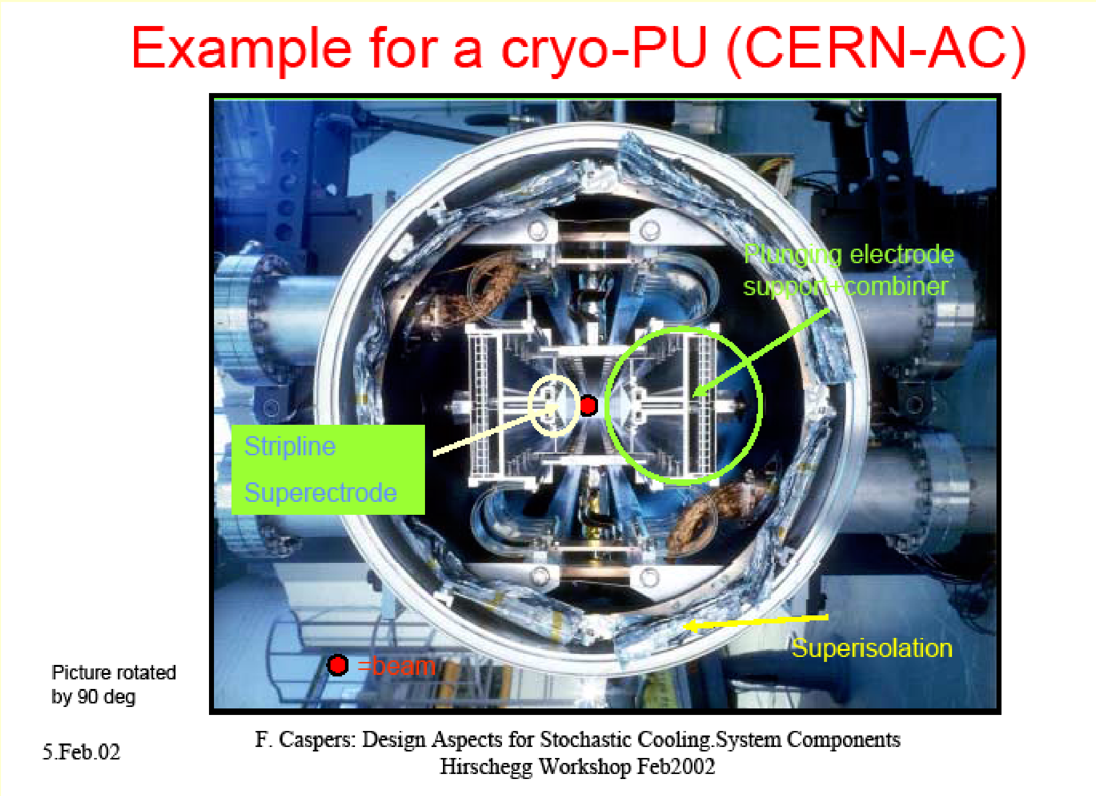


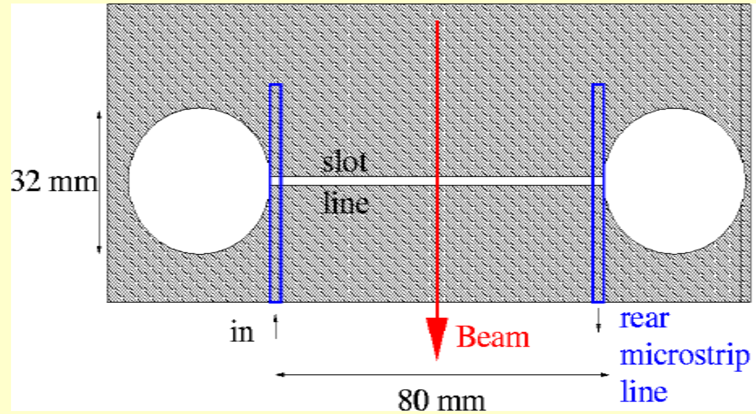
Faltin type slotline Coupler as used in the CERN AA
Ref:S.v.d.Meer Nobel lecture 1984

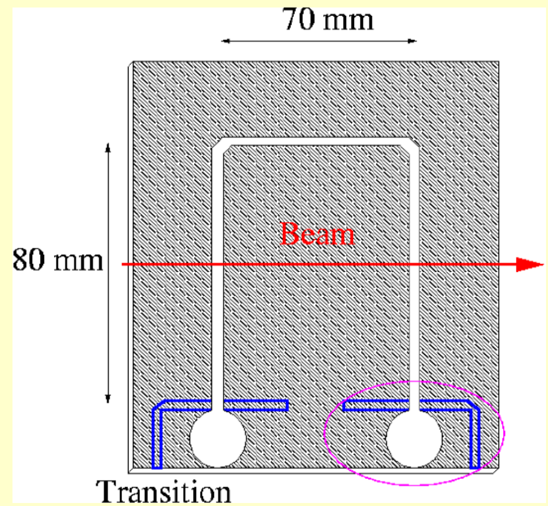


Coaxial lines, not waveguides

Discussion of pick-up structures (5)







AD pickup; plunging electrode structures for transverse sensitivity enhancement during the cooling process; the electrodes follow the beam envelope

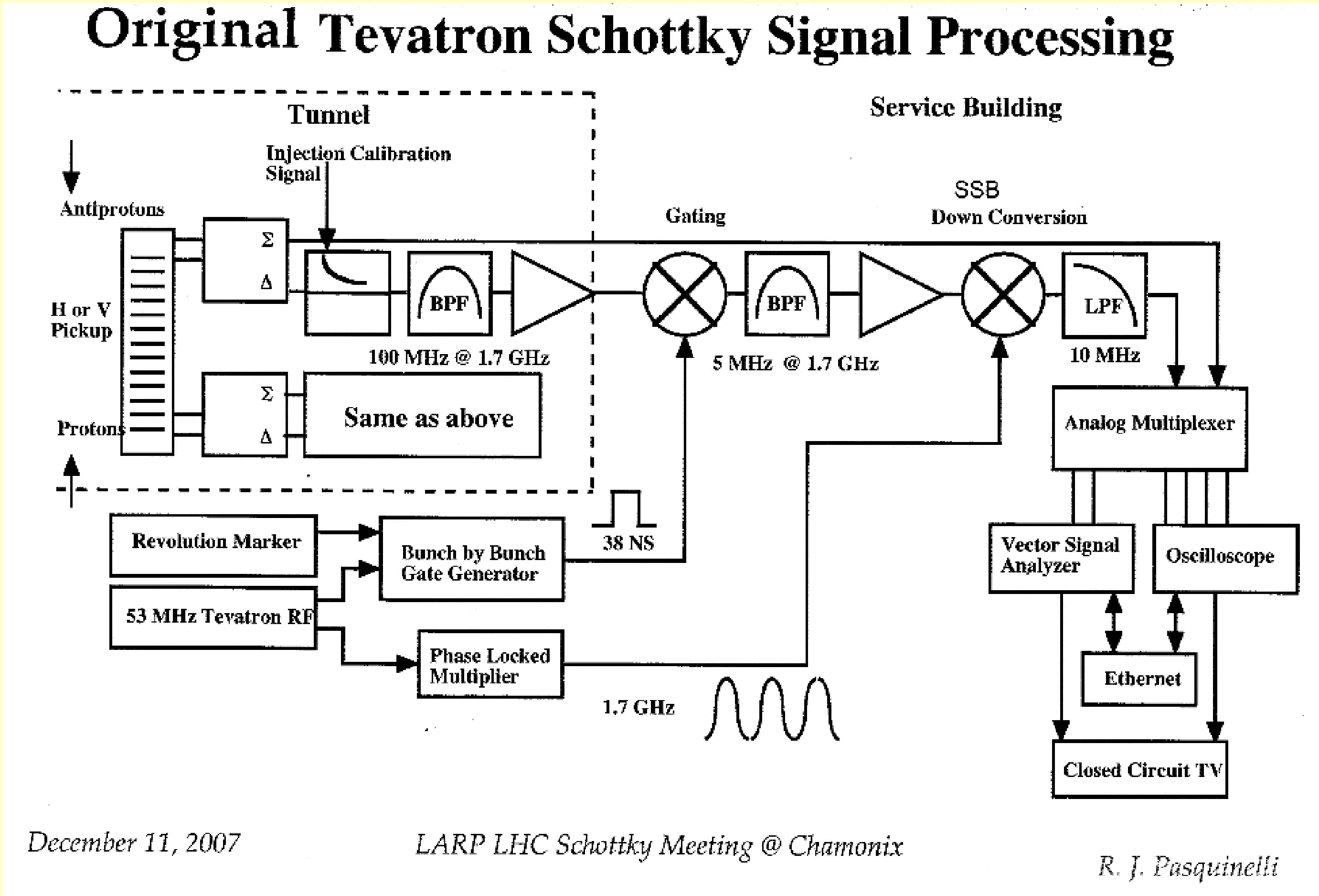
http://www-w2k.gsi.de/frs/meetings/hirschegg/program.asp

Printed slotline structure (on ceramic or dieletric substrate) used at GSI „Fritz bones“

Signal treatment (1)

- The classical method for signal treatment is done by using (several) stages of superheterodyne downmixing (like in old style spectrum analyzers)
  - This technique requires a considerable hardware investment, but still returns the highest dynamic range (in case of strong coherent signals present)
- However DSP type front ends are becoming more and more common and are in a way a copy of the architecture of modern real time signal processors.
  - One of the DSP related problems is to get a very high instantaneous dynamic range i.e. without range switching. For bunched beam Schottky applications this dynamic range may be up to 100 dB due to the presence of strong coherent signals in the revolution harmonics
- In practice one can often find a combination of both methods.
- Anyway, in the baseband FFT processing is nearly always used
- In certain cases fast RF gating close to the front end is required in order to separate individual bunches
- The very efficient and rather simple DDD (direct diode detection) method is excellent for detection of coherent signals , but may require beam excitation to see incoherent signals

Signal treatment (2)

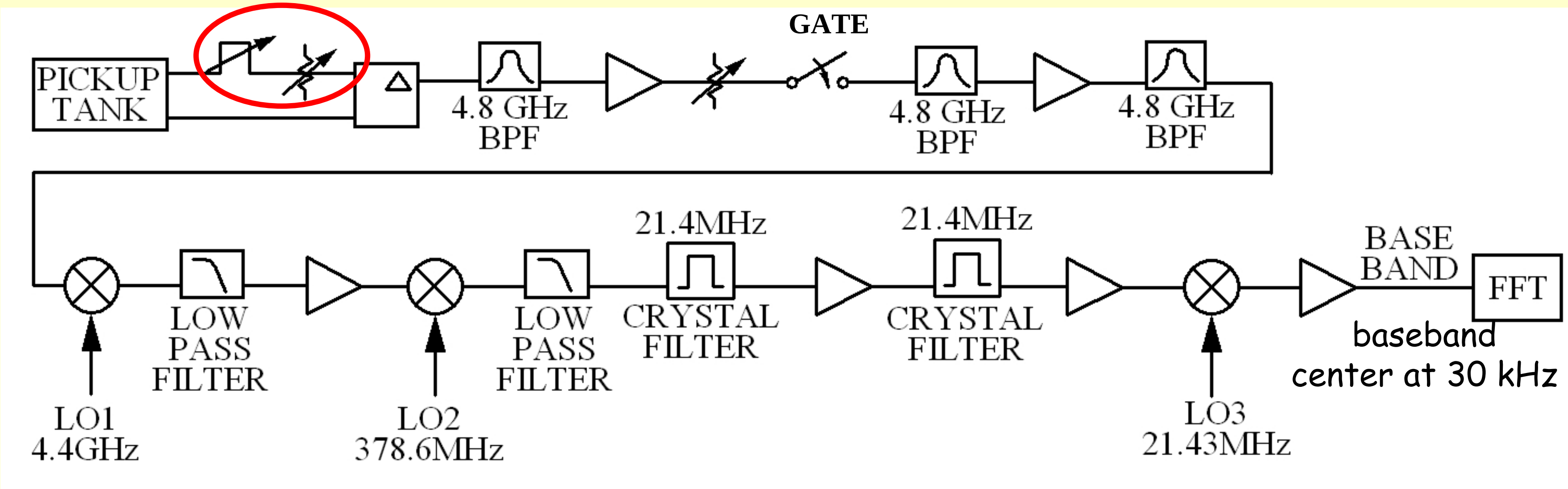


Example for Schottky signal treatment chain with gating and single downmixing

Many thanks to R. Pasquinelli

Signal treatment (3)

Adjustment of the electrical center of the PU



Example for the LHC Schottky signal treatment chain with gating and triple downmixing
the front end bandwidth is about 200 MHz for fast gating

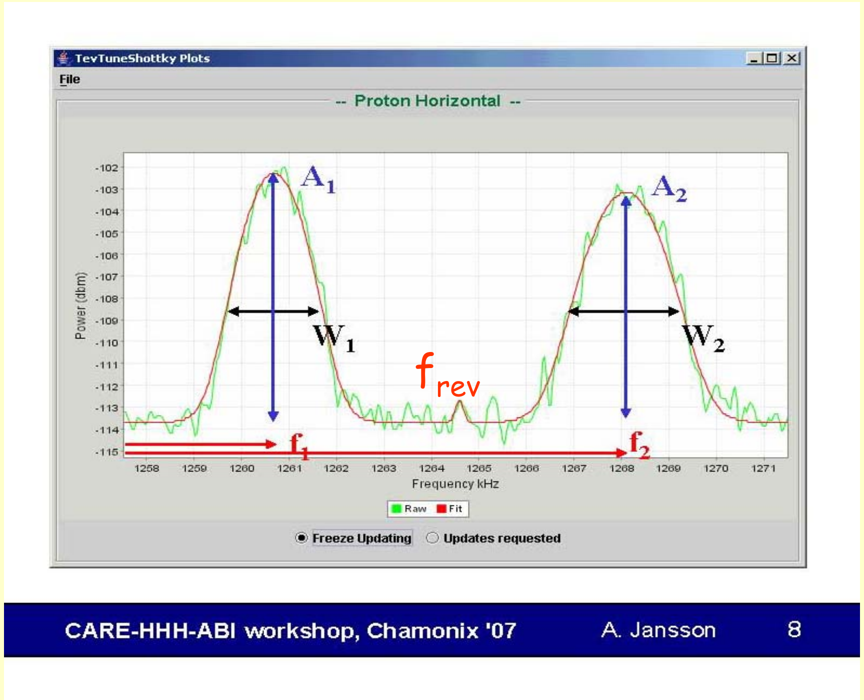
Measurement examples (1)

$$q = \frac{1}{2} + \frac{f_2 - f_1}{2\, f_{rev}}$$

$$\frac{\Delta p}{p} = \frac{1}{\eta} \cdot \frac{W_1 + W_2}{2 n f_0}$$

$$\xi \propto \frac{W_1 - W_2}{W_1 + W_2}$$

$$\varepsilon \propto A_1 W_1 + A_2 W_2$$



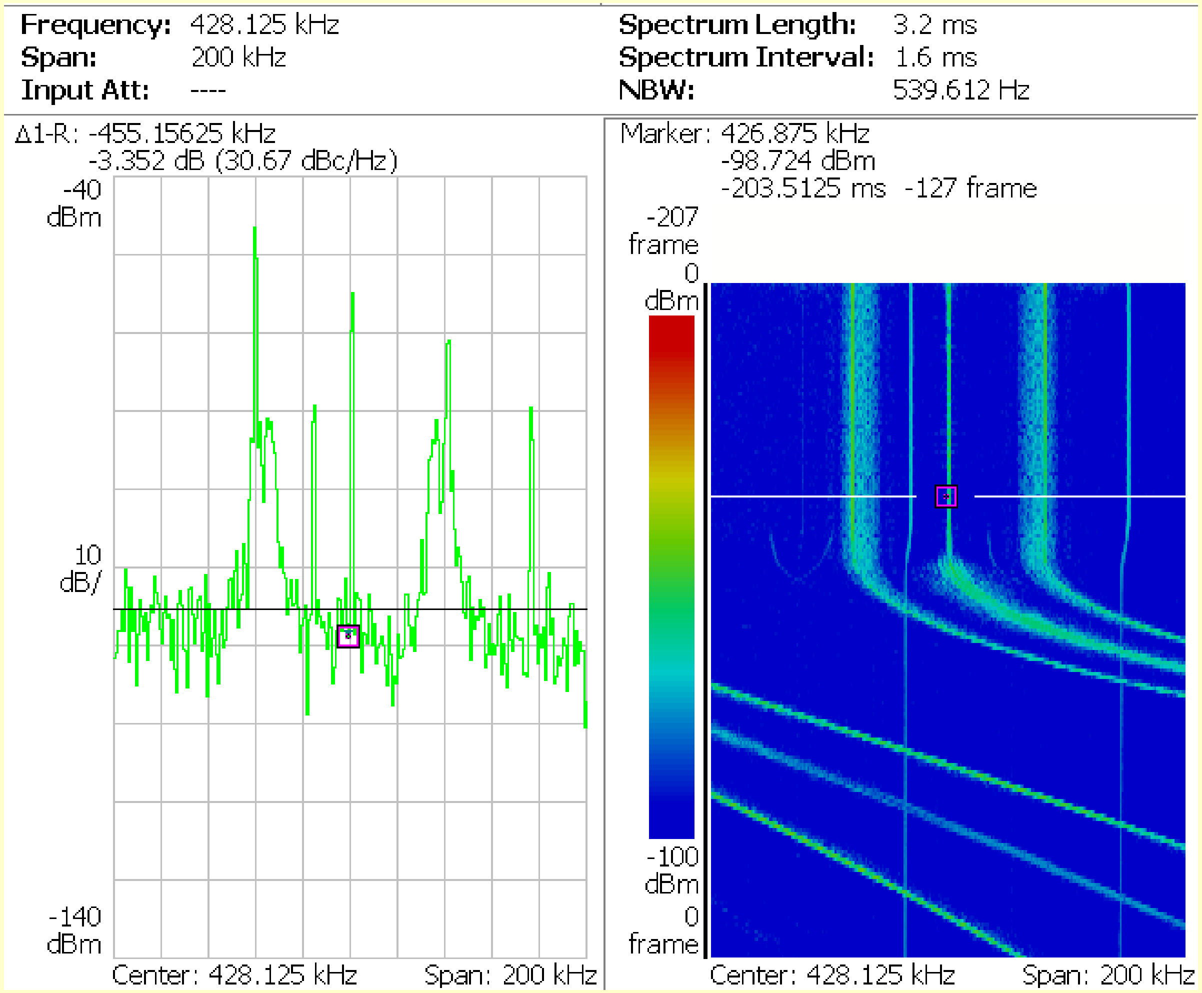
This slide shows how from the smoothed raw data the parameters of intererest (f_1,f_2 ,f_{rev}) are extracted for „q". The width W_1 and W_2 (e.g. as $\pm 1\sigma$ value) for Δf returns via η the value for $\Delta p/p$. Chromaticy ξ and emittance ε accordingly

many thanks to A. Jansson (FNAL)for those very nice Tevatron data !

Measurement examples (2)

- Evaluation of the (incoherent) tune and momentum spread is rather straight forward with a suitable fitting algorithm and data averaging (remember that in the end we would like to extract a RELIABLE number from a noisy trace).
- This is also possible during the ramp (acceleration) and often dispayed in a colour coded plot vs time.
- Since the emittance is proportional to the integrated power of the sidebands a rough relative measurment of the emittance is fairly simple. However for an absolute and precise result more effort is needed. [gain-drifts etc]
  - Calibration can be done e.g. via wire scanner profile and emittance meas.
  - Another complementary method (BN L, P.Cameron and K.Brown) takes advantage of the variation of signal strength in the revolution harmonic and the betatron sidebands as a function of the mechanical position of the PU (if it can be moved mechanically, which is not always the case).
  - The key ingredients to the method are moving the beam transversely in the detector, and measuring the ratio of the power in the rev. line to the power in the betatron lines (K.Brown, priv. comm)

Measurement examples (3)



A typical display showing a normal FFT result at a fixed moment in time (left)
and the evolution of the spectral lines vs time acceleration ramp going into a flattop in a colour coded plot (right)

From: P.Forck,W.Kaufmann, P. Kowina, P. Moritz,U.Rauch
GSI
Investigations on BaseBand tune measurements using direct digitized BPM signal;
Chamonix Schottky WS Dec 2007

A very short list of papers…

- D.Möhl, "*Stochastic cooling for beginners*", CAS Proceedings, CERN 84-15, 1984

- D.Boussard, "*Schottky noise and beam transfer function diagnostics*", CERN SPS 86-11 (ARF)

- S.van der Meer, "*Diagnostics with Schottky noise*", CERN PS 88-60 (AR)

- T.Linnecar, "*Schottky beam intrumentation*", Beam Instrumentation Proceedings, Chapter 6, CERN-PE-ED 001-92, Rev. 1994

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- T. Linnecar and R. Jones for many useful discussions and comments
- E. Ciapala and the CERN AB-RF group for support
- The CAS management for having given me the opportunity to join this school