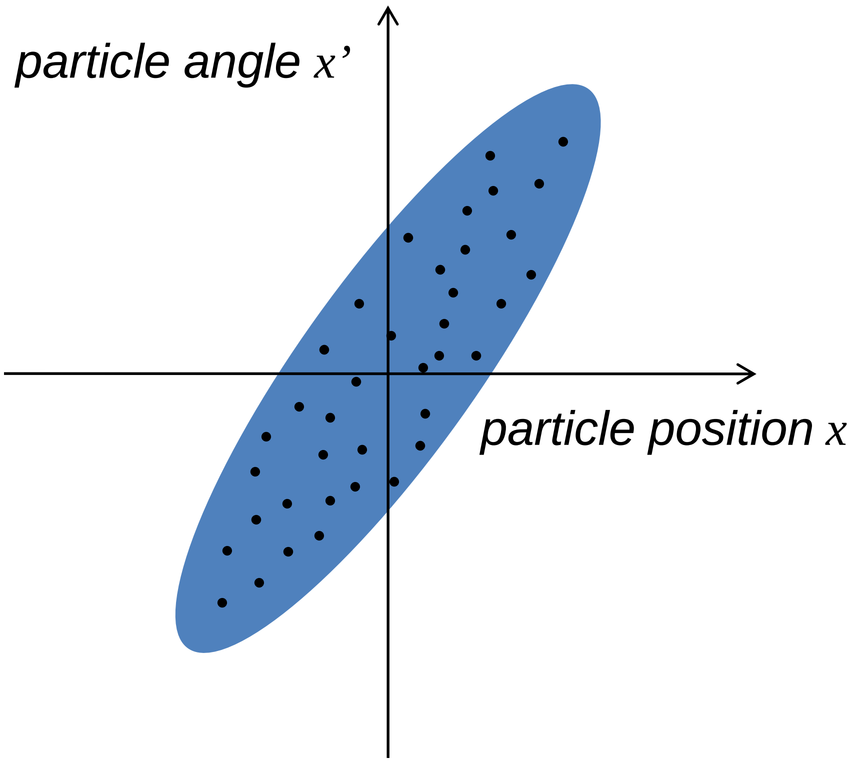
Emittance Diagnostics

- *Emittance, what is it ?*
- *Emittance, why measuring it ?*
- *How to measure it ?*
- *What can go wrong ?*

CAS Beam diagnostic
Dourdan, 2.6.2008

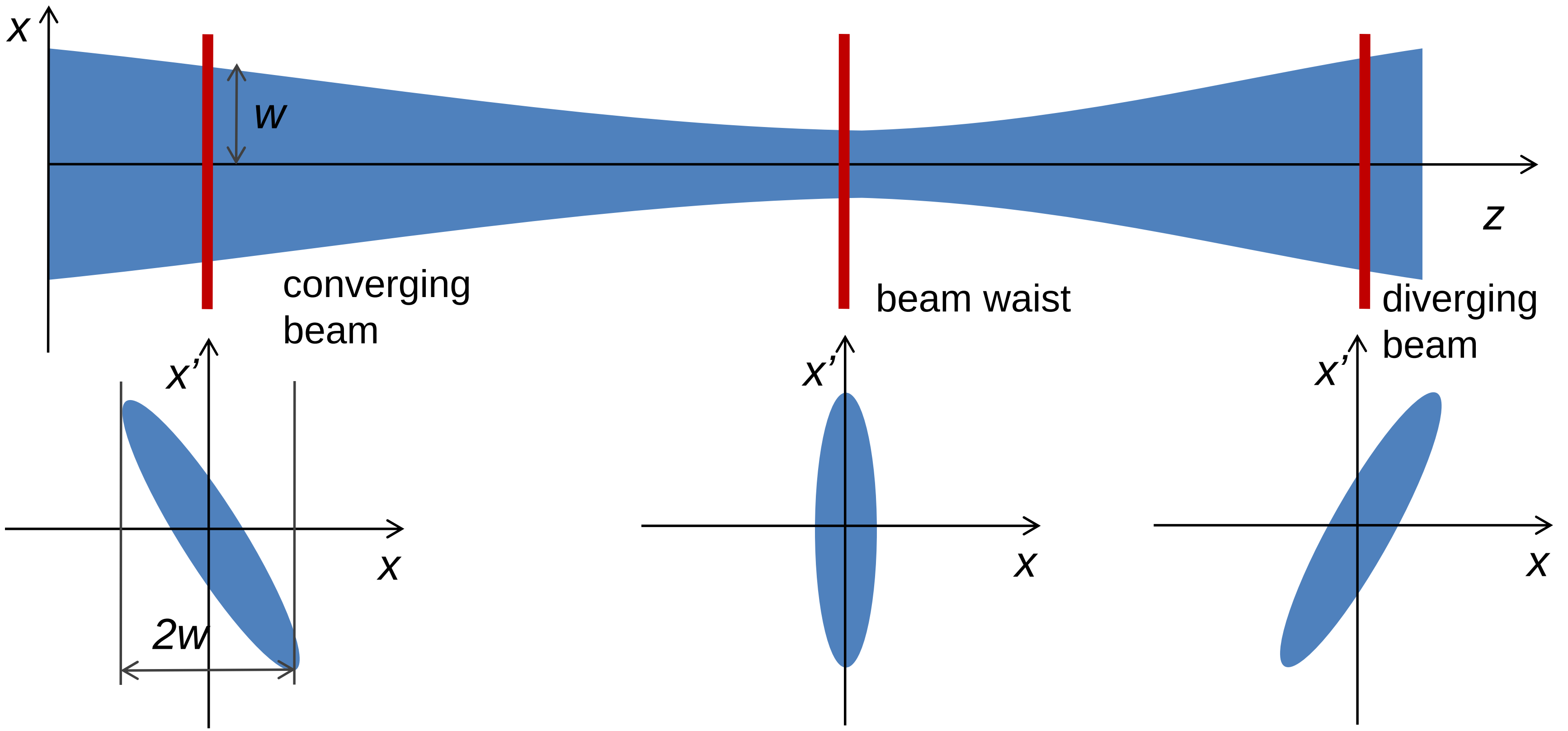
Hans Braun / CERN

Emittance, what is it ?



ε = Area in *x*, *x'* plane occupied by beam particles divided by π

Beam envelope along beamline



Along a beamline the orientation and aspect ratio of beam ellipse in *x, x'* plane varies, but area $\pi\varepsilon$ remains constant

Beam width along z is described with $w(z) = \sqrt{\beta(z)\,\varepsilon}$

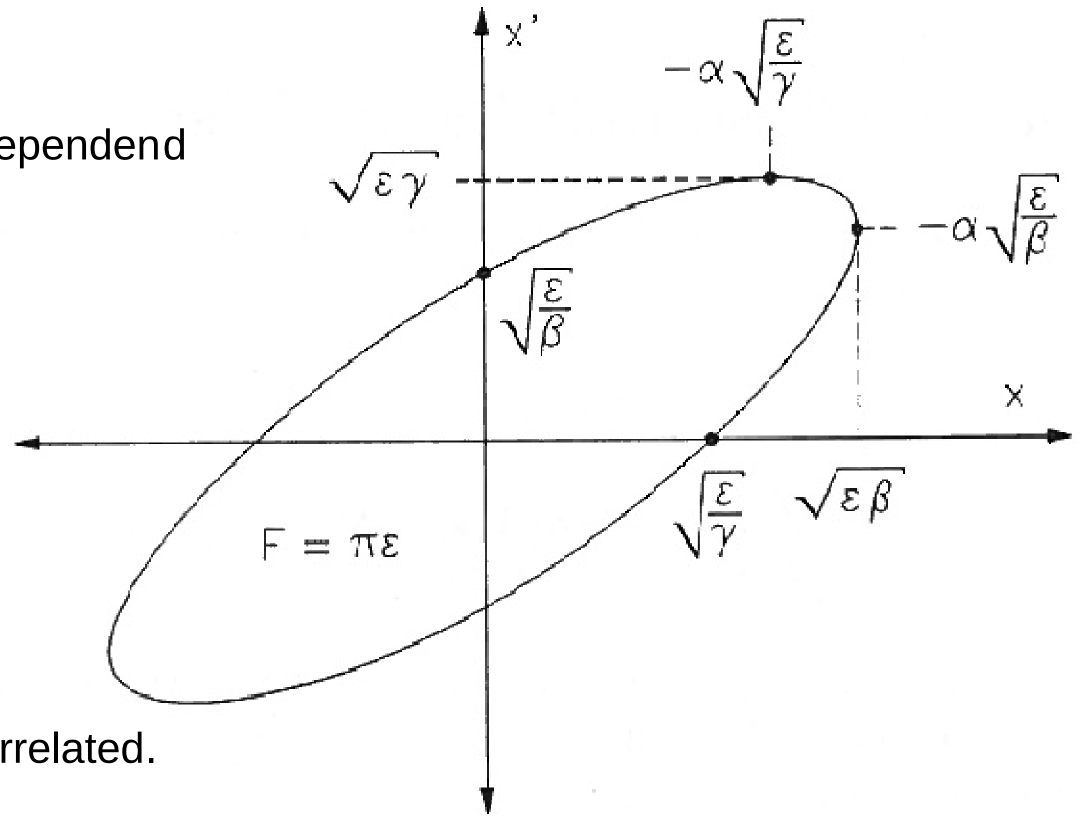
Beam ellipse and its orientation is described by 4 parameters $\varepsilon, \beta, \alpha, \gamma$
called Courant - Snyder or Twiss parameters

$$\varepsilon = \gamma x^2 + 2\,\alpha\; x\; x' + \beta\; x'^2$$

the three ellipse orientation parameters β, α, γ are connected by the relation

$$\gamma = \frac{1+\alpha^2}{\beta}$$

therefore only of these parameters are independend



$\sqrt{\beta\varepsilon}$ is the beam half width

$\sqrt{\gamma\varepsilon}$ is the beam half divergence

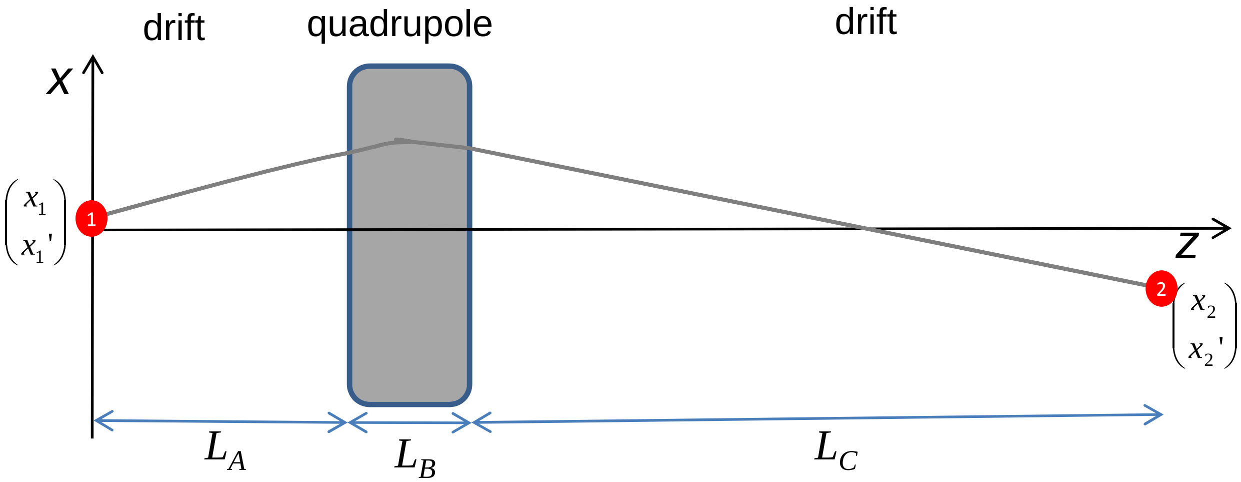
α describes how strong x and x' are correlated.

for $\alpha > 0$ beam is converging,

for $\alpha < 0$ beam is diverging.

for $\alpha = 0$ beam size has minimum (waist) or maximum (anti - waist)

Transport of single particle described with matrix algebra

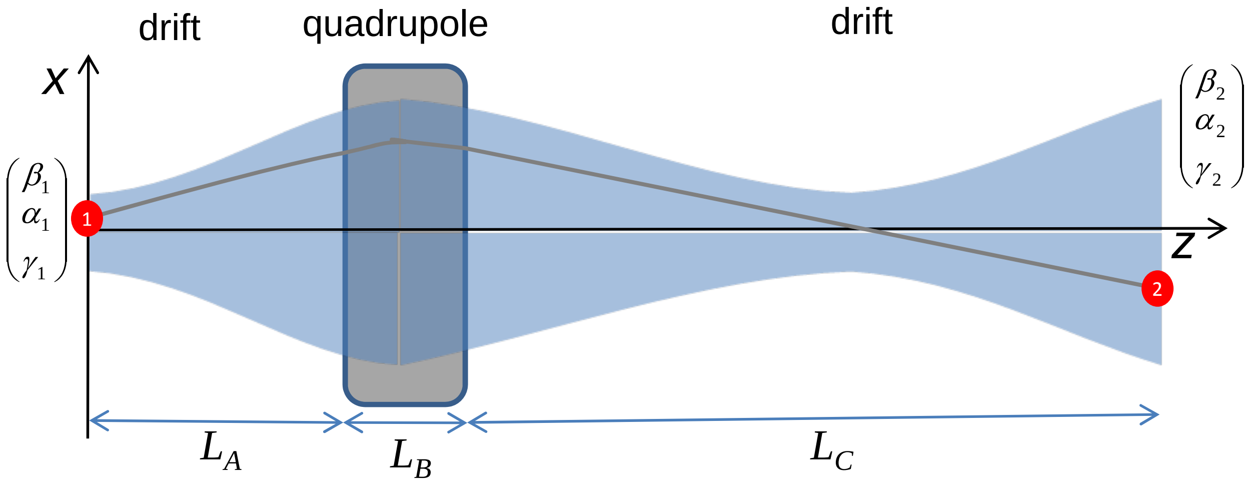


$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = M_C \cdot M_B \cdot M_A \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\sqrt{k} L_B\right) & 1/\sqrt{k} \sin\left(\sqrt{k} L_B\right) \\ -\sqrt{k} \sin\left(\sqrt{k} L_B\right) & \cos\left(\sqrt{k} L_B\right) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad M_{Quadrupole} = \begin{pmatrix} \cos\left(\sqrt{k} L\right) & 1/\sqrt{k} \sin\left(\sqrt{k} L\right) \\ -\sqrt{k} \sin\left(\sqrt{k} L\right) & \cos\left(\sqrt{k} L\right) \end{pmatrix}$$

generic names of matrix elements $\quad M = \begin{pmatrix} c & s \\ c' & s' \end{pmatrix}$

Transport of Twiss parameters β, α, γ of beam ellipse



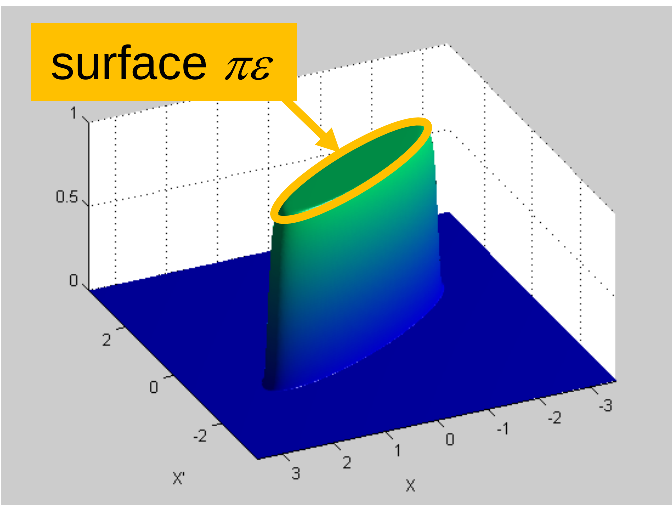
$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \cdot \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} c^2 & -2cs & s^2 \\ -cc' & cs'+c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix}$$

$$\beta_2 = c^2\, \beta_1 - 2cs\, \alpha_1 + s^2\, \gamma_1 \qquad | \cdot \varepsilon$$

$$\boxed{w_2^2 = c^2\, \beta_1 \varepsilon - 2cs\, \alpha_1 \varepsilon + s^2\, \gamma_1 \varepsilon}$$

So far we implicitly assumed that beam occupies
x, x' space area with a sharp boundary

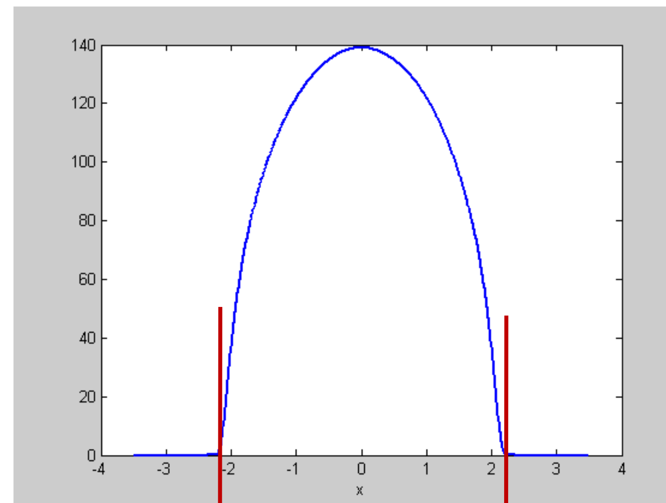
particle density in x, x' space



projection on x axis

$$f(x) = \int_{-\infty}^{\infty} \rho(x, x')\, dx'$$

transverse beam profile

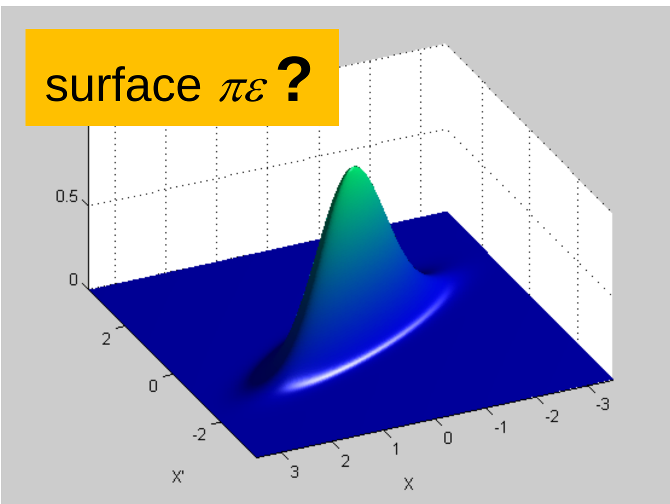


$2\ w$
beamwidth

$$w = \sqrt{\beta\varepsilon}$$

In reality beam density in x, x' space is rarely a constant on a well defined area but often like this

particle density in x, x' space



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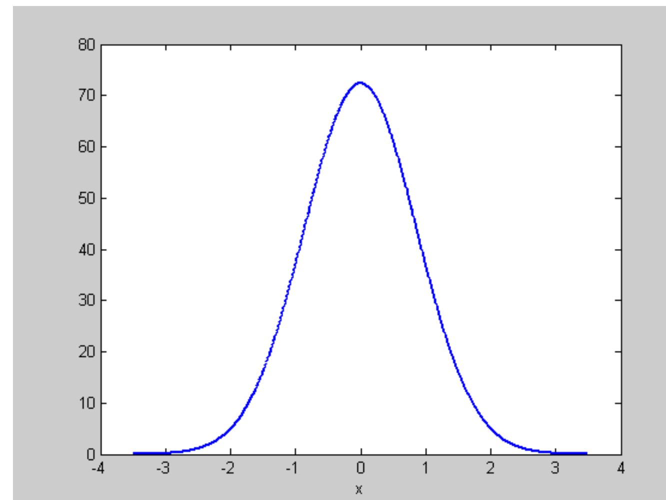


beamwidth ?

Definition of w_{RMS}

$$w_{RMS} = \sqrt{\frac{\int_{-\infty}^{+\infty} (x - x_{COG})^2 f(x)\, dx}{\int_{-\infty}^{+\infty} f(x)\, dx}}$$

$$x_{COG} = \frac{\int_{-\infty}^{+\infty} x \; f(x)\, dx}{\int_{-\infty}^{+\infty} f(x)\, dx}$$

Definition of w_{90}

$$\frac{\int_{-W_{90}}^{+W_{90}} f(x)\, dx}{\int_{-\infty}^{+\infty} f(x)\, dx} = 90\%$$

Commonly used definitions of emittance

ε_{rms} , 1 sigma r.m.s. emittance $\Leftrightarrow \quad w_{RMS} = \sqrt{\beta\varepsilon}$

ε_{2rms} , 2 sigma r.m.s. emittance $\Leftrightarrow \quad w_{RMS} = \dfrac{\sqrt{\beta\varepsilon}}{2}$

$\varepsilon_{90\%}$, 90% emittance $\Leftrightarrow \quad w_{90} = \sqrt{\beta\varepsilon}$

Relations between different emittance definitions

Always $\varepsilon_{2rms} = 4\ \varepsilon_{rms}$

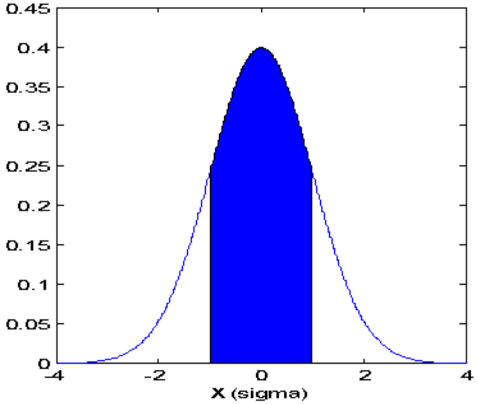
For Gauss beam distribution only

$$\varepsilon_{90\%} = 2.7055\ \varepsilon_{rms}$$

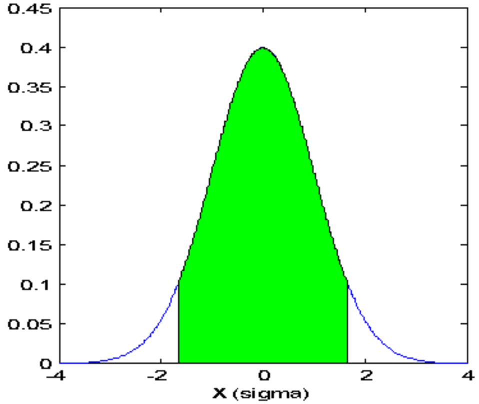
$$w_{rms} = \sigma_X$$

$$w_{90\%} = 1.6449\ \sigma_X$$

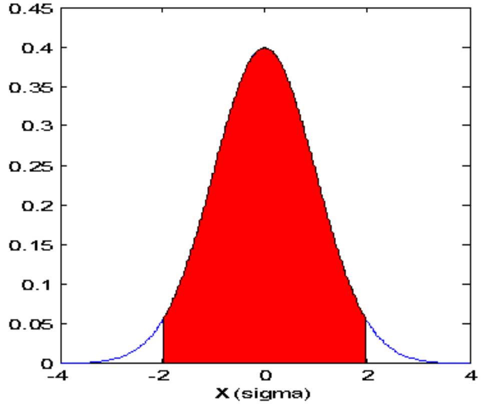
ε_{rms}, 68.3% of beam



$\varepsilon_{90\%}$, 90% of beam

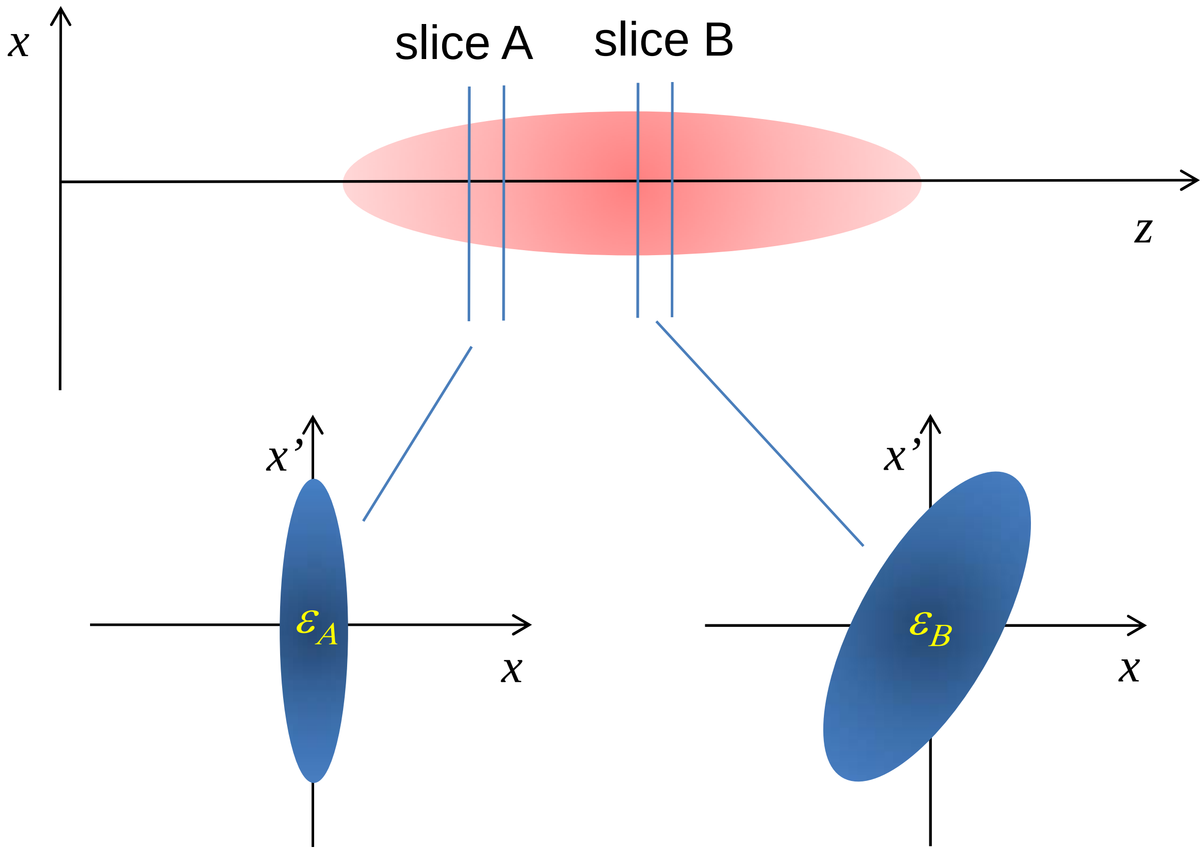


ε_{2rms}, 95.5% of beam

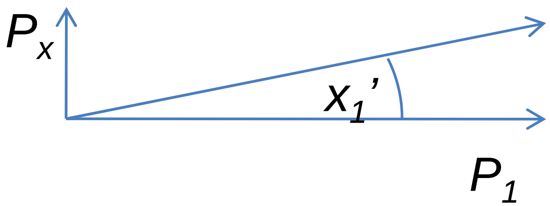


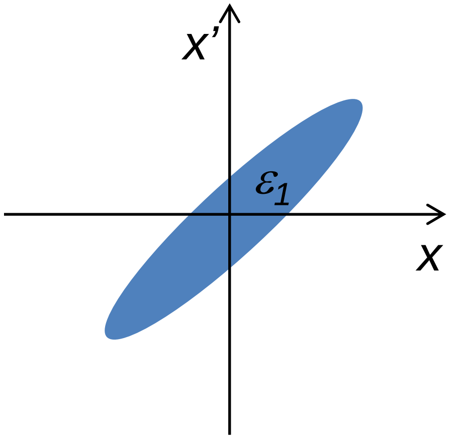
Slice emittance

important for free electron lasers !



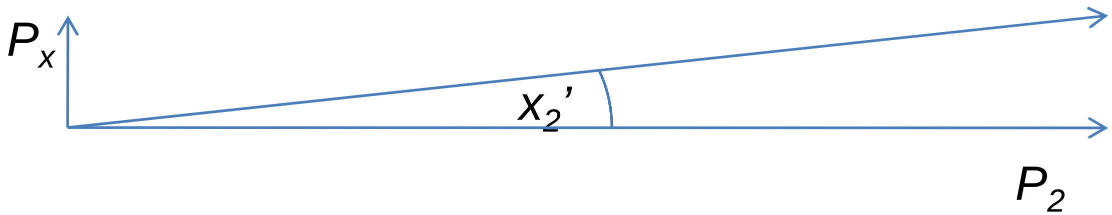
Emittance is only constant in beamlines without acceleration

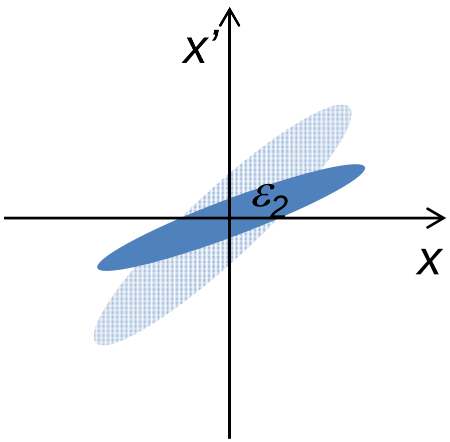




acceleration

$$\varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m\beta_{rel}\gamma_{rel}\ c$$





<u>normalised emittance</u> $\varepsilon_N = \beta_{rel}\gamma_{rel}\ \varepsilon$ preserved with acceleration !

To distinguish from normalised emittance ε_N , ε is quoted as “geometric emittance” !

Commonly used units for emittance

mm·mrad, m·rad, μm, m, nm

1 mm·mrad=10^{-6} m·rad=1μm=10^{-6} m=10^{3}nm

Often a π is added to the unit to indicate that the numerical value describes a surface in x, x' space divided by π, i.e. 1 π·mm·mrad

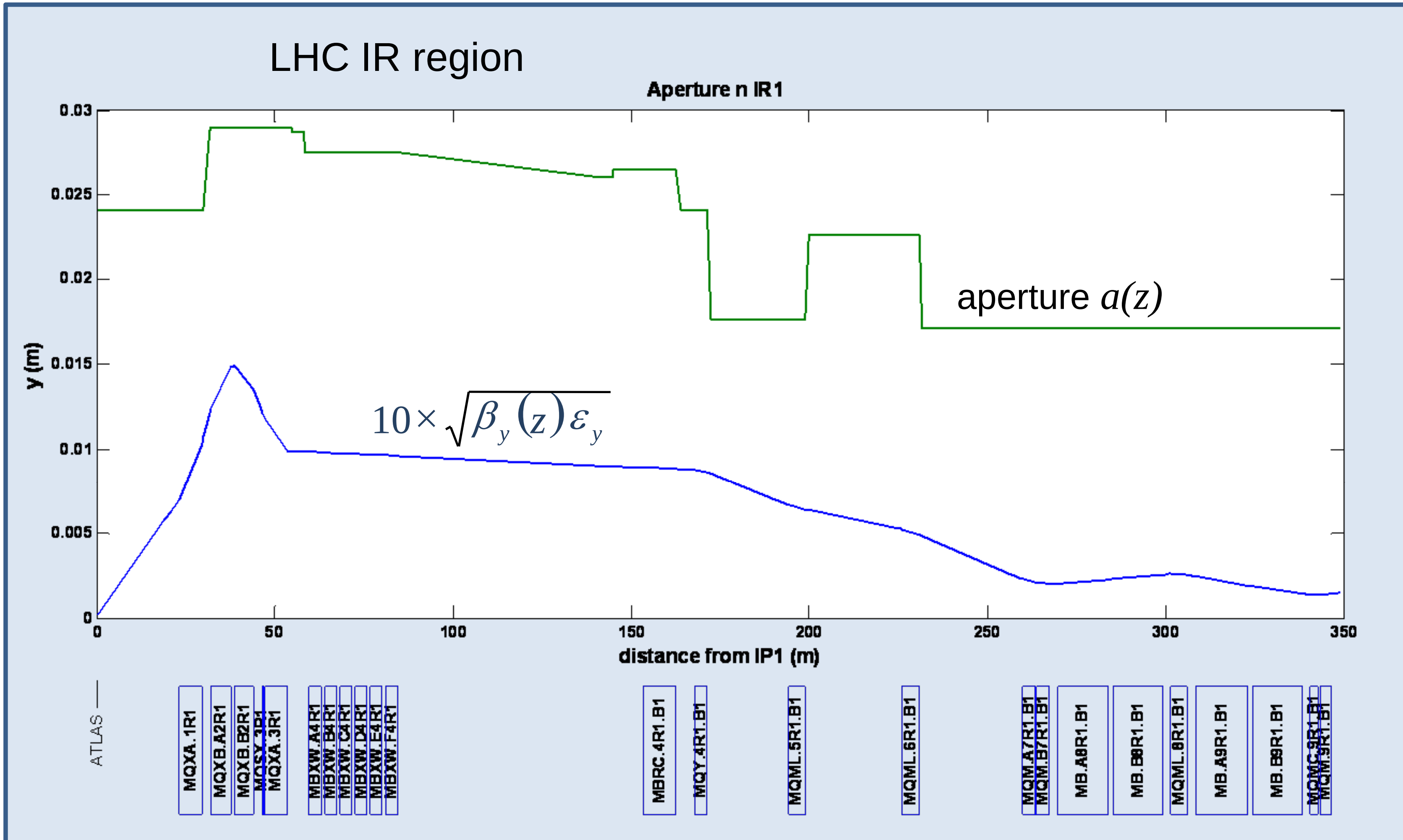
The units for normalised emittance are the same as for geometric emittance

Due to the various definitions it is recommended to always mention the emittance definition used when reporting measurement values !

Emittance, why measuring it ?

The emittance tells if a beam fits in the vacuum chamber or not

$$w(z) = \sqrt{\beta(z)\,\varepsilon} \quad < \quad a(z)$$



Emittance is one of key parameters for overall performance of an accelerator

- *Luminosity of colliders for particle physics*
- *Brightness of synchrotron radiation sources*
- *Wavelength range of free electron lasers*
- *Resolution of fixed target experiments*

Therefore emittance measurement is essential to guide tune-up of accelerator !

Measurement of emittance is closely linked with measurement of Twiss parameter β, α, γ .

- *Provide initial beam conditions for a beamline, thus allowing to compute optimum setting of focusing magnets in a beamline. Important to minimise beam losses in transfer line !*

- *Measurement of Twiss parameters at injection in synchrotron or storage ring allows to "match" Twiss parameters to circulating beam. This helps to minimise injection losses and to avoid emittance growth due to filamentation of unmatched beam.*

- *Measurement of emittance as function of time in synchrotrons and storage rings allows to identify, understand and possibly mitigate mechanism of emittance growth.*

- *Measurement of emittance at different locations in a beam line or in a linear accelerator allows to verify and/or improve beam optics model of this line. Moreover, it allows to identify, understand and possibly mitigate mechanism of emittance growth.*

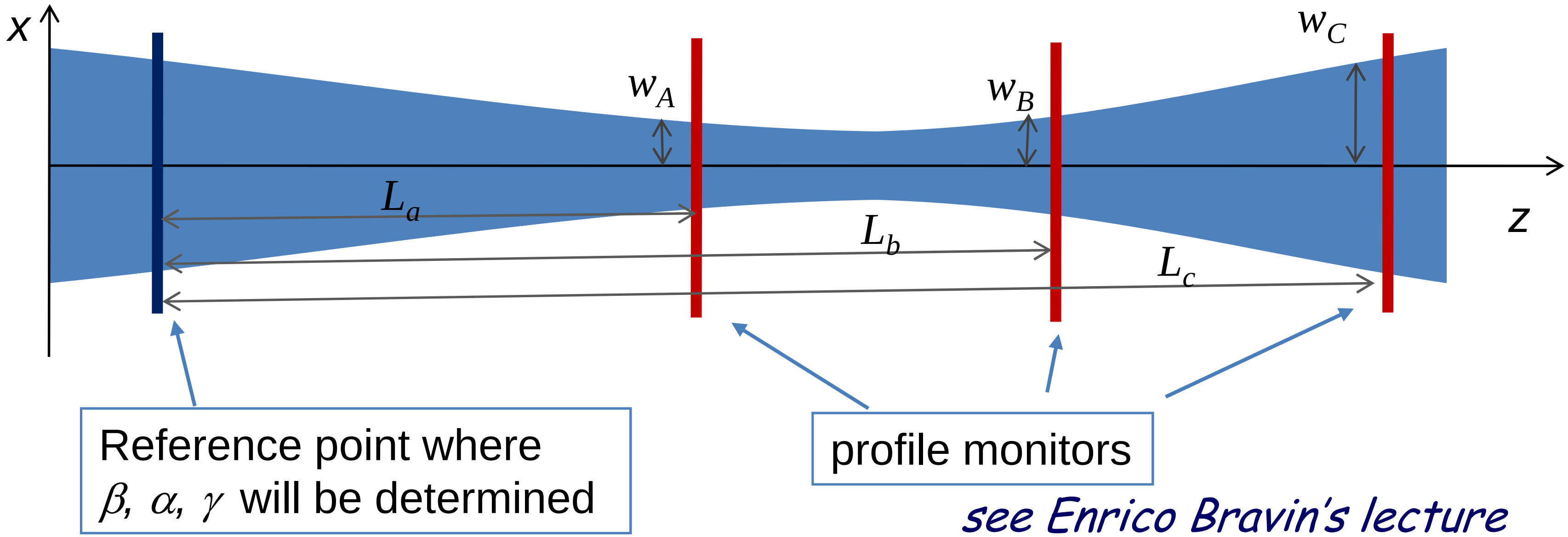
Emittance, how to measure it ?

- Methods based on transverse beam profile measurements
- Slit and pepperpot methods
- Methods based on Schottky signal analysis
  $\Rightarrow$ F. Caspers Lecture, June 3rd
- Direct methods to measure transverse profiles and beam divergence
  $\Rightarrow$ see literature list

<u>Emittance measurement in transfer line or linac</u>

Twiss parameter β, α, γ are a priori not known, they have to be determined together with ε.

Method A



see Enrico Bravin's lecture

$$w^2 = c^2\ \beta\,\varepsilon - 2\,c\,s\ \alpha\ \varepsilon + s^2\ \gamma\,\varepsilon, \text{ for drift } \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad w^2 = \beta\,\varepsilon - 2\ L\ \alpha\ \varepsilon + L^2\ \gamma\,\varepsilon$$

Derivation of emittance and Twiss parameters from beam width measurements

$$w_A^2 = \beta\,\varepsilon - 2\;L_A\;\alpha\;\varepsilon + L_A^2\;\gamma\,\varepsilon$$

$$w_B^2 = \beta\,\varepsilon - 2\;L_B\;\alpha\;\varepsilon + L_B^2\;\gamma\,\varepsilon$$

$$w_C^2 = \beta\,\varepsilon - 2\;L_C\;\alpha\;\varepsilon + L_C^2\;\gamma\,\varepsilon$$

$\Downarrow$ can be rewritten in Matrix notation

$$\begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} 1 & -2L_A & L_A^2 \\ 1 & -2L_B & L_B^2 \\ 1 & -2L_C & L_C^2 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 1 & -2L_A & L_A^2 \\ 1 & -2L_B & L_B^2 \\ 1 & -2L_C & L_C^2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}$$

$$\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta \cdot \gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Emittance measurements often require inversion of 3x3 Matrices.
Many Math and spreadsheet programs and programming languages have functions for this.

If you insist to code it yourself you can use :

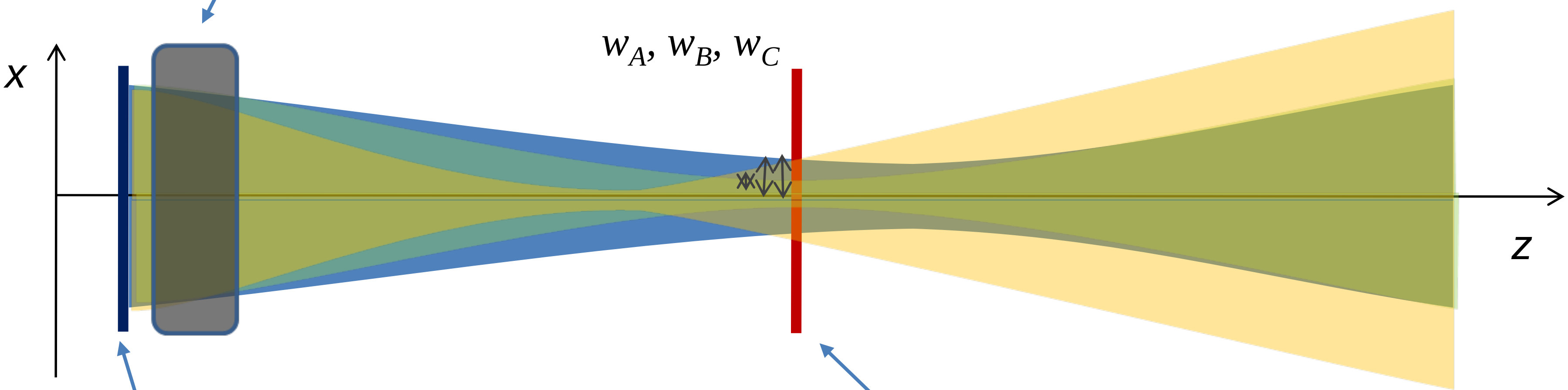
The inverse of a 3x3 matrix:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}^{-1} = 1/DET * \begin{vmatrix} a_{33}a_{22}-a_{32}a_{23} & a_{32}a_{13}-a_{33}a_{12} & a_{23}a_{12}-a_{22}a_{13} \\ a_{31}a_{23}-a_{33}a_{21} & a_{33}a_{11}-a_{31}a_{13} & a_{21}a_{13}-a_{23}a_{11} \\ a_{32}a_{21}-a_{31}a_{22} & a_{32}a_{11}-a_{31}a_{12} & a_{22}a_{11}-a_{21}a_{12} \end{vmatrix}$$

with $DET = a_{11}(a_{33}a_{22}-a_{32}a_{23})-a_{21}(a_{33}a_{12}-a_{32}a_{13})+a_{31}(a_{23}a_{12}-a_{22}a_{13})$

Emittance measurement in transfer line or linac, *Method B*

Adjustable magnetic lens with settings A,B,C
(quadrupole magnet, solenoid, system of quadrupole magnets…)



Reference point where
β, α, γ will be determined

profile monitor

$$w^2 = c^2\,\beta\varepsilon - 2cs\,\alpha\,\varepsilon + s^2\,\gamma\,\varepsilon, \qquad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

Derivation of emittance and Twiss parameters from beam width measurements

$$w_A^2 = c_A^2 \beta\,\varepsilon - 2\,c_A s_A\ \alpha\ \varepsilon + s_A^2\ \gamma\,\varepsilon$$
$$w_B^2 = c_B^2 \beta\,\varepsilon - 2\,c_B s_B\ \alpha\ \varepsilon + s_B^2\ \gamma\,\varepsilon$$
$$w_C^2 = c_C^2 \beta\,\varepsilon - 2\,c_C s_C\ \alpha\ \varepsilon + s_C^2\ \gamma\,\varepsilon$$

$\Downarrow$ can be rewritten in Matrix notation

$$\begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} c_A^2 & -2c_A s_A & s_A^2 \\ c_B^2 & -2c_B s_B & s_B^2 \\ c_C^2 & -2c_C s_C & s_C^2 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} c_A^2 & -2c_A s_A & s_A^2 \\ c_B^2 & -2c_B s_B & s_B^2 \\ c_C^2 & -2c_C s_C & s_C^2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}$$

$$\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2 \left(\beta \cdot \gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

To determine $\varepsilon, \beta, \alpha$ at a reference point in a beamline one needs at least three w measurements with different transfer matrices between the reference point and the w measurements location.

Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.

Three w measurements are in principle enough to determine $\varepsilon, \beta, \alpha$
In practice better results are obtained with more measurements.
However, with more than three measurements the problem is over-determined.
χ^2 formalism gives the best estimate of $\varepsilon, \beta, \alpha$
for a set of n measurements w_i , i=1-n with transfer matrix elements c_i, s_i .

χ^2 formalism to determine ε, β, α

Measured half beam width w_i

Predicted half beam width $\sqrt{c_i^2\beta\varepsilon - 2c_i s_i \alpha\varepsilon + s_i^2\gamma\varepsilon}$,
but $\beta\varepsilon, \alpha\varepsilon, \gamma\varepsilon$ a priori not known

Find set of $\beta\varepsilon, \alpha\varepsilon, \gamma\varepsilon$ values, which minimises

$$\chi^2 = \sum_{i=1}^{n}\left(c_i^2\beta\varepsilon - 2c_i s_i\alpha\varepsilon + s_i^2\gamma\varepsilon - w_i^{\,2}\right)^2$$

Conditions for χ^2 minimum

$$\frac{\partial\chi^2}{\partial\beta\varepsilon} = 0 \quad\Rightarrow\quad \beta\varepsilon\sum_{i=1}^{n} c_i^4 \quad - 2\alpha\varepsilon\sum_{i=1}^{n} c_i^3 s_i + \gamma\varepsilon\sum_{i=1}^{n} c_i^2 s_i^2 = \sum_{i=1}^{n} c_i^2 w_i^{\,2}$$

$$\frac{\partial\chi^2}{\partial\alpha\varepsilon} = 0 \quad\Rightarrow\quad \beta\varepsilon\sum_{i=1}^{n} c_i^3 s_i - 2\alpha\varepsilon\sum_{i=1}^{n} c_i^2 s_i^2 + \gamma\varepsilon\sum_{i=1}^{n} c_i s_i^3 = \sum_{i=1}^{n} c_i s_i w_i^{\,2}$$

$$\frac{\partial\chi^2}{\partial\gamma\varepsilon} = 0 \quad\Rightarrow\quad \beta\varepsilon\sum_{i=1}^{n} c_i^2 s_i^2 - 2\alpha\varepsilon\sum_{i=1}^{n} c_i s_i^3 + \gamma\varepsilon\sum_{i=1}^{n} s_i^4 \quad = \sum_{i=1}^{n} c_i^2 w_i^{\,2}$$

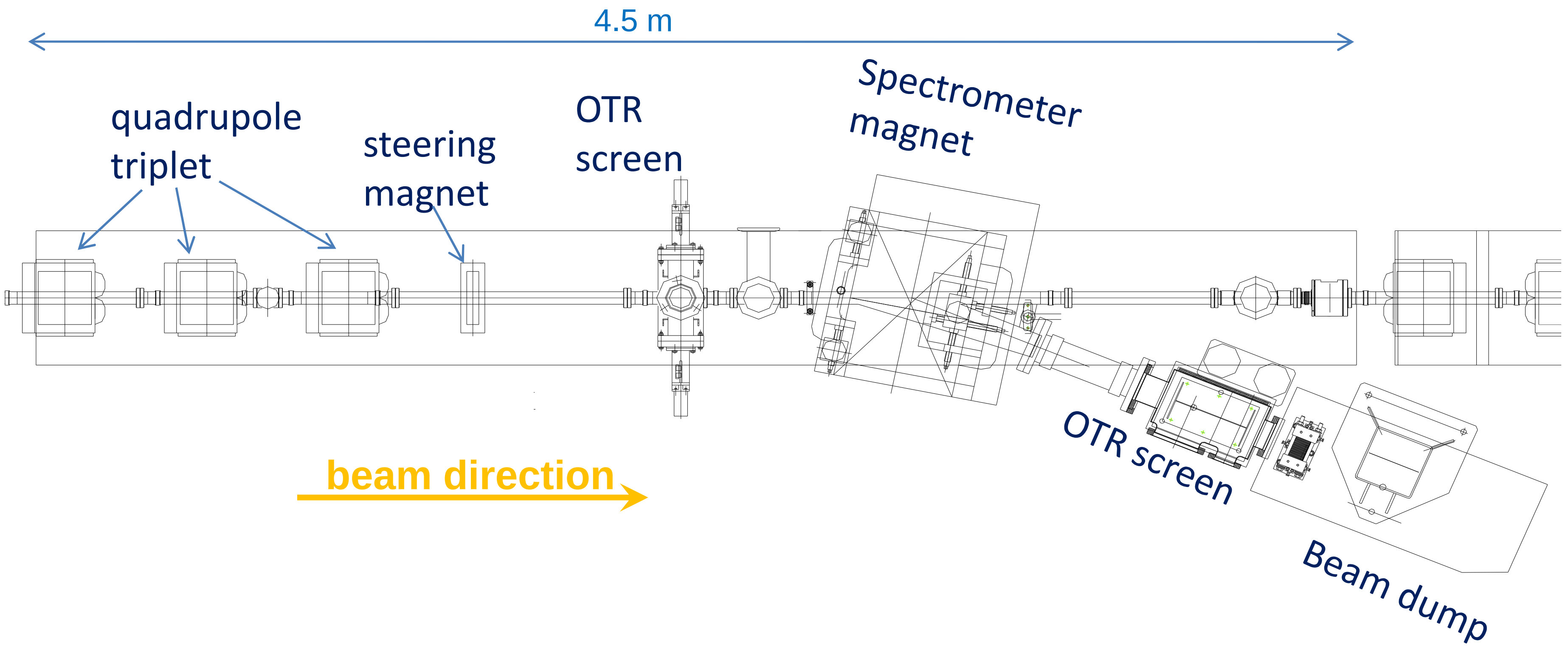
These conditions can be rewritten in matrix notation

$$\begin{pmatrix} \sum_{i=1}^{n} c_i^4 & -2\sum_{i=1}^{n} c_i^3 s_i & \sum_{i=1}^{n} c_i^2 s_i^2 \\ \sum_{i=1}^{n} c_i^3 s_i & -2\sum_{i=1}^{n} c_i^2 s_i^2 & \sum_{i=1}^{n} c_i s_i^3 \\ \sum_{i=1}^{n} c_i^2 s_i^2 & -2\sum_{i=1}^{n} c_i s_i^3 & \sum_{i=1}^{n} s_i^4 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} c_i^2 w_i^{\,2} \\ \sum_{i=1}^{n} c_i s_i w_i^{\,2} \\ \sum_{i=1}^{n} c_i^2 w_i^{\,2} \end{pmatrix},$$

$$\begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} c_i^4 & -2\sum_{i=1}^{n} c_i^3 s_i & \sum_{i=1}^{n} c_i^2 s_i^2 \\ \sum_{i=1}^{n} c_i^3 s_i & -2\sum_{i=1}^{n} c_i^2 s_i^2 & \sum_{i=1}^{n} c_i s_i^3 \\ \sum_{i=1}^{n} c_i^2 s_i^2 & -2\sum_{i=1}^{n} c_i s_i^3 & \sum_{i=1}^{n} s_i^4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \sum_{i=1}^{n} c_i^2 w_i^{\,2} \\ \sum_{i=1}^{n} c_i s_i w_i^{\,2} \\ \sum_{i=1}^{n} c_i^2 w_i^{\,2} \end{pmatrix}$$

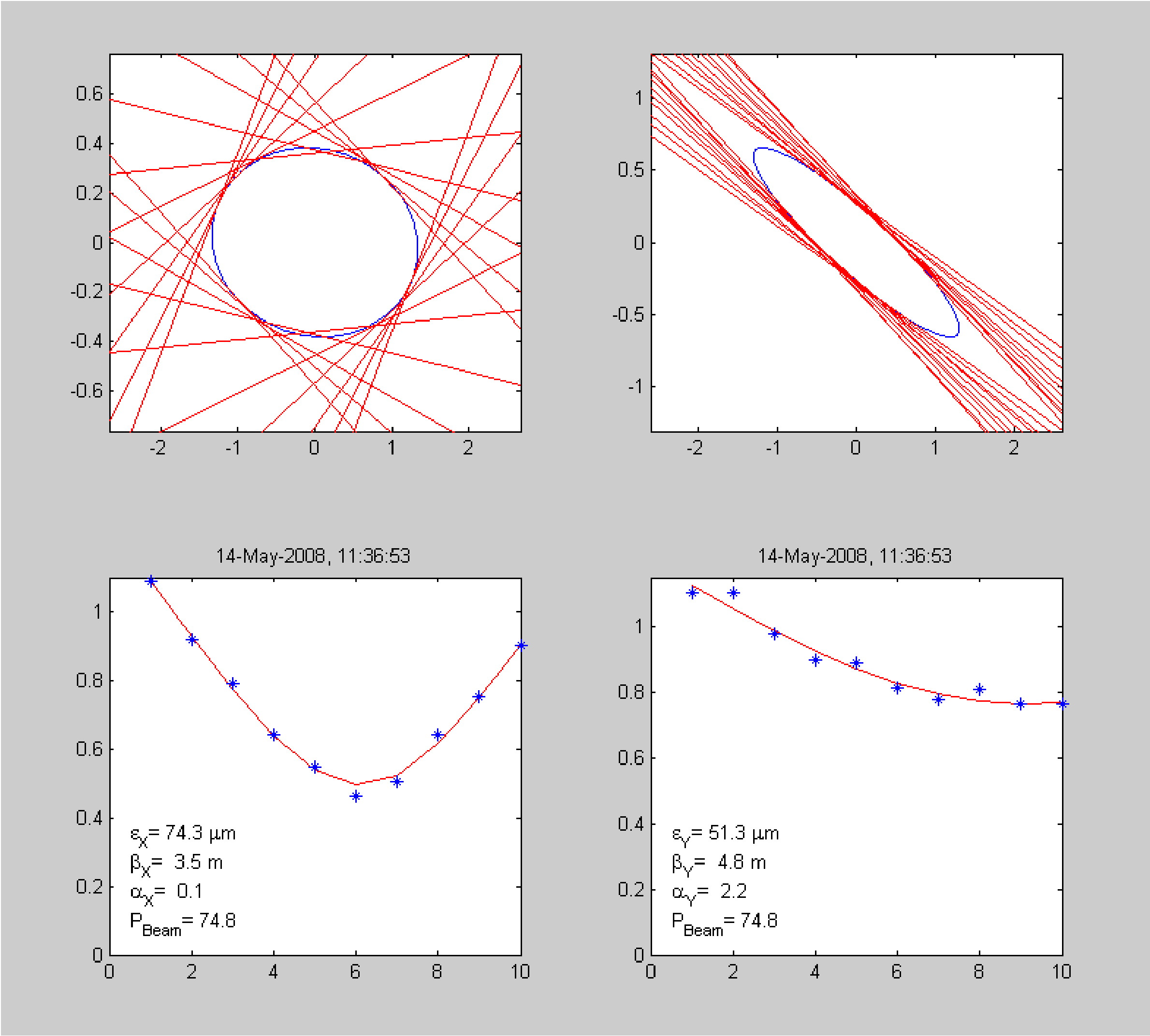
$$\sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Emittance measurement insertion in CTF3 linac

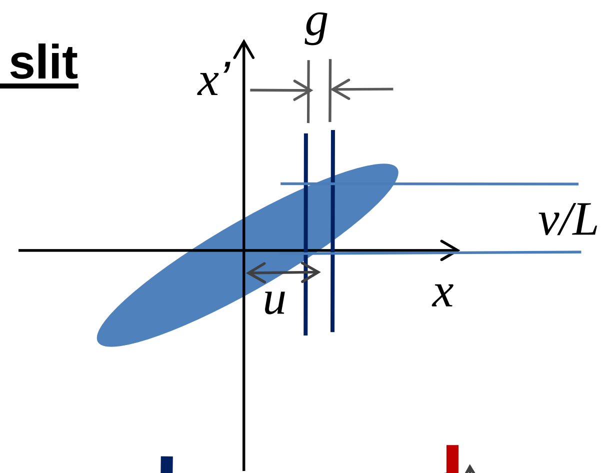


Girder 10

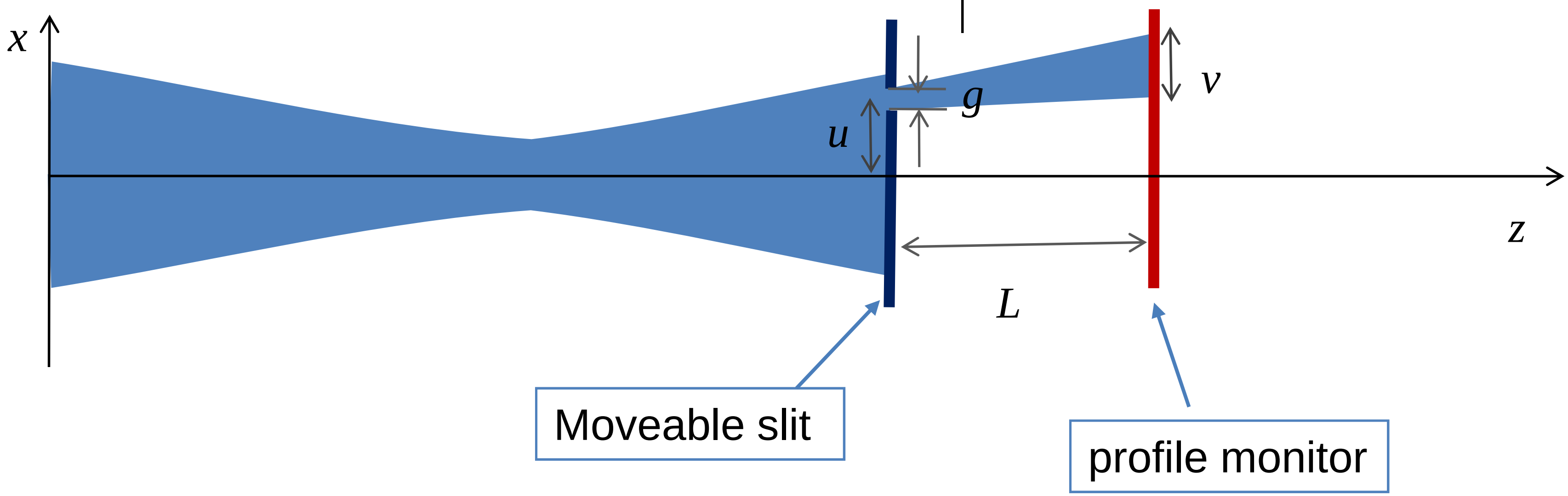
Example of ε measurement in CTF3 linac



Emittance measurement with moveable slit



Intensity at u, x' in x, x'-space is mapped to position $u + x' \cdot L$ on screen



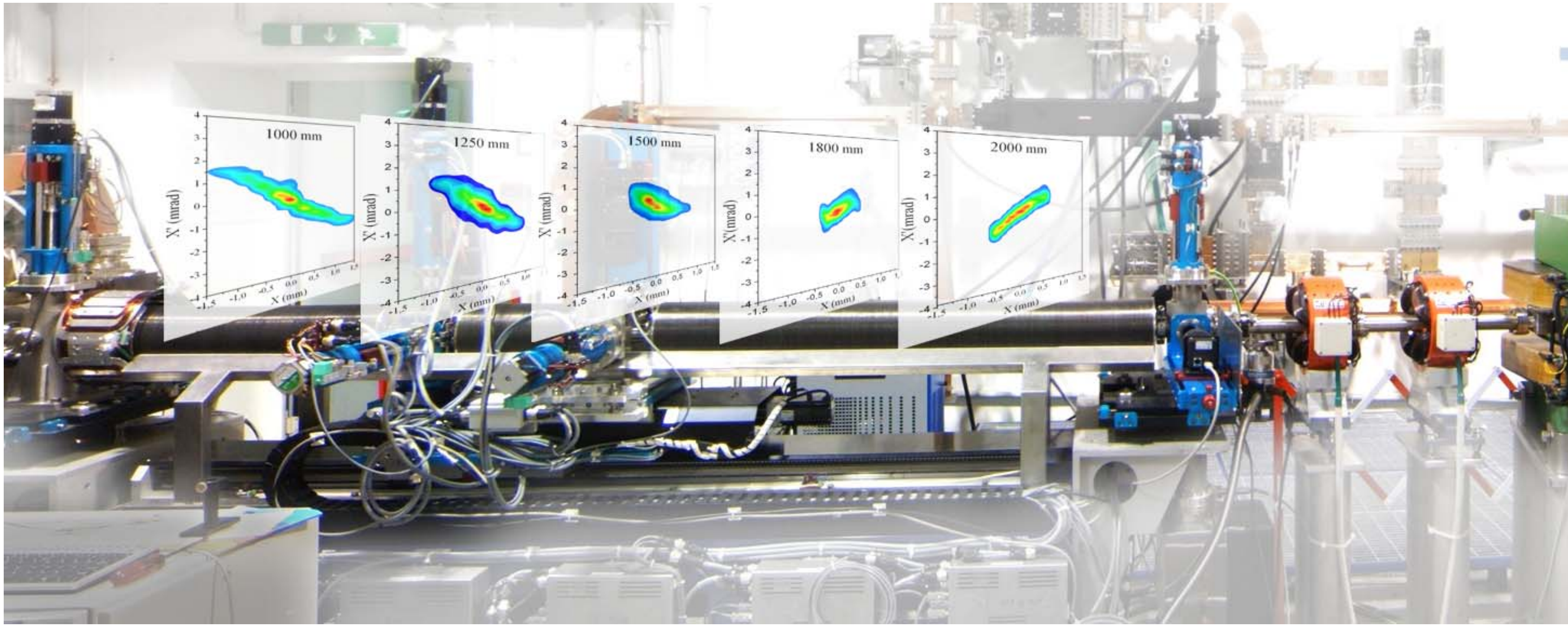
From width and position of slit image mean beam angle and divergence of slice at position u is readily computed.

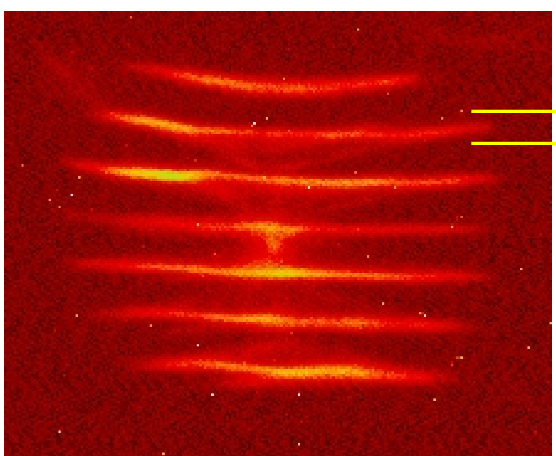
By moving slit across the beam complete distribution in x, x' space is reconstructed.

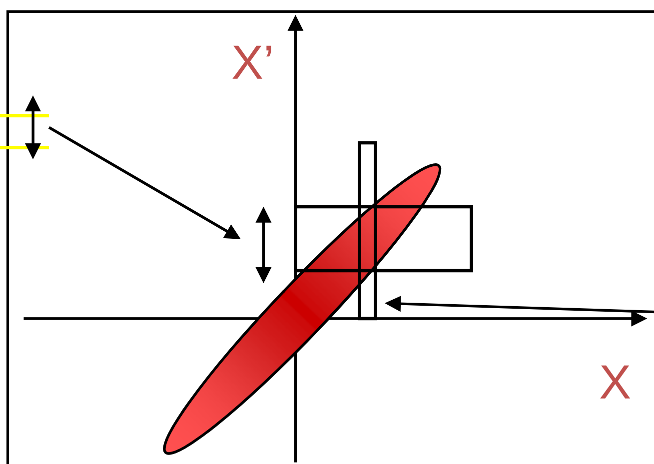
Conditions for good resolution: $v >> s$

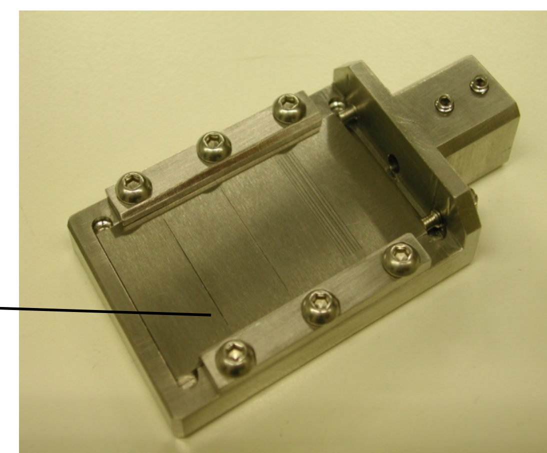
Phase space reconstruction at SPARC

courtesy of Massimo Ferrario / LNF

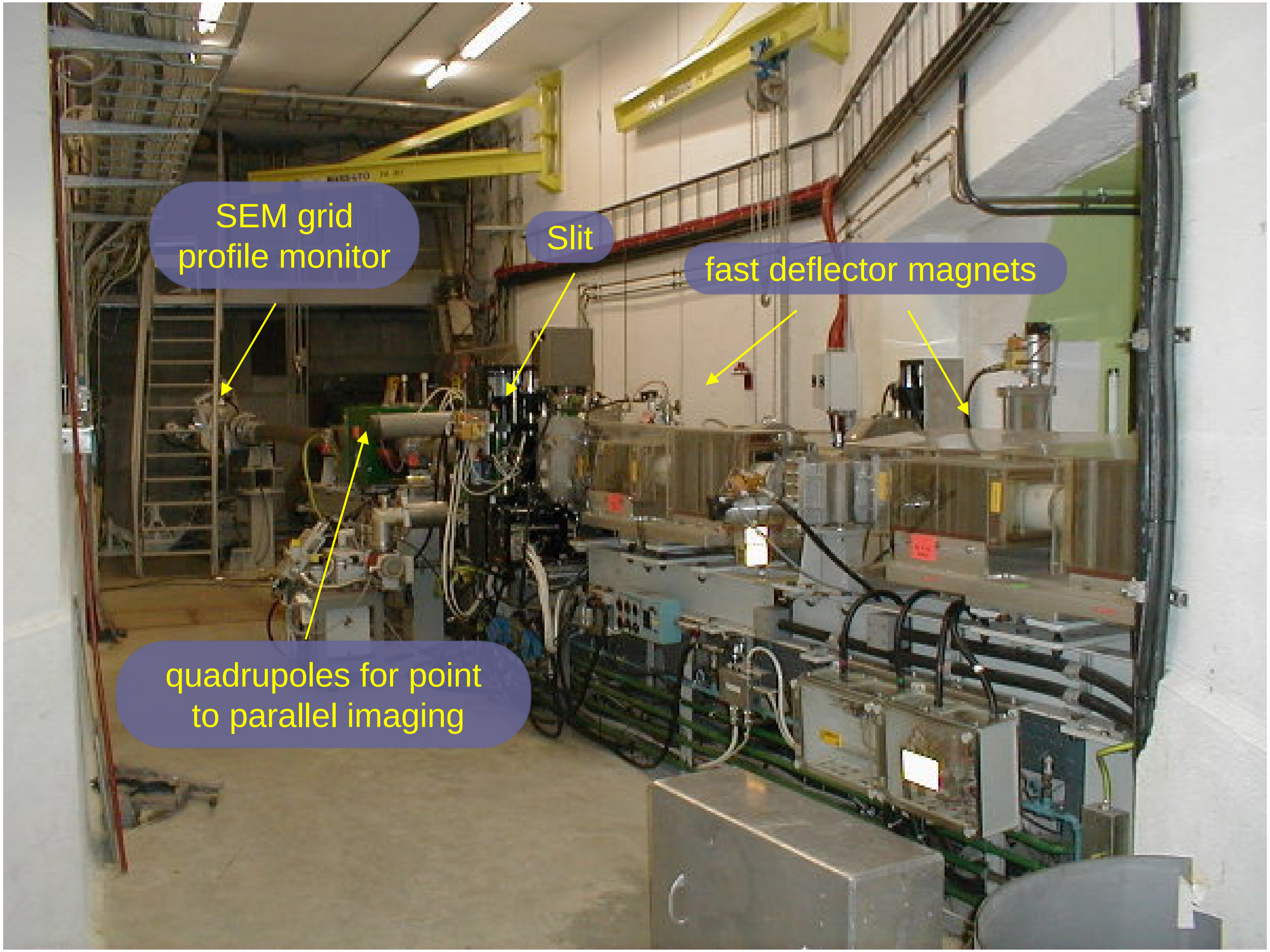




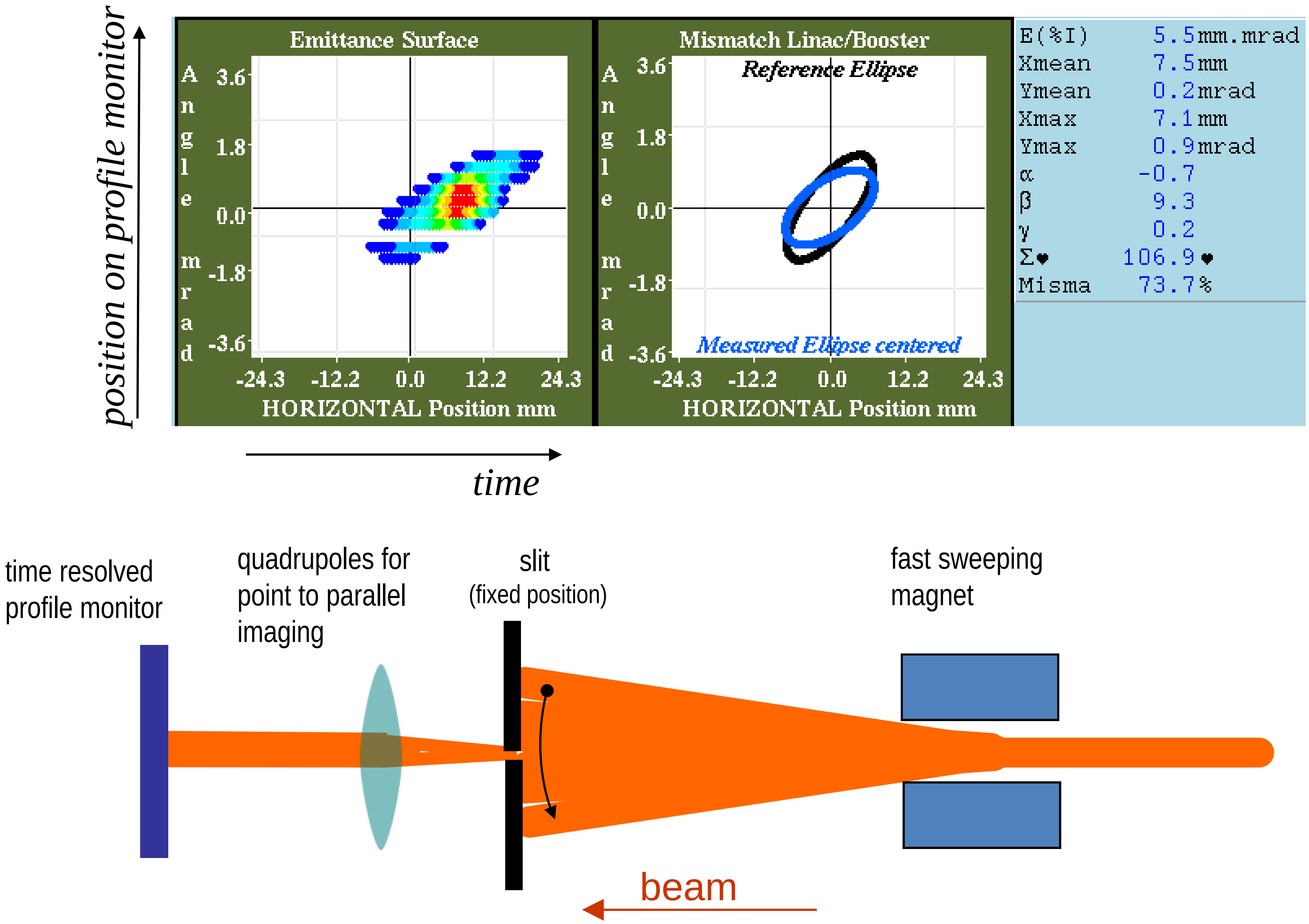




Single shot emittance measurement for 50 MeV proton beam at LINAC II / CERN



Single shot emittance measurement
for 50 MeV proton beam at LINAC II / CERN



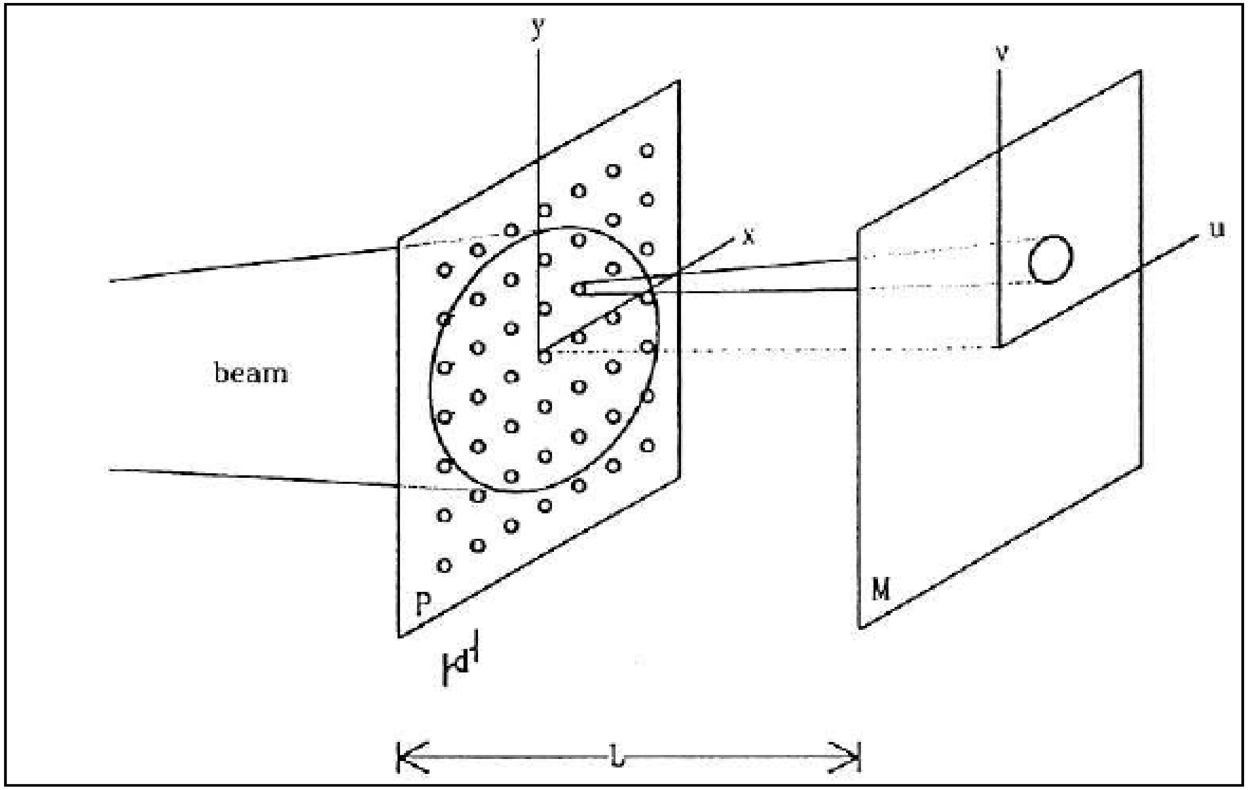
Pepperpot

similar to moving slit method

+ allows measurement in both planes simultaneously

- pepperpot dimensions have to be well adapted to beam conditions

hole spacing << beam widths
hole image >> hole diameter
hole image < hole spacing



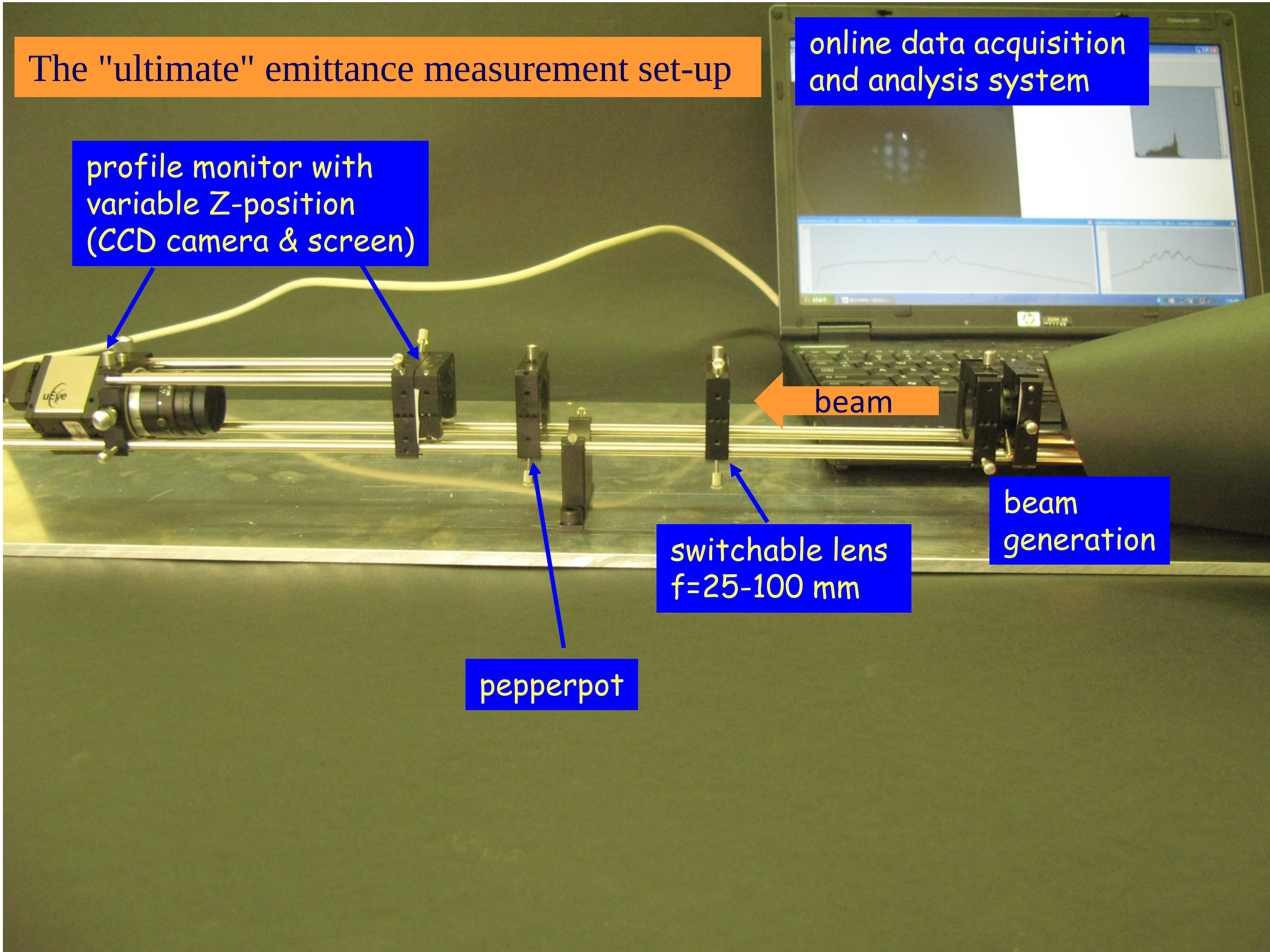
Where to use what ?

quadrupole scan for beamlines where linear beam optics is valid,
i.e. where space charge forces are small and energy spread is not too large

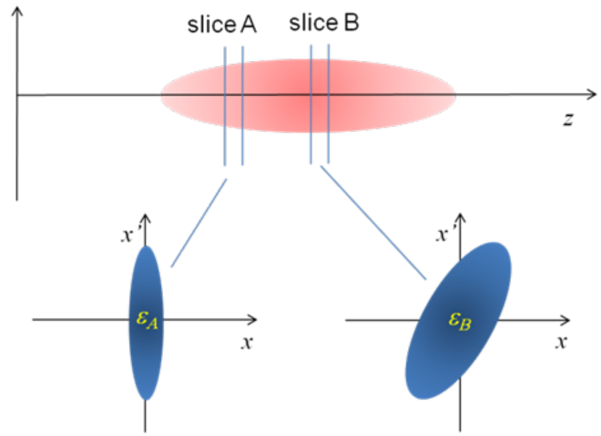
Slit / pepper-pot for low energy beams,
where beam can be stopped in slit / pepperpot mask

	high energy	high space charge forces	large energy spread
slit / pepperpot	-	+	+
quadrupole scan	+	-	-
drift with several monitors	+	-	+

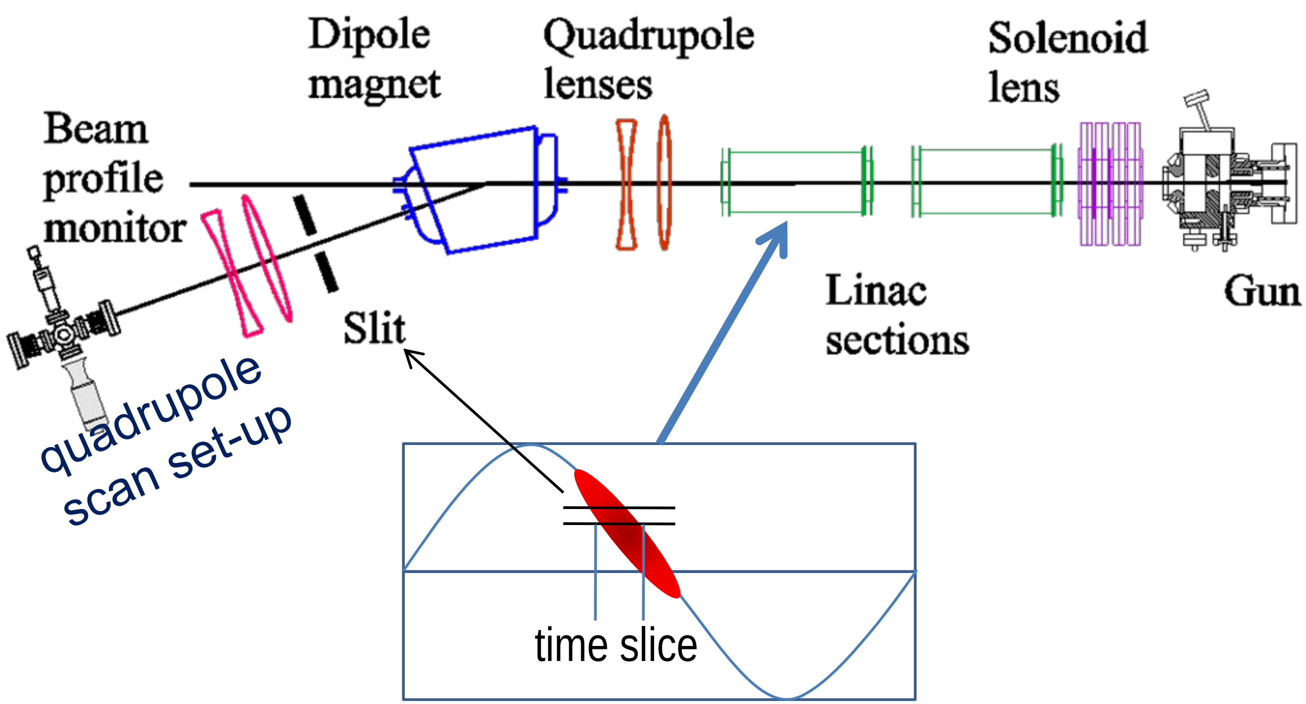
The "ultimate" emittance measurement set-up



Slice emittance measurement

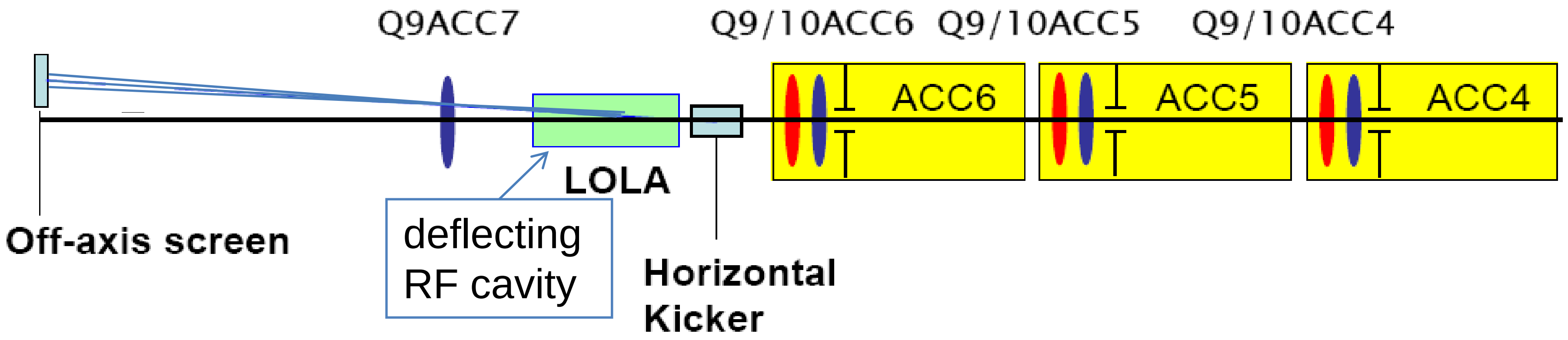


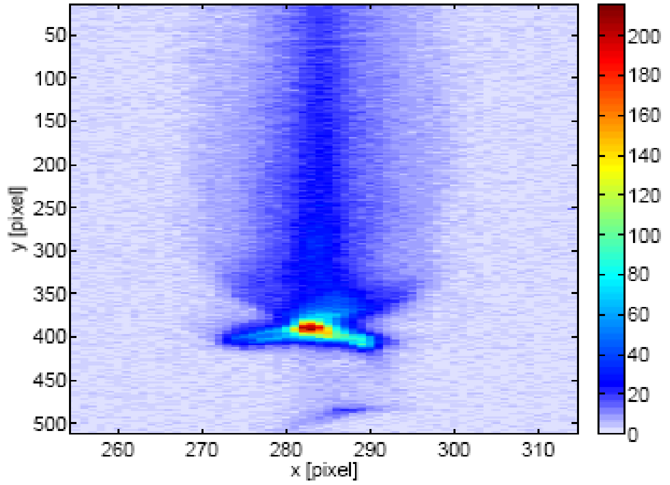
ATF at BNL



off crest acceleration

Slice emittance set-up at FLASH / DESY





Emittance measurement in storage ring or synchrotron

Beam profile measurement at one location of ring is sufficient to determine ε

$$M(z, z+C) = \begin{pmatrix} c & s \\ c' & s' \end{pmatrix}$$

is single particle transfer matrix for one ring revolution starting at profile monitor positon z.

$M(z, z+C)$ can be calculated from magnet parameters and position.

Twiss functions must be periodic with Circumference

$$\Rightarrow \quad \begin{pmatrix} \beta(z) \\ \alpha(z) \\ \gamma(z) \end{pmatrix} = \begin{pmatrix} \beta(z+C) \\ \alpha(z+C) \\ \gamma(z+C) \end{pmatrix} = \begin{pmatrix} c^2 & -2cs & s^2 \\ -cc' & cs'+c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta(z) \\ \alpha(z) \\ \gamma(z) \end{pmatrix}$$

Therefore $\begin{pmatrix} \beta(z) \\ \alpha(z) \\ \gamma(z) \end{pmatrix}$ is Eigenvector of matrix $\begin{pmatrix} c^2 & -2cs & s^2 \\ -cc' & cs'+c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{pmatrix}$.

Solving for the Eigenvector gives $\beta(z) = \dfrac{2s}{\sqrt{(2-c-s')(2+c+s')}}$

Measured beam halfwidth $\quad w = \sqrt{\beta(z)\,\varepsilon} \quad \Rightarrow \quad \boxed{\varepsilon = \dfrac{w^2}{\beta(z)}}$

Complication in storage ring

$$w(z) = \sqrt{\beta(z)\,\varepsilon + D(z)^2 \left\langle \frac{\Delta P}{P}^2 \right\rangle}$$

Emittance determination requires profile monitors at location with

$$\beta(z)\,\varepsilon \quad >> \quad D(z)^2 \left\langle \frac{\Delta P}{P}^2 \right\rangle$$

For measurement of $\beta(z)$ and $D(z)$ see Jörg Wenniger’s lecture.

For measurement of $\frac{\Delta P}{P}$ see Mario Ferianis’s lecture

Longitudinal emittance

$$\varepsilon_L = \sqrt{\langle z^2 \rangle \left\langle \frac{\Delta P}{P}^2 \right\rangle - \left\langle z \cdot \frac{\Delta P}{P} \right\rangle}$$

In synchrotron or storage rings with Q_s << 1

$$\varepsilon_L = \sqrt{\langle z^2 \rangle \left\langle \frac{\Delta P}{P}^2 \right\rangle}$$

If intensity dependend synchrotron tune shift is negligible it is sufficient

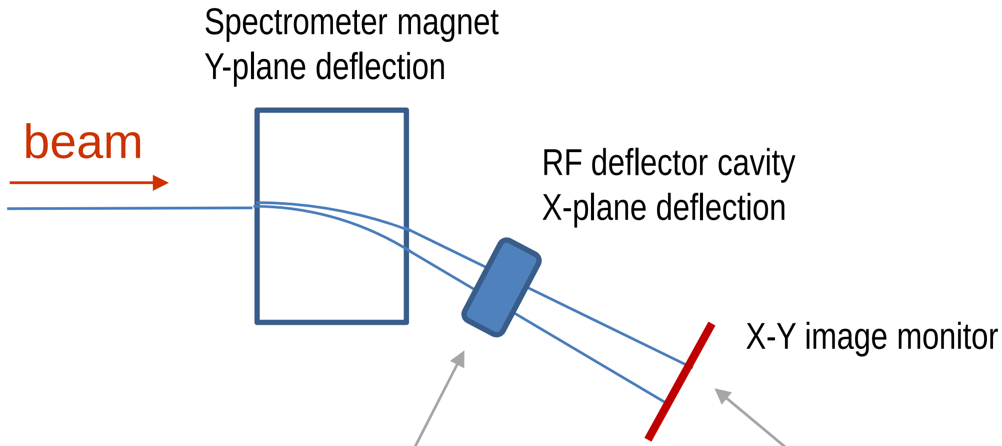
to measure either $\langle z^2 \rangle$ or $\left\langle \frac{\Delta P}{P}^2 \right\rangle$ and the synchrotron oscillation frequency ω_s

using

$$\sqrt{\langle z^2 \rangle} = \frac{c\left(\alpha - \frac{1}{\gamma^2}\right)}{\omega_s} \sqrt{\left\langle \frac{\Delta P}{P}^2 \right\rangle}$$

l

Longitudinal emittance measurement for Linacs I

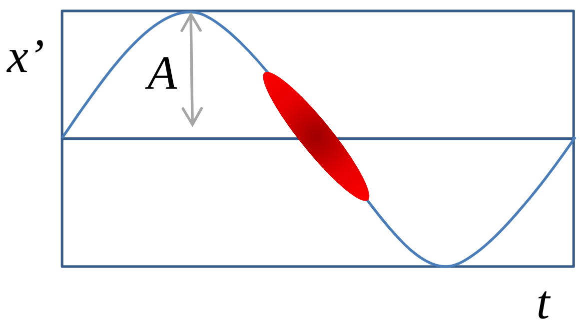


For good accuracy

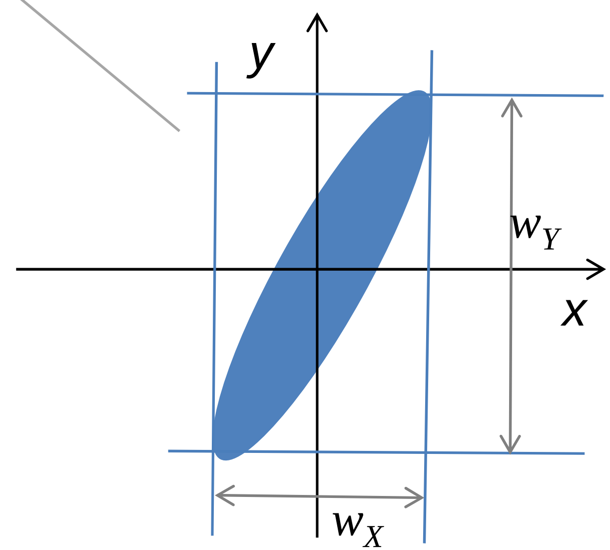
$$(AL\omega w_t)^2 >> \beta\varepsilon_X$$

$$(D w_{\frac{\Delta P}{P}})^2 >> \beta\varepsilon_Y$$

required



bunch centered on
zero cresting deflection



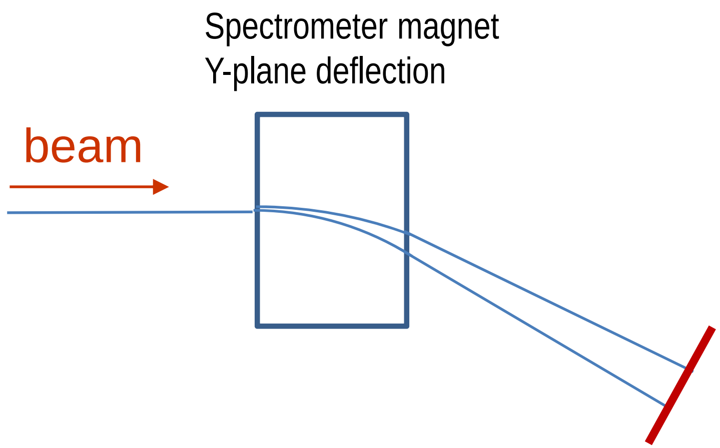
$$y = D_Y \frac{\Delta P}{P}$$

$$w_{\frac{\Delta P}{P}} = \frac{1}{D_Y} w_Y$$

$$x = AL\sin(\omega t)$$

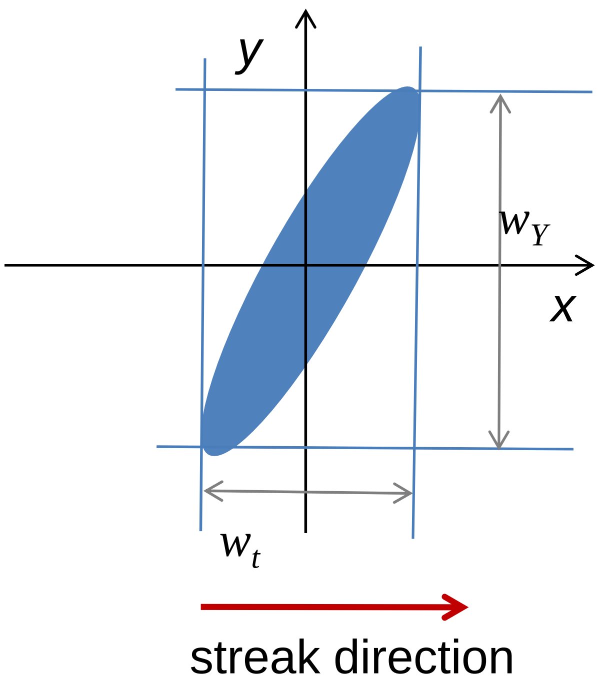
$$w_t \approx \frac{1}{AL\omega} w_x$$

Longitudinal emittance measurement for Linacs II



Profile monitor with preservation
of time information like
OTR or Cerenkov radiator

+ observation with streak camera



$$y = D_Y \frac{\Delta P}{P}$$

$$w_{\frac{\Delta P}{P}} = \frac{1}{D_Y} w_Y$$

Pitfalls with emittance measurements

- Beam width determination
- Space charge
- Chromatic effects
- Calibration errors