Measurements of Beam Energy

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Outline

- Introduction
  - Why do we need to know the beam energy?

- Methods of energy calibration
  - classical: spectrometers
  - exotic: from particle/nuclear physics processes
  - photon based measurements
  - energy measurements from central frequency
  - best precision: resonant depolarisation
  - energy from energy losses

- Applications and results
  - Beam parameters
  - LEP energy calibration

- Summary
What for?

- **Example 1 (LEP):**
  For precise measurements of Z mass/width and cross sections a beam energy needs to be known.

- **Example 2 (LEP2):**
  $E_0$ for determination of $m_W$.

- **Example 3 (Syn. Light Sources):**
  for insertion devices: $\varepsilon_\gamma \propto E^2$
  $1\% \Delta E/E \rightarrow 2\% \Delta \varepsilon_\gamma / \varepsilon_\gamma$

- **Example 4 (e.g. Tandem):**
  For resonances in nuclear physics in “ion beam on fixed target” configurations.

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“most precise measurement of the number 3”
Measurements of quadrupole gradients show systematic offset relative to the model if the beam energy is wrong....

This effects both lepton and hadron accelerators....
Beam Energy Determination using Spectrometers
Spectrometers measure the particle momentum by precisely determining the angle of deflection in a dipole magnet:

\[ \theta \propto \frac{1}{E_0} \int B ds \]

**Ingredients:**
- beam position \( (\mathcal{O}(1\mu m)) \) at entrance and exit of analysing magnet
- magnetic field \( (\mathcal{O}(10^{-5})) \) or better

**Single pass systems**
- position measurement with position sensitive detector at the beam stop (possible attenuation)

**Storage rings**
- no beam stop therefore position measurement with beam position monitors following the deflection
position measurement with 1 μm

magnetic fields
\[ \Delta B / B = \mathcal{O}(10^{-5}) \]

final energy resolution:
\[ \Delta E / E = \mathcal{O}(10^{-4}) \]
The LEP Spectrometer II

- Pickup positions monitored with a stretched wire system (behave of thermal effects due to synchrotron radiation etc.)
- Take into account the local magnetic field of the earth and fields generated by close-by power lines

Jurassic Limestone Block

Stretched Wire Position Sensors
Beam Energy from Particle Physics Processes
Radiative Returns to the Z

- X-check $E_{cm}$ using of the type $e^+e^- \rightarrow Z\gamma$, $Z \rightarrow ff$ where the fermion $f$ is a quark, electron, muon or $\tau$-lepton

- From knowledge of $m_Z$ at LEP1 invert problem and deduce initial collision energy of event

- $\Delta E/E = \mathcal{O}(3 \cdot 10^{-4})$

Beam Energy from Photon Based Methods
The spectrum of synchrotron radiation has a strong dependence on the electron beam energy.

- Measure synchrotron photon spectrum
- Determine $\epsilon_c$
- Measure $B$
- Derive $E_0$

Relative Power

$\epsilon_c [\text{eV}] = 665 E^2 [\text{GeV}^2] B [\text{T}]$

$\xi = \omega / \omega_c$

$\sim 2.1 \xi^{1/3}$

$\sim 1.3 (\xi)^{1/2} e^{-\xi}$

$50\%$

$\Delta E/E \sim O(10^{-3})$
Measure photon spectrum for different absorber (Al) thicknesses

Integrated spectra allow to reconstruct synchrotron radiation spectrum

Critical energy $\epsilon_c$ follows from fit according to $e^{-\epsilon/\epsilon_c}$

Laser photons of energy $\epsilon_1$ scatter with electrons of energy $E_0$ according to relativistic kinematics, resulting in the final photon energy $\epsilon_2$

$$\epsilon_2 = \epsilon_1 \frac{1 - \beta \cos \phi}{1 - \beta \cos \theta + \epsilon_1 (1 - \cos (\theta - \phi))/E_0}$$

For head-on collisions ($\phi = \pi$) and observation in direction of electron beam ($\theta = 0$)

$$\epsilon_{2\text{max}} = \epsilon_1 \frac{4\gamma^2}{1 + 4\gamma \epsilon_1/(m_e c^2)}$$

→ determine end of spectrum $\epsilon_{2\text{max}}$ at a detector (e.g. HPGe)

Relative uncertainty $\Delta E/E \propto \Delta \epsilon_{2\text{max}}/\epsilon_{2\text{max}}$ is of $\mathcal{O}(10^{-4})$
Compton Back Scattering II

Measurement at BESSY I (800 MeV)

\[ E_0 = 796.88(12) \text{ MeV} \]

R. Klein et al., NIM A (384) 1997.
Beam Energy from Measurements of the Central Frequency
The speed of particles relative to $c$ can be written as

$$\beta = \frac{C f_{\text{rev}}}{c} = \frac{C f_{\text{RF}}}{\hbar c}$$

To simultaneously determine $p$ and $C$, measure $f_{\text{rev}}$ for two particle types with different $Z/m$ in the same machine ($e^+ / p$, $p / \text{Pb}^{53+}$).

Momentum can be written as

$$p \approx m_p c \left[ \frac{f_{\text{RF},p}}{2\Delta f_{\text{RF}}} \left( \frac{m_i}{Zm_p} \right)^2 - 1 \right]$$

For high energies difficult since $\Delta f \propto (m_i/Zm_p)^2 / p^2$

- Maximise $m_i / Zm_p$ ($\text{Pb}^{53+}$)

$\Delta p / p$ is of $O(10^{-4})$
Beam Energy from Measurements of the Energy Loss
Energy from Energy Loss

- Energy loss per turn and particle:
  \[ U_0 = \frac{4\pi}{3} \frac{r_c}{(mc^2)^3} \frac{E_0^4}{\rho} \]
  
  → high sensitivity on \( E \)!

- Quantities that depend on \( U_0 \) are e.g. radiation damping, \( \Delta x_D \) and \( Q_s \)

<table>
<thead>
<tr>
<th>Circumference / m</th>
<th>Energy / GeV</th>
<th>( U_0 ) / MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANKA</td>
<td>110</td>
<td>2.5</td>
</tr>
<tr>
<td>ESRF</td>
<td>844</td>
<td>6.0</td>
</tr>
<tr>
<td>LEP</td>
<td>26700</td>
<td>94.5</td>
</tr>
</tbody>
</table>
Coherent damping at LEP

- composed of radiation and head-tail damping:

\[
\frac{1}{\tau_{\text{coh}}} = \frac{1}{\tau_0} + \frac{1}{\tau_{\text{head-tail}}}
\]

\[
\frac{1}{\tau_0} = \frac{1}{2} \frac{U_0}{E_0} f_{\text{rev}} J_x \sim E_0^3
\]

\[
\frac{1}{\tau_{\text{head-tail}}} \propto \frac{\sigma_s Q'}{E_0} I_b
\]

- Synchrotron radiation damping
  - due to emission of synchrotron $\gamma$ and RF gains

- Head-tail damping
  - depends on chromaticity and bunch current

- Beware of filamentation!
Measure for different chromaticities → contribution from head-tail damping varies, radiation damping stays the same as long as the energy stays the same (warning: better be finished before the tide turns....)

Energy loss from comparison to MAD:
\[ \mathcal{U}_0 = \mathcal{U}_0^{\text{MAD}} \frac{\tau_0^{\text{MAD}}}{\tau_0^{\text{meas}}} \]

Sources of uncertainty
→ \( J_x \) and central frequency
→ energy / frequency shifts due to tides

Finally:
✗ Energy uncertainty \( \mathcal{O}(1\%) \)
→ not sufficiently precise for \( E \)-calibration but still quite interesting

\[ E_0 = 45.625 \text{ GeV} \]
**$Q_s$ and RF Voltage**

- first pointed out by H. Burkhardt and A. Hofmann as a means to determine the energy loss at LEP
- Synchrotron tune $Q_s$ depends on total RF voltage and beam energy

$$Q_s^4 = \left( \frac{\alpha c \hbar}{2\pi E} \right)^2 \left\{ e^2 V_{RF}^2 - \left( \frac{C_y}{\rho} E^4 + K \right)^2 \right\} \sim \frac{a}{E^2} + bE^6$$

- X-calibrate at “low” energies with RDP and use method to extrapolate to energies where RDP doesn’t work
- Energy resolution: $\Delta E/E = \mathcal{O}(10^{-4})$
Beam Energy from Resonant Depolarisation
Transverse Polarisation

- Polarisation build-up by emission of synchrotron radiation:
  - asymmetry in spin flip probability leads to transverse polarisation
  - max. polarisation is given by size of asymmetry term:
    \[ \frac{8}{5\sqrt{3}} \approx 92.4\% \]

- Polarisation level increases exponentially with build up time
  - typical for electrons: several minutes to a few hours
  - LEP: 340’, ANKA: 10’

\[ \tau_p = \frac{8}{5\sqrt{3}} \frac{1}{\alpha} \left( \frac{m_0 c^2}{\hbar c} \right)^2 \frac{\rho^3}{c \gamma^5} \left( \frac{R}{\rho} \right) \]

\[ P(t) = P_0 \left( 1 - e^{-t/\tau_p} \right) \]
Spin Motion

The motion of the spin vector $\vec{s}$ of a relativistic electron in the presence of electric and magnetic fields $\vec{E}$ and $\vec{B}$ is described by the Thomas-BMT (Bargmann, Michel, Telegdi) equation:

$$\frac{d\vec{s}}{dt} = \vec{\Omega}_{\text{BMT}} \times \vec{s}$$

The spin precession frequency $\vec{\Omega}_{\text{BMT}}$ can be written as

$$\vec{\Omega}_{\text{BMT}} = -\frac{e}{\gamma m_0} \left[ (1 + a\gamma) \vec{B}_\perp + (1 + a) \vec{B}_\parallel - \left( a\gamma + \frac{\gamma}{1 + \gamma} \right) \vec{\beta} \times \frac{\vec{E}}{c} \right]$$

$$a = (g_e - 2)/2 = 0.001159652193(10)$$

The average over all particles of the number of spin oscillations per revolution is defined as the spin tune

$$\nu = f_{\text{spin}}/f_{\text{rev}} = \frac{(g_e - 2)/2}{m_0 c^2} E_0$$
Resonant Depolarisation

- Horizontal magn. field $B_x$ modulated with $f_{\text{dep}}$ is applied to the beam

- For a certain phase relation between the kicks of the depolariser and the spin tune the small spin rotations add up coherently from turn to turn and the polarisation is destroyed

  $\rightarrow$ resonance condition for spin rotations

  $$f_{\text{dep}} = (k \pm [\nu]) \cdot f_{\text{rev}}, \; k \in \mathbb{N}$$

- To determine the spin tune, the frequency of the depolariser field is slowly varied with time over a given frequency range

  $\rightarrow$ beam energy from $\gamma = \nu / a$

  $\rightarrow$ precision: $\Delta E / E \approx 10^{-6} - 10^{-5}$
To detect a change in polarisation, a polarimeter is needed.

- Touschek cross-section depends on electron beam polarisation.
  - **Use particle loss rate as a measure for polarisation level.**
- Set up a beam loss monitor in a Touschek sensitive region (low $\beta$ followed by large $D_x$) and monitor fast loss rate changes.

Test detector sensitivity and setup by moving tune from and to a resonance (known impact on lifetime / loss rate).

Touschek polarimetry even works for machines that are not limited by the Touschek effect.
Simple Polarimeter

- E.g. with scintillators wrapped in lead sheets to suppress the contribution of synchrotron radiation to the count rate

- Alternative: Pb-Glass block with photo multiplier

- Photo multiplier pulses are converted to NIM signals and counted using a custom made interface to a Linux PC
Absolute measurement of polarisation level with Compton scattering:

- Circ. polarized laser light collides with beam
- Measure vert. profile of $\gamma$s in Si-W calorimeter
- $P_\perp \propto$ vert. shift of $\gamma$ profiles for the two pol. states
- RDP works for $P > 5\% \Rightarrow$ extrapolation methods necessary for higher $E_0$
LEP Polarimeter

Si-W calorimeter

Movable absorber (Pb)

Synchrotron light monitor

Detectors

Nd-YAG laser (100 Hz)

Mirror

Soleil-Babinet compensator

Rotating λ/2 plate

Expander

Laser pulse

Electron detector

Backscattered Photons

Positron bunch (11 kHz)

Electron bunch (11 kHz)

Positron detector

Mirror

Focussing mirror for positron measurement

3 mrad

LIR

Movable absorber (Pb)

Focussing mirror

Electron bunch (11 kHz)

Mirror

Optical bench

313 m

313 m
Depolarisation Scans

- scan the depolariser frequency around the suspected beam energy equivalent
- get $f_{\text{dep}}$ from a fit

$$r = a - \frac{\partial r_{\text{L}}}{\partial t} t + \frac{\Delta r}{1 + \exp \left\{ - \frac{t-t_d}{\sigma_d} \right\}}$$

- depolarisation can also occur on synchrotron side bands
- due to the single particle nature of the depolarisation process, this happens at $f_{\text{dep}}/f_{\text{rev}} = [\nu] \pm Q_{s}^{\text{inc}}$
Some RDP Applications

- Measure $\alpha_c$ using RDP scans for different $f_{RF}$:
  \[
  \frac{\Delta f_{RF}}{f_{RF}} = \alpha_{c1} \delta + \alpha_{c2} \delta^2
  \]

- Relative precision $\Delta \alpha/\alpha \approx 10^{-3}$

- Measure simultaneously inc. (RDP) and coh. (stripline etc.)

- The coh./inc ratio is a measure for bunch lengthening with current (A. Hofmann, 1994)
  \[
  \frac{Q_{s}^{coh}}{Q_{s}^{inc}} = 1 - \lambda I_{bunch}
  \]
**LEP and the Moon**

- Tides affect both the oceans and the earth crust
- Local radius change $\Delta R$ due to mass $M$ at distance $d$ with zenith angle $\theta$:
  \[
  \Delta R \propto \frac{M}{d^3} (3 \cos^2 \theta - 1)
  \]

- Some facts:
  - Sun tides are 50% weaker than Moon tides
  - Full Moon tides at equator $\Delta R \approx \pm 50$ cm
  - Geneva region: vertical motion $\approx \pm 12.5$ cm
  - Change in LEP circumference of $\approx \pm 0.5$ mm
Circumference and Energy

- Impact on LEP beam energy because
  - length of actual orbit $L$ is determined by the frequency of the RF system
  - change in circumference $\Delta C$ $\Rightarrow$ beam has to move off-centre through the magnets
  - additional bending field in the quadrupoles changes the beam energy

\[ E \propto \int B \, ds \]

- Resulting change in beam energy:

\[ \frac{\Delta E}{E} = - \frac{1}{\alpha_c} \frac{(f_{RF} - f_{RF}^c)}{f_{RF}} = - \frac{1}{\alpha_c} \frac{\Delta C}{C} \]
Beam energy measurement with RDP during full moon compared to a prediction by a geological model

(L. Arnaudon et al.: Effects of Terrestrial Tides on the LEP Beam Energy. NIM A357, pages 249–252, 1995.)
Continuous monitoring of the beam energy by measurements of the bending field using $E_0 \propto \int dsB$

- NMR probes and flux loop cables
- Field measurements calibrated for low beam energies with RDP and then used for extrapolation to high beam energies
Measurements of the LEP dipole field with NMR probes in some of the bending magnets yield variations of the beam energy since

\[ E_0 \propto \int dsB \]

correlation with human activity?
about 20% of current does not flow back to power station over the railway tracks

in France some lines use DC currents: lower voltages ⇒ higher currents
The energy model of LEP:

\[ E_{\text{beam}}(t) = (E_{\text{initial}} + \Delta E_{\text{dipole}}(t)) \cdot (1 + C_{\text{tide}}(t)) \cdot (1 + C_{\text{orbit}}) \cdot (1 + C_{\text{RF}}(t)) \cdot (1 + C_{\text{hcorr}}(t)) \cdot (1 + C_{\text{QFQD}}(t)) \]

<table>
<thead>
<tr>
<th>Term</th>
<th>Physical Effect</th>
<th>( \sigma(\Delta E) ) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta E_{\text{dipole}} )</td>
<td>magnet temperature, parasitic currents on the vacuum chamber etc.</td>
<td>10</td>
</tr>
<tr>
<td>( C_{\text{tide}} )</td>
<td>tidal deformations (quadrupole contribution)</td>
<td>10</td>
</tr>
<tr>
<td>( C_{\text{orbit}} )</td>
<td>Circumference changes due to rainfall / underground water table height</td>
<td>10</td>
</tr>
<tr>
<td>( C_{\text{RF}} )</td>
<td>RF frequency change (~ 100 Hz) to reduce ( \sigma_x ) by increasing ( J_x )</td>
<td>100</td>
</tr>
<tr>
<td>( C_{\text{hcorr}} )</td>
<td>horizontal correctors change ( \int B ds ) and ( \Delta L )</td>
<td>10</td>
</tr>
<tr>
<td>( C_{\text{QFQD}} )</td>
<td>stray fields of power supply cables</td>
<td>1</td>
</tr>
</tbody>
</table>
The problems with electrical trains close to physics institutes are not exactly a recent discovery...

Journal article in 1895:

Central-Zeitung für Optik und Mechanik, XVI. Jg., No. 13, Page 151

“Nachtheile physikalischer Institute durch elektrische Bahnen”

(which roughly translates to “Disadvantages for Physics Institutes from Electrical Trains”)

What happened:
During first tests with electrical trams (instead of the horse-powered models) in Berlin it became clear that vagabond currents severely disturbed the delicate electrometers
Beam energy is an important parameter for control room and experiment

A variety of methods exists, choose according to design precision and energy range

To achieve highest precision, a combination of several measurements might be needed

Once achieved, the high precision measurements might have some surprises in store....

Advice: plan for long shifts...