Cryogenics for particle accelerators

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CAS Course in General Accelerator Physics
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• Low temperatures and liquefied gases
• Cryogenics in accelerators
• Properties of fluids
• Heat transfer & thermal insulation
• Cryogenic distribution & cooling schemes
• Refrigeration & liquefaction
• Low temperatures and liquefied gases
  • Cryogenics in accelerators
  • Properties of fluids
  • Heat transfer & thermal insulation
  • Cryogenic distribution & cooling schemes
  • Refrigeration & liquefaction
• **cryogenics**, that branch of physics which deals with the production of very low temperatures and their effects on matter

*Oxford English Dictionary*
2\textsuperscript{nd} edition, Oxford University Press (1989)

• **cryogenics**, the science and technology of temperatures below 120 K

*New International Dictionary of Refrigeration*
3\textsuperscript{rd} edition, IIF-IIR Paris (1975)
## Characteristic temperatures of cryogens

<table>
<thead>
<tr>
<th>Cryogen</th>
<th>Triple point [K]</th>
<th>Normal boiling point [K]</th>
<th>Critical point [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>90.7</td>
<td>111.6</td>
<td>190.5</td>
</tr>
<tr>
<td>Oxygen</td>
<td>54.4</td>
<td>90.2</td>
<td>154.6</td>
</tr>
<tr>
<td>Argon</td>
<td>83.8</td>
<td>87.3</td>
<td>150.9</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>63.1</td>
<td>77.3</td>
<td>126.2</td>
</tr>
<tr>
<td>Neon</td>
<td>24.6</td>
<td>27.1</td>
<td>44.4</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>13.8</td>
<td>20.4</td>
<td>33.2</td>
</tr>
<tr>
<td>Helium</td>
<td>2.2 (*)</td>
<td>4.2</td>
<td>5.2</td>
</tr>
</tbody>
</table>

(*): λ Point
Densification, liquefaction & separation of gases

LNG

130,000 m³ LNG carrier with double hull

Air separation by cryogenic distillation
Up to 4500 t/day LOX

LIN & LOX

Rocket fuels

Ariane 5
25 t LHY, 130 t LOX
What is a low temperature?

- The entropy of a thermodynamical system in a macrostate corresponding to a multiplicity $W$ of microstates is
  \[ S = k_B \ln W \]
- Adding reversibly heat $dQ$ to the system results in a change of its entropy $dS$ with a proportionality factor $T$
  \[ T = \frac{dQ}{dS} \]

⇒ high temperature: heating produces small entropy change
⇒ low temperature: heating produces large entropy change

L. Boltzmann’s grave in the Zentralfriedhof, Vienna, bearing the entropy formula
The average thermal energy of a particle in a system in thermodynamic equilibrium at temperature T is

\[ E \sim k_B T \]

\[ k_B = 1.3806 \times 10^{-23} \text{ J.K}^{-1} \]

1 K is equivalent to 10^{-4} eV or 10^{-23} J thermal energy
- a temperature is « low » for a given physical process when \( k_B T \) is small compared with the characteristic energy of the process considered
- cryogenic temperatures reveal phenomena with low characteristic energy and enable their application
### Characteristic temperatures of low-energy phenomena

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debye temperature of metals</td>
<td>few 100 K</td>
</tr>
<tr>
<td>High-temperature superconductors</td>
<td>~ 100 K</td>
</tr>
<tr>
<td>Low-temperature superconductors</td>
<td>~ 10 K</td>
</tr>
<tr>
<td>Intrinsic transport properties of metals</td>
<td>&lt; 10 K</td>
</tr>
<tr>
<td>Cryopumping</td>
<td>few K</td>
</tr>
<tr>
<td>Cosmic microwave background</td>
<td>2.7 K</td>
</tr>
<tr>
<td>Superfluid helium 4</td>
<td>2.2 K</td>
</tr>
<tr>
<td>Bolometers for cosmic radiation</td>
<td>&lt; 1 K</td>
</tr>
<tr>
<td>Low-density atomic Bose-Einstein condensates</td>
<td>~ μK</td>
</tr>
</tbody>
</table>
• Superconductivity only exists in a limited domain of temperature, magnetic field and current density

• Electrotechnical applications require transport current and magnetic field

• Operating temperature of the device must therefore be significantly lower than the critical temperature of the superconductor
Optimization of operating temperature for superconducting RF cavity

- Power per unit length: $P/L \sim R_s \frac{E^2}{\omega}$
- BCS theory: $R_{BCS} = (A \frac{\omega^2}{T}) \exp \left(-B \frac{T_c}{T}\right)$
- For practical materials: $R_s = R_{BCS} + R_0$
- Refrigeration (Carnot): $P_a = P \left(\frac{T_a}{T} - 1\right)$

⇒ optimum operating temperature for superconducting cavities is well below critical temperature of superconductor.
Useful range of cryogens & critical temperature of superconductors
Cryogens for superconducting devices

- **Helium** is the only practical cryogen for LTS devices
- **Subcooled nitrogen** is applicable to HTS devices at low and moderate current density
- Thanks to its general availability and low cost, **liquid nitrogen** is very often used for precooling and thermal shielding of helium-cooled devices
- In spite of its cost, **neon** can constitute an interesting alternative to subcooled nitrogen for operating HTS at high current density, and to helium for MgB₂ devices
  ⇒ *in the following, focus on helium and nitrogen*
• Low temperatures and liquefied gases
• **Cryogenics in accelerators**
  • Properties of fluids
  • Heat transfer & thermal insulation
  • Cryogenic distribution & cooling schemes
  • Refrigeration & liquefaction
Superconductivity and circular accelerators

- Beam energy, field in bending magnets and machine radius are related by:

\[ E_{\text{beam}} = 0.3 \quad B \quad r \quad [\text{GeV}] \quad [\text{T}] \quad [\text{m}] \]

At the LHC (r = 2.8 km), \( B = 8.33 \) T to reach \( E_{\text{beam}} = 7 \) TeV

- Superconductivity permits to produce high field and thus to limit size and electrical consumption of the accelerators

<table>
<thead>
<tr>
<th>Magnetic field</th>
<th>Normal conducting</th>
<th>Superconducting (LHC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field geometry</td>
<td>Defined by magnetic circuit</td>
<td>Defined by coils</td>
</tr>
<tr>
<td>Current density in windings</td>
<td>10 A/mm(^2)</td>
<td>400 A/mm(^2)</td>
</tr>
<tr>
<td>Electromagnetic forces</td>
<td>20 kN/m</td>
<td>3400 kN/m</td>
</tr>
<tr>
<td>Electrical consumption</td>
<td>10 kW/m</td>
<td>2 kW/m</td>
</tr>
</tbody>
</table>
Limiting energy stored in beam

- Energy $W$ stored in the beams of circular accelerators and colliders
  \[
  W \text{ [kJ]} = 3.34 \ E_{\text{beam}} \text{ [GeV]} \ I_{\text{beam}} \text{ [A]} \ C \text{ [km]}
  \]
  $C$ circumference of accelerator/collider
  \(\Rightarrow\) building compact machines, i.e. producing higher bending field $B$
  limits beam stored energy

- Example: the LHC
  \[
  E_{\text{beam}} = 7000 \text{ GeV} \\
  I_{\text{beam}} = 0.56 \text{ A} \\
  C = 26.7 \text{ km}
  \]
  \(\Rightarrow\) \quad W = 350 \text{ MJ!}
Low impedance for beam stability

- Transverse impedance
  \[ Z_T(\omega) \sim \rho \frac{r}{\omega} b^3 \]
  - \( \rho \) wall electrical resistivity
  - \( r \) average machine radius
  - \( b \) half-aperture of beam pipe

- Transverse resistive-wall instability
  - dominant in large machines
  - must be compensated by beam feedback, provided growth of instability is slow enough
  - maximize growth time \( \tau \sim 1/ Z_T(\omega) \) i.e. reduce \( Z_T(\omega) \)

\( \Rightarrow \) for a large machine with small aperture, low transverse impedance is achieved through low \( \rho \), i.e. low-temperature wall
Cryopumping maintains good vacuum

Saturation pressure of all gases except helium vanish at cryogenic temperature
CERN

Rationale for superconductivity & cryogenics in particle accelerators

Cryogenics

Superconductivity

Compactness

- Zero resistance
- Limit beam stored energy
- Reduce power consumption
- Reduce cost
- Beam stability
- Beam vacuum

Cryopumping

Beam impedance

Compactness

Zero resistance

Cryogenics

Superconductivity
Contents

- Low temperatures and liquefied gases
- Cryogenics in accelerators
- Properties of fluids
  - Heat transfer & thermal insulation
  - Cryogenic distribution & cooling schemes
  - Refrigeration & liquefaction
## Properties of cryogens compared to water

<table>
<thead>
<tr>
<th>Property</th>
<th>He</th>
<th>N₂</th>
<th>H₂O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal boiling point [K]</td>
<td>4.2</td>
<td>77</td>
<td>373</td>
</tr>
<tr>
<td>Critical temperature [K]</td>
<td>5.2</td>
<td>126</td>
<td>647</td>
</tr>
<tr>
<td>Critical pressure [bar]</td>
<td>2.3</td>
<td>34</td>
<td>221</td>
</tr>
<tr>
<td>Liq./Vap. density (*) [J.g⁻¹]</td>
<td>7.4</td>
<td>175</td>
<td>1600</td>
</tr>
<tr>
<td>Heat of vaporization (*) [J.g⁻¹]</td>
<td>20.4</td>
<td>199</td>
<td>2260</td>
</tr>
<tr>
<td>Liquid viscosity (*) [μPl]</td>
<td>3.3</td>
<td>152</td>
<td>278</td>
</tr>
</tbody>
</table>

(*) at normal boiling point
## Vaporization of normal boiling cryogens under 1 W applied heat load

<table>
<thead>
<tr>
<th>Cryogen</th>
<th>$[\text{mg.s}^{-1}]$</th>
<th>$[\text{l.h}^{-1}]$ (liquid)</th>
<th>$[\text{l.min}^{-1}]$ (gas NTP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>48</td>
<td>1.38</td>
<td>16.4</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>5</td>
<td>0.02</td>
<td>0.24</td>
</tr>
</tbody>
</table>

These numbers may be used for measuring heat load to a cryogen bath from boil-off flow measurements at constant liquid level.

At decreasing level, the escaping flow is lower than the vaporization rate and a correction must be applied.

\[
\dot{m}_{\text{out}} = \dot{m}_{\text{vap}} \left(1 - \frac{\rho_v}{\rho_l}\right) < \dot{m}_{\text{vap}}
\]
Amount of cryogens required to cool down 1 kg iron

<table>
<thead>
<tr>
<th>Using</th>
<th>Latent heat only</th>
<th>Latent heat and enthalpy of gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHe from 290 to 4.2 K</td>
<td>29.5 litre</td>
<td>0.75 litre</td>
</tr>
<tr>
<td>LHe from 77 to 4.2 K</td>
<td>1.46 litre</td>
<td>0.12 litre</td>
</tr>
<tr>
<td>LN2 from 290 to 77 K</td>
<td>0.45 litre</td>
<td>0.29 litre</td>
</tr>
</tbody>
</table>

⇒ recover enthalpy from cold gas (i.e. moderate flow of cryogen)
⇒ pre-cool with liquid nitrogen to save liquid helium
Cooldown of LHC sector (4625 t over 3.3 km)

600 kW precooling to 80 K with LIN (up to ~5 tons/h)

1260 tons LIN unloaded
Phase diagram of helium

Temperature [K]

Pressure [kPa]

- SOLID
- VAPOUR
- He I
- He II
- CRITICAL POINT
- PRESSURIZED He II (Subcooled liquid)
- SATURATED He I
- SATURATED He II
- \(\lambda\) LINE
- SUPER-CRITICAL

\[0, 1, 2, 3, 4, 5, 6\]

\[1, 10, 100, 1000, 10000\]
# Helium as a cooling fluid

<table>
<thead>
<tr>
<th>Phase domain</th>
<th>Advantages</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated He I</td>
<td>Fixed temperature, High heat transfer</td>
<td>Two-phase flow, Boiling crisis</td>
</tr>
<tr>
<td>Supercritical</td>
<td>Monophase, Negative J-T effect</td>
<td>Non-isothermal, Density wave instability</td>
</tr>
<tr>
<td>He II</td>
<td>Low temperature, High conductivity, Low viscosity</td>
<td>Second-law cost, Subatmospheric</td>
</tr>
</tbody>
</table>
He II cooling of LHC magnets allows to reach the 8 -10 T range using Nb-Ti superconductor

![Graph showing the Jc vs B for different materials and temperature conditions.](image)

+ 3 tesla
Enhancement of heat transfer

- Low viscosity $\Rightarrow$ permeation
- Very high specific heat $\Rightarrow$ stabilization
  - $10^5$ times that of the conductor per unit mass
  - $2 \times 10^3$ times that of the conductor per unit volume
- Very high thermal conductivity $\Rightarrow$ heat transport
  - $10^3$ times that of cryogenic-grade OFHC copper
  - Peaking at 1.9 K

Full benefit of these transport properties can only be reaped by appropriate design providing good wetting of the superconductors and percolation paths in the insulation, often in conflict with other technical requirements.
High thermal conductivity of the liquid suppresses boiling

*Electrical heater in saturated liquid helium*

He I (T=2.4 K)  
He II (T=2.1 K)
Heat transfer across electrical insulation of LHC superconducting cable

C. Meuris et al.
Calorimetry in isothermal He II bath

- For slow thermal transients, the He II bath is quasi-isothermal: a single temperature measurement allows to estimate heat deposition/generation $Q'$

\[ Q' = M_{\text{bath}} \frac{dH}{dt}|_1 \]

- $M_{\text{bath}}$ can be estimated by *in situ* calibration, using applied heating power $W'$

\[ W' = M_{\text{bath}} \frac{dH}{dt}|_2 \]
Measurement of electrical dissipation in LHC magnet subsector by He II calorimetry

The additional power measured by He II calorimetry is 10.0 W, corresponding to the applied electrical power

The method is validated and able to resolve < W

L. Tavian
Contents

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Typical heat transfer coefficients at cryogenic temperatures
Heat conduction in solids

Fourier’s law: \[ Q_{\text{con}} = k(T) \cdot S \cdot \frac{dT}{dx} \]

\( k(T) \): thermal conductivity \([\text{W/m.K}]\)

Integral form: \[ Q_{\text{con}} = \frac{S}{L} \int_{T_1}^{T_2} k(T) \cdot dT \]

\( \int k(T) \cdot dT \): thermal conductivity integral \([\text{W/m}]\)

Thermal conductivity integrals for standard construction materials are tabulated
## Thermal conductivity integrals of selected materials [W/m]

<table>
<thead>
<tr>
<th>Material</th>
<th>20 K</th>
<th>80 K</th>
<th>290 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFHC copper</td>
<td>11000</td>
<td>60600</td>
<td>152000</td>
</tr>
<tr>
<td>DHP copper</td>
<td>395</td>
<td>5890</td>
<td>46100</td>
</tr>
<tr>
<td>1100 aluminium</td>
<td>2740</td>
<td>23300</td>
<td>72100</td>
</tr>
<tr>
<td>2024 aluminium alloy</td>
<td>160</td>
<td>2420</td>
<td>22900</td>
</tr>
<tr>
<td>AISI 304 stainless steel</td>
<td>16.3</td>
<td>349</td>
<td>3060</td>
</tr>
<tr>
<td>G-10 glass-epoxy composite</td>
<td>2</td>
<td>18</td>
<td>153</td>
</tr>
</tbody>
</table>
Non-metallic composite support post with heat intercepts

- 5 K cooling line (SC He)
- Aluminium intercept plates glued to G-10 column
- Aluminium strips to thermal shield at 50-75 K
Thermal radiation

- **Wien’s law**
  - Maximum of black body power spectrum
    \[ \lambda_{\text{max}} T = 2898 \, [\mu\text{m.K}] \]
- **Stefan-Boltzmann’s law**
  - Black body
    \[ Q_{\text{rad}} = \sigma A T^4 \]
    \[ \sigma = 5.67 \times 10^{-8} \, \text{W/m}^2\text{K}^4 \]
    (Stefan Boltzmann’s constant)
  - “Gray” body
    \[ Q_{\text{rad}} = \varepsilon A T^4 \]
    \[ \varepsilon \text{ emissivity of surface} \]
  - “Gray” surfaces at \( T_1 \) and \( T_2 \)
    \[ Q_{\text{rad}} = E \sigma A (T_1^4 - T_2^4) \]
    \[ E \text{ function of } \varepsilon_1, \varepsilon_2, \text{ geometry} \]
# Emissivity of technical materials at low temperatures

<table>
<thead>
<tr>
<th></th>
<th>Radiation from 290 K Surface at 77 K</th>
<th>Radiation from 77 K Surface at 4.2 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless steel, as found</td>
<td>0.34</td>
<td>0.12</td>
</tr>
<tr>
<td>Stainless steel, mech. polished</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>Stainless steel, electropolished</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>Stainless steel + Al foil</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Aluminium, as found</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>Aluminium, mech. polished</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>Aluminium, electropolished</td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td>Copper, as found</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td>Copper, mech. Polished</td>
<td>0.06</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Residual gas conduction

\( \lambda_{\text{molecule}} \): mean free path of gas molecules

- **Viscous regime**
  - At high gas pressure \( \lambda_{\text{molecule}} \ll d \)
  - Classical conduction \( Q_{\text{res}} = k(T) A \frac{dT}{dx} \)
  - Thermal conductivity \( k(T) \) independant of pressure

- **Molecular regime**
  - At low gas pressure \( \lambda_{\text{molecule}} \gg d \)
  - Kennard’s law \( Q_{\text{res}} = A \alpha(T) \Omega P (T_2 - T_1) \)
  - Conduction heat transfer proportional to pressure, independant of spacing between surfaces
    \( \Omega \) depends on gas species
  - Accommodation coefficient \( \alpha(T) \) depends on gas species, \( T_1 \), \( T_2 \)
  and geometry of facing surfaces
Multi-layer insulation (MLI)

- Complex system involving three heat transfer processes
  - \( Q_{\text{MLI}} = Q_{\text{rad}} + Q_{\text{sol}} + Q_{\text{res}} \)
  - With \( n \) reflective layers of equal emissivity, \( Q_{\text{rad}} \sim 1/(n+1) \)
  - Due to parasitic contacts between layers, \( Q_{\text{sol}} \) increases with layer density
  - \( Q_{\text{res}} \) due to residual gas trapped between layers, scales as \( 1/n \) in molecular regime
  - Non-linear behaviour requires layer-to-layer modeling

- In practice
  - Typical data available from (abundant) literature
  - Measure performance on test samples
### Typical heat fluxes at vanishingly low temperature between flat plates [W/m²]

<table>
<thead>
<tr>
<th>Heat Flux Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black-body radiation from 290 K</td>
<td>401</td>
</tr>
<tr>
<td>Black-body radiation from 80 K</td>
<td>2.3</td>
</tr>
<tr>
<td>Gas conduction (100 mPa He) from 290 K</td>
<td>19</td>
</tr>
<tr>
<td>Gas conduction (1 mPa He) from 290 K</td>
<td>0.19</td>
</tr>
<tr>
<td>Gas conduction (100 mPa He) from 80 K</td>
<td>6.8</td>
</tr>
<tr>
<td>Gas conduction (1 mPa He) from 80 K</td>
<td>0.07</td>
</tr>
<tr>
<td>MLI (30 layers) from 290 K, pressure below 1 mPa</td>
<td>1-1.5</td>
</tr>
<tr>
<td>MLI (10 layers) from 80 K, pressure below 1 mPa</td>
<td>0.05</td>
</tr>
<tr>
<td>MLI (10 layers) from 80 K, pressure 100 mPa</td>
<td>1-2</td>
</tr>
</tbody>
</table>
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Transport of refrigeration in large distributed cryogenic systems

![Graph showing temperature difference vs. distance for various cryogenic systems. The graph includes points for different locations such as Tore Supra, Tevatron, HERA, LHC, and SSC (main Ring) and (HEB). The graph uses different markers and colors to represent Pressurised He II, Saturated LHe II, and He I.]
Cryogenic distribution scheme: design issues

- Monophase vs. two-phase
  - temperature control
  - hydrostatic head & flow instabilities
- Pumps vs. no pumps
  - efficiency & cost
  - reliability & safety
- Use of liquid nitrogen
  - cooldown and/or normal operation
  - capital & operating costs of additional fluid
  - safety in underground areas (ODH)
- Lumped vs. distributed cryoplants
- Separate cryoline vs. integrated piping
- Number of active components (valves, actuators)
- Redundancy of configuration
The Tevatron at Fermilab, USA
Central helium liquefier, separate ring cryoline and 24 satellite refrigerators
HERA proton ring at DESY, Germany
RHIC at Brookhaven National Lab, USA
RHIC distribution scheme

Central cryoplant and piping integrated in magnet cryostat
The LHC at CERN
LHC distribution scheme

8 x 18 kW @ 4.5 K
1,800 SC magnets
24 km and 20 kW @ 1.9 K
36,000 tons @ 1.9 K
96 tons of He

Cryogenic distribution line
Cryogenic plant

Cryoplants at five points, separate ring cryoline, 107 m long strings
Principle of He II cooling of LHC magnets

- saturated He II, flowing
- pressurized He II, static
- magnet
- sc bus bar connection
- helium vessel
- heat exchanger tube
Cryogenic operation of LHC sector

Sector temperature profile at 19 Feb 14:28

Temperature [K]

Point 4
RF cavities
Arc magnets
LSS magnets

Point 5
Mid Arc

Saturated Vapour Temperature: 1.79K

3.3 km
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Thermodynamics of cryogenic refrigeration

First principle [Joule]

\[ Q_0 = Q_i + W \]

Second principle [Clausius]

\[ \frac{Q_0}{T_0} \geq \frac{Q_i}{T_i} \]

(= for reversible process)

Hence,

\[ W \geq T_0 \cdot \frac{Q_i}{T_i} - Q_i \]

which can be written in three different ways:

1. \[ W \geq T_0 \cdot \Delta S_i - Q_i \] introducing entropy \( S \) as

   \[ \Delta S_i = \frac{Q_i}{T_i} \]

2. \[ W \geq Q_i \cdot \left( \frac{T_0}{T_i} - 1 \right) \] Carnot factor

3. \[ W \geq \Delta E_i \] introducing exergy \( E \) as

   \[ \Delta E_i = Q_i \cdot \left( \frac{T_0}{T_i} - 1 \right) \]
Consider the extraction of 1 W at 4.5 K, rejected at 300 K.
The minimum refrigeration work (equation 2) is:

\[
W_{\text{min}} = Q_i \cdot \left( \frac{T_0}{T_i} - 1 \right) = 1 \cdot \left( \frac{300}{4.5} - 1 \right) = 65.7 \text{ W}
\]

In practice, the most efficient helium refrigerators have an efficiency of about 30% w.r. to the Carnot limit.

\[
\Rightarrow W_{\text{real}} = \frac{W_{\text{min}}}{\eta} = \frac{65.7}{0.3} = 220 \text{ W}
\]
C.O.P. of large cryogenic helium refrigerators

Carnot
Refrigeration cycles and duties

Introduction to the T-S diagram

Thermodynamic transformation from A to B, if reversible:

\[ \Delta Q = \int_{A}^{B} T \cdot dS \]

To make a refrigeration cycle, need a substance, the entropy of which depends on some other variable than temperature.

Pressure of gas: Compression/expansion cycle
Magnetization of solid: magnetic refr. cycle

\[ \Delta Q_1: \text{heat absorbed at } T_1 \]
\[ \Delta Q_2: \text{heat rejected at } T_2 \]

\[ \rightarrow \text{Refrigeration cycle } A \ B \ C \ D \]
T-S diagram for helium
A Carnot cycle is not feasible for helium liquefaction

- It would need a HP of 613 kbar!
- There exists no true isothermal compressor
- There exists no true isentropic compressor or expander
A real cycle
internal heat exchange and para-isothermal compression

Practical compressors are adiabatic, need aftercooling and if multistage, intercooling

Heat exchanger between HP and LP streams
Elementary cooling processes on T-S diagram

- **B₁** (isobar, heat exchanger)
- **B₂** (adiabatic, expansion engine)
- **B₃** (isenthalpic, Joule-Thomson valve)
- **B'₂**

**Lines:**
- **P₁**
- **P₂ (< P₁)**
- **H**

**Labels:**
- **T**
- **S**
Brazed aluminium plate heat exchanger
Cryogenic turbo-expander

Cryogenic turboexpander
Self-acting gas bearing system
Maximum Joule-Thomson inversion temperatures

<table>
<thead>
<tr>
<th>Cryogen</th>
<th>Maximum inversion temperature [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>43</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>202</td>
</tr>
<tr>
<td>Neon</td>
<td>260</td>
</tr>
<tr>
<td>Air</td>
<td>603</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>623</td>
</tr>
<tr>
<td>Oxygen</td>
<td>761</td>
</tr>
</tbody>
</table>

While air can be cooled down and liquefied by JT expansion from room temperature, helium and hydrogen need precooling down to below inversion temperature by heat exchange or work-extracting expansion (e.g. in turbines)
Two-stage Claude cycle
Claude-cycle helium refrigerators/liquefiers
(Air Liquide & Linde)

<table>
<thead>
<tr>
<th></th>
<th>HELIAL SL</th>
<th>HELIAL ML</th>
<th>HELIAL LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Liquefaction capacity without LN2</td>
<td>25 L/h</td>
<td>70 L/h</td>
<td>145 L/h</td>
</tr>
<tr>
<td>Max. Liquefaction capacity with LN2</td>
<td>50 L/h</td>
<td>150 L/h</td>
<td>330 L/h</td>
</tr>
<tr>
<td>Compressor electrical motor</td>
<td>55 kW</td>
<td>132 kW</td>
<td>250 kW</td>
</tr>
<tr>
<td>Specific consumption for liquefaction w/o LN2</td>
<td>645 W/W</td>
<td>552 W/W</td>
<td>505 W/W</td>
</tr>
<tr>
<td>% Carnot</td>
<td>10%</td>
<td>12%</td>
<td>13%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Without LN, precooling</th>
<th>With LN, precooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>L70</td>
<td>20 – 35 l/h</td>
<td>40 – 70 l/h</td>
</tr>
<tr>
<td>L140</td>
<td>45 – 70 l/h</td>
<td>90 – 140 l/h</td>
</tr>
<tr>
<td>L280</td>
<td>100 – 145 l/h</td>
<td>200 – 290 l/h</td>
</tr>
<tr>
<td>LR70</td>
<td>100 – 145 Watt</td>
<td>130 – 190 Watt</td>
</tr>
<tr>
<td>LR140</td>
<td>210 – 290 Watt</td>
<td>255 – 400 Watt</td>
</tr>
<tr>
<td>LR280</td>
<td>445 – 640 Watt</td>
<td>560 – 900 Watt</td>
</tr>
</tbody>
</table>
Process cycle & T-S diagram of LHC 18 kW @ 4.5 K cryoplant
LHC 18 kW @ 4.5 K helium cryoplants

- 33 kW @ 50 K to 75 K
- 23 kW @ 4.6 K to 20 K
- 41 g/s liquefaction
- 4 MW compressor power
- C.O.P. 220-230 W/W @ 4.5 K

Air Liquide

Linde
Oil-injected screw compressor
Compressor station
of LHC 18 kW@ 4.5 K helium refrigerator
Challenges of power refrigeration at 1.8 K

- Compression of large mass flow-rate of He vapor across high pressure ratio ⇒ intake He at maximum density, i.e. cold
- Need contact-less, vane-less machine ⇒ hydrodynamic compressor
- Compression heat rejected at low temperature ⇒ thermodynamic efficiency
Cold compressors for 1.8 K refrigeration

Axial-centrifugal impeller

Cartridge 1st stage

4 cold compressor stages
Simplified flow-schemes of the 1.8 K refrigeration units of LHC

**Air Liquide Cycle**
- 4 K, 15 mbar, 124 g/s
- 20 K, 1.3 bar, 124 g/s

**IHI-Linde Cycle**
- 4 K, 15 mbar, 124 g/s
- 20 K, 1.3 bar, 124 g/s
C.O.P. of LHC 1.8 K units

Carnot Limit

Air Liquide

IHI-Linde

L. Tavian
Some references

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  - U. Wagner, *Refrigeration*
  - G. Vandoni, *Heat transfer*
  - Ph. Lebrun, *Design of a cryostat for superconducting accelerator magnet*
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