Introduction to Accelerators

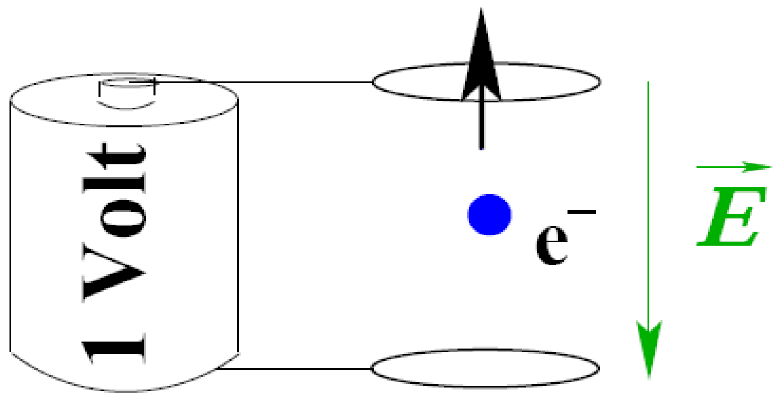
D. Brandt, CERN

Why this Introduction?

- During this school, you will learn about beam dynamics in a rigorous way…
- but some of you are completely new to the field of accelerator physics.
- It seemed therefore justified to start with the introduction of a few very basic concepts, which will be used throughout the course.

This is a completely intuitive approach (no mathematics) aimed at highlighting the physical concepts, without any attempt to achieve any scientific derivation.

Units: the electronvolt (eV)



The electronvolt (eV) is the energy gained by an electron travelling, in vacuum, between two points with a voltage difference of 1 Volt. $1\ eV = 1.602\ 10^{-19}$ Joule

We also frequently use the electronvolt to express masses from $E=mc^2$: $1\ eV/c^2 = 1.783\ 10^{-36}$ kg

Beam Dynamics (1)

In order to describe the motion of the particles, each particle is characterised by:

- Its azimuthal position along the machine: s
- Its Energy: E
- Its horizontal position: x
- Its horizontal slope: x'
- Its vertical position: y
- Its vertical slope: y'

i.e. a sixth dimensional vector

(s, E, x, x', y, y')

Beam Dynamics (2)

- In an accelerator designed to operate at the energy E_{nom}, all particles having $(s, E_{nom}, 0, 0, 0, 0)$ will happily fly through the center of the vacuum chamber without any problem. These are "ideal particles".

- The difficulties start when:

  - one introduces dipole magnets

  - the energy $E \neq E_{nom}$ or $(p-p_{nom}/p_{nom}) = \Delta p/p_{nom} \neq 0$

  - either of $x, x', y, y' \neq 0$

Basic problem:

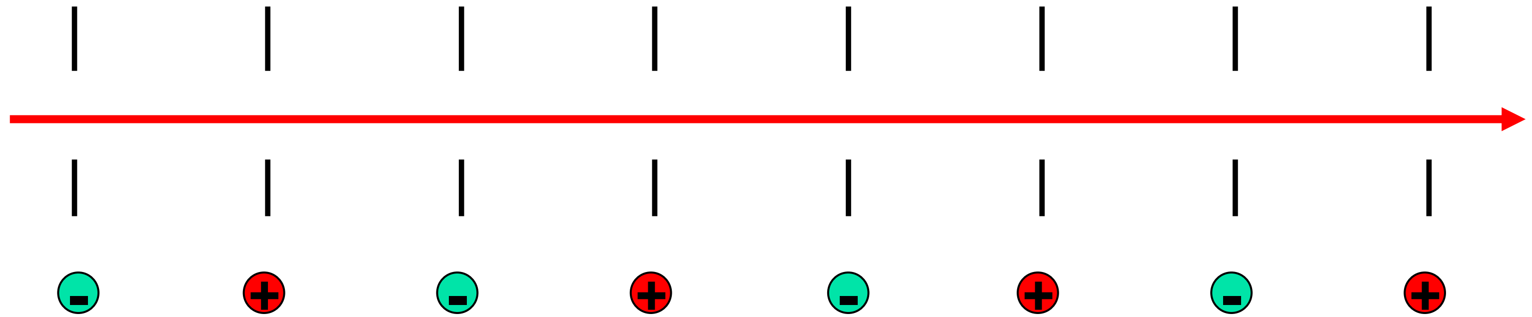
With more than 10^{10} particles per bunch, most of them will not be ideal particles, i.e. they are going to be lost !

Purpose of this lecture: how can we keep the particles in the machine ?

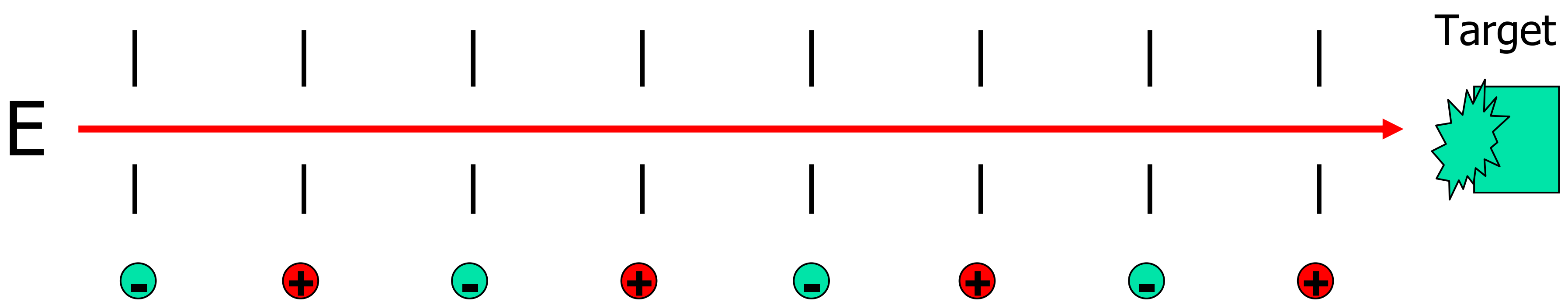
What is a Particle Accelerator?

➢ a machine to accelerate some particles ! How is it done ?

➢ Many different possibilities, but rather easy from the general principle:



Ideal linear machines (linacs)



Available Energy : $E_{c.m.} = m \cdot (2+2\gamma)^{1/2} = (2m.(m+E))^{1/2}$

with $\gamma = E/E_0$

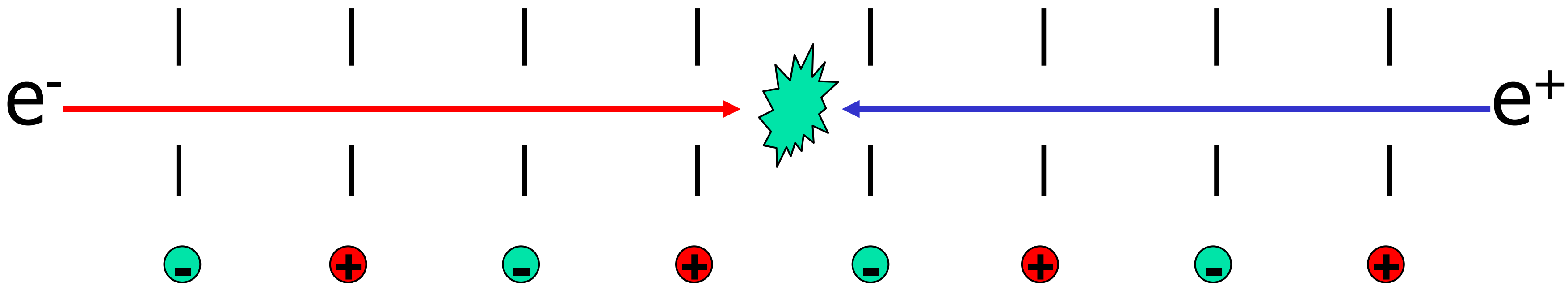
<u>Advantages</u>: Single pass

High intensity

<u>Drawbacks</u>: Single pass

Available Energy

Improved solution for $E_{c.m.}$



Available Energy : $E_{c.m.} = 2m\gamma = 2E$

with $\gamma = E/E_0$

<u>Advantages</u>: High intensity

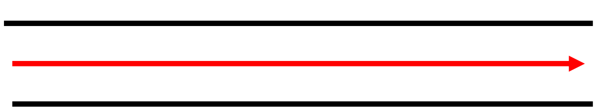
<u>Drawbacks</u>: Single pass

Space required

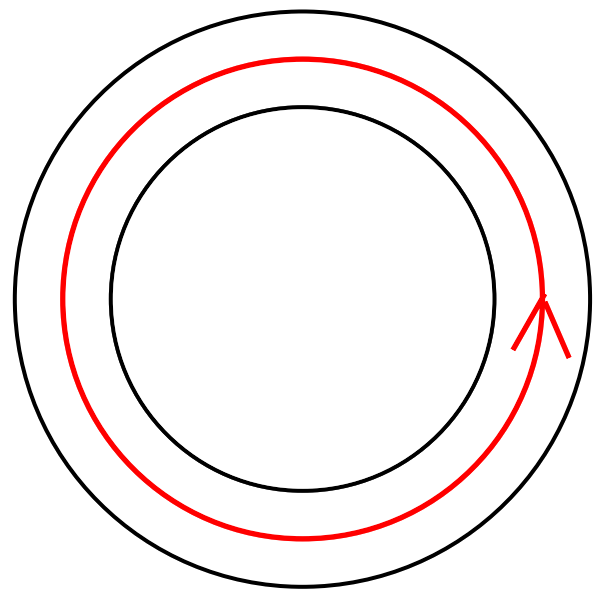
Keep particles: circular machines

Basic idea is to keep the particles in the machine for many turns.

<u>Move from the linear design</u>



<u>To a circular one:</u>

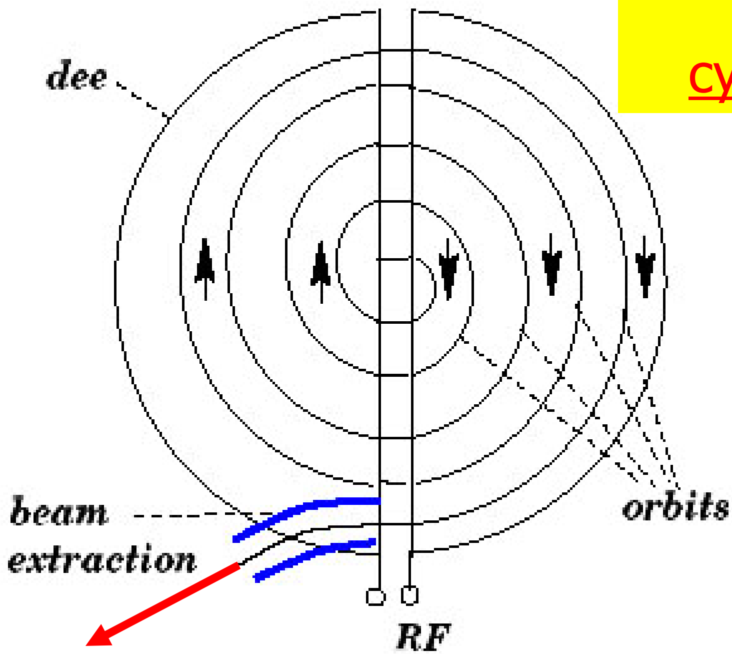


- Need Bending
- Need Dipoles!

Circular machines ($E_{c.m.} \sim (mE)^{1/2}$)

fixed target:

cyclotron



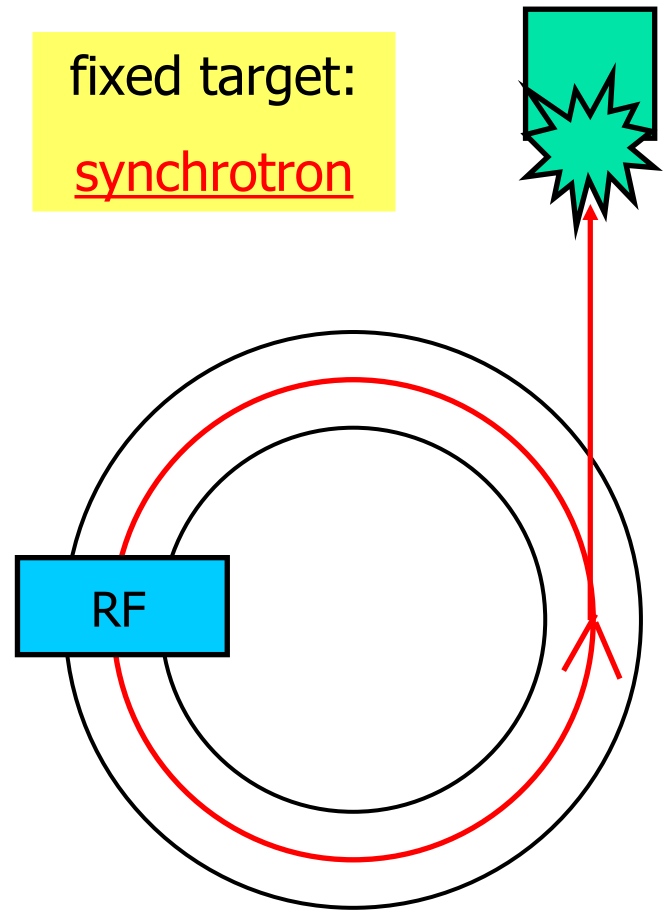
huge dipole, compact design,

B = constant

low energy, single pass.

fixed target:

synchrotron



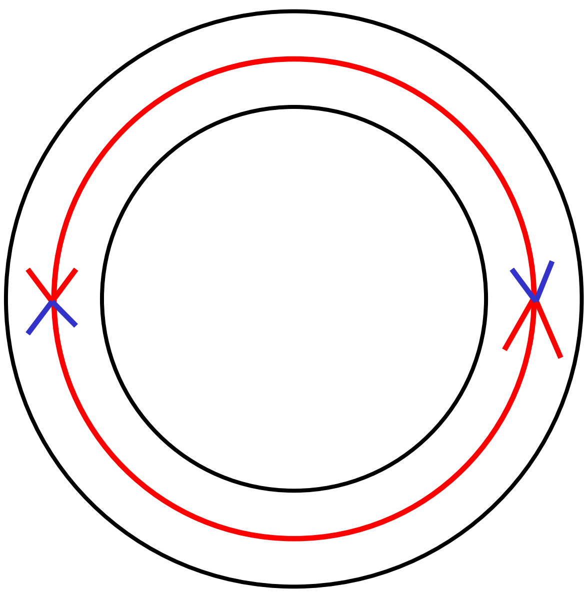
varying B, small magnets, high energy

Colliders ($E_{c.m.}$=2E)

<u>Colliders:</u>

electron – positron

proton - antiproton



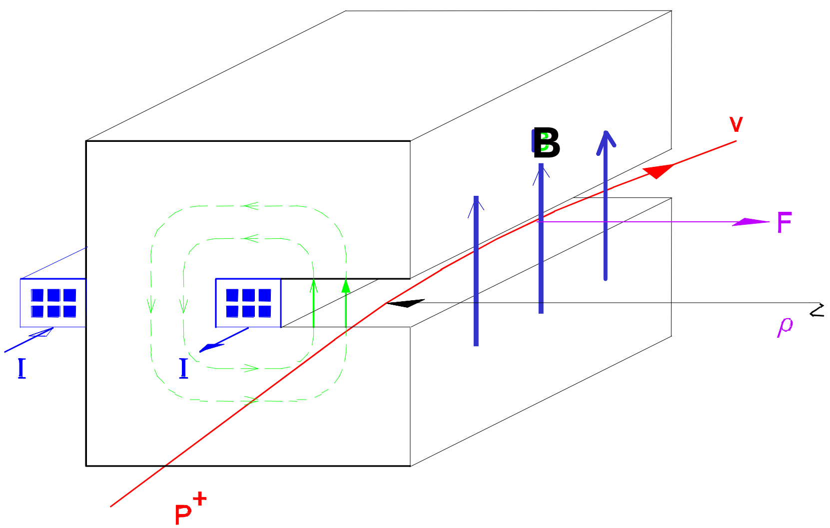
Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions

Ex: LHC

8 possible interaction regions

4 experiments collecting data

Circular machines: Dipoles



Classical mechanics:

Equilibrium between two forces

Lorentz force

$F = e.(\underline{v} \times \underline{B})$

Centrifugal force

$F = mv^2/\rho$

$evB = mv^2/\rho$

$p = m_0.c.(\beta\gamma)$

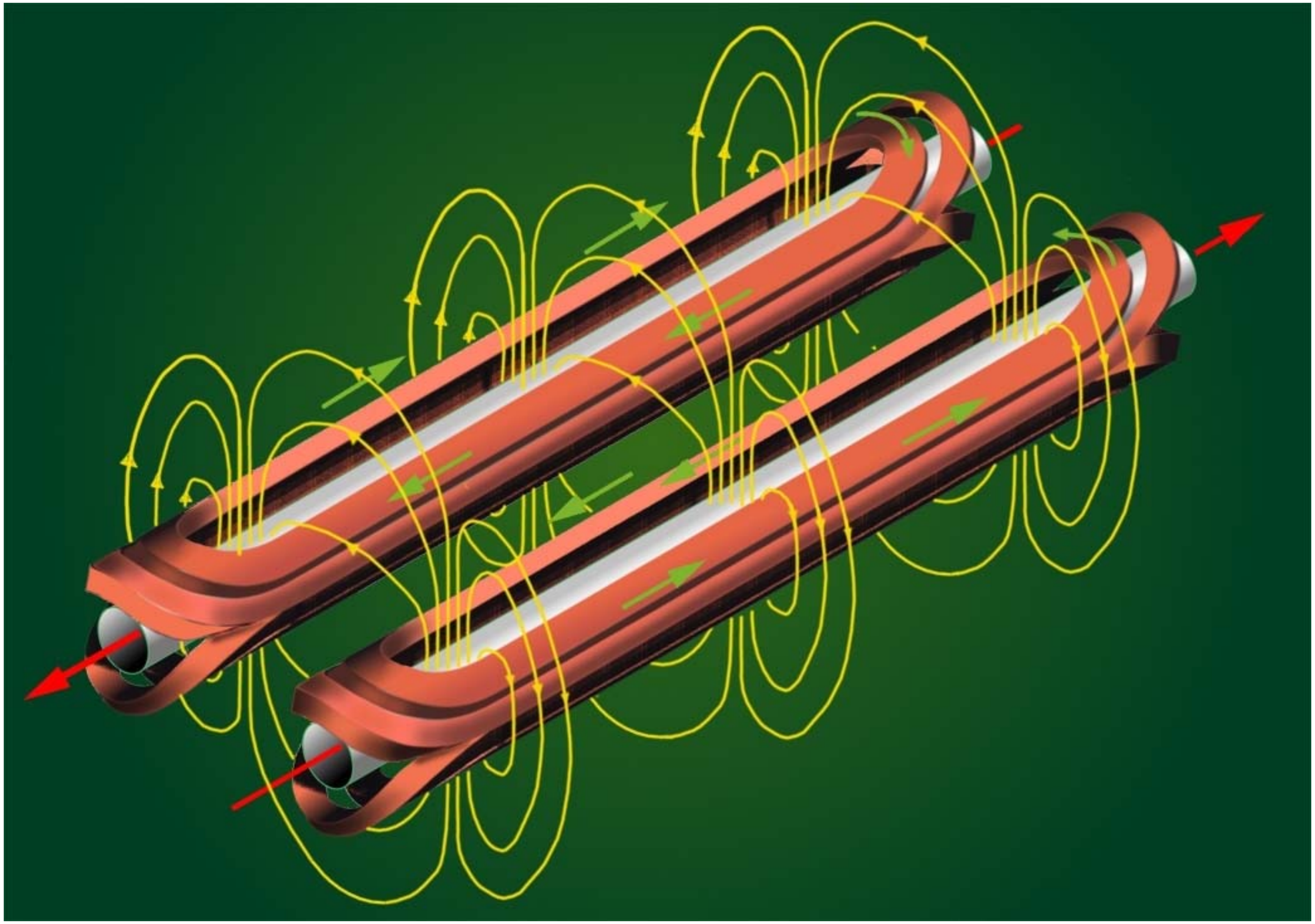
Magnetic rigidity:

$B\rho = mv/e = p/e$

Relation also holds for relativistic case provided the classical momentum mv is replaced by the relativistic momentum p

Dipoles (1):

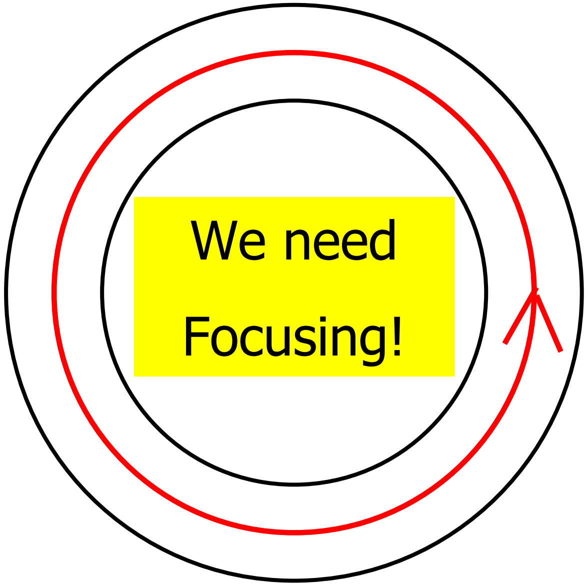
Dipoles (2):



Ideal circular machine:

- Neglecting radiation losses in the dipoles
- Neglecting gravitation

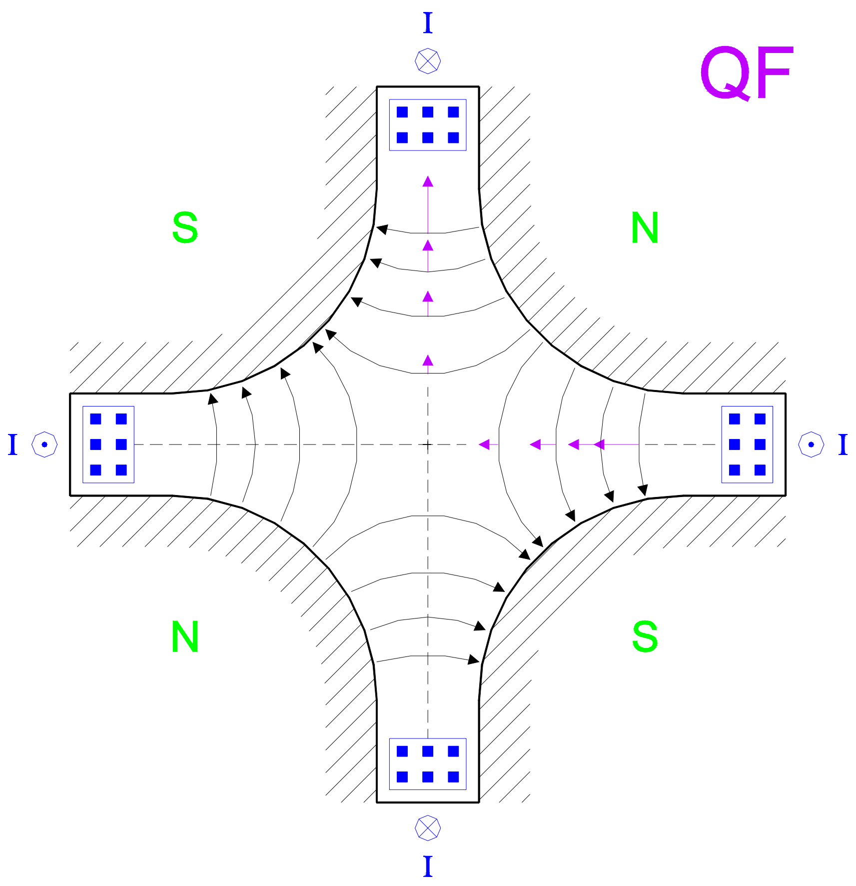
ideal particle would happily circulate on axis in the machine for ever!



Unfortunately: real life is different!

Gravitation: Δy = 20 mm in 64 msec!	
Alignment of the machine	Limited physical aperture
Ground motion	Field imperfections
Energy error of particles and/or $(x, x')_{inj} \neq (x, x')_{nominal}$	
Error in magnet strength (power supplies and calibration)	

Focusing with quadrupoles



$$F_x = -g.x$$

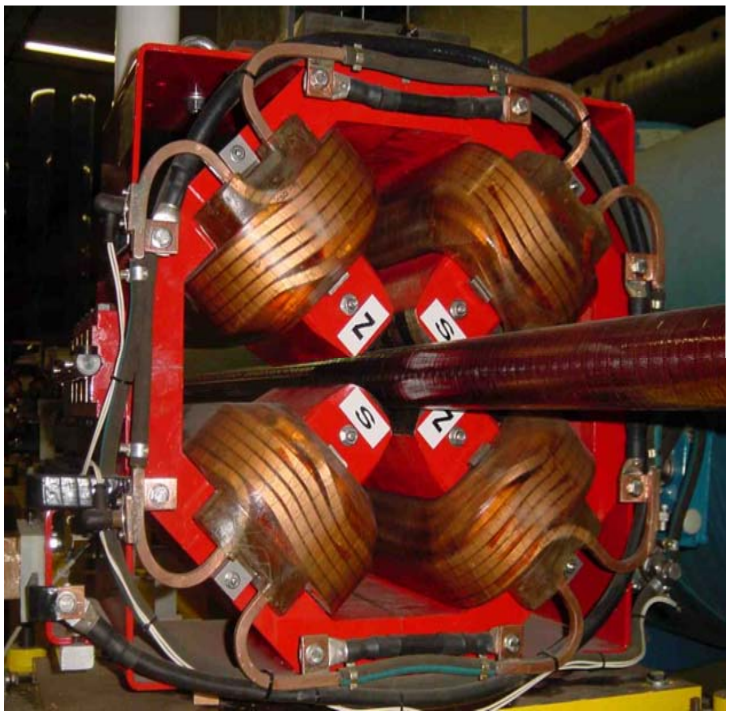
$$F_y = g.y$$

Force increases linearly with displacement.

Unfortunately, effect is opposite in the two planes (H and V).

Remember: this quadrupole is focusing in the horizontal plane but defocusing in the vertical plane!

Quadrupoles:



Focusing properties …

A quadrupole provides the required effect in one plane…

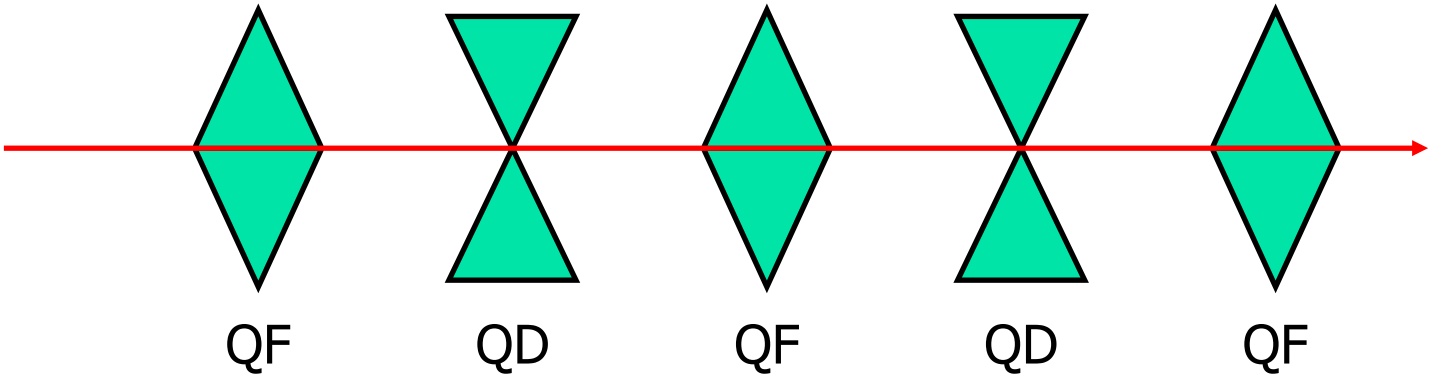
but the opposite effect in the other plane!

Is it really interesting ?

Alternating gradient focusing

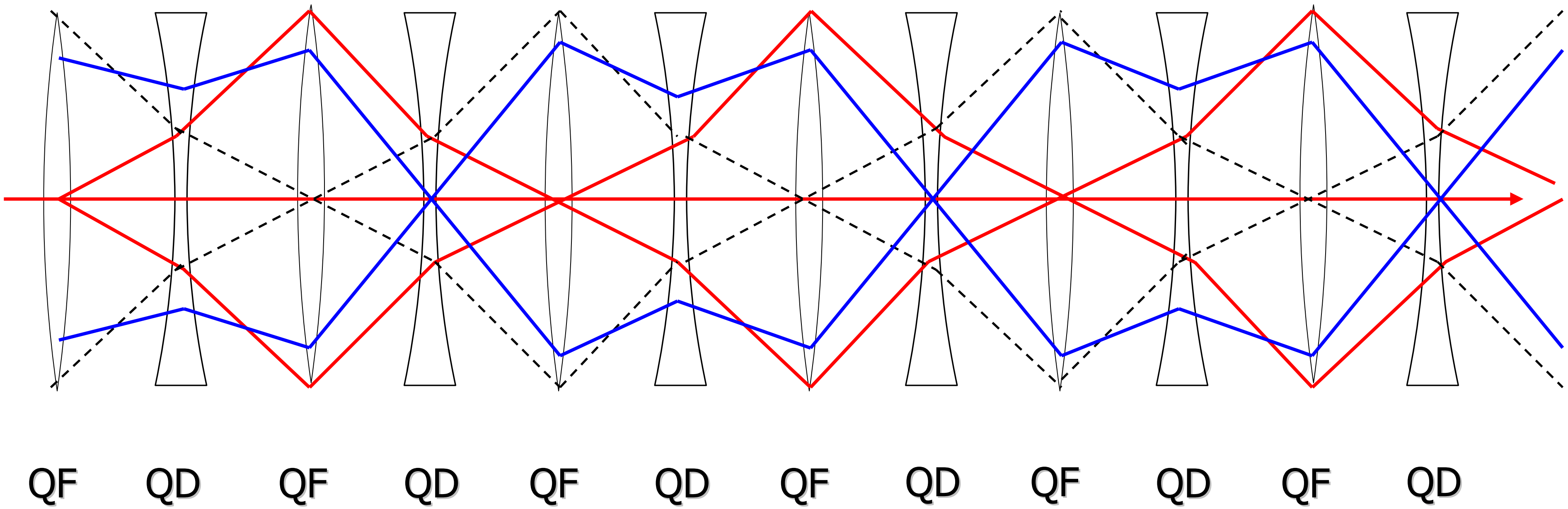
Basic new idea:

Alternate QF and QD



valid for one plane only (H or V) !

Alternating gradient focusing



Alternating gradient focusing:

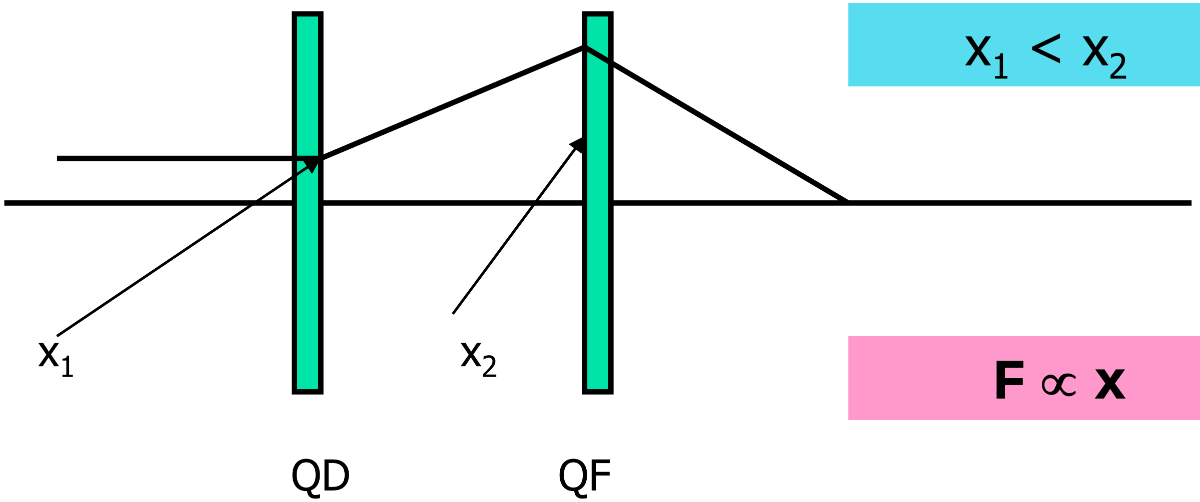
It can be shown that a section composed of alternating focusing and defocusing elements has a net focusing effect, provided the quadrupoles are correctly placed.

Particles for which $x, x', y, y' \neq 0$ thus oscillate around the ideal particle …

but the trajectories remain inside the vacuum chamber !

Why net focusing effect?

Purely intuitively:

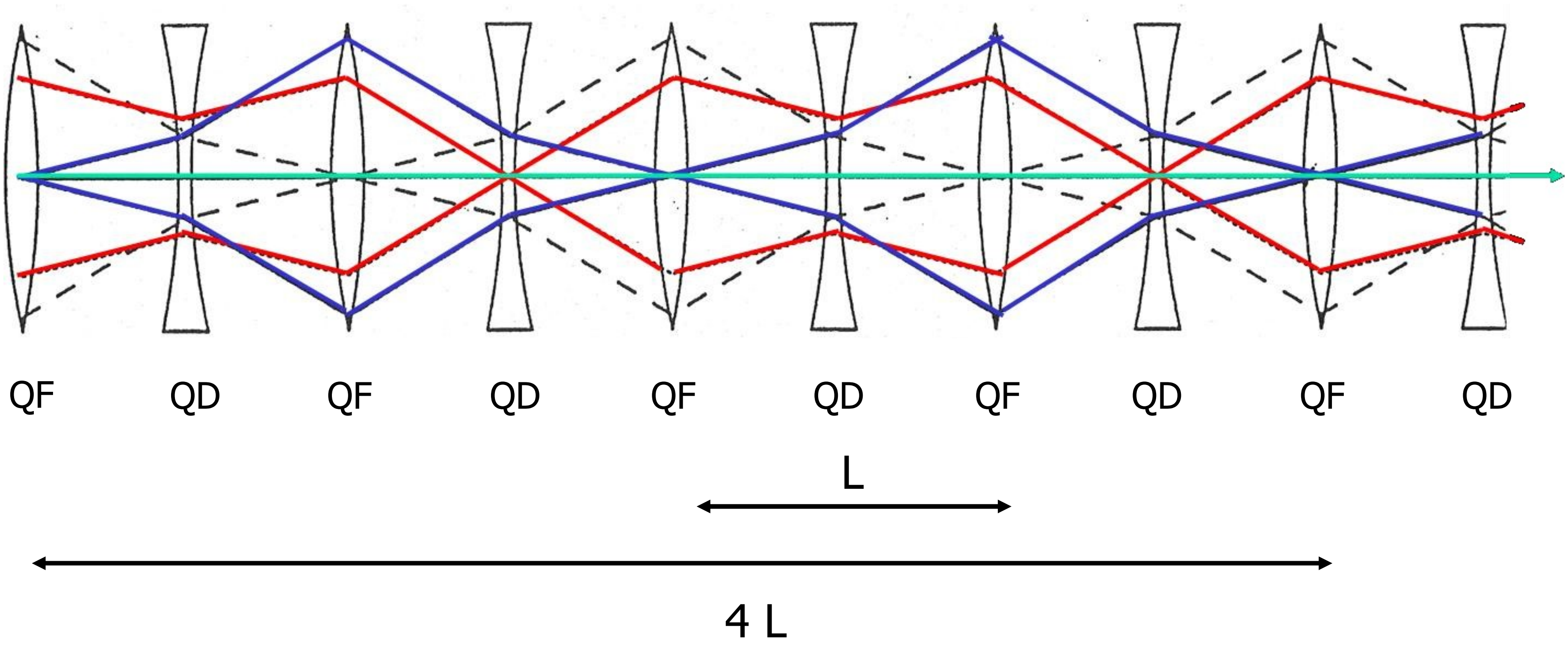


$x_1 < x_2$

$\mathbf{F \propto x}$

Rigorous treatment rather straightforward !

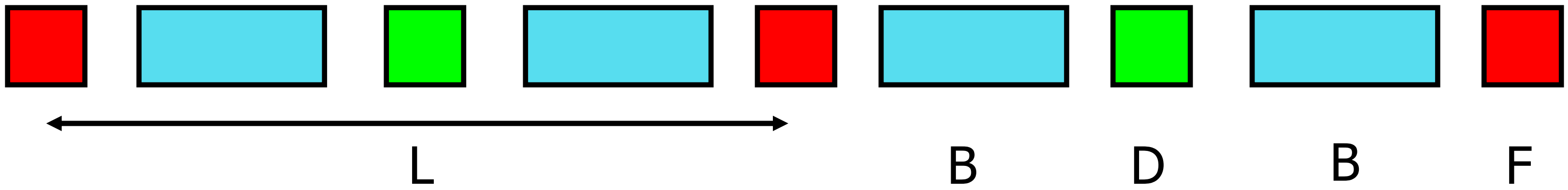
The concept of the « FODO cell »



One complete oscillation in 4 cells $\Rightarrow$ 90°/ cell $\Rightarrow$ **μ = 90°**

Real circular machines

The accelerator is composed of a periodic repetition of cells:



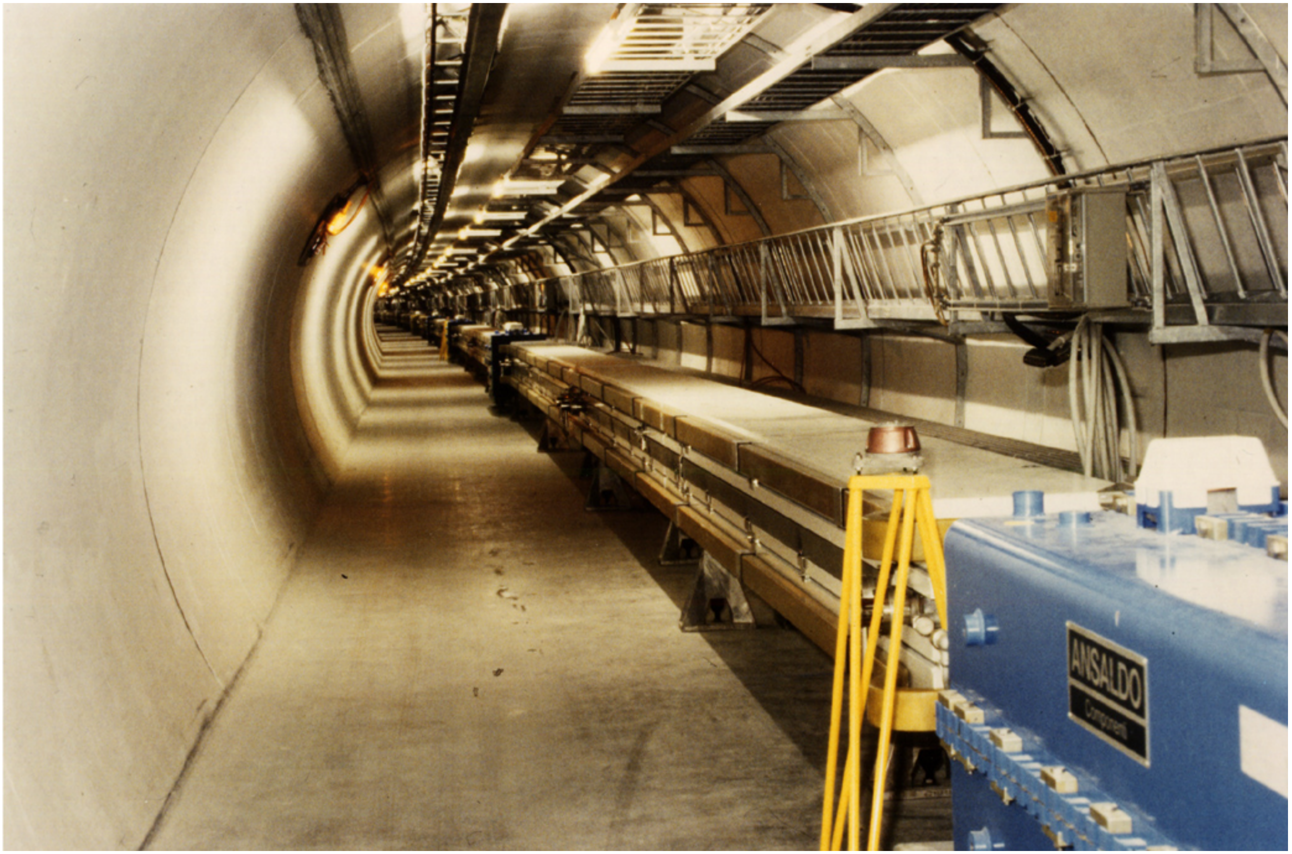
- The phase advance per cell μ can be modified, in each plane, by varying the strength of the quadrupoles.

- The ideal particle will follow a particular trajectory, which closes on itself after one revolution: the closed orbit.

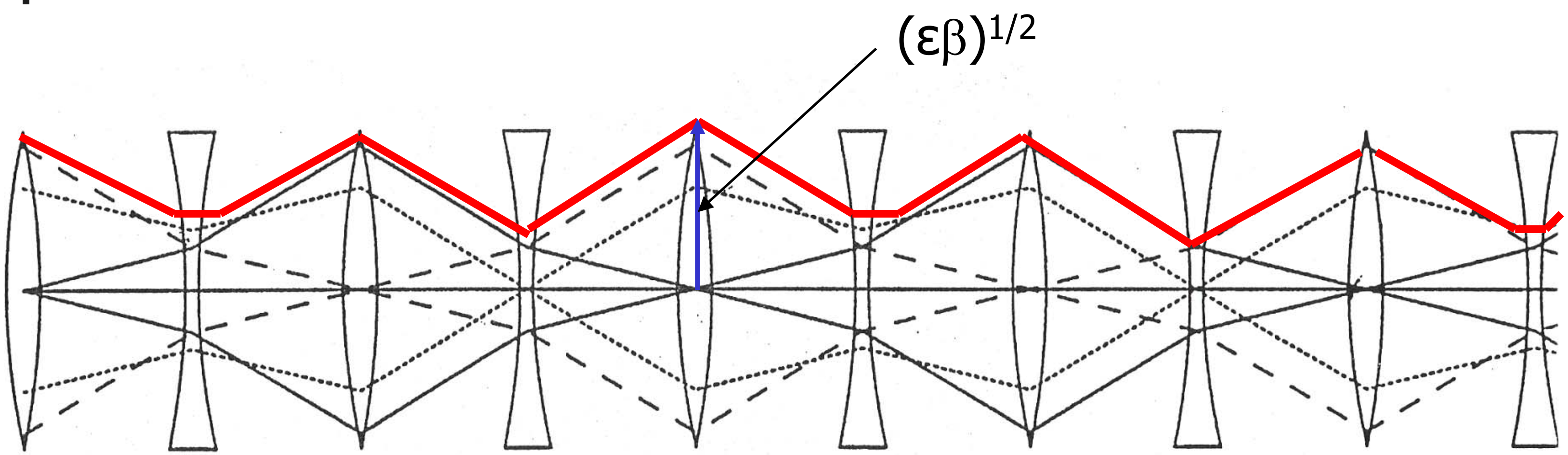
- The real particles will perform oscillations around the closed orbit.

- The number of oscillations for a **complete revolution** is called the **Tune Q** of the machine (Qx and Qy).

Regular periodic lattice: The Arc



The beta function $\beta(s)$



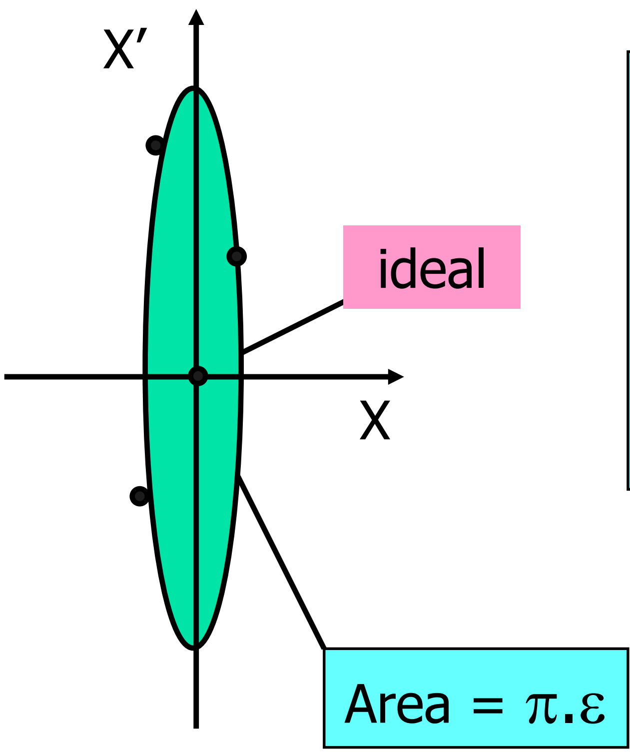
The β-function is the envelope around all the trajectories of the particles circulating in the machine.

The β-function has a minimum at the QD and a maximum at the QF, ensuring the net focusing effect of the lattice.

It is a periodic function (repetition of cells). The oscillations of the particles are called betatron motion or **betatron oscillations**.

Phase space at some position (s)

- Select the particle for which 65% of the particles (1 σ) have a smaller betatron motion and plot its position vs. its phase (x vs. x') at some location in the machine for many turns.

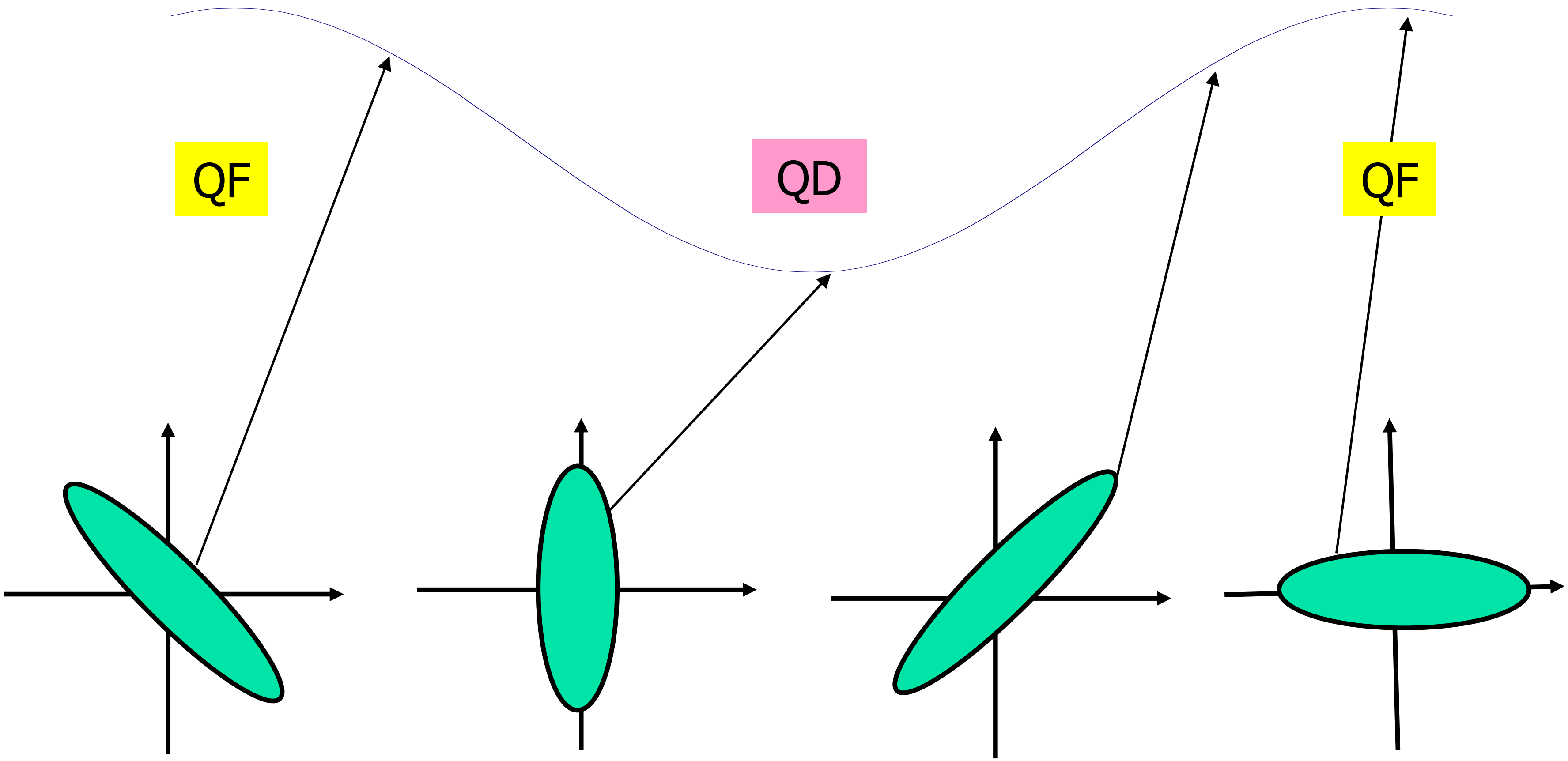


- ε Is the emittance of the beam [mm mrad]
- ε is a property of the beam (quality)
- Measure of how much particle depart from ideal trajectory.
- β is a property of the machine (quadrupoles).

Beam size [m]

$$\sigma(s) = (\varepsilon.\beta(s))^{1/2}$$

Emittance conservation



The shape of the ellipse varies along the machine, but its area (the emittance ε) remains constant for a given energy.

Recapitulation 1

- The **fraction** of the oscillation performed in a periodic cell is called the **phase advance μ per cell** (x or y).

- The total number of oscillations over one full turn of the machine is called the **betatron tune Q** (x or y).

- The envelope of the betatron oscillations is characterised by the **beta function $\beta(s)$**. This is a property of the quadrupole settings.

- The quality of the (injected) beam is characterised by the **emittance** ε. This is a property of the beam and is invariant around the machine.

- The r.m.s. beam size (measurable quantity) is $\sigma = (\beta.\varepsilon)^{1/2}$.

Off momentum particles:

- These are “non-ideal” particles, in the sense that they do not have the right energy, i.e. all particles with $\Delta p/p \neq 0$

What happens to these particles when traversing the magnets ?

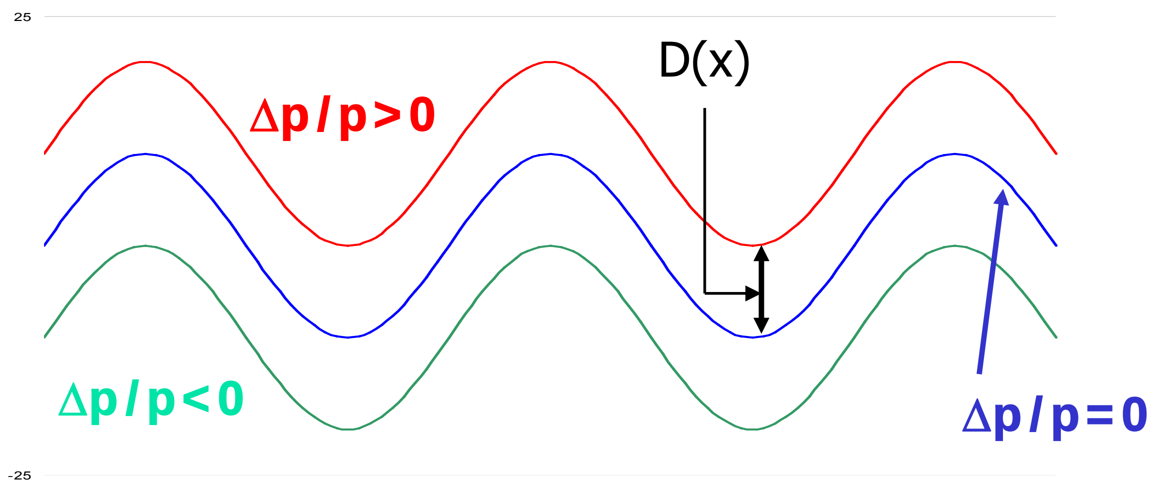
Off momentum particles ($\Delta p/p \neq 0$)

Effect from Dipoles

- If $\Delta p/p > 0$, particles are less bent in the dipoles ➔ should spiral out !
- If $\Delta p/p < 0$, particles are more bent in the dipoles ➔ should spiral in !

No!

There is an equilibrium with the restoring force of the quadrupoles

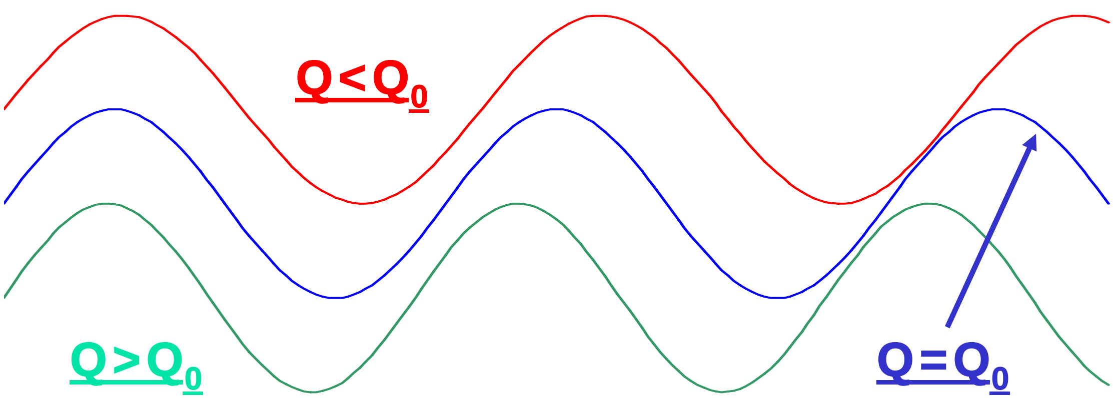


Off momentum particles ($\Delta p/p \neq 0$)

Effect from Quadrupoles

- If $\Delta p/p > 0$, particles are less focused in the quadrupoles ➔ **lower Q !**
- If $\Delta p/p < 0$, particles are more focused in the quadrupoles ➔ **higher Q !**

Particles with different momenta would have a different betatron tune $Q=f(\Delta p/p)$!



The chromaticity Q'

Particles with different momenta ($\Delta p/p$) would thus have different tunes Q.
So what ?

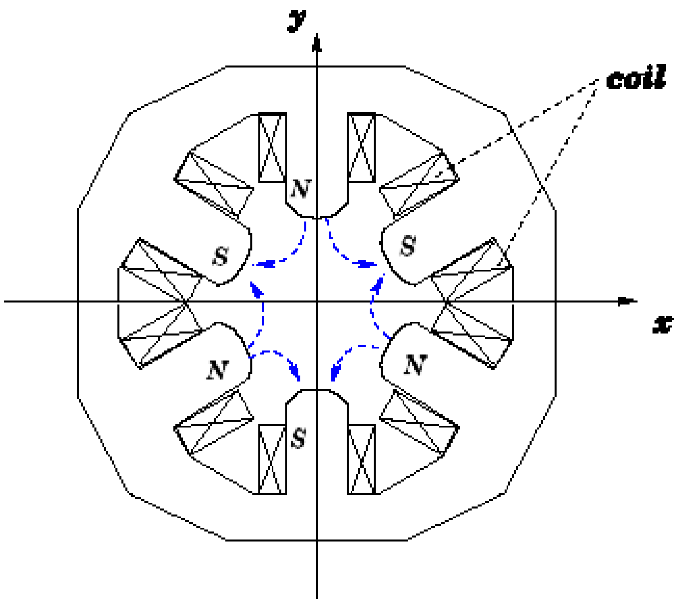
unfortunately

- The tune dependence on momentum is of fundamental importance for the stability of the machine. It is described by the chromaticity of the machine Q':

$$Q' = \Delta Q / (\Delta p/p)$$

The chromaticity has to be carefully controlled and corrected for stability reasons. This is achieved by means of sextupoles.

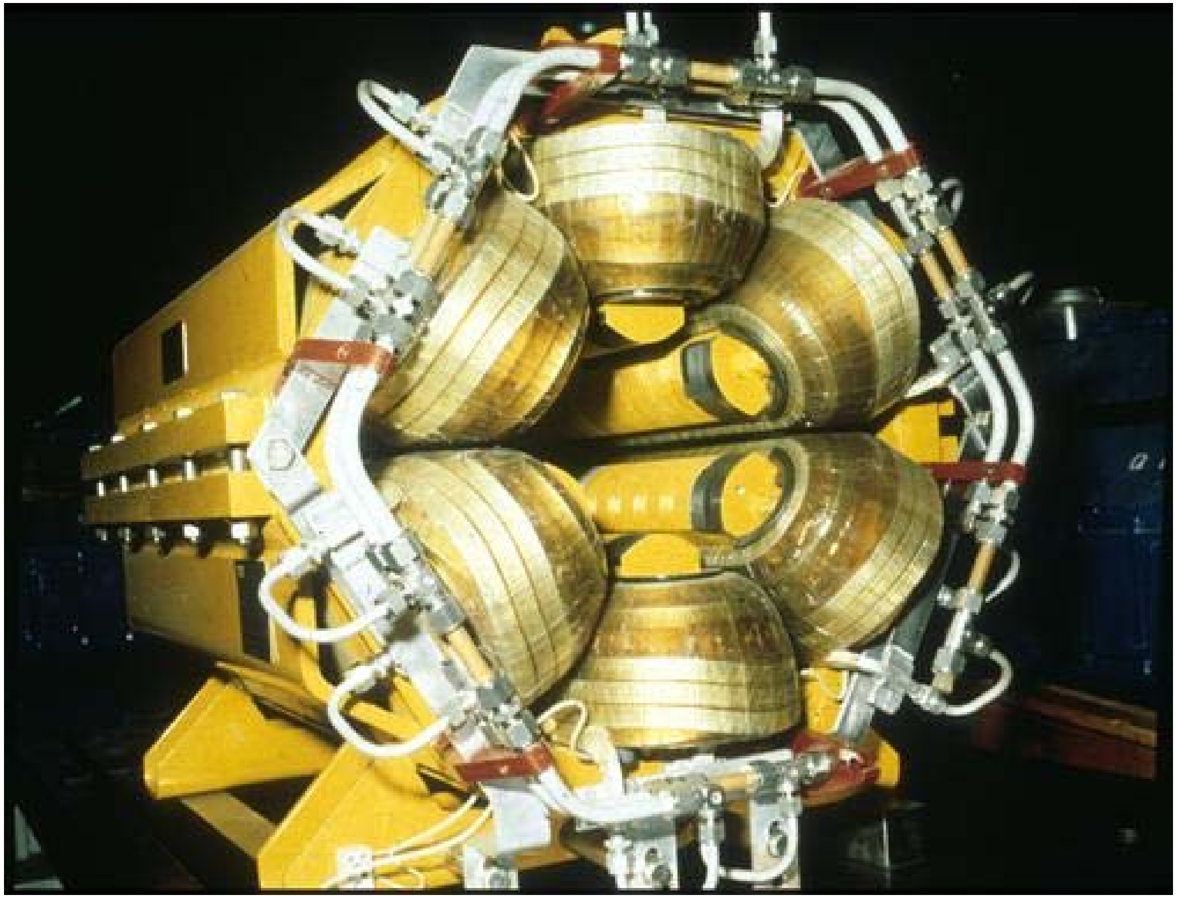
The sextupoles (SF and SD)



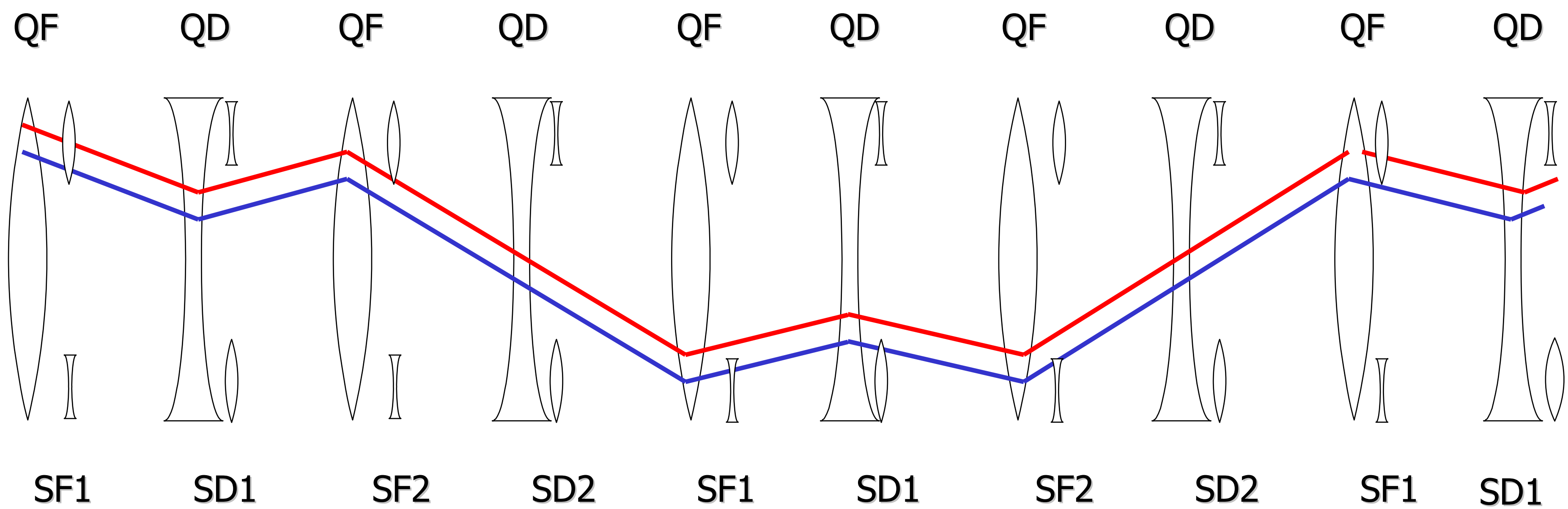
- $\Delta x' \propto x^2$
- A SF sextupole basically « adds » focusing for the particles with $\Delta p/p > 0$, and « reduces » it for $\Delta p/p < 0$.
- The chromaticity is corrected by adding a sextupole after each quadrupole of the FODO lattice.

Sextupoles:

SPS



Chromaticity correction



The undesired effect of sextupoles on particles with the nominal energy can be avoided by grouping the sextupoles into « families ».

Nr. of families:

$N = (k * 180\,°)/\mu = \text{Integer}$

e.g. 180 °/ 90 ° = 2

Recapitulation 2

- For off momentum particles ($\Delta p/p \neq 0$), the magnets induce other important effects, namely:

  - The dispersion (dipoles)
  - The chromaticity (quadrupoles)

Longitudinal plane

➢ So far, we considered only the motion in the transverse planes from an intuitive point of view. The corresponding rigorous treatment will be given in the lectures on "**Transverse Beam Dynamics**".

➢ The lectures on "**Longitudinal Beam Dynamics**" will explain the details of the corresponding longitudinal motion as well as the RF acceleration of the particles.

Natural chromaticity…

- Take a particle and slightly increase its momentum:

  □ $\Delta p/p > 0$ □ $\Delta Q < 0$ □ $Q' < 0$

- Take a particle and slightly decrease its momentum:

  □ $\Delta p/p < 0$ □ $\Delta Q > 0$ □ $Q' < 0$

Natural Q' is always negative !

Tunes of the machine

Why do we have to control the tunes (Q_x and Q_y) so accurately?

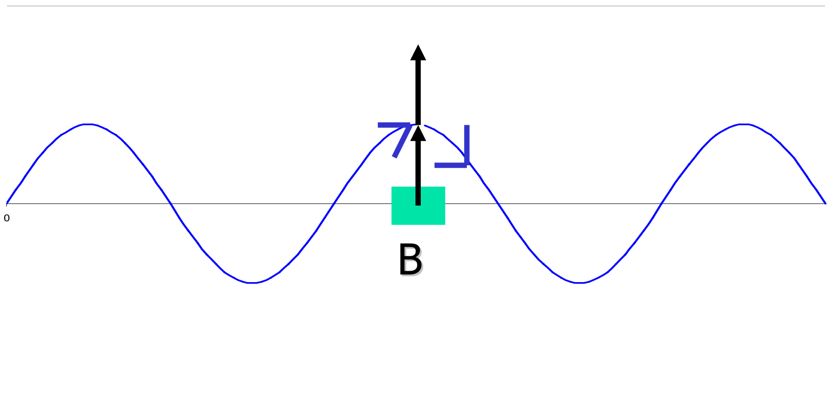
Because there are some (many !) forbidden values!

Forbidden values for Q

- An error in a dipole gives a kick which has always the same sign!

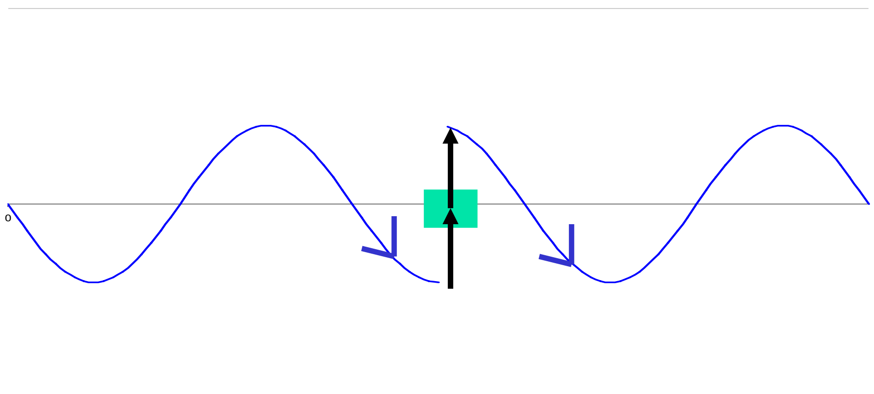
Integer Tune Q = N

Forbidden!
The perturbation adds up!



Half-integer Tune Q = N + 0.5

O.K. for an error in a dipole!
The perturbation cancels after each turn!

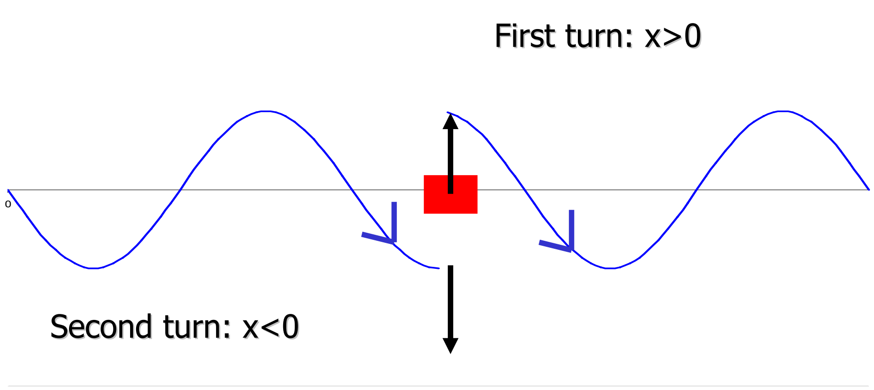


Forbidden values for Q

- An error in a quadrupole gives a kick whose sign depends on x

Half-integer Tune Q = N + 0.5

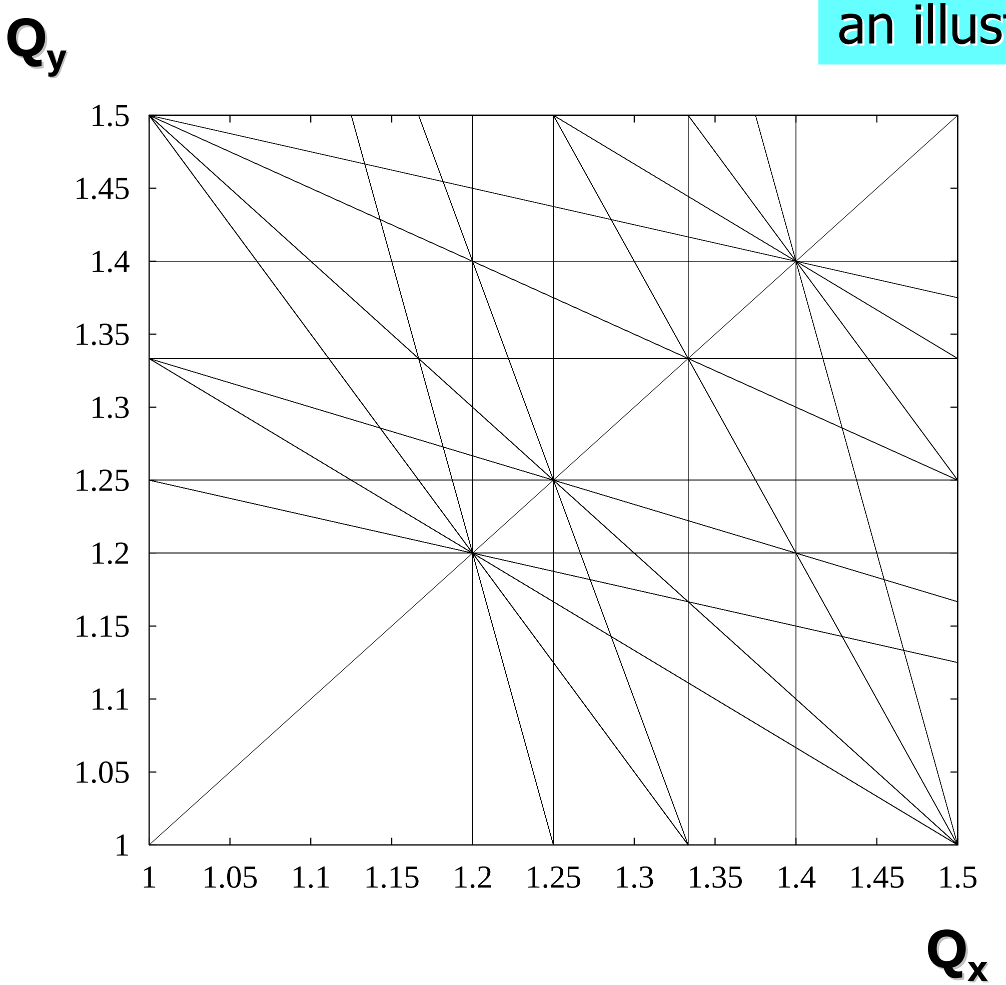
Forbidden !
The amplitude of the oscillation is steadily increasing!



Similar conclusions for 1/3, 1/4, 1/5, …

Tune diagram

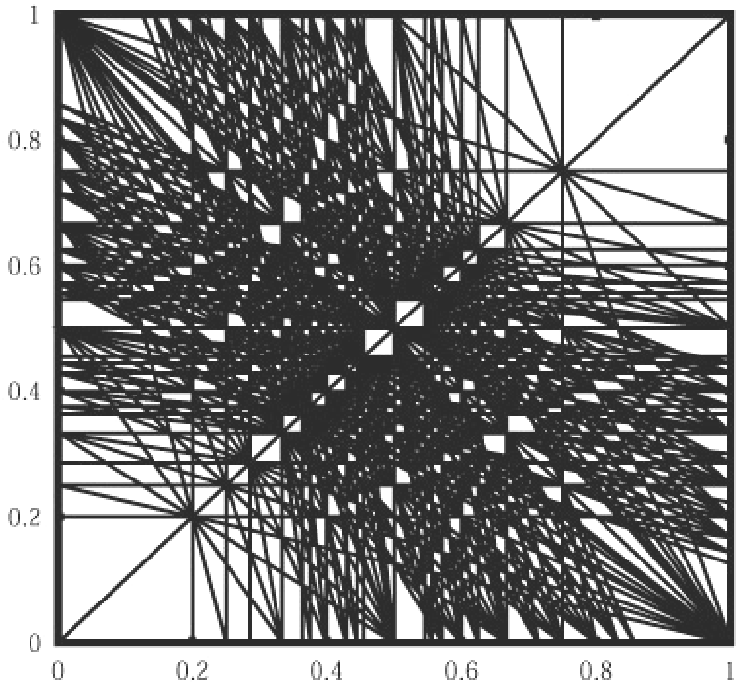
an illustration for a lepton machine:



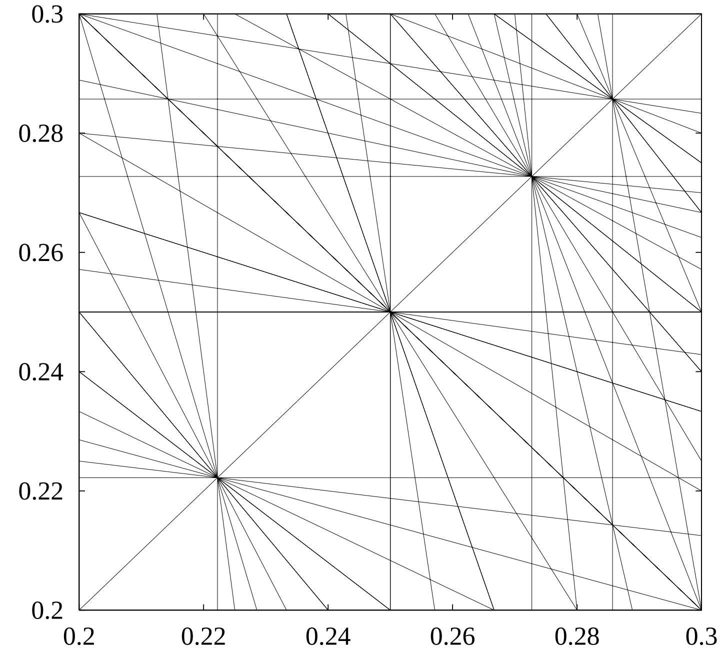
Tune values
(Qx and/or Qy)
which are forbidden
in order to avoid
resonances

The lowest the order of the resonance, the most dangerous it is.

Tune diagram for protons



The particles have a certain tune spread, the bunch thus represents a small area rather than a point in the tune diagram.



That's it for the Introduction…

Thank you very much for your attention !!!