Why this Introduction?

- During this school, you will learn about **beam dynamics** in a rigorous way…
- but some of you are completely new to the field of accelerator physics.
- It seemed therefore justified to start with the introduction of a few very basic concepts, which will be used throughout the course.

This is a completely intuitive approach (no mathematics) aimed at highlighting the physical concepts, without any attempt to achieve any scientific derivation.
Units: the electronvolt (eV)

The electronvolt (eV) is the energy gained by an electron travelling, in vacuum, between two points with a voltage difference of 1 Volt.  

\[ 1 \text{ eV} = 1.602 \times 10^{-19} \text{ Joule} \]

We also frequently use the electronvolt to express masses from \( E=mc^2 \):  

\[ 1 \text{ eV}/c^2 = 1.783 \times 10^{-36} \text{ kg} \]
In order to describe the motion of the particles, each particle is characterised by:

- Its azimuthal position along the machine: \( s \)
- Its Energy: \( E \)
- Its horizontal position: \( x \)
- Its horizontal slope: \( x' \)
- Its vertical position: \( y \)
- Its vertical slope: \( y' \)

i.e. a sixth dimensional vector 
\((s, E, x, x', y, y')\)
Beam Dynamics (2)

- In an accelerator designed to operate at the energy $E_{\text{nom}}$, all particles having $(s, E_{\text{nom}}, 0, 0, 0, 0)$ will happily fly through the center of the vacuum chamber without any problem. These are “ideal particles”.

- The difficulties start when:
  - one introduces dipole magnets
  - the energy $E \neq E_{\text{nom}}$ or $(p-p_{\text{nom}}/p_{\text{nom}}) = \Delta p/p_{\text{nom}} \neq 0$
  - either of $x, x', y, y' \neq 0$
Basic problem:

With more than $10^{10}$ particles per bunch, most of them will not be ideal particles, i.e. they are going to be lost!

Purpose of this lecture: how can we keep the particles in the machine?
What is a Particle Accelerator?

➢ a machine to accelerate some particles! How is it done?

➢ Many different possibilities, but rather easy from the general principle:
Ideal linear machines (linacs)

**Advantages:**
- Single pass
- High intensity

**Drawbacks:**
- Single pass

**Available Energy:**

\[ E_{\text{c.m.}} = m \cdot (2 + 2\gamma)^{1/2} = (2m(m+E))^{1/2} \]

with \( \gamma = E/E_0 \)

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Introduction to Accelerators

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Improved solution for $E_{c.m.}$.

Available Energy: $E_{c.m.} = 2m\gamma = 2E$

with $\gamma = E/E_0$

Advantages: High intensity

Drawbacks: Single pass

Space required
Keep particles: circular machines

Basic idea is to keep the particles in the machine for many turns. Move from the linear design.

- Need Bending
- Need Dipoles!

To a circular one:
Circular machines \( (E_{\text{c.m.}} \sim (mE)^{1/2}) \)

**fixed target:**
- **cyclotron**: huge dipole, compact design, \( B = \text{constant} \), low energy, single pass.

**fixed target:**
- **synchrotron**: varying \( B \), small magnets, high energy

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Colliders (E_{c.m.}=2E)

**Colliders:**
- electron – positron
- proton - antiproton

Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions.

**Ex: LHC**
- 8 possible interaction regions
- 4 experiments collecting data
Circular machines: Dipoles

Classical mechanics:
Equilibrium between two forces

Lorentz force
Centrifugal force

\[ F = e \cdot (v \times B) \]
\[ F = \frac{mv^2}{\rho} \]

\[ evB = \frac{mv^2}{\rho} \]

Magnetic rigidity:
\[ B\rho = \frac{mv}{e} = \frac{p}{e} \]

Relation also holds for relativistic case provided the classical momentum \( mv \) is replaced by the relativistic momentum \( p \)
Dipoles (1):
Dipoles (2):
Ideal circular machine:

- Neglecting radiation losses in the dipoles
- Neglecting gravitation

An *ideal particle* would happily circulate on axis in the machine for ever!

Unfortunately: real life is different!

<table>
<thead>
<tr>
<th>Gravitation: $\Delta y = 20$ mm in $64$ msec!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alignment of the machine</td>
</tr>
<tr>
<td>Ground motion</td>
</tr>
<tr>
<td>Energy error of particles and/or $(x, x')<em>{\text{inj}} \neq (x, x')</em>{\text{nominal}}$</td>
</tr>
<tr>
<td>Error in magnet strength (power supplies and calibration)</td>
</tr>
</tbody>
</table>

We need Focusing!
Focusing with quadrupoles

\[ F_x = -g.x \]
\[ F_y = g.y \]

Force increases linearly with displacement.

Unfortunately, effect is opposite in the two planes (H and V).

Remember: this quadrupole is focusing in the horizontal plane but defocusing in the vertical plane!
Quadrupoles:
Focusing properties ...

A quadrupole provides the required effect in one plane...

but the opposite effect in the other plane!

Is it really interesting?
Alternating gradient focusing

Basic new idea:
Alternate QF and QD

valid for one plane only (H or V)!
Alternating gradient focusing
Alternating gradient focusing:

It can be shown that a section composed of alternating focusing and defocusing elements has a net focusing effect, provided the quadrupoles are correctly placed.

Particles for which \( x, x', y, y' \neq 0 \) thus oscillate around the ideal particle ...

but the trajectories remain inside the vacuum chamber!
Why net focusing effect?

Purely intuitively:

\[ x_1 < x_2 \]

Rigorous treatment rather straightforward!
The concept of the « FODO cell »

One complete oscillation in 4 cells ⇒ 90°/ cell ⇒ μ = 90°
Real circular machines

The accelerator is composed of a periodic repetition of cells:

- The phase advance per cell $\mu$ can be modified, in each plane, by varying the strength of the quadrupoles.
- The ideal particle will follow a particular trajectory, which closes on itself after one revolution: the closed orbit.
- The real particles will perform oscillations around the closed orbit.
- The number of oscillations for a complete revolution is called the Tune $Q$ of the machine ($Q_x$ and $Q_y$).
Regular periodic lattice: The Arc
The beta function $\beta(s)$

The $\beta$-function is the *envelope* around all the trajectories of the particles circulating in the machine.

The $\beta$-function has a minimum at the QD and a maximum at the QF, ensuring the net focusing effect of the lattice.

It is a *periodic function* (repetition of cells). The oscillations of the particles are called betatron motion or *betatron oscillations*. 
Select the particle for which 65% of the particles (1 σ) have a smaller betatron motion and plot its position vs. its phase (x vs. x’) at some location in the machine for many turns.

- ε is the emittance of the beam [mm mrad]
- ε is a property of the beam (quality)
- Measure of how much particle depart from ideal trajectory.
- β is a property of the machine (quadrupoles).

Beam size [m]

$$\sigma(s) = (\varepsilon \beta(s))^{1/2}$$

Area = $\pi \cdot \varepsilon$
Emittance conservation

The shape of the ellipse varies along the machine, but its area (the emittance $\varepsilon$) remains constant for a given energy.
Recapitulation 1

- The **fraction** of the oscillation performed in a periodic cell is called the **phase advance** $\mu$ **per cell** (x or y).

- The total number of oscillations over one full turn of the machine is called the **betatron tune** $Q$ (x or y).

- The **envelope** of the betatron oscillations is characterised by the **beta function** $\beta(s)$. This is a property of the quadrupole settings.

- The quality of the (injected) beam is characterised by the **emittance** $\varepsilon$. This is a property of the beam and is **invariant** around the machine.

- The r.m.s. beam size (measurable quantity) is $\sigma = (\beta \cdot \varepsilon)^{1/2}$. 
Off momentum particles:

- These are “non-ideal” particles, in the sense that they do not have the right energy, i.e. all particles with $\Delta p/p \neq 0$.

What happens to these particles when traversing the magnets?
Off momentum particles ($\Delta p/p \neq 0$)

Effect from Dipoles

- If $\Delta p/p > 0$, particles are less bent in the dipoles ⇒ should spiral out!
- If $\Delta p/p < 0$, particles are more bent in the dipoles ⇒ should spiral in!

No!
There is an equilibrium with the restoring force of the quadrupoles
Off momentum particles ($\Delta p/p \neq 0$)

**Effect from Quadrupoles**

- If $\Delta p/p > 0$, particles are less focused in the quadrupoles $\Rightarrow$ lower Q!
- If $\Delta p/p < 0$, particles are more focused in the quadrupoles $\Rightarrow$ higher Q!

Particles with different momenta would have a different betatron tune $Q=f(\Delta p/p)$!
The chromaticity $Q'$

Particles with different momenta ($\Delta p/p$) would thus have different tunes $Q$. So what?

- The tune dependence on momentum is of fundamental importance for the stability of the machine. It is described by the chromaticity of the machine $Q'$:

$$Q' = \frac{\Delta Q}{\Delta p/p}$$

The chromaticity has to be carefully controlled and corrected for stability reasons. This is achieved by means of sextupoles.
The sextupoles (SF and SD)

• $\Delta x' \propto x^2$

• A SF sextupole basically « adds » focusing for the particles with $\Delta p/p > 0$, and « reduces » it for $\Delta p/p < 0$.

• The chromaticity is corrected by adding a sextupole after each quadrupole of the FODO lattice.
Sextupoles:

SPS

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The undesired effect of sextupoles on particles with the **nominal energy** can be avoided by grouping the sextupoles into «families».

Nr. of families:

\[ N = (k \times 180^\circ)/\mu = \text{Integer} \]

\[ \text{e.g. } 180^\circ/90^\circ = 2 \]
Recapitulation 2

- For off momentum particles ($\Delta p/p \neq 0$), the magnets induce other important effects, namely:
  - The dispersion (dipoles)
  - The chromaticity (quadrupoles)
So far, we considered only the motion in the transverse planes from an intuitive point of view. The corresponding rigorous treatment will be given in the lectures on "Transverse Beam Dynamics".

The lectures on "Longitudinal Beam Dynamics" will explain the details of the corresponding longitudinal motion as well as the RF acceleration of the particles.
Natural chromaticity…

- Take a particle and slightly **increase** its momentum:
  \[ \Delta p/p > 0 \quad \Delta Q < 0 \quad Q' < 0 \]

- Take a particle and slightly **decrease** its momentum:
  \[ \Delta p/p < 0 \quad \Delta Q > 0 \quad Q' < 0 \]

**Natural Q’ is always negative!**
Why do we have to control the tunes ($Q_x$ and $Q_y$) so accurately?

Because there are some (!) forbidden values!
Forbidden values for $Q$

- An error in a dipole gives a kick which has always the same sign!

**Integer Tune $Q = N$**

- Forbidden!
- The perturbation adds up!

**Half-integer Tune $Q = N + 0.5$**

- O.K. for an error in a dipole!
- The perturbation cancels after each turn!
Forbidden values for $Q$

- An error in a quadrupole gives a kick whose sign depends on $x$.

Half-integer Tune $Q = N + 0.5$

Forbidden!
The amplitude of the oscillation is steadily increasing!

Similar conclusions for $1/3, 1/4, 1/5, ...$
Tune diagram

Tune values (Qx and/or Qy) which are forbidden in order to avoid resonances

The lowest the order of the resonance, the most dangerous it is.

an illustration for a lepton machine:
The particles have a certain tune spread, the bunch thus represents a small area rather than a point in the tune diagram.
That’s it for the Introduction…

Thank you very much for your attention !!!!