Concept of Luminosity

(in particle colliders)

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http://cern.ch/Werner.Herr/CAS2009/lectures/Darmstadt_luminosity.pdf

http://cern.ch/Werner.Herr/CAS2009/proceedings/lum_proc.pdf

Why colliding beams ?

- **Two beams:** $E_1, \vec{p_1}, E_2, \vec{p_2}, m_1 = m_2 = m$
- $E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2}$
- **Collider versus fixed target:**

  Fixed target: $\vec{p_2} = \mathbf{0}$ ➡ $E_{cm} = \sqrt{2m^2 + 2E_1 m}$

  Collider: $\vec{p_1} = -\vec{p_2}$ ➡ $E_{cm} = E_1 + E_2$
- **LHC (pp): 14000 GeV versus ≈ 115 GeV**
- **LEP (e^+e^-): 210 GeV versus ?**

Collider performance issues

- **Available energy**
- **Number of interactions per second (useful collisions)**
- **Total number of interactions**
- **Secondary issues:**
  - **Time structure of interactions (how often and how many at the same time)**
  - **Space structure of interactions (size of interaction region)**
  - **Quality of interactions (background, dead time etc.)**

Luminosity:

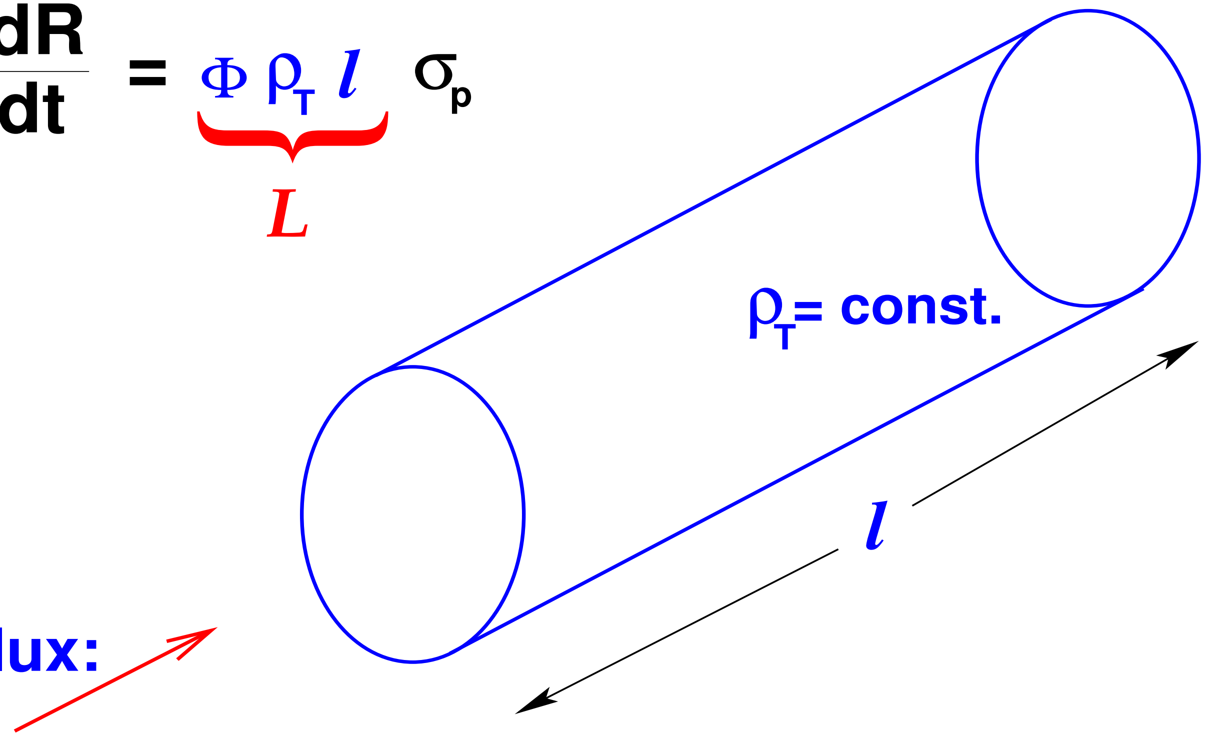
- **We want:**
  - **Proportionality factor between cross section σ_p and number of interactions per second $\frac{dR}{dt}$**

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p \qquad (\rightarrow \quad \mathrm{units:} \ \ \mathrm{cm}^{-2}\mathrm{s}^{-1})$$

  - **Relativistic invariant**
  - **Independent of the physical reaction**
  - **Reliable procedures to compute and measure**

Fixed target luminosity

$$\frac{dR}{dt} = \underbrace{\Phi \, \rho_T \, l}_{L} \, \sigma_p$$

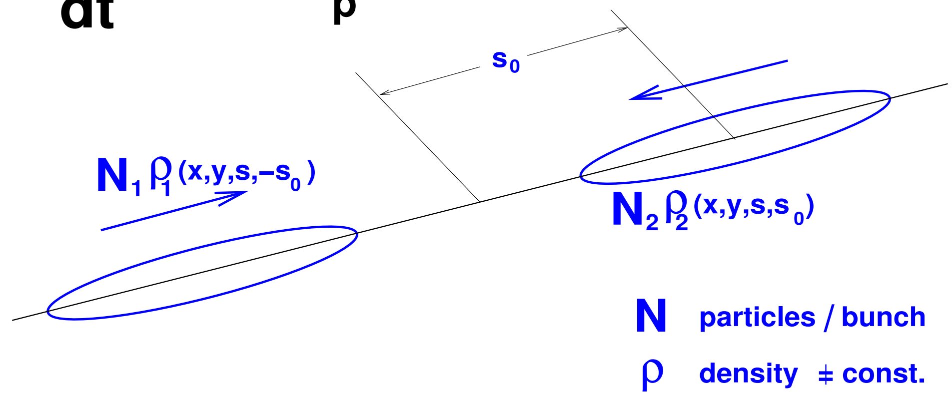


Flux:

$\Phi = N/s$

Collider luminosity (per bunch crossing)

$$\frac{dR}{dt} = L\,\sigma_p$$



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)\,dx\,dy\,ds\,ds_0$$

s_0 is "time"-variable: $s_0 = c \cdot t$

Kinematic factor: $K = \sqrt{(\vec{v_1} - \vec{v_2})^2 - (\vec{v_1} \times \vec{v_2})^2/c^2}$

Collider luminosity (per beam)

- **Assume uncorrelated densities in all planes**

→ **factorize:** $\rho(x, y, s, s_0) = \rho_x(x) \cdot \rho_y(y) \cdot \rho_s(s \pm s_0)$

- **For head-on collisions ($\vec{v_1} = -\vec{v_2}$) we get:**

$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot \int\int\int\int_{-\infty}^{+\infty} dx dy ds ds_0$$

$$\rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s - s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s + s_0)$$

- **In principle: should know all distributions**

→ **Mostly use Gaussian ρ for analytic calculation (in general: it is a good approximation)**

Gaussian distribution functions

- $$\rho_{iz}(z) = \frac{1}{\sigma_z\sqrt{2\pi}}\exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad i = 1,2, \quad z = x,y$$

- $$\rho_{is}(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}}\exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$$

- **For non-Gaussian profiles not always possible to find analytic form, need a numerical integration**

Luminosity for two beams (1 and 2)

- **Simplest case : equal beams**
  - $\sigma_{1x} = \sigma_{2x}, \quad \sigma_{1y} = \sigma_{2y}, \quad \sigma_{1s} = \sigma_{2s}$
  - **but:** $\sigma_{1x} \neq \sigma_{1y}, \quad \sigma_{2x} \neq \sigma_{2y}$ **is allowed**
- **Further: no dispersion at collision point**

Integration (head-on)

for $\sigma_1 = \sigma_2 \quad \rightarrow \quad \rho_1 \rho_2 \; = \; \rho^2$ **:**

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} dx dy ds ds_0$$

integrating over s **and** s_0**, using:**

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi / a}$$

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} dx dy$$

finally after integration over x and y: $\Longrightarrow \quad \mathcal{L} = \dfrac{N_1 N_2 f n_b}{4 \pi \sigma_x \sigma_y}$

Luminosity for two (equal) beams (1 and 2)

- **Simplest case :** $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

  or: $\sigma_{1x} \neq \sigma_{2x} \neq \sigma_{1y} \neq \sigma_{2y}$, $\quad$ *but* : $\sigma_{1s} \approx \sigma_{2s}$

$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \left(\mathcal{L} = \frac{N_1 N_2 f n_b}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} \right)$$

Examples

	Energy (GeV)	$\mathcal{L}_{max}$ cm^{-2}s^{-1}	**rate** s^{-1}	σ_x/σ_y μm/μm	**Particles per bunch**
SPS (p$\bar{p}$)	**315x315**	**6 10^{30}**	**4 10^{5}**	**60/30**	$\approx$ **10 10^{10}**
Tevatron (p$\bar{p}$)	**1000x1000**	**100 10^{30}**	**7 10^{6}**	**30/30**	$\approx$ **30/8 10^{10}**
HERA (e$^+$p)	**30x920**	**40 10^{30}**	**40**	**250/50**	$\approx$ **3/7 10^{10}**
LHC (pp)	**7000x7000**	**10000 10^{30}**	**10^{9}**	**17/17**	$\approx$ **11 10^{10}**
LEP (e$^+$e$^-$)	**105x105**	**100 10^{30}**	$\leq$ **1**	**200/2**	$\approx$ **50 10^{10}**
PEP (e$^+$e$^-$)	**9x3**	**8000 10^{30}**	**NA**	**150/5**	$\approx$ **2/6 10^{10}**

What else ?

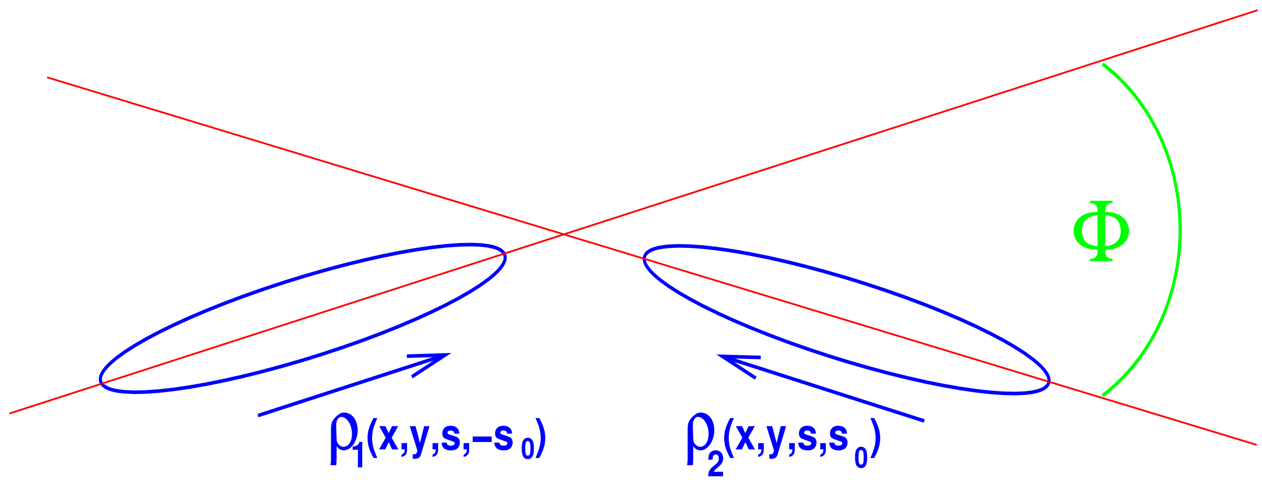
- **What about linear colliders ?**

→ **See later ...**

Complications

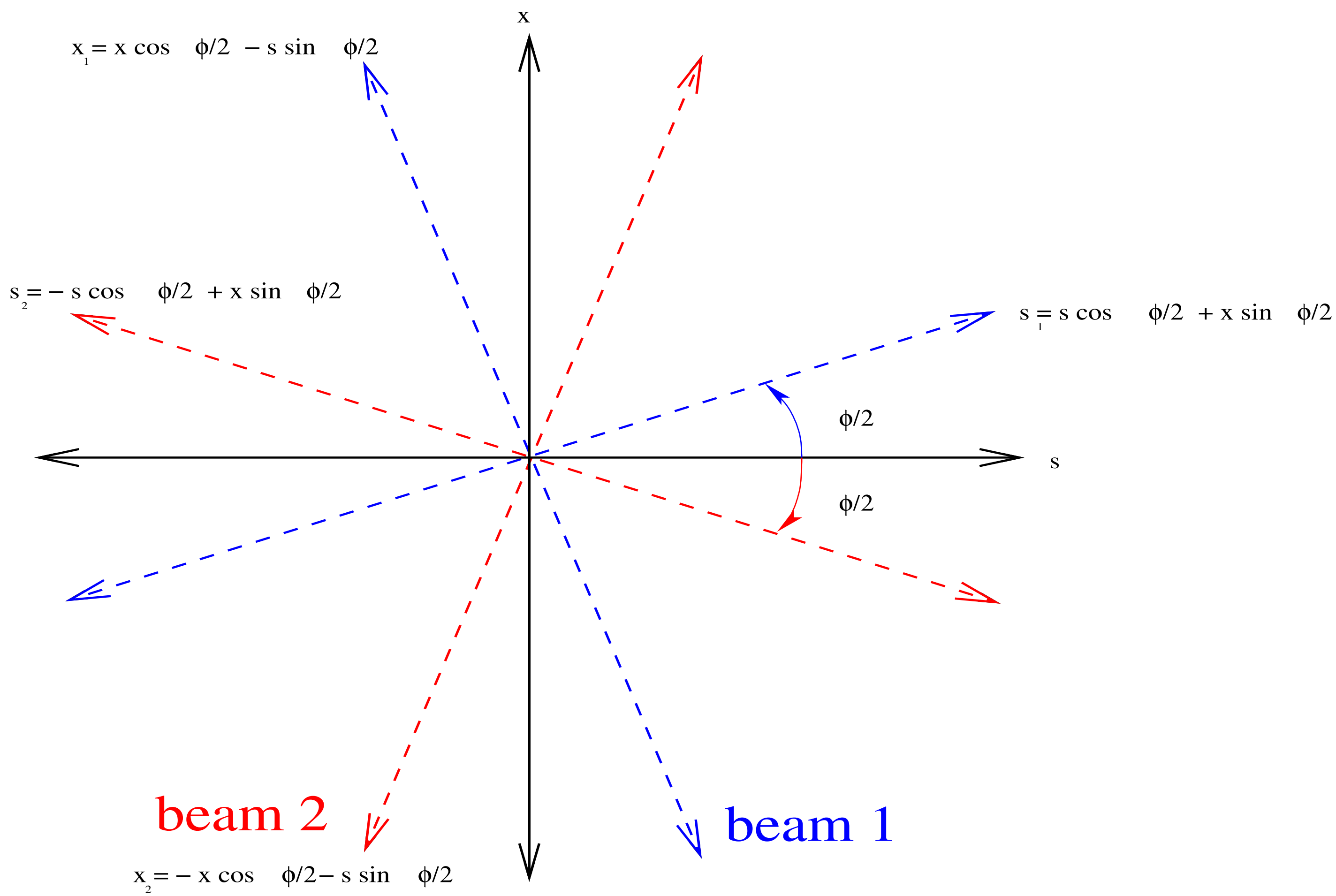
- **Crossing angle**
- **Hour glass effect**
- **Collision offset (wanted or unwanted)**
- **Non-Gaussian profiles**
- **Dispersion at collision point**
- $\delta\beta^*/\delta s \; = \; \alpha^* \; \neq \; 0$
- **Strong coupling**
- **etc.**

Collisions at crossing angle



- Needed to avoid unwanted collisions
  - For colliders with many bunches: LHC, CESR, KEKB
  - For colliders with coasting beams

Collisions angle geometry (horizontal plane)



Crossing angle

Assume crossing in horizontal (x, s)- plane.
Transform to new coordinates:

$$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\mathcal{L} = 2\cos^2\frac{\phi}{2} N_1 N_2 f n_b \int\int\int\int_{-\infty}^{+\infty} dx\,dy\,ds\,ds_0$$
$$\rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - s_0)\rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 + s_0)$$

Integration (crossing angle)

use as before:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

and:

$$\int_{-\infty}^{+\infty} e^{-(at^2+bt+c)} dt \quad = \quad \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{a}}$$

Further:

- **Since σ_x, x and sin($\phi/2$) are small:**
  - **drop all terms $\sigma^k{}_x sin^l(\phi/2)$ or $x^k sin^l(\phi/2)$ for all:** **k+l** $\geq$ 4
  - **approximate: sin($\phi/2$) $\approx$ tan($\phi/2$)$\approx$ $\phi/2$**

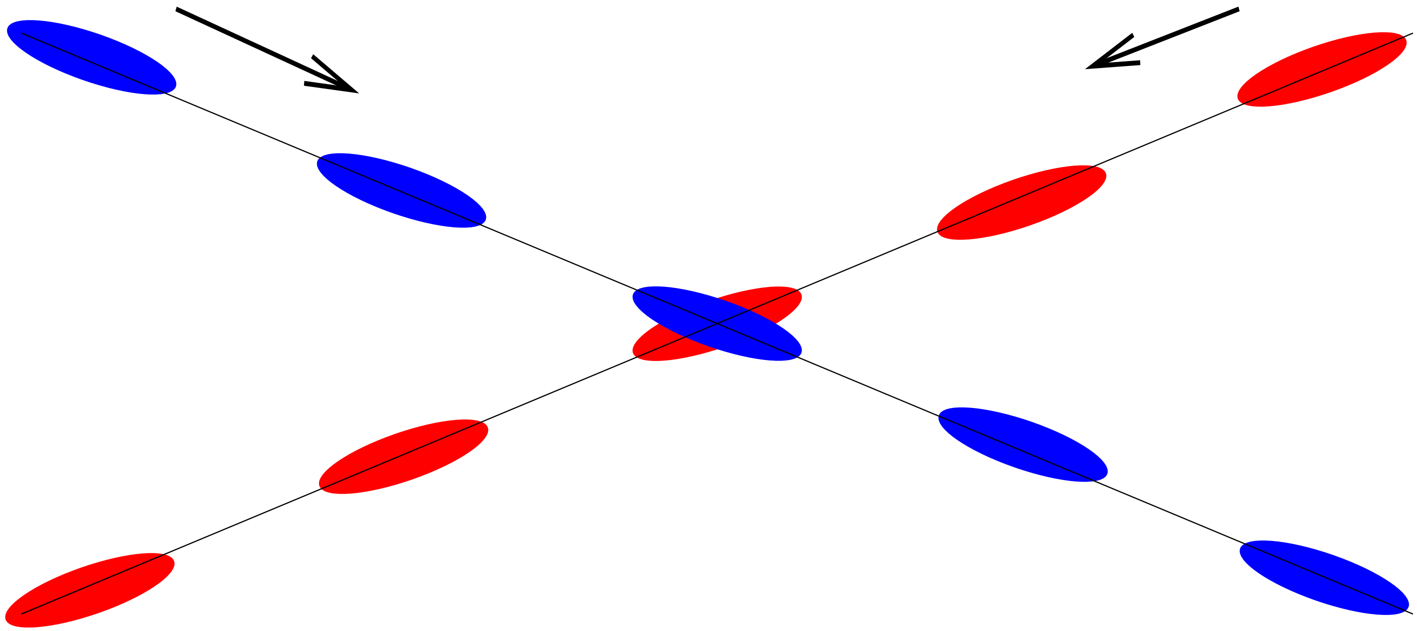
Crossing angle

- **Crossing Angle** $\Rightarrow$ $$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}\cdot S$$
- S **is the reduction factor**
- **For small crossing angles and** $\sigma_s \gg \sigma_{x,y}$
- $$\Rightarrow S = \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

Example LHC:

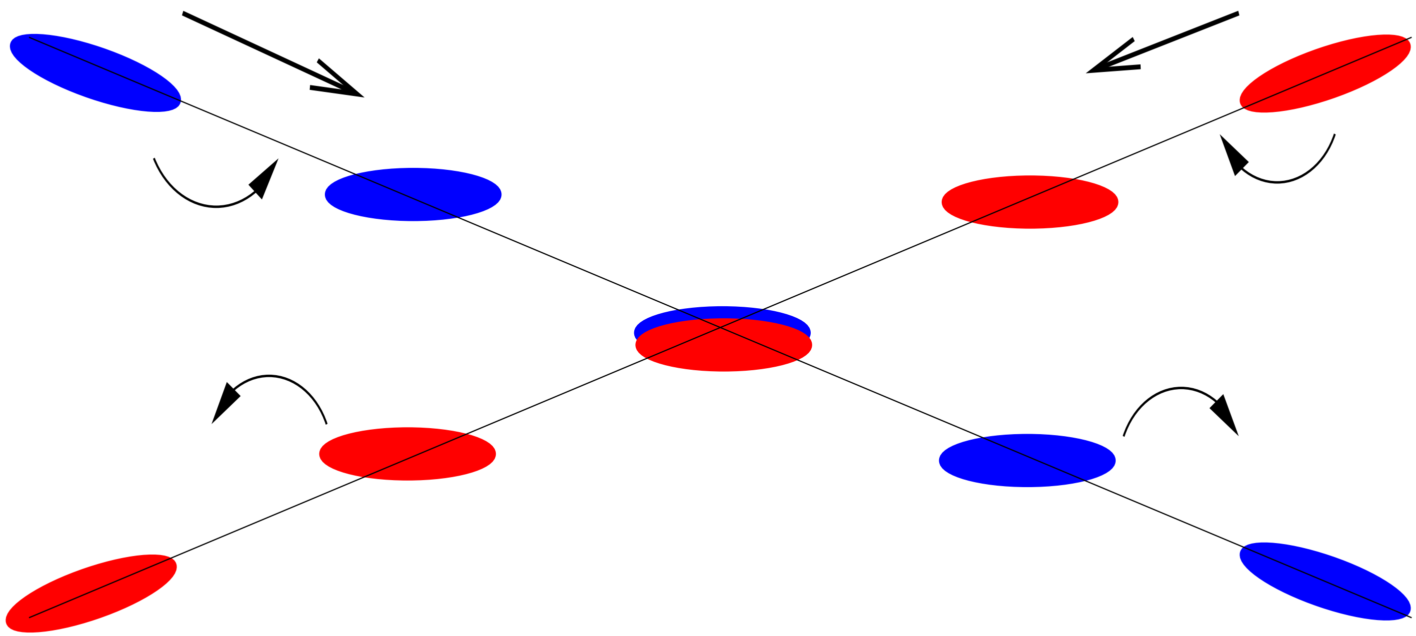
Φ **= 285** μ**rad,** σ_s **= 7.5 cm, S = 0.84**

Large crossing angle



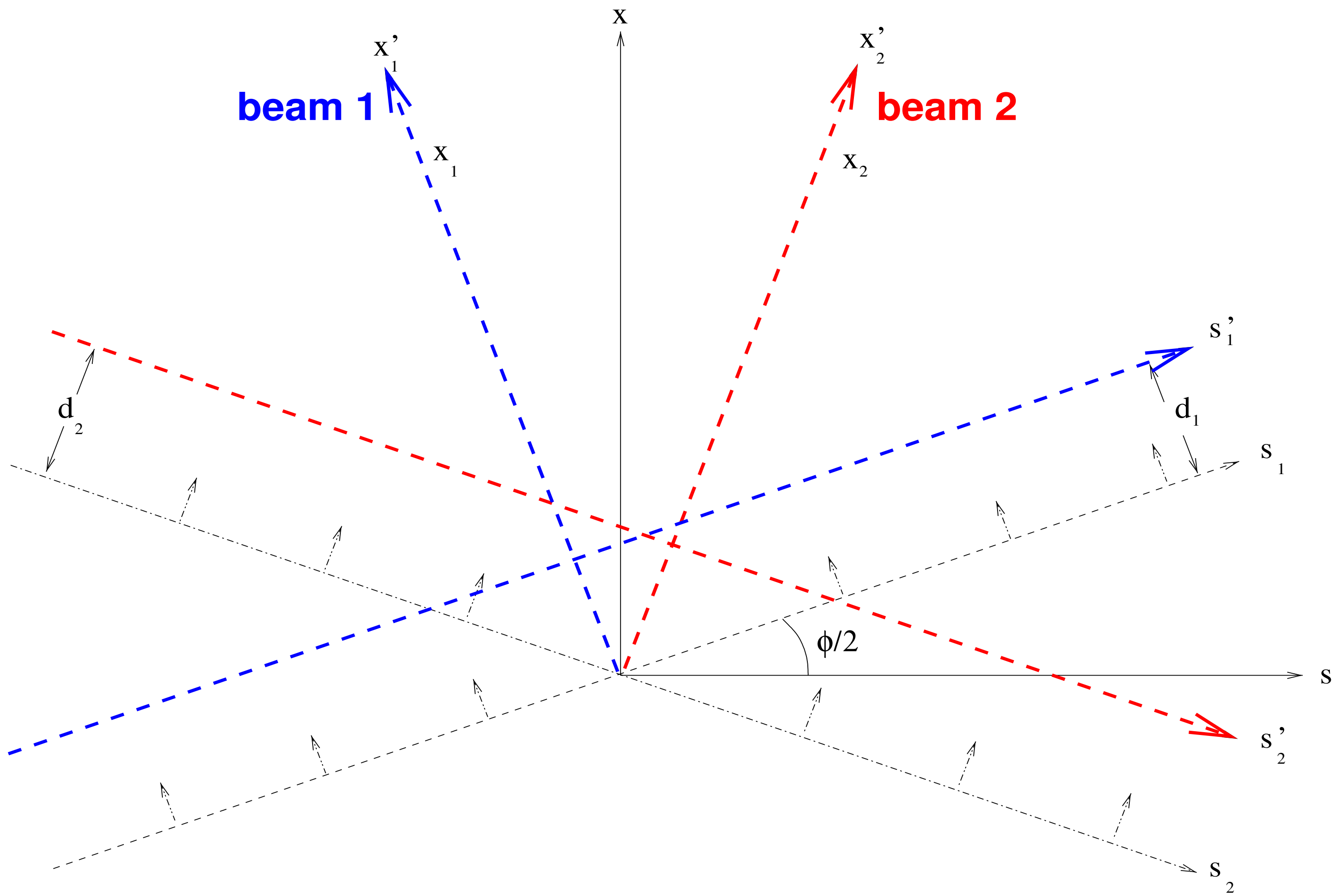
→ **Large crossing angle: large loss of luminosity**

"crab" crossing scheme



- "crab" crossing recovers geometric loss factor
- feasibility needs to be demonstrated

Offset and crossing angle



Offset and crossing angle

- **Transformations with offsets in crossing plane:**

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, \quad & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, \quad & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

- **Gives after integration over y and s_0:**

$$\mathcal{L} = \frac{\mathcal{L}_0}{2\pi\sigma_s\sigma_x} 2\cos^2\frac{\phi}{2} \int\int e^{-\frac{x^2\cos^2(\phi/2)+s^2\sin^2(\phi/2)}{\sigma_x^2}} e^{-\frac{x^2\sin^2(\phi/2)+s^2\cos^2(\phi/2)}{\sigma_s^2}}$$

$$\times \quad e^{-\frac{d_1^2+d_2^2+2(d_1+d_2)x\cos(\phi/2)-2(d_2-d_1)s\sin(\phi/2)}{2\sigma_x^2}} dxds.$$

Offset and crossing angle

After integration over x:

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}}\sigma_s} \quad 2\cos\frac{\phi}{2} \quad \int_{-\infty}^{+\infty} W \cdot \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds$$

with:

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} \qquad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2}$$

and $\quad W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}$

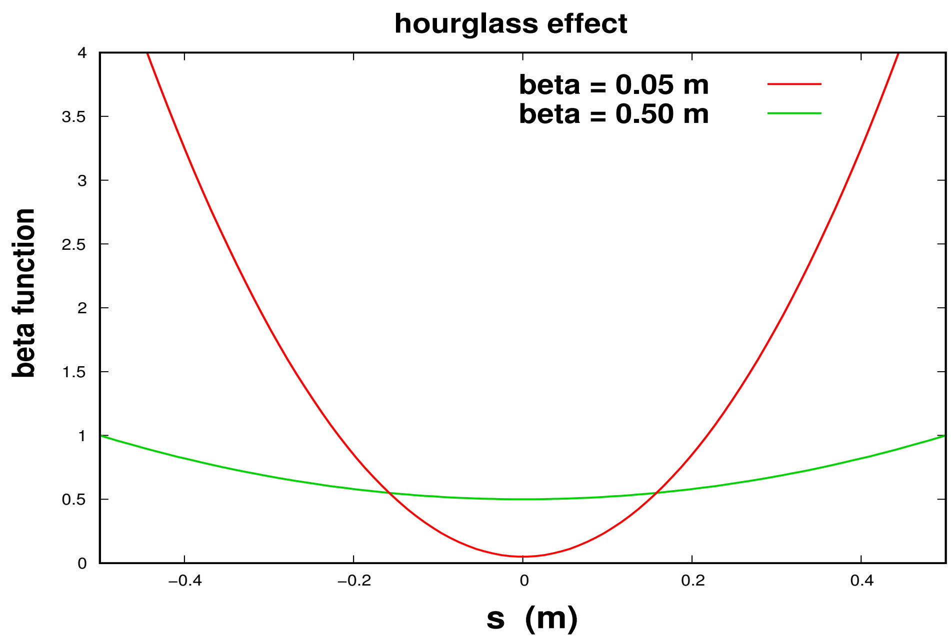
$\Longrightarrow$ **Luminosity with correction factors**

Luminosity with correction factors

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S$$

- W: **correction for beam offset**
- S: **correction for crossing angle**
- $e^{\frac{B^2}{A}}$: **correction for crossing angle and offset**

Hour glass effect

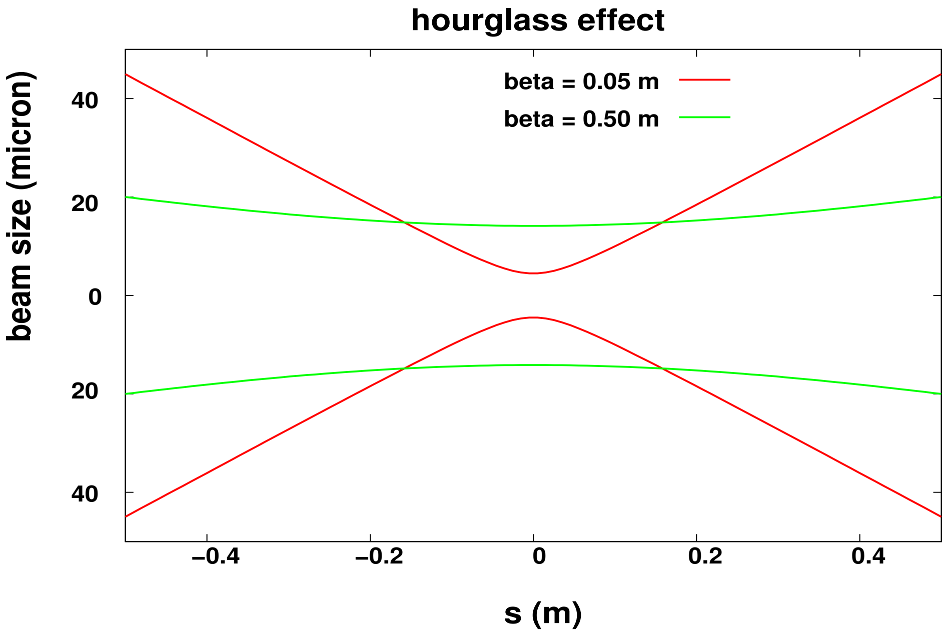


- β**-functions depends on position s**
- $\beta(s) \approx \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$

Hour glass effect

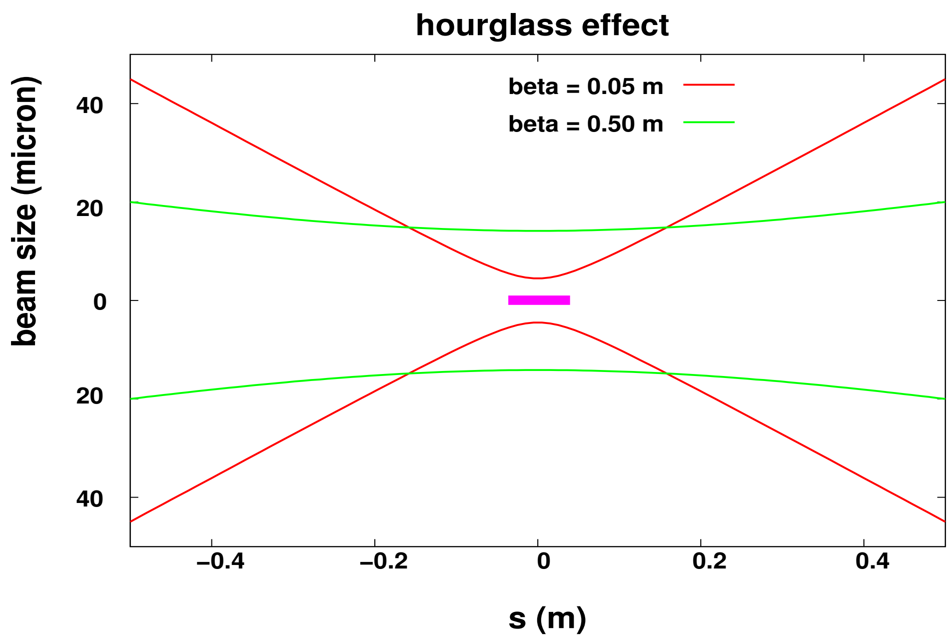
- β-functions depends on position s

Hour glass effect



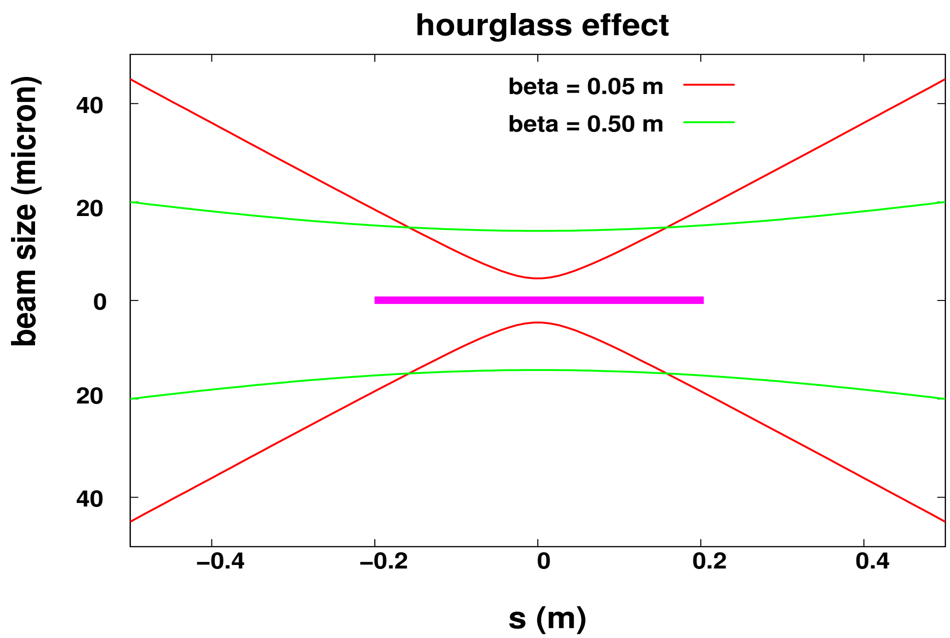
- **Beam size** $\sigma \quad (\propto \sqrt{\beta^*(s)})$ **depends on position s**

Hour glass effect - short bunches



- Small variation of beam size along bunch

Hour glass effect - long bunches



- Significant effect for long bunches and small β^*

Hour glass effect

- β**-functions depends on position s**
- **Usually:** $\beta(s) = \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$
  - **i.e.** $\sigma \implies \sigma(s) \neq$ **const.**
  - $\sigma(s) = \sigma^* \sqrt{(1 + \left(\frac{s}{\beta^*}\right)^2)}$
- **Important when** β^* **comparable to the r.m.s. bunch length** σ_s **(or smaller !)**

Hour glass effect

- Take it easy: $\beta_x^* = \beta_y^*$, crossing angle, but no offset
- Replace σ by σ(s) in standard formulae

$$\mathcal{L} = \left(\frac{N_1 N_2 f n_b}{8\pi}\right) \frac{2\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} \frac{e^{-s^2 A}}{\sigma_x^* \sigma_y^* [1 + (\frac{s}{\beta^*})^2]} ds$$

$$A = \frac{\sin^2\frac{\phi}{2}}{(\sigma_x^*)^2 [1 + (\frac{s}{\beta^*})^2]} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}$$

$\rightarrow$ **Numerical Integration**

Calculations for the LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ **particles/bunch**
- $n_b = 2808$ **bunches/beam**
- $f = 11.2455$ kHz, $\phi = 285$ μ**rad**
- $\beta_x^* = \beta_y^* = 0.55$ m
- $\sigma_x^* = \sigma_y^* = 16.6$ μm, $\sigma_s = 7.7$ cm

- **Simplest case (Head on collision):**

$$\mathcal{L} = 1.200 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle:**

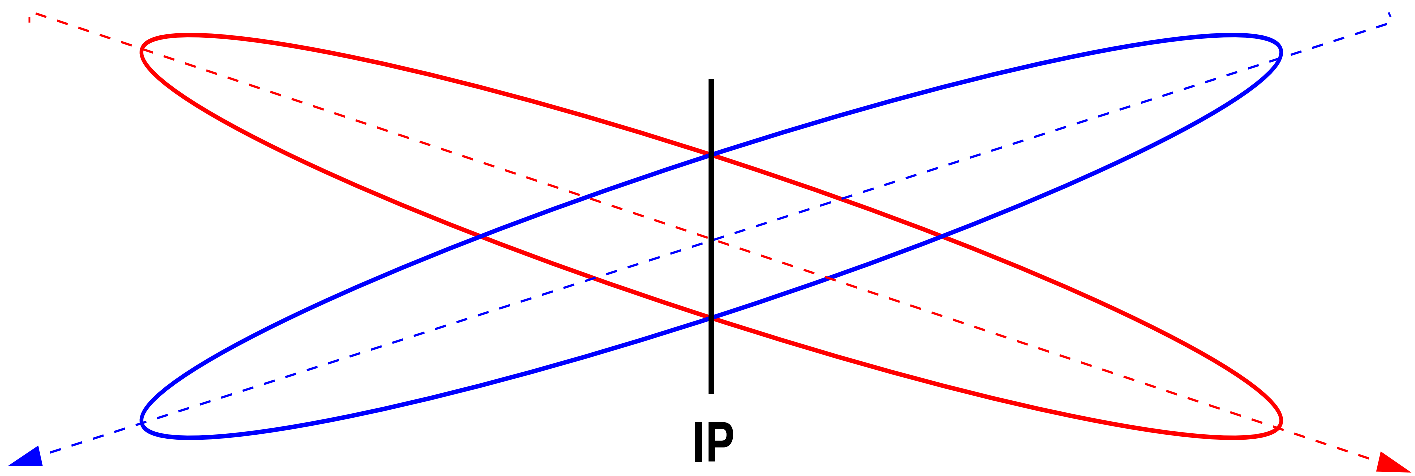
$$\mathcal{L} = 0.973 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle & Hourglass:**

$$\mathcal{L} = 0.969 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

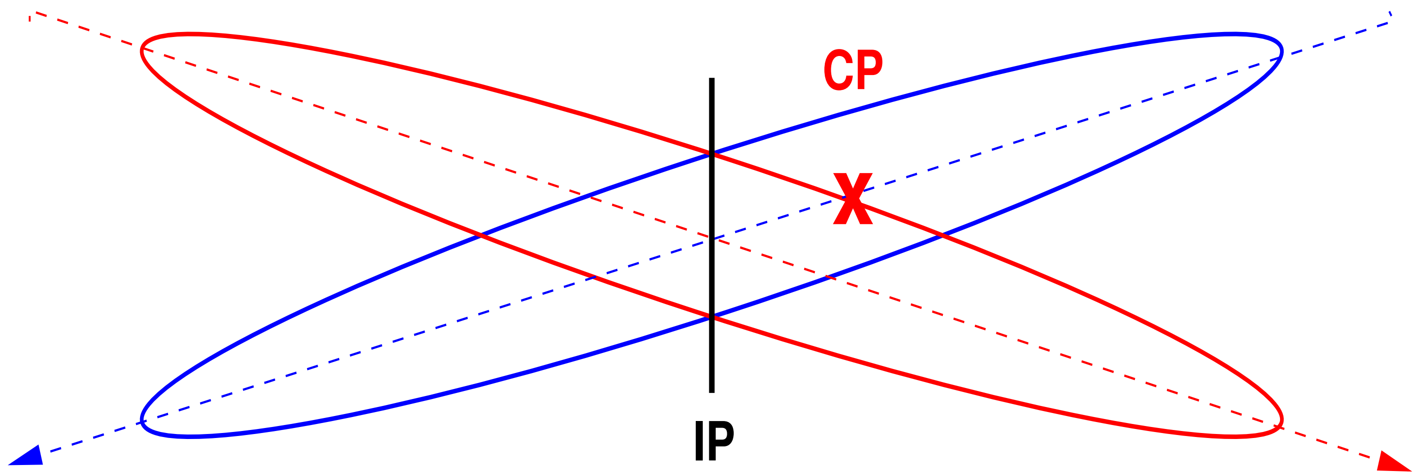
What about large crossing angle and long bunches ???

Large crossing angle - long bunches



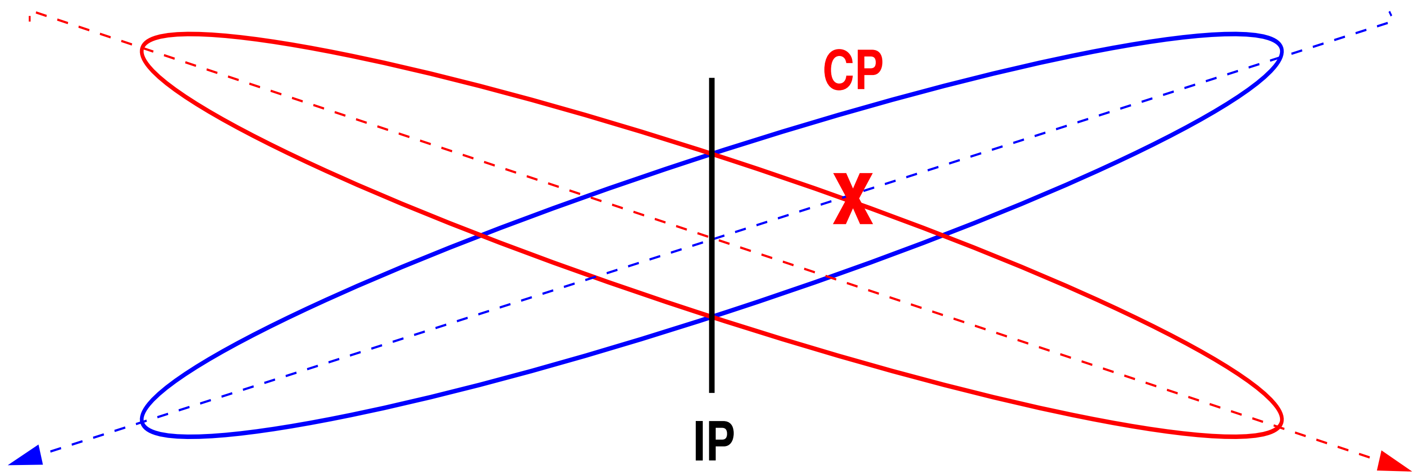
- Assume crossing angle in horizontal plane
- Large crossing angle: large loss of luminosity

Large crossing angle



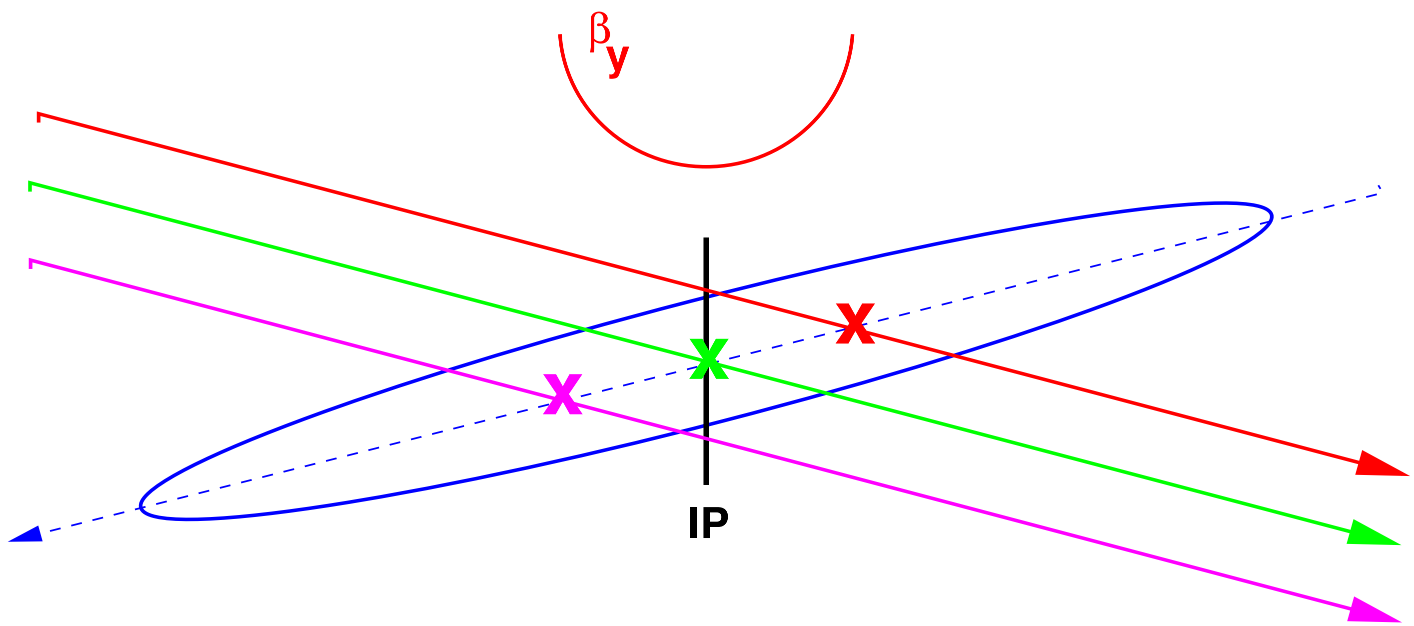
- For large amplitude particles: collision point longitudinally displaced

Large crossing angle



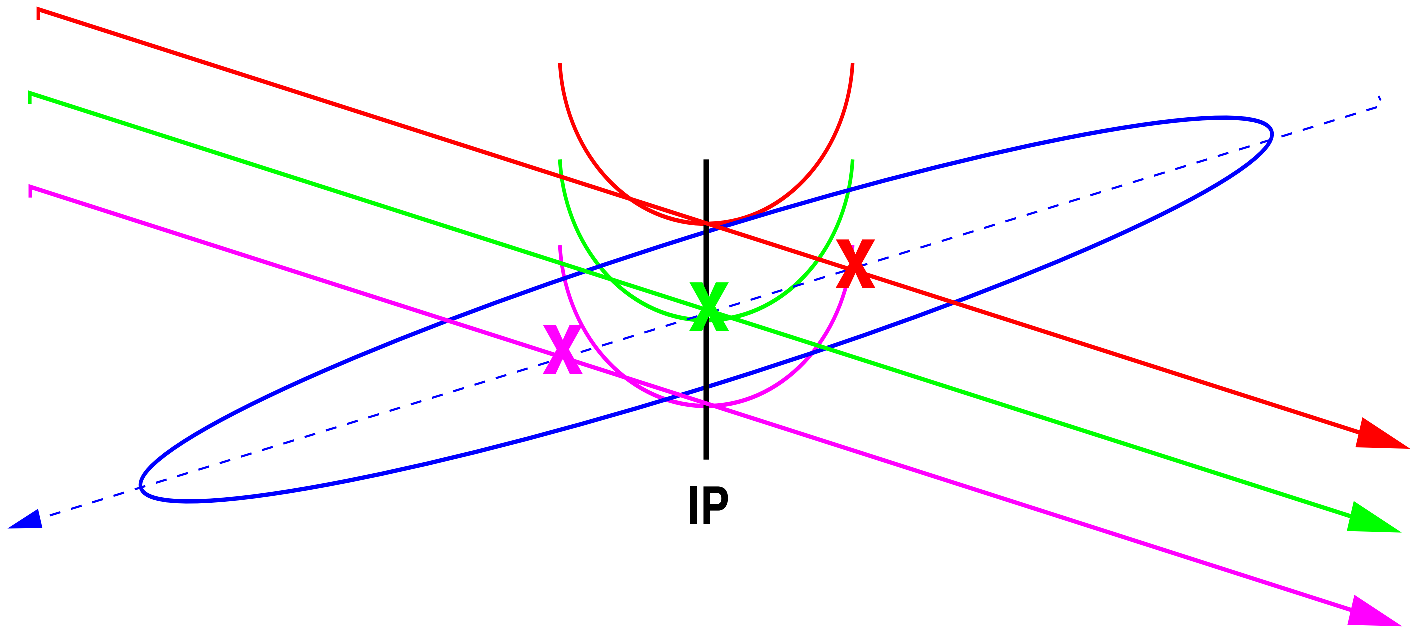
- For large amplitude particles: collision point longitudinally displaced
- Can introduce coupling (transverse and synchro betatron, bad for flat beams)

Large crossing angle



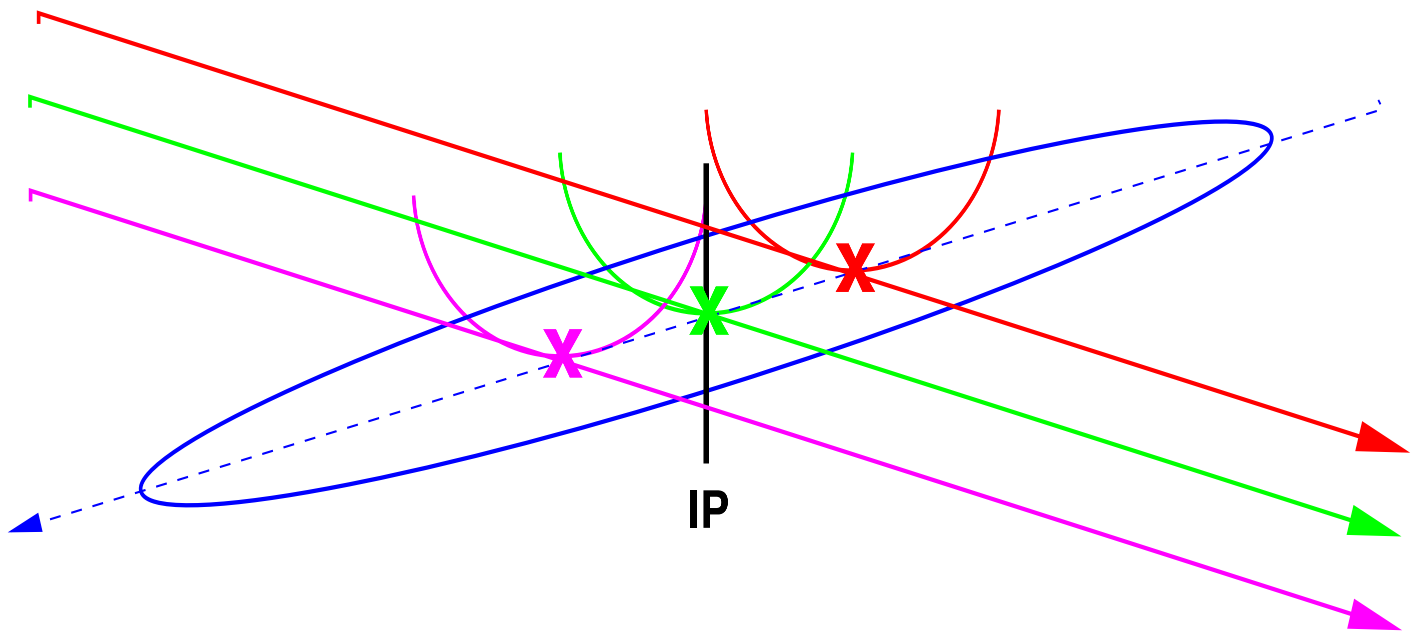
- A particle's collision point amplitude dependent
- Different (vertical) β functions at collision points

Large crossing angle



- A particle's collision point amplitude dependent
- Different β functions at collision points (hour glass !)

"crab waist" scheme



- **Make vertical waist (β_y^{min}) amplitude (x) dependent**
- **All particles in both beams collide in minimum β_y region**

”crab waist” scheme

- Make vertical waist (minimum of β) amplitude (x) dependent
- Without details: can be done with two sextupoles
- First tried at DAPHNE (Frascati) in 2008
- Geometrical gain small
- Smaller vertical tune shift as function of horizontal coordinate
  - Less betatron and synchrotron coupling
  - Good remedy for flat (i.e. lepton) beams with large crossing angle

If the beams are not Gaussian ??

Exercise:

- **Assume flat distributions (normalized to 1)**

  $\rho_1 = \rho_2 = \frac{1}{2a}$, **for** $[-a \leq z \leq a]$, $z = x, y$

  Calculate r.m.s. in x and y:

$$< (x,y)^2 > = \int_{-\infty}^{+\infty} (x,y)^2 \cdot \rho(x,y) dxdy$$

and

$$\mathcal{L} = \int_{-\infty}^{+\infty} \rho_1(x,y)\rho_2(x,y)\ dxdy$$

- **Compute:** $\mathcal{L} \cdot \sqrt{< x^2 > \cdot < y^2 >}$
- **Repeat for various distributions and compare**

Integrated luminosity

- $$\mathcal{L}_{\text{int}} = \int_0^T \mathcal{L}(t)dt$$
- **The figure of merit:**

  $\mathcal{L}_{\text{int}} \cdot \sigma_p =$ **number of events**
- **Experiments: continuous recording of** $\mathcal{L}$
- **For studies: assume some life time behaviour.**

  E.g. $\mathcal{L}(t) \longrightarrow \mathcal{L}_0 \exp\left(-\frac{t}{\tau}\right)$
- **Contributions to life time from: intensity decay, emittance growth etc.**

Integrated luminosity

- Knowledge of preparation time allows optimization of $\mathcal{L}_{\mathrm{int}}$

Integrated luminosity

- **Typical run times LEP:**
  t_r **≈ 8 - 10 hours**
- **For LHC long preparation time t_p expected**

→ **Optimum combination of t_r and t_p gives maximum luminosity**

→ t_r **is usually a "free" parameter, i.e. can be chosen**

Maximising Integrated Luminosity

- **Assume exponential decay of luminosity**
  $\mathcal{L}(t) = \mathcal{L}_0 \cdot e^{t/\tau}$

- **Average luminosity** $< \mathcal{L} >$
  $< \mathcal{L} >= \frac{\int_0^{t_r} dt \mathcal{L}(t)}{t_r + t_p} = \mathcal{L}_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$

- **(Theoretical) maximum for:**
  $t_r \approx \tau \cdot \ln(1 + \sqrt{2t_p/\tau} + t_p/\tau)$

- **Example LHC:** $t_p \approx$ **10h,** $\tau \approx$ **15h,** $\Rightarrow t_r \approx$ **15h**

- **Exercise: Would you improve** τ **(long** t_r**) or** t_p **?**

Interactions per crossing

- **Luminosity/$fn_b \propto$ N$_1$N$_2$**
- **In LHC: crossing every 25 ns**
- **Per crossing approximately 20 interactions**
- **May be undesirable (pile up in detector)**
- $\Longrightarrow$ **more bunches n_b, or smaller N ??**

Beware: maximum (peak) luminosity $\mathcal{L}_{max}$

is not the whole story ... !

Luminosity measurement

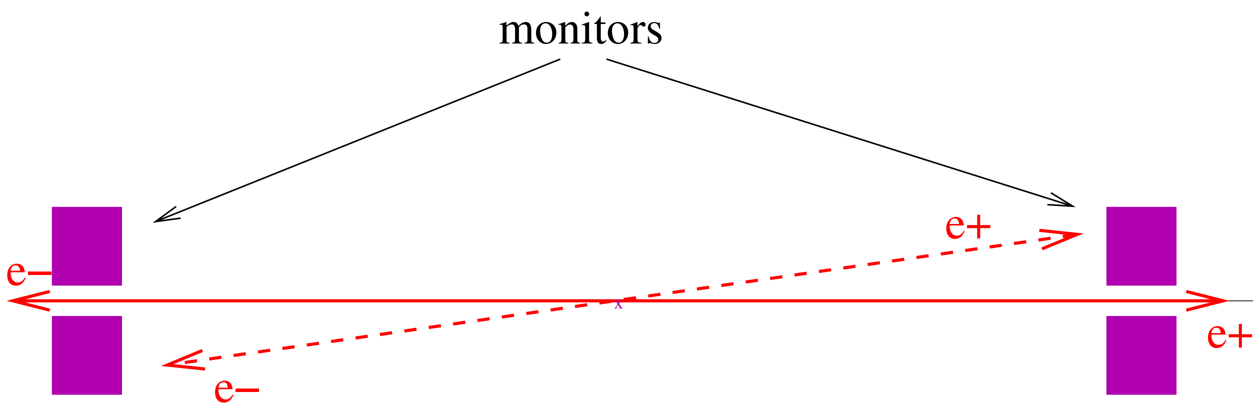
- **One needs to get a signal proportional to interaction rate ➡ Beam diagnostics**
- **Large dynamic range:**
  10^{27} cm^{-2}s^{-1} to 10^{34} cm^{-2}s^{-1}
- **Very fast, if possible for individual bunches**
- **Used for optimization**
- **For absolute luminosity need calibration**

Luminosity calibration

(e^+e^-)

- **Use well known and calculable process**
- $e^+e^- \rightarrow e^+e^-$ **elastic scattering (Bhabha scattering)**
- **Have to go to small angles** $(\sigma_{el} \propto \Theta^{-3})$
- **Small rates at high energy** $(\sigma_{el} \propto \frac{1}{E^2})$

Luminosity calibration



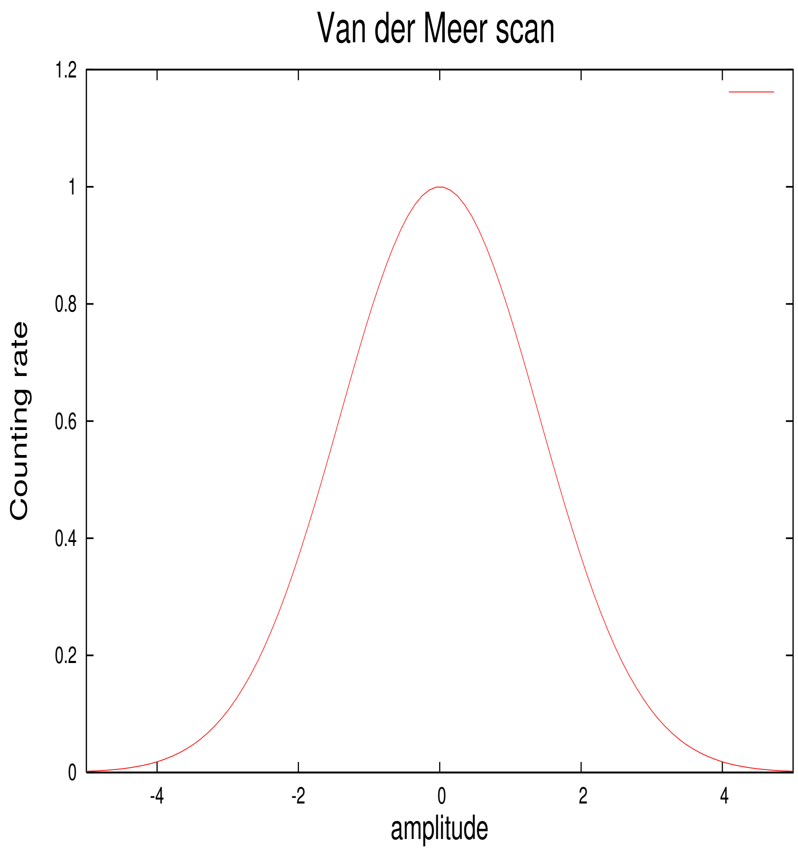
- Measure coincidence at small angles
- Low counting rates, in particular for high energy !
- Background may be problematic

Luminosity calibration

(hadrons, e.g. pp or $p\bar{p}$)

- **Must measure beam current and beam sizes**
- **Beam size measurement:**
  - **Wire scanner or synchrotron light monitors**
  - **Measurement with beam ... $\rightarrow$ remember luminosity with offset**
  - **Move the two beams against each other in transverse planes (van der Meer scan)**

Luminosity optimization



- Record counting rates ***R(d)*** as function of movement ***d***
- Since ***R(d)*** is proportional to luminosity ***L(d)***
- Get ratio of luminosity ***L(d)/L(0)***

Luminosity optimization

- **From ratio of luminosity $\mathcal{L}(\mathrm{d})/\mathcal{L}_0$**
- **Remember:** $W = e^{-\frac{1}{4\sigma^2}(d_2 - d_1)^2}$
- **Determines** σ
- **... and centres the beams !**
- **Others:**
  - **Beam-beam deflection scans LEP**
  - **Beam-beam excitation**

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By total rate and optical theorem**

  (also: luminosity independent determination of σ_{tot}):

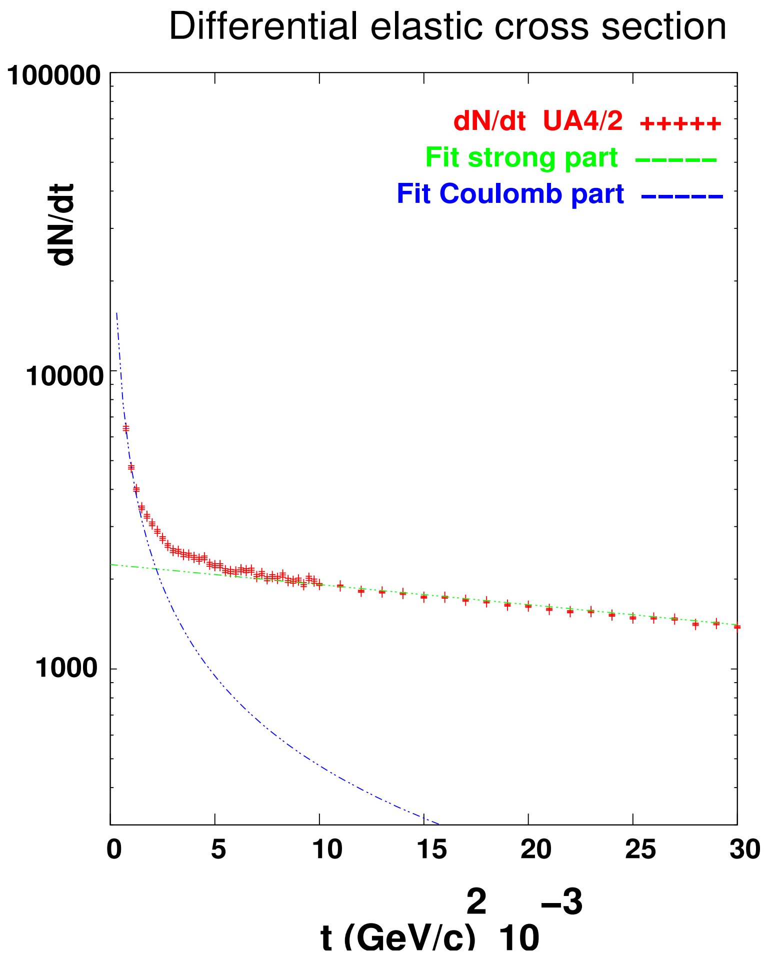
  - $\sigma_{tot} \cdot \mathcal{L} = N_{inel} + N_{el}$ **(Total counting rate)**
  - $$\lim_{t\to 0} \frac{d\sigma_{el}}{dt} = (1+\rho^2)\frac{\sigma_{tot}^2}{16\pi} = \frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}|_{t=0}$$

$$\mathcal{L} = \frac{(1+\rho^2)}{16\pi}\frac{(N_{inel}+N_{el})^2}{(dN_{el}/dt)_{t=0}}$$

- **Luminosity determined from experimental rates**

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By Coulomb normalization:**
  - **Coulomb amplitude exactly calculable:**
  - $$\lim_{t\to 0}\frac{d\sigma_{el}}{dt} = \frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}\Big|_{t=0} = \pi \mid f_C + f_N \mid^2$$
    $$\simeq \pi \mid \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi}(\rho + i)e^{b\frac{t}{2}} \mid^2 \simeq \frac{4\pi\alpha_{em}^2}{t^2} \mid_{|t|\to 0}$$
  - **Fit gives:** σ_{tot}, ρ, b **and** $\mathcal{L}$
- **Can be done measuring only elastic scattering (No $\mathbf{N}_{inel}$ needed !)**
- $\Longrightarrow$ **Roman pots**



- Measure dN/dt at small t (0.01 < (GeV/c)**2) and extrapolate to t = 0.0
- Needs special optics to allow measurement at very small t
- Measure total counting rate N_{el} + N_{inel}
  Needs good detector coverage
- Often use slightly modified method, precision 1 – 2 %

Luminosity in linear colliders

- Mainly (only) $e+e-$ colliders
- Past collider: SLC (SLAC)
- Under consideration: CLIC, ILC
- Special issues:
  - Particles collide only once (dynamics) !
  - Particles collide only once (beam power) !

→ Must be taken into account

Luminosity in linear colliders

- **Basic formula:**

$$\text{From}: \quad \mathcal{L} = \frac{N^2 \ f \ n_b}{4\pi\sigma_x\sigma_y} \quad \text{to}: \quad \mathcal{L} = \frac{N^2 \ f_{rep} \ n_b}{4\pi\sigma_x\sigma_y}$$

- **Replace frequency f by repetition rate f_{rep}.**
- **And introduce effective beam sizes $\overline{\sigma_x}, \overline{\sigma_y}$:**

$$\mathcal{L} = \frac{N^2 \ f_{rep} \ n_b}{4\pi\overline{\sigma_x} \ \overline{\sigma_y}}$$

Luminosity in linear colliders

- **Using the enhancement factor H_D:**

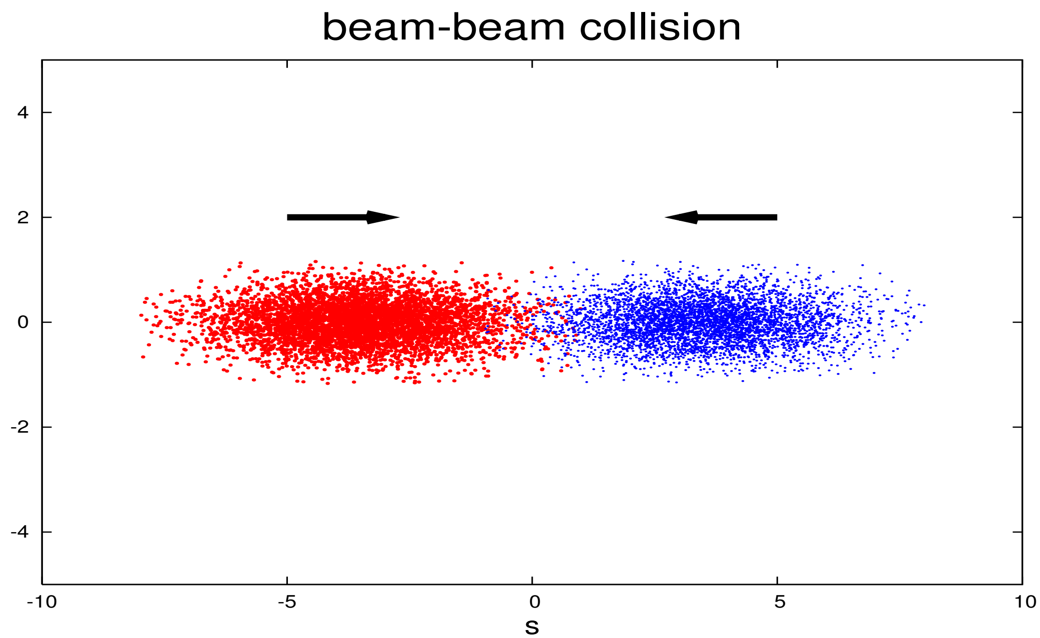
$$\mathcal{L} = \frac{N^2 \; f_{rep} \; n_b}{4\pi\overline{\sigma_x} \; \overline{\sigma_y}} \quad \Rightarrow \quad \mathcal{L} = \frac{H_D \cdot N^2 \; f_{rep} \; n_b}{4\pi\sigma_x \; \sigma_y}$$

- **Enhancement factor H_D takes into account reduction of nominal beam size by the disruptive field (pinch effect)**

- **Related to disruption parameter $\mathcal{D}$:**

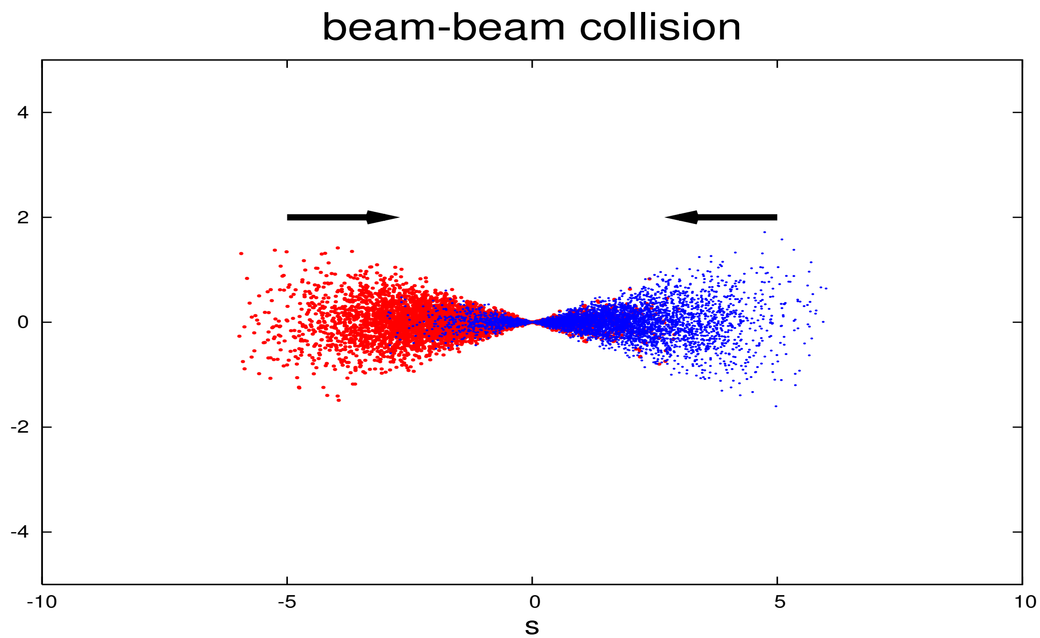
$$\mathcal{D}_{x,y} = \frac{2 r_e N \sigma_z}{\gamma \sigma_{x,y}(\sigma_x + \sigma_y)}$$

Pinch effect - disruption



- **Additional focusing by opposing beams**

Pinch effect - disruption



- **Additional focusing by opposing beams**

Luminosity in linear colliders

- **For weak disruption $\mathcal{D} \ll 1$ and round beams:**

$$H_D = 1 \; + \; \frac{2}{3\sqrt{\pi}}\mathcal{D} \; + \; \mathcal{O}(\mathcal{D}^2)$$

- **For strong disruption and flat beams: computer simulation necessary, maybe can get some scaling**

Beamstrahlung

- Disruption at interaction point is basically a strong "bending"
- Results in strong synchrotron radiation: beamstrahlung
- This causes (unwanted):
  - Spread of centre-of-mass energy
  - Pair creation and detector background
- Again: luminosity is not the only important parameter

Beamstrahlung Parameter Y

- **Measure of the mean field strength in the rest frame normalized to critical field B_c:**

$$Y = \frac{< E + B >}{B_c} \approx \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

with:

$$B_c = \frac{m^2 c^3}{e\hbar} \approx 4.4 \times 10^{13} G$$

Energy loss and power consumption

- **Average fractional energy loss δ_E:**

$$\delta_E = 1.24 \frac{\alpha \sigma_z m_e}{\lambda_C E} \frac{Y}{(1 + (1.5Y)^{2/3})^{1/2}}$$

where E is beam energy at interaction point and λ_C the Compton wavelength.

Luminosity in linear colliders

- **Using the beam power P_b and beam energy E in the luminosity:**

$$\mathcal{L} = \frac{H_D \cdot N^2 \; f_{rep} \; n_b}{4\pi\sigma_x \; \sigma_y} \quad \Rightarrow \quad \mathcal{L} = \frac{H_D \cdot N \cdot P_b}{eE \cdot 4\pi\sigma_x \; \sigma_y}$$

- **Beam power P_b related to AC power consumption P_{AC} via efficiency η_b^{AC}**

$$P_b \quad = \quad \eta_b^{AC} \quad \cdot \quad P_{AC}$$

Figure of merit in linear colliders

- **Luminosity at given energy normalized to power consumption and momentum spread due to beamstrahlung:**

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b}P_{AC}}$$

- **With previous definition (and reasonably small beamstrahlung) this becomes:**

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b}P_{AC}} \quad \propto \quad \frac{\eta_b^{AC}}{\sqrt{\epsilon_y^*}}$$

- **These are optimized in the linear collider design**

Not treated :

- **Coasting beams (e.g. ISR)**
- **Asymmetric colliders (e.g. PEP, HERA, LHeC)**

How to cook high Luminosity ?

- **Get high intensity**
- **Get small beam sizes (small ϵ and β^*)**
- **Get many bunches**
- **Get small crossing angle (if any)**
- **Get exact head-on collisions**
- **Get short bunches**