THE CERN ACCELERATOR SCHOOL

Sources of emittance growth (Hadrons)

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Summary:

- Introduction
- Emittance growth in single-passage systems
  - Scattering through thin foils
- Emittance growth in multi-passage systems
  - Injection process
  - Scattering processes
  - Others
- Emittance manipulation
  - Longitudinal
  - Transverse

Acknowledgements:
D. Brandt and D. Möhl

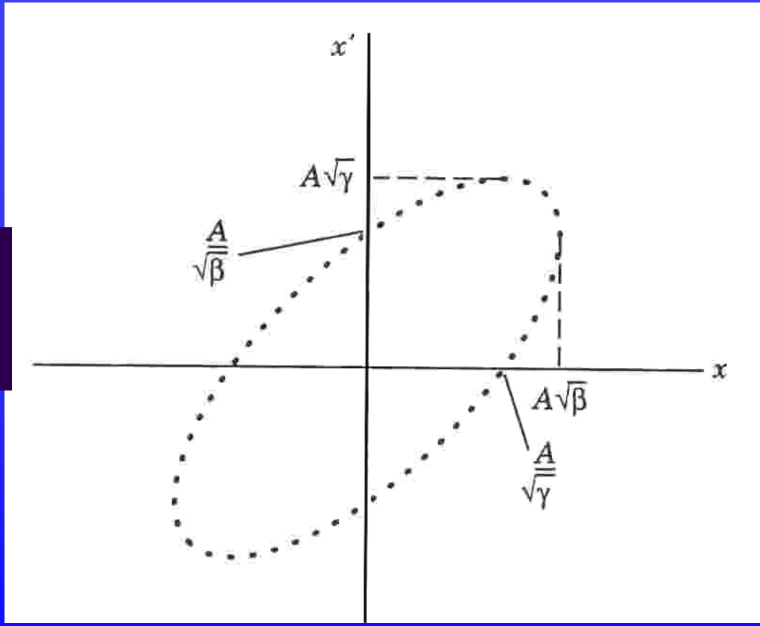
Introduction - I

- The starting point is the well-known Hill's equation

$$x(s)'' + K(s)\, x(s) = 0$$

- Such an equation has an invariant (the so-called Courant-Snyder invariant)

$$A = \gamma\, x^2 + 2\alpha\, x\, x' + \beta\, x'^2$$



Parenthetically: in a bending-free region the following dispersion invariant exists

$$A = \gamma\, D^2 + 2\alpha\, D\, D' + \beta\, D'^2$$

Introduction - II

In the case of a beam, i.e. an ensemble of particles:

- Emittance: value of the Courant-Snyder invariant corresponding to a given fraction of particles.
- Example: rms emittance for Gaussian beams.

Why emittance can grow?

- Hill equation is linear -> in the presence of nonlinear effects emittance is no more conserved.

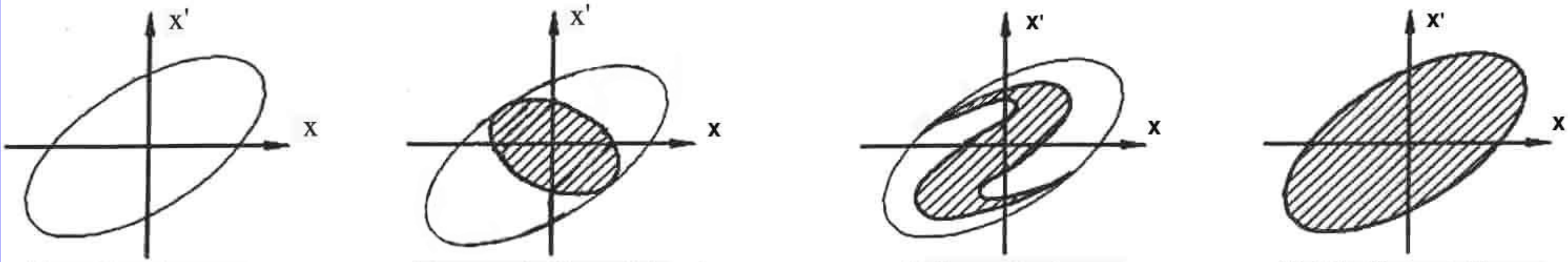
Introduction - III

Why emittance growth is an issue?

- Machine performance is limited or reduced, e.g.
  - Beam losses can generated
  - In the case of a collider the luminosity (i.e. the rate of collisions per unit time) is reduced.

Introduction - IV

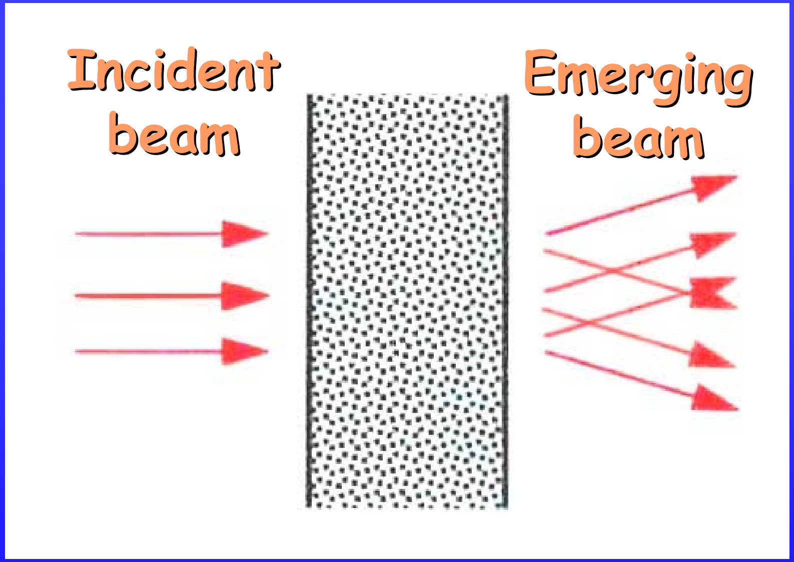
- Filamentation is one of the key concepts for computing emittance growth
  - Due to the presence of nonlinear imperfections, the rotation frequency in phase space is amplitude-dependent.
  - After a certain time the initial beam distribution is smeared out to fill a phase space ellipse.

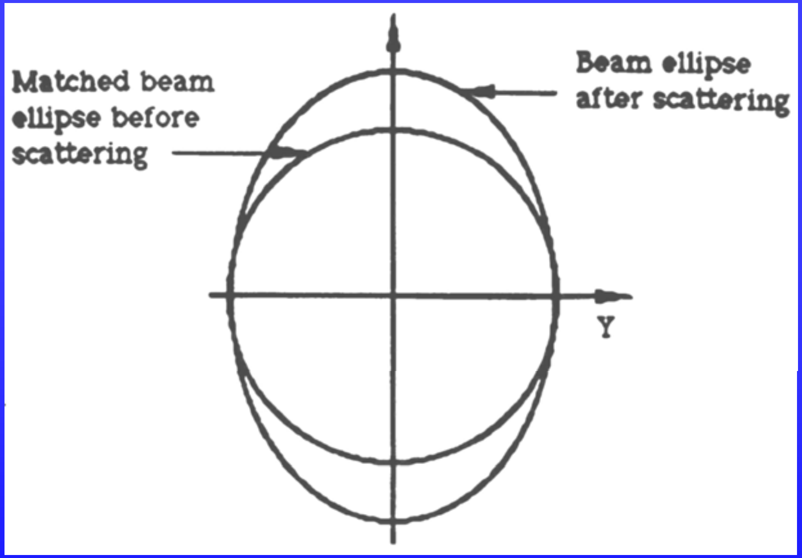


(a) machine phase space (b) unmatched beam injected (c) filamenting beam (d) fully filamented beam

Scattering through thin foil - I

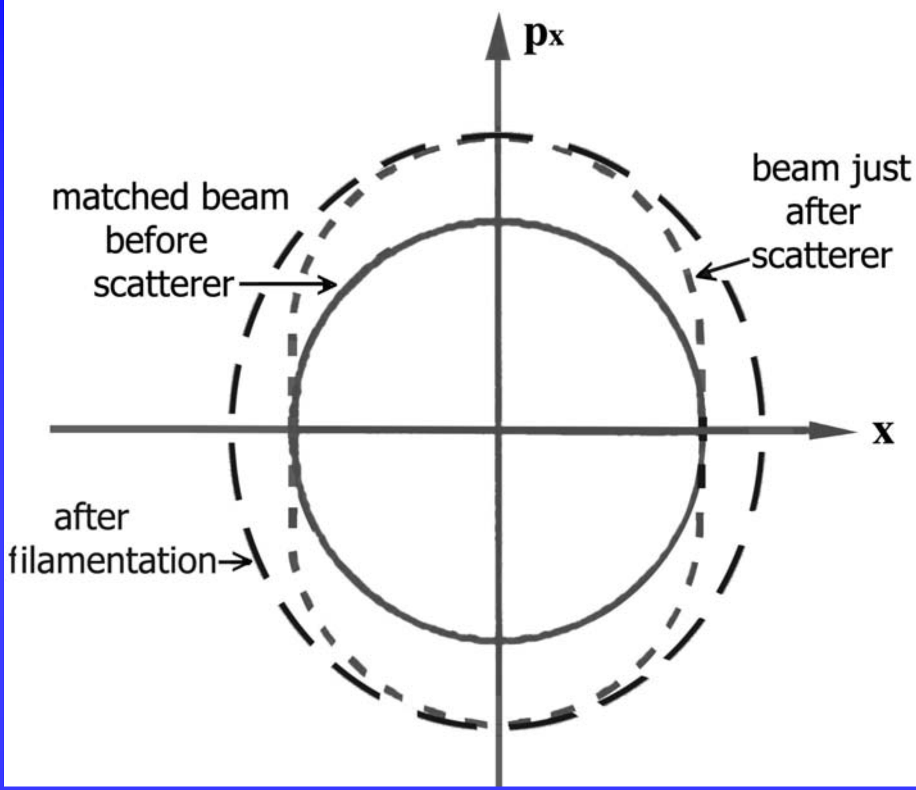
- **Typical situation:**
  - Vacuum window between the transfer line and a target (in case of fixed target physics)
  - Vacuum window to separate standard vacuum in transfer line from high vacuum in circular machine





The particles receive an angular kick

Scattering through thin foil - II



- Multiple Coulomb scattering due to beam-matter interaction is described by means of the rms scattering angle:

$$\theta_{rms} = \frac{14\ MeV/c}{p\beta_p} q_p \sqrt{\frac{L}{L_{rad}}} \left(1+\varepsilon_{corr}\right)$$

- Downstream of the foil the transformed coordinates are given by

$$x_i \rightarrow x_i = A_{i0} \sin(\psi_i)$$
$$p_{xi} \rightarrow p_{xi} + \Delta p = A_{i0} \cos(\psi_i) + \beta_x \theta_i$$

Normalised coordinate

$$p_x = \alpha_x x + \beta_x x'$$

α = 0 at the location of the foil

Scattering through thin foil - III

- By assuming that:
  - Scattering angle and betatronic phase are uncorrelated
  - Averaging over betatronic phase (due to filamentation) is possible

$$\left\langle A_i^2 \right\rangle = \left\langle x_i^2 + p_{xi}^2 \right\rangle = \left\langle A_{i0}^2 \right\rangle + \left\langle \beta_x^2 \theta_i^2 \right\rangle$$

  - Using the relation

$$\varepsilon_{rms} = \pi \frac{\left\langle A^2 \right\rangle}{2}$$

  - The final result reads

$$\Delta\varepsilon_{rms} = \frac{\pi}{2} \theta_{rms}^2 \beta_x$$

Scattering through thin foil - IV

- Few remarks
  - The special case with α = 0 at the location of the thin foil is discussed -> it can be generalised.
  - The correct way of treating this problem is (see next slides):
    - Compute all three second-order moments of the beam distribution downstream of the foil
    - Evaluate the new optical parameters and emittance using the statistical definition
  - The emittance growth depends on the beta-function!

THE SMALLER THE VALUE OF THE BETA-FUNCTION AT THE LOCATION OF THE FOIL THE SMALLER THE EMITTANCE GROWTH

Scattering through thin foil - V

- Correct computation (always for α=0):

$$x = A_o\sqrt{\beta_{xo}}\cos(\psi_o) = A_1\sqrt{\beta_{x1}}\cos(\psi_1)$$
$$p_x = -A_o\sqrt{1/\beta_{xo}}\sin(\psi_o) + \theta = -A_1\sqrt{1/\beta_{x1}}\sin(\psi_1)$$

- By squaring and averaging over the beam distribution

$$< x^2 > = \frac{< A_0^2 > \beta_{x0}}{2} = \frac{< A_1^2 > \beta_{x1}}{2}$$
$$< p_x^2 > = \frac{< A_{x0}^2 >}{2\beta_{x0}} + \theta_{rms}^2 = \frac{< A_1^2 >}{2\beta_{x1}}$$

- Using the relation

$$\varepsilon_{rms} = \pi\frac{\left\langle A^2 \right\rangle}{2}$$

Scattering through thin foil - VI

- The solution of the system is given by

$$\frac{\langle A_1^2\rangle^2 - \langle A_0^2\rangle^2}{\langle A_0^2\rangle} = 2\beta_{x0}\theta_{rms}^2$$

$$\frac{\beta_{x1}}{\beta_{x0}} = \frac{\langle A_0^2\rangle}{\langle A_1^2\rangle}$$

- This can be solved exactly, or by assuming that the relative emittance growth is small, then

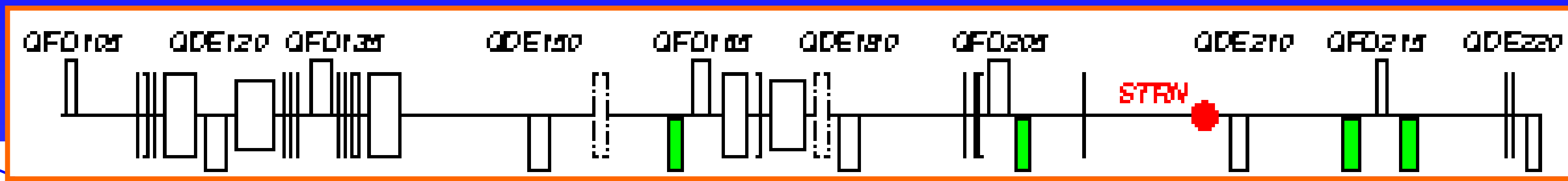
NB: the emittance growth is now only half of the previous estimate!

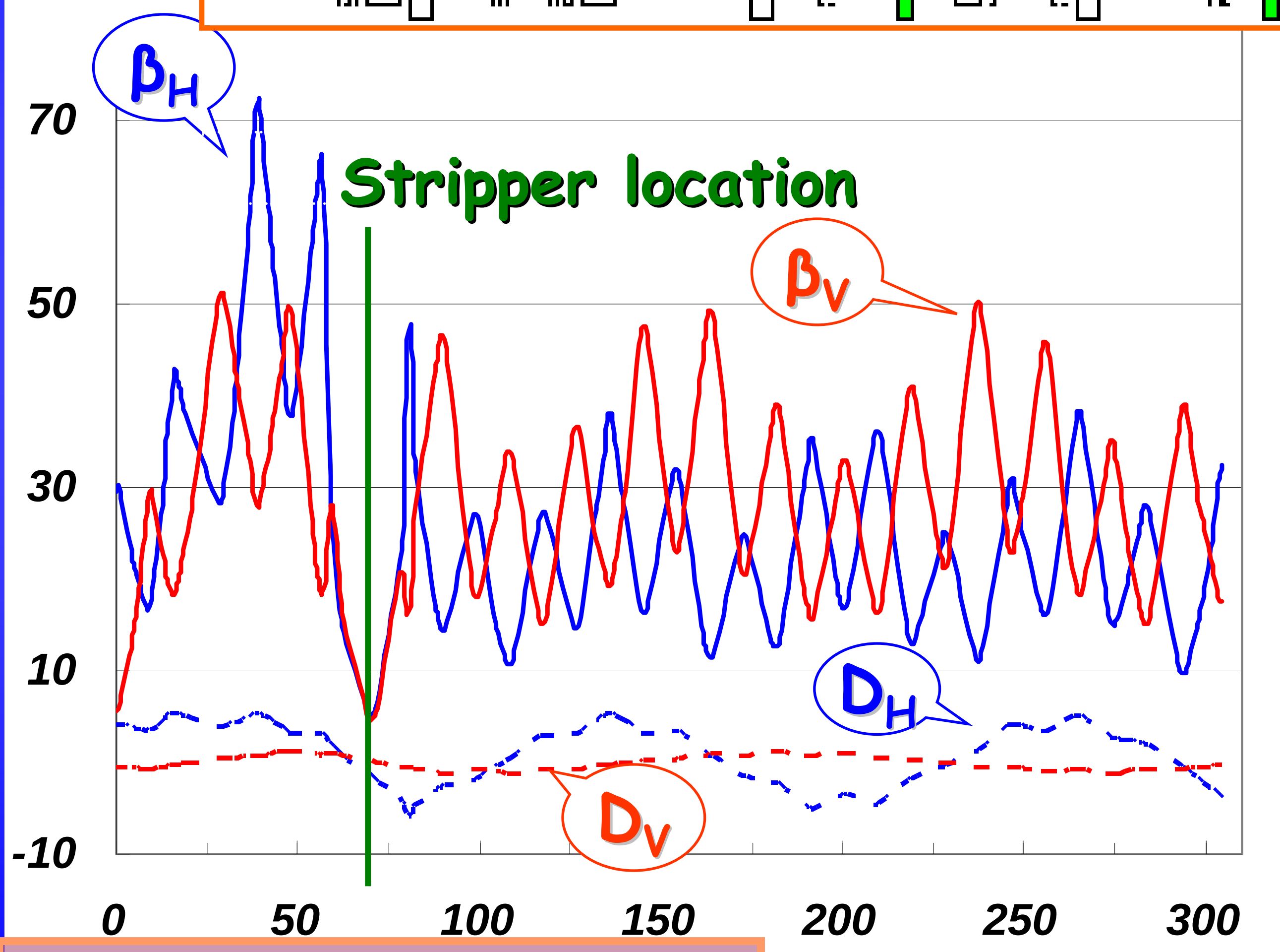
Downstream of the foil the transfer line should be matched using the new Twiss parameters α_{x1}, β_{x1}

$$\Delta\varepsilon_{rms} = \frac{\pi}{4}\theta_{rms}^2\beta_{x0}$$

$$\beta_{x1} = \beta_{x0}\left[1 - \frac{\pi}{4}\frac{\theta_{rms}^2}{\varepsilon_{0rms}}\right]$$

Example: ion stripping for LHC lead beam between PS and SPS





- Pb^{54+} -> Pb^{82+}
- A low-beta insertion is designed (beta reduced by a factor of 5)
- A stripping foil, **0.8 mm thick Al**, is located in the low-beta insertion

Courtesy M. Martini - CERN

Exercises

- Compute the Twiss parameters and emittance growth for a THICK foil
  - Hint: slice the foil assuming a sequence of drifts and thin scatterers.
- Compute the Twiss parameters and emittance growth for a THICK foil in a quadrupolar field
  - Hint: same as before, but now the drifts should be replaced by quadrupoles.

Injection process - I

- Two main sources of errors:
  - Steering.
  - Optics errors (Twiss parameters and dispersion).

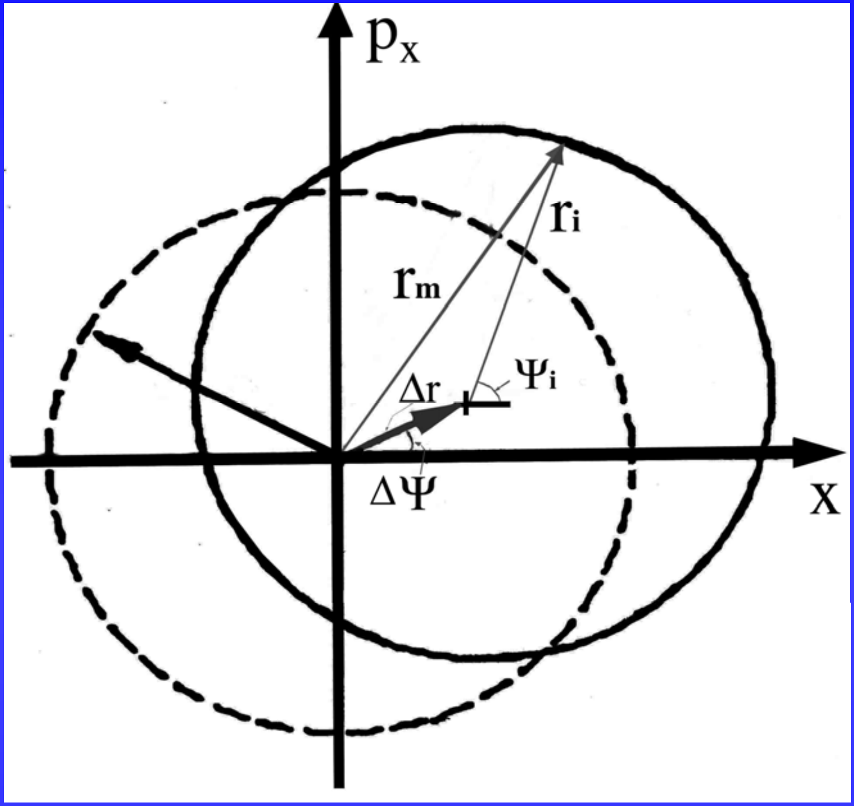
- In case the incoming beam has an energy error, then the effect will be a combination of the two.

- In all cases filamentation, i.e. nonlinear imperfections in the ring, is the source of emittance growth.

Injection process - II

- Steering errors:
  - Injection conditions, i.e. position and angle, do not match position and angle of the closed orbit.
- Consequences:
  - The beam performs betatron oscillations around the closed orbit. The emittance grows due to filamentation
- Solution:
  - Change the injection conditions, either by steering in the transfer line or using the septum and the kicker.
  - In practice, slow drifts of settings may require regular tuning. In this case a damper (see lecture on feedback systems) is the best solution.

Injection process - III



- Analysis in normalised phase space (i stands for injection m for machine):

$$x_m = r_i \cos\psi_i + \Delta r \cos\psi$$
$$p_{xm} = r_i \sin\psi_i + \Delta r \sin\psi$$

- Squaring and averaging gives

$$<r_m^2> = <x_m^2 + p_{xm}^2>$$
$$<r_m^2> = <r_i^2> + \Delta r^2$$

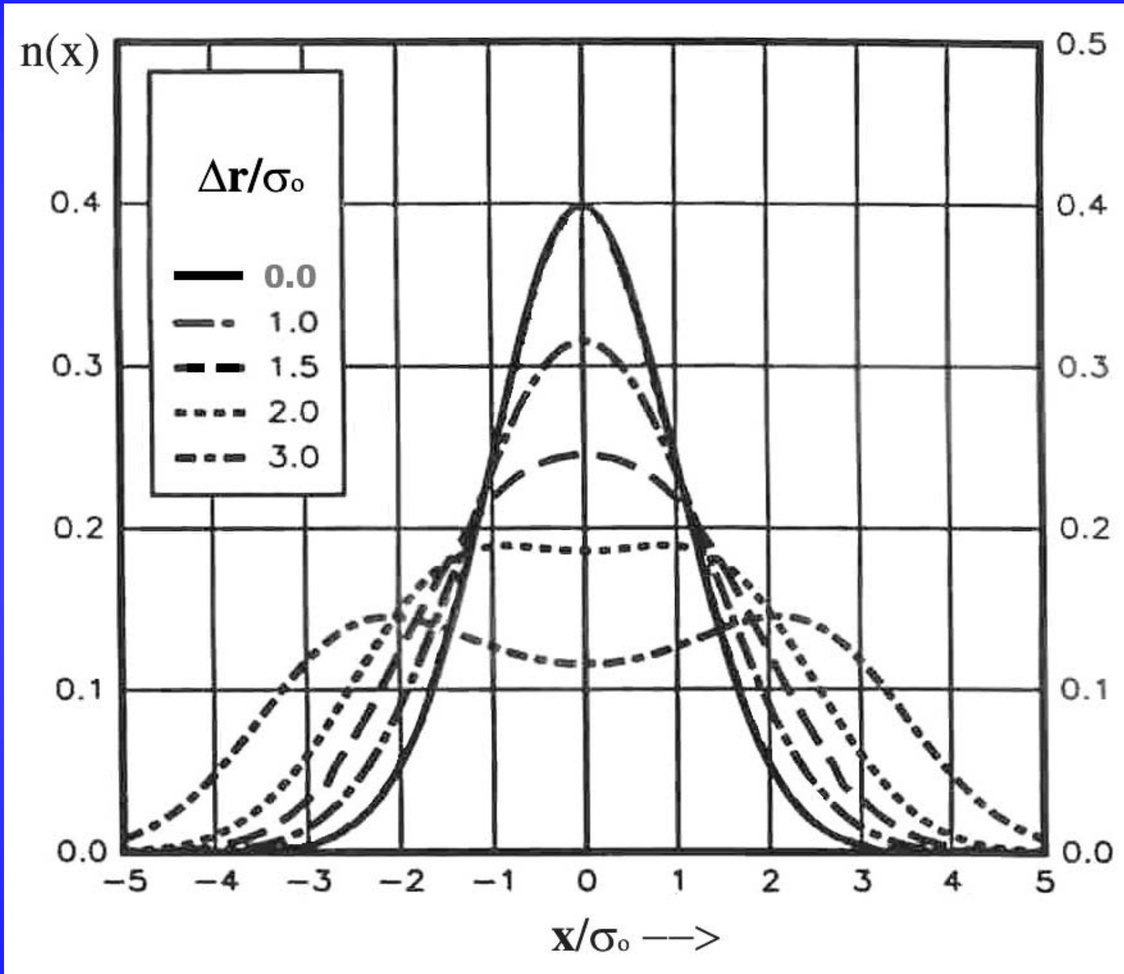
- After filamentation

$$<x^2_{after\ fil.}> = \frac{1}{2}<r_m^2> = \frac{1}{2}<r_i^2> + \frac{1}{2}\Delta r^2 \Rightarrow \varepsilon_{rms}^{after\ fil.} = \varepsilon_{rms} + \frac{\pi}{2}\Delta r^2$$

Injection process - IV

Example of beam distribution generated by steering errors and filamentation.

The beam core is displaced -> large effect on emittance



Injection process - V

- Dispersion mismatch: analysis is similar to that for steering errors.
- A particle with momentum offset Δp/p will have injection conditions given by

$$x_i = D_{ix}\,\Delta p/p$$
$$p_{xi} = D'_{ix}\,\Delta p/p$$

While the machine requires injection conditions given by

$$x_m = D_{mx}\,\Delta p/p$$
$$p_{xm} = D'_{mx}\,\Delta p/p$$

- The vector Δr is obtained by: taking difference of injection conditions; transforming in normalised phase space. Then after squaring and averaging over the beam distribution the final result is

$$\Delta r^2 = \left[\Delta D^2 + \left(\beta \Delta D' + \alpha \Delta D\right)^2\right]\sigma_p^2$$

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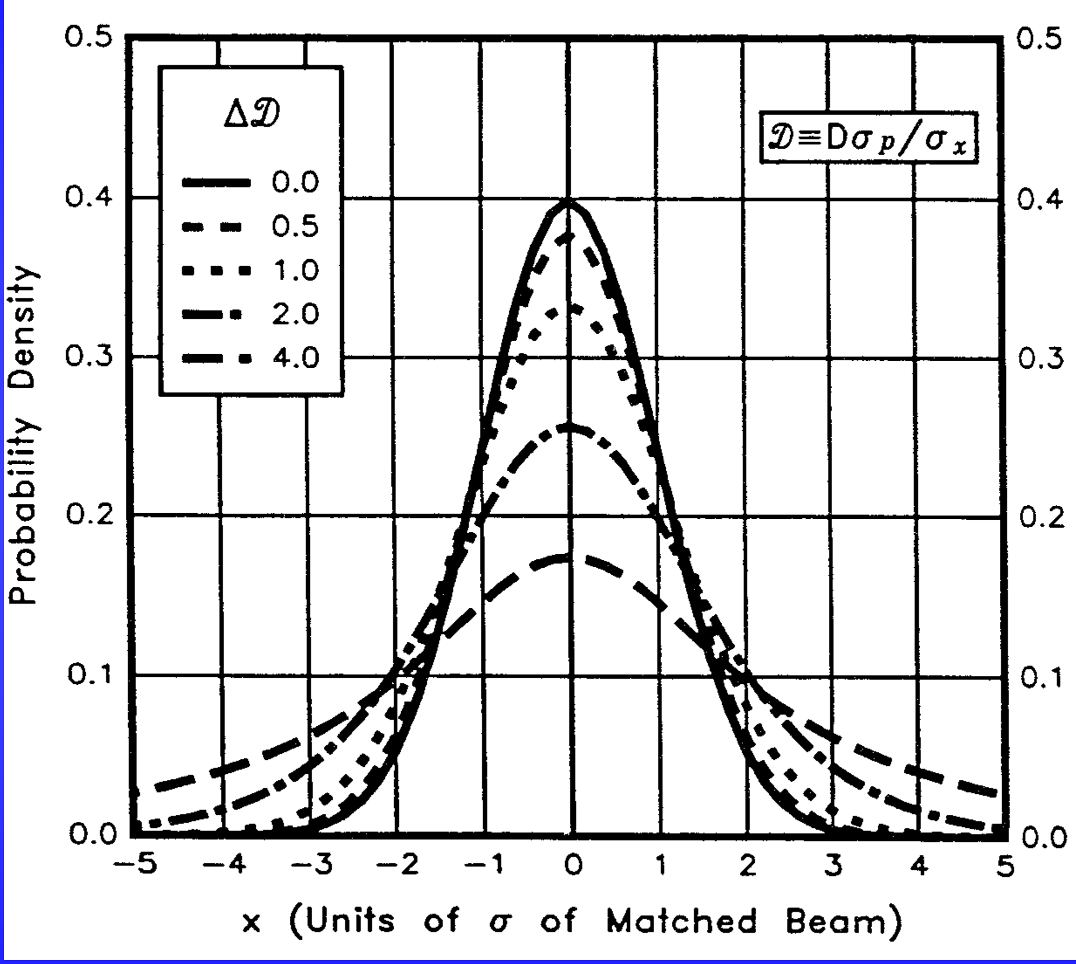
Injection process - VI

Example of beam distribution generated by dispersion mismatch and filamentation.

The effect is on the tails of the beam distribution.



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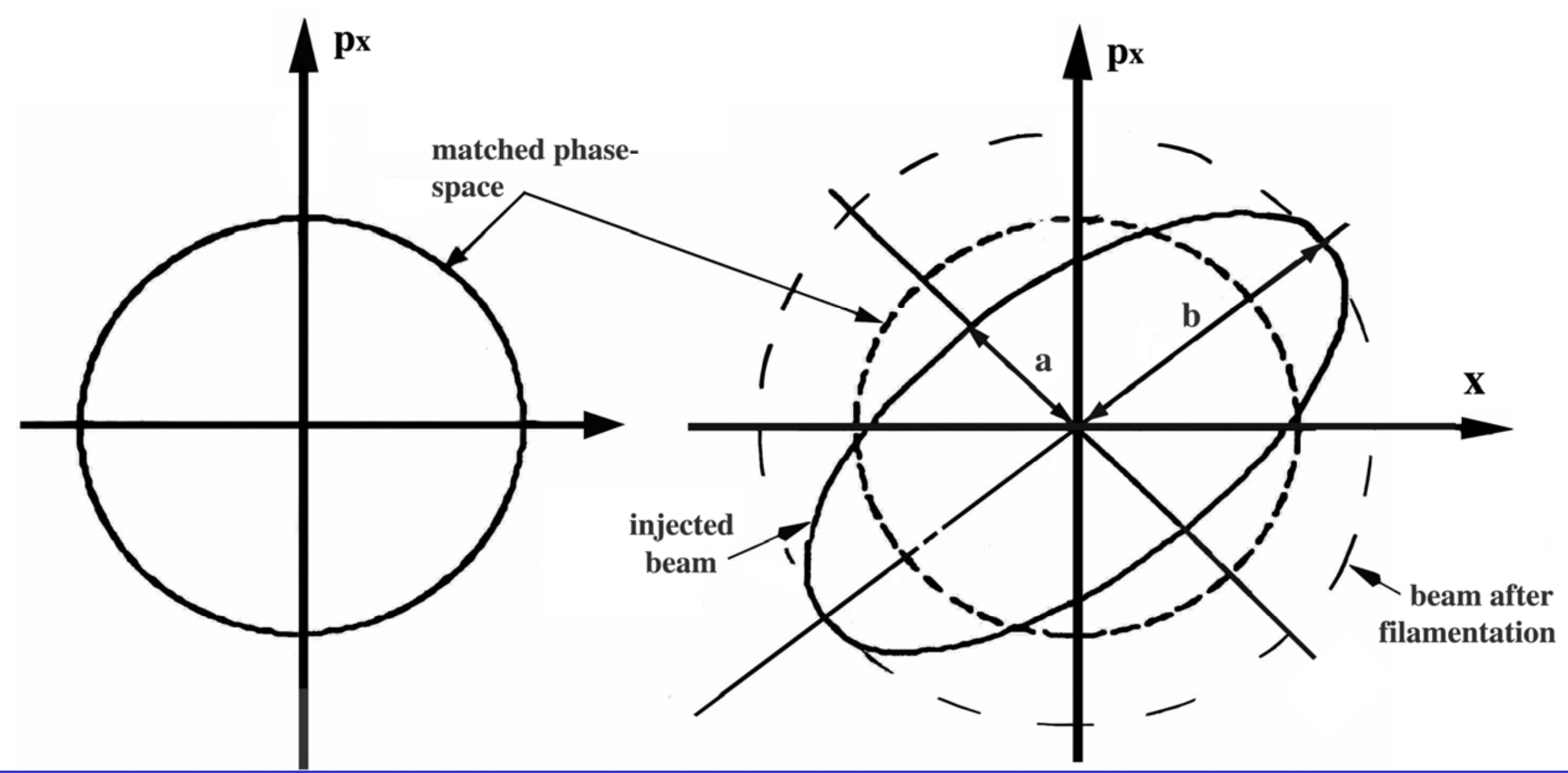
Injection process - VII

- **Optics errors:**
  - Optical parameters of the transfer line at the injection point are different from those of the ring.
- **Consequences:**
  - The beam performs quadrupolar oscillations (size changes on a turn-by-turn basis). The emittance grows due to filamentation.
- **Solution:**
  - Tune transfer line to match optics of the ring

Injection process - VIII



In normalised phase space (that of the ring) the injected beam will fill an ellipse due to the mismatch of the optics

Injection process - IX

The computation of the emittance blow-up due to optics errors is very similar to previous cases.

The final result reads:

$$\varepsilon_{rms}^{after\ fil.} = \varepsilon_{rms} F$$

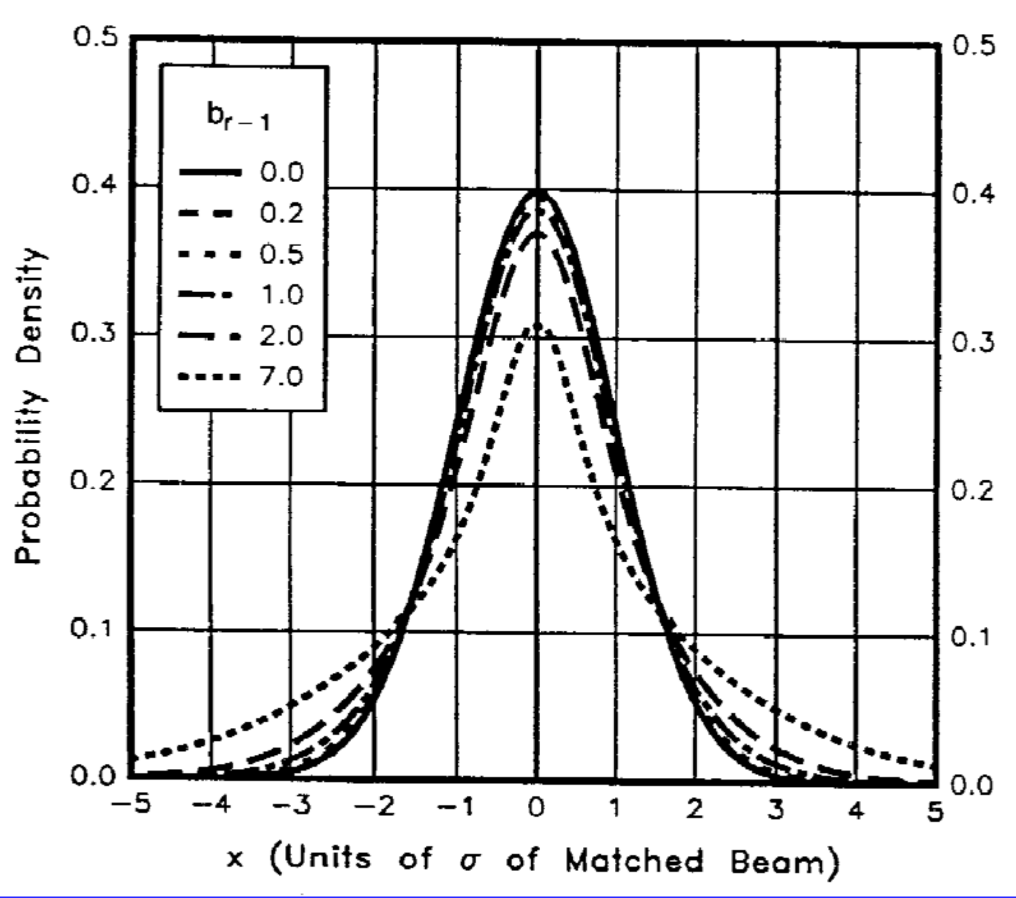
$$F = \frac{1}{2}\left(\frac{\beta_i}{\beta_m} + \frac{\beta_m}{\beta_i} + \left(\frac{\alpha_m}{\beta_m} - \frac{\alpha_i}{\beta_i}\right)^2 \beta_m \beta_i\right)$$

NB: in this case the emittance growth is proportional to the initial value of emittance

Injection process - X

Example of beam distribution generated by optics mismatch and filamentation.

The beam core is also affected as well as the tails.



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Scattering processes - I

- Two main categories considered:
  - Scattering on residual gas -> similar to scattering on a thin foil (the gas replaces the foil)

$$\Delta\varepsilon_{k\sigma} = \frac{\pi}{2} k^2 q_p^2 \left(\frac{14\ MeV/c}{p\beta_p} \right)^2 \bar{\beta}\ \frac{\beta_p c\, t}{L_{rad}} \quad where \quad \bar{\beta} \quad is\ the\ average\ beta$$

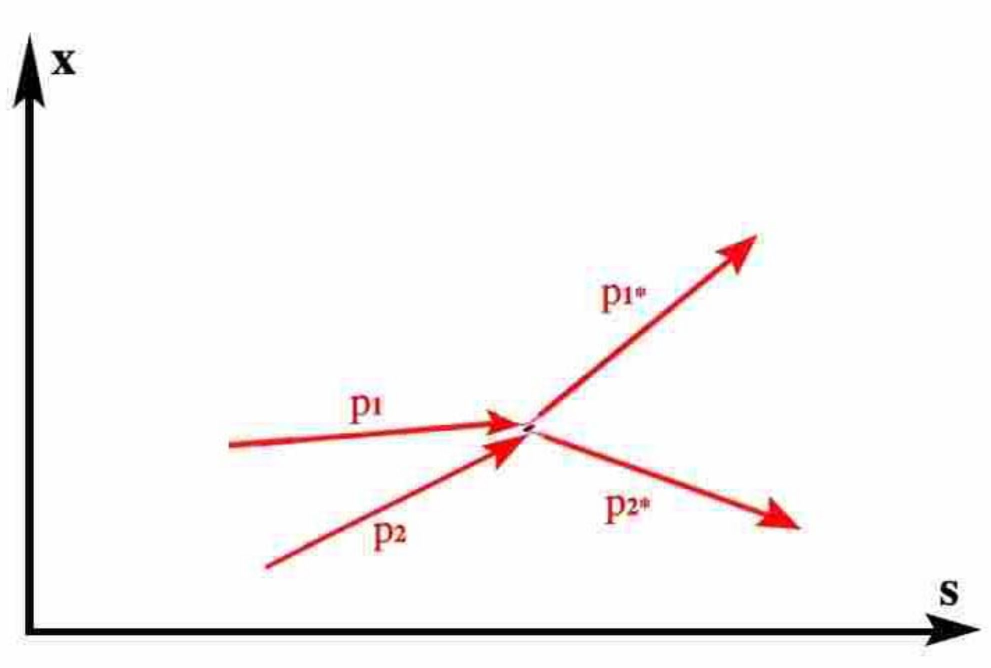
NB: $\beta_p ct$ represents the scatterer length until time t.

That is why good vacuum is necessary!

  - Intra-beam scattering, i.e. Coulomb scattering between charged particles in the beam.

Scattering processes - II

- Intra-beam scattering
  - Multiple (small angle) Coulomb scattering between charged particles.
  - Single scattering events lead to Touscheck effect.
  - All three degrees of freedom are affected.



Scattering processes - III

Features of IBS

- For constant lattice functions and below transition energy, the sum of the three emittances is constant.
- Above transition the sum of the emittances always grows.
- In any strong focusing lattice the sum of the emittances always grows.
- Even though the sum of emittances grows, emittance reduction in one plane is predicted by simulations, but never observed in real machines.

Scattering processes - IV

Scaling laws of IBS

- Accurate computations can be performed only with numerical tools.
- However, scaling laws can be derived.
- Assuming

$$\frac{1}{\tau_{x,y,l}} = \frac{1}{\tau_0} F_{x,y,l}$$

- Then

$$1/\tau_0 = \frac{N_b r_0^2 \left(\frac{q^2}{A}\right)^2}{(4/\pi^2)\,\gamma\,\varepsilon_x^* \varepsilon_y^* \varepsilon_l^* / E_0} \propto \frac{N_b \left(\frac{q^2}{A}\right)^2}{\gamma\,\varepsilon_x^* \varepsilon_y^* \varepsilon_l^*}$$

Strong dependence on charge

$\varepsilon_{x,y,l}^*$ are normalised emittances

N_b of particles/bunch

r_0 classical proton radius

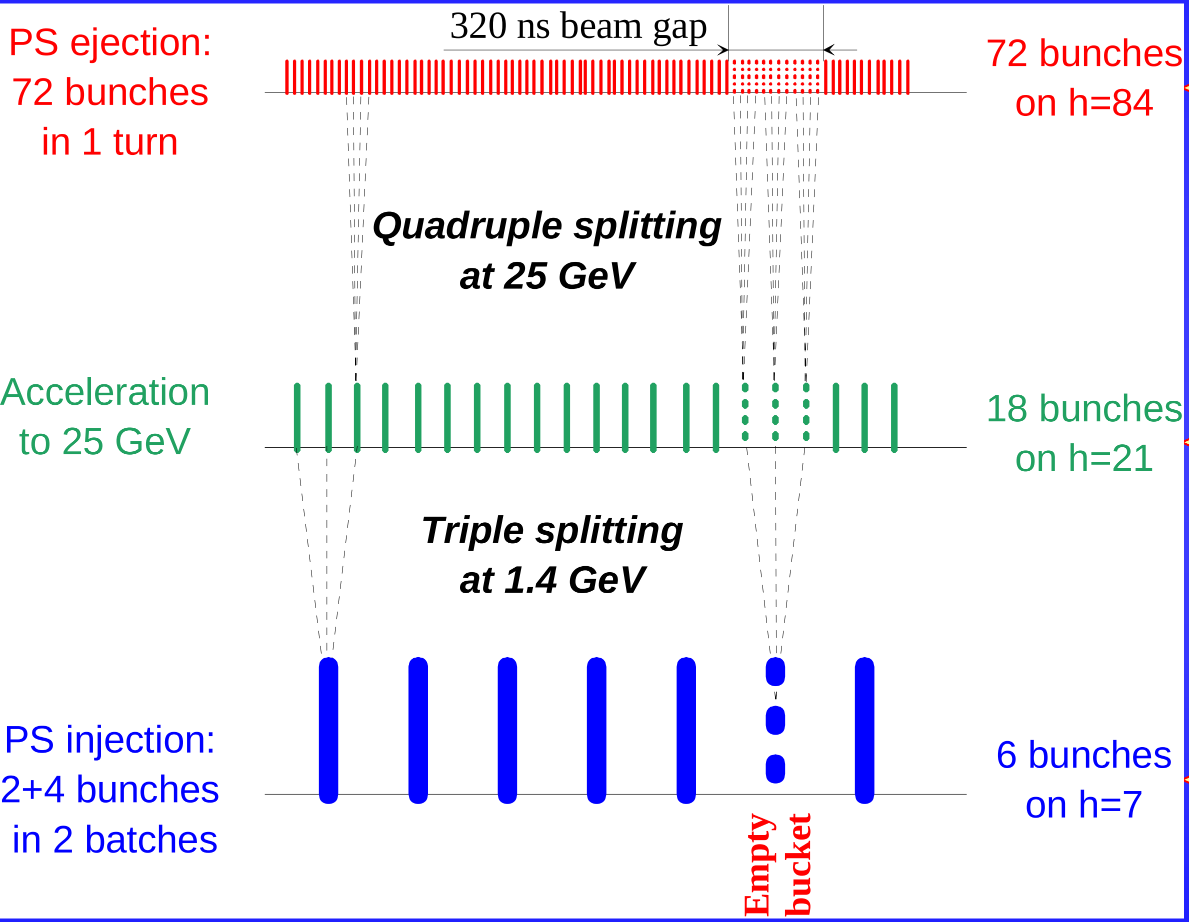
Others

- Diffusive phenomena:
  - Resonance crossings
- Collective effects
  - Space charge (soft part of Coulomb interactions between charged particles in the beam) -> covered by a specific lecture.
  - Beam-beam -> covered by a specific lecture.
  - Instabilities -> covered by a specific lecture.

Emittance manipulation

- Emittance is normally preserved.
- Sometimes, however, it is necessary to manipulate the beam so to reduce its emittance.
  - Standard techniques: electron cooling, stochastic cooling -> covered by a specific lecture.
  - Less standard techniques: longitudinal or transverse beam splitting.

Longitudinal manipulation: LHC beam in PS machine - I



0.35 eVs (bunch)
1.1 × 10^{11} ppb

40 % Blow-up (pessimistic)

1 eVs (bunch)
4.4 × 10^{11} ppb

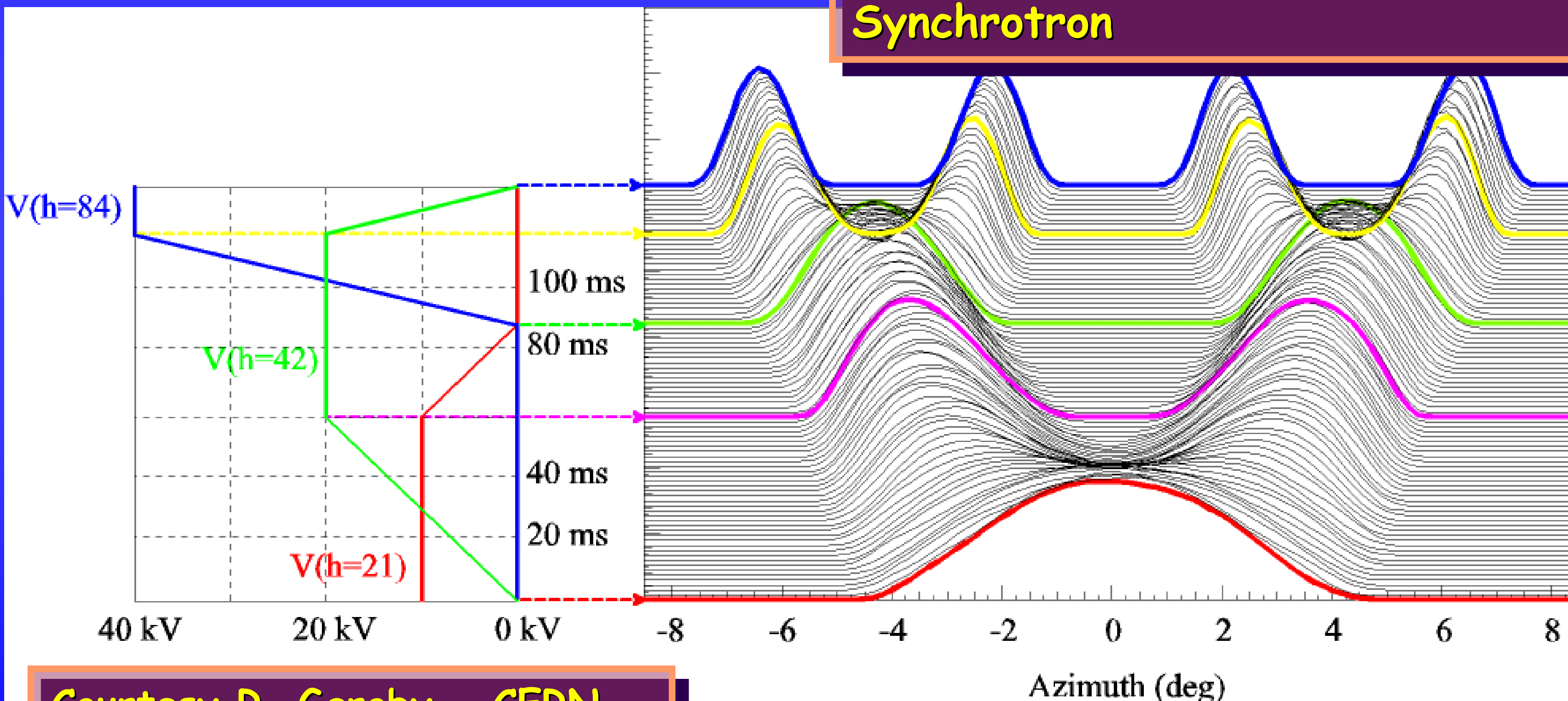
115 % Blow-up (voluntary)

1.4 eVs (bunch)
13.2 × 10^{11} ppb

Courtesy R. Garoby - CERN

Longitudinal manipulation: LHC beam in PS machine - II

Measurement results obtained at the CERN Proton Synchrotron



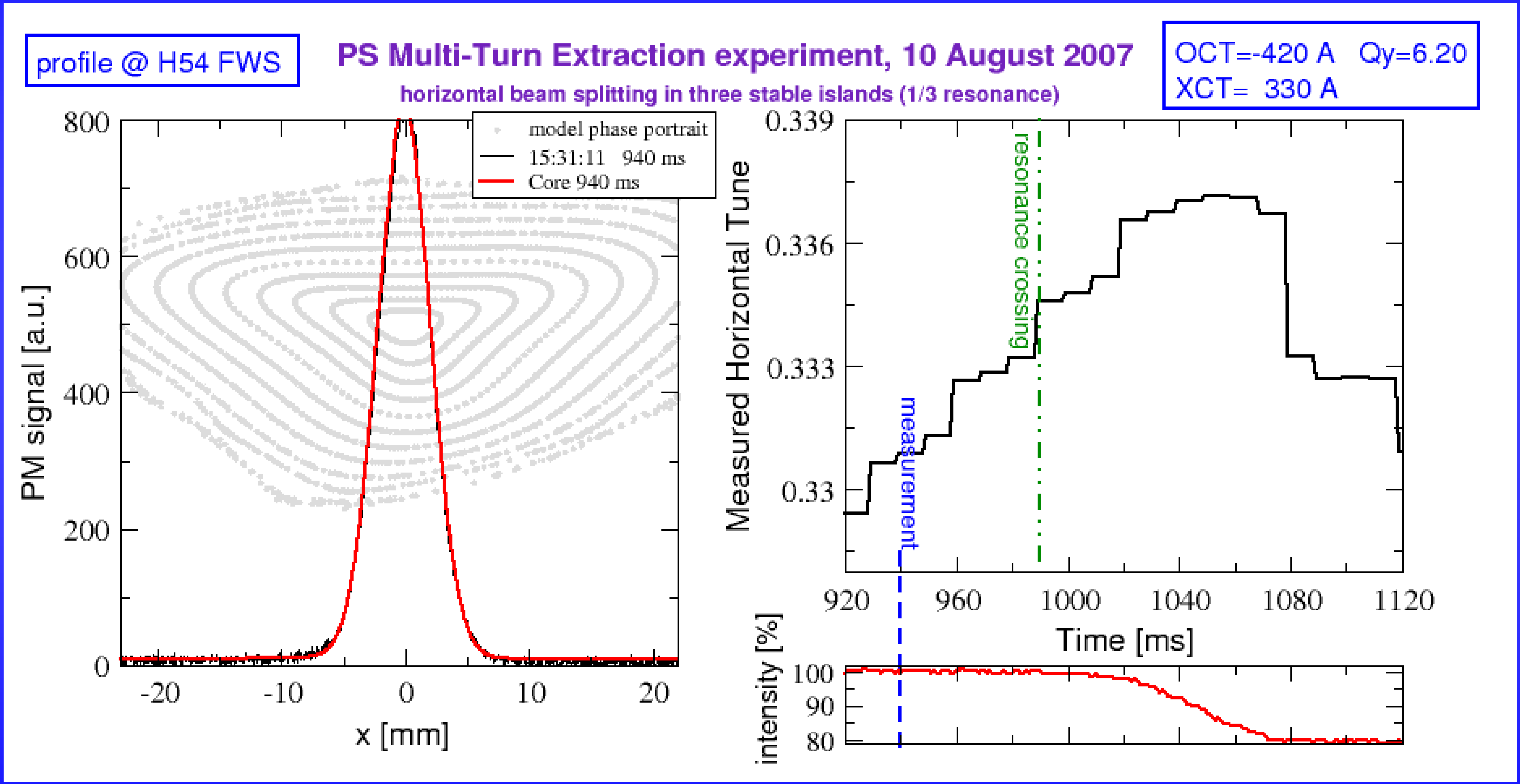
Courtesy R. Garoby - CERN

Transverse manipulation: CERN PS multi-turn extraction - I

The main ingredients are:

- The beam is split in the transverse phase space using
  - Nonlinear magnetic elements (sextupoles ad octupoles) to create stable islands.
  - Slow (adiabatic) tune-variation to cross an appropriate resonance.

Transverse manipulation: crossing third-order resonance



Transverse manipulation: crossing fourth-order resonance

A series of horizontal beam profiles have been taken when crossing the fourth-order resonance.

Measurement results obtained at the CERN Proton Synchrotron

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Some references

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