Introduction to Insertion Devices

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What is an Insertion Device?

- Oscillating Magnetic field create a beam undulation
- Also called Undulators and Wigglers
- Can be 1 to 20 m long, with period 15 to 200 mm
- Operated with a small magnetic gap (5 to 15 mm)
- Use:
  - **Intense Source of Radiation in electron storage rings**
  - Control of damping times in Electron Colliders (LEP, CESR, …)
  - Reduce emittance in advanced light sources (Petra III, NSLS II)
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Electron beam dynamics
Electron Trajectory in an Insertion Device

Consider Orthogonal Frame Oxzs

Electron velocity: \( \vec{v} = (v_x, v_z, v_s) \)

Electron position: \( \vec{R} = (x, z, s) \)

Magnetic field: \( \vec{B} = (B_x, B_z, B_s) \)

Define: \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \)

\[ \text{Lorentz Force} \]
\[ \gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B} \]

\[ \Rightarrow \gamma m \frac{dv}{dt} = -e(v_s B_z - v_z B_s) \]

Assume: \( v_x, v_z \ll v_s \approx c \)

\[ \frac{v_x(s)}{c} = -\frac{e}{\gamma mc} \int_{-\infty}^{s} B_z(s')ds' \]

\[ x(s) = -\frac{e}{\gamma mc} \int_{-\infty}^{s} \int_{-\infty}^{s'} B_z(s'')ds''ds' \]

and similar expression for \( v_z(s) \) and \( z(s) \)
Electron Trajectory in a Planar Sinusoidal Undulator

Consider \( \vec{B} = (0, B_0 \sin(2\pi \frac{s}{\lambda_0}), 0) \)

\[
\begin{align*}
\frac{v_x}{c} &= \frac{K}{\gamma} \cos(2\pi \frac{s}{\lambda_0}) \\
\frac{v_z}{c} &= 0 \\
\frac{v_x}{c} &= 1 - \frac{1}{2\gamma^2} (1 + K^2 \cos^2(2\pi \frac{s}{\lambda_0})) \\
x &\approx -\frac{\lambda_0}{2\pi \gamma} K \sin(2\pi \frac{s}{\lambda_0})
\end{align*}
\]

with \( K = \frac{eB_0\lambda_0}{2\pi mc} = 0.0934 \ B_0[T] \lambda_0[mm] \)

K is a fundamental parameter called : **Deflection Parameter**

Example: ESRF, Energy = 6GeV, Undulator \( \lambda_0 = 35 \ mm, B_0 = 0.7 \ T \)

\( K = 2.3, \ \frac{K}{\gamma} = 200 \ \mu rad, \ \frac{\lambda_0}{2\pi \gamma} K = 1.1 \ \mu m!! \)
\[ B_x = 0 \]
\[ B_z = B_0 \cosh(2\pi \frac{z}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0}) \]
\[ B_s = -B_0 \sinh(2\pi \frac{z}{\lambda_0}) \sin(2\pi \frac{s}{\lambda_0}) \]

Undulator Field Satisfying Maxwell Equation

\[ \gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B} \]
Lorentz Force Equation

2nd Order in \( \gamma^{-1} \)

\[ \frac{d^2 x}{ds^2} = 0 \]
\[ \frac{d^2 z}{ds^2} = -\frac{1}{2} \left( \frac{eB_0}{\gamma mc} \right)^2 \frac{\lambda_0}{4\pi} \sinh(4\pi \frac{z}{\lambda_0}) \approx -\frac{1}{2} \left( \frac{eB_0}{\gamma mc} \right)^2 z \]

\[ K_z = \frac{1}{2} \left( \frac{eB_0}{\gamma mc} \right)^2 \]

A vertical Field Undulator is Vertically Focusing!

\[ \frac{1}{F_z} = \int_{ID} K_z ds = \frac{1}{2} \left( \frac{eB_0}{\gamma mc} \right)^2 L \]
Interference with the beam dynamics in the ring lattice

• An Insertion Device is the first component of a photon beamline. Its field setting is fully controlled by the users of the beamline. The change of field results in a change of beam dynamics in the whole ring.

• As far as the lattices are concerned, **Insertion devices** should ideally behave like **drift space** but the reality is different:
  
  – Closed Orbit distortion (by non zero field integrals=dipole error)
  – Betatron tune shift (by nominal field and by quadrupole errors)
  – Coupling (skew quadrupole errors)
  – Reduction of dynamic aperture (=>Lifetime reduction & reduced injection efficiency)
    • Through a break of the lattice periodicity
    • A change of the focusing versus injection point x,z
  – Very high field IDs may change the damping time, emittance, energy spread …

• By combining carefull manufacture, magnetic field shimming and local active corrections, many perturbations can be compensated.

• The problem of the reduction of dynamic aperture is most severe on low energy rings with many insertion devices.
Synchrotron Radiation from an Insertion Device
Synchrotron Radiation is emitted tangentially to the trajectory inside a cone of angle $1/\gamma$. 

Bending Magnet

Electron bunch

Synchrotron light

Electrons
Radiation by a single electron

Computed for 6 GeV, I = 200 mA, B = 1 tesla

\[ E_c = \frac{3hc}{4\pi} \frac{\gamma^3}{\rho} = \frac{3he}{4\pi m} \gamma^2 B \]

\[ E_c [keV] = 0.665 E^2 [GeV] B [T] \]
Undulator

Source Points

A

B

C

D

\( \lambda_0 \)

e-

Electron trajectory

Electric field in time domain

Elleaume, CAS  Darmstadt Sept 28 - Oct 9, 2009
Electric field and Spectrum vs K

Computed for 6 GeV, I=200 mA, 35 mm period

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Fundamental wavelength of the radiation field

\[ \lambda = \lambda_0 \cos \theta - c t_{AC} \approx \lambda_0 \left(1 - \frac{\theta^2}{2}\right) - \frac{\lambda_0}{c \left(1 - \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2}\right)\right)} \approx \frac{\lambda_0}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right) \]
Wavelength of the Harmonics

\[ \lambda_n = \frac{\lambda_0}{2n\gamma^2} (1 + \frac{K^2}{2} + \gamma^2\theta^2) \]

Equivalently, the energy \( E_n \) of the harmonics are given by

\[ E_n[keV] = \frac{9.5 \, n \, E^2[GeV]}{\lambda_0[mm](1 + \frac{K^2}{2} + \gamma^2\theta^2)} \]

\( \lambda_n, E_n \): Wavelength, Energy of the \( n^{th} \) harmonic

\( n = 1, 2, 3, \ldots \): Harmonic number

\( \lambda_0 \): Undulator period

\( E = \gamma mc^2 \): Electron Energy

\( K \): Deflection Parameter = 0.0934 \( B_0[T] \lambda_0[mm] \)

\( \theta \): Angle between observer direction and e–beam
Undulator Emission by a Filament Electron Beam

\[ \frac{\Delta E}{E} \approx \frac{1}{nN} \]

\[ \Delta \theta \approx \frac{1}{\gamma \sqrt{nN}} \]

- \( n \): Harmonic number
- \( N \): Number of Periods

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E}{mc^2} \]

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What happens if the beam presents a finite emittance (size and divergence) and finite energy spread?
\[ \lambda = \frac{\lambda_0}{2n\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right) \]

Radiation from a filament e- Beam at a wavelength \( \lambda \)

e- Beam with Finite Divergence
Radiation Spectrum through a narrow aperture

- Electron Beam
  - Energy = 6 GeV
  - Current = 200 mA

- Undulator
  - Period = 35 mm
  - K = 2.2
  - Length = 3.2 m

- Collection
  - 10 x 10 µm

- Filament
  - Mono energetic Beam

- ESRF
  - Emittance and Energy Spread
  - (4, nm, 0.02 nm, 0.1%)
Broadening of the Harmonics by the Electron Emittance

Electron Beam
Energy = 6 GeV
Current = 200 mA
Vert. Emitt= 0.04 nm
Energy Spread = 0.1%

Undulator
Period = 35 mm
K = 2.2
Length = 3.2 m

Collection
10 x 10 µm

Photon Energy

Horiz Emitt.= 2 nm
4 nm
8 nm
Broadening of the Harmonics by Electron Energy Spread

**Electron Beam**
- Energy = 6 GeV
- Current = 200 mA
- Hor.z. Emitt. = 4 nm
- Vert. Emitt= 0.04 nm

**Undulator**
- Period = 35 mm
- $K = 2.2$
- Length = 3.2 m

**Collection**
- $10 \times 10 \mu m$

Energy Spread = 0 %
- 0.1 %
- 0.2 %
To make optimum use of the Undulators, the magnet lattice of synchrotron light sources should be designed to produce the smallest emittance and smallest energy spread.
Collecting Undulator Radiation in a variable Aperture

Electron Beam
Energy = 6 GeV
Current = 200 mA
Horiz. Emitt. = 4 nm
Vert. Emitt= 0.04 nm
Energy Spread = 0.1 %

Undulator
Period = 35 mm
K = 2.2
Length = 3.2 m
Maximum Spectral Flux On-axis on odd harmonics

\[ F_n \ [Ph/sec/\ 0.1\%] = 1.431 \times 10^{14} \ N \ I[A] \ Q_n(K) \]
Brilliance (or Brightness)

\[ B_n = \frac{F_n}{(2\pi)^2 \sum_x \sum'_x \sum_z \sum'_z} \]

\[ \Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{(2\pi)^2}} \]

\[ \Sigma'_x = \sqrt{\sigma'_{x}^2 + \frac{\lambda_n}{2L}} \]

Electron beam Single electron emission
Brilliance vs Photon Energy

\[ \lambda = \frac{\lambda_0}{2n\gamma^2} \left(1 + \frac{K^2}{2}\right) \]

\[ B_n = \frac{F_n}{(2\pi)^2 \sum_x \sum'_x \sum_z \sum'_z} \]

\[ \Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{(2\pi)^2}} \]

\[ \Sigma'_x = \sqrt{\sigma'_x^2 + \frac{\lambda_n}{2L}} \]

ESRF
Energy = 6 GeV
Current = 200 mA
Emittances = 4 & 0.02 nm
Low Beta Straight

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Angle Integrated Flux

For Large K, the angle integrated spectrum from an Undulator tends toward that of a bending magnet \( x \times 2N \)

\( \Rightarrow \) Such Devices are called **Wigglers**
Technology
The fundamental issue in the magnetic design of a planar undulator or wiggler is to produce a periodic field with a high peak field $B$ and the shortest period $\lambda_0$ within a given aperture (gap).

- Three type of technologies can be used:
  - Permanent magnets (NdFeB, Sm$_2$Co$_{17}$)
  - Room temperature electromagnets
  - Superconducting electromagnets
Magnetization is equivalent to a surface current

\[ M \quad \leftrightarrow \quad \text{Air coil with Surface Current Density}[\text{A/m}] \approx \frac{B_r[T]}{\mu_0} \]
Periodic Array of Magnets

Surface Current Density \([A/m]\) \(\approx\) \(\frac{2B_r[T]}{\mu_0}\)

or Current Density \([A/m^2]\) \(\approx\) \(\frac{4B_r[T]}{\mu_0 \lambda_0}\)

Examples:

- \(B_r = 1 \text{T} \), \(\lambda_0 = 20 \text{mm}\) \(\Rightarrow\) Equiv. Current Density=160A/mm\(^2\) !!
- \(B_r = 1 \text{T} \), \(\lambda_0 = 400 \text{mm}\) \(\Rightarrow\) Equiv. Current Density=8 A/mm\(^2\)

> 95 % of Insertion Devices are made of Permanent Magnets !!
Permanent Magnet Undulator

Pole (Steel)

Hybrid

Pure Permanent Magnet

Magnet (NdFeB, Sm$_2$Co$_{17}$,...)

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<table>
<thead>
<tr>
<th>Material</th>
<th>$B_c$ [T]</th>
<th>$\mu_{r,1}$</th>
<th>$\mu_{r,\perp}$</th>
<th>$H_{cj}$ [kA/m]</th>
<th>$10^{-2} / ^\circ C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SmCo$_5$</td>
<td>0.9–1.01</td>
<td>1.05</td>
<td></td>
<td>1500–2400</td>
<td>–0.04</td>
</tr>
<tr>
<td>Sm$<em>2$CO$</em>{17}$</td>
<td>1.04–1.12</td>
<td>1.05–1.08</td>
<td></td>
<td>800–2000</td>
<td>–0.03</td>
</tr>
<tr>
<td>NdFeB</td>
<td>1.0–1.4</td>
<td>1.04–1.06</td>
<td>1.15–1.17</td>
<td>1000–3000</td>
<td>–0.10</td>
</tr>
</tbody>
</table>
Support Structure

Must Handle:
- Magnetic Force: 1-20 Tons
- Gap Resolution: < 1 µm
- Parallellism: < 20 µm
Undulators are Fundamentally Small Gap Devices

- For a permanent magnet undulator, shrinking all dimensions maintains the field unchanged.

- The peak field 
  \[ B_0 \propto B_r \exp\left(-\pi \frac{\text{gap}}{\lambda_0}\right) \]

- Benefits of using small gaps Insertion Devices:
  - Decrease the volume of material (cost driving) \( \sim \text{gap}^3 \)
  - The lower the gap, the higher the energy of the harmonics of the undulator emission \( \Rightarrow \) the lower the electron energy required to reach the same photon energy

\[
\lambda = \frac{\lambda_0}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) \quad \text{with} \quad K = \frac{eB_0\lambda_0}{2\pi mc}
\]

- The most advanced undulators have magnet blocks in the vacuum with an operating magnetic gap of 4–6 mm!!
In Vacuum Permanent Magnet Undulators
**Application:** Build a pure permanent magnet undulator with NdFeB Magnets ($B_r = 1.2$ T)

**Undulator with K=1 with 6 GeV energy**

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>5</td>
<td>0.72</td>
<td>15</td>
<td>15.2</td>
<td>6.0</td>
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<tr>
<td>10</td>
<td>0.49</td>
<td>22</td>
<td>10.3</td>
<td>7.3</td>
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<tr>
<td>15</td>
<td>0.38</td>
<td>28</td>
<td>8.2</td>
<td>8.2</td>
</tr>
</tbody>
</table>
Electro-Magnet Undulator

Current Densities $< 20 \text{ A/mm}^2$

Lower field than permanent magnet
For small period / gap
Superconducting Wigglers

- High field: up to 10 T => Shift the spectrum to higher energies
- Complicated engineering & High costs
Magnetic Field Errors in Permanent Magnet Insertion Devices:

• Field errors originate from:
  – Non uniform magnetization of the magnet blocks (poles).
  – Dimensional and Positional errors of the poles and magnet blocks.
  – Interaction with environmental magnetic field (iron frame, earth field,…)

• Important to use highly uniform magnetized blocks
  – perform a systematic characterization of the magnetization
  – Perform a pairing of the blocks to cancel errors
  – Still insufficient …

• Two main type of field errors remain
  – Multipole Field Errors (Normal and skew dipole, quadrupole, sextupole,…).
  – Phase errors which reduce the emission on the high harmonic numbers
  – Further corrections:
    • Correction magnets
    • Shimming
Shimming

• **Mechanical Shimming** :
  – Moving permanent magnet or iron pole vertically or horizontally
  – Best when free space and mechanical fixation make it possible.

• **Magnetic Shimming** :
  – Add thin iron piece at the surface of the blocks
  – Reduce minimum gap and reduce the peak field
Magnetic shims

HYB

Multipole shims

Phase Shim

Magnet

Pole

PPM

Multipole shims

Phase Shim

Magnet V

Magnet H
Field Integral and Multipole Shimming

Horizontal Deflection
- Quadrupole
- Sextupole …

Vertical Deflection
- Skew Quadrupole
- Skew sextupole …

Gap/2 [mm]

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\( T_p : \) time distance between successive peaks

\[
T_p = \frac{c \gamma^2}{\lambda_0 (1 + \frac{K^2}{2})}
\]

\( T_p \) varies from one pole to the next due to period and peak field fluctuations

The \textbf{Phase shimming} consists of a set of local magnetic field corrections, which make \( T_p \) always identical.
Phase Shimming and the single electron spectrum

![Graph showing before and after shimming]
• Learning more about Insertion Devices.

  – J.D. Jackson, Classical Electrodynamics, Chapter 14, John Wiley


