High Brilliance Beam Diagnostic

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Outline

- Brightness and Brilliance
- Fundamental parameters
- Transverse and longitudinal measurements
- Intercepting and non intercepting diagnostic
Brightness and Brilliance

- Several authors give different definitions
- Brilliance is sometimes used, especially in Europe, instead of brightness
- There is also confusion because the same words apply both to particle beams and photon beams
- The best way is to look to units, which should be unambiguous
Some references

Definitions of Brightness

\[ B = \frac{dI}{dSd\Omega} \]

For many practical applications, it is more meaningful to know the total beam current that can be in a 4 dimensional trace space \( V_4 \).

\[ \bar{B} = \frac{I}{V_4} \]

For particle distribution whose boundary in 4D trace space is defined by an hyperellipsoid

\[ \bar{B}_n = \frac{2I}{\pi^2 \varepsilon_x \varepsilon_y} \quad \text{Normalized Brightness} \]

\[ \bar{B} = \frac{2I}{\pi^2 \varepsilon_n \varepsilon_{ny}} \quad \text{[A/(m-rad)^2]} \]
But

- Often the factor $2/\pi^2$ is left out in literature
- Often the rms emittance is used in place of effective emittance and so there is another factor to take into the account
- So it is important to agree on the brightness definition, but the difference can be only in numerical factors
Brilliance

\[ B = \frac{d^4 N}{dtd\Omega dSd\lambda / \lambda} \] Photons/ (s mm\(^2\) mrad\(^2\) 0.1% of bandwidth)

Wiedeman uses the name of spectral brightness but for photons
Parameters to measure

- High brightness can be achieved with small emittance, high charge or both
- **Longitudinal** and **transverse** parameters must be measured
- High charge and small emittance -> high power density beam
- We focus our attention on linac or transfer line where it is possible to use intercepting diagnostic
- For some applications, it is needed to measure also the transverse parameters in different longitudinal positions (correlation)
Transverse parameters

- The most important parameter is the transverse emittance
- To obtain high brightness beam it is of paramount importance to keep emittance growth under control
- Different methods apply for beams with or without space charge contribution
- Mainly the space charge is relevant at the exit of the RF GUN (few MeV)
Intercepting devices

- OTR monitors
  - High energy (>tens of MeV)
  - High charge (>hundreds of pC)
  - No saturation
  - Resolution limit closed to optical diffraction limit
  - Surface effect

- Scintillator (like YAG:CE)
  - Large number of photons
  - Resolution limited to grain dimension (down to few microns)
  - Saturation depending of the doping level
  - Bulk effect
  - Thin crystal to prevent blurring effect

- Wire scanner
  - Multiple scattering reduced
  - Higher beam power
  - Multishot measurement
  - 1 D
  - Complex hardware installation
To measure the emittance for a space charge dominated beam the used technique is known as the 1-D pepper-pot method. The emittance can be reconstructed from the second momentum of the distribution:

\[
\varepsilon = \sqrt{\langle x'^2 \rangle - \langle xx' \rangle^2} \]

Examples
Design issues

- The beamlets must be emittance dominated

- Assuming a round beam

\[ \sigma_x'' = \frac{\varepsilon_n^2}{\gamma^2 \sigma_x^3} + \frac{I}{\gamma^3 I_0 (\sigma_x + \sigma_y)} \]

*Martin Reiser, Theory and Design of Charged Particle Beams (Wiley, New York, 1994)*

- \( d \) must be chosen to obtain \( R_0 << 1 \), in order to have a beam emittance dominated

\[ R_0 = \frac{I \sigma_0^2}{2 \gamma I_0 \varepsilon_n^2} \]

\[ \sigma_x = \frac{d}{\sqrt{12}} \]
Design issues (2)

- The contribution of the slit width to the size of the beamlet profile should be negligible.
- The material thickness (usually tungsten) must be long enough to stop or heavily scatter beam at large angle.
- But the angular acceptance of the slit cannot be smaller of the expected angular divergence of the beam.

\[
\sigma = \sqrt{L \cdot \sigma' + \left(\frac{d^2}{12}\right)}
\]

\[
L \gg \frac{d}{\sigma' \cdot \sqrt{12}}
\]

\[
l < \frac{d}{2\sigma'}
\]
Phase space mapping
Phase space evolution

The most used techniques for emittance measurements are quadrupole scan and multiple monitors

\[ \gamma x^2 + 2\alpha xx' + \beta x'^2 = \varepsilon = \gamma_0 x_0^2 + 2\alpha_0 x_0 x' + \beta_0 x'_0 \]

\[
M(s_1, s_2) = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & S'C + SC' & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}
\]
Beam Matrix

\[ \sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} \]

\[ \sigma_{11} x^2 + 2 \sigma_{12} xx' + \sigma_{22} x'^2 = 1 \]

\[ \sigma_1 = M \sigma_0 M^T \]
Multiple screens

\[ \sigma_{i,11} = C_i^2 \sigma_{11} + 2S_i C_i \sigma_{12} + S_i^2 \sigma_{22} \]

- There are 3 unknown quantities
- \( \sigma_{i,11} \) is the rms beam size
- \( C_i \) and \( S_i \) are the element of the transport matrix
- We need 3 measurements in 3 different positions to evaluate the emittance
Example: FLASH @ DESY

- DESY-Technical Note 03-03, 2003 (21 pages) Monte Carlo simulation of emittance measurements at TTF2 P. Castro
Quadrupole scan

\[ \sigma_{11} = C^2(k)\sigma_{11} + 2C(k)S(k)\sigma_{12} + S^2(k)\sigma_{22} \]

- It is possible to measure in the same position changing the optical functions
- The main difference respect to the multi screen measurements is in the beam trajectory control and in the number of measurements
Source of errors

- Usually the largest error is in the determination of the RMS beam size (Mini Workshop on "Characterization of High Brightness Beams", Desy Zeuthen 2008, https://indico.desy.de/conferenceDisplay.py?confId=806)

- Systematic error comes from the determination of the quadrupole strength, mainly for hysteresis. So a cycling procedure is required for accurate measurements

- Thin lens model is not adequate

- Energy

- Large energy spread can gives chromatic effect

- Assumption: transverse phase space distribution fills an ellipse
Tomography is related to the Radon theorem: a n-dimensional object can be reconstructed from a sufficient number of projection in (n-1) dimensional space.
Tomography

\[ \hat{f}(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) \]

Radon Transform

\[ f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{i2\pi (ux + vy)} dudv. \quad f(x, y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} |w| F(w, \theta) e^{i2\pi w \rho} dw d\theta. \]

Fourier transform of the Radon transform

\[ S(w, \theta) = \int_{-\infty}^{\infty} \hat{f}(\rho, \theta) e^{-i2\pi \rho \rho} d\theta. \]

\[ f(x, y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} |w| S(w, \theta) e^{i2\pi w \rho} dw d\theta, \quad f(x, y) = \int_{0}^{\pi} Q(\rho, \theta) d\theta, \]

- D. Stratakis et al, “Tomography as a diagnostic tool for phase space mapping of intense particle beam”, Physical Review Special Topics – Accelerator and Beams 9, 112801 (2006)
Tomography measurements

\[
\begin{pmatrix}
  x_1 \\
  x'_1
\end{pmatrix} = M_1 \begin{pmatrix}
  x_0 \\
  x'_0
\end{pmatrix}
\]

\[s = \sqrt{M_{11}^2 + M_{12}^2},\]  
Scaling factor

\[
cos(\theta) = \frac{M_{11}}{\sqrt{M_{11}^2 + M_{12}^2}},\]  
Rotation angle

\[
sin(\theta) = \frac{M_{12}}{\sqrt{M_{11}^2 + M_{12}^2}}.\]

- C can be easily obtained from beam spatial distribution
- s can be calculated from the beam line optics
- The accuracy of the result depends from the total angle of the rotation and from the number of the projections
Longitudinal parameters

- Fundamental parameter for the brightness
- Bunch lengths are on ps (uncompressed) or sub-ps time scale
- Several methods
  - Streak Camera
  - Coherent radiations
  - RFD
  - EOS
  - Others
Streak camera

- Expensive device
- Resolution limited to 200-300 fs FWHM
- It is better to place the device outside the beam tunnel so a light collection and transport line is needed
- Reflective optics vs lens optics
- Intercepting device
The transverse voltage introduces a linear correlation between the longitudinal and the transverse coordinates of the bunch.
\[
\Delta x'(z) = \frac{eV_0}{pc} \sin(kz + \phi) \approx \frac{eV_0}{pzc} \left[ \frac{2\pi}{\lambda} z \cos \phi + \sin \phi \right] \quad |z| \ll \frac{\lambda}{2\pi}
\]

\[
\Delta x(z) = \frac{eV_0}{pc} \sqrt{\beta_d \beta_s} \sin \Delta \Psi \left[ \frac{2\pi}{\lambda} z \cos \phi + \sin \phi \right]
\]

\[
\sigma_x = \sqrt{\sigma_{x0}^2 + \sigma_z^2 \beta_d \beta_s \left( \frac{2\pi eV_0}{\lambda pc} \sin \Delta \Psi \cos \phi \right)^2}
\]

\[
eV_0 >> \frac{\lambda}{2\pi \sigma_z} \frac{1}{\sin \Delta \Psi \cos \phi} \sqrt{\frac{\epsilon_N pcme^2}{\beta_d}}
\]

- P. Emma, J. Frisch, P. Krejcik, ”A Transverse RF Deflecting Structure for Bunch Length and Phase Space Diagnostics “, LCLS-TN-00-12, 2000
Longitudinal phase space

- Using together a RFD with a dispersive element such as a dipole
- Fast single shot measurement
Slice parameters are important for linac driving FEL machines.

- Emittance can be defined for every slice and measured.
- Also the slice energy spread can be measured with a dipole and a RFD.
RFD conclusions

- Self calibrating
- Easy to implement
- Single shot
- Resolution down to tens of fs
- Intercepting device
- As energy increases some parameter must be increased:
  - Frequency
  - Voltage or length

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Coherent radiation

Any kind of radiation can be coherent and usable for beam diagnostics

- Transition radiation
- Diffraction radiation
- Synchrotron radiation
- Undulator radiation
- Smith-Purcell radiation
- Cherenkov radiation
Power Spectrum

\[ I_{\text{tot}}(\omega) = I_{\text{sp}}(\omega)[N + N^*(N-1) F(\omega)] \]

\[ F(\omega) = \left| \int_{-\infty}^{\infty} dz \rho(z)e^{i(\omega/c)z} \right|^2 \quad \rho(z) = \frac{1}{\pi c} \int_{0}^{\infty} d\omega \sqrt{F(\omega)} \cos \left( \frac{\omega z}{c} \right) \]

- From the knowledge of the power spectrum is possible to retrieve the form factor
- The charge distribution is obtained from the form factor via Fourier transform
- The phase terms can be reconstructed with Kramers-Kronig analysis (see R. Lai, A.J. Sievers, NIM A 397 (1997) 221-231)
Martin-Puplett Interferometer

\[ I(\delta) \propto \int_{-\infty}^{\infty} |E(t) + E(t + \delta / c)|^2 dt \]

\[ I(\omega) \propto \int_{-\infty}^{\infty} I(\delta) \cos \left( \frac{\omega \delta}{c} \right) d\delta \]

Incident radiation with an arbitrary intensity distribution \( I(\omega) \)

Golay cells or Pyroelectric detector

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Experimental considerations

- Spectrum cuts at low and high frequencies can affect the beam reconstruction
  - Detectors
  - Windows
  - Transport line
  - Finite target size

- For this reason usually the approach is to test the power spectrum with the Fourier transform of a guess distribution

- Coherent synchrotron radiation or diffraction radiation can be generated by totally not intercepting devices and so they are eligible for high brightness beams diagnostic
Electro Optical Sample (EOS)

- Totally non intercepting device and not disturbing device
- It is based on the change of the optical properties of a non linear crystal in the interaction with the Coulomb field of the moving charges
- Several schemes has been proposed and tested
- Very promising technique

Spectral vs temporal decoding

a) Spectral decoding

b) Temporal decoding
The problems of intercepting diagnostic

- High charge
- Small beam dimension (between 50 μm down to tens of nm)
- High repetition rate
- All the intercepting devices are damaged or destroyed from these kind of beams
- No wire scanners, no OTR screens, no scintillators
- There are good candidates for longitudinal diagnostic
- It is difficult to replace intercepting devices for transverse dimensions
- There are a lot of ideas in testing
Laser Wire

- Not intercepting device
- Multi shot measurement (bunch to bunch position jitter, laser pointing jitter, uncertainty in the laser light distribution at IP)
- Setup non easy
- Resolution limited from the laser wavelength
- Several effects to take into account

Rayleigh range of the laser beam: distance between the focus and the point where the laser spot-size has diverged to $\sqrt{2}$ of its minimum value.
Laser interferometry

Diffraction Radiation

- Similar to transition radiation but without intercepting the target
- Transverse size of the EM field is in the order of $\gamma\lambda/2\pi$
- If the gap is comparable with this value DR is emitted
- Angular distribution of the radiation contains valuable information of the beam size and the beam divergence
- The main limit is the small number of photons


Conclusions

- High brightness beam demands particular diagnostic techniques
- Especially non intercepting diagnostics are strongly recommended
- Some of them are already state of the art
- Some others are still developing
- New ideas are daily tested, so if you want your part of glory start to think about today!