Introduction to Feedback

Outline

- Feedback Systems in Accelerators
- Concept of Feedback
- Modelling Dynamic Systems
- Analysis of Feedback
- Design of Feedback

Feedback Systems in Accelerators

Publications 2006/2007 in JACOW

- Commissioning of the LEP Transverse Feedback System
- Multi-bunch Feedback Activities at Photon Factory advanced Ring.
- State of the SLS Multi-bunch Feedback
- Real Time feedback on Beam parameters
- Computation of Wake fields and Impedances for the PETRA III Longitudinal Feedback Cavity
- The design and Performance of the prototype Digital Feedback RF Control System For the PLS Storage
- The P0 feedback Control System Blurs the Line between IOC and FPGA

Publications 2006/07 (C'tnd)

- Operation Experiences of the bunch feedback system for TLS
- Compensation of BPM Chamber Motion in PLS Orbit Feedback System
- LCLS RF Gun Feedback Control
- Comparison of ILC Fast Beam-Beam Feedback Performance in the e-e- and e+e- Modes of Operation
- A Digital Ring Transverse feedback Low-Level RF Control System
- Performance of the New Coupled Bunch Feedback System at HERA-p
- Transverse Feedback Development at SOLEIL

Reference: http://cernsearch.web.cern.ch/cernsearch/Default.aspx?query=doctype:application/pdf%20url:abstract%20url:accelconf/jacow%20url:accelconf%20title:feedback

Examples (C'tnd)

Elettra fast orbit feedback:

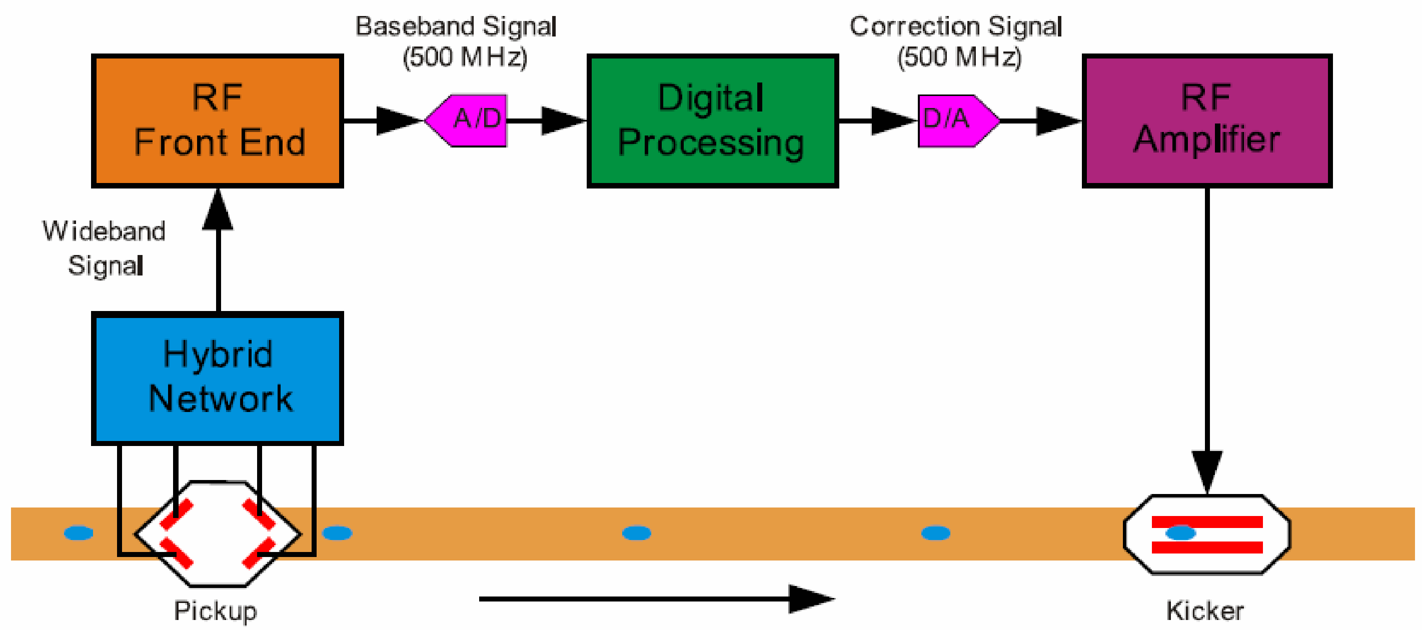


Figure: Functional blocks of a transverse bunch by bunch feedback system

Examples (C'tnd)

Beam based feedback for tune, coupling and chromaticity

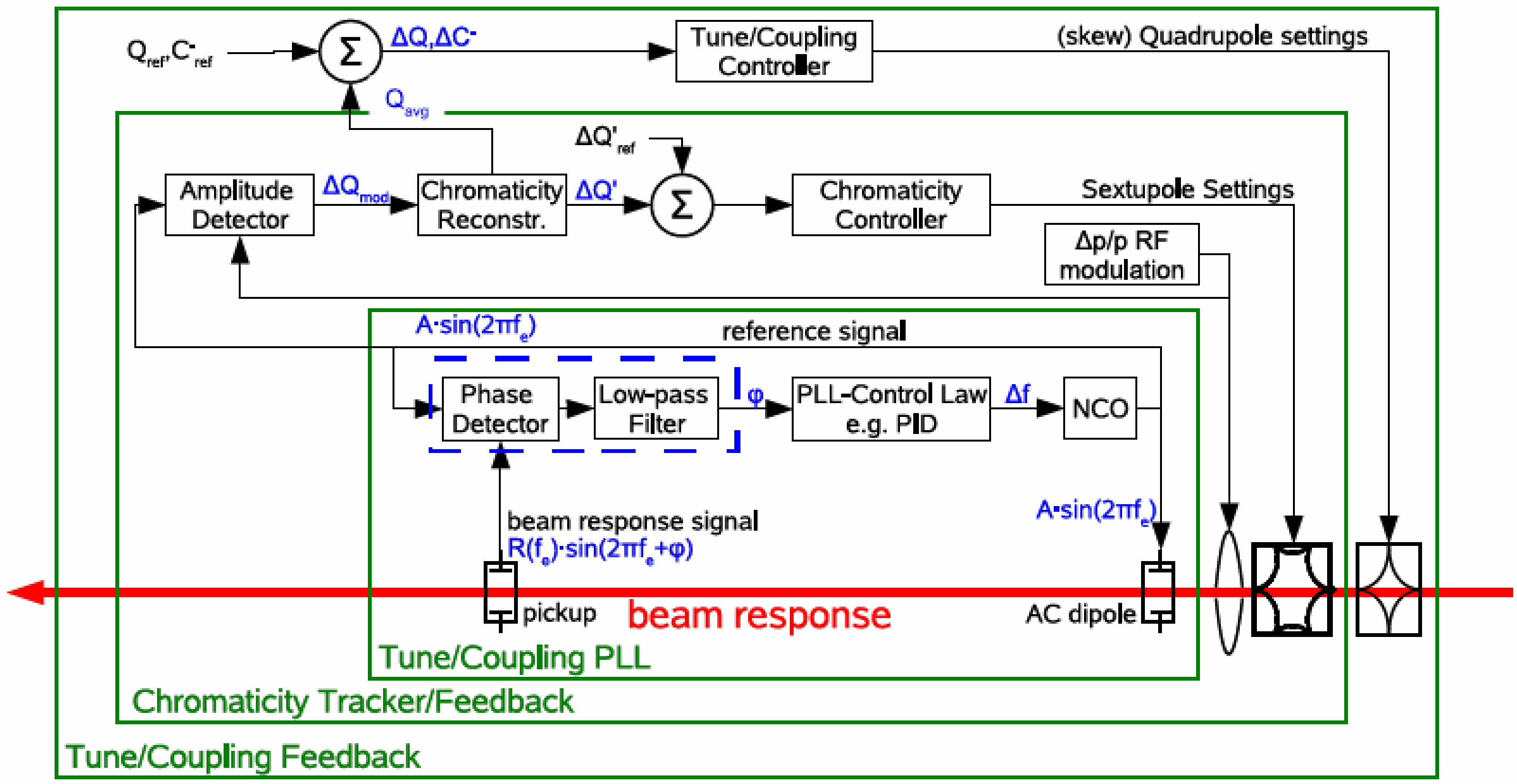


Figure : Nested loop scheme required for a coherent control of tune, coupling and chromaticity.

Examples (C'tnd)

SLS Multi bunch Feedback :

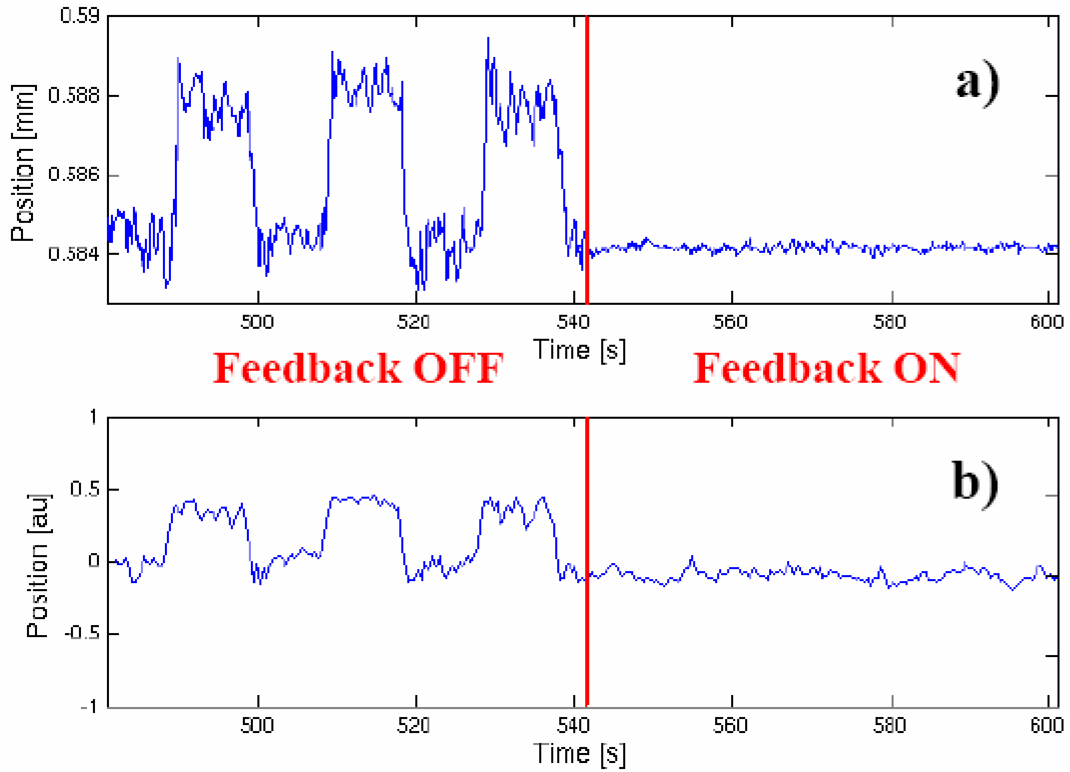


Figure : Effect of the feedback in the vertical plane measured by an electron (a) and a photon (b) BPM. The beam orbit is perturbed by the operation of the electromagnetic elliptical wiggler.

1. Feedback Systems

Objective:

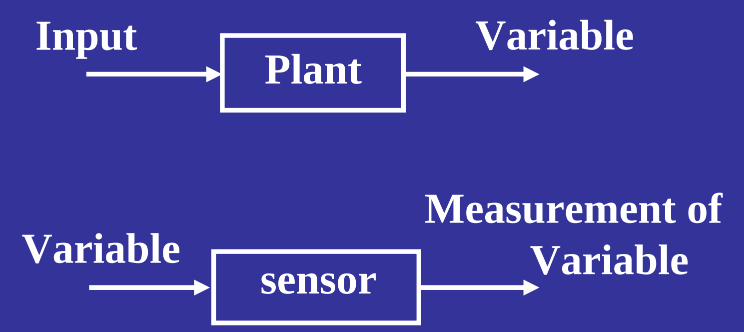
The Introduction of Feedback is concerned with the analysis and design of closed loop control systems.

Analysis:

Closed loop system is given ⟶ determine characteristics or behavior.

Design:

Desired system characteristics or behavior are specified ⟶ configure or synthesize closed loop system.

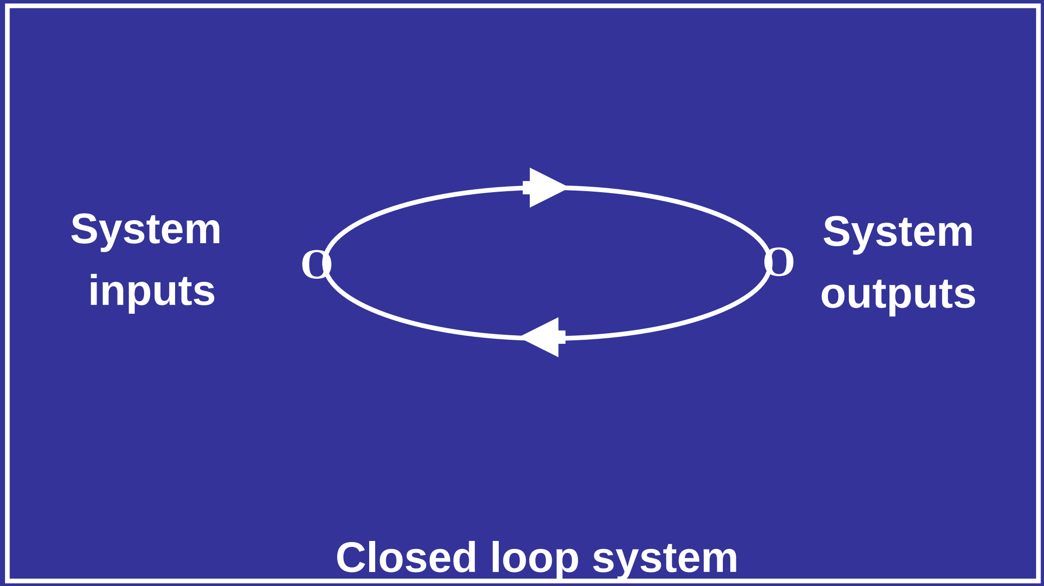


Control-system components

1. Feedback Systems

Definition:

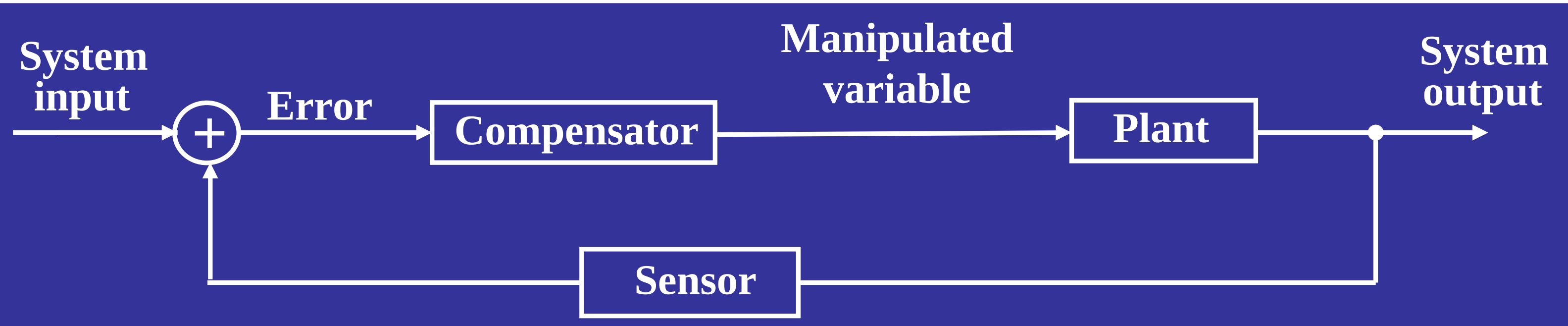
A closed-loop system is a system in which certain forces (we call these inputs) are determined, at least in part, by certain responses of the system (we call these outputs).



1. Feedback Systems

Definitions:

- ❖ The system for measurement of a variable (or signal) is called a ***sensor.***
- ❖ A ***plant*** of a control system is the part of the system to be controlled.
- ❖ The ***compensator*** (or controller or simply filter) provides satisfactory characteristics for the total system.



Closed loop control system

Two types of control systems:

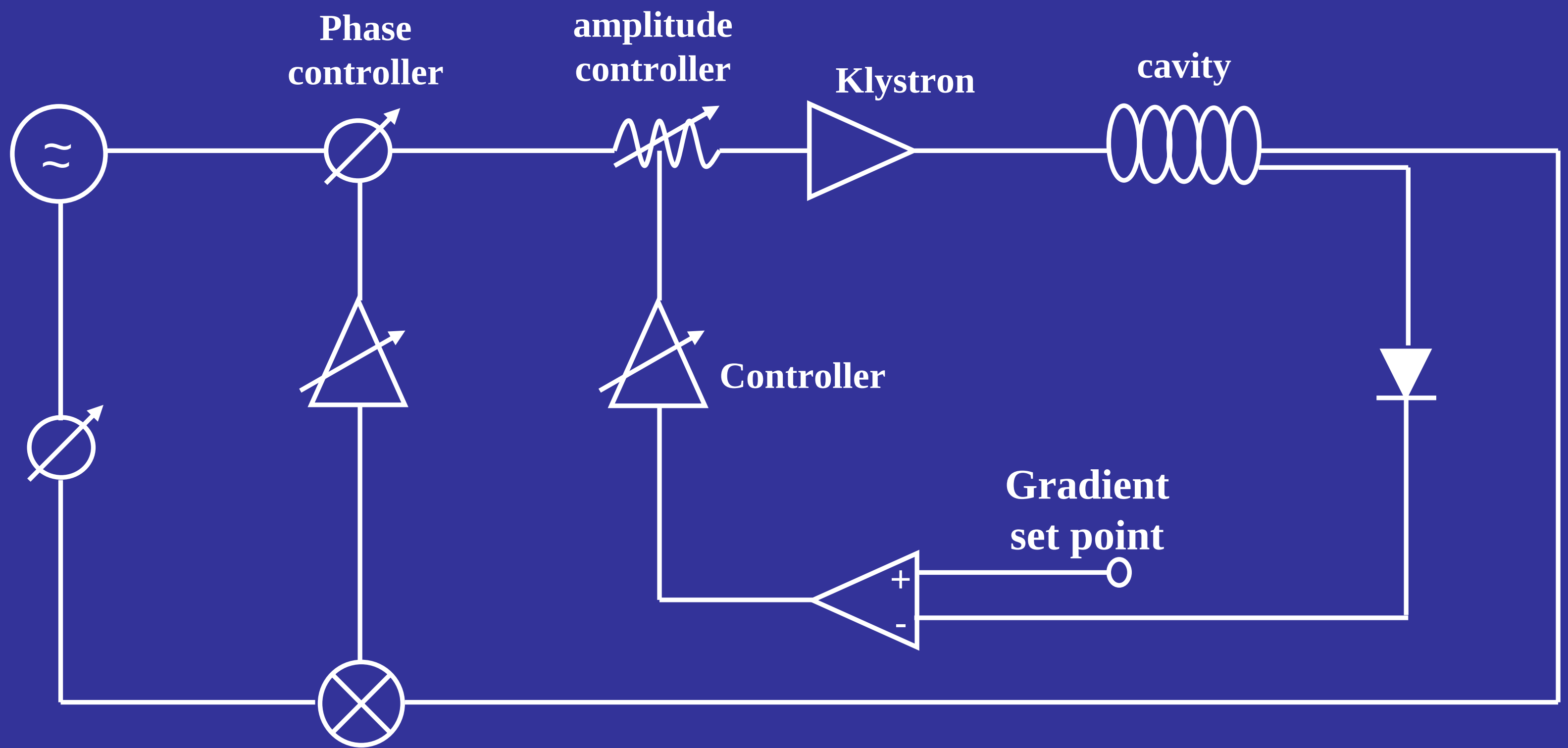
- ❖ A ***regulator*** maintains a physical variable at some constant value in the presence of perturbances.
- ❖ A ***servomechanism*** describes a control system in which a physical variable is required to follow, or track some desired time function (originally applied in order to control a mechanical position or motion).

1. Feedback Systems

Example 1: RF control system

Goal:
Maintain stable gradient and phase.

Solution:
Feedback for gradient amplitude and phase.



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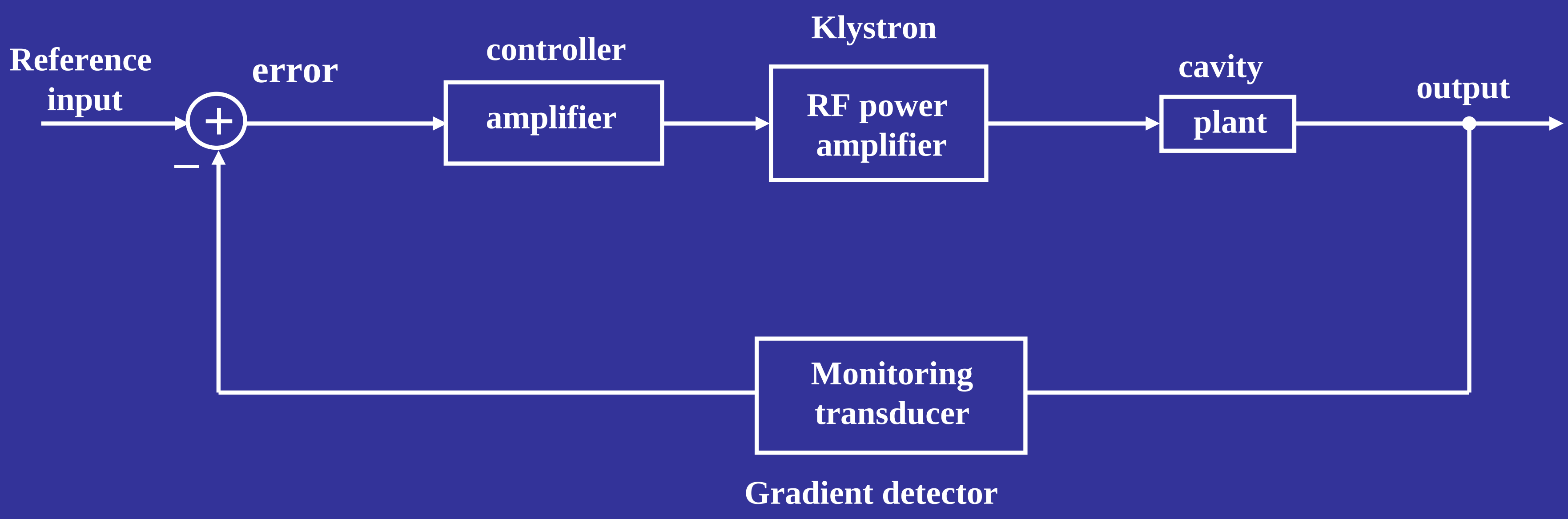
1. Feedback Systems

Model:

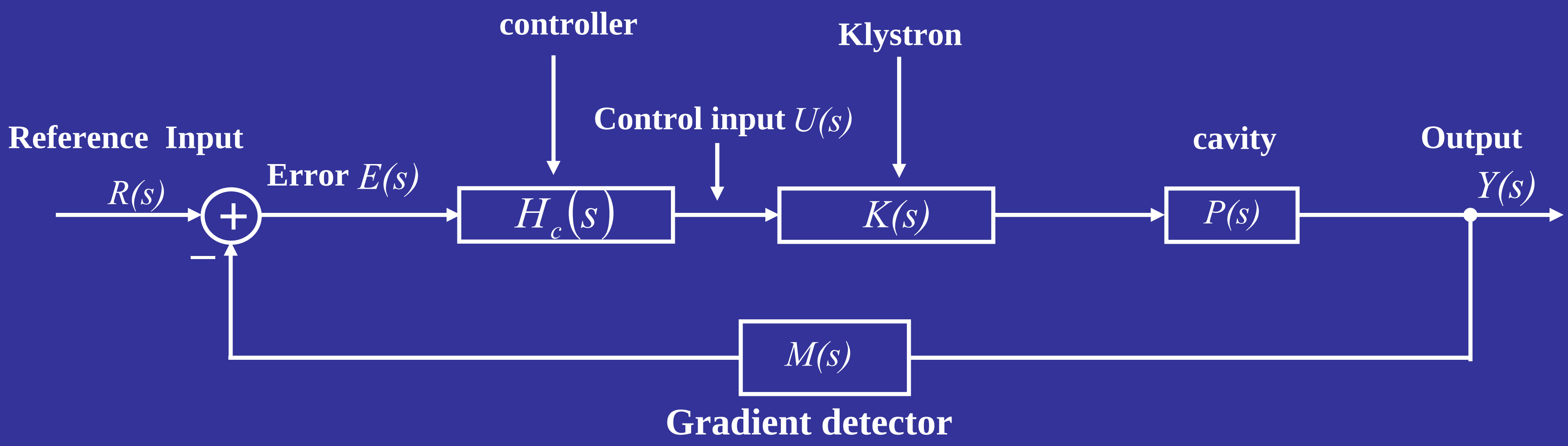
Mathematical description of input-output relation of components combined with block diagram.

Amplitude loop (general form):



1. Feedback Systems

RF control model using *"transfer functions"*



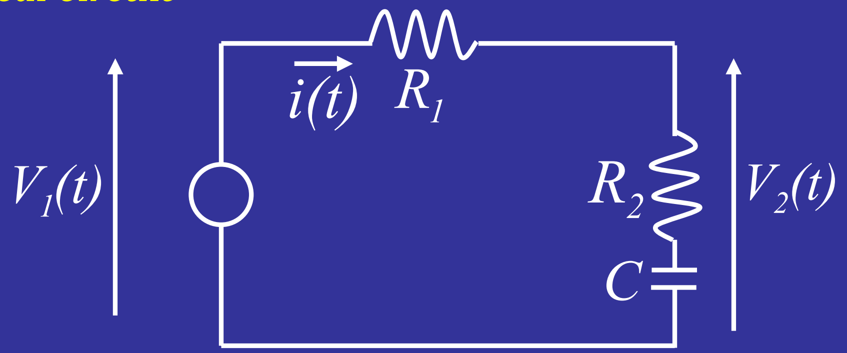
A transfer function of a **linear** system is defined as the ratio of the Laplace transform of the output and the Laplace transform of the input with I. C .'s =zero.

Input-Output Relations

Input	Output	Transfer Function
$U(s)$	$Y(s)$	$G(s) = P(s)K(s)$
$E(s)$	$Y(s)$	$L(s)=G(s)H_c(s)$
$R(s)$	$Y(s)$	$T(s) = (1+L(s)M(s))^{-1}L(s)$

1. Feedback Systems

Example2: Electrical circuit



Differential equations:

$$R_1\, i(t) + R_2\, i(t) + \frac{1}{C}\int_0^t i(\tau)\, d\tau = v_1(t)$$

$$R_2\, i(t) + \frac{1}{C}\int_0^t i(\tau)\, d\tau = v_2(t)$$

Laplace Transform:

$$R_1\, I(s) + R_2\, I(s) + \frac{1}{s \cdot C} I(s) = V_1(s)$$

$$R_2\, I(s) + \frac{1}{s \cdot C} I(s) = V_2(s)$$

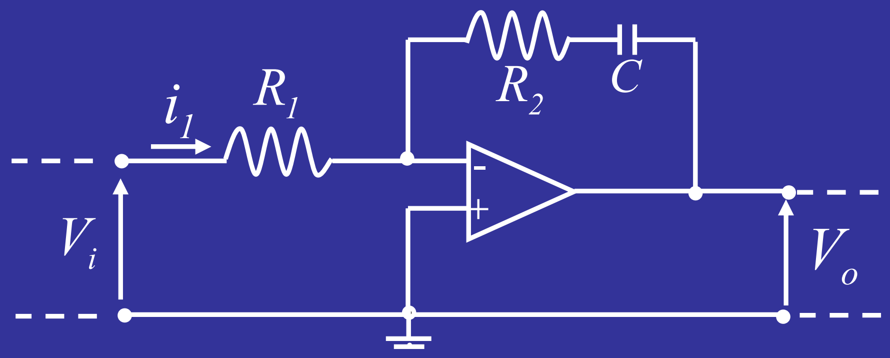
Transfer function:

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 \cdot C \cdot s + 1}{(R_1 + R_2) C \cdot s + 1}$$

Input V_1 ,output V_2

1. Feedback Systems

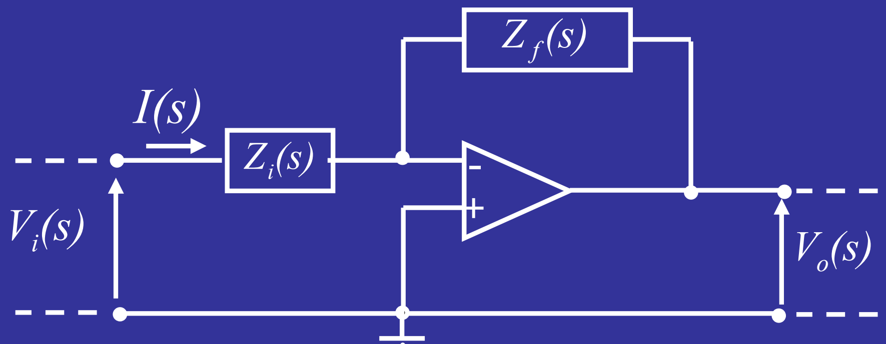
Example 3: Circuit with operational amplifier



$$V_i(s) = R_1 I_1(s) \quad \text{and} \quad V_o(s) = -\left(R_2 + \frac{1}{s \cdot C}\right) I_1(s)$$

$$G(s) = \frac{V_0(s)}{V_i(s)} = -\frac{R_2 \cdot C \cdot s + 1}{R_1 \cdot C \cdot s}$$

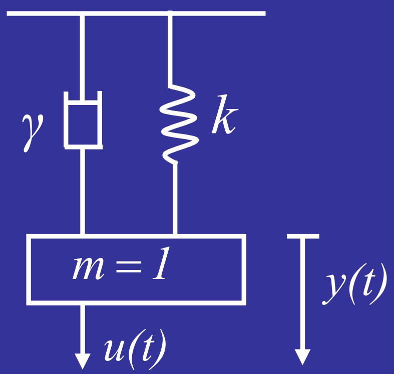
It is convenient to derive a transfer function for a circuit with a single operational amplifier that contains input and feedback impedance:



$$V_i(s) = Z_i(s)\, I(s) \quad \text{and} \quad V_o(s) = -Z_f(s)\, I(s) \longrightarrow G(s) = \frac{V_o(s)}{V_i(s)} = -\frac{Z_f(s)}{Z_i(s)}$$

2. Model of Dynamic System

We will study the following dynamic system:



Parameters:

k : spring constant

γ : damping constant

$u(t)$: force

Quantity of interest:

$y(t)$: displacement from equilibrium

Differential equation: Newton's third law $(m = 1)$

$$\ddot{y}(t) = \sum F_{ext} = -k\, y(t) - \gamma\, \dot{y}(t) + u(t)$$

$$\ddot{y}(t) + \gamma\, \dot{y}(t) + k\, y(t) = u(t)$$

$$y(0) = y_0\ ,\ \dot{y}(0) = \dot{y}_0$$

-Equation is linear (i.e. no $\dot{y}^2$ like terms).

-Ordinary (as opposed to partial e.g. $= \frac{\partial}{\partial x}\frac{\partial}{\partial t} f(x,t) = 0$)

-All coefficients constant: $k(t) = \kappa\ , \gamma(t) = \gamma$ for all t

2. Model of Dynamic System

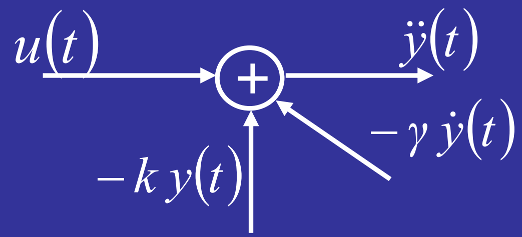
Stop calculating, let's paint!!!

Picture to visualize differential equation

1. Express highest order term (put it to one side)

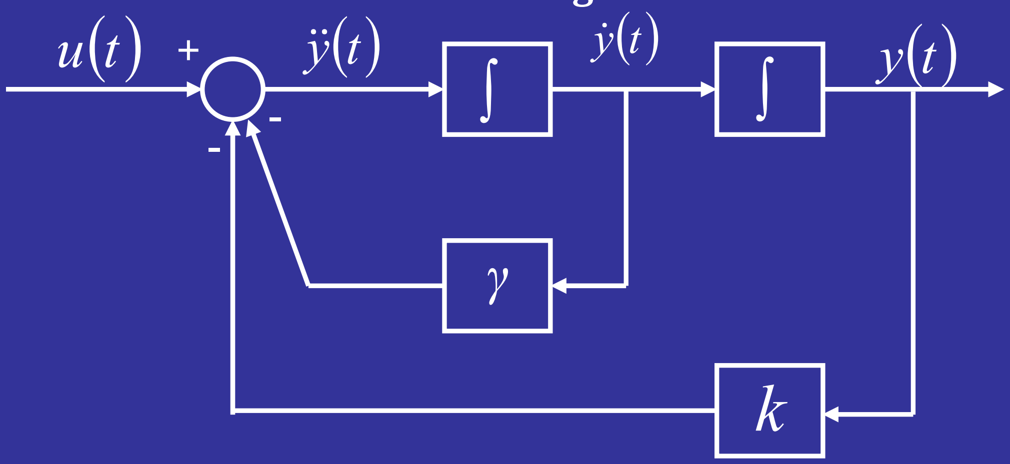
$$\ddot{y}(t) = -k\,y(t) - \gamma\,\dot{y}(t) + u(t)$$

2. Putt adder in front



3. Synthesize all other terms using integrators!

Block diagram



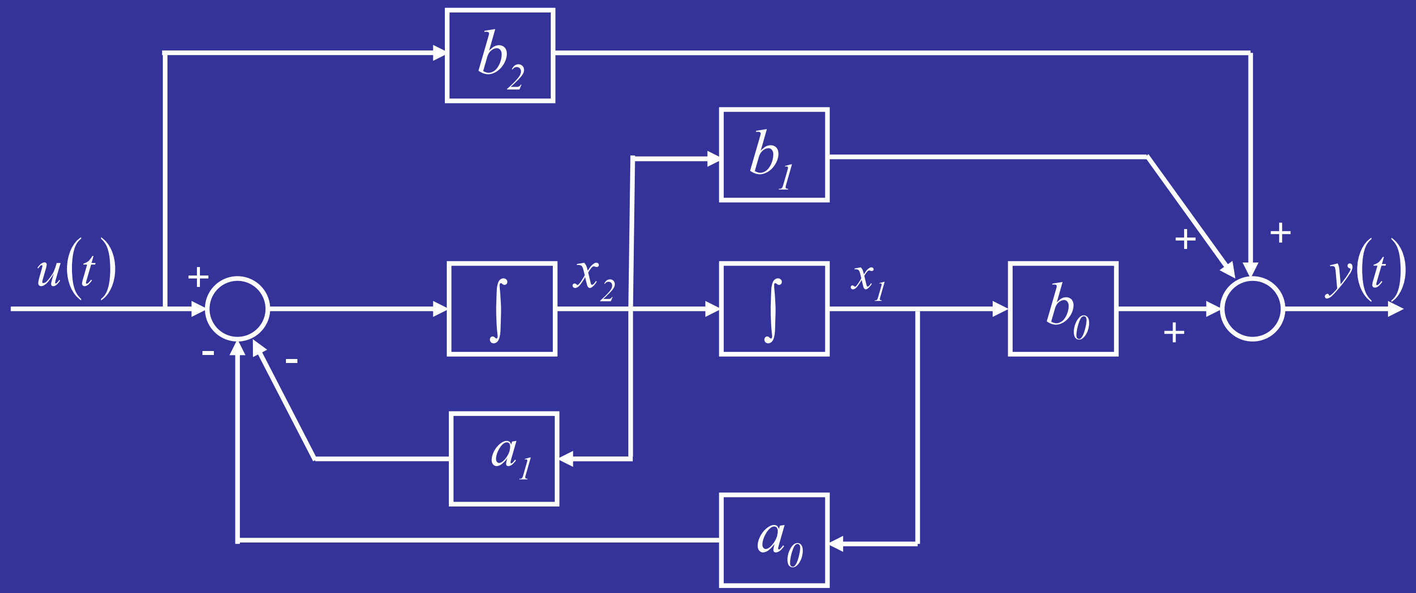
2.1 Linear Ordinary Differential Equation (LODE)

Most important for control system/feedback design:

$$y^{(n)}(t)+a_{n-1}\,y^{(n-1)}(t)+\ldots+a_1\,\dot{y}(t)+a_0\,y(t)=b_m\,u^{(m)}(t)+\ldots+b_1\,\dot{u}(t)+b_0\,u(t)$$

In general: given any linear time invariant system described by LODE can be realized/simulated/easily visualized in a block diagram $(n=2,\ m=2)$

Control-canonical form



Very useful to visualize *<u>interaction</u>* between variables!
What are x_1 and x_2 ????

More explanation later, for now: please simply accept it!

2.2 State Space Equation

Any system which can be presented by LODE can be represented in ***State space form*** (matrix differential equation).

What do we have to do ???

Let's go back to our first example (Newton's law):

$$\ddot{y}(t) + \gamma\,\dot{y}(t) + k\,y(t) = u(t)$$

1. STEP: Deduce set off first order differential equation in variables

$x_j(t)$ (so-called states of system)

$x_1(t) \cong$ Position : $y(t)$

$x_2(t) \cong$ Velocity : $\dot{y}(t)$:

$$\dot{x}_1(t) = \dot{y}(t) = x_2(t)$$

$$\dot{x}_2(t) = \ddot{y}(t) = -k\,y(t) - \gamma\,\dot{y}(t) + u(t)$$
$$= -k\,x_1(t) - \gamma\,x_2(t) + u(t)$$

One LODE of order *n* transformed into *n* LODEs of order 1

2.2 State Space Equation

2. STEP:

Put everything together in a matrix differential equation:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -\gamma \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

State equation

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$y(t) = C\,x(t) + D\,u(t)$$

Measurement equation

<u>Definition:</u>

The **system state** x of a system at any time t_0 is the "amount of information" that, together with all inputs for $t \geq t_0$, uniquely determines the behaviour of the system for all $t \geq t_0$.

2.2 State Space Equation

The linear time-invariant (LTI) analog system is described via

Standard form of the State Space Equation

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad \text{State equation}$$

$$y(t) = C\,x(t) + D\,u(t) \qquad \text{Measurement equation}$$

Where $\dot{x}(t)$ is the time derivative of the vector $x(t) = \begin{bmatrix} x_1(t) \\ \cdots \\ x_n(t) \end{bmatrix}$. And starting conditions $x(t_0)$

Declaration of variables

System completely described by state space matrixes A, B, C, D (in the most cases $D = 0$).

Variable	***Dimension***	***Name***
$X(t)$	$n \times 1$	state vector
A	$n \times n$	system matrix
B	$n \times r$	input matrix
$u(t)$	$r \times 1$	input vector
$y(t)$	$p \times 1$	output vector
C	$p \times n$	output matrix
D	$p \times r$	matrix representing direct coupling between input and output

2.2 State Space Equation

Why all this work with state space equation? Why bother with?

BECAUSE: Given any system of the LODE form

$$y^{(n)}(t)+a_{n-1}\,y^{(n-1)}(t)+\ldots+a_1\,\dot{y}(t)+a_0\,y(t)=b_m\,u^{(m)}(t)+\ldots+b_1\,\dot{u}(t)+b_0\,u(t)$$

Can be represented as

$$\dot{x}(t)=A\,x(t)+B\,u(t)$$
$$y(t)=C\,x(t)+D\,u(t)$$

with e.g. ***Control-Canonical Form*** (case $n=3\,,m=3$):

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a_0 & -a_1 & -a_2\end{bmatrix},\ B=\begin{bmatrix}0\\0\\1\end{bmatrix},\ C=\begin{bmatrix}b_0 & b_1 & b_2\end{bmatrix},\ D=b_3$$

or ***Observer-Canonical Form*:**

$$A=\begin{bmatrix}0 & 0 & -a_0\\ 1 & 0 & -a_1\\ 0 & 1 & -a_2\end{bmatrix},B=\begin{bmatrix}b_0\\b_1\\b_2\end{bmatrix},C=\begin{bmatrix}0 & 0 & 1\end{bmatrix},D=b_3$$

Notation is very compact, But: not unique!!!
Computers love state space equation! (Trust us!)
Modern control (1960-now) uses state space equation.
General (vector) block diagram for easy visualization.

2.2 State Space Equation

Now: Solution of State Space Equation in the time domain. Out of the hat…et voila:

$$x(t) = \Phi(t)\, x(0) + \int_0^t \Phi(\tau)\, B\, u(t-\tau)\, d\tau$$

Natural Response **+** ***Particular Solution***

$$\begin{aligned} y(t) &= C\, x(t) + D\, u(t) \\ &= C\, \Phi(t)\, x(0) + C \int_0^t \Phi(\tau)\, B\, u(t-\tau)\, d\tau + D\, u(t) \end{aligned}$$

With the ***<u>state transition matrix</u>***

$$\Phi(t) = I + At + \frac{A^2}{2!} t^2 + \frac{A^3}{3!} t^3 + \ldots = e^{At}$$

Exponential series in the matrix A (time evolution operator) properties of $\Phi(t)$ (state transition matrix).

1. $\dfrac{d\Phi(t)}{dt} = A\, \Phi(t)$
2. $\Phi(0) = I$
3. $\Phi(t_1 + t_2) = \Phi(t_1) \cdot \Phi(t_2)$
4. $\Phi^{-1}(t) = \Phi(-t)$

Example:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; \Phi(t) = I + At = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} = e^{At}$$

Matrix A is a nilpotent matrix.

2.4 Transfer Function G (s)

Continuous-time state space model

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad \text{State equation}$$

$$y(t) = C\,x(t) + D\,u(t) \qquad \text{Measurement equation}$$

Transfer function describes input-output relation of system.

$$s\,X(s) - x(0) = A\,X(s) + B\,U(s)$$

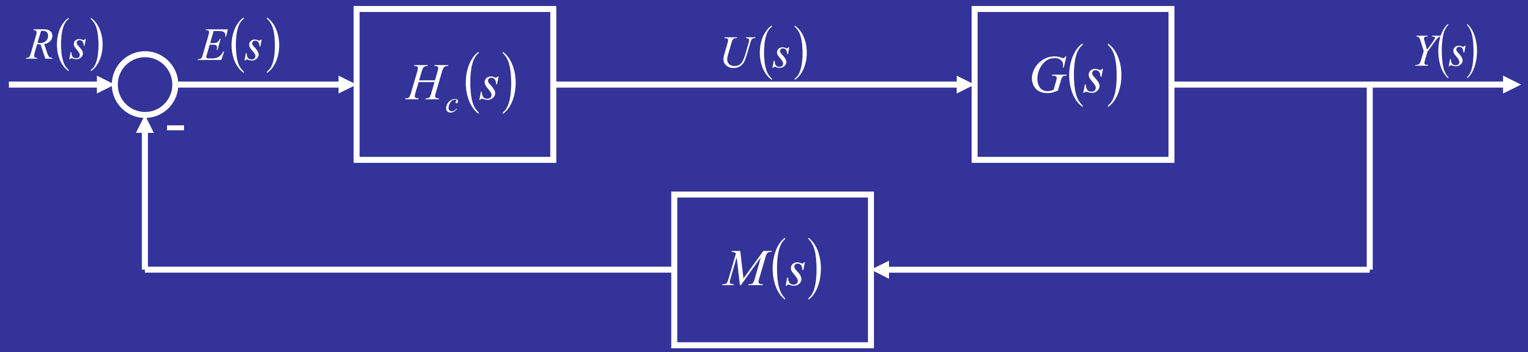
$$X(s) = (sI - A)^{-1} x(0) + (sI - A)^{-1} B\,U(s)$$
$$= \Phi(s)\,x(0) + \Phi(s)\,B\,U(s)$$

$$Y(s) = C\,X(s) + D\,U(s)$$
$$= C[(sI - A)^{-1}]x(0) + [c(sI - A)^{-1} B + D]U(s)$$
$$= C\,\Phi(s)\,x(0) + C\,\Phi(s)\,B\,U(s) + D\,U(s)$$

Transfer function $G(s)$ (pxr) (case: x(0)=0):

$$G(s) = C(sI - A)^{-1} B + D = C\,\Phi(s)\,B + D$$

2.4 Transfer Function of a Closed Loop System



We can deduce for the output of the system.

$$
\begin{aligned}
Y(s) &= G(s)\,U(s) = G(s)\,H_c(s)\,E(s) \\
&= G(s)\,H_c(s)\left[R(s) - M(s)\,Y(s)\right] \\
&= L(s)\,R(s) - L(s)\,M(s)\,Y(s)
\end{aligned}
$$

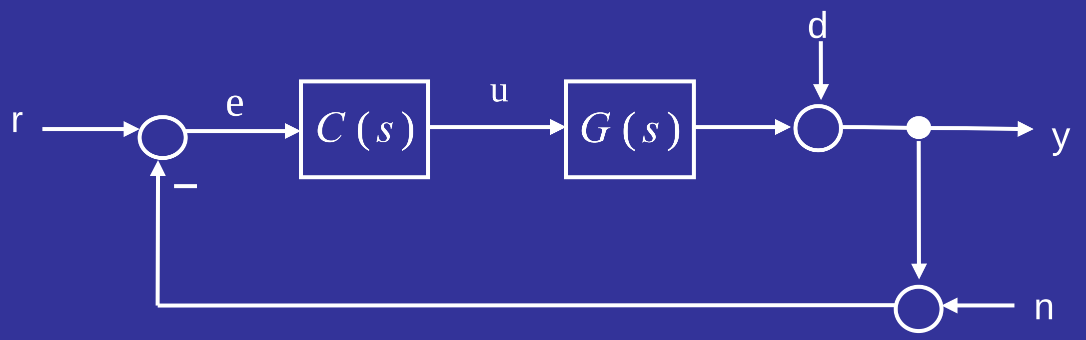
With $L(s)$ the transfer function of the open loop system (controller plus plant).

$$
\begin{aligned}
\left(I + L(s)\,M(s)\right) Y(s) &= L(s)\,R(s) \\
Y(s) &= \left(I + L(s)\,M(s)\right)^{-1} L(s)\,R(s) \\
&= T(s)\,R(s)
\end{aligned}
$$

$T(s)$ is called : Reference Transfer Function

2.5 Sensitivity and Disturbance Rejection

Consider the following closed loop system:



r(t) - reference input
d(t) – output disturbance
n(t) – measurement noise
y(t) – controlled output

The controlled output : $y = Gu + d$

substituting : $u = C(r - y - n)$

yields : $y = (I + GC)^{-1} GC(r - n) + (I + GC)^{-1} d$

Now define the transfer functions for:

Sensitivity: $S(s) = (I + G(s)C(s))^{-1}$

Complementary Sensitivity: $T(s) = (I + G(s)C(s))^{-1} G(s)C(s)$

$$T + S = I$$

2.5 Sensitivity and Disturbance Rejection (C'tnd)

Controller Design objectives are:

- Tracking: output should follow reference
  $y(t) = r(t)$ $\quad T(s) = I \quad S(s) = 0$
- Disturbance rejection: controller should keep the controlled output at it desired value $d(t) \cong 0$ $\quad T(s) = I \quad S(s) = 0$
- Noise rejection: suppressing measurement noise
  $n(t) \cong 0$ $\quad T(s) = 0 \quad S(s) = I$
- Reasonable control effort: must achieve given constraints of the actuator system

It turns out that perfect tracking and disturbance rejection on the one hand, and noise rejection on the other hand are conflicting design objectives.

Goal: Find the best trade-off between all to find the optimal controller for your application!

2.7 Poles and Zeroes

Can stability be determined if we know the TF of a system?

$$G(s) = C\,\Phi(s)\,B + D = C\,\frac{[sI - A]_{adj}}{\chi(s)}\,B + D$$

Coefficients of Transfer function G(s) are rational functions in the complex variable **s**

$$g_{ij}(s) = \alpha \cdot \frac{\prod_{k=1}^{m}(s - z_k)}{\prod_{l=1}^{n}(s - p_l)} = \frac{N_{ij}(s)}{D_{ij}(s)}$$

z_k zeroes. p_l poles, α real constant, and it is $m \le n$ (we assume common factors have already been canceled!)

What do we know about the zeros and the poles?

Since numerator $N(s)$ and denominator $D(s)$ are polynomials with real coefficients, Ploes and zeroes must be real numbers or must arise as complex conjugated pairs!

2.7 Poles and Zeroes

Stability directly from state-space

$$Recall : H(s) = C(sI - A)^{-1} B + D$$

Assuming D=0 (D could change zeros but not poles)

$$H(s) = \frac{C(sI - A)_{adj} B}{\det(sI - A)} = \frac{b(s)}{a(s)}$$

Assuming there are no common factors between the poly $Cadj(sI - A)B$ and $det(sI - A)$
i.e. no pole-zero cancellations (usually true, system called " minimal") then we can identify

and

$$b(s) = C(sI - A)_{adj} B$$

$$a(s) = det(sI - A)$$

i.e. poles are root of $det(sI - A)$

Let λ_i be the i^{th} eigenvalue of A

if $\mathrm{Re}\{\lambda_i\} \leq 0$ for all i => System stable

So with computer, with eigenvalue solver, can determine system stability directly from coupling matrix A.

2.8 Stability Criteria

A system is BIBO stable if, for every bounded input, the output remains bounded with Increasing time.

For a LTI system, this definition requires that all poles of the closed-loop transfer-function (all roots of the system characteristic equation) lie in the left half of the complex plane.

Several methods are available for stability analysis:

1. Routh Hurwitz criterion

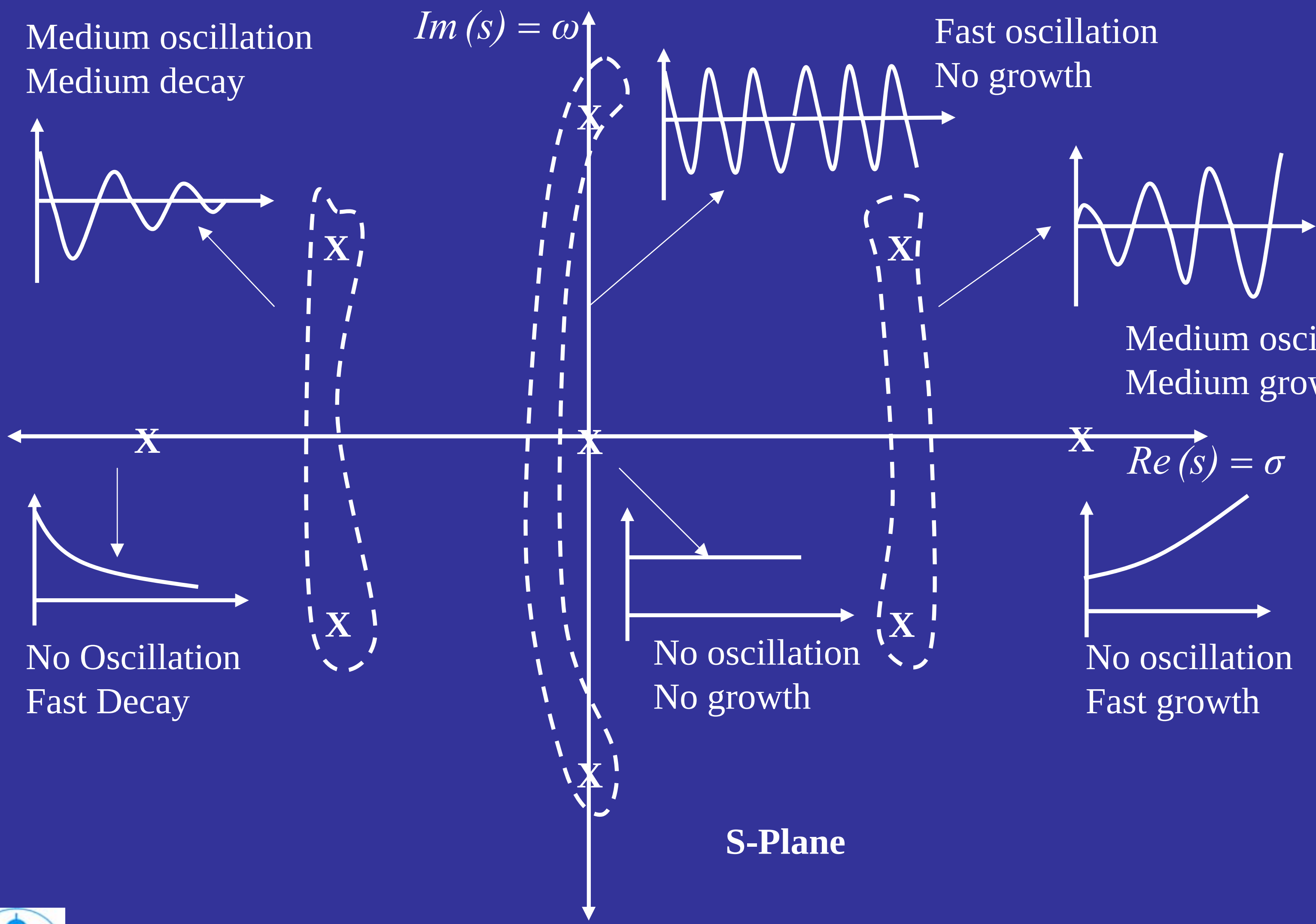
2. Calculation of exact locations of roots
    a. Root locus technique
    b. Nyquist criterion
    c. Bode plot

3. Simulation (only general procedures for nonlinear systems)

While the first criterion proofs whether a feedback system is stable or unstable,
the second Method also provides information about the setting time (damping term).

2.8 Poles and Zeroes

Pole locations tell us about impulse response i.e. also stability:



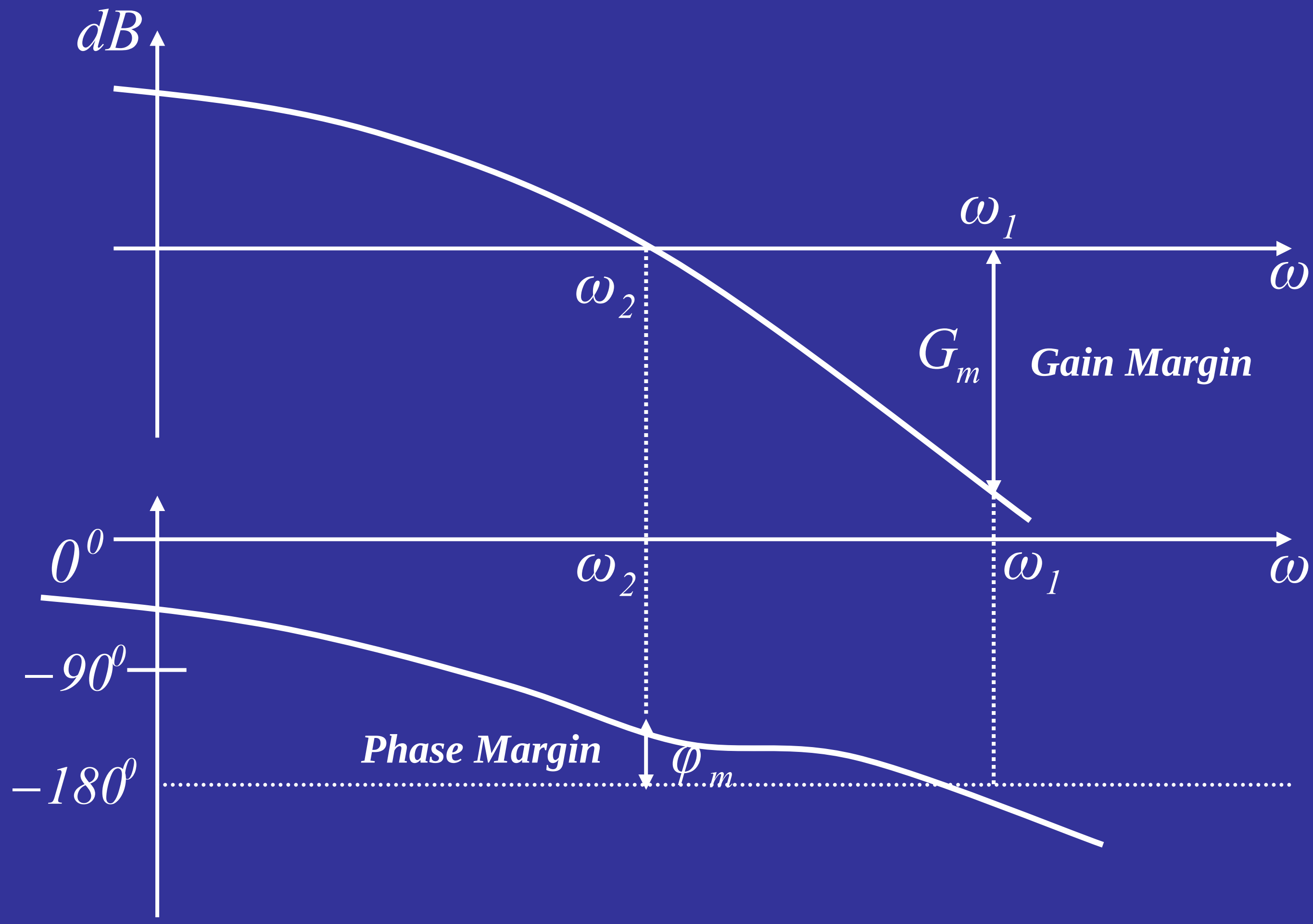
2.8 Poles and Zeroes

Furthermore: Keep in mind the following picture and facts!

- Complex pole pair: Oscillation with growth or decay.
- Real pole: exponential growth or decay.
- Poles are the Eigenvalues of the matrix A.
- Position of zeros goes into the size of c_j....

In general a complex root must have a corresponding conjugate root (N(s), D(S) polynomials with real coefficients.

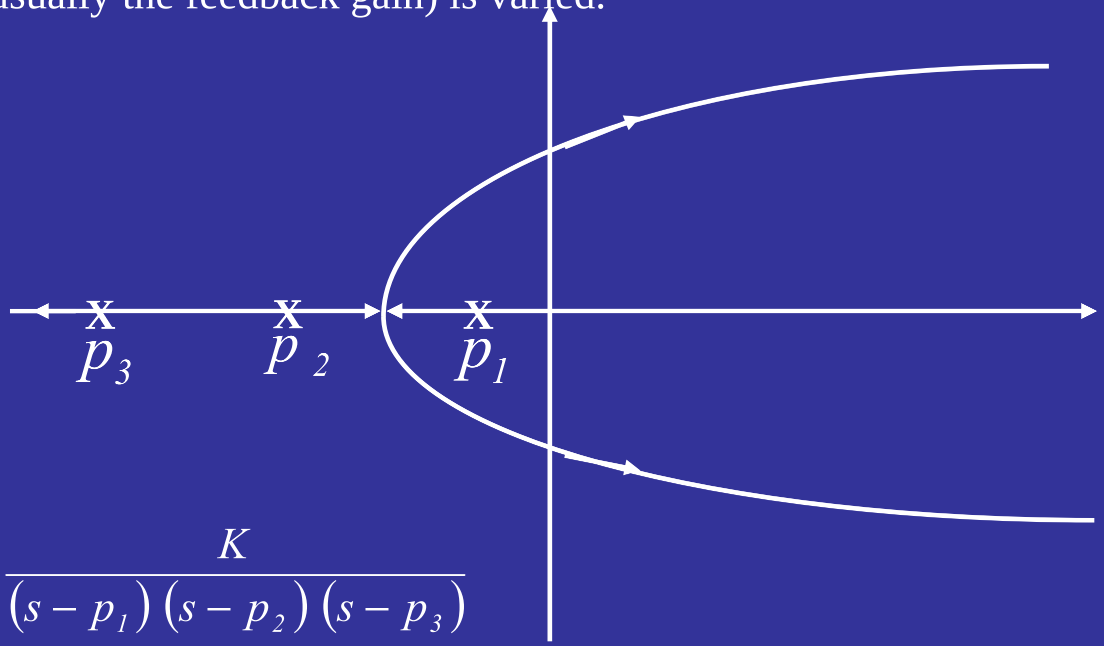
2.8 Bode Diagram



The closed loop is stable if the phase of the unity crossover frequency of the OPEN LOOP Is larger than-180 degrees.

2.8 Root Locus Analysis

Definition: A root locus of a system is a plot of the roots of the system characteristic Equation (the poles of the closed-loop transfer function) while some parameter of the system (usually the feedback gain) is varied.

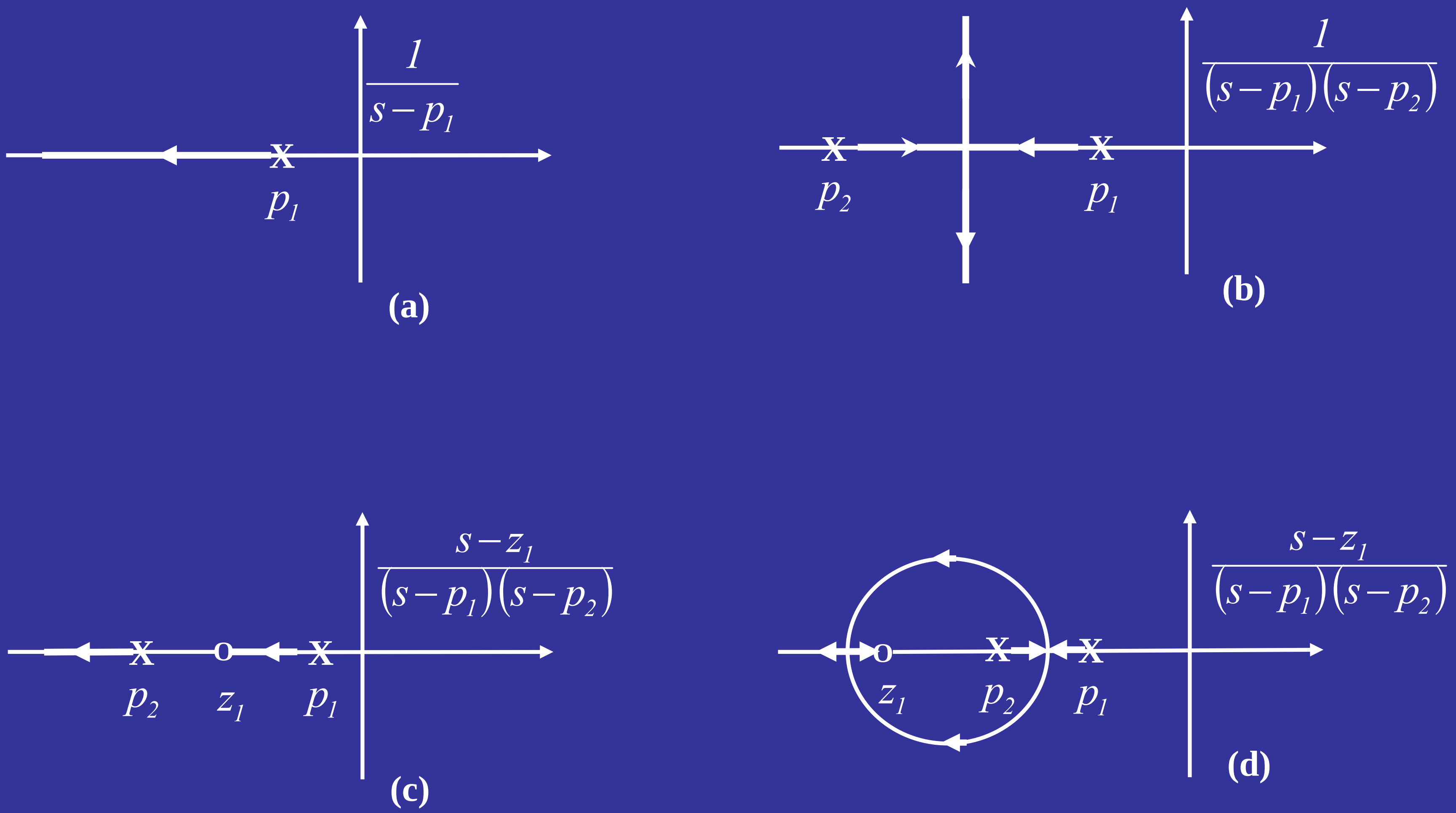


$$K\,H(s) = \frac{K}{(s-p_1)\,(s-p_2)\,(s-p_3)}$$

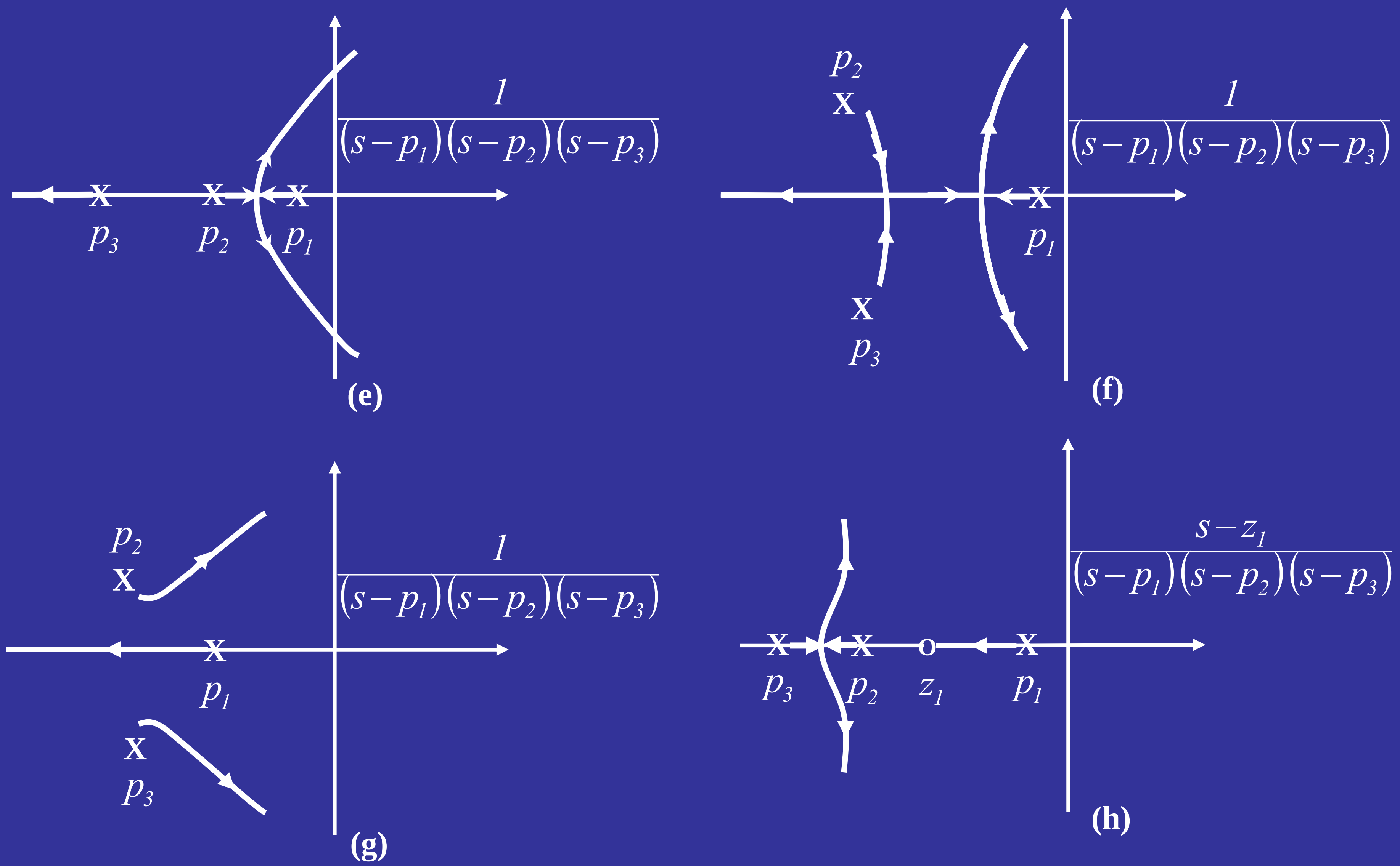
$$G_{CL}(s) = \frac{K\,H(s)}{1+K\,H(s)} \text{ roots at } 1+K\,H(s)=0.$$

How do we move the poles by varying the constant gain K?

2.8 Root Locus Analysis



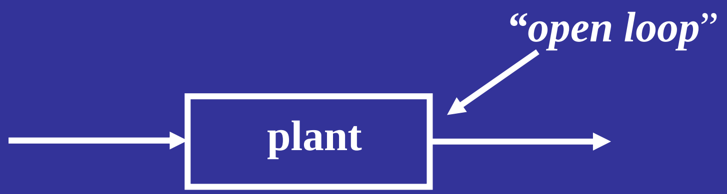
2.8 Root Locus Analysis (Cnt'd)



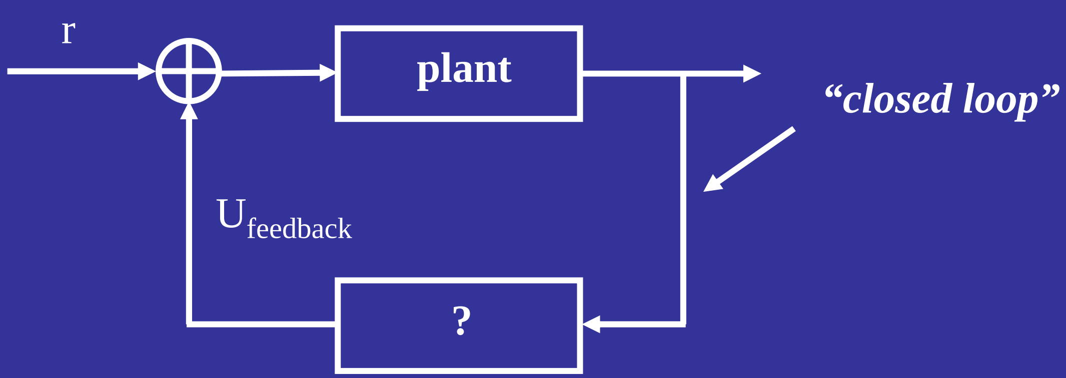
3. Feedback

The idea:

Suppose we have a system or "plant"



We want to improve some aspect of plant's performance by observing the output and applying a appropriate "correction" signal. ***This is feedback***



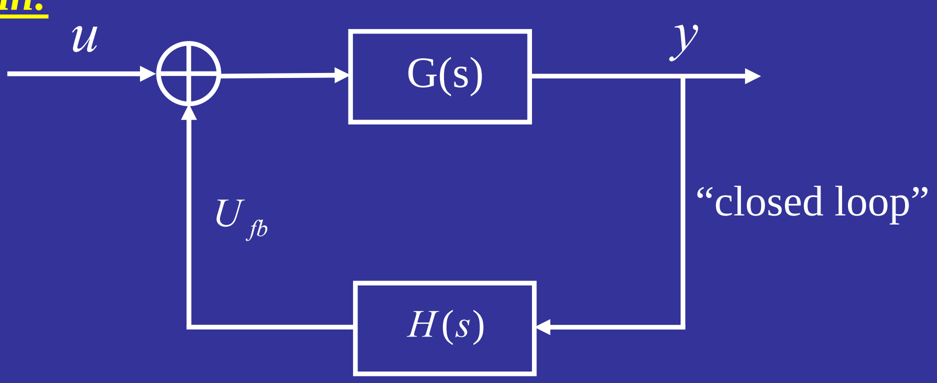
Question: What should this be?

3. Feedback

Open loop gain:

$$G^{O.L}(s) = G(s) = \left(\frac{u}{y}\right)^{-1}$$

Closed-loop gain:



$$G^{C.L}(s) = \frac{G(s)}{1 + G(s)\,H(s)}$$

$$Proof:\; y = G(u - u_{fb})$$

$$= G\,u - G\,u_{fb} \qquad \Rightarrow y + G\,H_y = G\,u$$

$$= G\,u - G\,Hy \qquad \Rightarrow \frac{y}{u} = \frac{G}{(1 + G\,H)}$$

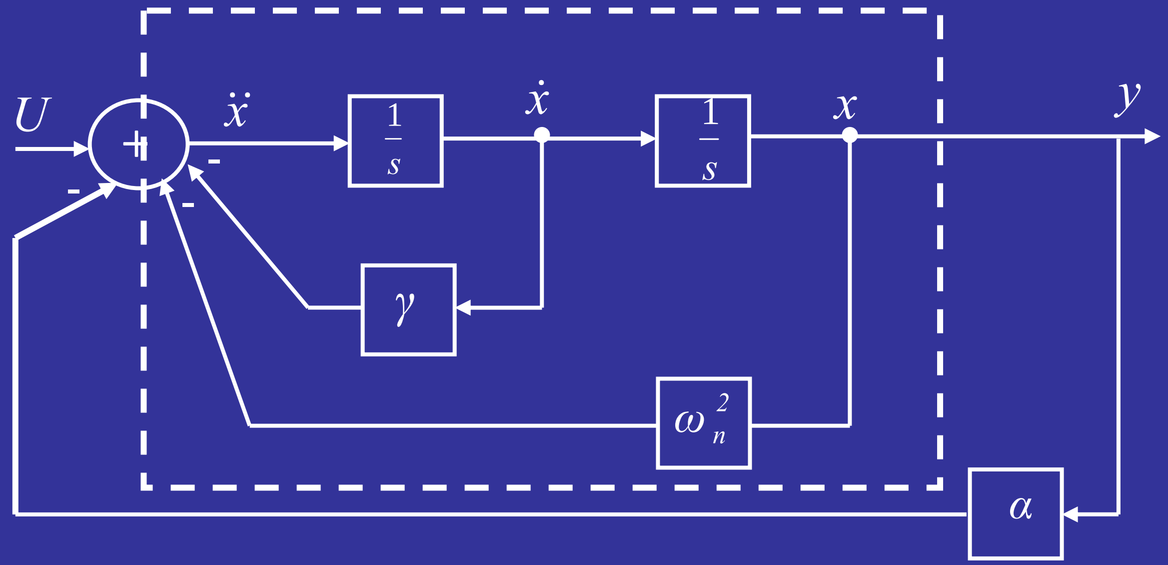
3.1 Feedback-Example 1

Consider S.H.O with feedback proportional to x i.e.:

Where

$$\ddot{x} + \gamma\,\dot{x} + \omega_n^2 x = u + u_{fb}$$

$$u_{fb}(t) = -\alpha\,x\,(t)$$



Then

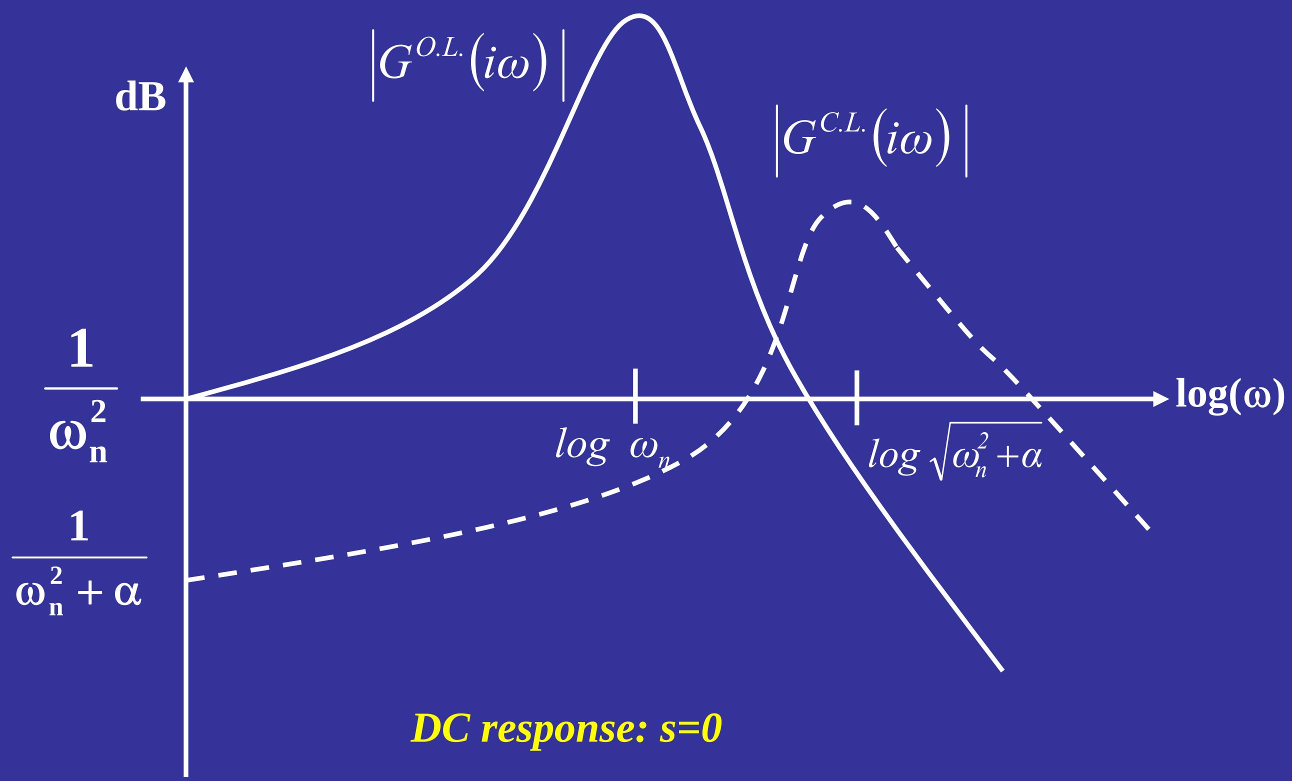
$$\ddot{x} + \gamma\,\dot{x} + \omega_n^2 x = u - \alpha\,x$$

$$==> \ddot{x} + \gamma\,\dot{x} + \left(\omega_n^2 + \alpha\right) x = u$$

Same as before, except that new “natural” frequency $\omega_n^2 + \alpha$

3.1 Feedback-Example 1

Now the closed loop T.F. is: $G^{C.L.}(s) = \frac{1}{s^2 + \gamma s + (\omega_n^2 + \alpha)}$



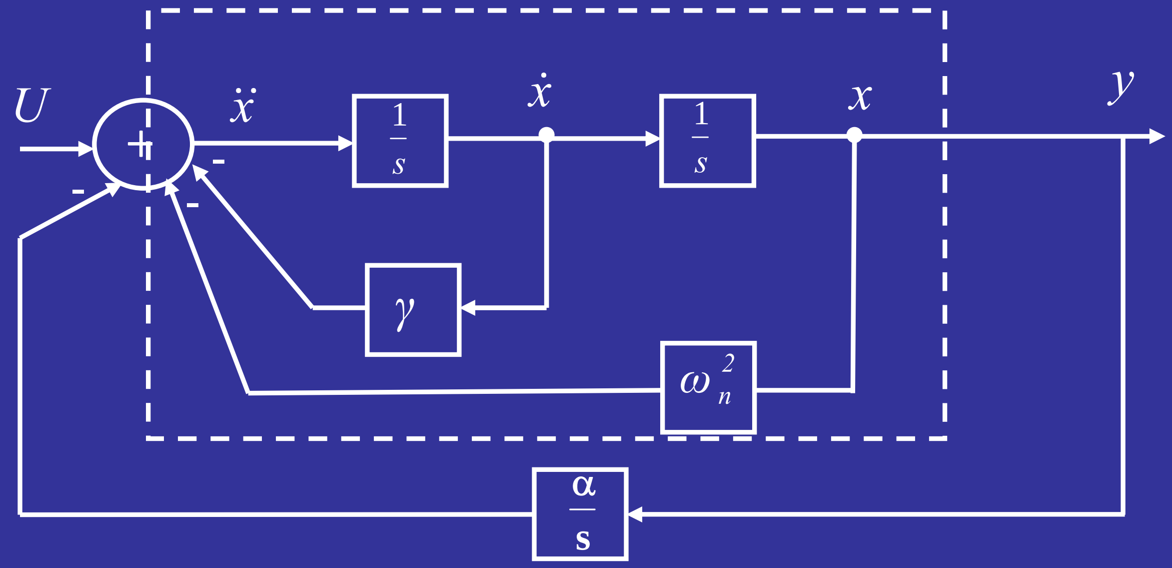
So the effect of the proportional feedback in this case is ***to increase the bandwidth of the system***
(and reduce gain slightly, but this can easily be compensated by adding a constant gain in front…)

3.1 Feedback-Example 2

In S.H.O. suppose we use *integral feedback*:

$$u_{fb}(t) = -\alpha \int_0^t x(\tau)\, d\tau$$

$$i.e \quad \ddot{x} + \gamma \dot{x} + \omega_n^2 x = u - \alpha \int_0^t x(\tau)\, d\tau$$



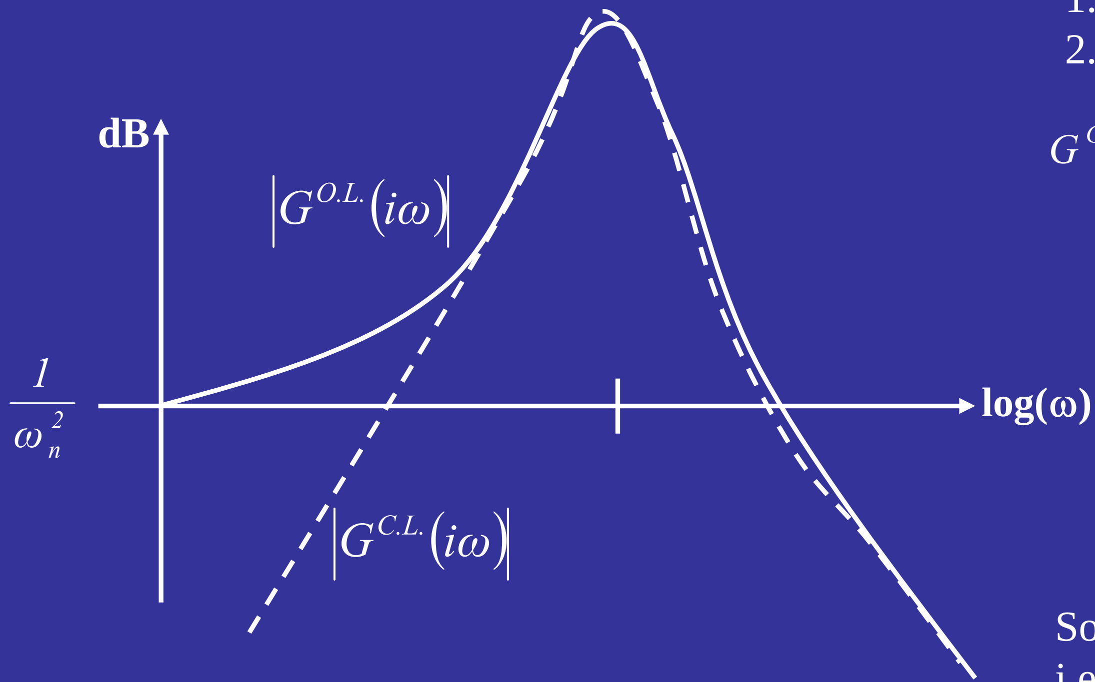
Differentiating once more yields: $\dddot{x} + \gamma \ddot{x} + \omega_n^2 \dot{x} + \alpha x = \dot{u}$

No longer just simple S.H.O., add another state

3.1 Feedback-Example 2

$$G^{C.L.}(s) = \frac{\frac{1}{s^2+\gamma s+\omega_n^2}}{1+\left(\frac{\alpha}{s}\right)\left(\frac{1}{s^2+\gamma s+\left(\omega_n^2+\alpha\right)}\right)}$$

$$= \frac{s}{s\left(s^2+\gamma s+\omega_n^2\right)+\alpha}$$



Observe that

1. $G^{C.L.}(0=0)$
2. For large s (and hence for large ω)

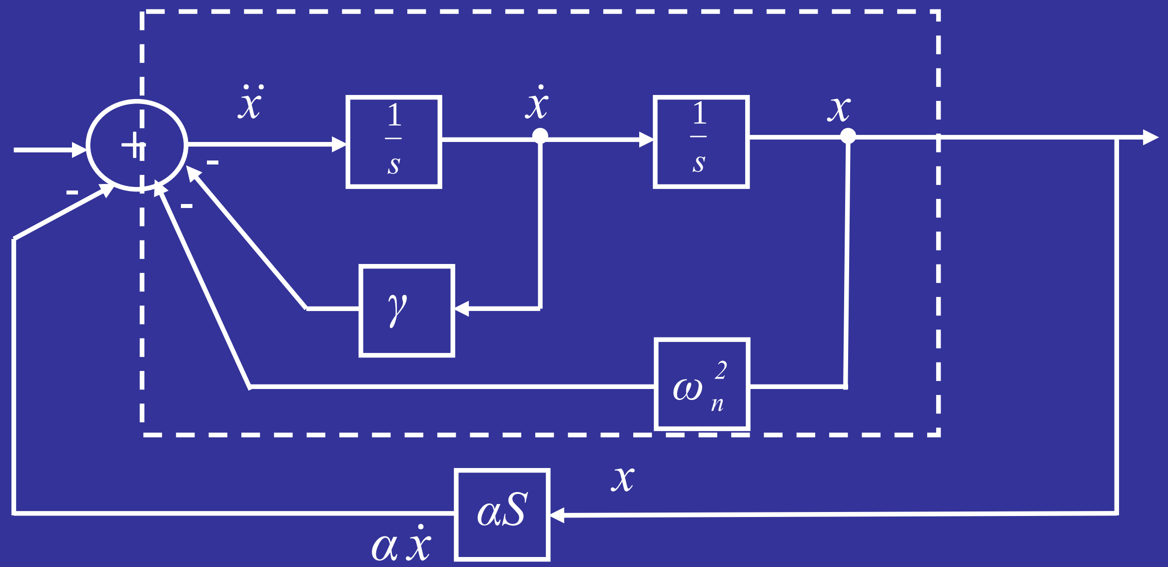
$$G^{C.L.}(s) \approx \frac{1}{\left(s^2+\gamma s+\omega_n^2\right)} \approx G^{O.L.}(s)$$

So integral feedback has killed DC gain
i.e system *rejects constant* disturbances

3.1 Feedback-Example 3

Suppose S.H.O now apply differential feedback i.e.

$$u_{fb}(t) = -\alpha\,\dot{x}(t)$$



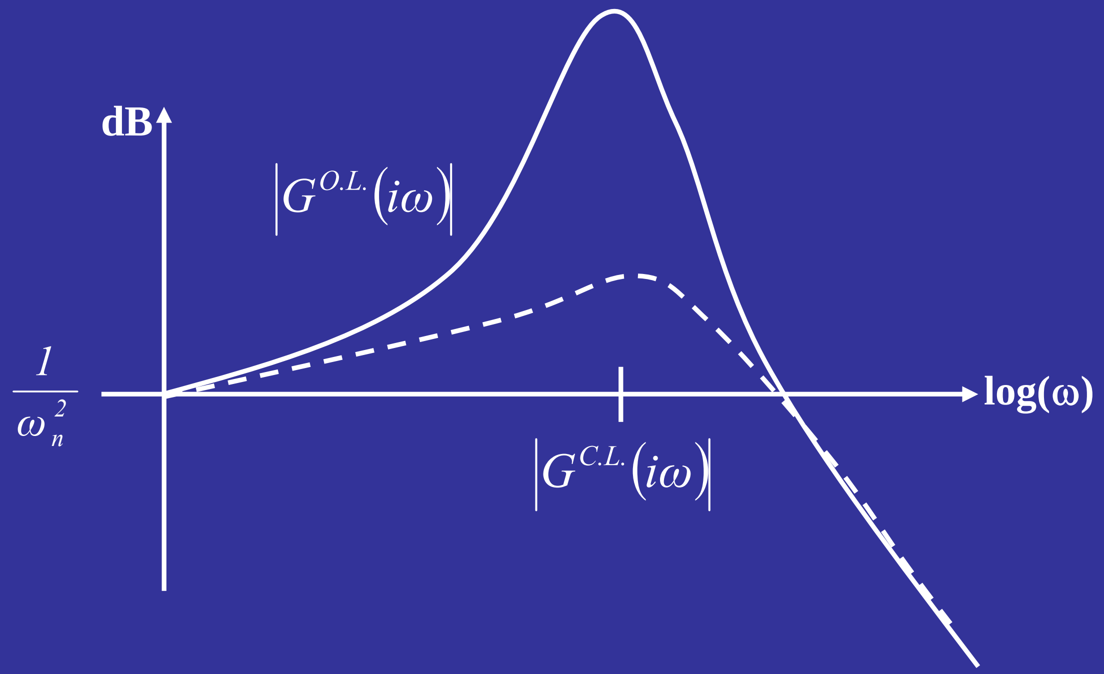
Now have

$$\ddot{x} + (\gamma + \alpha)\,\dot{x} + \omega_n^2 x = u$$

So effect off differential feedback is to increase damping

3.1 Feedback-Example 3

Now $$G^{C.L.}(s) = \frac{1}{s^2 + (\gamma + \alpha)s + \omega_n^2}$$

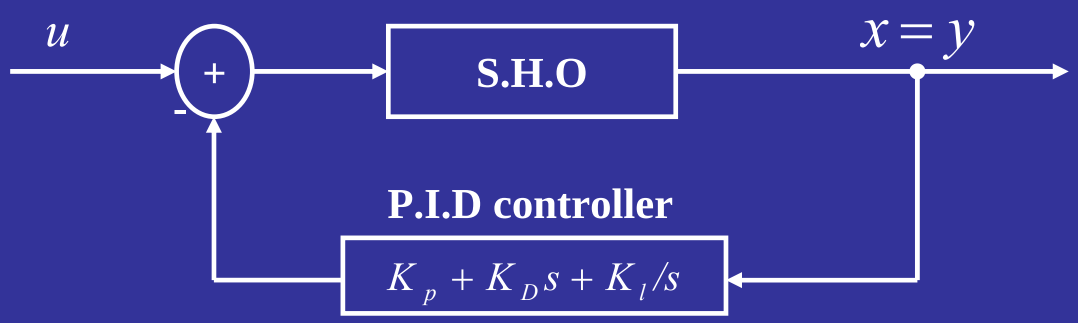


So the effect of differential feedback here is to "flatten the resonance" i.e. ***damping is increased***.

Note: Differentiators can never be built exactly, only approximately.

3.1 PID Controller

(1) The latter 3 examples of feedback can all be combined to form a P.I.D. controller (prop.-integral-diff).



$$u_{fb} = u_p + u_d + u_I$$

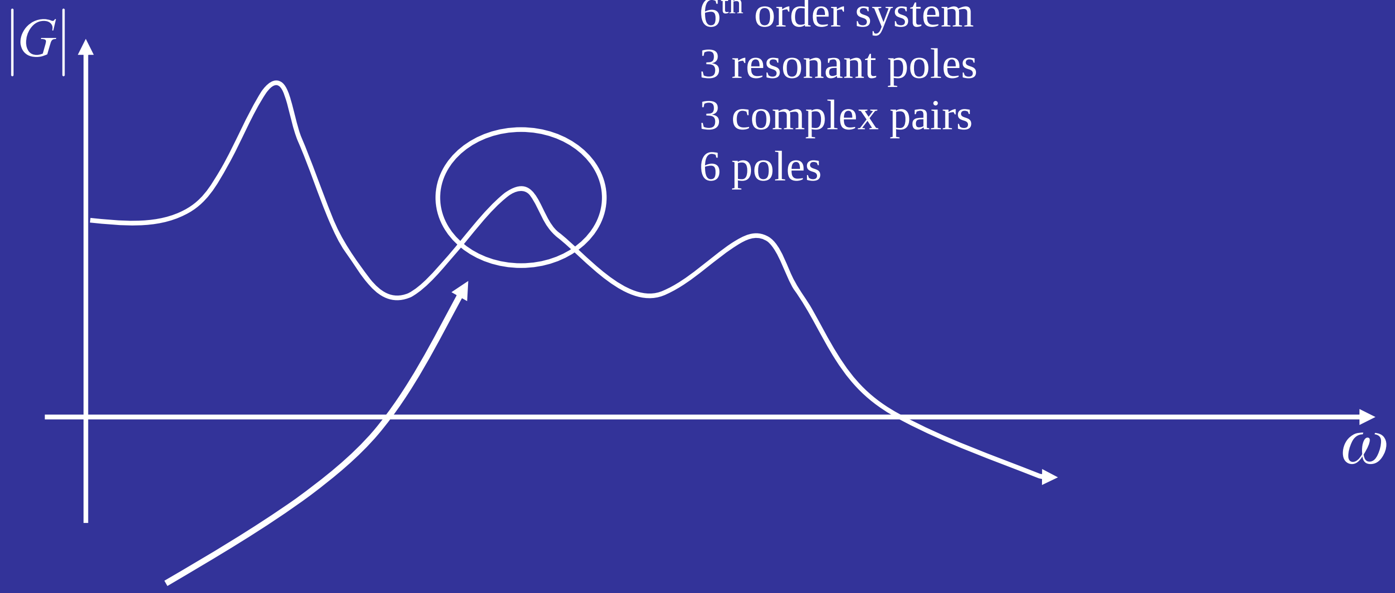
(2) In example above S.H.O. was a very simple system and it was clear what *physical interpretation* of P. or I. or D. did. But for *large complex systems* not obvious

==> **Require arbitrary “ tweaking ”**

That’s what we’re trying to avoid

3.1 PID Controller

For example, if you are so smart let's see you do this with your P.I.D. controller:



6th order system
3 resonant poles
3 complex pairs
6 poles

Damp this mode, but leave the other two modes undamped, just as they are.

This could turn out to be a tweaking nightmare that'll get you nowhere fast!

With modern control theory this problem can be solved easily.