Electron dynamics with Synchrotron Radiation

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Curved orbit of electrons in magnet field

Accelerated charge → Electromagnetic radiation
Electromagnetic waves
Crab Nebula
6000 light years away

First light observed
1054 AD

GE Synchrotron
New York State

First light observed
1947
Synchrotron radiation: some dates

- 1873 Maxwell’s equations
- 1887 Hertz: electromagnetic waves
- 1898 Liénard: retarded potentials
- 1900 Wiechert: retarded potentials
- 1908 Schott: Adams Prize Essay

... waiting for accelerators …
1940: 2.3 MeV betatron, Kerst, Serber
Was it a God whose inspiration
Led him to write these fine equations
Nature’s fields to me he shows
And so my heart with pleasure glows.

translated by John P. Blewett
Synchrotron radiation: some dates

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THEORETICAL UNDERSTANDING

1873 Maxwell’s equations

→ made evident that changing charge densities would result in electric fields that would radiate outward

1887 Heinrich Hertz demonstrated such waves:

It's of no use whatsoever[...] this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.
Synchrotron radiation: some dates

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... waiting for accelerators ...

1940: 2.3 MeV betatron, Kerst, Serber
Donald Kerst: first betatron (1940)

"Ausserordentlich hochgeschwindigkeitelektronenentwickelndenschwerarbeitsbeigollitron"
Synchrotron radiation: some dates

- 1946: Blewett observes energy loss due to synchrotron radiation from a 100 MeV betatron.
- 1947: First visual observation of synchrotron radiation (SR) from a 70 MeV synchrotron at GE Lab.
Paul Scherrer Institute, Switzerland

SwissFEL

Swiss Light Source
60,000 SR users world-wide
A larger view
Why do they radiate?
Synchrotron Radiation is not as simple as it seems

... I will try to show that it is much simpler
Charge at rest
Coulomb field, no radiation
Uniformly moving charge does not radiate

\[ v = \text{constant} \]

But! Cerenkov!
Free isolated electron cannot emit a photon

Easy proof using 4–vectors and relativity

- momentum conservation if a photon is emitted

\[ P_i = P_f + P_\gamma \]

- square both sides

\[ m^2 = m^2 + 2P_f \cdot P_\gamma + 0 \Rightarrow P_f \cdot P_\gamma = 0 \]

- in the rest frame of the electron

\[ P_f = (m, 0) \quad P_\gamma = (E_\gamma, p_\gamma) \]

this means that the photon energy must be zero.
We need to separate the field from charge
Bremsstrahlung or “braking” radiation
Transition Radiation

\[ c_1 = \frac{1}{\sqrt{\varepsilon_1 \mu_1}} \quad \text{and} \quad c_2 = \frac{1}{\sqrt{\varepsilon_2 \mu_2}} \]
Liénard–Wiechert potentials

\[ \varphi(t) = \frac{1}{4\pi \varepsilon_0} \frac{q}{r(1 - \mathbf{n} \cdot \beta)} \quad \text{ret} \]

\[ \vec{A}(t) = \frac{q}{4\pi \varepsilon_0 c^2} \left[ \frac{\vec{v}}{r(1 - \mathbf{n} \cdot \beta)} \right] \text{ret} \]

and the electromagnetic fields:

\[ \nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0 \quad \text{(Lorentz gauge)} \]

\[ \vec{B} = \nabla \times \vec{A} \]

\[ \vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \]
Fields of a moving charge

\[
\mathbf{E}(t) = \frac{q}{4\pi\varepsilon_0} \left[ \frac{\mathbf{n} - \beta}{(1 - \mathbf{n} \cdot \beta)^3 \gamma^2} \right] \cdot \frac{1}{r^2} + \text{“near field”}
\]

\[
\mathbf{B}(t) = \frac{1}{c} [\mathbf{n} \times \mathbf{E}]
\]

\[
\mathbf{B}(t) = \frac{q}{4\pi\varepsilon_0c} \left[ \mathbf{n} \times \left[ (\mathbf{n} - \beta) \times \beta \right] \right] \cdot \frac{1}{r} \text{“far field”}
\]
Energy flow integrated over a sphere

**Power** \( \sim E^2 \cdot \text{Area} \)

\[ A = 4\pi r^2 \]

Near field

\[ P \propto \frac{1}{r^4} \quad r^2 \propto \frac{1}{r^2} \]

Far field

\[ P \propto \frac{1}{r^2} \quad r^2 \propto \text{const} \]

Radiation = constant flow of energy to infinity
Transverse acceleration

Radiation field quickly separates itself from the Coulomb field
Longitudinal acceleration

Radiation field cannot separate itself from the Coulomb field
Synchrotron Radiation
Basic Properties
Beams of ultra-relativistic particles: e.g. a race to the Moon

An electron with energy of a few GeV emits a photon... a race to the Moon!

\[ \Delta t = \frac{L}{\beta c} - \frac{L}{c} = \frac{L}{\beta c} \left(1 - \beta\right) \sim \frac{L}{\beta c} \cdot \frac{1}{2\gamma^2} \]

Electron will lose
- by only 8 meters
- the race will last only 1.3 seconds

\[ \Delta L = L \left(1 - \beta\right) \approx \frac{L}{2\gamma^2} \]
Moving Source of Waves: Doppler effect

“redshift”

“blueshift”

Cape Hatteras, 1999
**Time compression**

Electron with velocity $\beta$ emits a wave with period $T_{\text{emit}}$ while the observer sees a different period $T_{\text{obs}}$ because the electron was moving towards the observer.

$$T_{\text{obs}} = (1 - n \cdot \beta) T_{\text{emit}}$$

The wavelength is shortened by the same factor

$$\lambda_{\text{obs}} = (1 - \beta \cos \theta) \lambda_{\text{emit}}$$

in ultra-relativistic case, looking along a tangent to the trajectory

$$\lambda_{\text{obs}} = \frac{1}{2\gamma^2} \lambda_{\text{emit}}$$

since

$$1 - \beta = \frac{1 - \beta^2}{1 - \beta^2} \approx \frac{1}{2\gamma^2}$$
Radiation is emitted into a narrow cone.

\[ \theta = \frac{1}{\gamma} \cdot \theta_e \]
Sound waves (non-relativistic)

Angular collimation

\[ \theta = \frac{v_{s\perp}}{v_{s\parallel}} + v = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{v}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{v}{v_s}} \]

Doppler effect (moving source of sound)

\[ \lambda_{\text{heard}} = \lambda_{\text{emitted}} \left( 1 - \frac{v}{v_s} \right) \]
Synchrotron radiation power

Power emitted is proportional to:

\[ P \propto E^2 B^2 \]

\[
P_\gamma = \frac{c C_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}
\]

\[
C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[ \frac{m}{\text{GeV}^3} \right]
\]
The power is all too real!

![Image of damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.]
Synchrotron radiation power

Power emitted is proportional to:

\[ P \propto E^2 B^2 \]

\[ P_\gamma = \frac{c C_\gamma \cdot E^4}{2\pi \rho^2} \]

\[ C_\gamma = \frac{4\pi r_e}{3 (m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[ \frac{m}{\text{GeV}^3} \right] \]

Energy loss per turn:

\[ U_0 = C_\gamma \cdot \frac{E^4}{\rho} \]

\[ U_0 = \frac{4\pi}{3} \alpha \hbar c \frac{\gamma^4}{\rho} \]

\[ \alpha = \frac{1}{137} \]

\[ \hbar c = 197 \text{ Mev} \cdot \text{fm} \]
Typical frequency of synchrotron light

Due to extreme collimation of light observer sees only a small portion of electron trajectory (a few mm)

\[ l \sim \frac{2\rho}{\gamma} \]

Pulse length: difference in times it takes an electron and a photon to cover this distance

\[ \Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c} (1 - \beta) \]

\[ \omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0 \]

\[ \Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2} \]
Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every $T_0$ (revolution period).

- The spectrum consists of harmonics of

- Flashes are extremely short: harmonics reach up to very high frequencies.

- At high frequencies the individual harmonics overlap, resulting in a continuous spectrum.

$$\omega_0 = \frac{1}{T_0}$$

$$\omega_{typ} \approx \gamma^3 \omega_0$$

$$\omega_0 \sim 1 \text{ MHz}$$

$$\gamma \sim 4000$$

$$\omega_{typ} \sim 10^{16} \text{ Hz}!$$
Wavelength continuously tunable!
\[
\frac{dP}{d\omega} = \frac{P_{\text{tot}}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)
\]

\[
S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{3/2}(x') dx' \quad \int_0^\infty S(x') dx' = 1
\]

\[
P_{\text{tot}} = \frac{2}{3} \hbar c^2 \alpha \frac{\gamma^4}{\rho^2}
\]

\[
\omega_c = \frac{3}{2} \frac{c \gamma^3}{\rho}
\]

\[
\varepsilon_c [\text{eV}] = 665 E^2 [\text{GeV}] B [\text{T}]
\]

\[
G_1(x) = x \int_x^\infty K_{3/2}(x') dx'
\]

\[
\sim 2.1 x^{1/3}
\]

\[
\sim 1.3 \sqrt{x} e^{-x}
\]
Synchrotron radiation flux for different electron energies

![Graph showing synchrotron radiation flux for different electron energies](image-url)

- **LEP Dipole Flux**
  - $I = 1 \text{ mA}$

Flux [photons/s/mrad/0.1%BW] vs Photon energy [eV]
Angular divergence of radiation

The rms opening angle $R'$

- at the critical frequency:
  \[ \omega = \omega_c \quad R' \approx \frac{0.54}{\gamma} \]

- well below
  \[ \omega \ll \omega_c \quad R' \approx \frac{1}{\gamma} \left( \frac{\omega_c}{\omega} \right)^{1/3} \approx 0.4 \left( \frac{\lambda}{\rho} \right)^{1/3} \]

- well above
  \[ \omega \gg \omega_c \quad R' \approx \frac{0.6}{\gamma} \left( \frac{\omega_c}{\omega} \right)^{1/2} \]

independent of $\gamma$!
Synchrotron light polarization
An electron in a storage ring

**Polarization:**
- **Linear** in the plane of the ring
- The electric field vector

**Elliptical** out of the plane
Angular distribution of SR

$\gamma \theta$
Synchrotron light based electron beam diagnostics
Seeing the electron beam (SLS)

X rays

visible light, vertically polarised

\[ \sigma_x \sim 55 \mu m \]
Seeing the electron beam (SLS)

Making an image of the electron beam using the vertically polarised synchrotron light
High resolution measurement

Wavelength used: 364 nm

For point-like source the intensity on axis is zero

Peak-to-valley intensity ratio is determined by the beam height

Present resolution: 3.5 µm
Useful books and references

H. Wiedemann, *Synchrotron Radiation*
Springer-Verlag Berlin Heidelberg 2003

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Cambridge University Press 2004

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