# Electron dynamics with Synchrotron Radiation

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*and*
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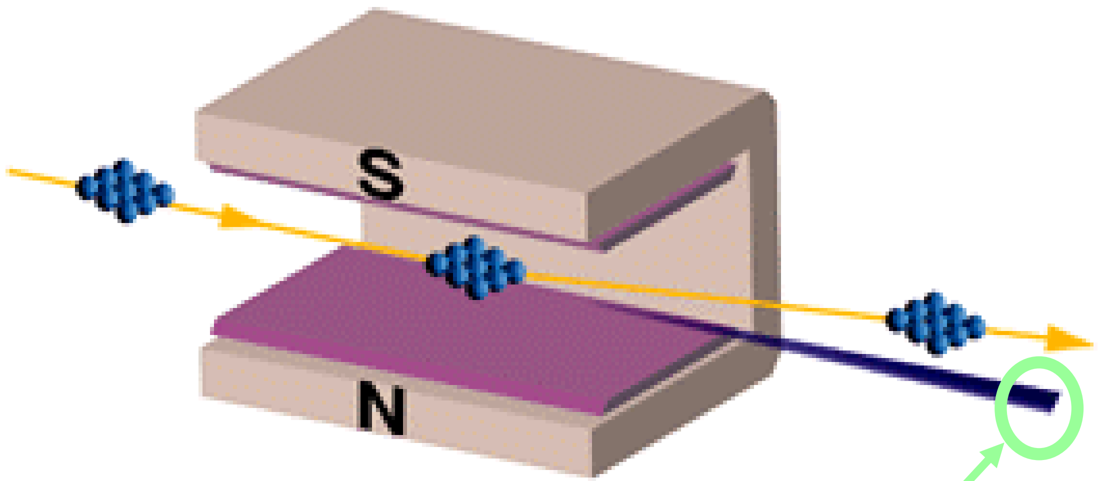
# Curved orbit of electrons in magnet field



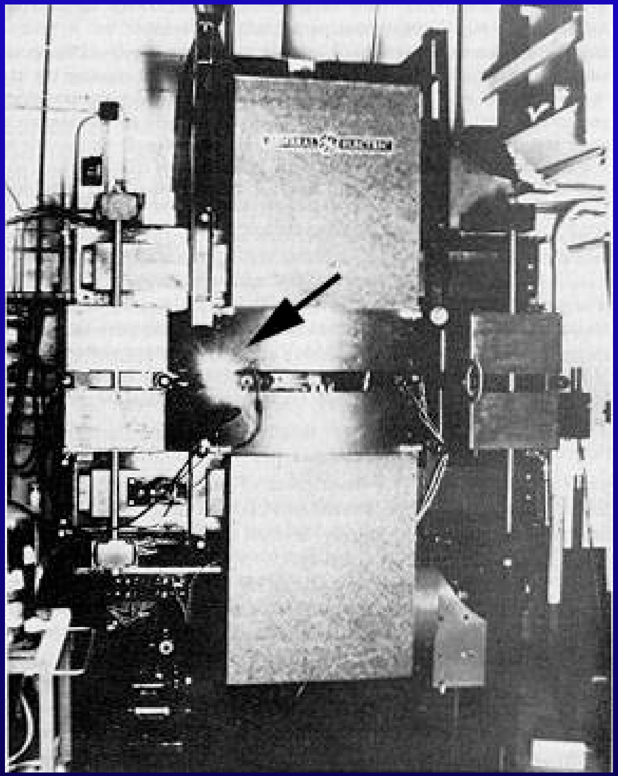

Accelerated charge → Electromagnetic radiation

# Electromagnetic waves

Crab Nebula
6000 light years away

First light observed
1054 AD

GE Synchrotron
New York State



First light observed
1947

# Synchrotron radiation: some dates

- 1873 Maxwell's equations
- 1887 Hertz: electromagnetic waves
- 1898 Liénard: retarded potentials
- 1900 Wiechert: retarded potentials
- 1908 Schott: Adams Prize Essay

... waiting for accelerators …
1940: 2.3 MeV betatron,Kerst, Serber

# Maxwell equations (poetry)

*War es ein Gott, der diese Zeichen schrieb*
*Die mit geheimnisvoll verborg'nem Trieb*
*Die Kräfte der Natur um mich enthüllen*
*Und mir das Herz mit stiller Freude füllen.*
Ludwig Boltzman

*Was it a God whose inspiration*
*Led him to write these fine equations*
*Nature's fields to me he shows*
*And so my heart with pleasure glows.*
translated by John P. Blewett

# Synchrotron radiation: some dates

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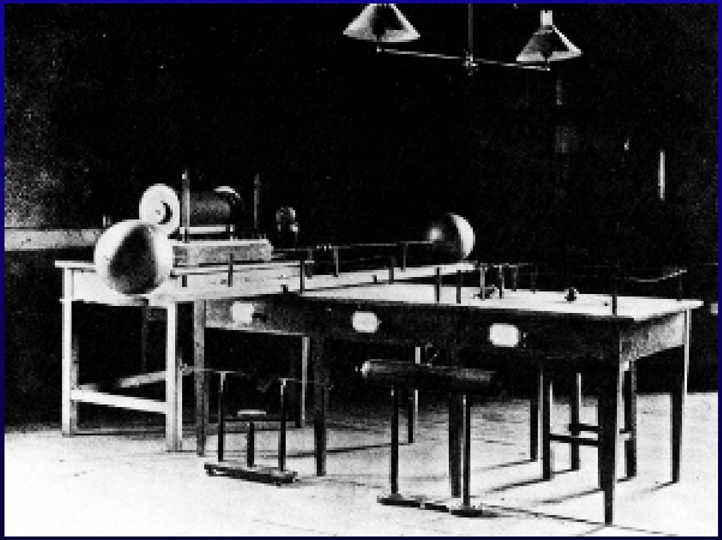
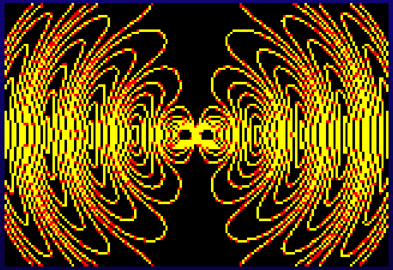
# THEORETICAL UNDERSTANDING →

1873 Maxwell's equations

→ made evident that changing charge densities would result in electric fields that would radiate outward

1887 Heinrich Hertz demonstrated such waves:





*It's of no use whatsoever*[...] *this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.*

# Synchrotron radiation: some dates

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# Donald Kerst: first betatron (1940)

*"Ausserordentlichhochgeschwindigkeitelektronenent wickelndenschwerarbeitsbeigollitron"*

# Synchrotron radiation: some dates

- 1946 Blewett observes energy loss due to synchrotron radiation
  100 MeV betatron
- 1947 First visual observation of SR
  70 MeV **synchrotron**, GE Lab

NAME!

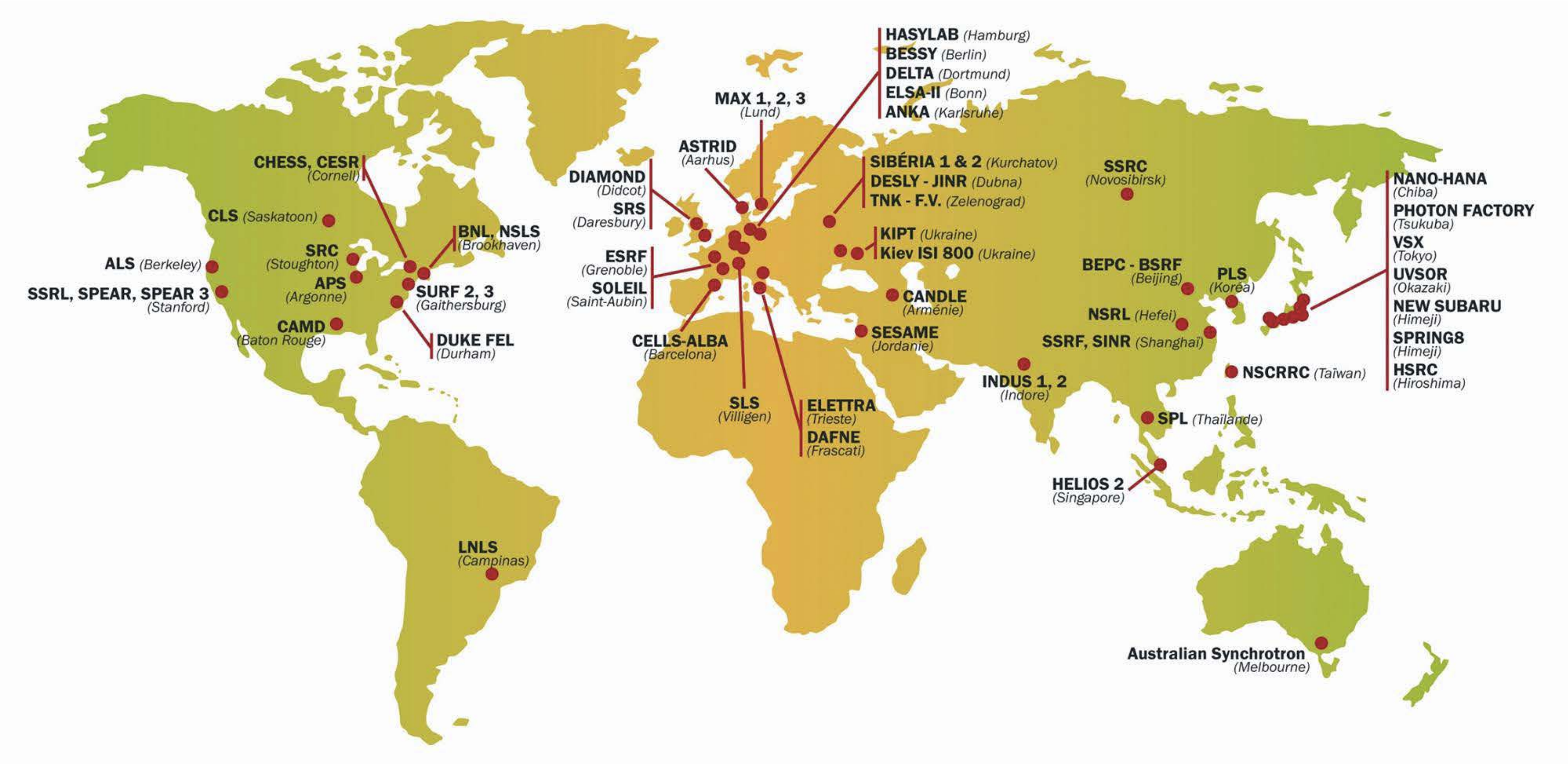
- 1949 Schwinger PhysRev paper

…

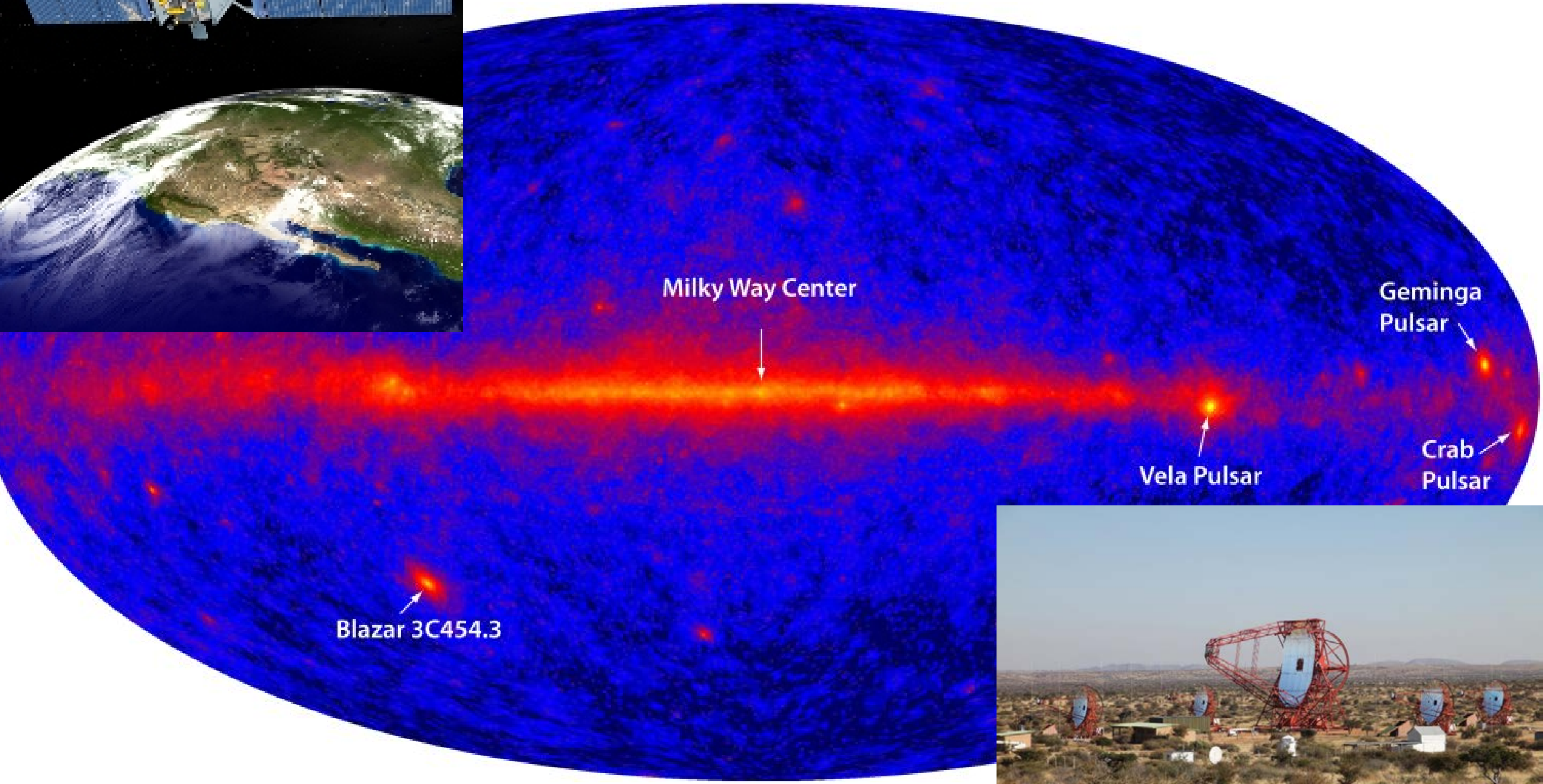
- 1976 Madey: first demonstration of Free Electron laser

# Paul Scherrer Institute, Switzerland

# 60‘000 SR users world-wide



# A larger view

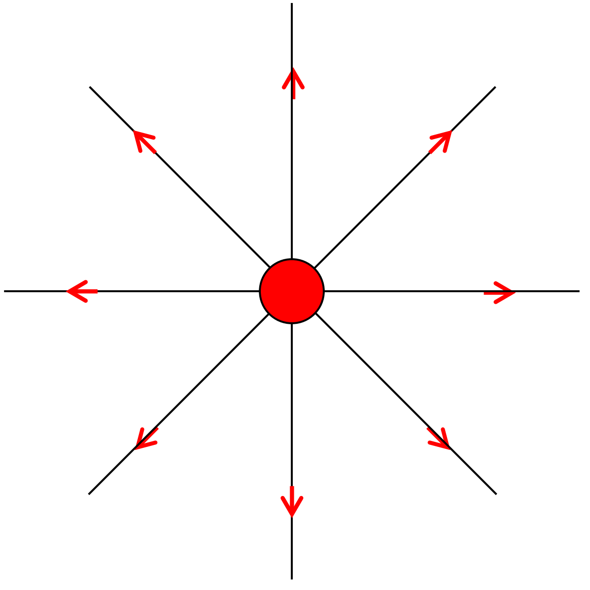


# Why do they radiate?

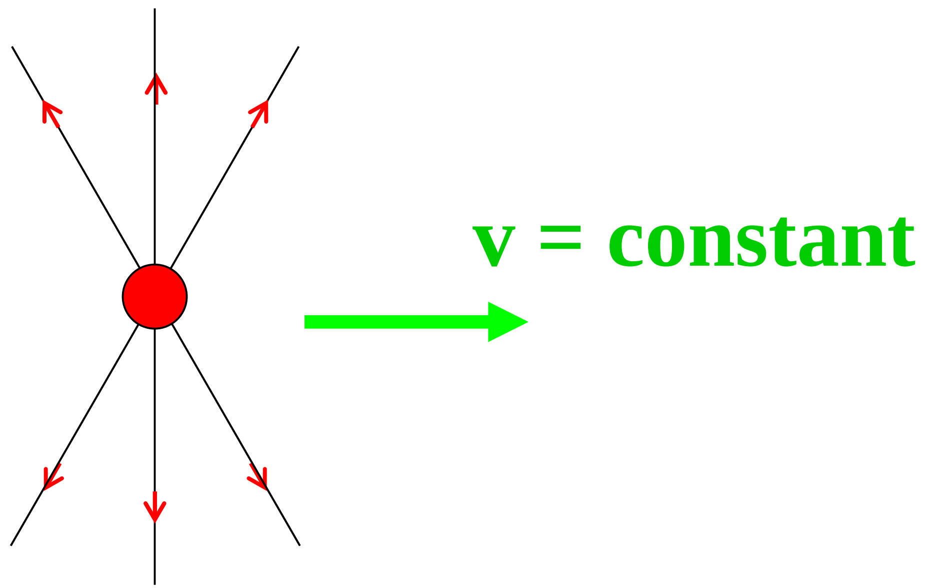
Synchrotron Radiation is not as simple as it seems

… I will try to show that it is much simpler

# Charge at rest
# Coulomb field, no radiation



# Uniformly moving charge does not radiate



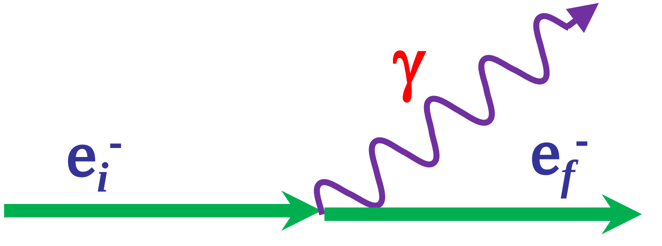

But! Cerenkov!

# Free isolated electron cannot emit a photon

Easy proof using 4-vectors and relativity

- momentum conservation if a photon is emitted

$$\boldsymbol{P}_i = \boldsymbol{P}_f + \boldsymbol{P}_\gamma$$




- square both sides

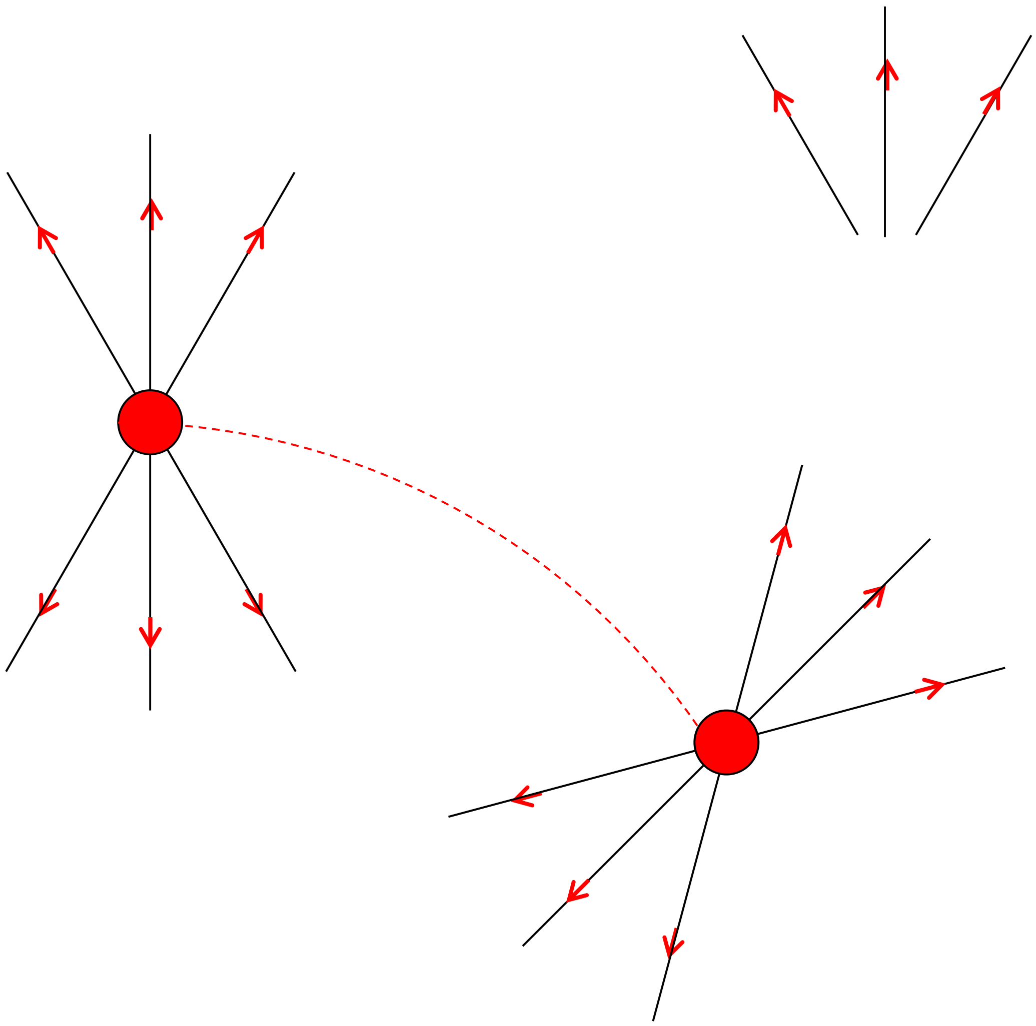
$$m^2 = m^2 + 2\boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma + 0 \;\Rightarrow\; \boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma = 0$$

- in the rest frame of the electron

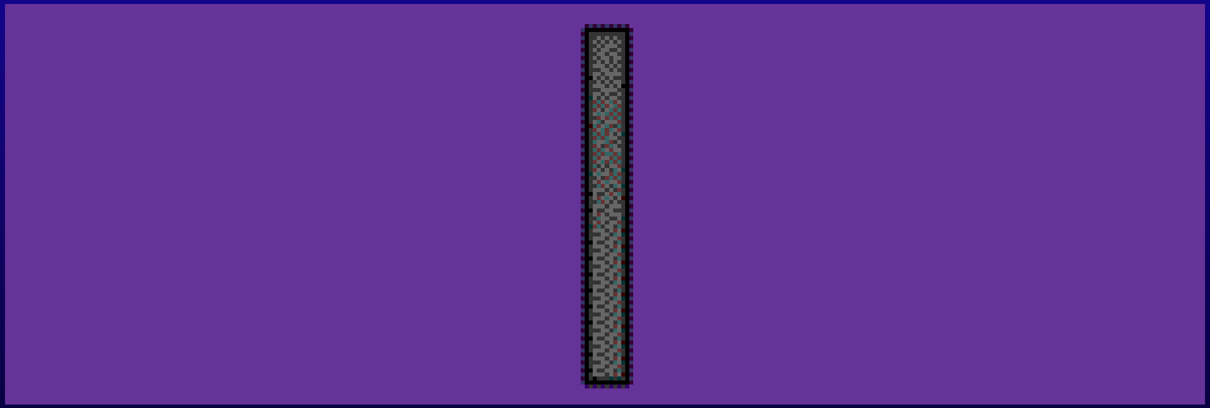
$$\boldsymbol{P}_f = (m, 0) \qquad \boldsymbol{P}_\gamma = (E_\gamma, p_\gamma)$$

this means that the photon energy must be zero.

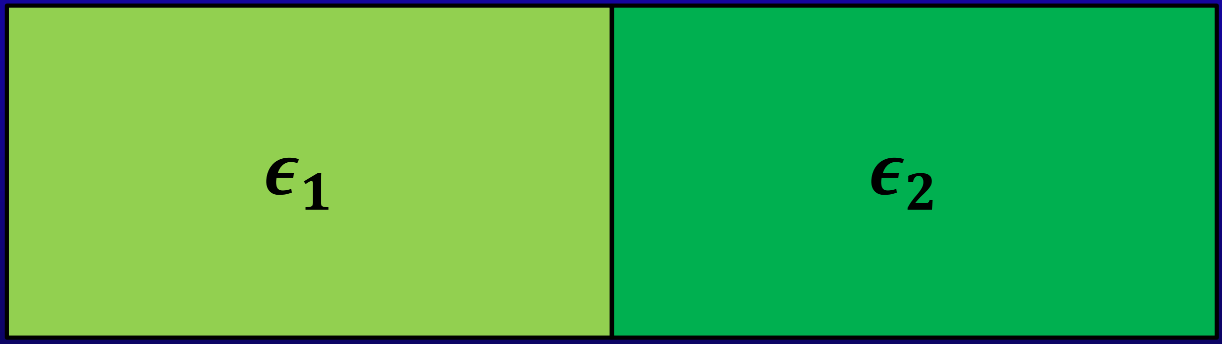
# We need to separate the field from charge



# Bremsstrahlung
# or
# “braking” radiation



# Transition Radiation




$$c_1 = \frac{1}{\sqrt{\epsilon_1 \mu_1}} \qquad c_2 = \frac{1}{\sqrt{\epsilon_2 \mu_2}}$$

# Liénard–Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)\right]_{ret}} \qquad \vec{\mathbf{A}}(t) = \frac{q}{4\pi\varepsilon_0 c^2}\left[\frac{\vec{\mathbf{v}}}{r\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)}\right]_{ret}$$

and the electromagnetic fields:

$$\nabla \cdot \vec{\mathbf{A}} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$$

(Lorentz gauge)

$$\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\nabla\varphi - \frac{\partial\vec{\mathbf{A}}}{\partial t}$$

# Fields of a moving charge

$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \boxed{\frac{\mathbf{1}}{\mathbf{r}^2}}\right]_{ret} +$$

"near field"

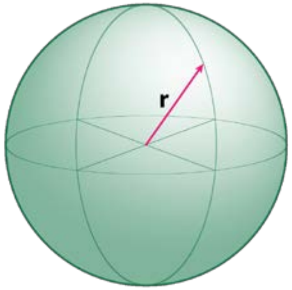
$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}} \times \left[\left(\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}\right) \times \boxed{\dot{\vec{\boldsymbol{\beta}}}}\right]}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \boxed{\frac{\mathbf{1}}{\mathbf{r}}}\right]_{ret}$$

"far field"

$$\vec{\mathbf{B}}(\mathrm{t}) = \frac{1}{\mathrm{c}}\left[\vec{\mathbf{n}} \times \vec{\mathbf{E}}\right]$$

# Energy flow integrated over a sphere

$$\boxed{Power \sim E^2 \cdot \text{Area}}$$



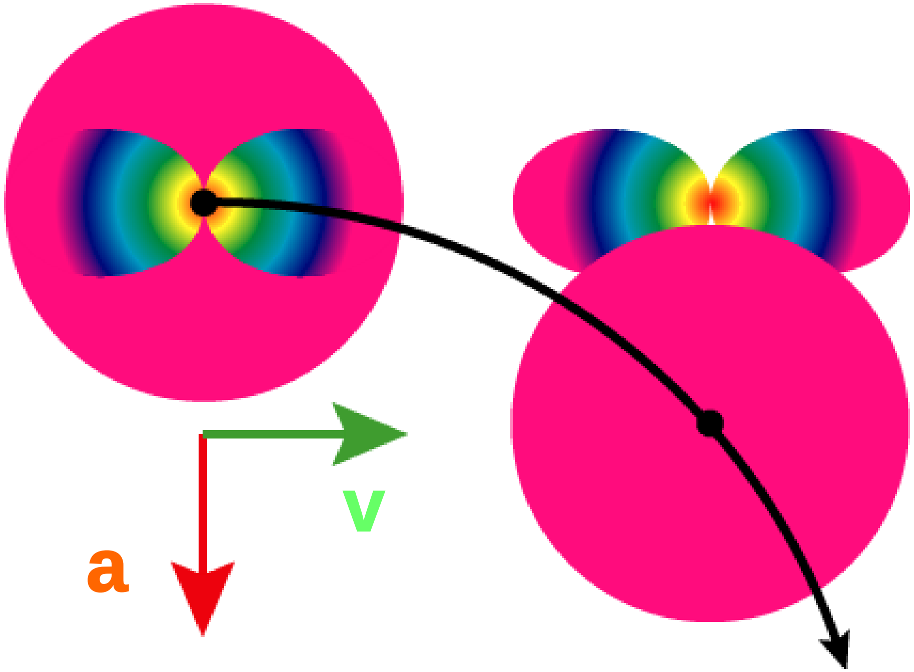

$$A = 4\pi r^2$$

Near field $$P \propto \frac{1}{r^4} r^2 \propto \frac{1}{r^2}$$

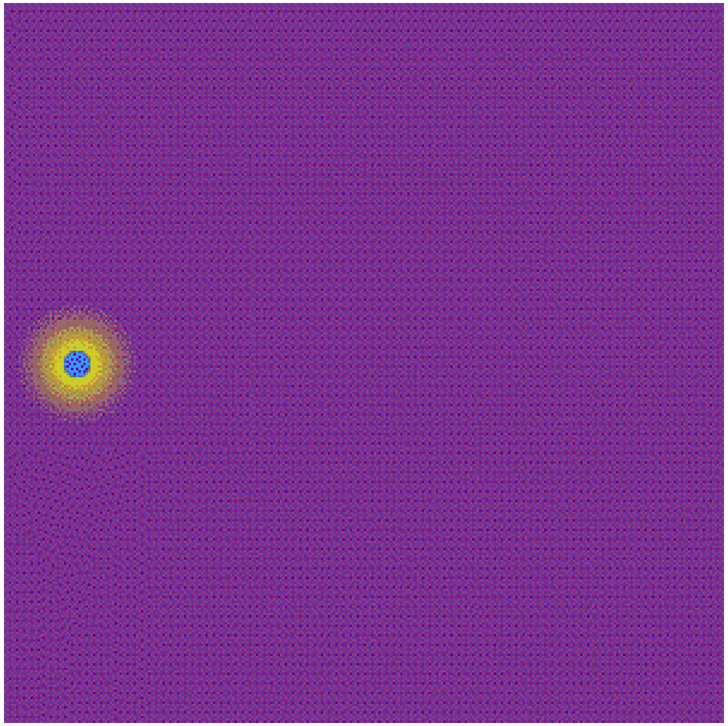
Far field $$\boxed{P \propto \frac{1}{r^2} r^2 \propto const}$$

*Radiation = constant flow of energy to infinity*

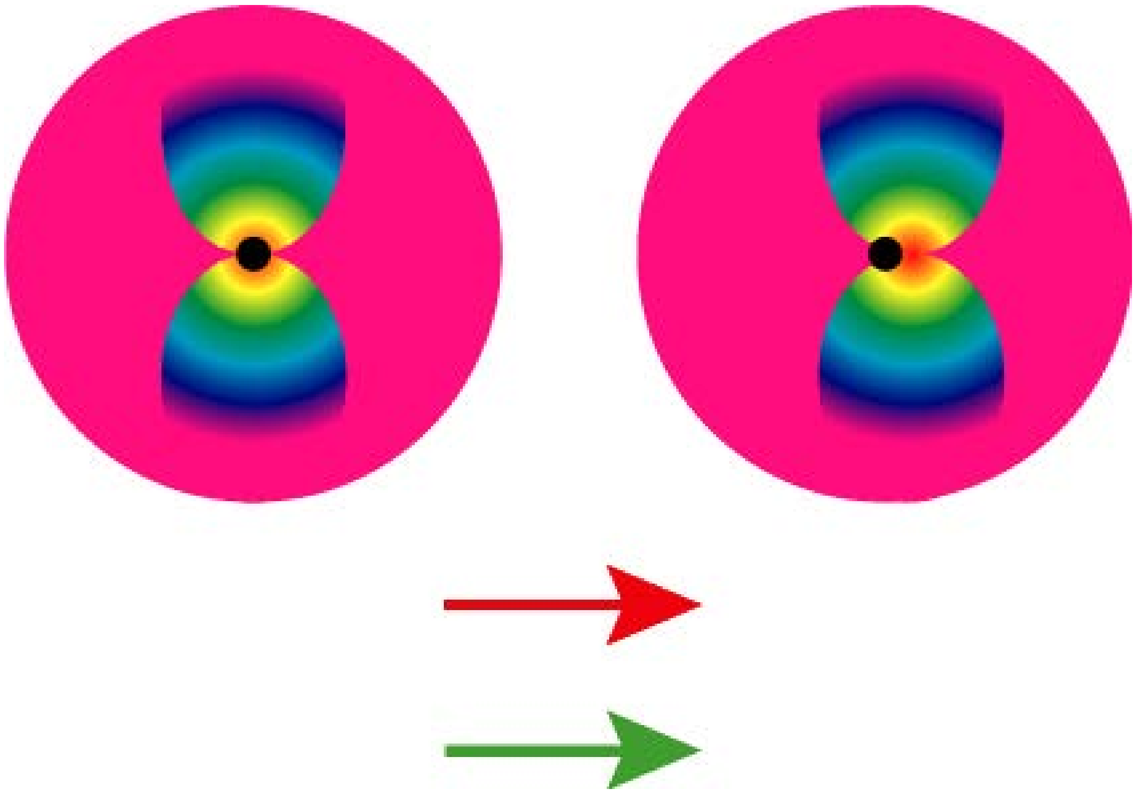
# Transverse acceleration




**Radiation field quickly separates itself from the Coulomb field**



# Longitudinal acceleration

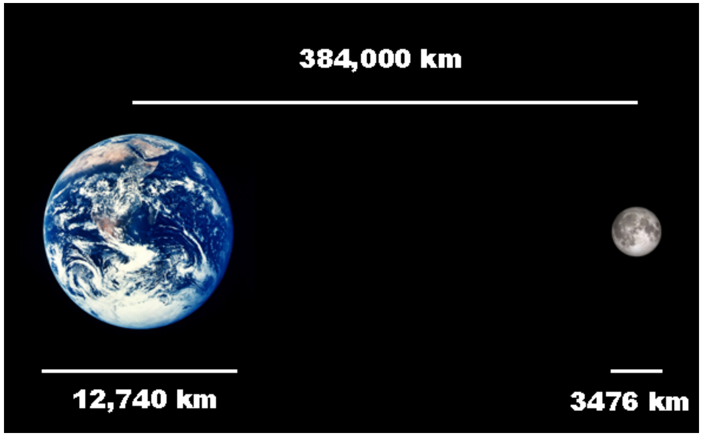


Radiation field cannot separate itself from the Coulomb field

# Synchrotron Radiation Basic Properties

# Beams of ultra-relativistic particles: e.g. a race to the Moon

An electron with energy of a few GeV emits a photon… a race to the Moon!



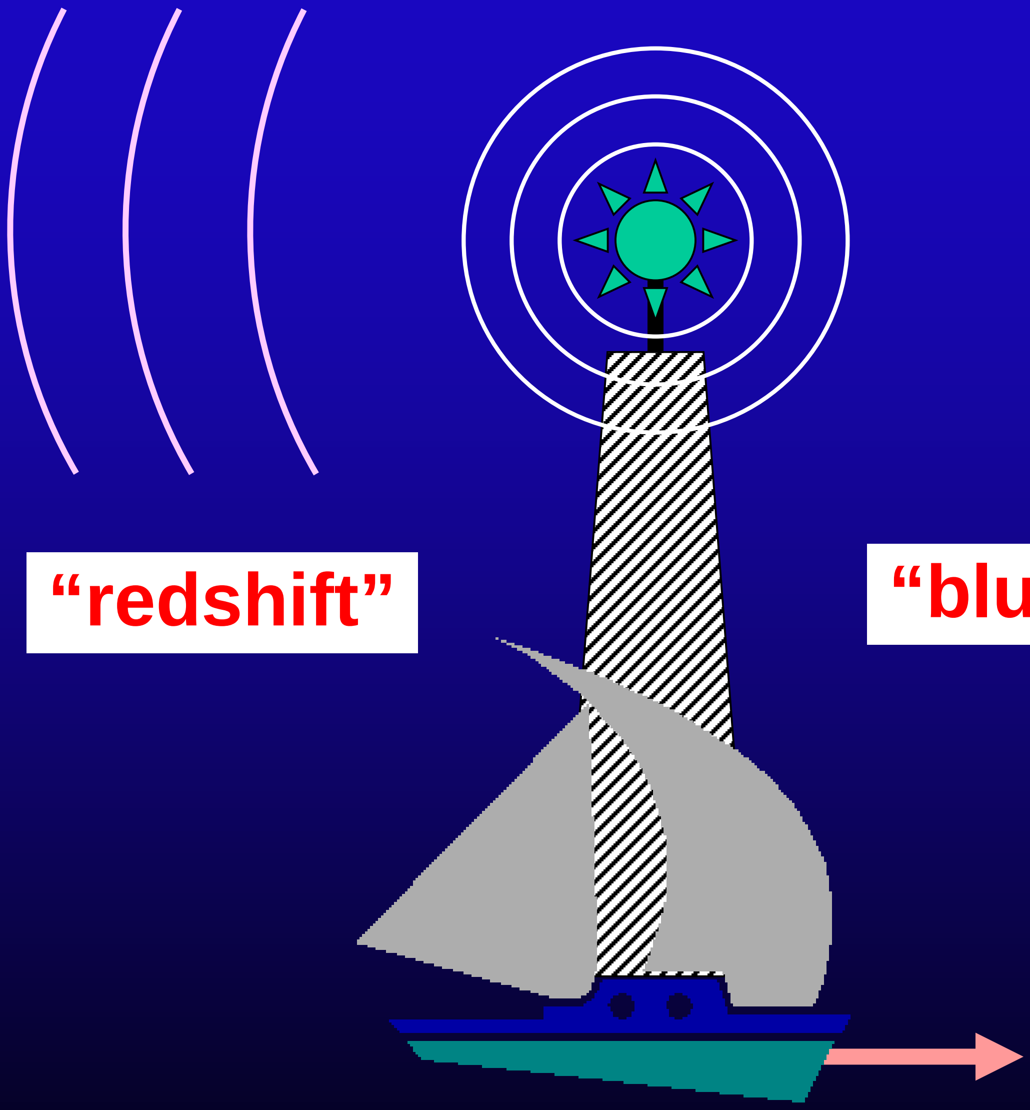

$$\Delta t = \frac{L}{\beta c} - \frac{L}{c} = \frac{L}{\beta c}(1-\beta) \sim \frac{L}{\beta c} \cdot \frac{1}{2\gamma^2}$$

Electron will lose

- by only 8 meters
- the race will last only 1.3 seconds

$$\Delta L = L(1-\beta) \cong \frac{L}{2\gamma^2}$$

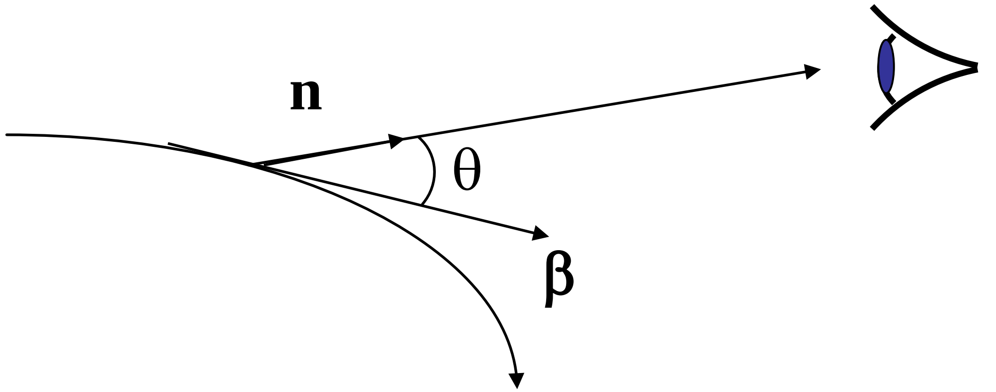
# Moving Source of Waves: Doppler effect




Cape Hatteras, 1999

# Time compression

**Electron with velocity β emits a wave with period $T_{emit}$ while the observer sees a different period $T_{obs}$ because the electron was moving towards the observer**




$$T_{obs} = (1 - \mathbf{n} \cdot \boldsymbol{\beta}) T_{emit}$$

**The wavelength is shortened by the same factor**

$$\lambda_{obs} = (1 - \beta \cos\theta)\, \lambda_{emit}$$

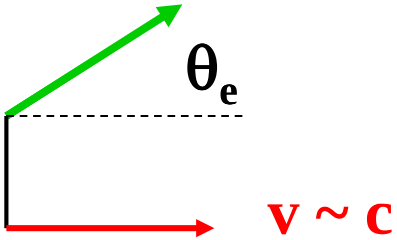
**in ultra-relativistic case, looking along a tangent to the trajectory**

$$\boxed{\lambda_{obs} = \frac{1}{2\gamma^2} \lambda_{emit}}$$

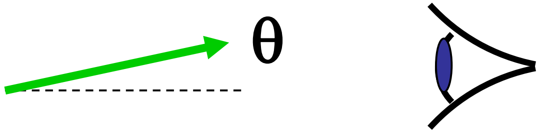
**since**

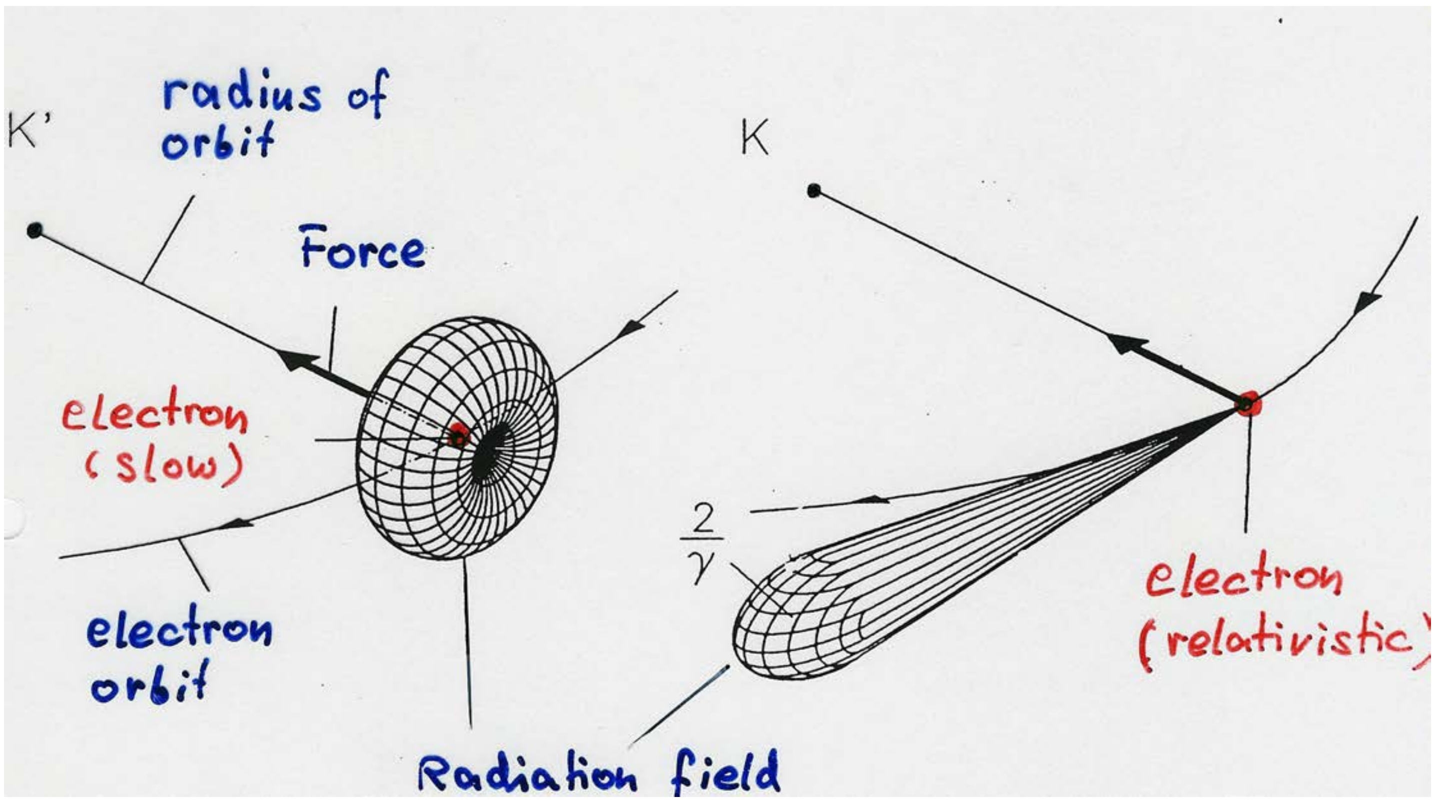
$$\boxed{1 - \beta = \frac{1 - \beta^2}{1 + \beta} \cong \frac{1}{2\gamma^2}}$$

# Radiation is emitted into a narrow cone




$$\theta = \frac{1}{\gamma} \cdot \theta_e$$



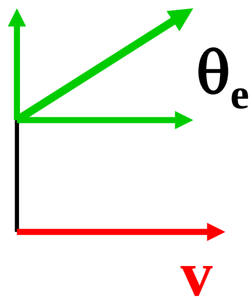



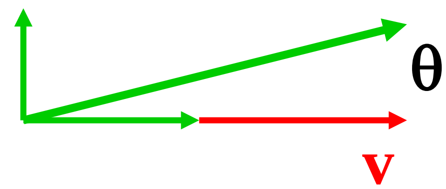

**v << c**

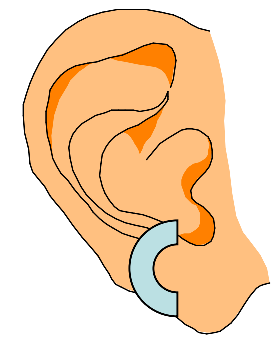
**v ≈ c**

# Sound waves (non-relativistic)

## Angular collimation



$$\theta = \frac{v_{s\perp}}{v_{s\parallel} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

## Doppler effect (moving source of sound)

$$\lambda_{heard} = \lambda_{emitted} \left( 1 - \frac{\mathbf{v}}{v_s} \right)$$

# Synchrotron radiation power

Power emitted is proportional to:

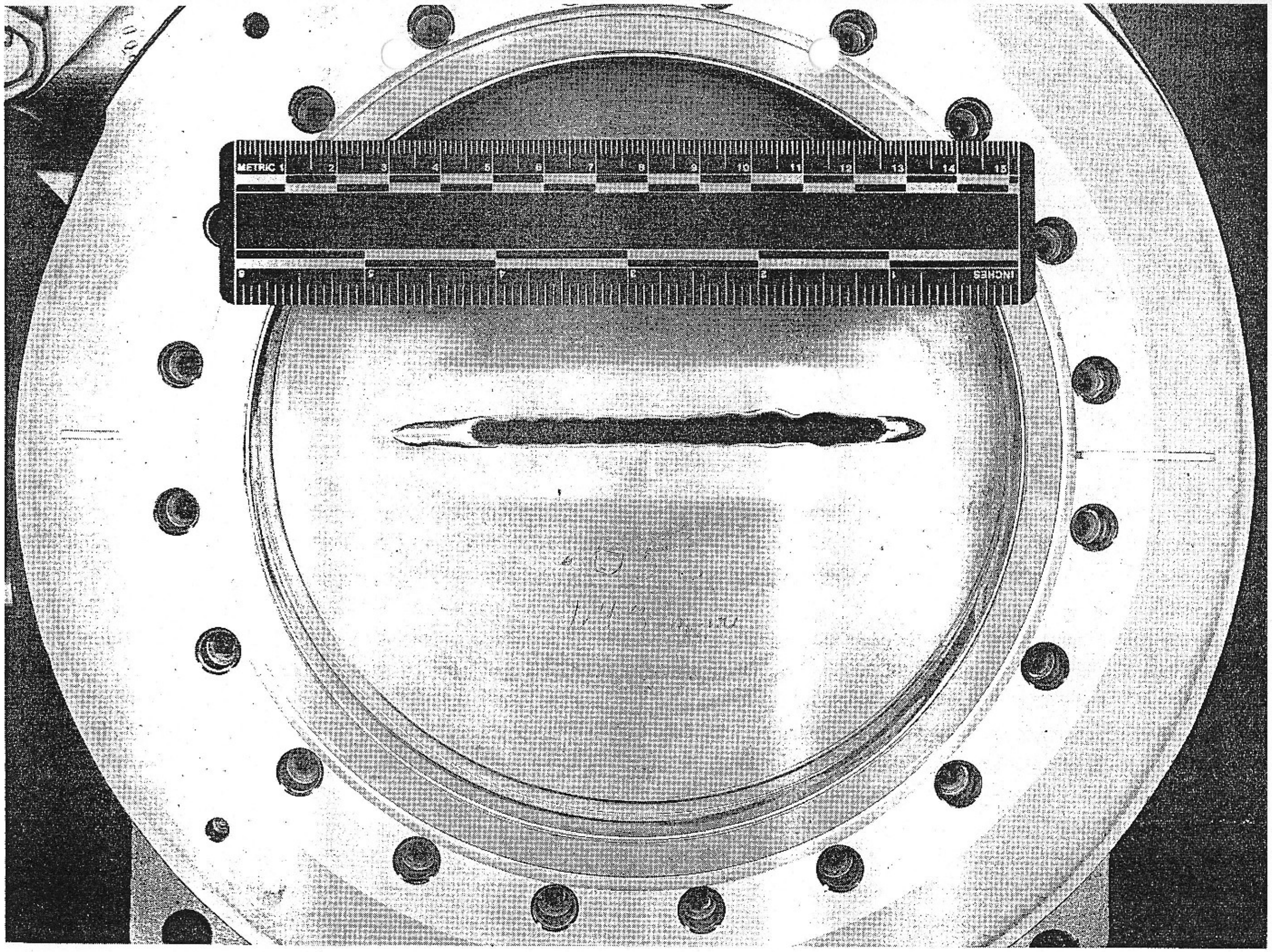
$$\boxed{P \propto E^2 B^2}$$

$$\boxed{P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}}$$

*E* = Energy!

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{\left(m_e c^2\right)^3} = 8.858 \cdot 10^{-5}\left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

# The power is all too real!




ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

# Synchrotron radiation power

Power emitted is proportional to:

$$P \propto E^2 B^2$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3} \alpha \hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197 \text{ Mev} \cdot \text{fm}$$

Energy loss per turn:

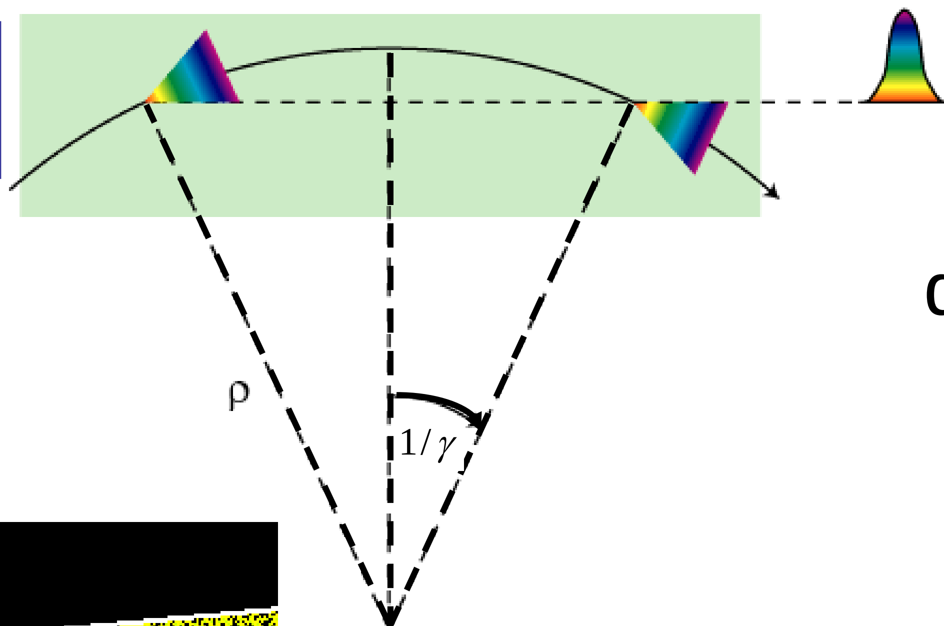
$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3} \alpha \hbar c \frac{\gamma^4}{\rho}$$

# Typical frequency of synchrotron light

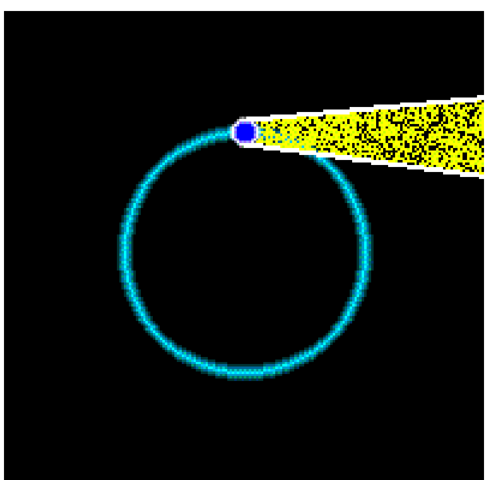
Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

$$l \sim \frac{2\rho}{\gamma}$$




Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$

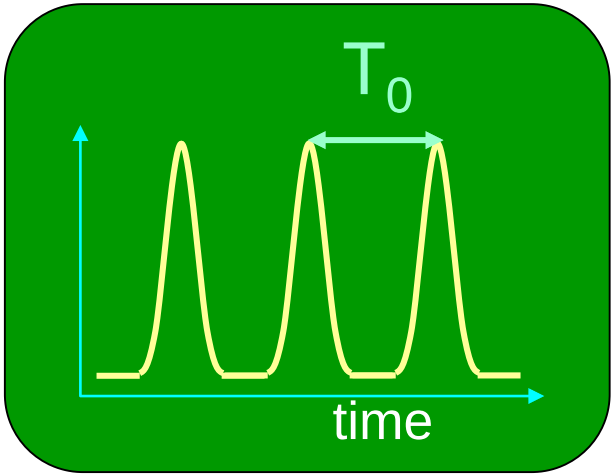


$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$

$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$

# Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every $T_0$ (revolution period)




- the spectrum consists of harmonics of

$$\omega_0 = \frac{1}{T_0}$$

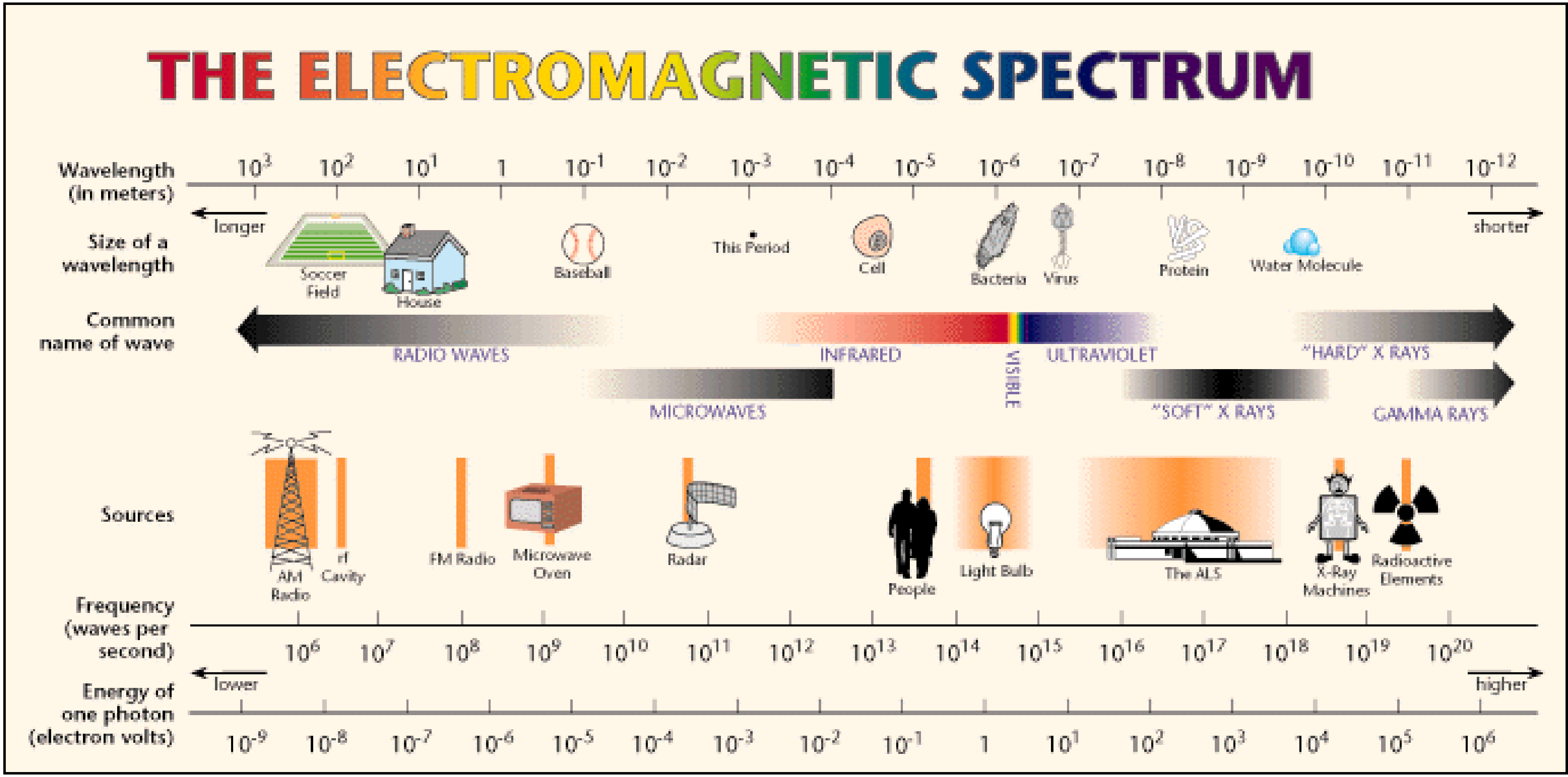
- flashes are extremely short: harmonics reach up to very high frequencies

$$\omega_{typ} \cong \gamma^3 \omega_0$$

$$\omega_0 \sim 1\ \mathrm{MHz}$$
$$\gamma \sim 4000$$
$$\omega_{\mathrm{typ}} \sim 10^{16}\ \mathrm{Hz}\,!$$

- At high frequencies the individual harmonics overlap

continuous spectrum !




Wavelength continuously tunable !

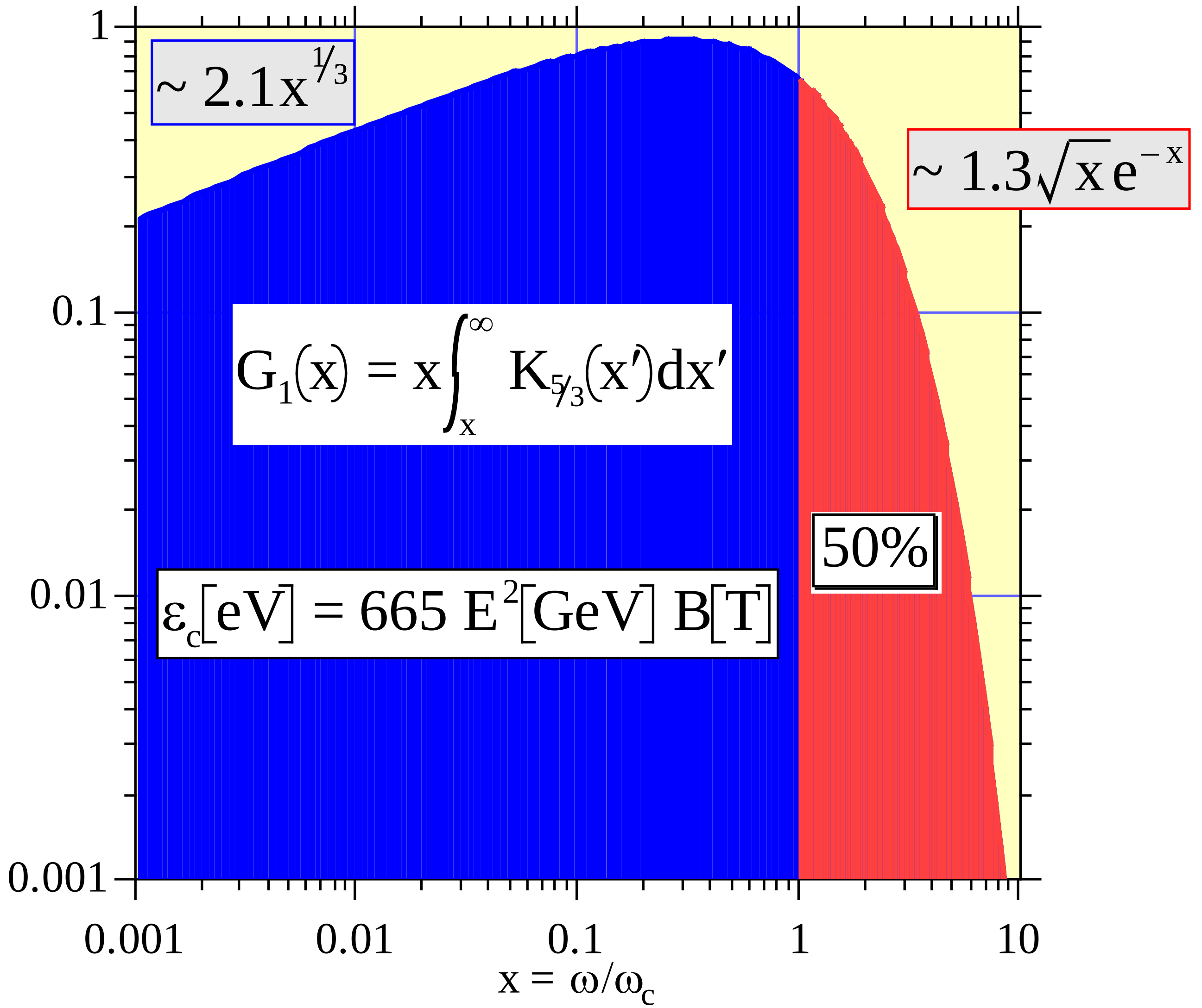
$$\boxed{\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)}$$

$$S(x) = \frac{9\sqrt{3}}{8\pi}\, x \int_x^\infty K_{5/3}(x')\,dx'$$

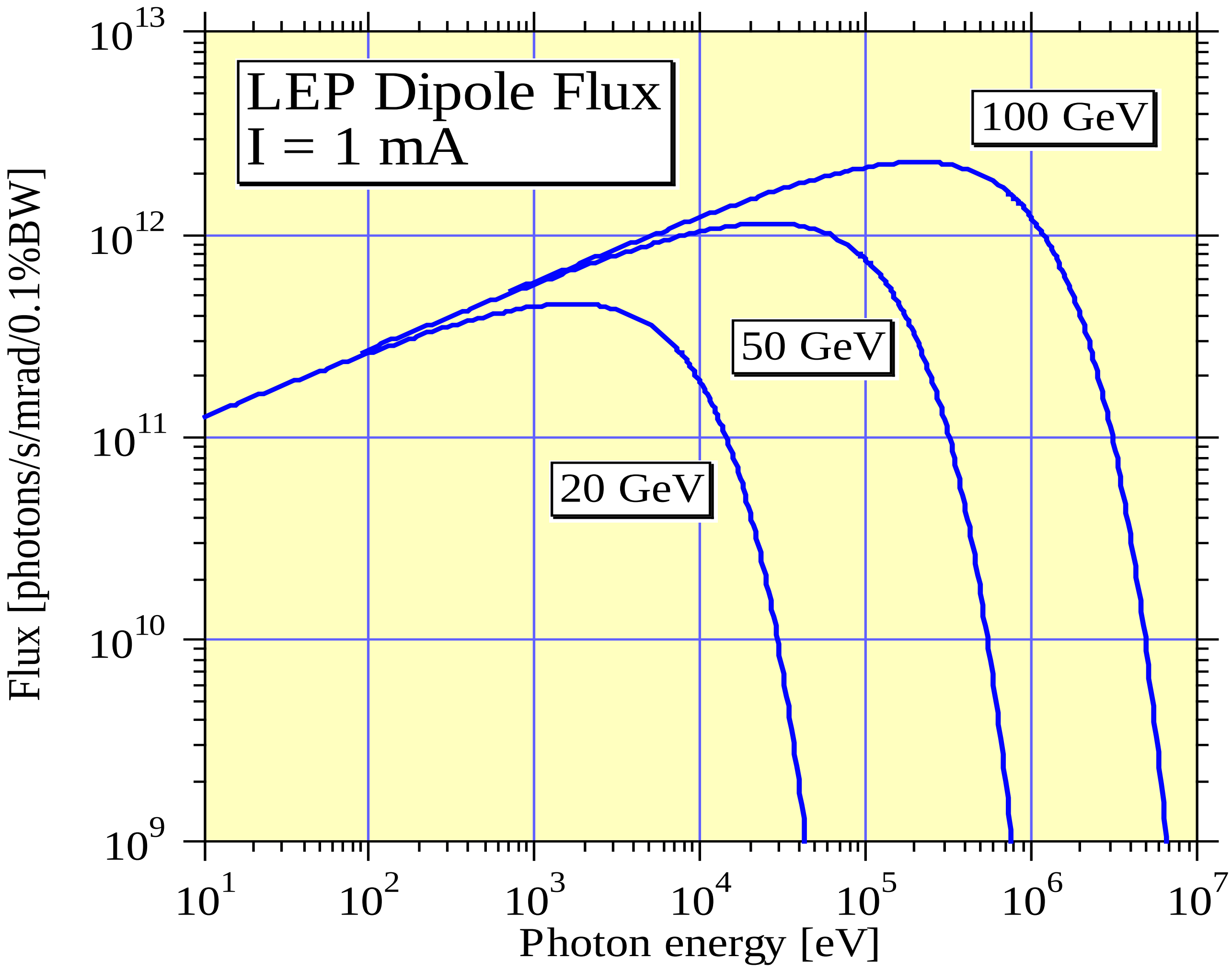
$$\int_0^\infty S(x')\,dx' = 1$$

$$P_{tot} = \frac{2}{3}\hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$



# Synchrotron radiation flux for different electron energies



# Angular divergence of radiation

**The rms opening angle R'**

- **at the critical frequency:**

$$\omega = \omega_c \qquad R' \approx \frac{0.54}{\gamma}$$

- **well below**

$$\omega \ll \omega_c \qquad R' \approx \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \approx 0.4\left(\frac{\lambda}{\rho}\right)^{1/3}$$

**independent of $\gamma$ !**

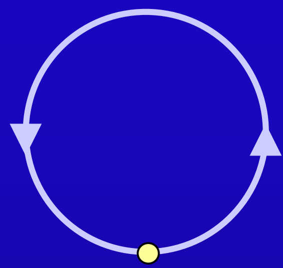
- **well above**

$$\omega \gg \omega_c \qquad R' \approx \frac{0.6}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2}$$

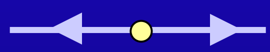
# Synchrotron light polarization

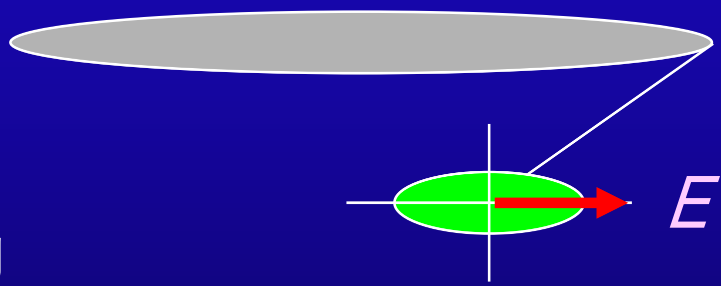
# An electron in a storage ring

**TOP VIEW**



**SIDE VIEW**



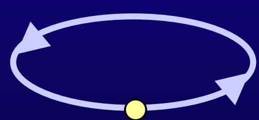



**<u>Polarization:</u>**

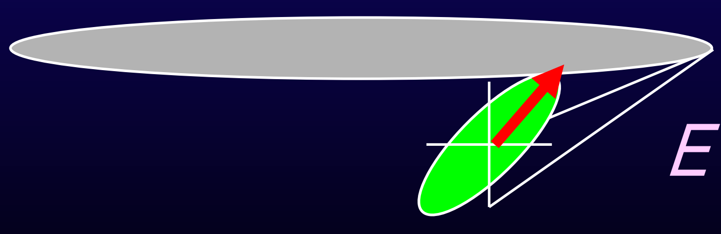
**Linear in the plane of the ring**

**the electric field vector**

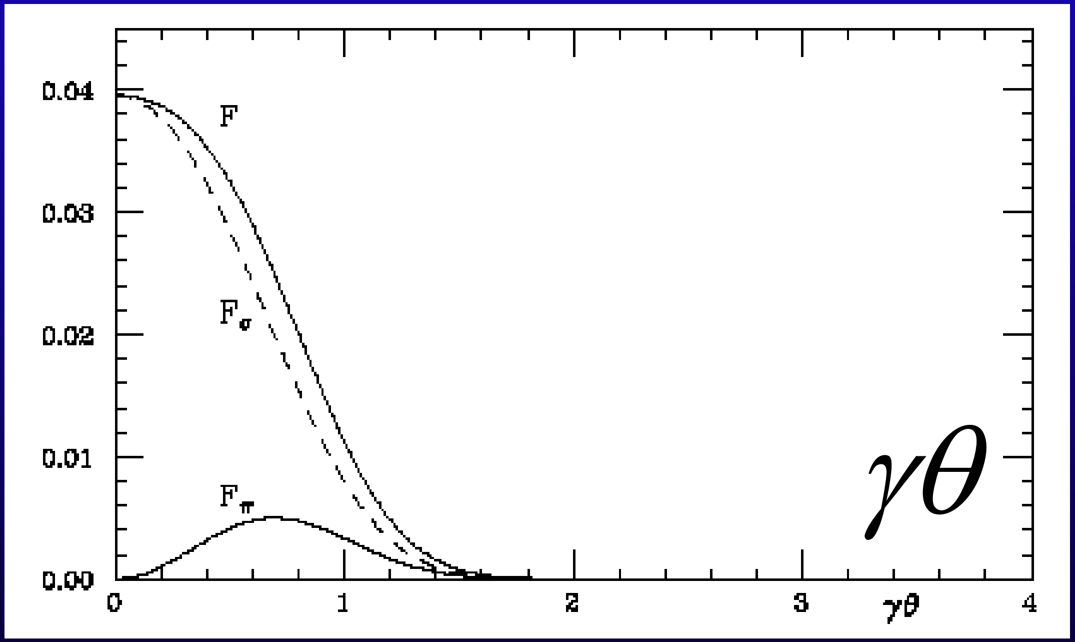
**TILTED VIEW**

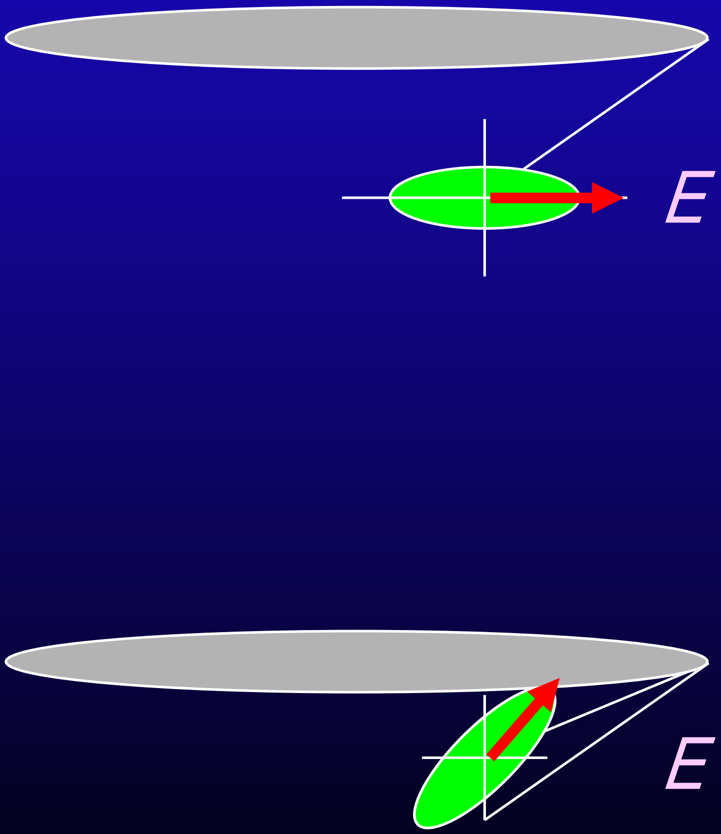


**elliptical out of**
**the plane**



# Angular distribution of SR

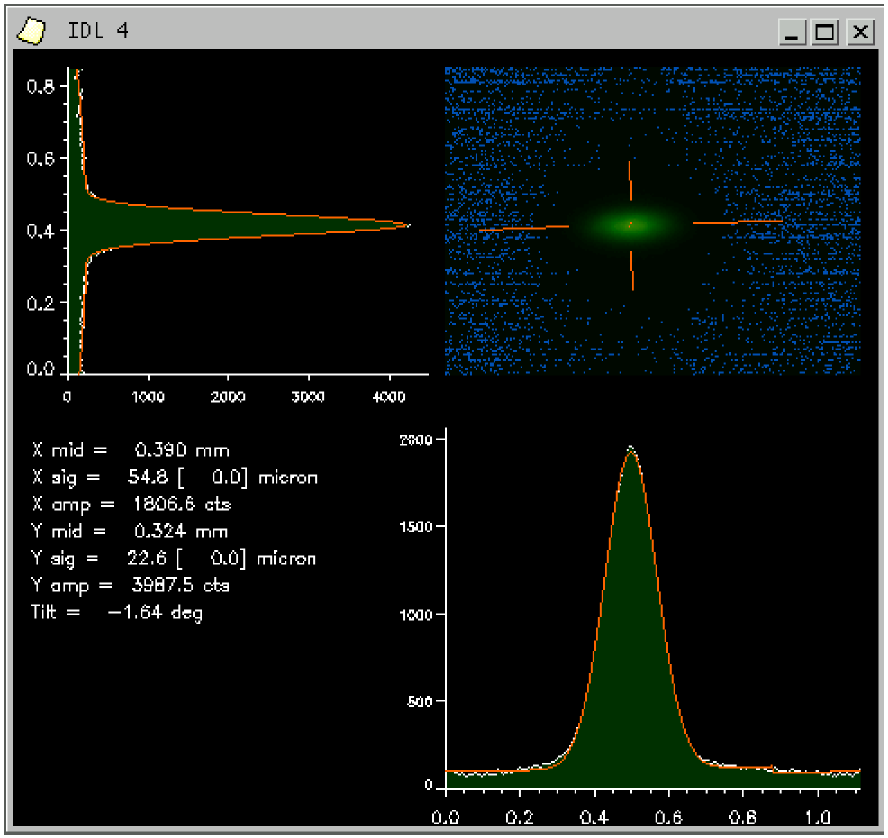


# Synchrotron light based electron beam diagnostics

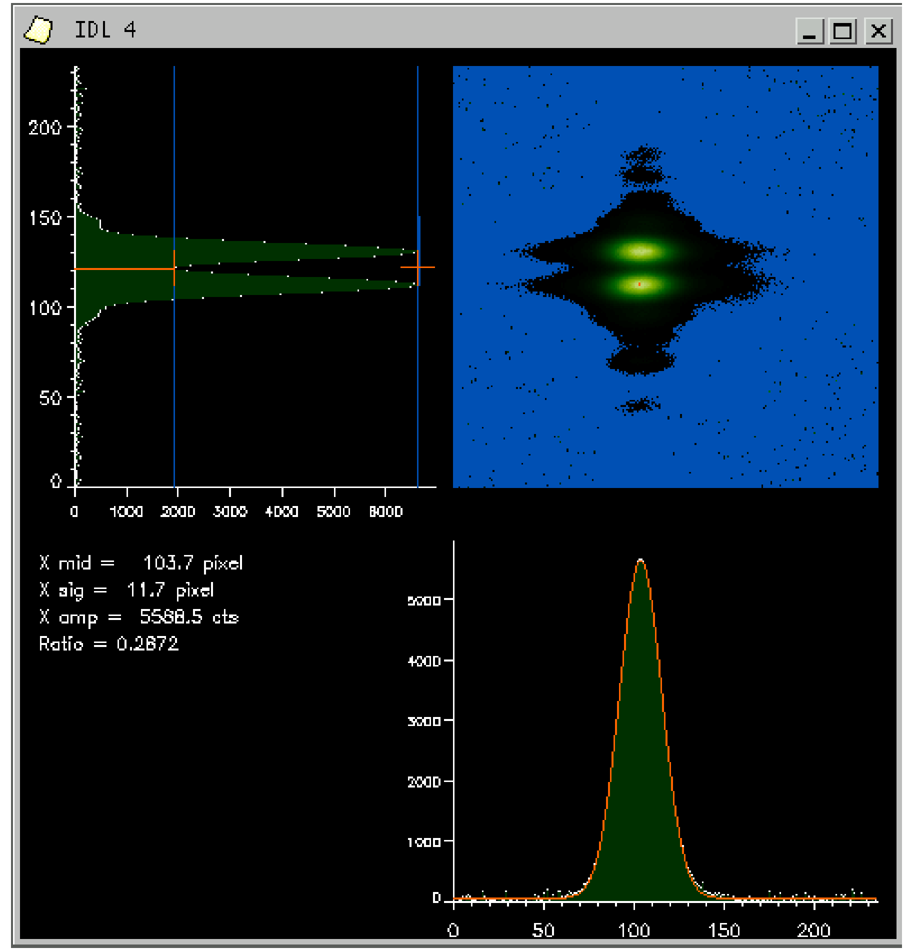
# Seeing the electron beam (SLS)

**X rays**



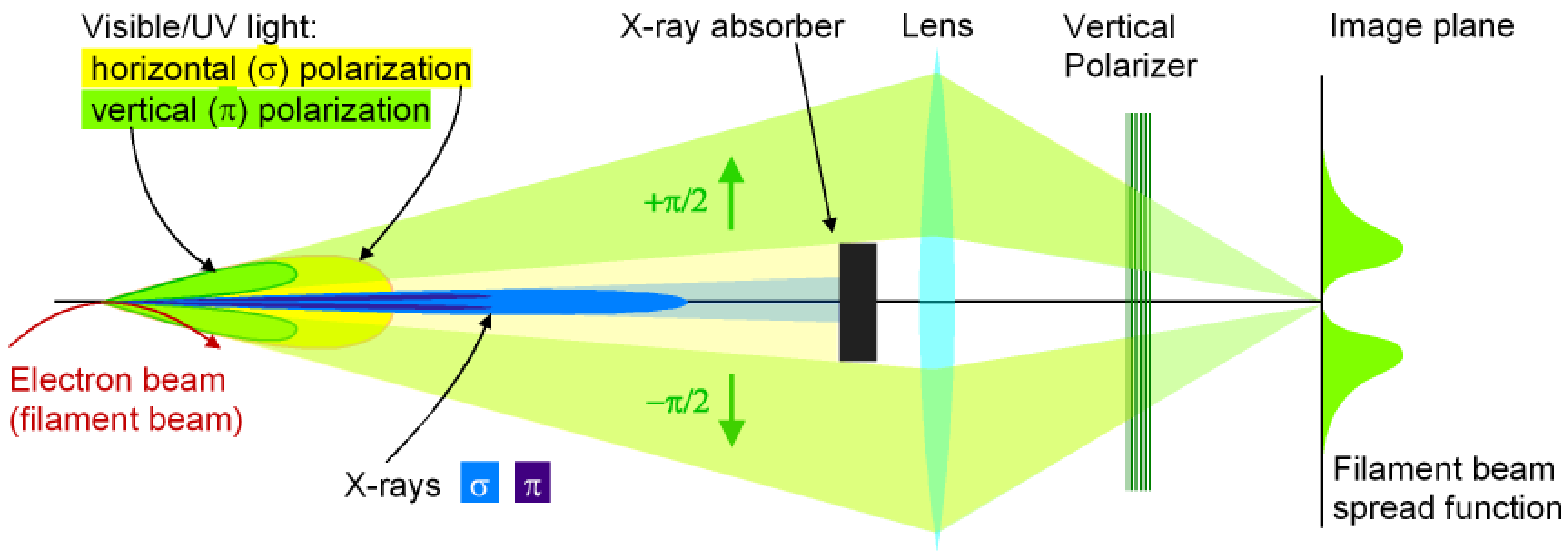

$\sigma_x \sim 55\mu m$

visible light, vertically polarised



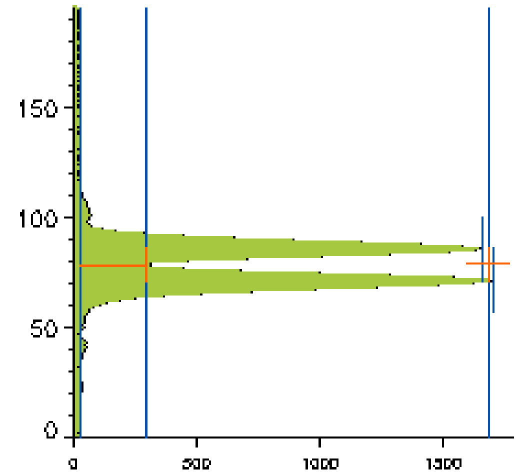
# Seeing the electron beam (SLS)

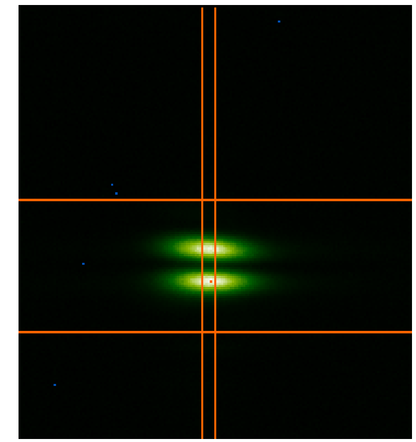
Making an image of the electron beam using the vertically polarised synchrotron light



# High resolution measurement

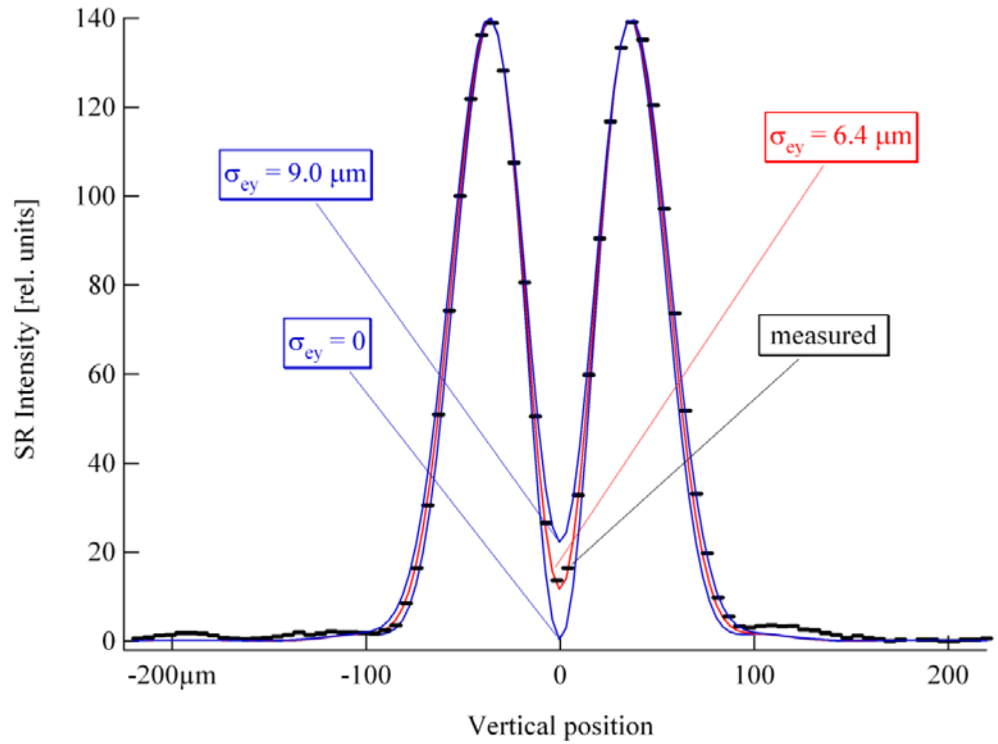
Wavelength used: 364 nm



For point-like source the intensity on axis is zero

Peak-to-valley intensity ratio is determined by the beam height




Present resolution: **3.5 μm**

# Useful books and references

H. Wiedemann, *Synchrotron Radiation*
Springer-Verlag Berlin Heidelberg 2003

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A.Hofmann, *The Physics of Synchrotron Radiation*
Cambridge University Press 2004

A. W. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific 1999

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## Synchrotron Radiation and Free Electron Lasers

Grenoble, France, 22 - 27 April 1996
(A. Hofmann's lectures on synchrotron radiation)
CERN Yellow Report 98-04

Brunnen, Switzerland, 2 – 9 July 2003
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