Outline

• Introduction

• Choice of parameters
  • Frequency and voltage

• RF cavity parameters
  • Shunt impedance, beam loading, power coupling

• Power amplifiers
  • Tube or solid state
  • Local feedbacks

• Longitudinal beam control system
  • Building blocks: RF source and receiver
  • Phase, radial and synchronization loops

• Summary
Introduction
Introduction

• The radiofrequency (RF) system transforms a string of magnets into an accelerator

• Cavity most is the most visible part of an RF system
  → On top of the RF system food chain
  → Interacts directly with beam

→ What is below?
→ How are RF signals generated which make the beam feel comfortable?
Frequency and wavelength ranges

PS longitudinal damper

SPS 200 MHz

PS main RF system

CLIC 12 GHz

100 kHz
3 km

1 MHz
300 m

10 MHz
30 m

100 MHz
3 m

1 GHz
30 cm

10 GHz
3 cm

100 GHz
3 mm

Long wave

Medium/short wave

VHF

Microwave links
Amplitude ranges

Signals from beam pick-ups
- LLRF systems
- Low/Medium energy hadron RF
- SLS
- LHC: 16 MV
- LEP: 3.6 GV total

Cooled hadron beams (ELENA)

Electron light sources
- LHC
- ILC and CLIC: several TV

- LHC
- ILC
- CLIC
- SLS
- LEP
- LHC
- ELENA
Particle velocity

• Particle velocity depends on its type:
  \[ \beta = \frac{v}{c} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2} \]

• Old television set (30 kV):
  - Electrons at 30% of \( c_0 \)
  - Protons just at 0.7%

• Small synchrotron (500 MeV):
  - Electrons at 99.99995%
  - Protons at 75.8%

→ Most electron accelerators at ‘fixed’ frequency
Parameter choices
RF system for high-energy accelerators

Accelerator type

- Linear (single pass)
  - Electron: Maximum RF voltage, constant velocity
  - Hadrons: Maximum RF voltage, variable velocity

- Circular (multi-pass)
  - Electron: Compensate synchrotron radiation losses
  - Hadrons: Sweep frequency with beam

\[ f_{RF} = n \cdot f_{rev} \]

*Exceptions (rare) exist*
Choice of frequency (range)
## Why chose a **low** RF frequency?

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Large beam aperture</td>
<td>• Bulky cavities, size scales $\propto f$, volume $\propto f^3$</td>
</tr>
<tr>
<td>• Long RF buckets, large acceptance</td>
<td>• Lossy material to downsize cavities</td>
</tr>
<tr>
<td>• <strong>Wide-band or wide range tunable cavities</strong> possible</td>
<td>• Moderate or low acceleration gradient</td>
</tr>
<tr>
<td>• Power amplification and transmission straightforward</td>
<td>• Short particle bunches difficult to generate</td>
</tr>
</tbody>
</table>

**RF frequencies below ~200 MHz** for

→ Some hadron linear accelerators
→ Cyclotrons
→ Low- and medium energy hadron synchrotrons
# Why choose a high RF frequency?

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Cavity size scales $\propto f$, volume $\propto f^3$</td>
<td>• Maximum beam available aperture scales $\propto 1/f$</td>
</tr>
<tr>
<td>• Break down voltage increases</td>
<td>• No technology for wide-band or tunable cavities</td>
</tr>
<tr>
<td>• High gradient per length</td>
<td>• Power amplifiers more difficult</td>
</tr>
<tr>
<td>• Particle bunches are short</td>
<td>• Power transmission losses</td>
</tr>
</tbody>
</table>

RF frequencies **above** ~200 MHz **used for**

→ Linear accelerators
→ Electron storage rings
→ High energy hadron storage rings
Limits to maximum gradient

- **Surface electric field in vacuum**

  Kilpatrick 1957,
  \[ f \text{ in GHz, } E_{\text{crit}} \text{ in MV/m} \]

  \[ 24.67 \sqrt{f} = E_{c} e^{\frac{E_{c}}{E_{\text{crit}}}} \]

  → High frequencies preferred for large gradient

  Wang & Loew, SLAC-PUB-7684, 1997
### Some standard frequencies

If exact RF frequency not critical, **chose standard value**

<table>
<thead>
<tr>
<th>Accelerator</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hadron synchrotrons (PSB, PS, JPARC RCS, MR)</td>
<td>&lt;10 MHz</td>
</tr>
<tr>
<td>Hadron accelerators and storage rings (RHIC, SPS)</td>
<td>~200 MHz</td>
</tr>
<tr>
<td>Electron storage rings (LEP, ESRF, Soleil)</td>
<td>352 MHz</td>
</tr>
<tr>
<td>Electron storage rings (DORIS, BESSY, SLS,...)</td>
<td>499.6...499.8 MHz</td>
</tr>
<tr>
<td>Supraconducting electron linacs and FELs (X-FEL, ILC)</td>
<td>1300 MHz</td>
</tr>
<tr>
<td>Normal conducting electron linacs (SLAC)</td>
<td>2856 MHz</td>
</tr>
<tr>
<td>High-gradient electron linac (CLIC)</td>
<td>11.99 GHz</td>
</tr>
</tbody>
</table>

→ **Off-the-shelf RF components easily available in frequency ranges used by industry**

→ **Exchange of developments and equipment amongst research laboratories**
RF voltage
Minimum voltage requirement

• RF system expected to provide given energy gain

\[ qV = \Delta E \]

→ On-crest acceleration
→ Used in some linear accelerators
→ Insufficient in a circular accelerator

• More voltage provided to avoid on-crest acceleration

\[ qV > \Delta E \rightarrow qV \sin(\phi_0) = E \]

→ Off-crest acceleration
→ Needed for circular accelerator
→ Higher voltage for given energy gain
Bucket area dependence on stable phase

- In a circular accelerator the area in energy-time phase space (bucket area) depends on the stable phase.

Below transition, \( \phi_0 = 0\ldots90^\circ \)

- Typical synchronous phase with respect to \( 0^\circ \) or \( 180^\circ \)
  - Hadron accelerators: \( < 40^\circ \)
  - Electron storage rings: \( \sim 20^\circ \)
Minimum voltage requirement (circular)

The RF system must compensate

1. Energy gain per turn due to changing magnetic field

   \[ F_Z = F_L \rightarrow \frac{p}{q} = \rho B \rightarrow \dot{p} = q \rho \dot{B} \]

   \[ \dot{p} = \frac{\Delta p}{\Delta t} = \frac{m_0 c^2 \beta}{2\pi R} (\beta \Delta \gamma + \gamma \Delta \beta) = \frac{\Delta E_{\text{turn}}}{2\pi R} \]

   \[ \Delta E_{\text{turn}} = 2\pi q \rho R \dot{B} \]

2. Energy loss, e.g., due to synchrotron radiation (electrons)

   \[ \Delta E_{\text{turn}} = \frac{e^2}{3\epsilon_0 (m_0 c^2)^4 \rho} E^4 \]

   \[ \Delta E_{\text{turn}}[\text{keV}] = 88.5 \cdot \frac{E^4[\text{GeV}]^4}{R[\text{m}]} \quad P_{\text{loss}}[\text{kW}] = 88.5 \cdot \frac{E^4[\text{GeV}]^4}{R[\text{m}]} \cdot I_B[\text{A}] \]

   \[ \rightarrow (m_p/m_e)^4 = 1836^4 \sim 1.1 \cdot 10^{13} \text{ times less for protons} \]
RF system overview

Beam

Cavity

→ Convert RF power into longitudinal electric field

Power amplifier

→ Amplify low-power signal from beam control to kW, MW or GW

Low-level RF system

→ Provide RF signals with correct frequency, amplitude and phase

Beam
RF system overview

→ Convert RF power into longitudinal electric field

→ Amplify low-power signal from beam control to kW, MW or GW

→ Provide RF signals with correct frequency, amplitude and phase
RF cavity
Cavity parameters

• The resonance of a cavity can be understood as simple parallel resonant circuit described by $R$, $L$, $C$

\[
\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C
\]

with \( \omega_0 = \frac{1}{\sqrt{LC}} \)

\[
Q = \omega_0 \frac{\text{Stored energy}}{\text{Average power loss}} = \frac{\omega_0 E}{P} = \frac{\omega_0}{\omega_0 L}
\]

\[
Q = \omega_0 RC = \frac{R}{\omega_0 L}
\]

\[
E = \frac{1}{2} CV^2 = \frac{1}{2} LI^2
\]

\[
P = \frac{1}{2} \frac{U^2}{R} = \frac{1}{2} I^2 R
\]
Cavity parameters

- The resonance of a cavity can be understood as simple parallel resonant circuit described by \( R, L, C \)

\[
\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C
\]

with \( \omega_0 = \frac{1}{\sqrt{LC}} \)

\[
Q = \omega_0 RC = \frac{R}{\omega_0 L} \quad Z(\omega) = \frac{R}{1 + iQ \left( \frac{\omega^2 - \omega_0^2}{\omega \omega_0} \right)} \approx \frac{R}{1 + 2iQ \frac{\Delta\omega}{\omega_0}}
\]

→ Resonant circuit can also be described by \( R, R/Q, \omega_0 \) or any other set of three parameters
Cavity parameters

- The resonance of a cavity can be understood as simple parallel resonant circuit described by $R, L, C$

\[
Q = \omega_0 RC' = \frac{R}{\omega_0 L}
\]

\[
Z(\omega) = \frac{R}{1 + iQ \left(\frac{\omega^2 - \omega_0^2}{\omega \omega_0}\right)} \approx \frac{R}{1 + 2iQ \frac{\Delta\omega}{\omega_0}}
\]

→ Resonant circuit can also be described by $R, R/Q, \omega_0$ or any other set of three parameters
Cavity parameters

• Most common choice by cavity designers $\omega_0$, $R$, $R/Q$ – why?

• Resonance frequency, $\omega_0$
  → Exactly defined for given application, e.g. $hf_{rev}$

• Shunt impedance, $R$
  → Power required to produce a given voltage without beam

• “R-upon-Q”, $R/Q$
  → Defined only by the cavity geometry
  → Criterion to optimize a geometry
  → Detuning with beam proportional to $R/Q$
Why R/Q?

→ Charged particle experiences cavity gap as capacitor

\[ q = V_{\text{ind}} C \]

\[ Q = \omega_0 R C \quad \rightarrow \quad \frac{1}{C} = \left( \frac{R}{Q} \right) \omega_0 \]

\[ V_{\text{ind}} = \frac{q}{C} \propto \frac{R}{Q} \]

→ Cavity geometry with small \( R/Q \) to reduce beam loading
RF cavities in low frequency range

- RF wavelength large below ~10 MHz: >30 m
  - Would need huge cavities → too large for accelerators
  - Line resonators: $\lambda/4$ resonator

Why is this resonator so common in particle accelerators?
RF cavities in low frequency range

- Coaxial structure with inner conductor as beam pipe

\[ Z(\omega) \rightarrow \text{Beam axis} \rightarrow \text{Short-circuit} \]

\[ \text{Accelerating gap, isolated} \]

→ Still rather long geometry, 7.5 m at 10 MHz

→ Add capacitive or inductive shortening

- Plate capacitor
- Ferrite inductivity
Capacitive loading

→ Add capacitor at gap of cavity to shorten the resonator

NSLS, 52.88 MHz

DESY PIA, 10.4 MHz, inner cond.

Outer cond.

ACOL, 9.53 MHz

→ Significantly reduces cavity size
→ Fixed frequency only
→ Small losses due to capacitor
→ Cavity in vacuum

M. Nagl
Inductive loading

→ Inductive loading with magnetic material shortens resonator from tens of meters to a device, lossy though

CERN PSB Finemet cav., 0.6-18 MHz
CERN PS, double gap, 2.8-10 MHz

• Additional advantage: permeability of ferrite can be controlled by DC bias current → variable inductivity
→ Cavity with programmable resonance frequency
→ Essential for hadron acceleration in low-energy accelerators
Tunable cavities at higher frequencies

→ Remove inductive or capacitive loading

SSC Low Energy Booster, 
~47 MHz to 60 MHz

FNAL Booster 2\textsuperscript{nd} harmonic, 
76 MHz – 106 MHz, 100 kV

→ Upper frequency limit for cavities with large tuning range
Further increase frequency

→ Remove inner conductor from coaxial set-up

→ The resonator becomes a pill-box cavity

DORIS cavity

Electric field, $\text{TM}_{010}$-mode

→ The basis for cavity resonators

E. Jensen
Example: 400 MHz cavities in LHC

→ Reduce beam loading in RF cavities
→ Shunt impedance, $R$, low for small $R/Q$ with normal conducting cavities → superconducting cavities in LHC

Bell shape: \( R/Q \sim 44 \, \Omega, \, 400 \, MHz \)

→ 2×8 cavities, 5.3 MV/m

\[
\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}
\]
RF cavities in linear accelerators

- Beam only passes once – Maximize gradient
- Many accelerating cells to best reuse RF voltage

SuperHILAC, ~70 MHz, Berkley

CLIC, 12 GHz, ~100 MV/m

→ Cavity is the contrary to ‘one size fits all’
→ Many, many more variants
Coupling power into a cavity
Coupling power into a cavity

- Attack inductivity or capacitance of resonator, or combined

\[
\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}
\]

→ Coupling loop forms transformer with resonator inductivity

- Main coupler
  PSI cyclotron
  → ~1 MW at 50 MHz
Coupling power into a cavity

- Attack inductivity or capacitance of resonator, or combined

\[
\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}
\]

→ **Capacitive divider** to gap to transform generator impedance to cavity shunt impedance

→ **Beam also couples capacitively via the gap**
Coupling power into a cavity

- Attack inductivity or capacitance of resonator, or combined

\[ \frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}} \]

→ Combined electromagnetic coupling
→ Antenna radiating into cavity
Capacitive or combined coupling

- Some examples of capacitive and antenna couplers

Capacitive coupler of CERN PS 40 MHz

Antenna coupler of LHC cavities

→ Coupler forms one half of capacitor with the gap

→ Coupler antenna transmits directly into the cavity
RF system overview

Konverter RF power into longitudinal electric field

→ Amplify low-power signal from beam control to kW, MW or GW

→ Provide RF signals with correct frequency, amplitude and phase
Power amplifiers
How much power is required?

1. Power to accelerate beam → Wanted
2. Compensate beam-induced voltage → Refl. $P$
3. Compensate electrical losses in cavity → Heat
4. Compensate electrical losses in distribution → Heat

\[ P_{\text{amplifier}} = P_{\text{dist}} + P_{\text{cavities}} + P_{\text{BL}} \]

\[ P_{\text{cavities}} = n \frac{(V/n)^2}{2R} \]

\[ P_{\text{BL}} = I_B \cdot V_{\text{ind}} \text{ (ideally)} \]

\[ P_B = I_B \cdot \Delta E_{\text{turn}} \]
Power amplifiers

• Basically

\[ P_{\text{out}} = g \cdot P_{\text{in}} \quad \text{or} \quad V_{\text{out}} = \sqrt{g} \cdot V_{\text{in}} \]

• The ideal power amplifier
  → Large bandwidth: amplifies all frequencies equally
  → No saturation, infinite power
  → Zero delay
  → No added noise
  → Unconditionally stable and resistant to reverse power
  → Radiation-hard

→ Unfortunately such a device has not been invented yet
→ Let us have a look at some real amplifiers
Basics of grid tube

• From diode to tetrode amplifier

• Vacuum tube
• Heater + Cathode
• Heated cathode
  • Coated metal, carbides, borides,...
• thermionic emission
• Electron cloud
• Anode
→ Diode

*For tube amplifier designs voltages are named $U$ instead of $V*

E. Montesinos
Basics of grid tube

• From diode to tetrode amplifier

- Vacuum tube
- Heater + Cathode
  - Heated cathode
    - Coated metal, carbides, borides,...
  - thermionic emission
- Electron cloud
- Anode
  → Diode

E. Montesinos
Basics of grid tube

- From diode to tetrode amplifier

→ Triode

- Modulating the grid voltage proportionally modulates the anode current
- Transconductance
  - Voltage at grid
  → Current at anode
- Limitations
  - Parasitic capacitor from anode to control grid (g1)
  - Tendency to oscillate

E. Montesinos
Basics of grid tube

- From diode to tetrode amplifier

→ Tetrode

- Screen grid
  - Positive (lower anode)
  - Decouple anode and g₁
  - Higher gain

- Limitations
  - Secondary electrons
  - Anode treated to reduce secondary emission
Tetrode based power amplifier

- Example of SPS 200 MHz amplifier, tetrode RS2004

→ Very simplified block diagram

E. Montesinos
Example: Tetrode amplifier driving SPS RF

• Two transmitters, $2 \times 1$ MW at 200 MHz (almost continuous)
• Eight tetrodes per amplifier

→ In operation since 1976

RS2004 tetrode
Amplifier trolley
Complete transmitter

E. Montesinos
Tetrode amplifier driving PS RF

→ Frequency range 2.8...10 MHz, ~60 kW per cavity, 11 units
→ Space constraints to have amplifier installed below cavity

→ Tetrode is obvious choice
  → High power in small volume
  → Operates in radioactive environment
Basics of linear beam tube

- **Klystron:** a complete mini-accelerator

- **Klystron's velocity modulation**
  - Converts the kinetic energy into RF power

- **Vacuum tube**

- **Electron gun**
  - Thermionic cathode
  - Anode

- **Electron beam**

- **Drift space**

- **Collector**

- **e- constant speed until the collector**

E. Montesinos
Basics of linear beam tube

- **Klystron:** a complete mini-accelerator

- **Cavity resonators and drift**
- **RF input cavity (Buncher)** → Modulates electron velocity
- **Drift space** → Faster electrons catch up → Slower electrons fall behind
- **RF output cavity (Catcher)**
  - Resonating at same frequency as input cavity
  - At place where electrons are maximally bunched
  - Kinetic energy converted into voltage and extracted
Basics of linear beam tube

• Klystron: a complete mini-accelerator

• Cavity resonators and drift

• RF input cavity (Buncher)
  → Modulates electron velocity

• Drift space
  → Faster electrons catch up
  → Slower electrons fall behind

• RF output cavity (Catcher)
  • Resonating at same frequency as input cavity
  • At place where electrons are maximally bunched
  • Kinetic energy converted into voltage and extracted

E. Montesinos
Example: Klystrons driving the LHC

- 2 x 8 cavities, each driven by separate 400 MHz klystron, 330 kW
  → First klystron amplifiers powering a hadron collider

- 12 GHz pulsed klystron for CLIC
  → 50 MW in 1.5 μs

- Significantly more power was required to feed LEP (until 2000)
  → About 50 MW CW was installed at 352 MHz
In a push-pull circuit the RF signal is applied to two devices:

- One of the devices is active on the positive voltage swing and off during the negative voltage swing.
- The other device works in the opposite manner so that the two devices conduct half the time.

→ The full RF signal is then amplified.

→ Needs two different type of devices.
Another **push-pull configuration** is to use a balun (balanced-unbalanced)

- Power splitter, equally dividing the input power between the two transistors
- Balun keeps one port in phase and inverts the second port in phase

Since the signals are out of phase only one device is On at a time

→This configuration is easier to manufacture since only one type of device is required
Example: Soleil 45 kW, 352 MHz

Electron storage ring running at 352 MHz

330 W amplifier module

600 W, 300 V\text{DC}/30 V\text{DC} converter
Example: Soleil 45 kW, 352 MHz

Large scale solid state amplifier installations

- 45 kW per tower (2004 and 2007)
- 150 kW per tower (2012)

→ Requires a series of power combiners to moderate power per amplifier module to several tens of kilowatts

E. Montesinos
RF power amplifier

Power capability of commercially available amplifier types

Typical ranges (commercially available)

- Grid tubes
- Klystrons
- IOT
- CCTWTs
- Transistors (solid state x32)

E. Jensen
How to chose the right RF amplifier?

<table>
<thead>
<tr>
<th>Prefer tube amplifier, when</th>
<th>Prefer solid-state amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Amplifier must <strong>be installed in the accelerator tunnel</strong></td>
<td>• Amplifier can be located in <strong>non-radioactive environment</strong></td>
</tr>
<tr>
<td>• Expecting important <strong>spikes from beam induced voltage</strong></td>
<td>• <strong>Circulator</strong> can be installed to protect the amplifier</td>
</tr>
<tr>
<td>• <strong>Large output power of a single device is required, without combiners</strong></td>
<td>• <strong>Delay due to unavoidable combiner stages is little issue</strong></td>
</tr>
<tr>
<td>• Not much space is available</td>
<td>• Sufficient space can be made available</td>
</tr>
<tr>
<td>• <strong>High peak power in pulsed mode</strong></td>
<td>• <strong>Continuous operation</strong></td>
</tr>
<tr>
<td>• Amplifier must be <strong>compact and/or close to cavity</strong></td>
<td>• Amplifier can be <strong>separate from the cavity</strong></td>
</tr>
</tbody>
</table>

→ Mostly no hard criteria → decide on case by case basis
Summary

• RF system parameters
  → Chose frequency and voltage wisely

• Parameters of RF cavities
  → $R$, $R/Q$
  → No ‘one-size fits’ all

• Power amplifier
  → Ideal amplifier does not (yet) exist
  → Tube or solid-state based

• Feedbacks and longitudinal beam control
  → Make the beam feel comfortable in bucket
  → Beam phase, radial and synchronization loops
RF Systems II

H. Damerau
CERN

Introduction to Accelerator Physics

28 September 2018
Outline

• Introduction

• Choice of parameters
  • Frequency and voltage

• RF cavity parameters
  • Shunt impedance, beam loading, power coupling

• Power amplifiers
  • Tube or solid state
  • Local feedbacks

• Longitudinal beam control system
  • Building blocks: RF source and receiver
  • Phase, radial and synchronization loops

• Summary
Local feedbacks
Reduction of cavity impedance

- Energy transfer from cavity to beam, but from beam to cavity
  → Both, RF generator and beam can induce voltage in cavity

1. Reduce beam induced voltage by reducing $R$, but not efficient
   → Obviously needs more power → $$$

2. Feedback to decrease the apparent impedance for the beam
   → Use amplifier to counteract beam induced voltage
Reduction of cavity impedance

- Energy transfer from cavity to beam, but from beam to cavity
  → Both, RF generator and beam can induce voltage in cavity

1. Compare drive signal (no beam) with gap (beam and generator)
2. Amplify inverted difference

\[ Z_{eq}(\omega) = \frac{dV}{dI_B} = \frac{Z(\omega)}{1 + g_{OL}} \]
Example: 10 MHz RF system in CERN PS

Transfer function with and without feedback

- Feedback gain of 24 dB
  → Equivalent impedance, $Z_{eq}(\omega)$ reduced
  → Impedance for amplifier remains unchanged, $Z(\omega)$

Why not further reduction with more gain?
- Subtraction of gap voltage and drive signal imperfect due to
  1. Delay of cables and amplifier
  2. Parasitic resonances of amplifier and cavity system

Bandwidth $\uparrow$ ↔ Achievable gain $\downarrow$
Example: 10 MHz RF system in CERN PS

- 10 + 1 ferrite loaded cavities, tunable from 2.8...10 MHz

- Fast wide-band feedback around amplifier (internal)
  → Gain limited by delay
Main 10 MHz RF system

- 10 + 1 ferrite loaded cavities, tunable from 2.8...10 MHz

**Drive**

- Fast wide-band feedback around amplifier (internal) → Gain limited by delay

- 1-turn delay feedback → High gain at $n \times f_{rev}$
Feedbacks with 1-turn delay

→ Reduce cavity impedance beyond stability limit of wide-band FB

Open/closed loop transfer functions

Spectrum at cavity gap return

Feedback off

Feedback on

→ Important additional impedance reduction

→ Clever usage of beam periodicity in circular accelerator
RF system overview

→ Convert RF power into longitudinal electric field

→ Amplify low-power signal from beam control to kW, MW or GW

→ Provide RF signals with correct frequency, amplitude and phase
Global feedbacks
Low-level RF beam control
Longitudinal beam control

- **Local** feedbacks → Act on individual RF stations
- **Global** feedbacks → Act on all RF stations simultaneously

→ RF distribution to compensate time of flight between stations
→ Beam control drives all stations *like a single one*
Basic building blocks
**Mixer or multiplier**

- **Example:** analogue 4 quadrant multiplier and low pass filter

\[
\begin{align*}
\sin(\omega_1 t + \phi_1) & \quad \rightarrow \quad \frac{1}{2} \left\{ \cos[(\omega_1 - \omega_2) t + (\phi_1 - \phi_2)] \\
& \quad - \cos[(\omega_1 + \omega_2) t + (\phi_1 + \phi_2)] \right\} \\
\sin(\omega_2 t + \phi_2) \quad \rightarrow \quad 2
\end{align*}
\]

- **Signals:**

![Graph showing signals](image)
Mixer or multiplier

- Example: analogue 4 quadrant multiplier and low pass filter

\[ \sin(\omega_1 t + \phi_1) \]

\[ \sin(\omega_2 t + \phi_2) \]

\[ \frac{1}{2} \{ \cos[(\omega_1 - \omega_2) t + (\phi_1 - \phi_2)] \} \]

\[ \cos[(\omega_1 + \omega_2) t + (\phi_1 + \phi_2)] \}

Remove ripple → Low-pass filter

- Signals:
How to detect phase differences?

- Example: analogue 4 quadrant multiplier and low pass filter

\[ \sin(\omega_1 t + \phi_1) \]

\[ \frac{1}{2}\left\{ \cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] \right\} \]

\[ \cos[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)] \]

Remove ripple → Low-pass filter

Relative: arbitrary shift by 90°

- Signals:

- Phase discriminator in approximately +/-90° range
Two signals at different frequencies $\omega_1$ and $\omega_2$ change linearly in phase difference, $Df$, between both signals.

- **Phase difference, $\Delta \phi$, between both signals changes linearly.**
- **Ambiguity** to distinguish between $\Delta \phi = -\pi, \pi, -3\pi, 3\pi, ...$.
- **Saw-tooth in phase** means constant frequency difference.

**Equivalence of frequency and phase** 

$\omega = \frac{d\phi}{dt} \iff \phi = \int \omega \, dt$
RF sources
What finally generates the RF signal to power amplifier and cavity?
→ Need an RF source!

- **Electron accelerators**
  - Off-the-shelf high-performance laboratory generators as reference: BESSY SR, CERN CTF$_3$
  - Dedicated commercial fixed-frequency sources with low phase noise: free electron lasers, CERN AWAKE

- **Proton accelerators**
  - Special sweeping RF sources, controlled by beam-based loops: mostly in-house developments
Noisy RF signals

• Degradation of signal quality due to noise
  • Amplitude and/or phase jitter
• What is the difference between a coherent signal and noise?

→ Amplitude of **coherent**, quasi monochromatic signal (at 200 MHz) is **independent of observation bandwidth**

→ Incoherent **noise power** (dominated by spectrum analyzer front-end amplifier/mixer) is **proportional to bandwidth**

→ Thermal noise power \( \frac{P}{\Delta f} = k_B T = 1.38 \cdot 10^{-23} \text{ J/K} \cdot 296 \text{ K} \simeq -174 \text{ dBm/Hz} \)
Analysis of phase noise

• Compare noise power with carrier power as reference

\[ \text{Ratio of carrier to noise: } \text{dBc} \]

\[ \text{Bandwidth } \Delta f = 1 \text{ Hz for normalization} \]

• Noise power density

\[ \mathcal{L}(f) = \frac{\text{Power density}}{\text{Carrier power}} \left[ \frac{\text{dBc}}{\text{Hz}} \right] = \frac{1}{2} S_{\phi}(f) \]

→ Its integral is the phase jitter and using

\[ \Delta t = \frac{\Delta \phi}{2\pi f_c} \]

the jitter in time becomes

\[ \Delta t_{\text{rms}} = \frac{1}{2\pi f_c} \sqrt{\int_{f_1}^{f_2} S_{\phi}(f) \, df} \]
Typical phase noise plots

- Measure phase noise of a synthesized lab generator

→ Note: jitter values can be added as square root of quadratic sum

$$\Delta t_{\text{rms}} = \sqrt{\Delta t_{\text{rms},1}^2 + \Delta t_{\text{rms},2}^2 + \cdots}$$

→ Convenient split to relevant ranges

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>$\Delta t_{\text{rms}}$ [fs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10...100 Hz</td>
<td>12.4</td>
</tr>
<tr>
<td>100 Hz ...1 kHz</td>
<td>5.4</td>
</tr>
<tr>
<td>1...10 kHz</td>
<td>5.4</td>
</tr>
<tr>
<td>10...100 kHz</td>
<td>11.1</td>
</tr>
<tr>
<td>100 kHz...1 MHz</td>
<td>13.0</td>
</tr>
<tr>
<td>Total</td>
<td>31.0</td>
</tr>
</tbody>
</table>
Variable frequency: direct digital synthesis

- Generate (almost) any frequency starting from a given clock frequency, $f_{clk}$
- Digitally programmable in frequency

\[ f_{out} = \frac{\text{Frequency word}}{2^n} \cdot f_{clk} \]
Direct digital synthesis

• Generate (almost) any frequency starting from a given clock frequency, $f_{clk}$
• Digitally programmable in frequency and phase

$\phi \quad t$

2^n adder

Phase accumulator

$\phi \quad t$

Phase offset word

CORDIC

$\sin \quad t$

$\cos \quad t$

$\sin \quad t$

$\cos \quad t$

$\phi \quad t$

$\phi \quad t$

$f_{\text{out}} = \frac{\text{Frequency word}}{2^n} \cdot f_{\text{clk}}$

→ Two output signals with 90° ideal phase shift
→ Output signals are digital data streams
Receivers
I/Q representation of signals

- Any signal can be represented by amplitude $A$ and phase $\phi$

\[ I = A \cos \phi \]
\[ Q = A \sin \phi \]
\[ A = \sqrt{I^2 + Q^2} \]
\[ \phi = \arctan \frac{Q}{I} \]

→ In phase, $I$ and quadrature, $Q$ also describe the same signal
→ Avoids phase discontinuities at 0, $2\pi$, ...

A narrow peak is shown with $A$ and $\phi$. A circle with $A$ and $\phi$ is also shown. Another circle with $Q$ and $\phi$ is shown. Dashed lines are shown.
Signal receivers

- Radio with listens to beam or cavity signals
- Listens to amplitude and phase

\[ A \sin(\omega_{in} t + \phi_{in}) \cdot \sin(\omega_{LO} t + \phi_{LO}) \]

\[ \frac{A}{2} \cos[(\omega_{in} - \omega_{LO}) t + (\phi_{in} - \phi_{LO})] \]

\[ A \sin(\omega_{in} t + \phi_{in}) \cdot \cos(\omega_{LO} t + \phi_{LO}) \]

\[ \frac{A}{2} \sin[(\omega_{in} - \omega_{LO}) t + (\phi_{in} - \phi_{LO})] \]

→ With \( \omega_{in} \approx \omega_{LO} \) input signal is down-converted to base-band
→ Resulting I/Q vector rotates slowly with \( \omega_{in} - \omega_{LO} \)
Digital receivers

- No conceptual difference between analogue and digital
- Digitization can be performed at any level

$\sin(\omega_{\text{LO}}t + \phi_{\text{LO}})$

$\cos(\omega_{\text{LO}}t + \phi_{\text{LO}})$

$\frac{A}{2} \cos[(\omega_{\text{in}} - \omega_{\text{LO}})t + (\phi_{\text{in}} - \phi_{\text{LO}})]$

$\frac{A}{2} \sin[(\omega_{\text{in}} - \omega_{\text{LO}})t + (\phi_{\text{in}} - \phi_{\text{LO}})]$

→ Analog down-conversion, then digital processing
→ High input frequencies beyond ADC sampling rates
Digital receivers

- No conceptual difference between analogue and digital
- Digitization can be performed at any level

→ Analogue mixers become digital multipliers
→ All digital receiver
→ Theoretically perfect I/Q symmetry
Cascaded integrator-comb filter (CIC)

- Efficient implementation of low pass filter
- Standard form with sampling rate decimation: $f_{\text{clk}} \rightarrow f_{\text{clk}}/d$

$H(z) = \left(\frac{1 - z^{-d}}{1 - z^{-1}}\right)^n$

- $n$: filter order
- $d$: decimation ratio

→ Easy to implement in programmable logic: no multipliers
→ Only adders and shift registers
Cascaded integrator-comb filter (CIC)

Why particularly interesting for circular accelerators?

- Chose clock frequency, $f_{clk} = 2^m f_{rev}$ and decimation $d = 2^m$
  → Notches at all multiples of $f_{rev}$ except zero
  → Linear phase $\phi(f)$ → filter behaves like a fix delay

Example:

$f_{clk} = 128 f_{rev}$, 
$d = 128$, 
$n = 3$

Ideal low-pass filter in digital receivers
  → Filter selected multiple of $f_{rev}$ while suppressing all others
Vector modulator
Invers receiver: vector modulator

- Convert I/Q data into modulated RF signal

\[ I \cos(\omega_{LO}t + \phi_{LO}) + Q \sin(\omega_{LO}t + \phi_{LO}) \]
Inverse receiver: vector modulator

- Convert I/Q data into modulated RF signal

[Diagram showing the process of converting I/Q data into a modulated RF signal with I and Q signals going through DACs and mixing with a LO signal.]

→ Perfect I/Q symmetry difficult to achieve
→ Up-conversion of digital signal to a high RF frequency
Beam phase loop
Electronic phase-locked loop

- Frequency re-generation and multiplication
- Voltage controlled oscillator (VCO) locked in phase to input

\[ \omega_{VCO} = 2\pi f_{VCO} \]
\[ = \frac{d\phi}{dt} = K_{VCO} V_{in} \]

\[ f_{out} \]
\[ \phi_{out} \]
\[ n \]

→ Fixed phase relationship: \[ \phi_{out}/n - \phi_{in} = \text{const.} \]

→ Optional divider: \[ f_{out} = n \cdot f_{in} \]
Beam phase loop

Phase pick-up

Beam phase

\[ \Delta \phi \]

Cavity phase

Synchronous phase, \( \phi_s \)

\[ - + \]

Loop filter

\[ \phi_{err} \sim \Delta f \]

RF cavity

Power amplifier

Digital synthesizer

\[ h \cdot f_{rev}, \text{ from } B, p \]

RF

Slow signal

\( h f_{rev} \) (digital)
Beam phase loop

Phase pick-up

Beam phase $\Delta \phi$

Cavity phase

Synchronous phase, $\phi_s$

Loop filter

$\phi_{err} \sim \Delta f$

$\pm\Delta f$

DDS

Digital synthesizer

$f_{out} = f_{in} \pm \Delta f$

Power amplifier

RF cavity

RF

$\pm h f_{rev}$ (digital)

$h \cdot f_{rev}$, from $B$, $p$

→ Phase-locked loop with beam phase as reference for RF system
Beam phase loop

- Phase pick-up
- Beam phase
- Cavity phase
- Synchronous phase, $\phi_s$
- Loop filter
- $\Delta \phi$
- DDS
- Power amplifier
- Digital synthesizer
- Loop corr.
- RF cavity
- RF
- Slow signal
- $h f_{\text{rev}}$ (digital)
- $f_{\text{out}} = f_{\text{in}} \pm \Delta f$
- $h \cdot f_{\text{rev}}$, from $B$, $p$
- Fast control of RF frequency to cavities, but no slow corrections
Effect of beam phase loop at injection

- Example: Injection of a bunch from PS Booster into PS

$90^\circ$ error, phase loop off

$90^\circ$ error, phase loop on

→ Essential in hadron accelerators to keep RF locked to beam
→ How does this look like in longitudinal phase space?
Effect of beam phase loop at injection

→ Essential in hadron accelerators to keep RF locked to beam

Bunch in rigid bucket, no loop

Injection with phase loop

→ Even large transients (injection, transition) can be controlled
→ Small longitudinal emittance blow-up
Beam phase loop during acceleration

→ What happens with phase loop during acceleration?

→ During plateaus the phase between RF and beam either $0^\circ$ or $180^\circ$

→ Fast phase changes well handled, but need slow frequency correction

→ Radial or synchronization loop
Radial loop
Radial loop

Reference magnet

Δ Hybrid Σ

Δ/Σ

ΔR

Beam

Reference magnet

RF
Slow signal
Digital signal

DDS

Δf

B+ B-

Frequency program

B · frev

→ Slow correction of average RF frequency
Radial loop

• Slow correction of RF frequency to keep beam centred

Why needed at all with arbitrary precision synthesizers driving the RF system?

• At transition energy
  → The longer path of higher energy particle compensated by higher velocity
  → No revolution frequency change for energy offset

\[
\frac{\Delta R}{R} = \frac{\gamma^2}{\gamma_{tr}^2 - \gamma^2} \frac{\Delta f}{f}
\]

→ Need beam-based frequency correction
Synchro(nization) loop
Beam phase loop

Phase pick-up

Beam phase

\( \Delta \phi \)

Cavity phase

RF cavity

Power amplifier

Synchronous phase, \( \phi_s \)

Loop filter

Loop corr.

DDS

Precision volt. controlled oscillator

\[ f_{out} = f_{in} \pm \Delta f \]

\( h \cdot f_{rev} \), from \( B, p \)

→ Fast control of RF frequency to cavities, but no slow corrections
Synchronization loop, internal reference

- Phase pick-up
- Beam phase
- Cavity phase
- RF cavity
- Power amplifier
- DDS
- Synchronous phase, $\phi_s$
- Loop filter
- $\phi_{err} \sim \Delta f$
- Loop corr.
- Loop filter
- Ref. DDS
- $h \cdot f_{rev}$, from $B$, $p$

→ Avoids noise from radial detection when not crossing transition
**Synchronization loop, internal reference**

- **Phase pick-up**
  - Beam phase
  - Cavity phase

- **RF cavity**
  - Power amplifier
  - Loop filter

- **DDS**
  - Loop filter
  - Loop corr.

- **Synchronous phase, $\phi_s$**
  - +
  - AC coupling

- **Loop filter**
  - $\phi_{err}$ ~ $\Delta f$

- **Power amplifier**
  - $f_{RF}$

- **RF**
  - $h f_{rev}$ (digital)

- **Synchronize between accelerators for transfer**

$h \cdot f_{rev}$, from $B$, $p$
Before synchronization

- Simple test case of circumference ratio 2: \( C_2 = 2C_1 \)

\[ \rightarrow \text{Synchronize both accelerator to force: } f_{\text{rev},1} = 2f_{\text{rev},2} \]
Simple synchronization process

1. Move beam to off-momentum ($B$ const.): 
   \[ \frac{df}{f} = \frac{\gamma^2_{tr} - \gamma^2}{\gamma^2 \gamma^2_{tr}} \frac{dp}{p} \]
   → Well defined frequency difference between accelerators

2. Measure azimuth error, when beam at correct azimuth
   → Close synchronization loop
   → Moves beam to ref. momentum

---

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Beam azimuth (from phase loop) → Ref. azimuth (from master divider) → Act on $f_{RF}$ of slave → Locked! 200 ms
After synchronization

- Simple test case of circumference ratio 2: \( C_2 = 2C_1 \)
  
  Source or target accelerator is master at transfer

\[ f_{\text{rev,1}} = 2f_{\text{rev,2}} \]

→ Revolution frequencies coupled: \( f_{\text{rev,1}} = 2f_{\text{rev,2}} \)
→ Ready to extract during every turn of the target accelerator
Summary

• RF system parameters
  → Chose frequency and voltage wisely

• Parameters of RF cavities
  → $R, \frac{R}{Q}$
  → No ‘one-size fits’ all

• Power amplifier
  → Ideal amplifier does not (yet) exist
  → Tube or solid-state based

• Feedbacks and longitudinal beam control
  → Make the beam feel comfortable in bucket
  → Beam phase, radial and synchronization loops
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References

• W. D. Kilpatrick, Criterion for vacuum sparking designed to include both rf and dc, Rev. Sci. Instrum. 28 (1957), 1957, http://inspirehep.net/record/44645
Normalized Hamiltonian representation

- For a single harmonic RF system

\[ H(\phi, \dot{\phi}) = \frac{1}{2} \dot{\phi}^2 + \frac{\omega_s^2}{\cos \phi_0} \left[ \cos \phi_0 - \cos \phi + (\phi - \phi_0) \sin \phi_0 \right] \]

with \( \phi = \phi_0 + \Delta \phi \) it becomes

\[ H(\Delta \phi, \dot{\phi}) = \frac{1}{2} \dot{\phi}^2 + \frac{\omega_s^2}{\cos \phi_0} \left[ \cos \phi_0 - \cos(\phi_0 + \Delta \phi) - \Delta \phi \sin \phi_0 \right] \]

using \( \cos(\phi_0 + \Delta \phi) = \cos \phi_0 \cos \Delta \phi - \sin \phi_0 \sin \Delta \phi \)

\[ \approx \cos \phi_0 \left( 1 - \frac{1}{2} \Delta \phi^2 \right) - \sin \phi_0 \Delta \phi \]

this simplifies to

\[ H(\Delta \phi, \dot{\phi}) \approx \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \omega_s^2 \Delta \phi^2 \]
Transmission of reference signals

- Thermal drift of long coaxial cables or optical fibres

- Thermal coefficient of delay:
  \[ TCD = \frac{\Delta \tau}{\tau} \cdot \frac{1}{\Delta T} = \frac{\Delta \phi}{\phi} \cdot \frac{1}{\Delta T} \]

- Example: 2 km long RG223 cable with \(~10\ \mu s\) delay
  \[ \Delta T \text{ of only } 1^\circ C \text{ (room temperature) changes delay by } \sim 0.5\ \text{ns} \]
  \[ 1.8^\circ \text{ at } 10\ \text{MHz (CERN PS), but } 73^\circ \text{ at } 400\ \text{MHz (LHC)} \]

- Optical fibres are typically 10...100 times more stable

- What to do if this is still not sufficient?