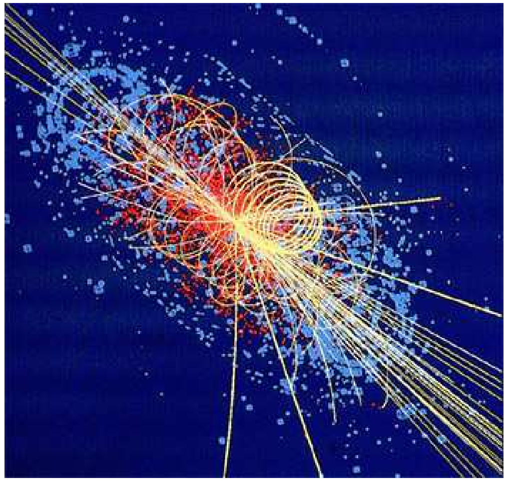
Particle Colliders and Concept of Luminosity

(or: explaining the jargon $^{*)}$...)

http://cern.ch/Werner.Herr/CAS2018_Romania/lectures/luminosity.pdf

$^{*)}$ **(beta*, cross section, femtobarn, inverse femtobarn, crossing angle, luminosity measurement, filling schemes, pile-up, hour glass effect, crab crossing, dynamic beta, beam-beam effects ...)**

Particle colliders ?



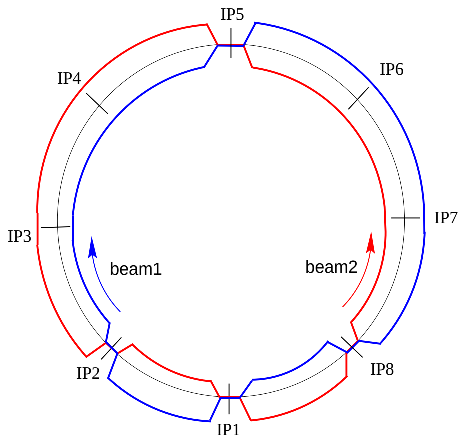
- Used in particle physics
- Look for rare interactions
- Many interactions (events)
- Want highest energies

Collider versus fixed target (once more ..):

Fixed Target: $\vec{p_2} = \mathbf{0}$ ➡ $\sqrt{s} = \sqrt{2m^2 + 2E_1 m}$

Symmetric Collider: $\vec{p_1} = -\vec{p_2}$ ➡ $\sqrt{s} = E_1 + E_2$

Circular Colliders (mostly synchrotrons):



double ring colliders (e.g. LHC, ISR)

can accelerate any type of particle

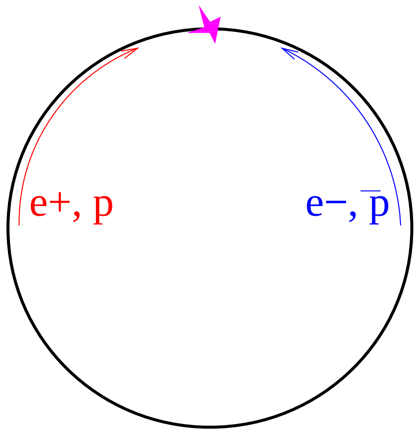
some may have to be produced first : $\mu, \gamma, \ldots$

usually with crossing angles

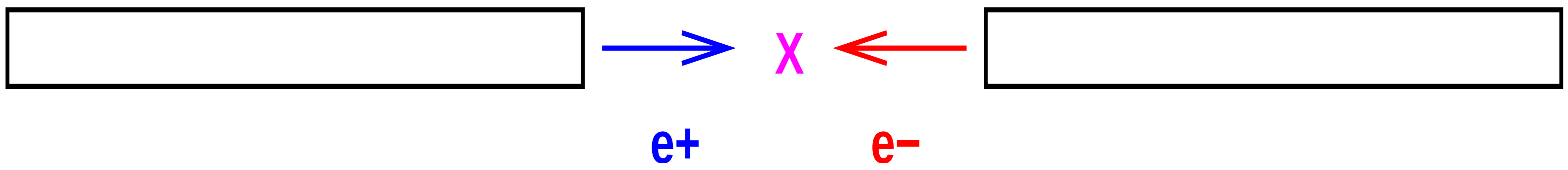
single ring colliders (e.g. LEP, SppS)

collides particles – antiparticles

some sort of separation required



Linear colliders (SLC, $CLIC$, ILC, ...)



- Mainly used (proposed) for leptons

  (reduced synchrotron radiation)

- With or without crossing angle

- Need very small beam sizes

(maybe treated if time permits ..)

Rare interactions and cross section

- **Cross section σ measures the <u>likelihood</u> a particular process occurs:** **nothing to do with its size ! (imagine e^+e^- or $\gamma\gamma$ collisions)**

  Characteristic for a given process

- **Measured in: barn = b = 10^{-24} cm^2 (picobarn = 10^{-36}cm^2)**

  Some examples for the LHC energy:

$$\sigma(pp \rightarrow X) \quad \approx \quad 0.1 \text{ b}$$

$$\sigma(pp \rightarrow X + H) \quad \approx \quad 1 \cdot 10^{-11} \text{ b}$$

$$\sigma(pp \rightarrow X + H \ \rightarrow \ \gamma\gamma) \quad \approx \quad 50 \cdot 10^{-15} \text{ b} \ = \ 50 \text{ fb (femtobarn)}$$

  VERY rare (one in $2\ 10^{12}$), need many collisions ...

(traditionally: cm^2 instead of m^2)

Collider performance issues

Luminosity:

Number of interactions

Number of interactions per second

More: they have to be useful, some issues

- **Time structure of interactions (how often and how many at the same time: pile-up)**
- **Space structure of interactions (size of interaction region: vertex density)**
- **Quality of interactions (background, dead time etc.)**

Luminosity - we want:

➡ **Relates cross section σ_p and number of interactions per second $\frac{dR}{dt}$**

$$\frac{dR}{dt} = L \times \sigma_p \qquad (\rightarrow \quad \mathrm{units:} \;\; \mathrm{cm}^{-2}\mathrm{s}^{-1})$$

Typically: $\frac{dR}{dt}$ **measured and σ_p wanted**

<u>Must</u> be:

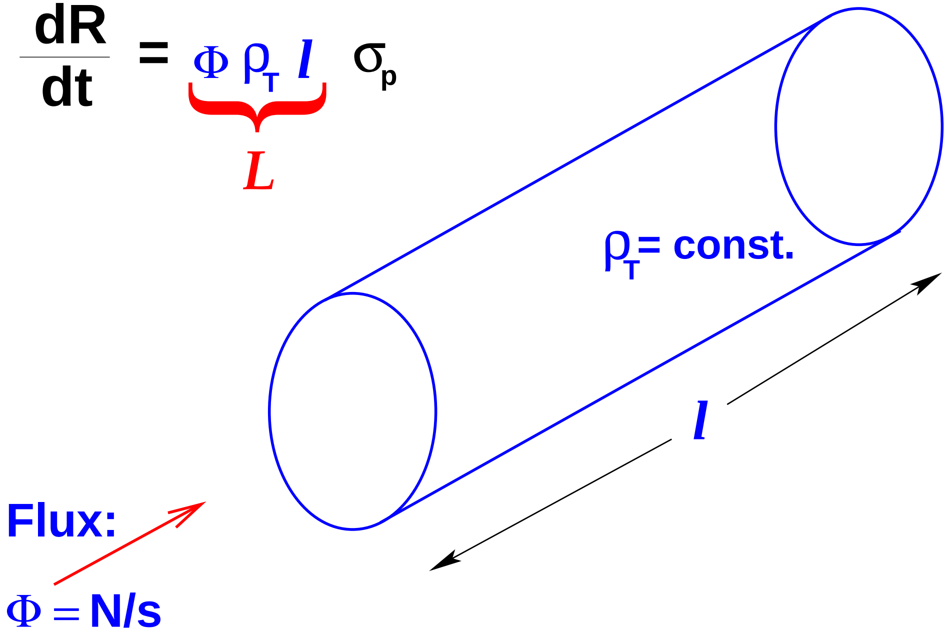
➡ **Relativistic invariant (see lecture on "Relativity")**

➡ **A property of the collider: Independent of the physical reaction, i.e. σ_p**

➡ **Reliable procedures to compute and measure**

Fixed target "luminosity" L

$$\frac{dR}{dt} = \underbrace{\Phi\, \rho_T\, l}_{L}\ \sigma_p$$



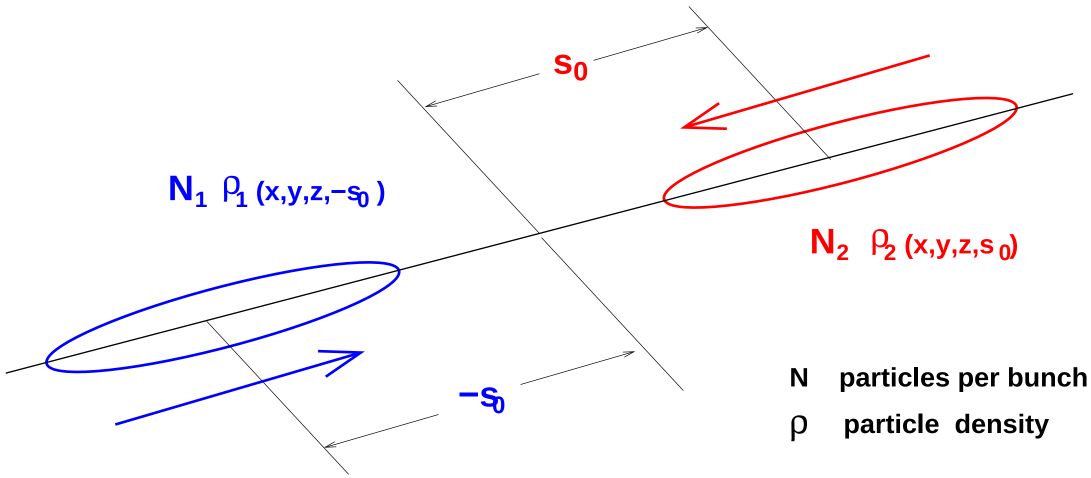
Interaction rate from:

flux N/s

target density ρ

size

Collider luminosity: now target (bunched beam) is moving



$$L \quad \propto \quad N_1 \; N_2 \int\int\int\int \rho_1(x,y,s,-s_0)\; \rho_2(x,y,s,s_0)\; dxdydsds_0$$

s_0 **is "time"-variable:** $s_0 = c \cdot t$ (at : $s_0 = 0$ and $t = 0$ bunch <u>centres</u> collide)

Assume uncorrelated densities in all planes, then they factorize:

$$\rho(x,y,s,s_0) = \rho_x(x) \cdot \rho_y(y) \cdot \rho_s(s \pm s_0)$$

Moving beams: requires a <u>Kinematic Factor</u> →

$$\mathsf{K} = \sqrt{((\vec{v_1} - \vec{v_2})^2 - (\vec{v_1} \times \vec{v_2})^2)/c^2}$$

For head-on collisions: $\vec{v_1} = -\vec{v_2}$ → $\mathsf{K}_{bb} = 2$ **(Space charge:** $\mathsf{K}_{sc} = 1 - \beta$**)**

With revolution frequency f and number of bunches n_b the luminosity L becomes:

$$\boxed{L = \mathsf{K} \cdot N_1 N_2 \cdot f \cdot n_b \int_{-\infty}^{\infty} \rho_x(x)\rho_y(y)\rho_s(s - s_0) \cdot \rho_x(x)\rho_y(y)\rho_s(s + s_0)}$$

In principle: should know all distributions ρ and ρ, but Gaussian distributions are usually a good approximation, tails can be ignored

transverse : $\rho(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_x^2}\right)$ $\rho(y) = \frac{1}{\sigma_y\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$

Plugging it in:

For beams of equal size: $\sigma_1 = \sigma_2 \quad \rightarrow \quad \rho_1 \; \rho_2 \; = \; \rho^2$ **:**

$$L = \frac{2 \cdot N_1 N_2 \; f \; n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int e^{-\frac{x^2}{\sigma_x^2}} \; e^{-\frac{y^2}{\sigma_y^2}} \; e^{-\frac{s^2}{\sigma_s^2}} \; e^{-\frac{s_0^2}{\sigma_s^2}} \quad dx dy ds ds_0$$

Integrating over s **and** s_0 **(means during of passage), using:**

$$\int_{-\infty}^{\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a} \qquad \text{(the happy Gaussian)}$$

$$L = \frac{2 \cdot N_1 N_2 \; f \; n_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int e^{-\frac{x^2}{\sigma_x^2}} \; e^{-\frac{y^2}{\sigma_y^2}} \quad dx dy$$

Finally after integration over x and y: $\Longrightarrow$ $$\boxed{L = \frac{N_1 \; N_2 \; f \; n_b}{4\pi \; \sigma_x \; \sigma_y}}$$

The more general case:

$\sigma_x \neq \sigma_x$ and $\sigma_y \neq \sigma_y$

$$\Longrightarrow \quad \boxed{L = \frac{N_1 \;\; N_2 \quad f \;\, n_b}{2\;\pi\;\sqrt{\sigma_x^2 + \sigma_x^2}\;\sqrt{\sigma_y^2 + \sigma_y^2}}}$$

What if the distributions are not Gaussian ?

Using r.m.s. of an arbitrary (but realistic) distribution for σ to compute Luminosity the errors are typically 5%

➡ This has consequences for luminosity measurements (makes it easier) ...

Examples: some circular colliders

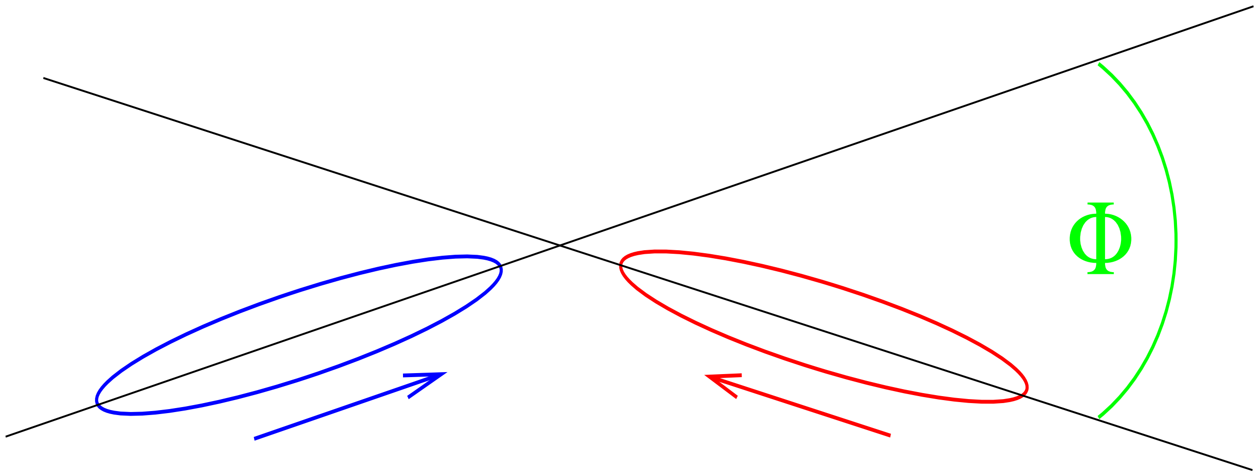
	Energy (GeV)	L_{max} $cm^{-2}s^{-1}$	rate s^{-1}	σ_x/σ_y $\mu m/\mu m$	Particles per bunch
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$	$4\ 10^{5}$	60/30	$\approx 10\ 10^{10}$
Tevatron ($p\bar{p}$)	1000x1000	$100\ 10^{30}$	$7\ 10^{6}$	30/30	$\approx 30/8\ 10^{10}$
HERA (e^+p)	30x920	$40\ 10^{30}$	40	250/50	$\approx 3/7\ 10^{10}$
LHC (pp)	7000x7000	$10000\ 10^{30}$	10^{9}	17/17	$\approx 16\ 10^{10}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$	≤ 1	200/2	$\approx 50\ 10^{10}$

I will concentrate on elephants

Complications

- **Crossing angle**
- **Hour glass effect**
- **Collision offset (wanted or unwanted)**
- **Displaced waist (minimum beam size not where we collide)**
- **Non-Gaussian profiles**
- **Dispersion at collision point**
- **Strong coupling**
- **etc.**

Collisions at crossing angle

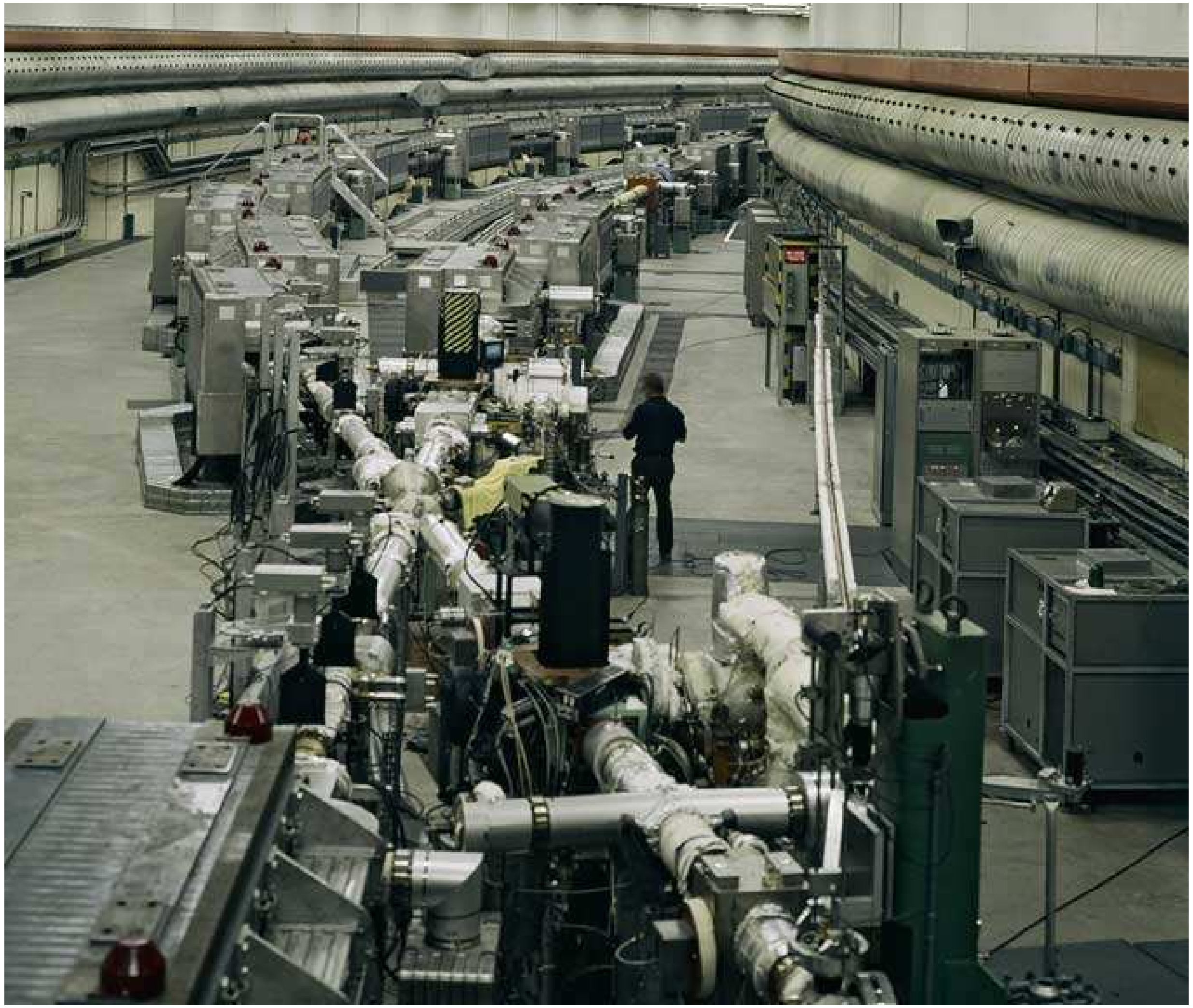


Needed to avoid unwanted collisions

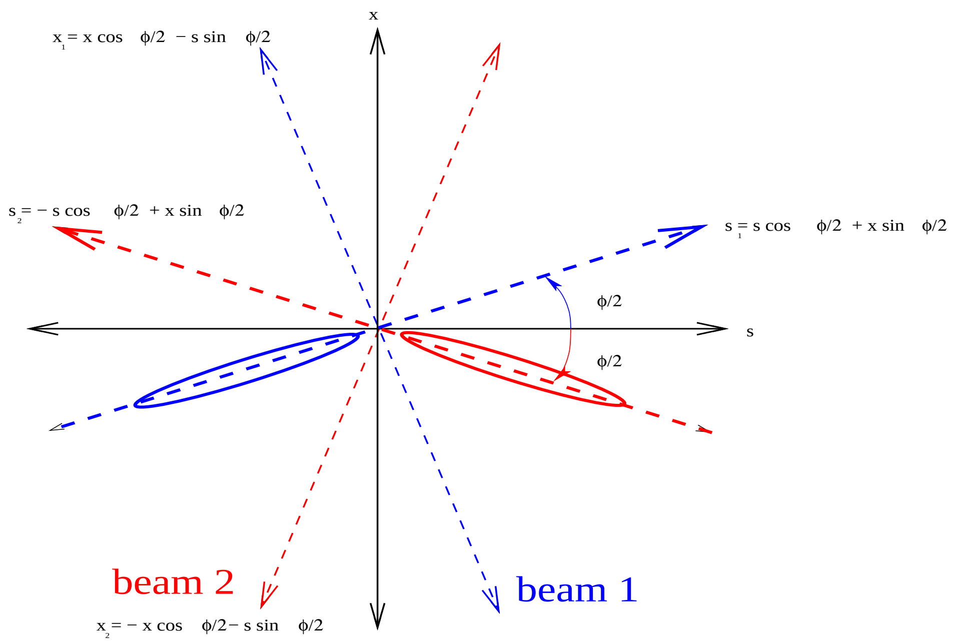
- **For colliders with many bunches: e.g. LHC, CESR, KEKB**
- **For colliders with coasting beams: e.g. the late ISR**

Some numbers:

- **LHC: 0.300 mrad**
- **ISR: 300 mrad**



Collisions angle geometry (horizontal plane)



For the calculation of the integral:

The coordinate systems for the two beams are tilted (by half the crossing angle and in opposite directions)

Assume crossing in horizontal (x, s)- plane. Transform to new coordinates (now different coordinate systems for the two beams):

$(x, s) \rightarrow (x_1, s_1, x_2, s_2)$

$$\begin{pmatrix} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{pmatrix}$$

After longitudinal integration:

$$\boxed{L = 2 \cdot \cos^2\frac{\phi}{2} \cdot N_1 \cdot N_2\ f\ n_b \int_{-\infty}^{\infty} \rho_x(x_1)\rho_y(y_1)\ \rho_x(x_2)\rho_y(y_2)\ dxdy}$$

The Integration with crossing angle:

use as before :

$$\int_{-\infty}^{\infty} e^{-au^2} du = \sqrt{\pi/a}$$

and :

$$\int_{-\infty}^{\infty} e^{-(au^2+bu+c)} du = \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{a}}$$

A simplification here: since σ_x, x and $\sin(\phi/2)$ are small:

1. **drop all terms** $\sigma_x{}^k \sin^l(\phi/2)$ or $x^k \sin^l(\phi/2)$ when $k + l \geq 4$
2. **approximate** $\sin(\phi/2) \approx \tan(\phi/2) \approx \phi/2$

(not a good approximation for the ISR, but it had coasting beams ...)

Correction for crossing angle

Crossing Angle $\Rightarrow$ $$\boxed{L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot S}$$

- S **is called the "geometric factor"**
- **For small crossing angles and** $\sigma_s \gg \sigma_{x,y}$

$$\Rightarrow S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

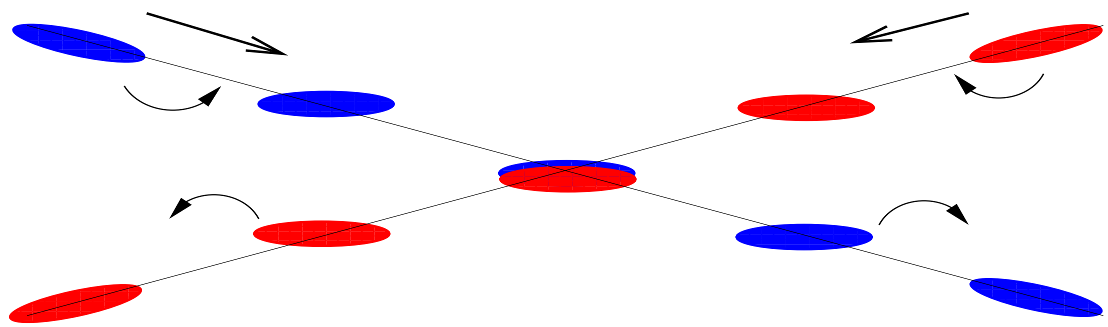
Example nominal LHC (at 7 TeV):

Φ **= 285** μ**rad,** σ_x $\approx$ **17** μ**m,** σ_s **= 7.5 cm, S = 0.84**

For large crossing angle Φ **and small beam size** σ_x **the loss can be large, maybe too large**

A proposed fix: "crab" crossing scheme

- crossing angle: loss of luminosity can be large for long bunches or small β^* (small beam sizes)
- "crab" crossing can recover geometric factor



- Done with transversely deflecting cavities (if you wondered what they can be used for)
- Foreseen for the LHC luminosity upgrade (lower β^* planned)

Crossing angle plus : Offset

Transformations with offsets d_1 and d_2 in crossing plane:

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, \quad s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, \quad s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

Gives after integration over y and s_0:

$$L = \frac{L_0}{2\pi\sigma_s\sigma_x} 2\cos^2\frac{\phi}{2} \int\int dx ds\ e^{-\frac{x^2\cos^2(\frac{\phi}{2}) + s^2\sin^2(\phi/2)}{\sigma_x^2}}\ e^{-\frac{x^2\sin^2(\phi/2) + s^2\cos^2(\phi/2)}{\sigma_s^2}}$$

$$\times\ e^{-\frac{d_1^2 + d_2^2 + 2(d_1+d_2)x\cos(\phi/2) - 2(d_2-d_1)s\sin(\phi/2)}{2\sigma_x^2}}.$$

After integration over x:

$$L = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}}\sigma_s} \quad 2\cos\frac{\phi}{2} \quad \int W \cdot \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds$$

with:

$$A = \frac{\sin^2\phi/2}{\sigma_x^2} + \frac{\cos^2\phi/2}{\sigma_s^2} \qquad\qquad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2}$$

and $W = e^{-\frac{(d_2-d_1)^2}{4\sigma_x^2}}$ **(important, see later !)**

$\Longrightarrow$ **After integration: Luminosity with correction factors**

Luminosity with correction factors

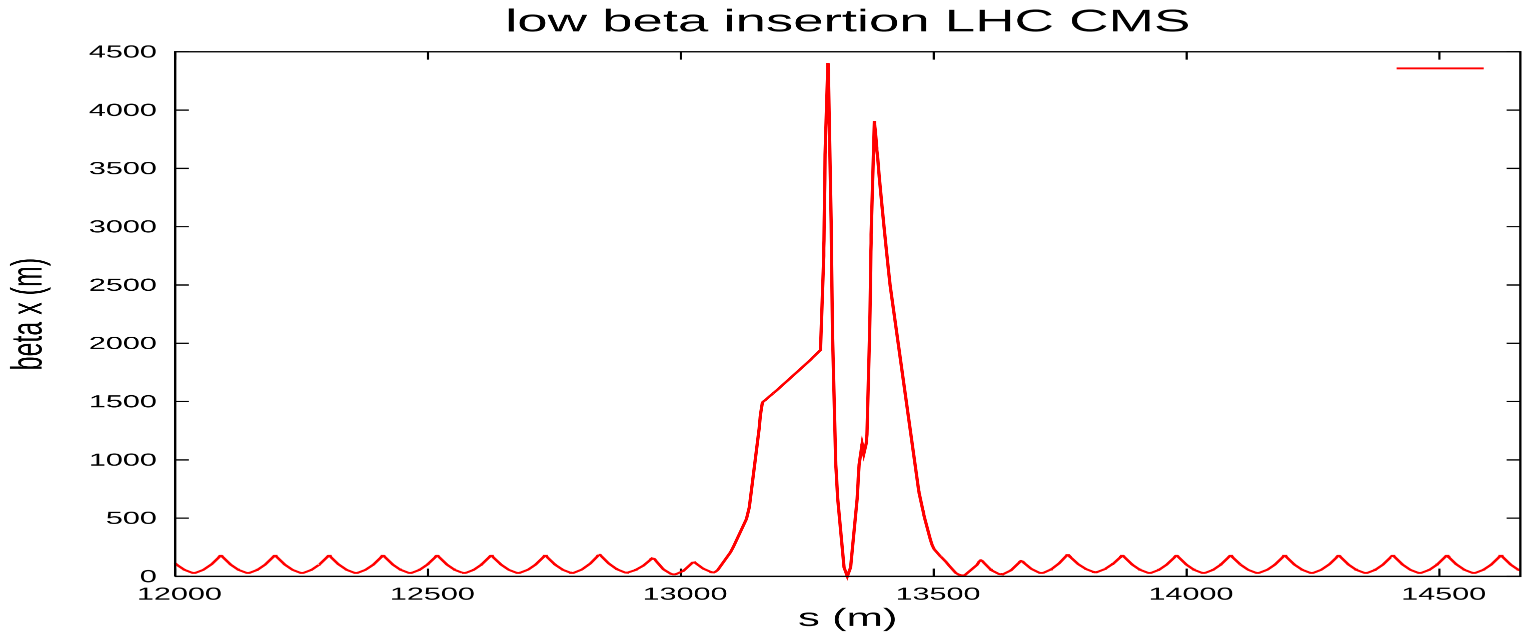
$$\boxed{L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S}$$

- W: correction for beam offset (one per plane)
- S: correction for crossing angle
- $e^{\frac{B^2}{A}}$: correction for crossing angle and offset

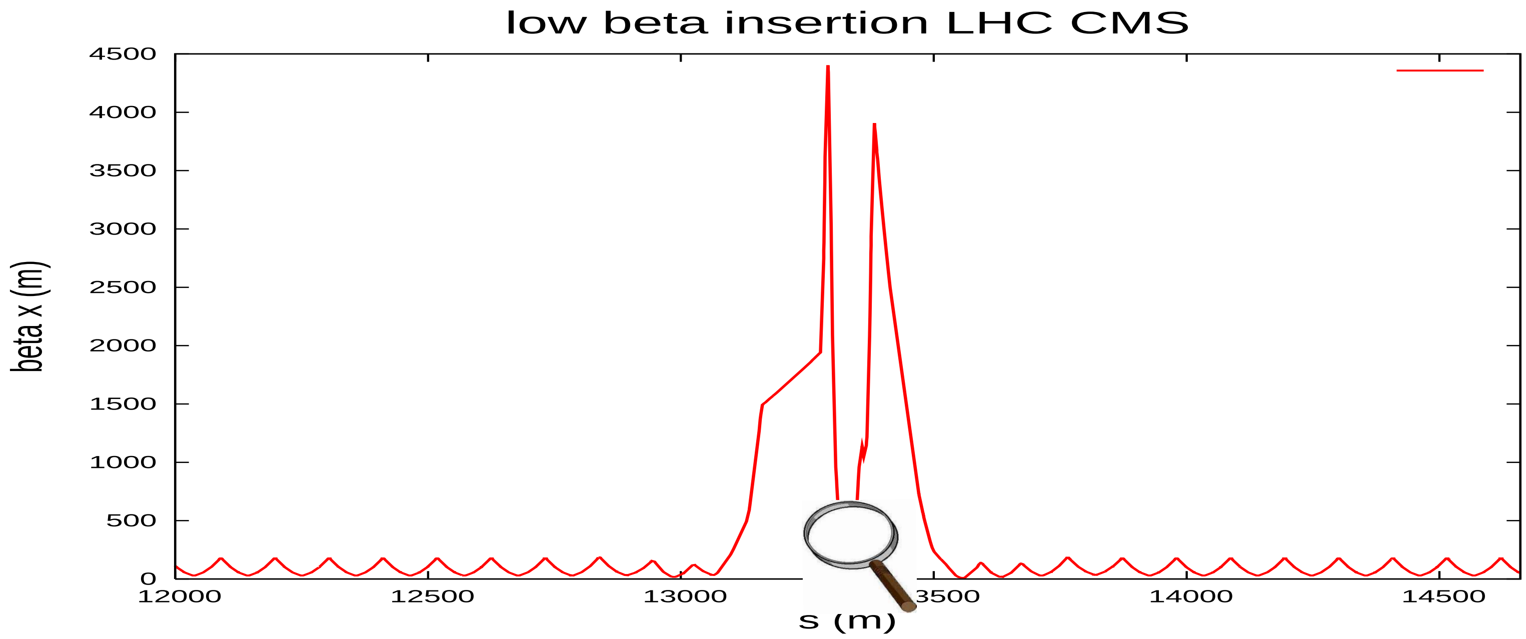
  (if in the <u>same</u> plane)

What about crossing in both planes (e.g. LHCb in the LHC) ???

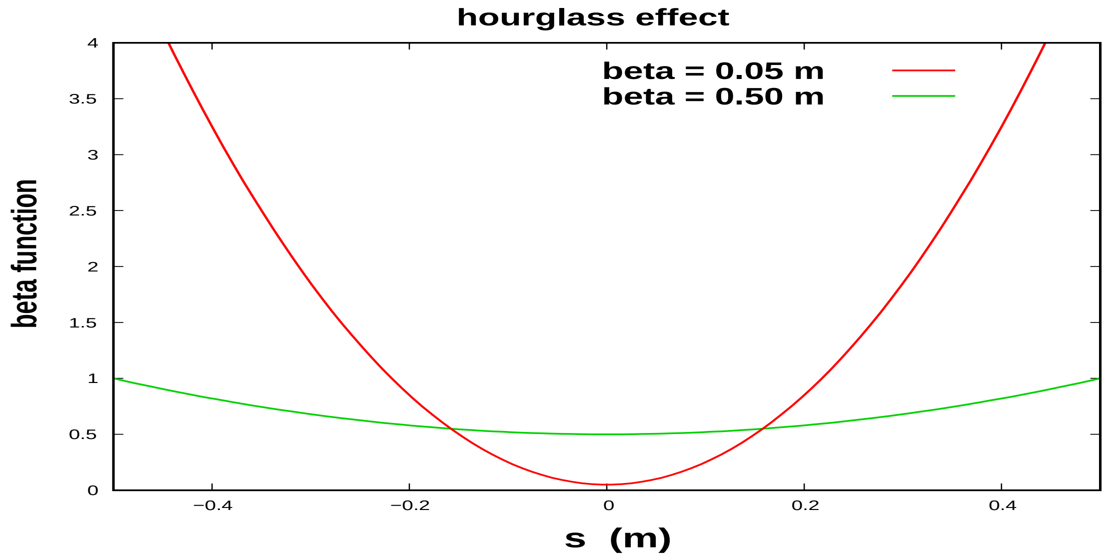
Next: Hour glass effect



Remember the insertion: β-functions depends on longitudinal position s



Remember the insertion: β-functions depends on longitudinal position s



In our low β insertion we have: $\beta(s) \approx \beta^* \left(1 + \left(\frac{s}{\beta^*}\right)^2\right)$

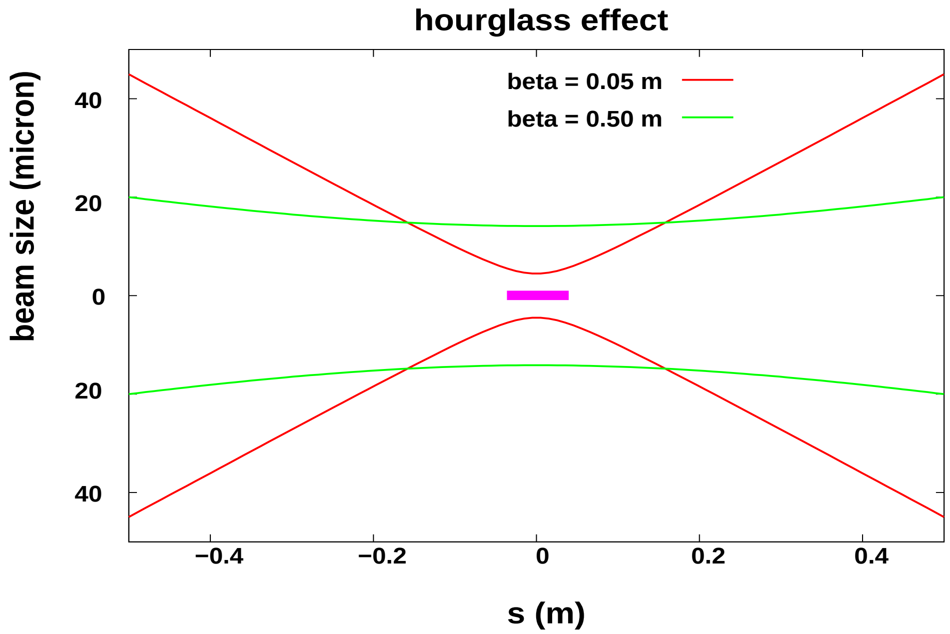
For small β^* the beam size grows very fast: $\approx \frac{s^2}{\beta^*}$

Beam size σ depends on longitudinal position s

Contribution to luminosity depends on longitudinal position s !

Beam size has shape of an Hour Glass

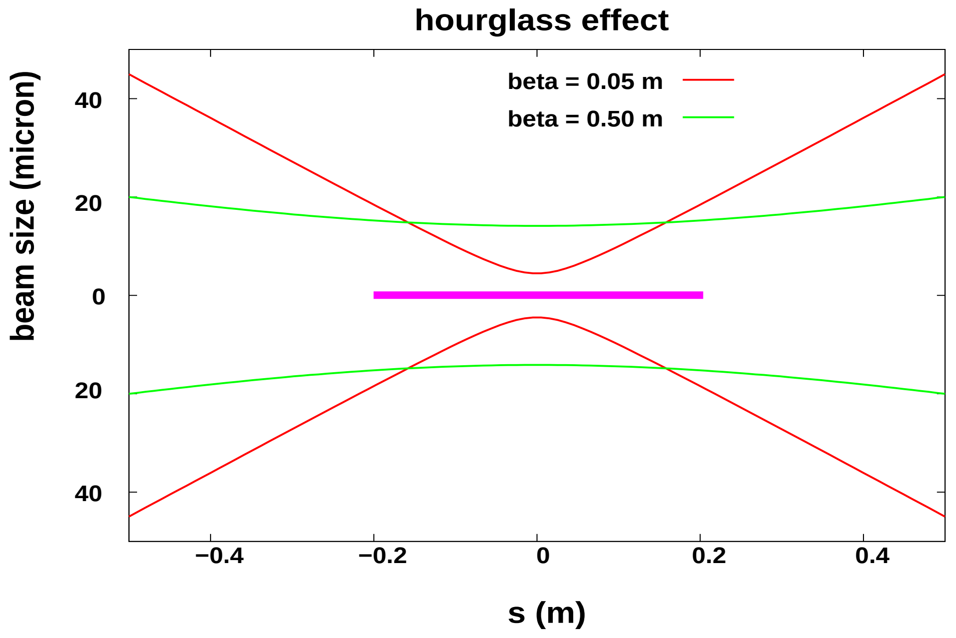
Hour glass effect - short bunches



Small variation of beam size along bunch

(Picture shows LHC values)

Hour glass effect - long bunches



Significant variation for long bunches and small β^*

- β**-functions depends on position s**
- **Need modification to the overlap integral**
- **Usually:** $\beta(s) = \beta^* \left(1 + \left(\frac{s}{\beta^*}\right)^2\right)$

→ **i.e.** $\sigma \implies \sigma(s) \neq$ **const.**

→ $\sigma(s) = \sigma^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2}$

Then the same procedure as before, but watch out for the longitudinal integration now.

- **Important when** β^* **comparable to the r.m.s. bunch length** σ_s **(or smaller !)**

Here just for one plane, becomes more laborious for flat beams (see literature)

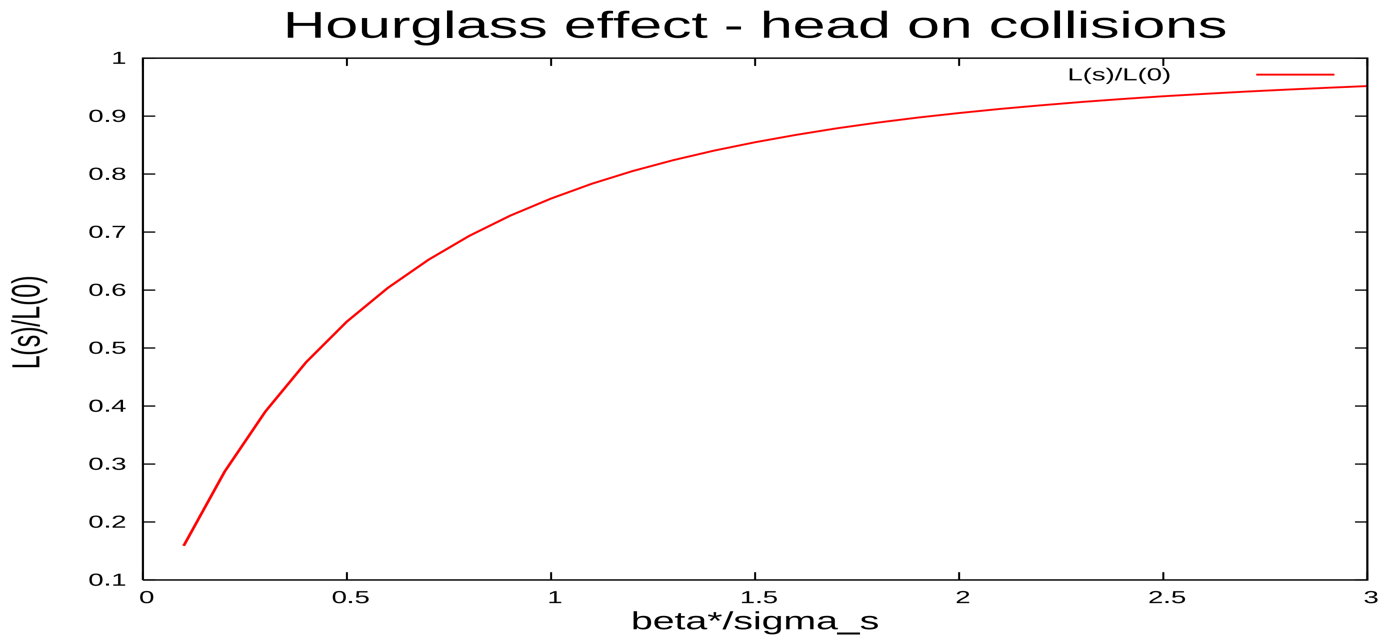
Using the expression: $u_x = \beta^* / \sigma_s$

Without crossing angle and for symmetric, round Gaussian beams we get the relative luminosity reduction as:

$$\frac{L(\sigma_s)}{L(0)} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-u^2}}{[1 + (\frac{u}{u_x})^2]} du = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

$$L(\sigma_s) = L(0) \cdot H \quad \mathrm{with:} \quad H = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

Complicated situations may need numerical intengration



- **Hourglass reduction factor as function of ratio β^*/σ_s.**
- **A lesson: small β^* does not always lead to high luminosity !**

Now LHC works with β^*/σ_s larger than 4 (nominal above 7)

Luminosity with (more) correction factors

$$\boxed{L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

- W: **correction for beam offset**
- S: **correction for crossing angle**
- $e^{\frac{B^2}{A}}$: **correction for crossing angle and offset**
- H: **correction for hour glass effect**

Calculations for the (nominal) LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ **particles/bunch**

- $n_b = 2808$ **bunches/beam**

- $f = 11.2455 \text{ kHz}, \quad \phi = 285 \ \mu$**rad**

- $\beta_x^* = \beta_y^* = 0.55 \text{ m}$

- $\sigma_x^* = \sigma_y^* = 16.6 \ \mu\text{m}, \quad \sigma_s = 7.7 \text{ cm}$

- **Simplest case L_0 (Head on collision)**

$$L = 1.200 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle:**

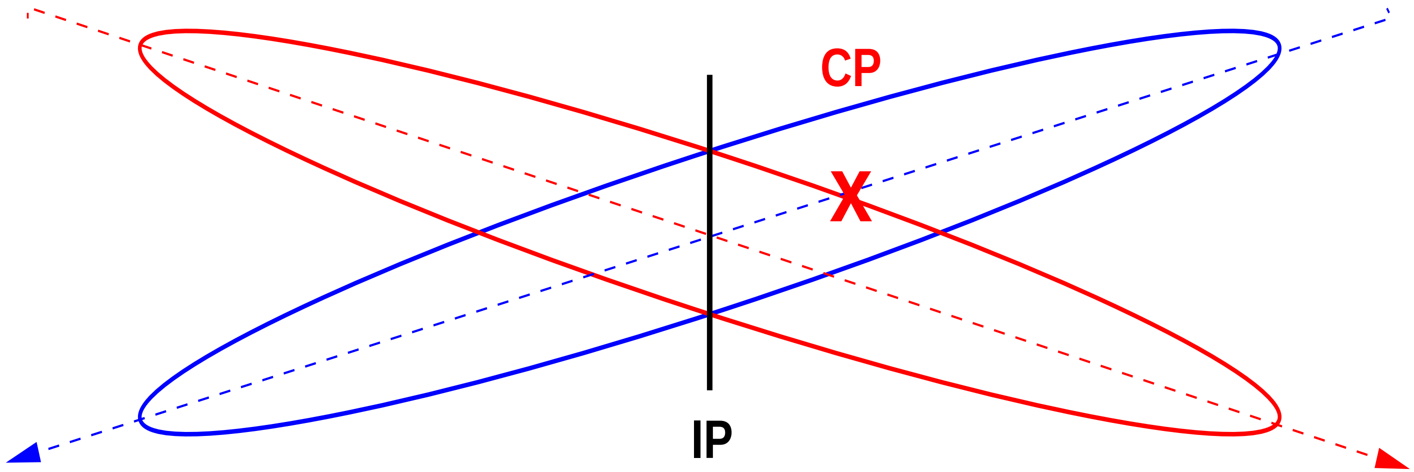
$$L = 0.973 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle & Hourglass:**

$$L = 0.969 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

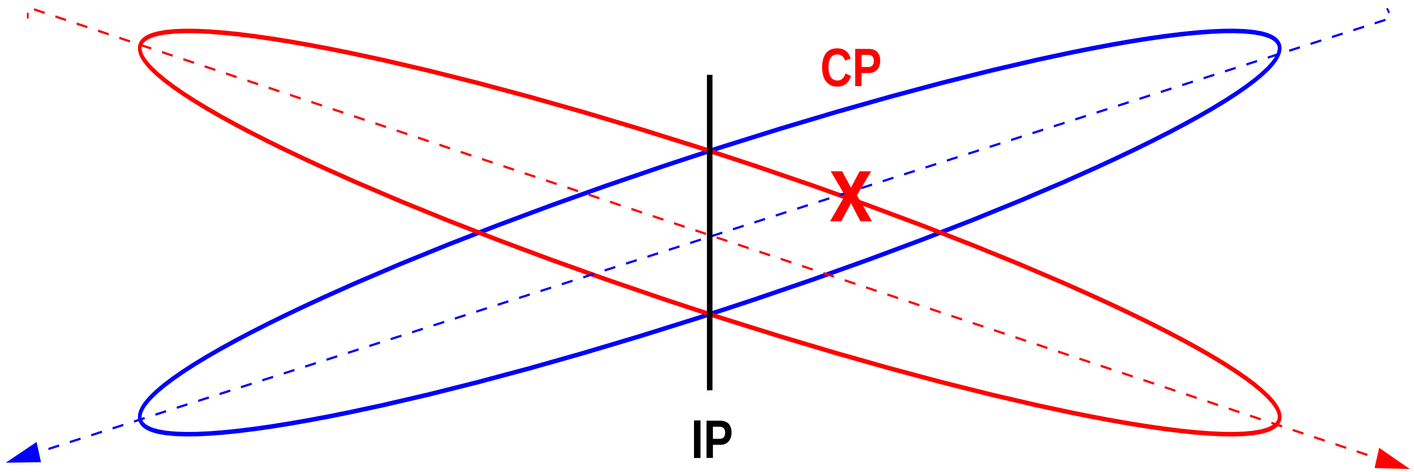
→ **Most important: effect of crossing angle**

But there is more:



For large amplitude particles: collision point CP longitudinally displaced

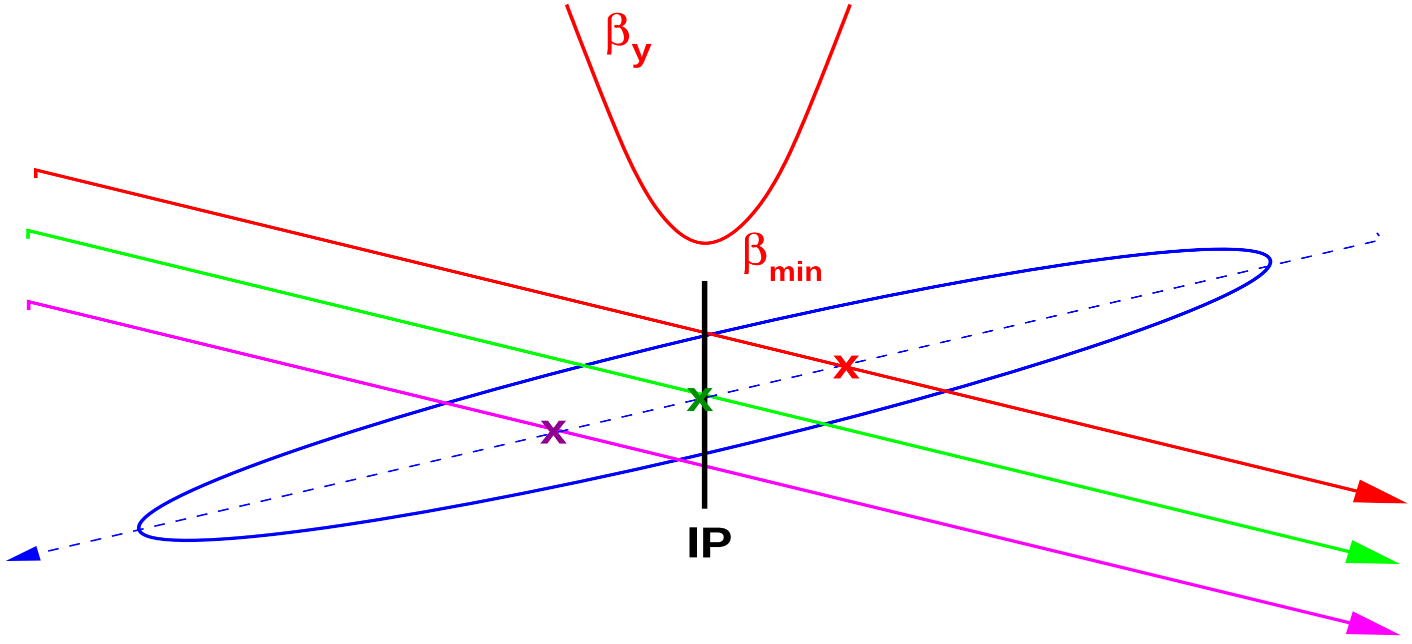
they do not meet the centre of the other beam at the smallest β^* (at IP)



- **For large amplitude particles: collision point CP longitudinally displaced**

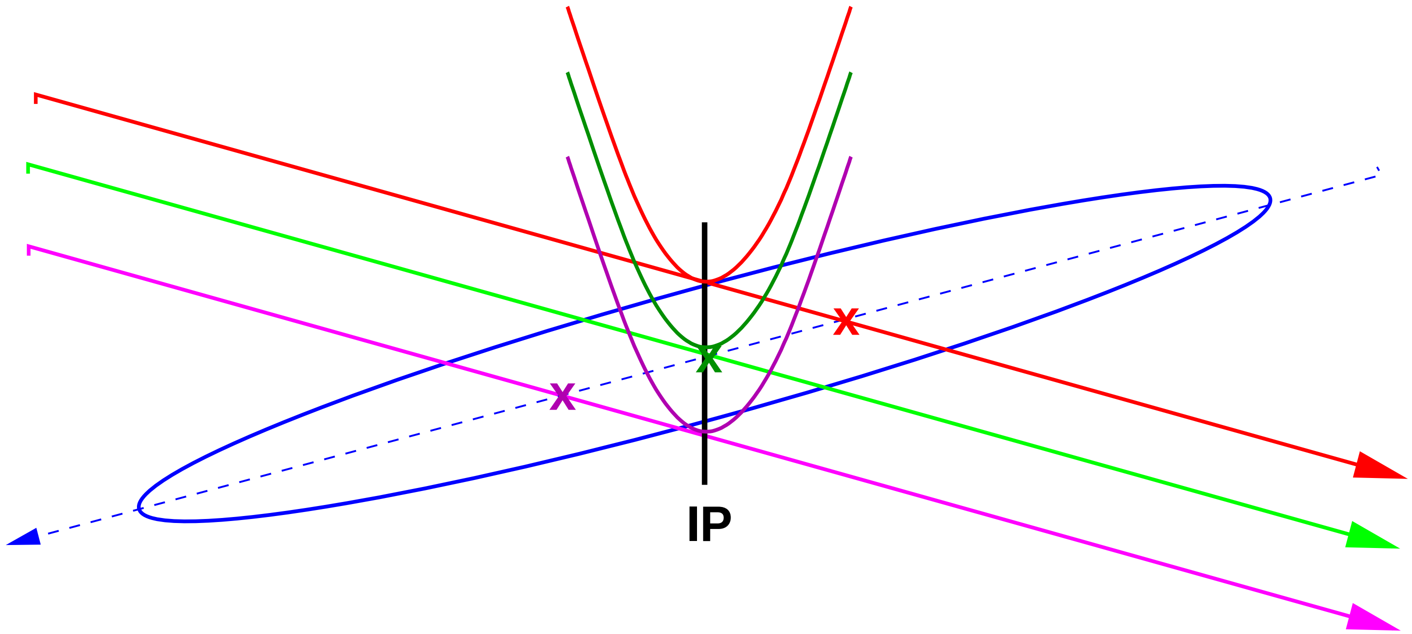
  they do not meet the centre of the other beam at the smallest β^* (at IP)

- **Can introduce coupling (transverse and synchro betatron, bad for flat beams)**



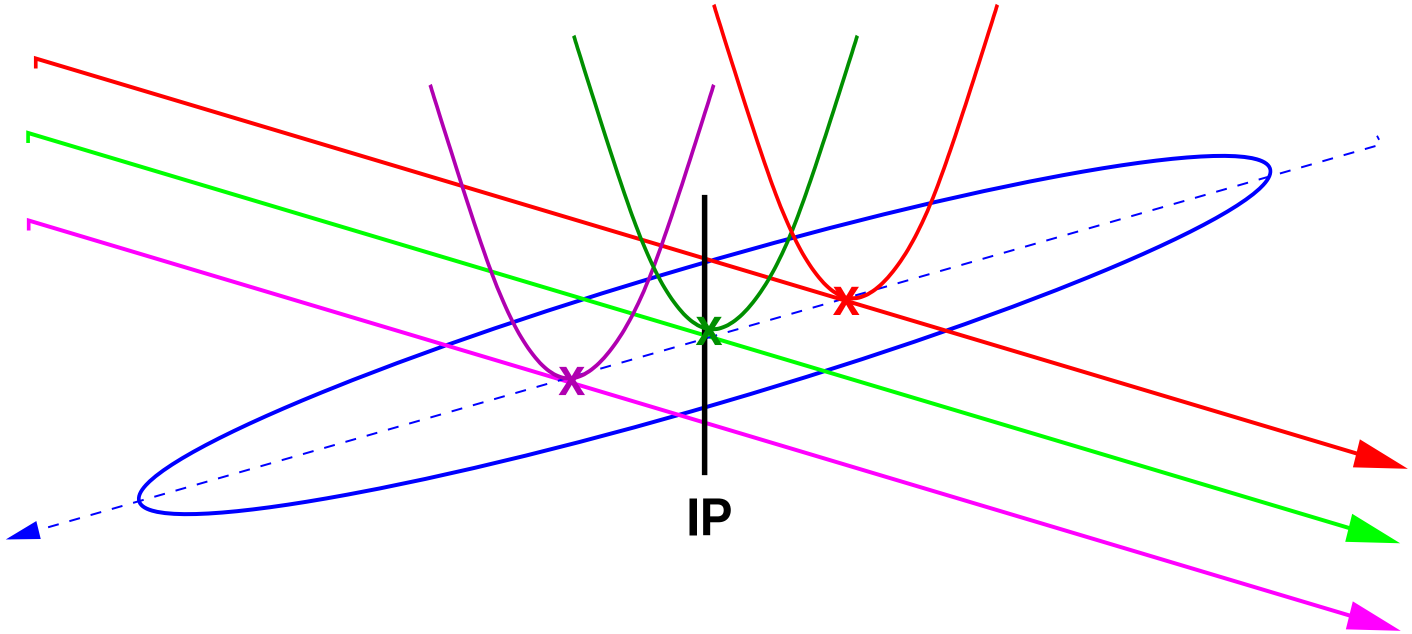
- A particle's collision point (centre of other beam) amplitude dependent
- Different (vertical) β functions at collision points (not to scale)

Only small (zero) amplitude particle collide at minimum β^*



- **A particle's collision point (centre of other beam) amplitude dependent**
- **Different β functions at collision points (hour glass like !)**

A fix: crossing angle plus "crab waist" scheme



- **Make vertical waist (β_y^{min}) also amplitude (x) dependent**

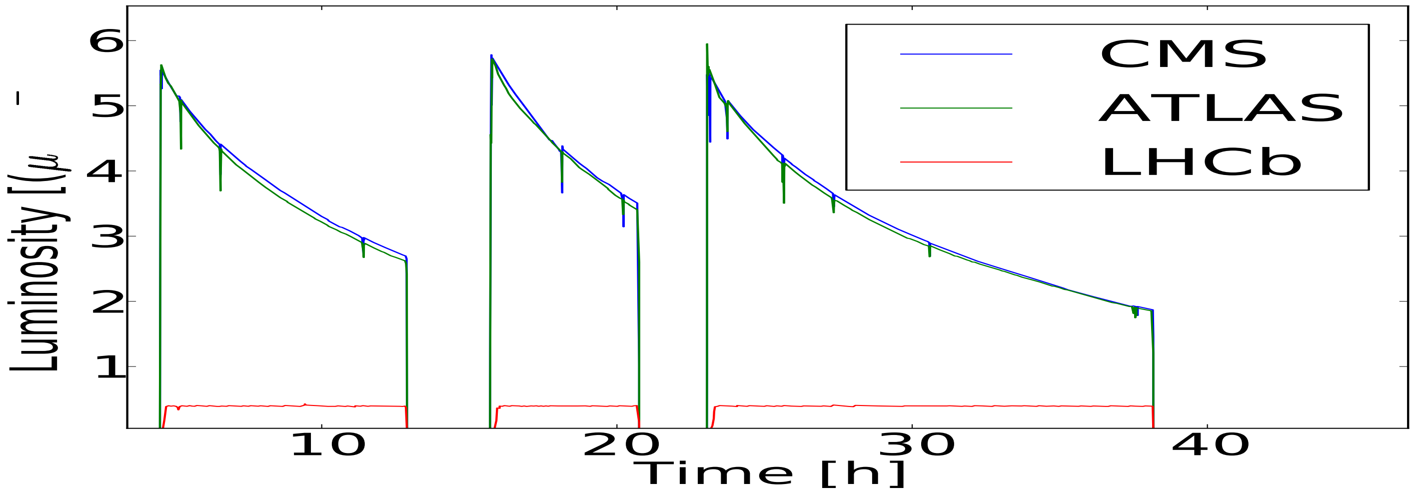
  "Different particles" have diferent waists

- **All particles in both beams collide in minimum β_y region**

"crab waist" (or "crabbed waist") scheme

- **Make vertical waist (minimum of β) amplitude (x) dependent**
- **Without details: can be done with two sextupoles**
- **First tried at DAPHNE (Frascati) in 2008**
- **Geometrical gain small, it is <u>not</u> the issue**
  - **Less betatron and synchrotron coupling**
  - **Good remedy for flat (i.e. lepton) beams with large crossing angle**

Luminosity in Operation



(Courtesy X. Buffat)

- Luminosity evolution as function of time in LHC during 2 typical days

- Run time up to 15 hour

- Preparation time 3 - 4 hours

What really counts: Integrated luminosity

$$L_{\rm int} = \int_0^T L(t)dt$$

$L_{\rm int} \cdot \sigma_p$ **= total number of events observed of process** p

Unit is: cm^{-2}**, i.e. inverse cross-section**

Often expressed in inverse barn

1 fb^{-1} (inverse femtobarn) is $10^{39} cm^{-2}$

for 1 fb^{-1}: requires 10^5 **s running at L =** $10^{34} cm^{-2} s^{-1}$

What does it mean ?

Assume:

- **You are interested in** $\sigma(pp \rightarrow X + H \rightarrow \gamma\gamma)$ **≈ 50 fb (femtobarn)**
- **You have: accumulated 20 fb^{-1} (inverse femtobarn)**
- **You have: 20 fb^{-1} · 50 fb = 1000**
- **You have: 1000 events of interest in your data !!**

But you have to <u>find</u> them !

A popular story: Clean and Dirty machines ...

pp → Cross section into hadrons : $\approx$ 100 mb $\approx$ $const.$

	E_{beam} (GeV)	L	events/s	events/d	events/year
LHC	7000	$1.0\ 10^{34}$	$1.0\ 10^{9}$	$1.4\ 10^{14}$	$4.5\ 10^{16}$
LHC	7000	$5.0\ 10^{34}$	$5.0\ 10^{9}$	$7.0\ 10^{14}$	$22.5\ 10^{16}$

e^+e^- → Cross section into hadrons : $\dfrac{22\ \text{nb GeV}^2}{E^2_{beam}}$

	E_{beam} (GeV)	L	events/s	events/d	events/year
LEP	55	$1.0\ 10^{34}$	0.07	6000	$2\ 10^{6}$
LEP	100	$1.0\ 10^{34}$	0.02	2000	$7\ 10^{5}$
...	1000	$1.0\ 10^{34}$	0.0002	20	$7\ 10^{3}$

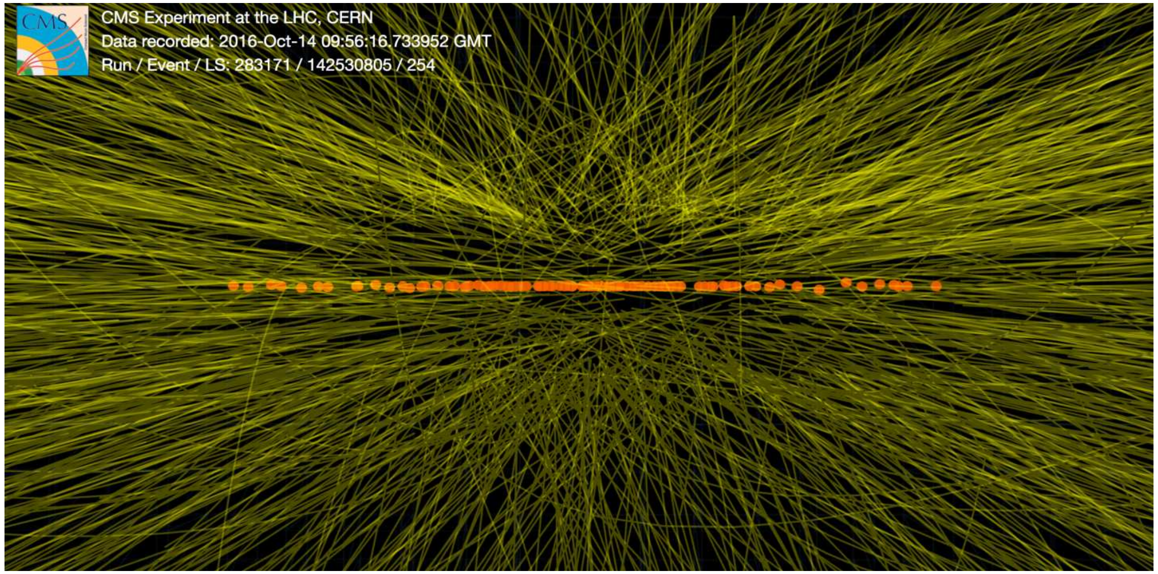
Simultaneous interactions per crossing - pile-up

➡ Only an issue for hadron (pp) colliders (see previous slide)

LHC at nominal luminosity: Per bunch crossing more than 20 interactions pile-up (much more in the future)

Bunch crossing every 25 ns   (can you think of another problem ?)

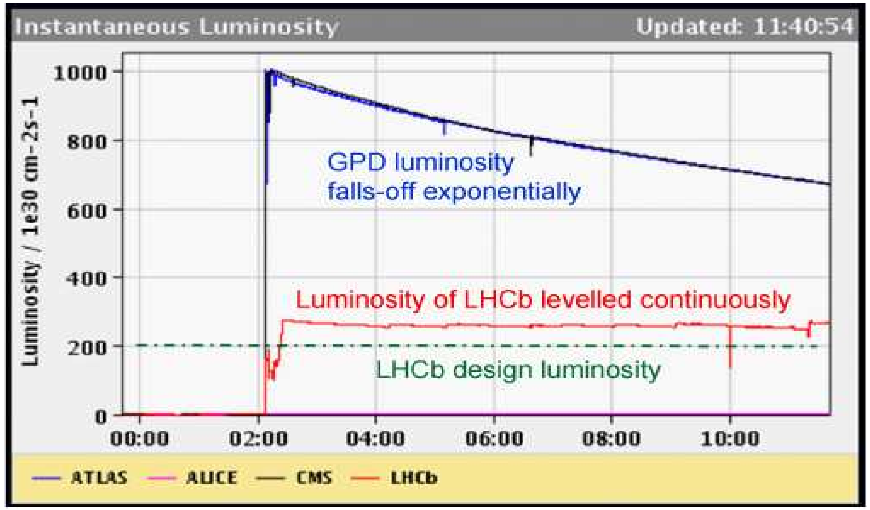
Very difficult to handle by the detectors:



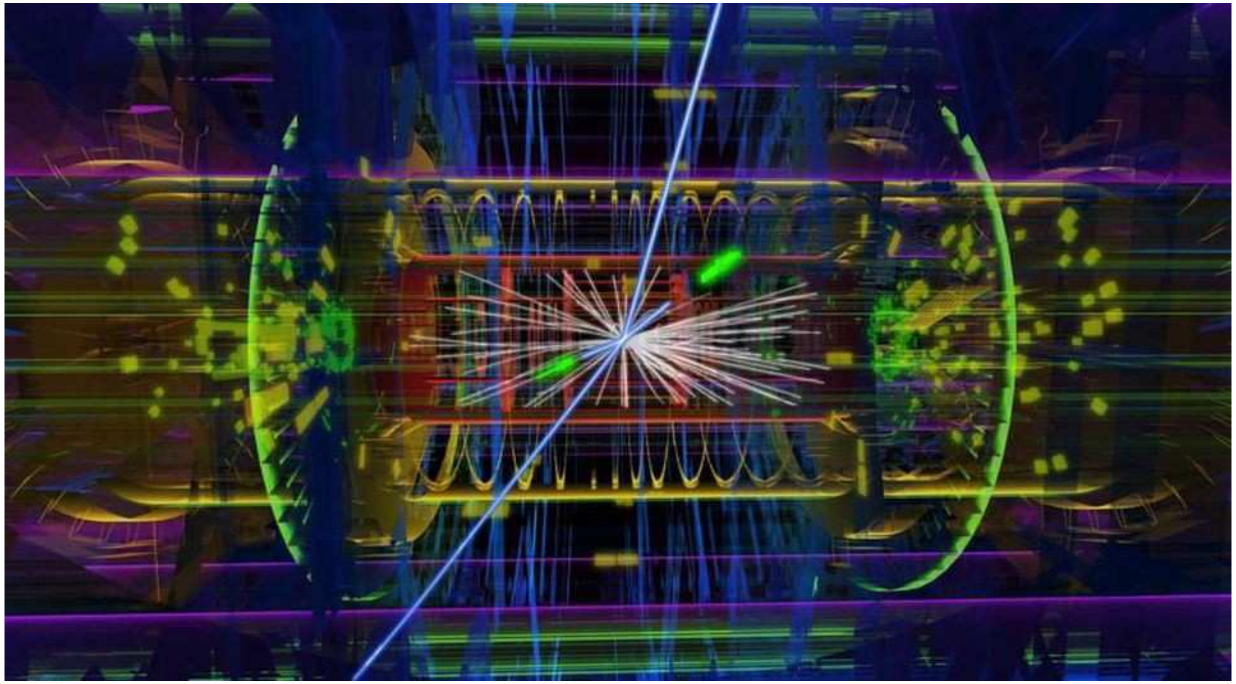
Ideal: Operate all the time at maximum digestible luminosity

A possible fix - Luminosity Levelling:

- at the start adjust the luminosity to ideal level
- keep it constant during all data taking, options are:
- decreasing β^* during a run to maintain this level (was done at SPS collider)
- separate the beams and adjust to get the desired luminosity (remember the W): already done in LHCb

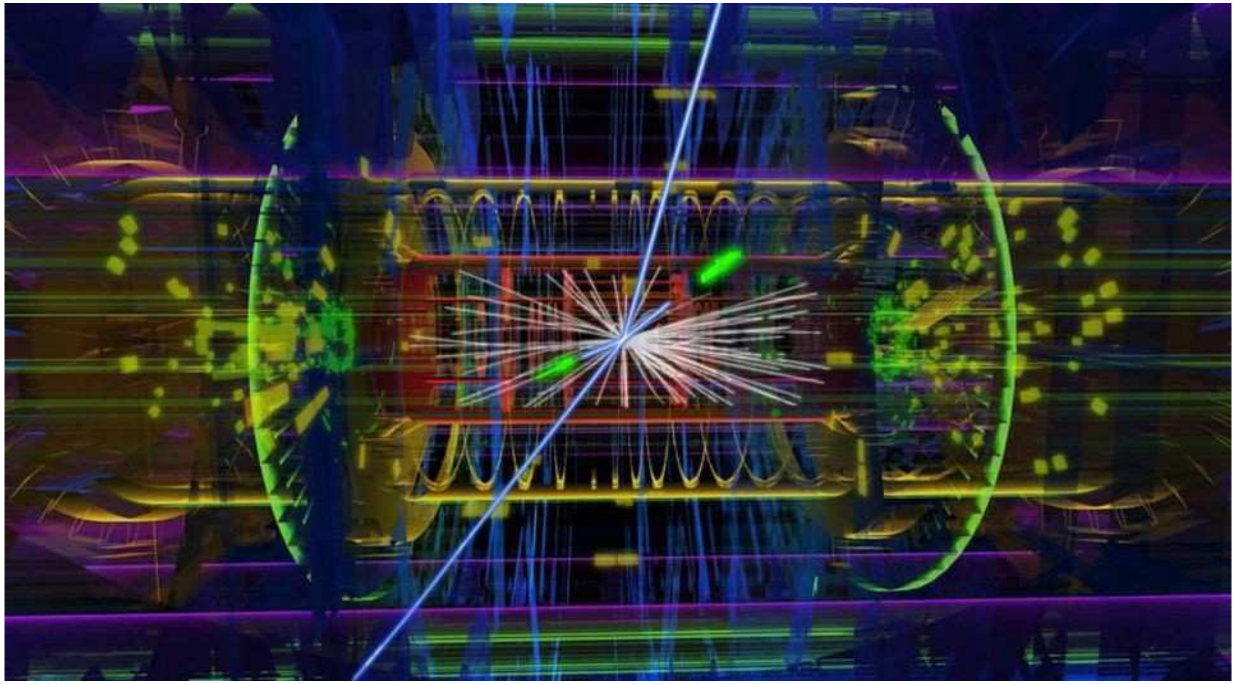


The relevance of integrated luminosity:



A very popular picture (shown many times at CAS and elsewhere)
Find the Higgs !

The relevance of integrated luminosity:

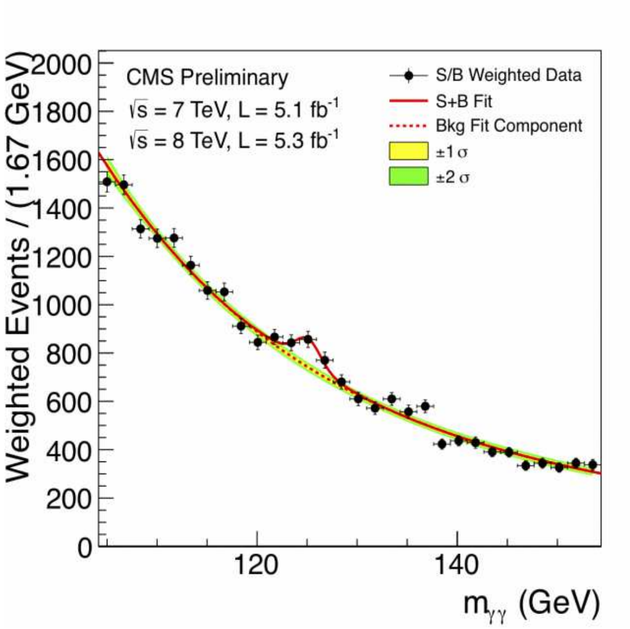


A very popular picture (shown many times at CAS and elsewhere)
Find the Higgs !

➡ **Although sometimes claimed to be obvious:**

Nobody knows whether it is a Higgs and where !!!

Why (and this is one reason why we want high integrated luminosity)?



A more relevant picture

Number of events at $m_{\gamma\gamma}$

all have the same characterics

only excess is evidence for Higgs

→ some Higgs in the slot

The "excess" increases proportionally to integrated luminosity - the background does not !

Higher integrated luminosity increases signal over background ratio !
(when particlists take about $3\sigma,\ 4\sigma,\ 5\sigma, \ldots$ signals)

Luminosity measurement

- **One needs to get a signal proportional to interaction rate**
  → Beam diagnostics

  Dynamic range can be very large:
  10^{27} cm^{-2}s^{-1} to 10^{34} cm^{-2}s^{-1}

  Should be very fast, if possible for individual bunches

  Should also be used for optimization

  But for absolute luminosity needs calibration

Luminosity calibration

Remember the basic definition:

$$\frac{dR}{dt} = L \; \times \; \sigma_p$$

- **For a well known and calculable process we know** σ_p

- **The experiments measure the counting rate** $\frac{dR}{dt}$ **for this process**

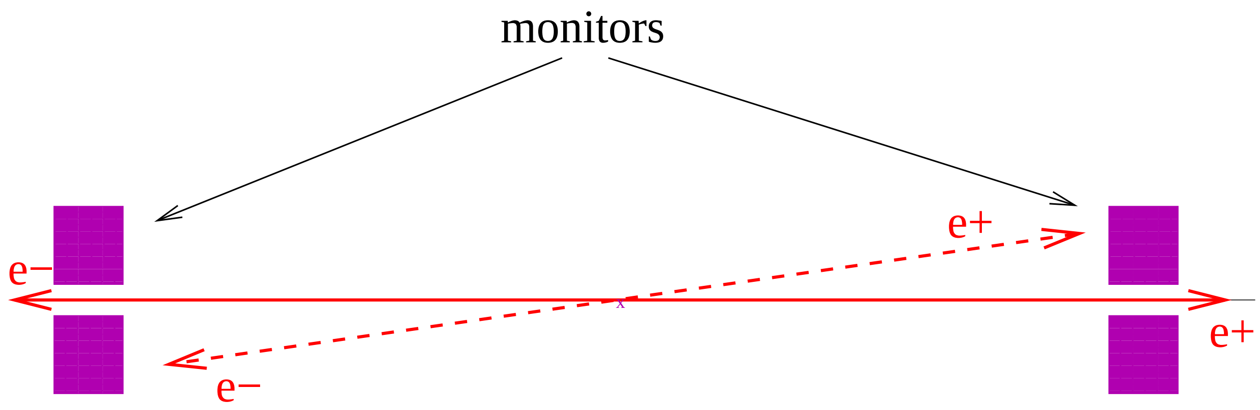
➡ **Get the absolute, calibrated luminosity**

But: hadron and lepton colliders are very different !

Luminosity calibration - e^+e^-

Use exactly calculable (QED) process:

$e^+e^- \to e^+e^-$ **elastic scattering (Bhabha scattering)**



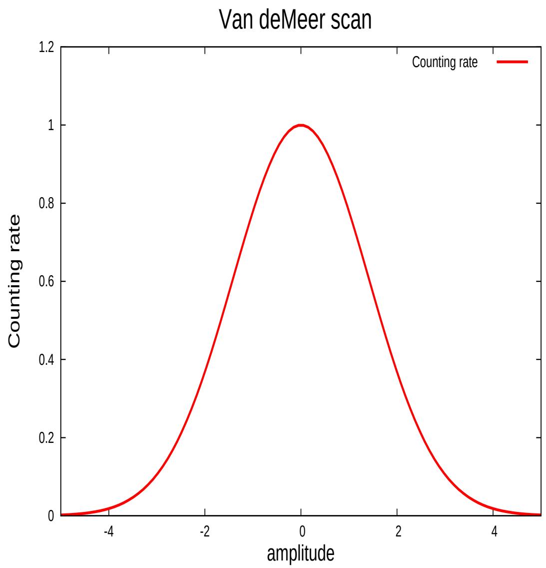
Measure coincidence at small angles ($\sigma_{el} \propto \Theta^{-3}$)

Low counting rates at high energy ($\sigma_{el} \propto \frac{1}{E^2}$)

Background may be problematic

Luminosity calibration
(hadrons, e.g. pp or $p\bar{p}$)

- **Must measure beam current and beam sizes**
- **Beam size measurement:**
  - **Wire scanner or synchrotron light monitors**
  - **Measurement with beam ... $\rightarrow$ remember luminosity with offset**
  - **Move the two beams against each other in transverse planes (remember the W) (van der Meer scan, ISR 1973 - LHC 2012)**



Record counting rates $R(d)$

as function of movement d

Since R(d) is proportional to $L(d)$

get ratio $\frac{L}{L_0}$

From ratio of luminosity L(d)/L_0 = $W = e^{-\frac{1}{4\sigma^2}(d_2-d_1)^2}$

one obtains σ

A problem for very high bunch intensities:
size of bunches can change during the scan (caused by beam-beam effects)

For LHC acceptable, for LEP it changed by a factor 2 !

Absolute value of L (pp or $p\bar{p}$) by Coulomb normalization

Look at elastic scattering $pp \longrightarrow pp$ which has 2 contributions

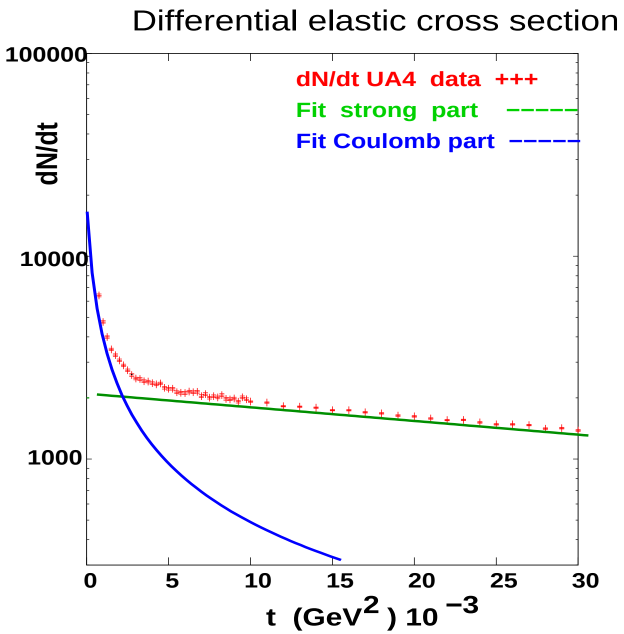
The Coulomb contribution f_C exactly calculable, however the nuclear part f_N is not, try to separate them:

- $$\lim_{t\to 0} \frac{d\sigma_{el}}{dt} = \frac{1}{L}\frac{dN_{el}}{dt}|_{t=0} = \pi \mid f_C + f_N \mid^2$$

$$\simeq \pi \mid \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi}(\rho + i)e^{-\frac{b\,t}{2}} \mid^2 \simeq \underbrace{\frac{4\pi\alpha_{em}^2}{t^2}\,|_{|t|\to 0}}_{calculable}$$

- **Coulomb contribution strongly dominates at small scattering angles**
- **Measure $\frac{d\sigma_{el}}{dt}$ at very small angles and you get: L**

(t measures the momentum transfer (related to the scattering angle) for elastic scattering)



Measure dN/dt at small t :

(t $<$ 0.001 (GeV/c)^2)

and extrapolate to t $=$ 0.0

Needs special optics to go

to small t : very large β^*

To measure at small t (e.g. close to beam):
beam divergence σ' must be very small, i.e. particle trajectories almost parallel

➡ **since $\sigma' = \sqrt{\epsilon/\beta^*}$ one should have a very large β^* (≥ 2000 m)**

Rule of thumb: σ' more than 5 times smaller than typical scattering angle

Can hope for a precision of 1 - 2 %

First glance at beam-beam effects - almost verbatim

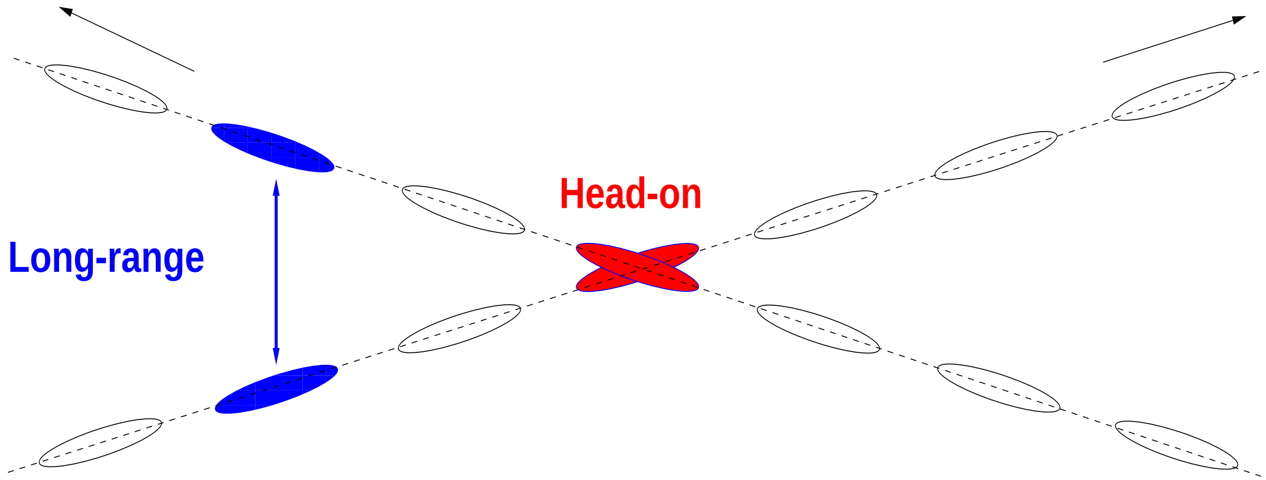
Remember: $$L = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y} \cdot W \cdot S \cdot H \quad = \quad \frac{N_1 N_2 f n_B}{4\pi \cdot \ \sigma_x\sigma_y} \cdot W \cdot S \cdot H$$

High luminosity is not good for beam-beam effects ...
Beam-beam effects are not good for high luminosity ...

It will cause (amongst many others):

- **VERY large tune spread ($\approx$ 4 times for uncorrected chromaticity)!**
- **Not only tune spread but also excites nonlinear betatron and synchrobetatron resonances**
- **Emittance growth and bad life time**
- **Sudden, total beam loss, Multi bunch coherent modes**
- **Orbit, Tune and Chromaticity changes, also different from bunch to bunch (further increase of total tune/orbit/chromaticity spread)**
- **...**

LHC beam-beam interactions



- **Two types: head on and long range interactions**

Beams separated, but still same vacuum chamber

Particles experience distant (weak) forces

Separation typically 6 - 12 σ (weak, but many: 120)

Head on first: Force for round Gaussian beams

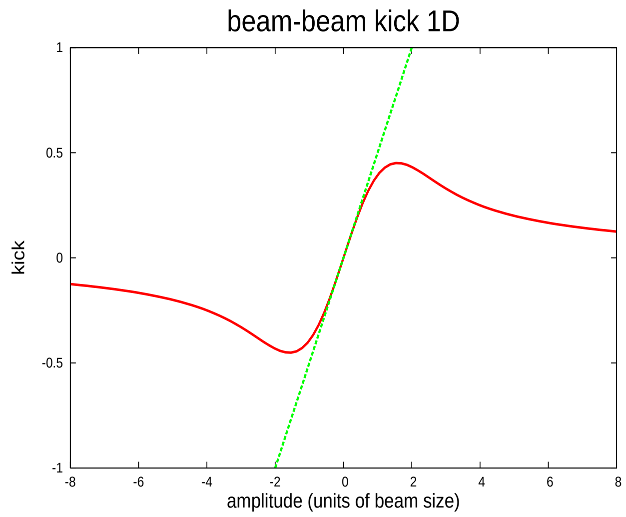
Simplification 1: $\sigma_x = \sigma_y = \sigma$**,** $Z_1 = -Z_2$ **= 1**

Simplification 2: **very relativistic** $\rightarrow$ $\beta \approx$ **1**

- **Force has only radial component, i.e. for round beams depends only on distance r from bunch centre where:** $r^2 = x^2 + y^2$

$$\boxed{F_r(r) = -\frac{Ne^2(1 + \beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]}$$

Form of the kick (as function of amplitude)

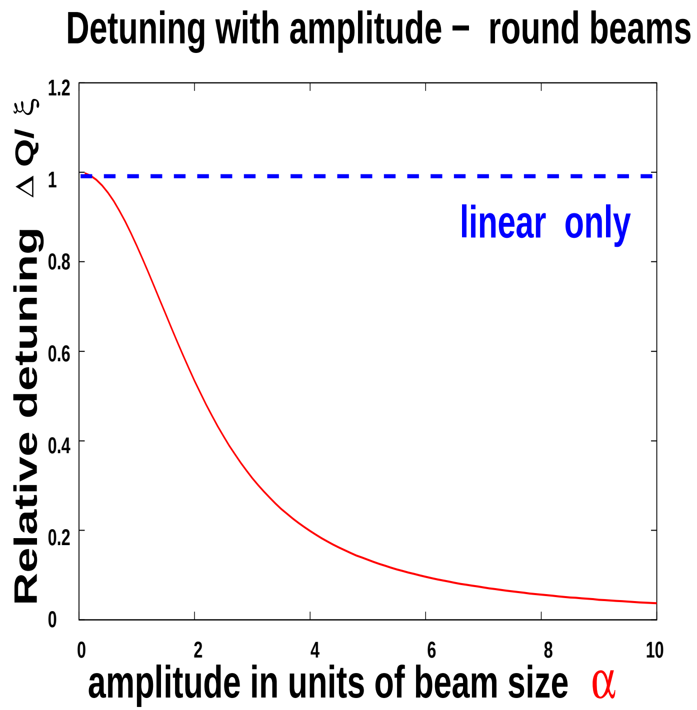


- **For small amplitudes: linear force (like quadrupole), the same in both planes ! Slope is $\frac{N}{\epsilon n}$ independent of beta* and energy**
- **Focusing (or defocusing) in both planes !! But:**

  For large amplitudes: very non-linear force

Non-linear force: Amplitude detuning

- $\Delta\ Q$ depends on amplitude
  - Different particles have different tunes
  - Largest effect for small amplitudes



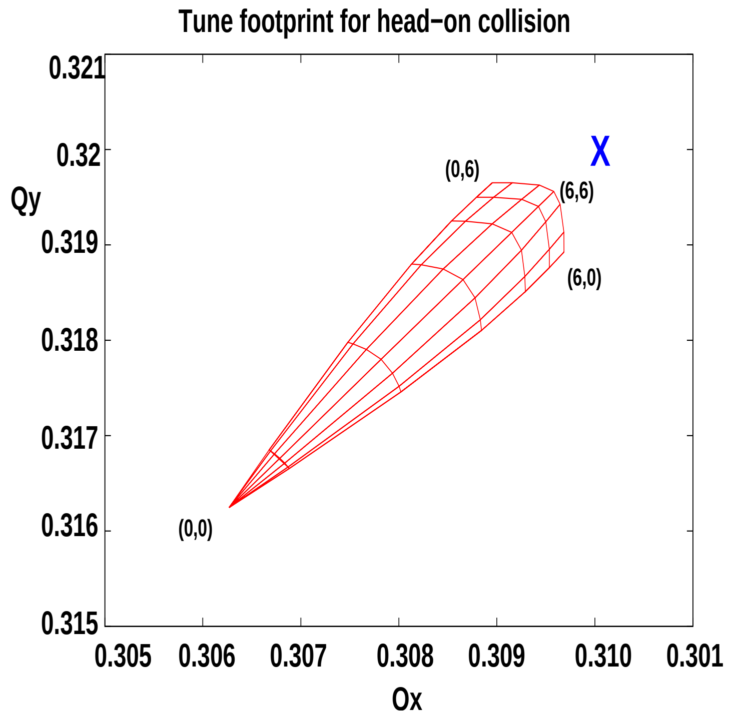
→ with $\xi = \frac{N}{\epsilon_n}$ we get: $\Delta Q = \xi \frac{4}{\alpha^2}\left[1 - I_0(\frac{\alpha^2}{4}) \cdot e^{\frac{-\alpha^2}{4}}\right]$

Non-Linear tune shift - two dimensions

- Start with standard working point
- Tunes depend on x and y amplitudes
- No single tune in the beam:

  Tunes are "spread out"

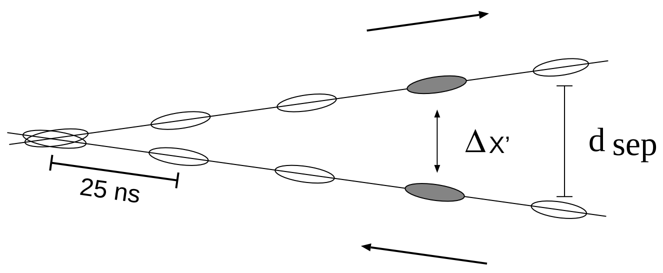
  Point becomes a footprint



Tune (of beam centre) shifted to "injection working point"

The spread is ≈ 0.004 (one IP) !   Are we worried ??

Quantitatively: Long range kick



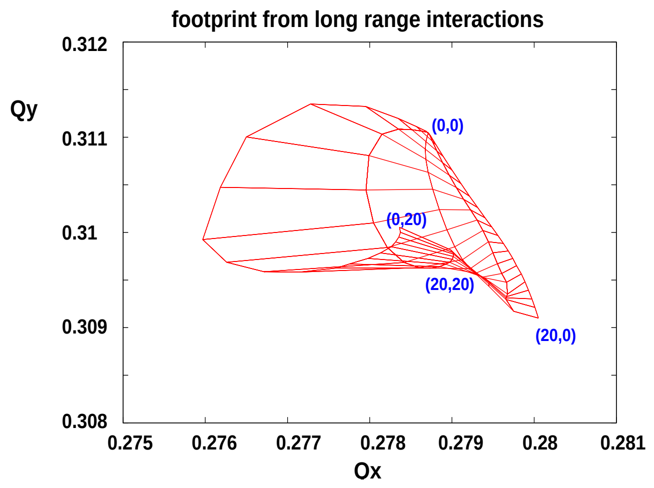
➡ **Modified "kick" with horizontal separation d:**

$$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\cdot\frac{(x+d)}{r^2}\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]$$

(with: $r^2 = (x+d)^2 + y^2$**)**

Red flag: to use this expression, e.g. in a simulation, there is a small complication, was used incorrectly in the past (before 1990 and in Chao Handbook), if interested ask offline

- Tune shift large for largest amplitudes (where non-linearities are strong)
- Size proportional to $\frac{1}{d^2}$
- We should expect problems at small separation
- Footprint is very asymmetric



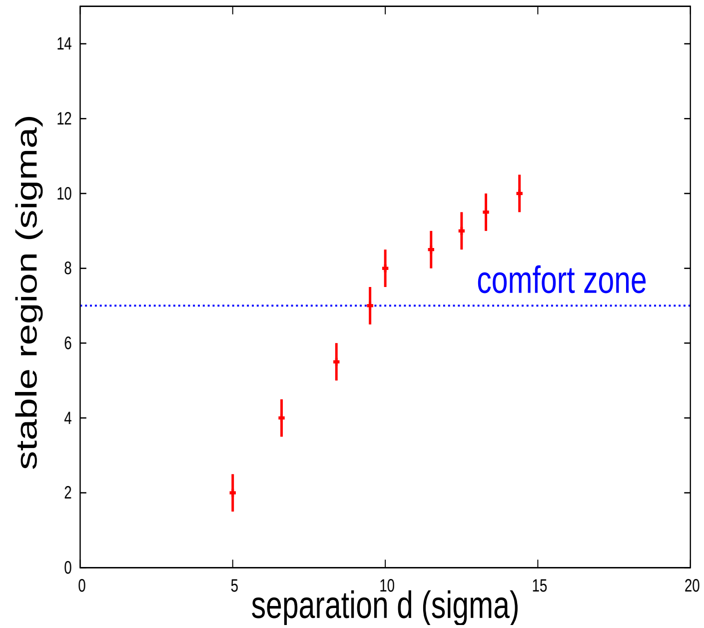
One observes a "folding" (can easily be understood from the picture)

For small separation, the size of the footprint can be large ➡ particle losses

Small crossing angle $\Longleftrightarrow$ small separation $\Longleftrightarrow$ big problem ?

Stable region (a.k.a Dynamic Aperture) versus separation in units of beam size σ

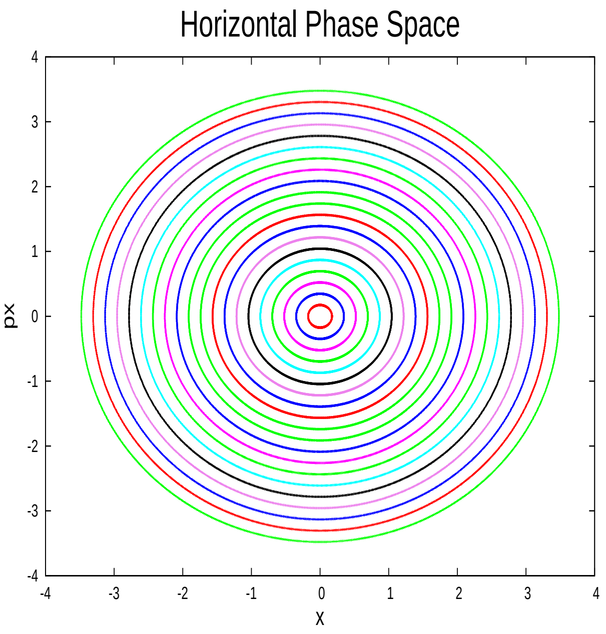
(from simulations)

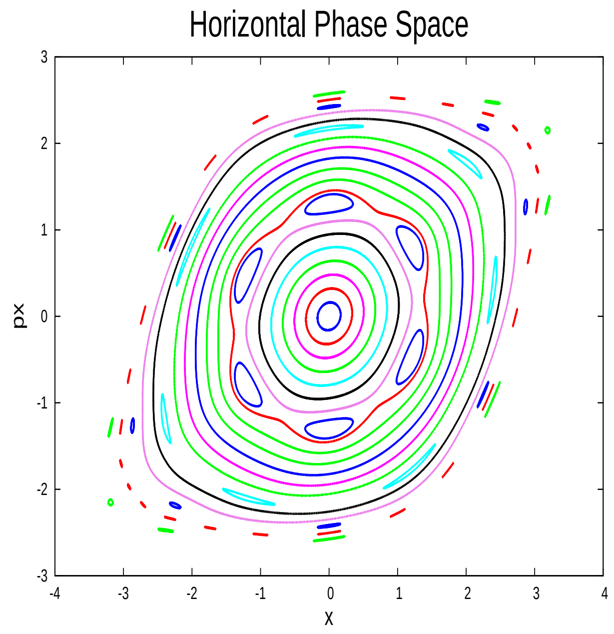


Minimum separation for LHC:
$\approx$ 10 σ (design value)

For too small separation: particles may be lost and/or bad liftime

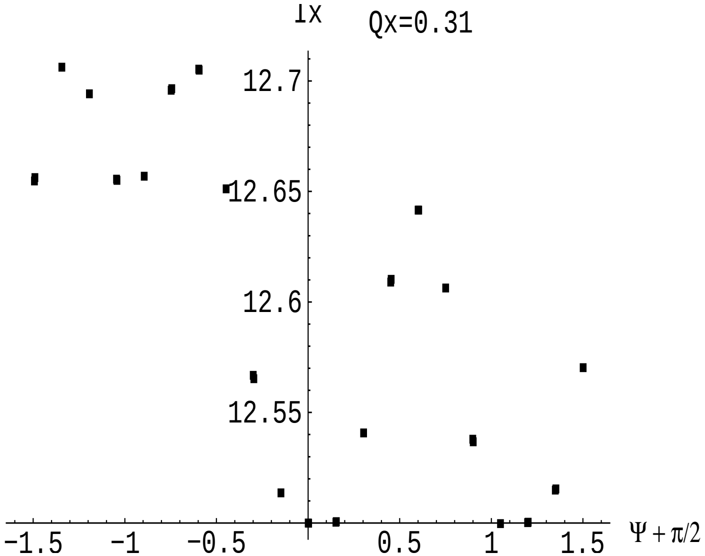
Long range interactions are the bad guys !

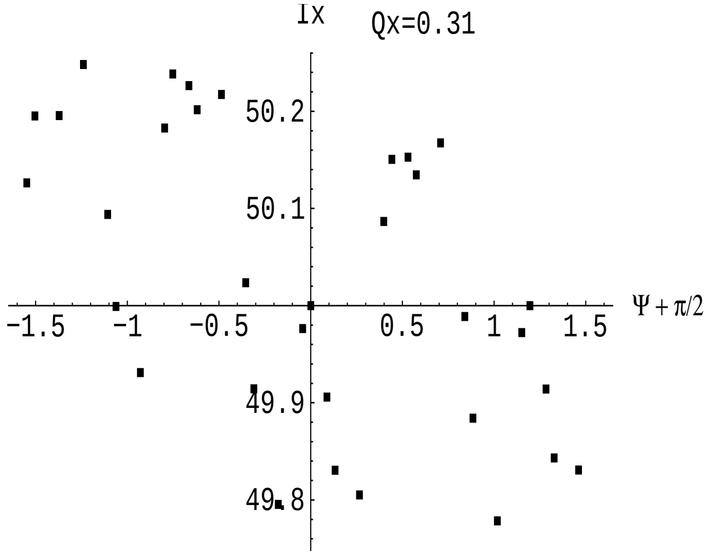




- **Here one head on beam-beam interaction, many resonances (6th, 8th, 10th, 13th, 26th, ..) seen ...(note: no losses !!)**
- **Can we reproduce (analytically) this features ??**
- **Are Hamiltonians good for something ?**
- **Try a comparison with tracking:**

Invariant from tracking: Poincaré section of one IP

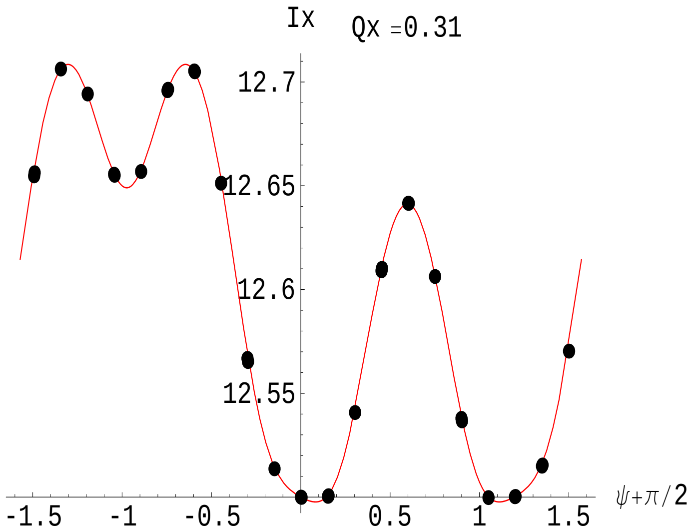


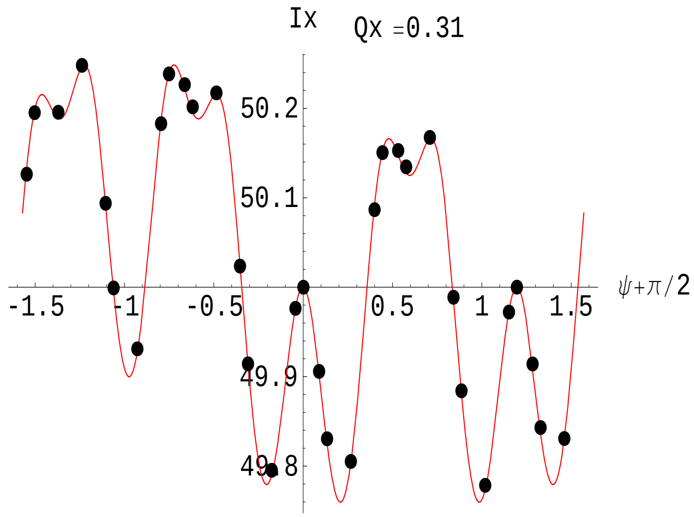


- **Phase space coordinates (action-angle) plotted each turn**
- **Shown for particle amplitudes of 5σ_x and 10σ_x**

  Without beam-beam: a straight line
- **Try to use Hamiltonian treatment:**

Invariant versus tracking: one IP





One can reproduce and analyse the motion ...

Used for optimization

➡ **Buzzword: effective Hamiltonians (maybe 2019) ...**

Summary I

- Colliders are used exclusively for particle physics experiments
- Colliders are the only tools to get highest centre of mass energies
- Type of collider is decided by the type of particles and its purpose
- Design and performance must take into account the needs of the experiments

- Most likely beam dynamics problem: beam-beam effects ....

Summary Ib

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- Most likely beam dynamics problem: beam-beam effects ....
- But if you have to fight elephants: Hamiltonians are you gun
- Maybe something on that: Danmark 2019

Bibliography

Luminosity lectures and basics:

W. Herr and B. Muratori, ***Concept of Luminosity***, CERN Accelerator School, Zeuthen 2003, in: CERN 2006-002 (2006).

A. Chao and M. Tigner, ***Handbook of Accelerator Physics and Engineering***, World Scientific, (1998).

T. Pieloni, ***Beam-beam effects***, CAS, Advanced Accelerator Physics, 2017, Egham, UK.

W. Herr, ***Nonlinear Dynamics: Methods and Tools***, CAS, Advanced Accelerator Physics, 2017, Egham, UK.

Linear colliders

- Mainly (only) e^+e^- colliders
- Past collider: SLC (SLAC)
- Under consideration: CLIC, ILC
- Special issues:
  - Interaction cross section low for e^+e^- collisions requires very high luminosity
  - Particles collide only once (dynamics) !

→ Must be taken into account

Luminosity in linear colliders

Single pass: replace frequency f by repetition rate f_{rep}.

$$L = \frac{N^2\ f\ n_b}{4\pi\sigma_x\sigma_y} \quad \Rightarrow \quad L = \frac{N^2\ f_{rep}\ n_b}{4\pi\sigma_x\sigma_y}$$

Effective beam sizes $\overline{\sigma}_x, \overline{\sigma}_y$

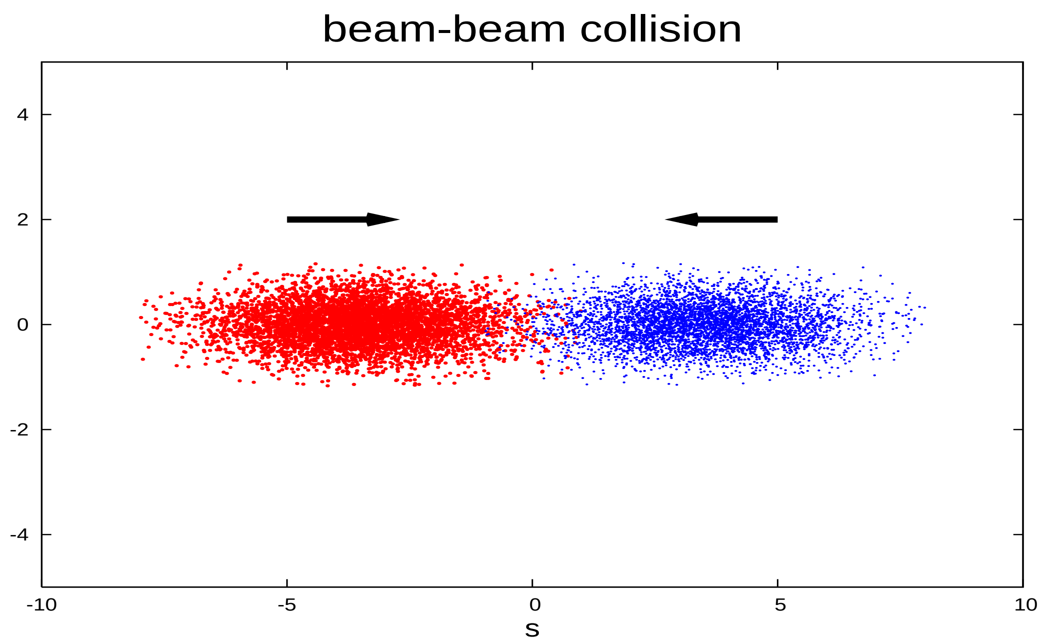
Effective beam sizes $\overline{\sigma}_x, \overline{\sigma}_y$

$$\Rightarrow \quad L = \frac{N^2\ f_{rep}\ n_b}{4\pi\overline{\sigma}_x\ \overline{\sigma}_y}$$

Enhancement factor H_D due to "pinch effect"

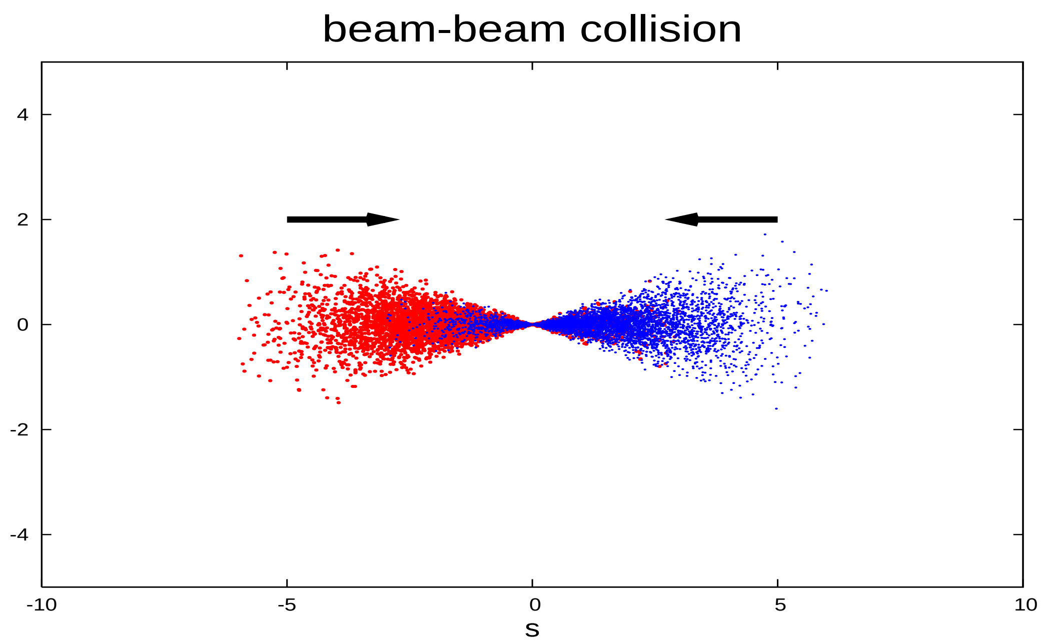
$$\Rightarrow \quad L = \frac{H_D \cdot N^2\ f_{rep}\ n_b}{4\pi\overline{\sigma}_x\ \overline{\sigma}_y}$$

Pinch effect - disruption



- **Additional focusing by opposing beams**

Pinch effect - disruption



- **Additional focusing by opposing beams**

It is usually described by the "Disruption Parameter":

$$D_{x,y} = \frac{2r_e N \sigma_z}{\gamma \sigma_{x,y}(\sigma_x + \sigma_y)}$$

Meaning: ratio of the r.m.s. bunch length to the focal length of the interaction

For weak disruption $D \ll 1$ and round beams:

$$H_D = 1 \; + \; \frac{2}{3\sqrt{\pi}} D \; + \; O(D^2)$$

For strong disruption and flat beams: computer simulation necessary, (maybe can get some scaling)

Some numbers: **electric field** $\vec{E} \; \geq \; 10^{12} \; \frac{V}{m}$ $\Rightarrow$ $\vec{B} \; \geq \; 3 \; kT$

Beamstrahlung

- Disruption at interaction point is basically a strong "bending"
- Results in strong synchrotron radiation: beamstrahlung
- This causes (unwanted):
  - Spread of centre-of-mass energy
  - Pair creation and detector background
- Again: luminosity is not the only important parameter

Not treated :

- Coasting beams (e.g. ISR)
- Asymmetric colliders (e.g. PEP, HERA, LHeC)

→ All concepts can be formally extended ...

Luminosity in a nutshell

$$\boxed{L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

Are there limits to what we can do ?

Yes, there are beam-beam effects

In LHC: $\approx 10^{11}$ collisions with the other beam per fill !!

$$\boxed{L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H}$$

Summary

- **Colliders are used exclusively for particle physics experiments**
- **Colliders are the only tools to get highest centre of mass energies**
- **Type of collider is decided by the type of particles and its purpose**
- **Design and performance must take into account the needs of the experiments**

- **Not the highest, but highest <u>useful</u> Luminosity**

- **Most likely a mean saboteur: beam-beam effects**

Bibliography

Luminosity lectures and basics:

W. Herr and B. Muratori, ***Concept of Luminosity***, CERN Accelerator School, Zeuthen 2003, in: CERN 2006-002 (2006).

A. Chao and M. Tigner, ***Handbook of Accelerator Physics and Engineering***, World Scientific, (1998).

- BACKUP SLIDES -

If the beams are not Gaussian ??

- **Assume flat distributions (normalized to 1)**

  $\rho_1 = \rho_2 = \frac{1}{2a} = \rho,$ **for** $[-a \leq z \leq a]$, $z = x, y$

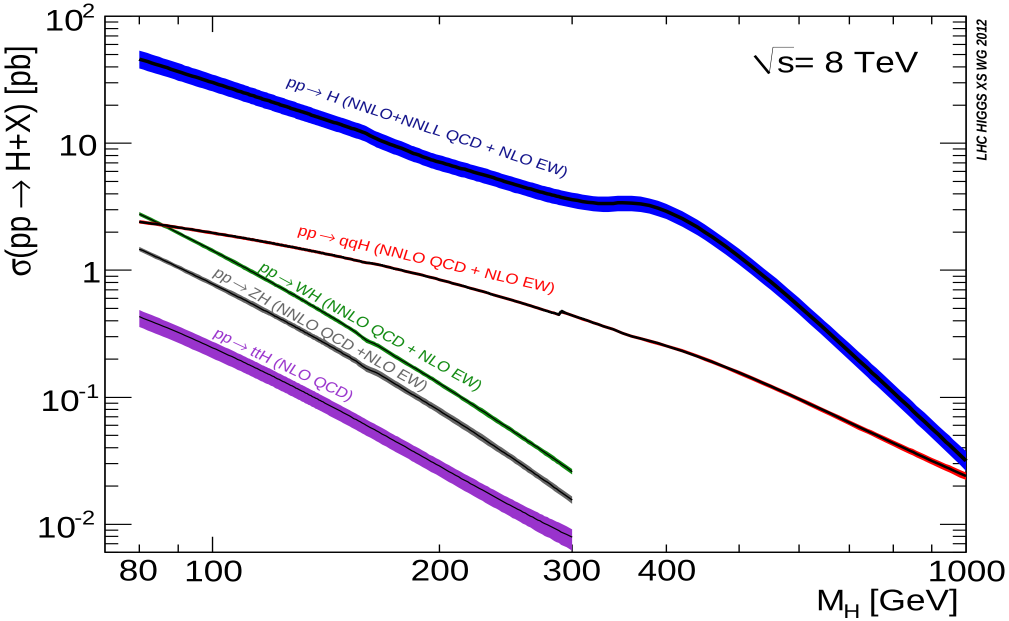
  Calculate r.m.s. in x and y:

$$< x^2 > = \int_{-\infty}^{+\infty} x^2 \cdot \rho(x,y) dxdy \qquad < y^2 > = \int_{-\infty}^{+\infty} y^2 \cdot \rho(x,y) dxdy$$

and $$L_p = \int_{-\infty}^{+\infty} \rho^2(x,y) \; dxdy$$

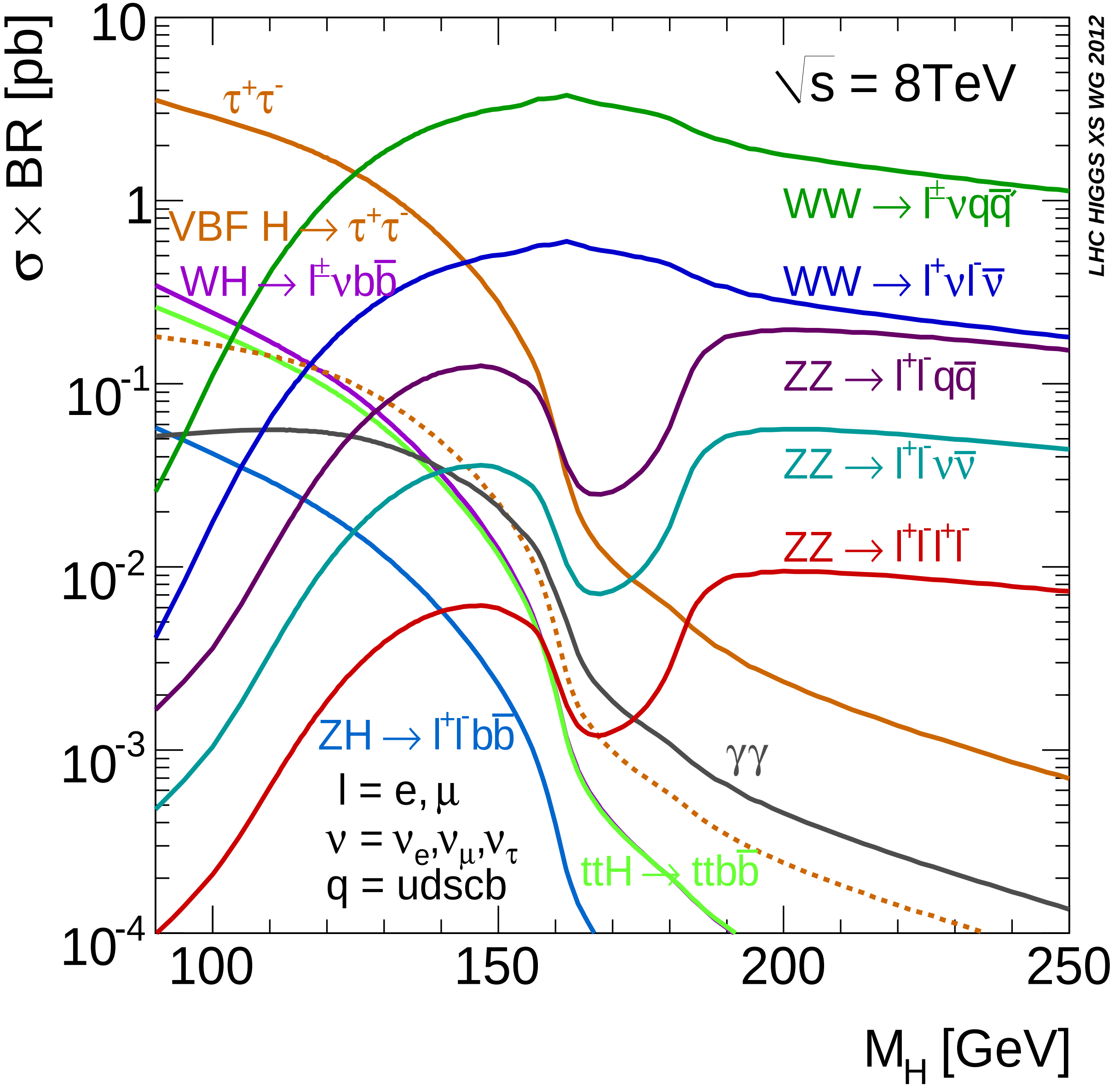
- **Compute:** $L_p \cdot \sqrt{< x^2 > \cdot < y^2 >}$
- **Repeat for various distributions and compare**

Rare interactions and high energy



- **Often seen: cross section σ for Higgs particle**
- **Typical channels**

Rare interactions and high energy



σ × BR [pb]
10
1
10^-1
10^-2
10^-3
10^-4
100
150
200
250
M_H [GeV]
√s = 8TeV
LHC HIGGS XS WG 2012
τ+τ-
VBF H → τ+τ-
WH → l±νbb̄
WW → l±νqq̄′
WW → l+νl-ν̄
ZZ → l+l-qq̄
ZZ → l+l-νν̄
ZZ → l+l-l+l-
ZH → l+l-bb̄
γγ
ttH → ttbb̄
l = e, μ
ν = νe,νμ,ντ
q = udscb

- **Often seen: cross section σ for Higgs particle**
- **Typical channels**

Maximising Integrated Luminosity

- **Assume exponential decay of luminosity** $L(t) = L_0 \cdot e^{t/\tau}$
- **Average (integrated) luminosity** $< L >$
  $< L >= \frac{\int_0^{t_r} dt L(t)}{t_r + t_p} = L_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$
- **(Theoretical) maximum for:** $t_r \approx \tau \cdot \ln(1 + \sqrt{2t_p/\tau} + t_p/\tau)$
- **Example LHC:** $t_p \approx$ **10h,** $\tau \approx$ **15h,** $\Rightarrow t_r \approx$ **15h**
- **Exercise: Would you improve** τ **(long** t_r**) or** t_p **?**