# Kickers, septa and beam transfer lines

- Beam transfer devices
  - Kickers
  - Septa
  - Protection devices
- Beam transfer lines
  - Distinctions between transfer lines and circular machines
  - Linking machines/experiments together
  - Emittance blow-up from mismatch
  - Measure beam parameters (measurement lines)

Matthew Fraser, CERN (TE-ABT-BTP) based on lectures by
M.J. Barnes, W. Bartmann, J. Borburgh, B. Goddard, V. Kain and M. Meddahi

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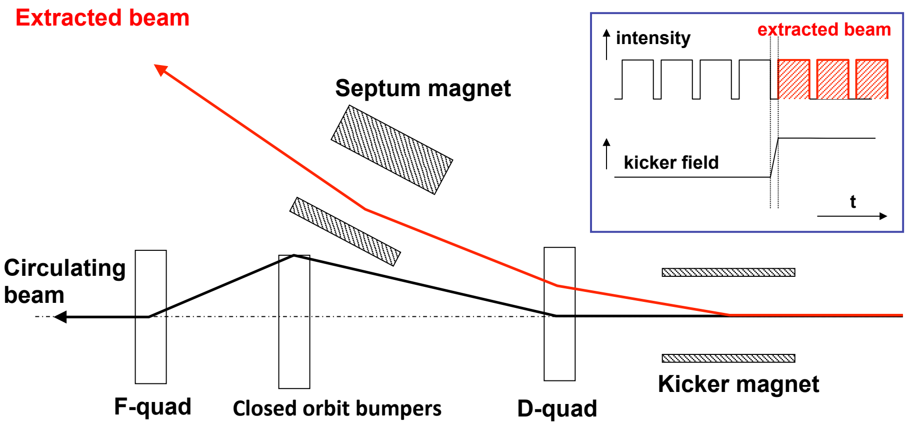
# Reminder: extraction, septum and kicker



- Kickers produce fast pulses, rising their field within the particle-free gap in the circulating beam (**temporal separation**)
- Septa compensate for the relatively low kicker strength, and approach closely the circulating beam (**spatial separation**)

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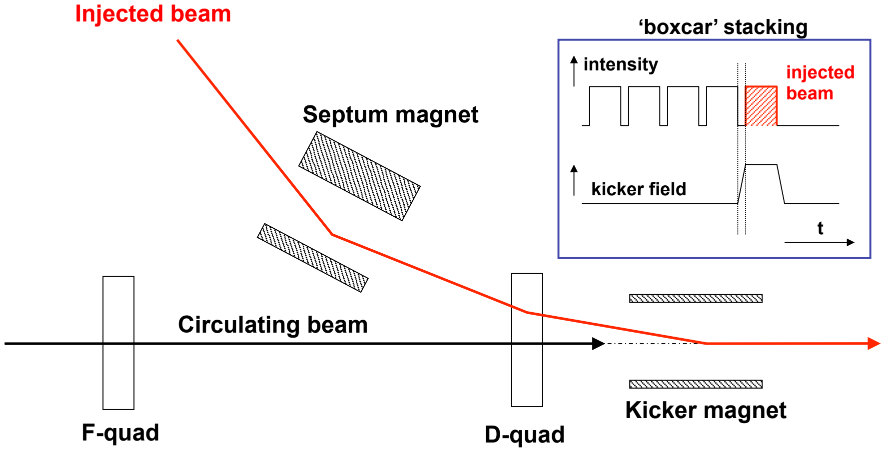
# Reminder: injection, septum and kicker



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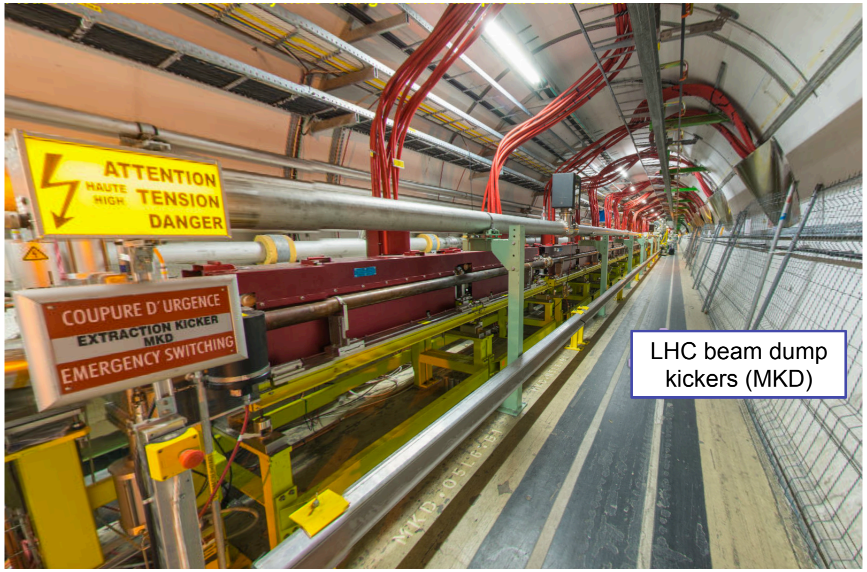
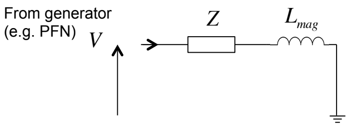
# Kickers

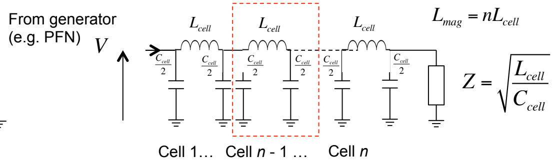


LHC beam dump kickers (MKD)

# Magnets – design options

- Type: “lumped inductance” or “distributed inductance” (**transmission line**)





- simple magnet design
- magnet must be nearby the generator to minimise inductance
- exponential field rise-time:

$$I = \frac{V}{Z}(1 - e^{-t/\tau}) \qquad \tau = \frac{L_{mag}}{Z}$$

- slow: rise-times ~ 1 µs

- complicated magnet design
- impedance matching important
- field rise-time depends on propagation time of pulse through magnet:

$$\tau = n\sqrt{L_{cell} \cdot C_{cell}} = n\frac{L_{cell}}{Z} = \frac{L_{mag}}{Z}$$

- **fast: rise-times << 1 µs**

- Other considerations:
  - **Machine vacuum**: kicker in-vacuum or external
  - **Aperture**: geometry of ferrite core
  - **Termination**: matched impedance or short-circuit

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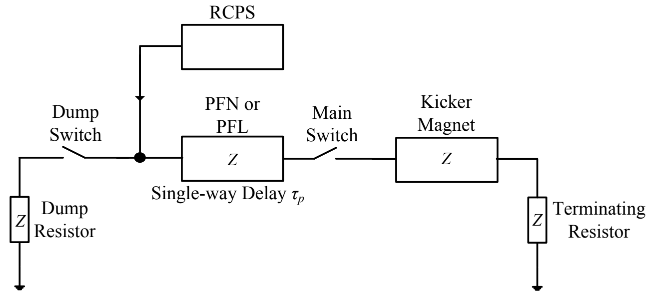
# Simplified kicker system schematic



- PFL/PFN charged to voltage $V_0$ by the RCPS
- Main switch is closed…

  …voltage pulse of $V_0/2$ flows through kicker
- Once the pulse reaches the (matched) terminating resistor full-field has been established in the kicker magnet
- Pulse length controlled between t = 0 and $2\tau_p$ with dump switch

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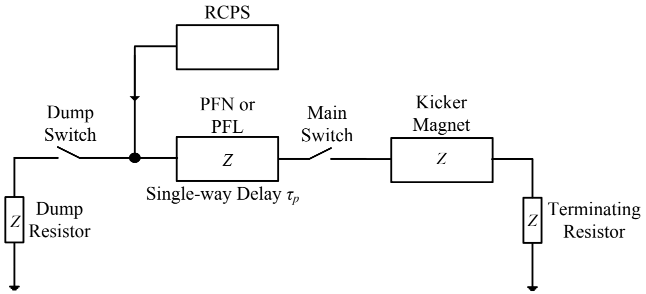
# Simplified kicker system schematic



- Main sub-systems (“components”) of kicker system;
  - **RCPS** = Resonant Charging Power Supply
  - **PFL** = Pulse Forming Line (coaxial cable) or **PFN** = Pulse Forming Network (lumped elements)
  - Fast high power **switch(es)**
  - **Transmission line(s)**: coaxial cable(s)
  - **Kicker Magnet**
  - **Terminators** (resistive)

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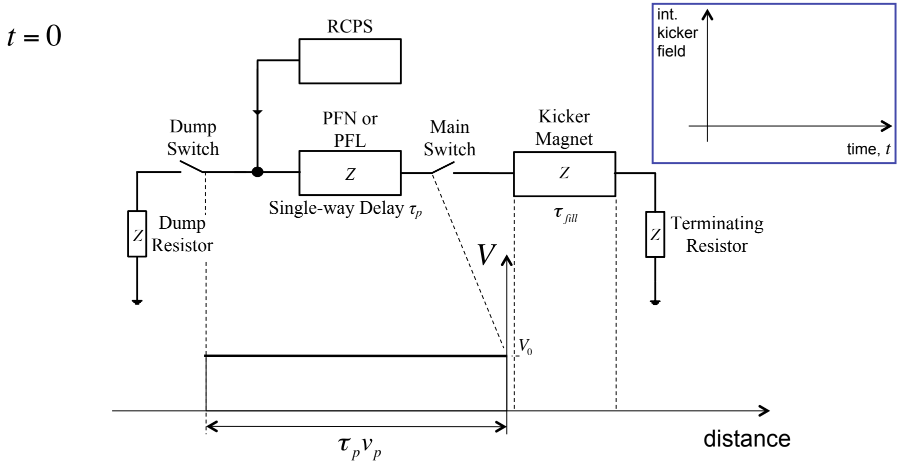
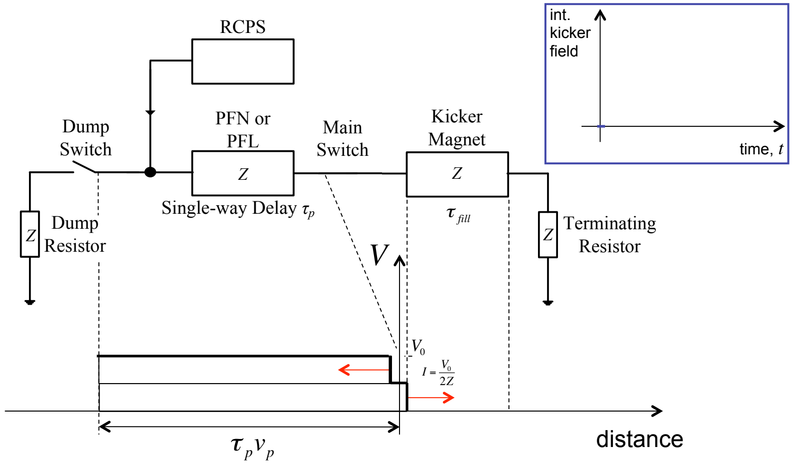
# Simplified kicker system schematic



- Pulse forming network or line (PFL/PFN) charged to voltage $V_0$ by the resonant charging power supply (RCPS)
  - RCPS is de-coupled from the system through a diode stack

# Simplified kicker system schematic

$t \approx 0$



- Pulse forming network or line (PFL/PFN) charged to voltage $V_0$ by the resonant charging power supply (RCPS)
  - RCPS is de-coupled from the system through a diode stack
- At t = 0, main switch is closed and current starts to flow into the kicker

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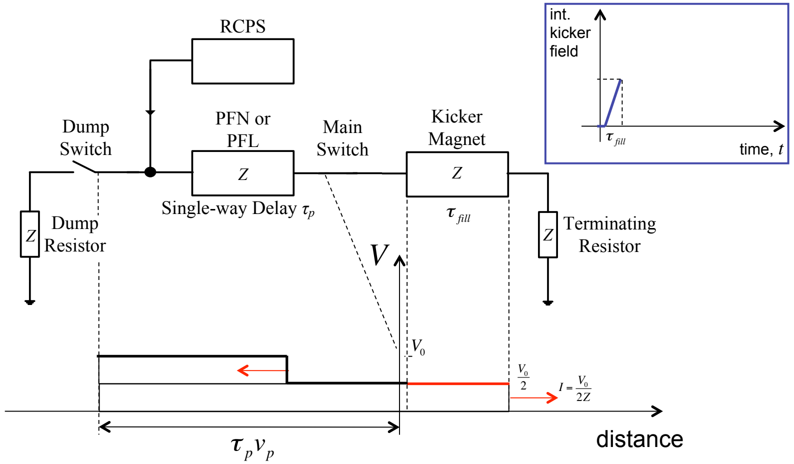
# Simplified kicker system schematic

$t \approx \tau_{fill}$



- At t = $\tau_{fill}$, the voltage pulse of magnitude $V_0/2$ has propagated through the kicker and nominal field achieved with a current $V_0/2Z$
  - typically $\tau_p >> \tau_{fill}$ (schematic for illustration purposes)

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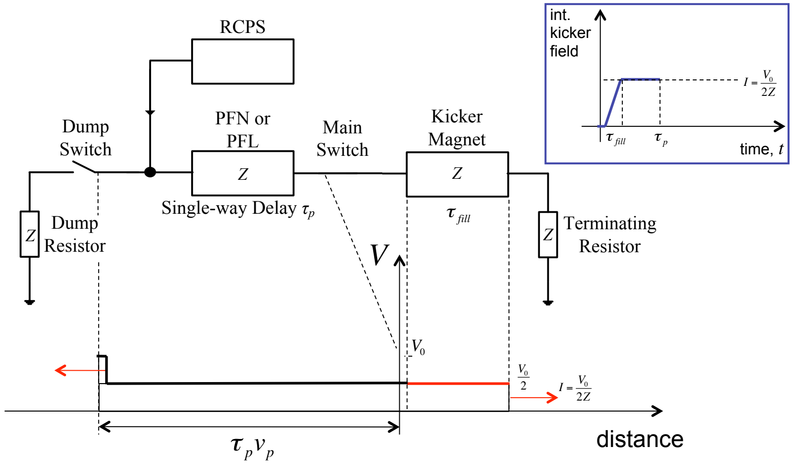
# Simplified kicker system schematic

$t \approx \tau_p$



- PFN continues to discharge energy into kicker magnet and matched terminating resistor

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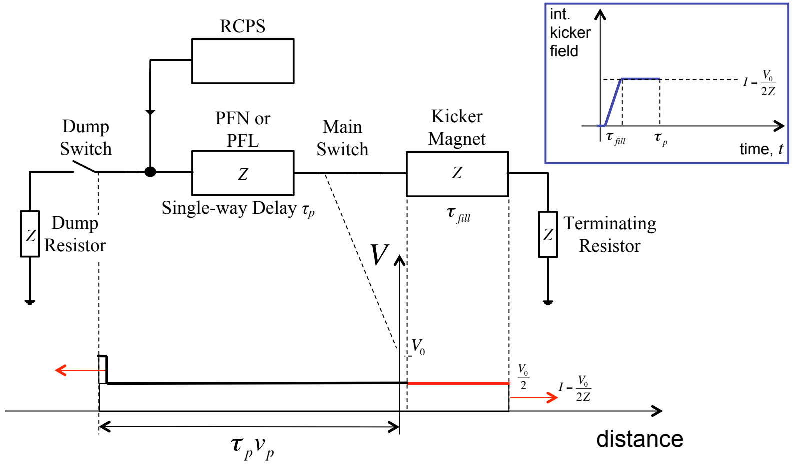
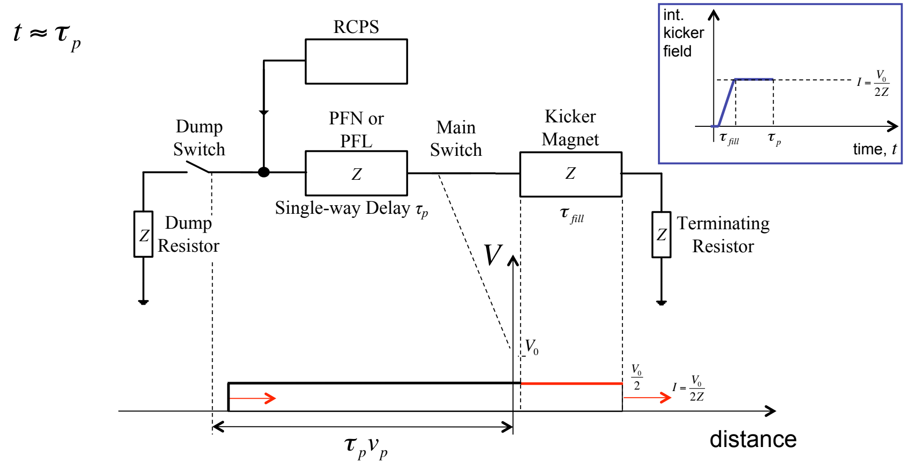


- PFN continues to discharge energy into kicker magnet and matched terminating resistor
- At t ≈ $\tau_p$ the negative pulse reflects off the open end of the circuit (dump switch) and back towards the kicker

# Simplified kicker system schematic

$t \approx \tau_p$



- PFN continues to discharge energy into matched terminating resistor
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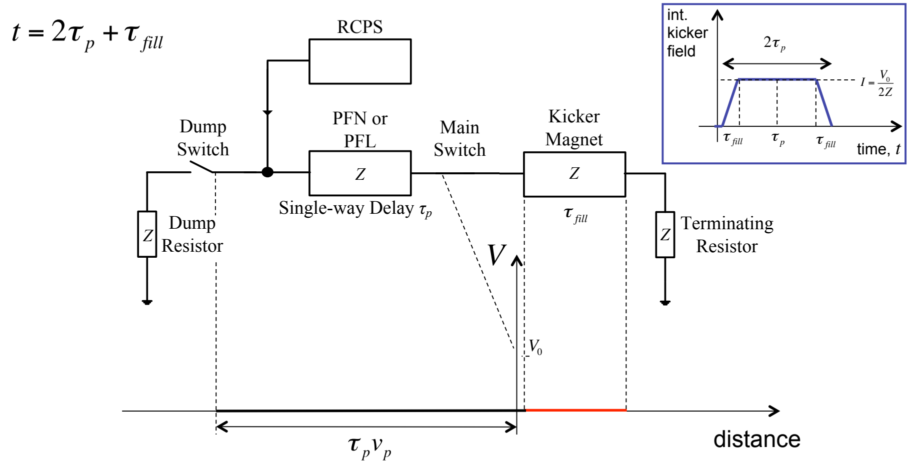
# Simplified kicker system schematic

$t = 2\tau_p + \tau_{fill}$



- A kicker pulse of approximately $2\tau_p$ is imparted on the beam and all energy has been emptied into the terminating resistor

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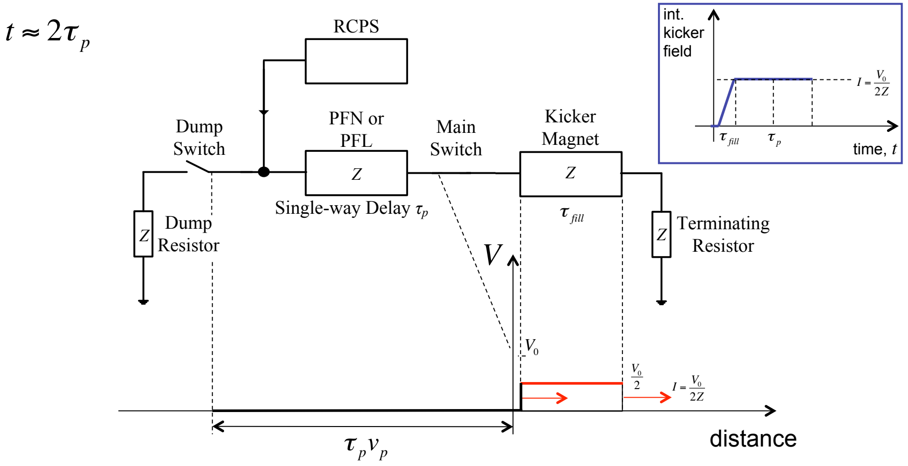
# Simplified kicker system schematic

$t \approx 2\tau_p$



- At t ≈ $2\tau_p$ the pulse arrives at the kicker and field starts to decay

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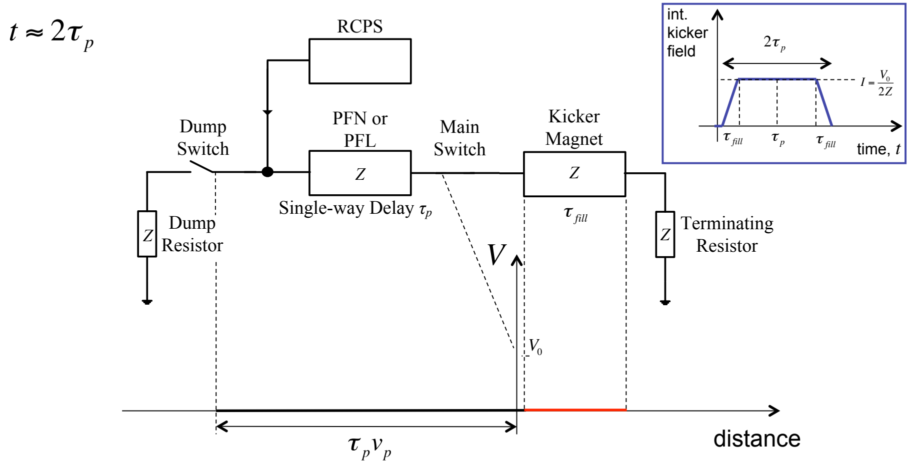
# Simplified kicker system schematic

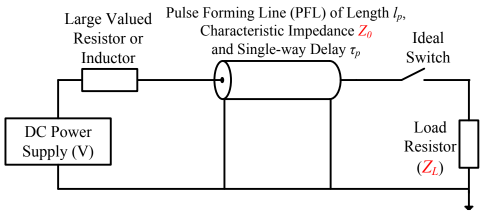
$t \approx 2\tau_p$



- Kicker pulse length can be changed by adjusting the relative timing of dump and main switches:
  - e.g. if the dump and main switches are fired simultaneously the pulse length will be halved and energy shared on dump and terminating resistors

# Reflections

- A simplified pulse forming circuit:



Match impedances to avoid reflections!

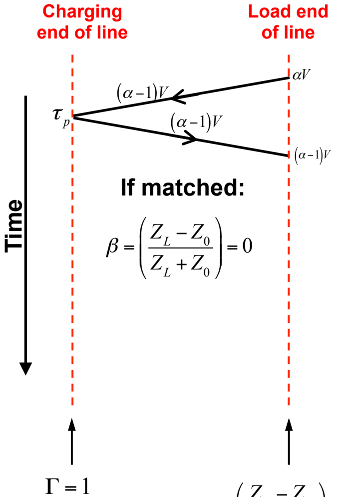
- When the switch is fired the voltage is divided as:

$$V_L = V \cdot \left(\frac{Z_L}{Z_0 + Z_L}\right) = \alpha V$$

- In the matched case:

$$Z_0 = Z_L \qquad \alpha = \frac{1}{2},\ \beta = 0$$

- Mismatches will ring in the circuit causing ripples on the pulse, or post-pulse.



**If matched:**

$$\beta = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) = 0$$

$$\Gamma = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) = \beta$$

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# Reflections

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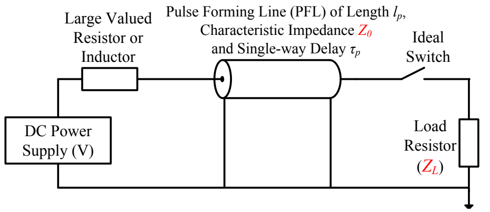

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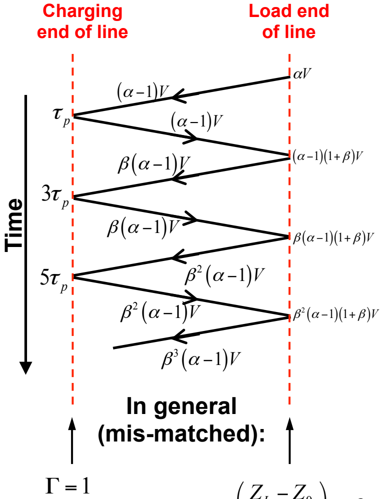


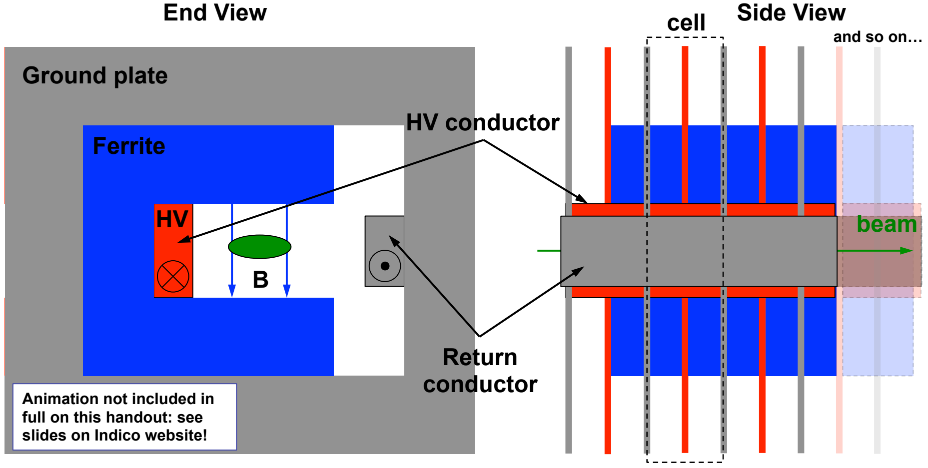
**In general (mis-matched):**

$$\Gamma = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) = \beta$$

---

# Magnets – transmission line

- Today's fast (rise-times of < few hundred ns) kicker magnets are generally **ferrite loaded** transmission lines:
  - Ferrite C-cores are sandwiched between HV plates
  - Grounded plates are interleaved to form a capacitor to ground



Animation not included in full on this handout: see slides on Indico website!

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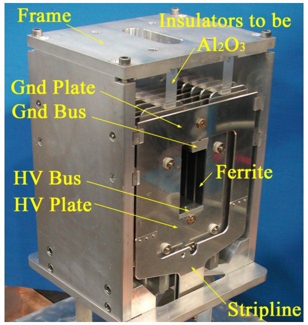
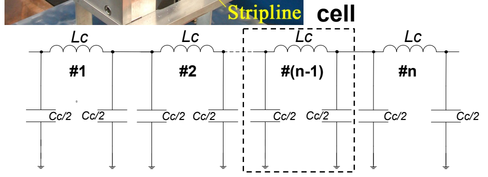
# Magnets – transmission line

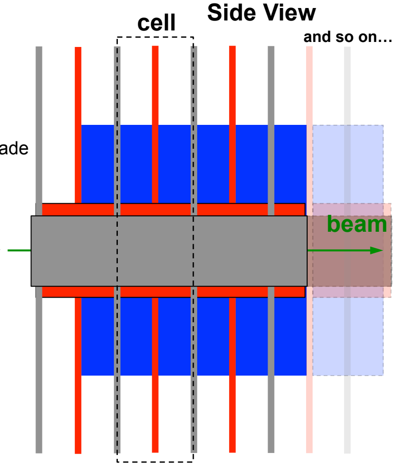
- Today's fast (rise-times of < few hundred ns) kicker magnets are generally **ferrite loaded** transmission lines:
  - Kicker magnets consists of many, relatively short, cells to approximate a broadband coaxial cable



A real kicker…

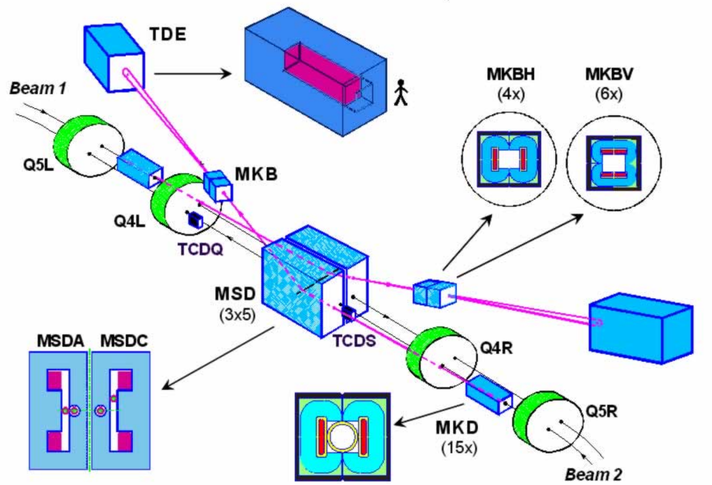
Prototype for AGS injection kicker upgrade





# Magnets – lumped inductance

- Lumped inductance kicker magnets are robust and reliable, and suitable for applications where the rise-time is typically > 1 μs:
  - e.g. LHC beam dump extraction and dilution kicker magnets



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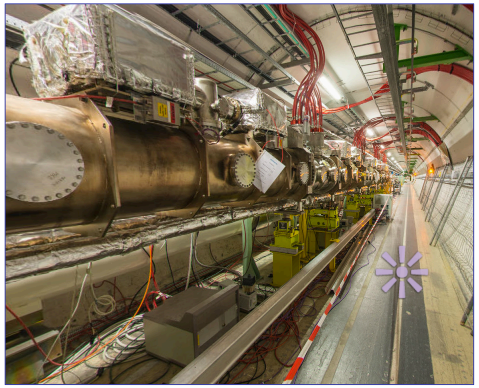
Generators nearby in gallery next to LHC tunnel



MKB dilution magnets in the LHC tunnel

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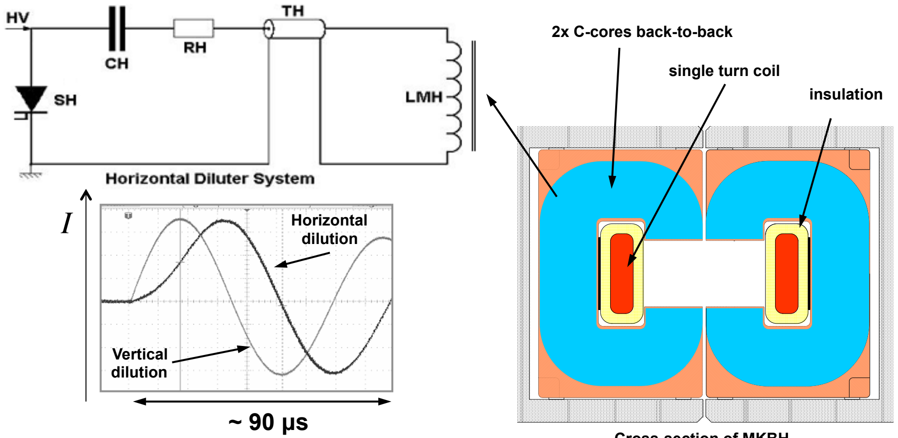

*A damped series RLC circuit (switch closed after capacitor charged)*

**Cross-section of MKBH (horizontal dilution magnet)**

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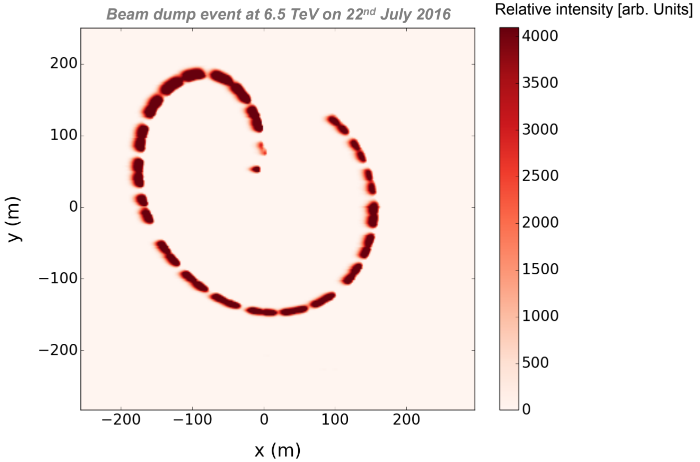
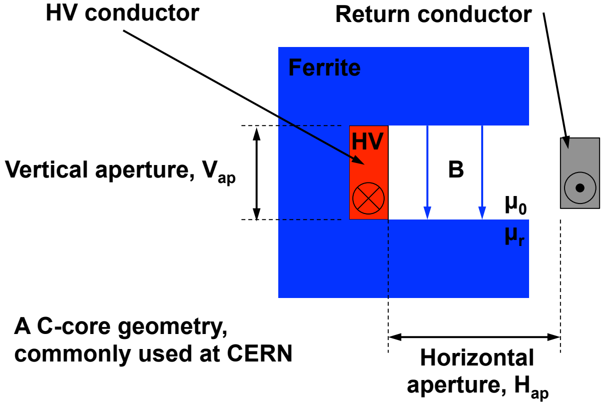


# Magnetic parameters



**Magnetic field**

$$\boxed{B_y \cong \mu_0 \left( \frac{N \cdot I}{V_{ap}} \right)}$$

*Derivation: remember Ampère's Law:*

$$\oint_C \vec{B}.d\vec{l} = \mu_0 I_{enc}$$

**Magnet inductance [per unit length]**

$$\boxed{L_{mag/\mathrm{m}} \cong \mu_0 \left( \frac{N^2 \cdot H_{ap}}{V_{ap}} \right)}$$

*Derivation: remember Faraday's Law:*

$$\Phi_B = \int V\,dt \quad \text{and} \quad V = L\,dI/dt$$

- Dimensions $H_{ap}$ and $V_{ap}$ specified by beam parameters at kicker location
- Ferrite ($\mu_r \approx 1000$) reinforces magnetic circuit and uniformity of the field in the gap
- For fast rise-times the inductance must be minimised: typically the number of turns, $N = 1$.
- Kickers are often split into several magnet units, powered independently.

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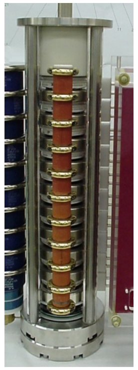
# Switches

**Thyratrons**

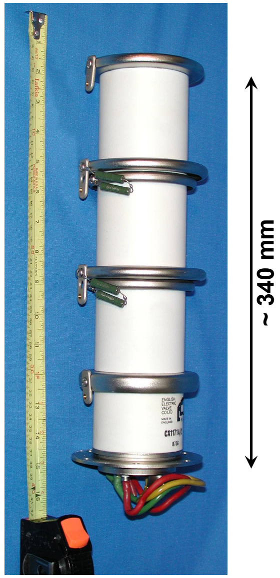
- Deuterium gas thyratrons are commonly used
- Hold off 80 kV and switch up to 6 kA
- **Fast switching** ~ 30 ns (~150 kA/μs)
- Erratic turn-on: use with RCPS to reduce hold-off time

**Power semiconductor switches**

- Suitable for scenarios where erratic turn-on is not allowed
- For example, LHC beam dump kickers held at nominal voltage throughout operation (>10h) ready to fire and safely abort at any moment.
- Hold off up to 30 kV and switch up to 18 kA
- **Slower switching** > 1 μs (~18kA/μs)
- Low maintenance



Stack of high-power semiconductor switches (GTOs)



Thyratron

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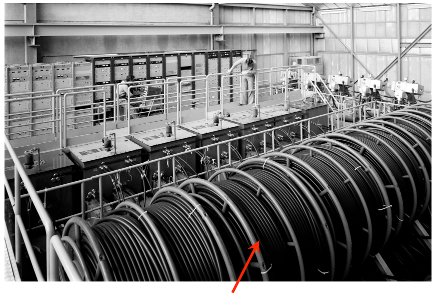
# PFL/PFN

**Pulse Forming Line (PFL)**

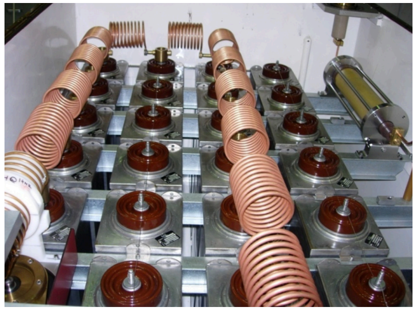
- Low-loss coaxial cable
- Fast and ripple-free pulses
- Attenuation (droop ~1%) becomes problematic for pulses > 3 μs
- Above 50 kV SF6 pressurized PE tape cables are used
- Bulky: 3 μs pulse ~ 300 m of cable

**Pulse Forming Network (PFN)**

- Artificial coaxial cable made of lumped elements
- For low droop and long pulses > 3 μs
- Each cell individually adjustable: adjustment of pulse flat-top difficult and time consuming.



Reels of PFL used at the PS complex (as old as the photograph!)



SPS extraction kicker (MKE) PFN (17 cells)

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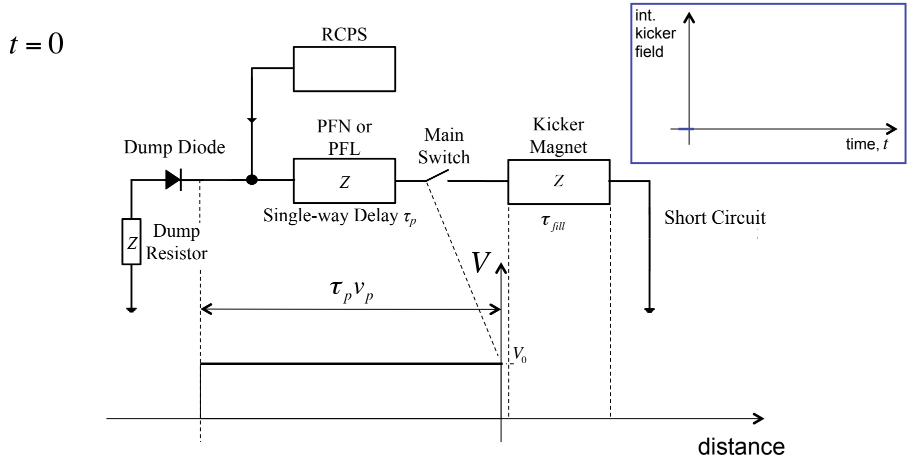
# Terminated vs. short circuit

- Short-circuiting the termination offers twice the kick (for a given kicker magnet):
  - Fill time of kicker magnet is doubled
  - Diode as dump switch provides solution for fixed pulse length



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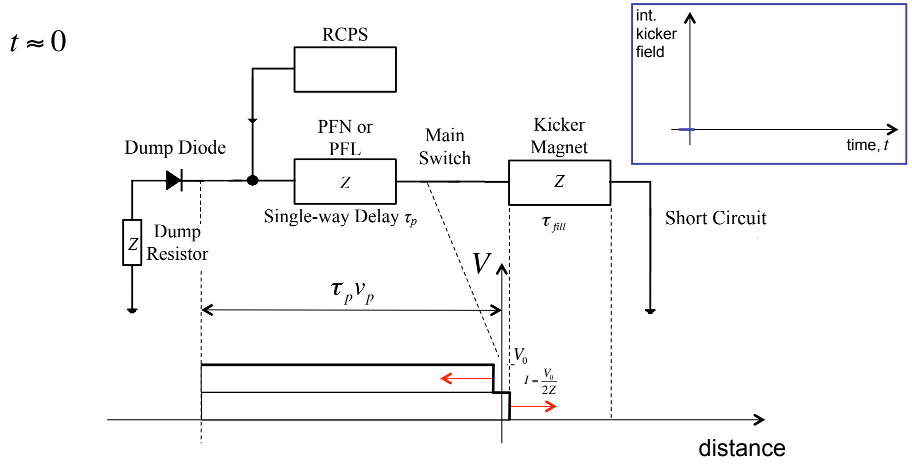

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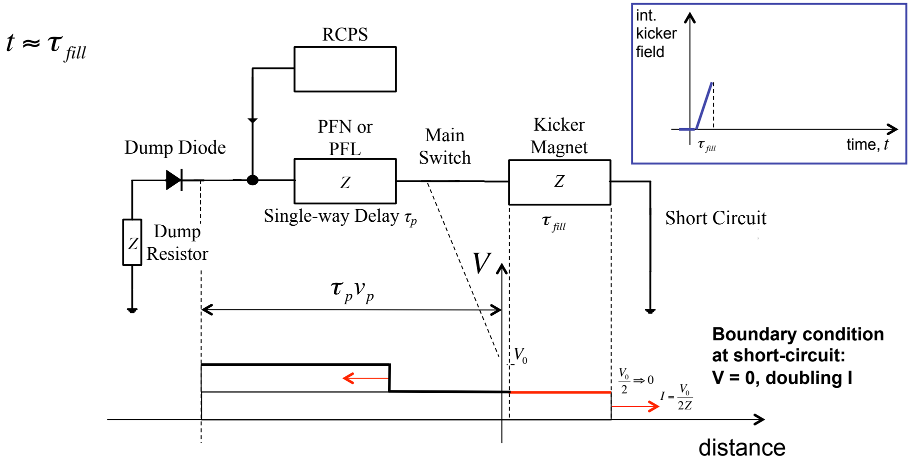

**Boundary condition at short-circuit: V = 0, doubling I**

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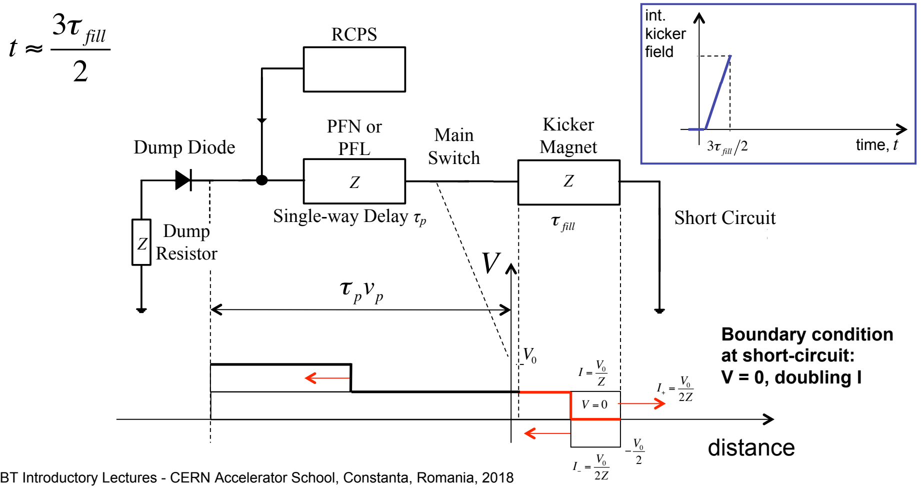

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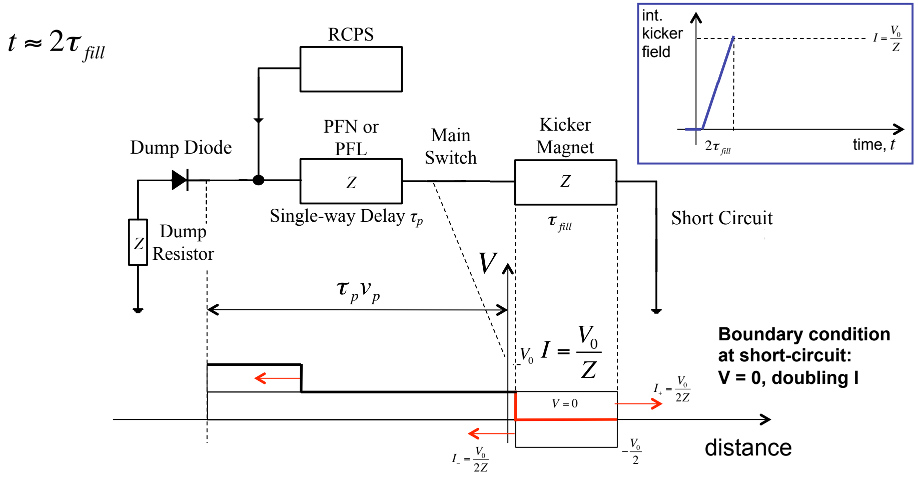

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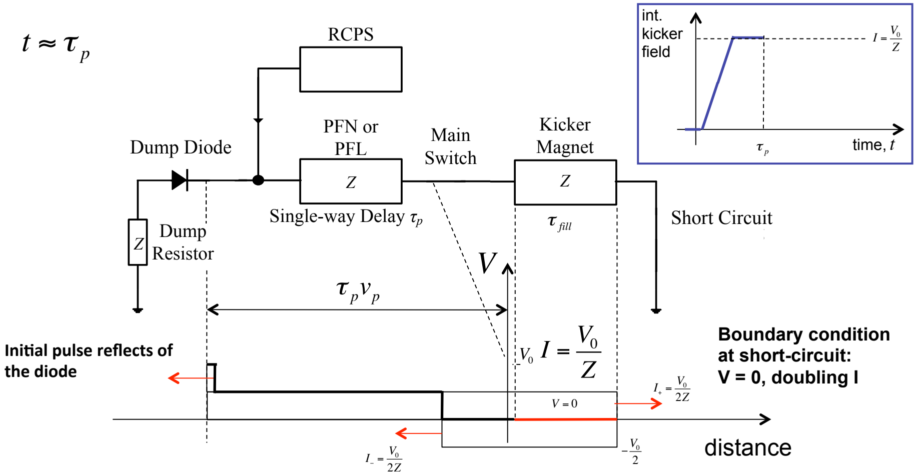

**Initial pulse reflects of the diode**

**Boundary condition at short-circuit: V = 0, doubling I**

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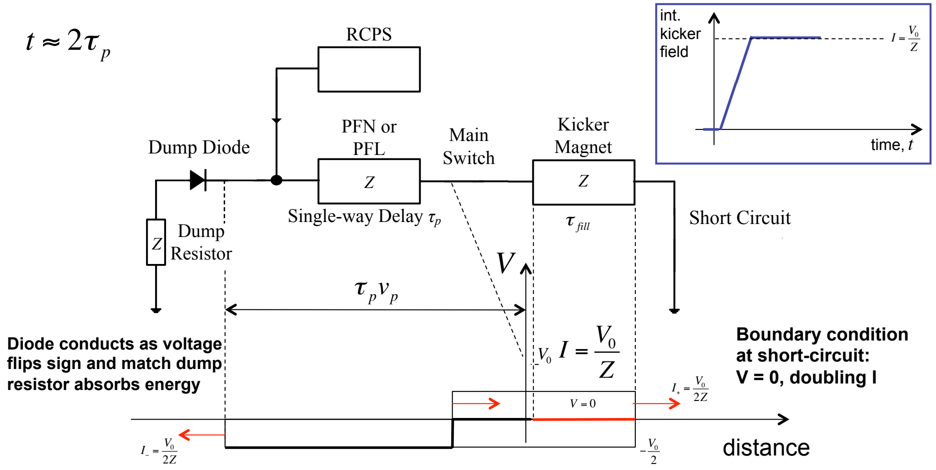

**Diode conducts as voltage flips sign and match dump resistor absorbs energy**

**Boundary condition at short-circuit: V = 0, doubling I**

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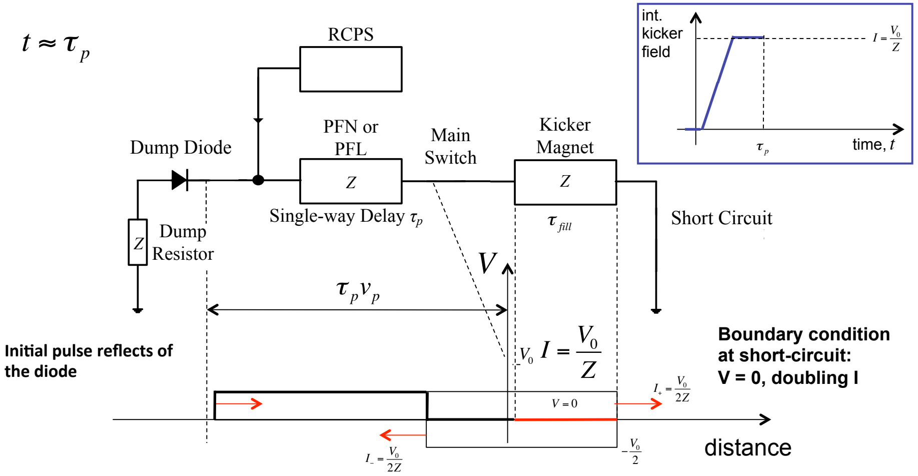

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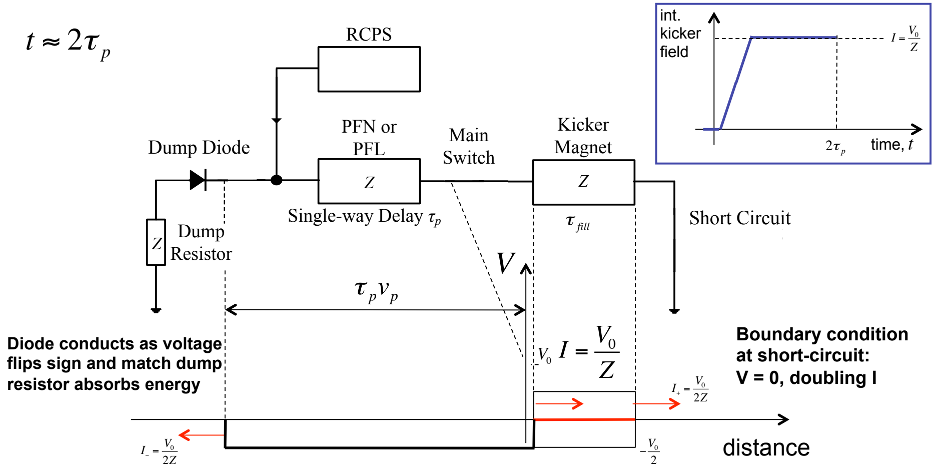

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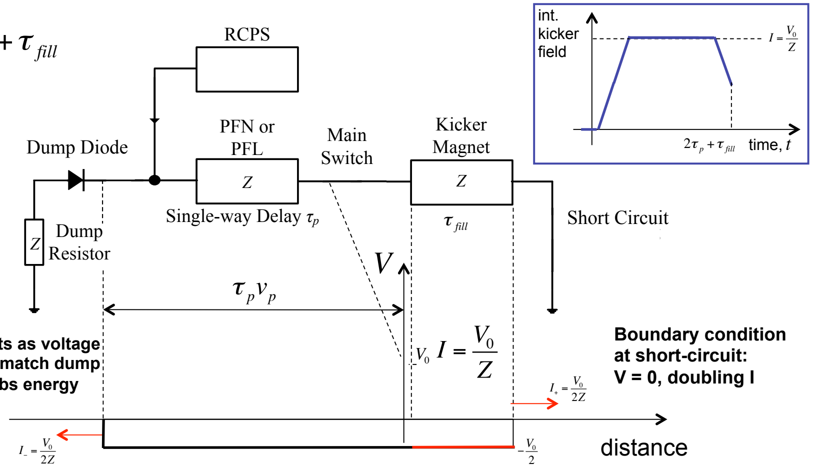
  - Diode as dump switch provides solution for fixed pulse length

$t \approx 2\tau_p + \tau_{fill}$



**Diode conducts as voltage flips sign and match dump resistor absorbs energy**

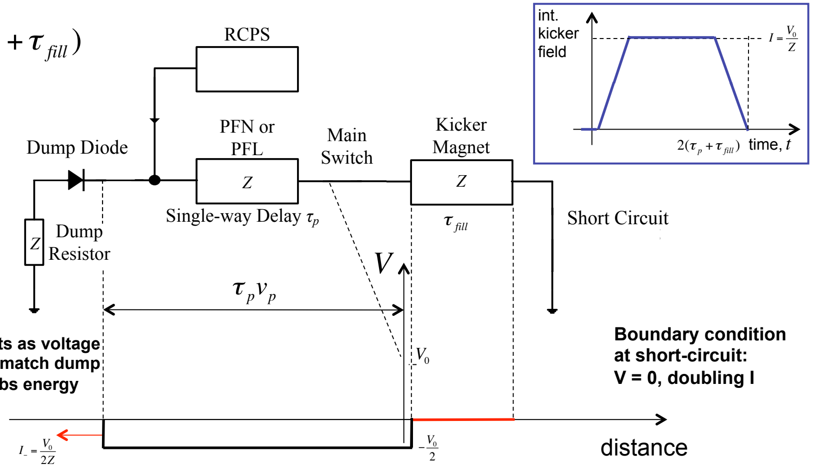
**Boundary condition at short-circuit: V = 0, doubling I**

---

# Terminated vs. short circuit

- Short-circuiting the termination offers twice the kick (for a given kicker magnet):
  - Fill time of kicker magnet is doubled
  - Diode as dump switch provides solution for fixed pulse length

$t \approx 2(\tau_p + \tau_{fill})$



**Diode conducts as voltage flips sign and match dump resistor absorbs energy**

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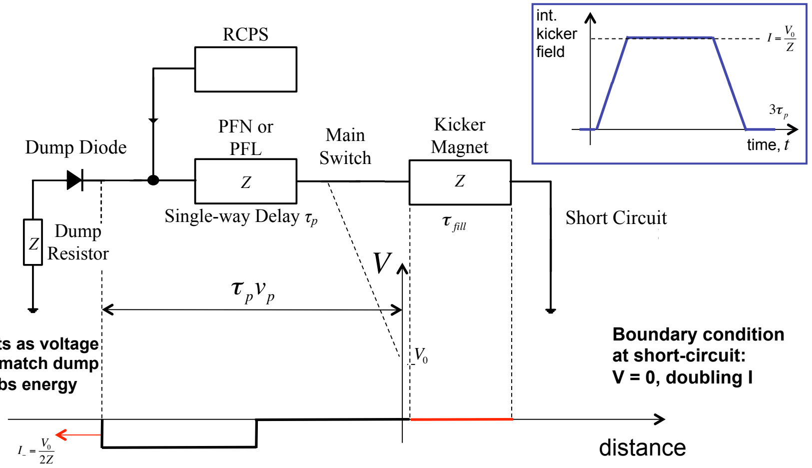
---

# Terminated vs. short circuit

- Short-circuiting the termination offers twice the kick (for a given kicker magnet):
  - Fill time of kicker magnet is doubled
  - Diode as dump switch provides solution for fixed pulse length

$t \approx 3\tau_p$



**Diode conducts as voltage flips sign and match dump resistor absorbs energy**

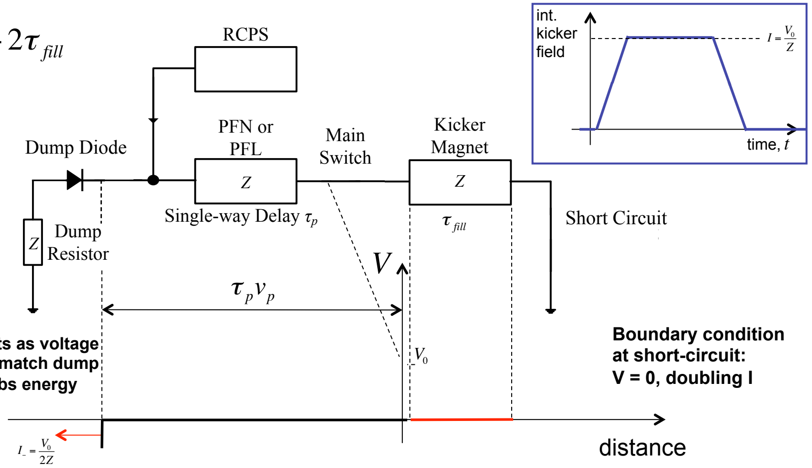
**Boundary condition at short-circuit: V = 0, doubling I**

---

# Terminated vs. short circuit

- Short-circuiting the termination offers twice the kick (for a given kicker magnet):
  - Fill time of kicker magnet is doubled
  - Diode as dump switch provides solution for fixed pulse length

$t = 3\tau_p + 2\tau_{fill}$



**Diode conducts as voltage flips sign and match dump resistor absorbs energy**

**Boundary condition at short-circuit: V = 0, doubling I**

# Terminated vs. short circuit

- Short-circuiting the termination offers twice the kick (for a given kicker magnet):
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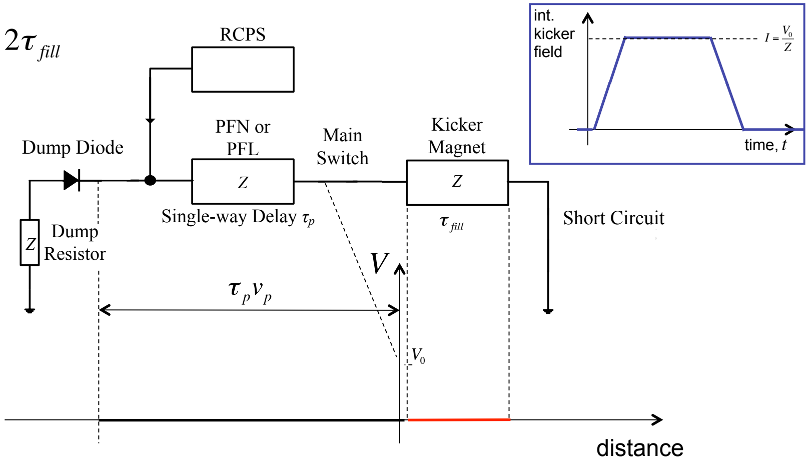
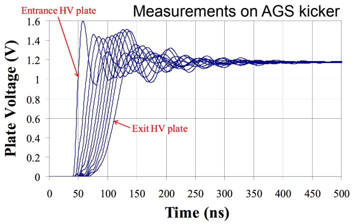


---

# Other topics and considerations

- **Ripple:** cells of a transmission line kicker have a cut-off frequency that introduces dispersion in pulse:
  - Cut-off frequency:

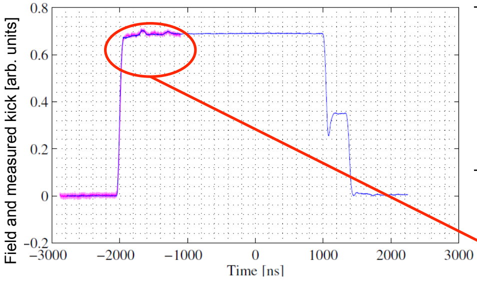
$$\omega_c = \frac{2}{\sqrt{L_{cell} C_{cell}}} = \frac{Z}{L_{cell}}$$

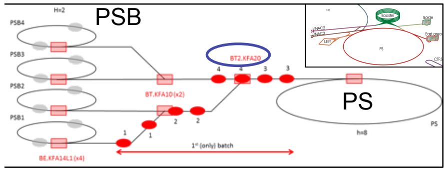


- **In vacuum:** aperture dimensions ($H_{ap}$ and $V_{ap}$) minimised if in vacuum:
  - For given B, lower I and L can be achieved with smaller $H_{ap}$ and $V_{ap}$
  - Machine vacuum is a reliable dielectric, recovers after flashover
  - Costly and time consuming to construct/maintain (cleanliness, bake-out)
- **Beam coupling impedance:** kickers are a source of beam impedance in accelerators (wakefields and beam instabilities)
  - Ferrite is shielded from beam with beam screens or serigraphy by permitting a smooth conducting path for beam induced image charges
  - Beam induced heating of ferrite yoke can heat it above the Curie temp.

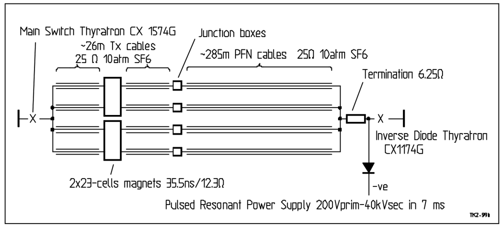
---

# An example of reflections

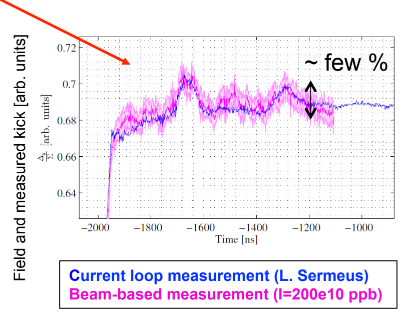




Recombination kicker KFA.20 deflects PS booster rings 1 and 2 vertically into plane of PS at CERN



BT.KFA20 circuit schematic: operates in “virtual” short-circuit



Beam-based kicker measurements at higher intensities, V. Forte
BT + PS injection kicker meeting, CERN (15th August 2016)

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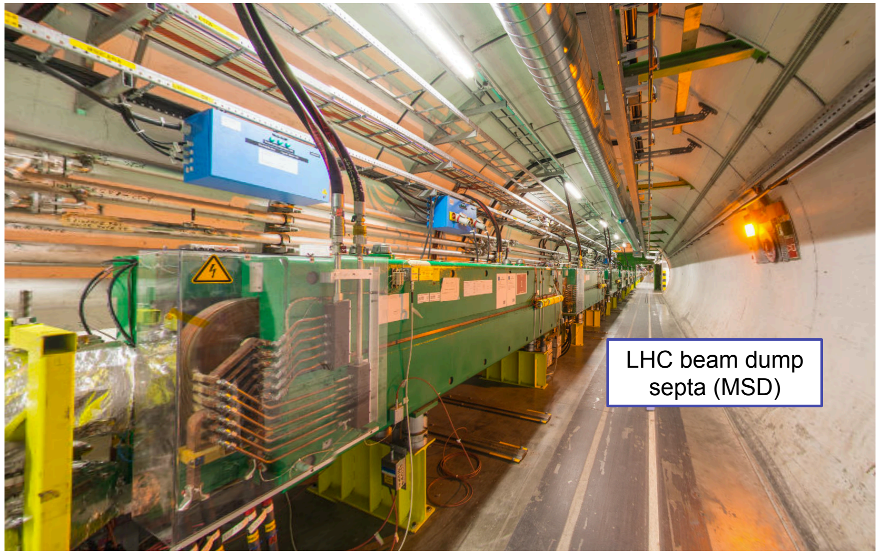
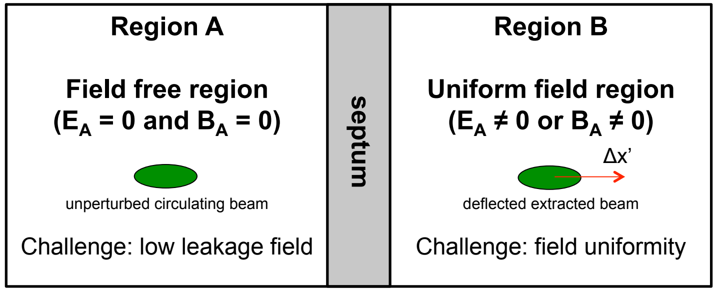
# Septa



# Septa

- Two main types:
  - Electrostatic septa (DC)
  - Magnetic septa (DC and pulsed):
    - Direct drive septum
    - Eddy current septum (pulsed only)
    - Lambertson septum (deflection parallel to septum)



**Region A**

**Field free region**
**($E_A = 0$ and $B_A = 0$)**

unperturbed circulating beam

Challenge: low leakage field

**septum**

**Region B**

**Uniform field region**
**($E_A \neq 0$ or $B_A \neq 0$)**

$\Delta x'$

deflected extracted beam

Challenge: field uniformity

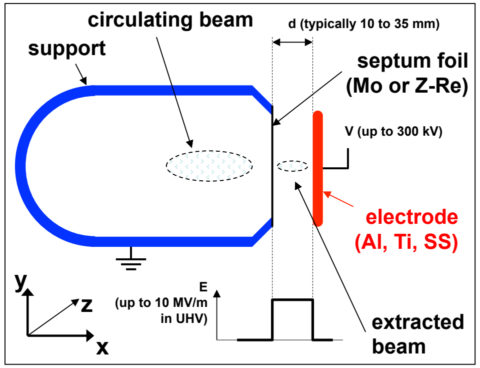
---

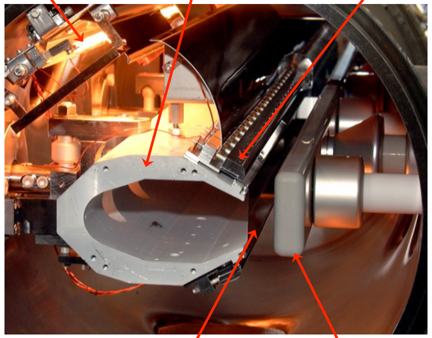
# Electrostatic septum

- At SPS we slow-extract 400 GeV protons using approximately 15 m of septum split into 5 separate vacuum tanks each over 3 m long:
  - Alignment of the 60 - 100 μm wire array over 15 m is challenging!

---

# Electrostatic septum

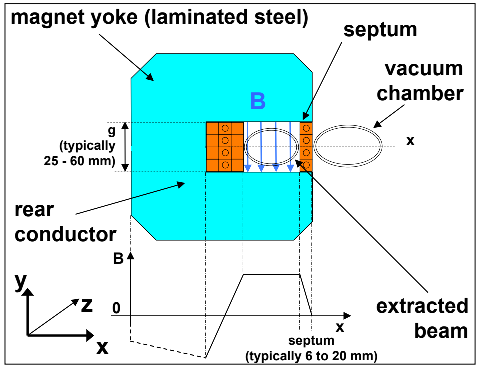


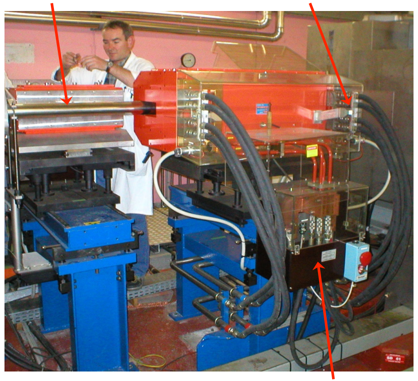


- Thin septum ~ 0.1 mm needed for high extraction efficiency:
  - Foils typically used
  - Stretched wire arrays provide thinner septa and lower effective density
- Challenges include conditioning and preparation of HV surfaces, vacuum in range of $10^{-9}$ – $10^{-12}$ mbar and in-vacuum precision position alignment

---

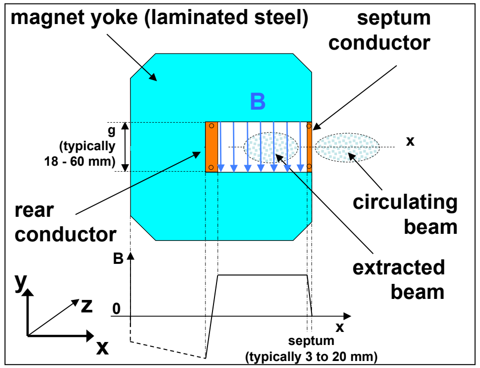
# DC direct drive magnetic septum

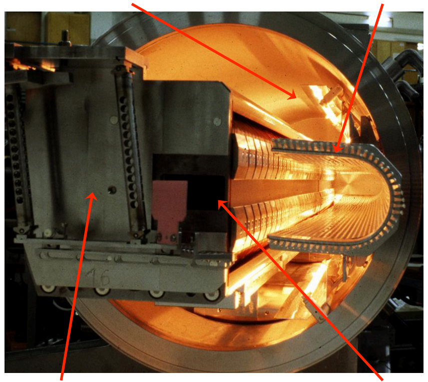




- Continuously powered, rarely under vacuum
- Multi-turn coil to reduce current needed but cooling still an issue:
  - Cooling water circuits flow rate typically at 12 – 60 l/min
  - Current can range from 0.5 to 4 kA and power consumption up to 100 kW!

# Direct drive **pulsed** magnetic septum

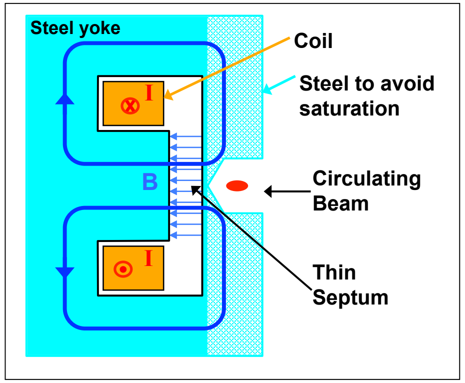


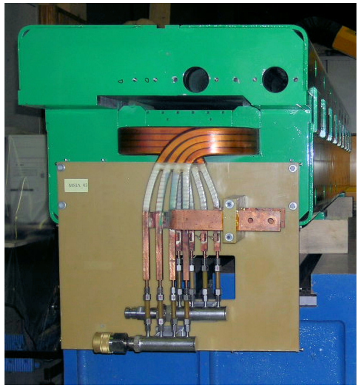


- In vacuum, to minimise distance between circulated and extracted beam
- Single-turn coil to minimise inductance, bake-out up to 200 °C (~$10^{-9}$ mbar)
- Pulsed by capacitor discharge (third harmonic flattens the pulse):
  - Current in range 7 – 40 kA with a few ms oscillation period
  - Cooling water circuits flow rate from 1 – 80 l/min

---

# Lambertson septum

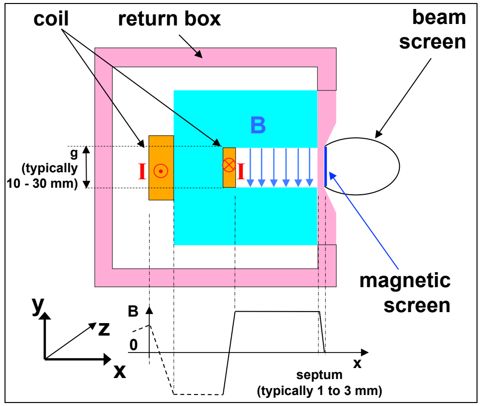




- Magnetic field in gap orthogonal to previous examples of septa:
  - Lambertson deflects beam orthogonal to kicker: dual plane injection/extraction
- Rugged design: conductors safely hidden away from the beam
- Thin steel yoke between aperture and circulating beam – however extra steel required to avoid saturation, magnetic shielding often added

---

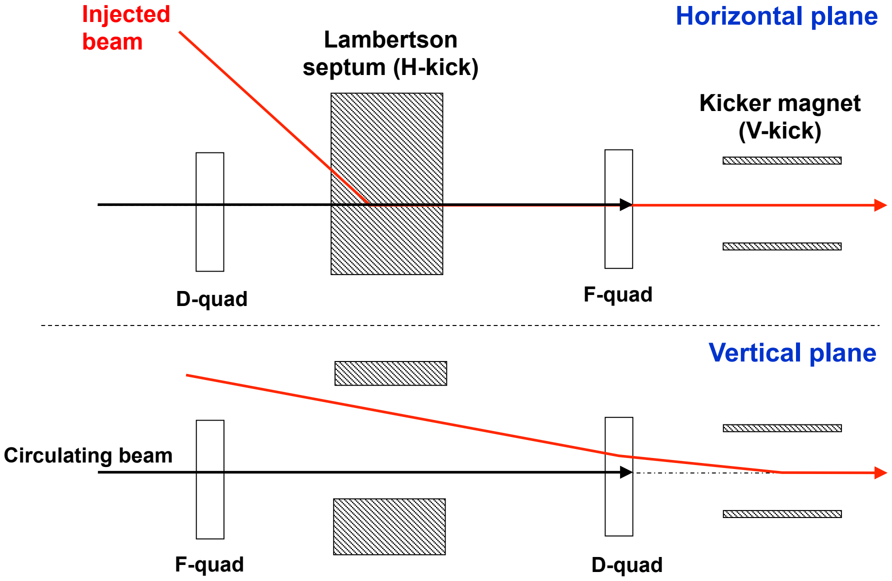
# Eddy current septum



- Coil removed from septum and placed behind C-core yoke:
  - Coil dimension not critical
  - Very thin septum blade
- Magnetic field pulse induces eddy currents in septum blade
- Eddy currents shield the circulating beam from magnetic field
- Return box and magnetic screen reduce fringe field seen by circulating beam
- In or out of vacuum, single-turn coil
- Pulsed by capacitor discharge (third harmonic flattens the pulse):
  - Current ~10 kA fast pulsed with ~ 50 µs oscillation period
  - Cooling water circuits flow rate from 1 – 10 l/min

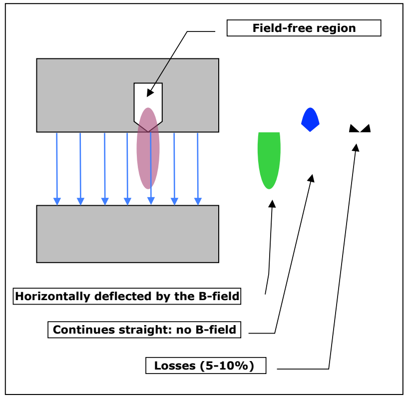
---

# Two plane injection with Lambertson



# Lambertson septum

- At SPS we use Lambertson septa to split the 400 GeV slow-extracted proton spill (~ seconds) to different target stations simultaneously:
  - These devices are radioactive: critical that coils are located away from the septum



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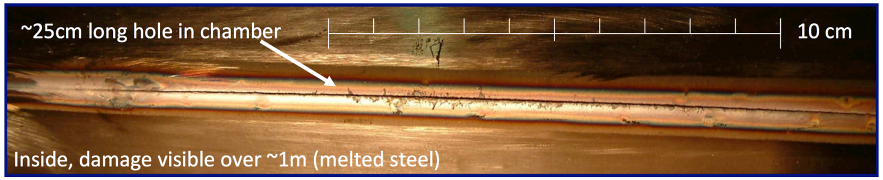
# Protection devices

- When beam energy exceeds damage limit for machine equipment one has to design for certain failure scenarios
- Critical beam transfer systems have redundancy and multiple layers of protection:
  - Passive protection devices form the last layer of this security
- Protection devices are designed to dilute and absorb beam energy safely
- Failures associated with beam transfer equipment are typically very fast and difficult to catch, for example:
  - No turn-on of kicker: injection protection
  - Erratic turn-on of kicker: sweep circulating beam in the machine
  - Flash-over (short-circuit) in kicker: impart the wrong kicker angle
  - Transfer line magnet failure: steering beam onto aperture of downstream machine

---

# Protection devices

- When things go wrong…!
  - SPS extraction septum power supply tripped during setting-up of LHC beam, 25th October 2004:

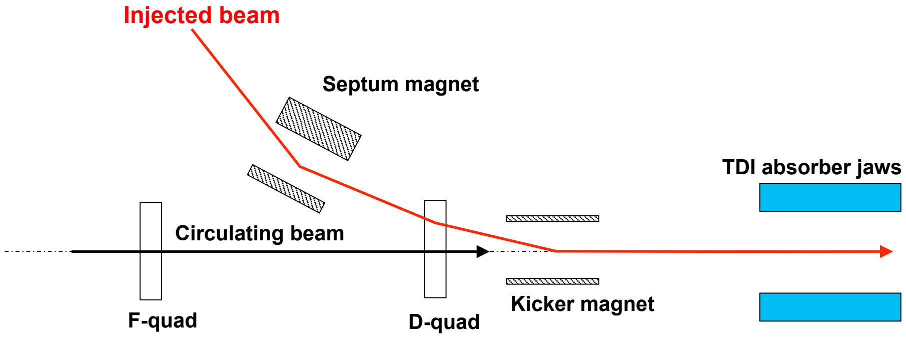


  - Septum field dropped by 5% in 11 ms
  - 3.4 x $10^{13}$ protons at 450 GeV, i.e. 2.5 MJ of beam energy dissipated on the aperture of the transfer line
  - Vacuum chamber and quadrupole magnet damaged requiring replacement
  - Upgraded fast interlock system was implemented to protect against such fast failures

---

# Injection protection: e.g. LHC injection

- LHC has a dedicated injection dump (TDI) to protect against fast failures on the injection kicker



*In reality the LHC injection is dual plane: Lambertson septum kick orthogonal to kicker*

## Injection protection: e.g. LHC injection

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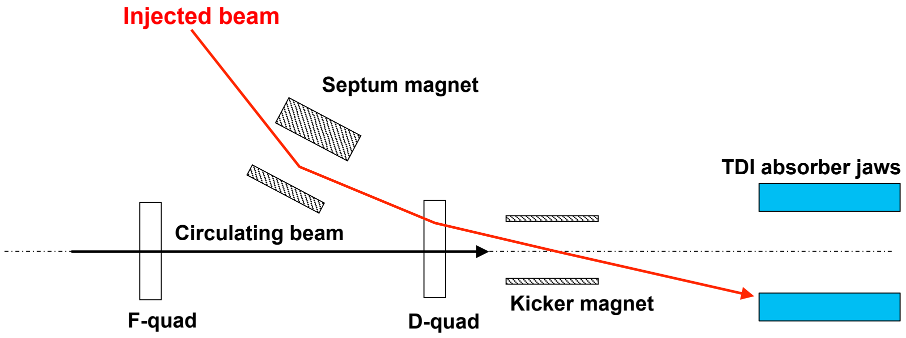
  - No turn-on of kicker: beam steered safely onto absorber:



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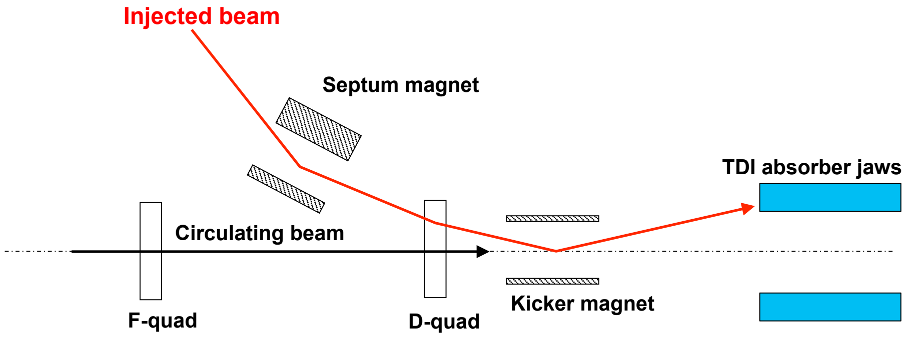
---

## Injection protection: e.g. LHC injection

- LHC has a dedicated injection dump (TDI) to protect against fast failures on the injection kicker
  - No turn-on of kicker: beam steered safely onto absorber
  - Erratic turn-on of kicker: circulating beam steered safely onto absorber
  - Flash-over (short-circuit) in kicker: “worst-case” gives twice deflection:



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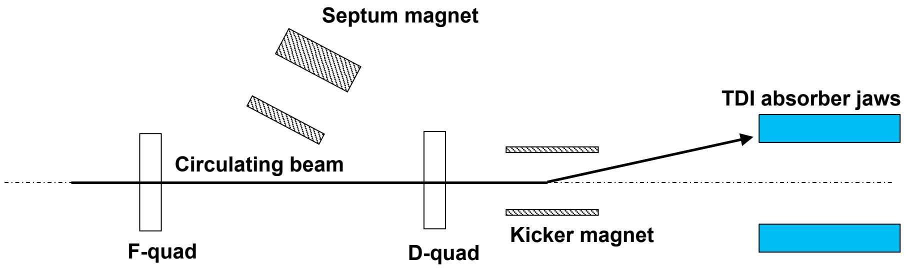
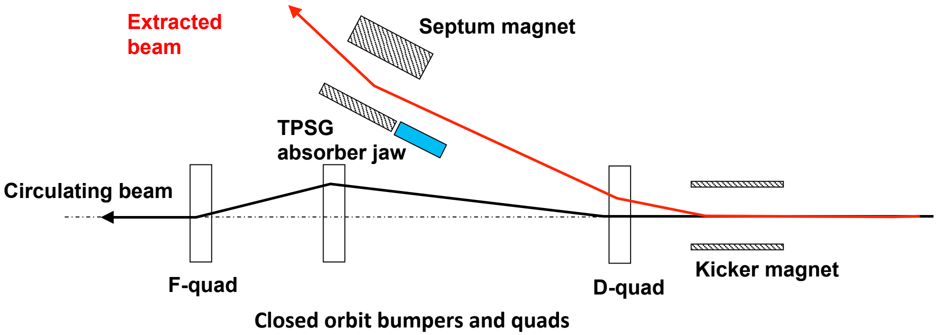


*In reality the LHC injection is dual plane: Lambertson septum kick orthogonal to kicker*

---

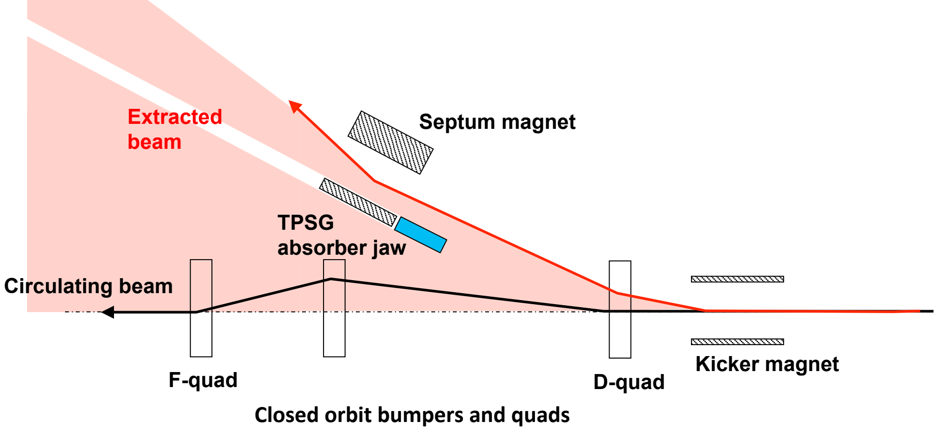
## Extraction protection: e.g. SPS extraction

- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker



# Extraction protection: e.g. SPS extraction

- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker
  - Erratic turn-on of kicker: asynchronous timing with particle-free gap and circulating beam swept across TPSG into transfer line:



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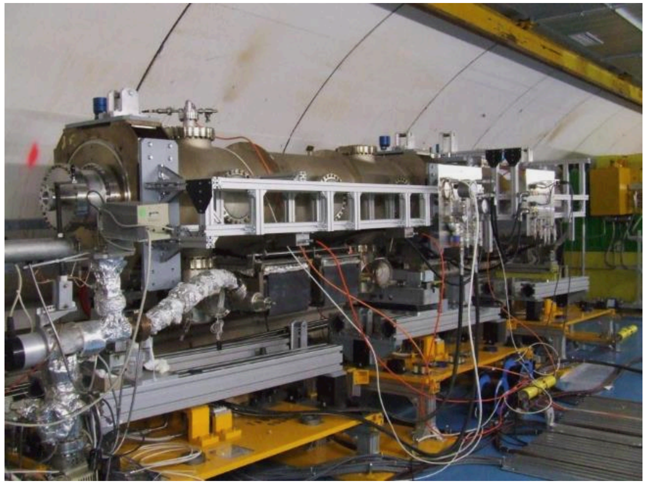
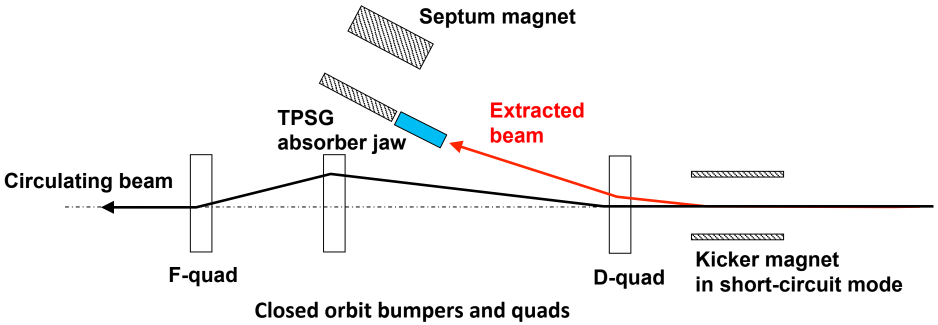


*TPSG and MSE (magnetic septum) installed at HIRADMAT irradiation test facility in 2012: impacted with LHC nominal intensity (288b and $1.1\times10^{11}$ p/b): both devices survived!*

---

# Extraction protection: e.g. SPS extraction

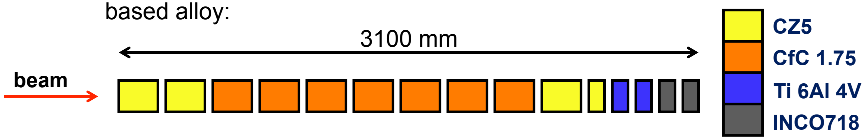
- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker
  - Erratic turn-on of kicker: asynchronous timing with particle-free gap and circulating beam swept across TPSG into transfer line
  - Flash-over (short-circuit) in kicker: worst-case amplitude places the extracted beam onto the absorber jaw:



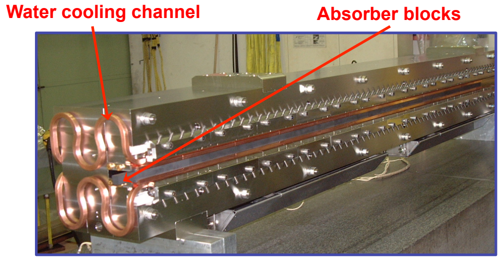
---

# Extraction protection: e.g. TPSG

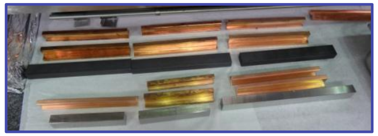
- SPS has a dedicated absorber (TPSG) to protect the extraction septum in case of fast failures of the extraction kicker
  - Diluter made of graphite, 2D carbon composite, titanium alloy and nickel based alloy:



  - Designed to protect downstream septum from direct impact of 450 GeV LHC ultimate beam (288 bunches at $1.7\times10^{11}$ protons per bunch, 3.5 MJ)



*TPSG assembly without vacuum tank*

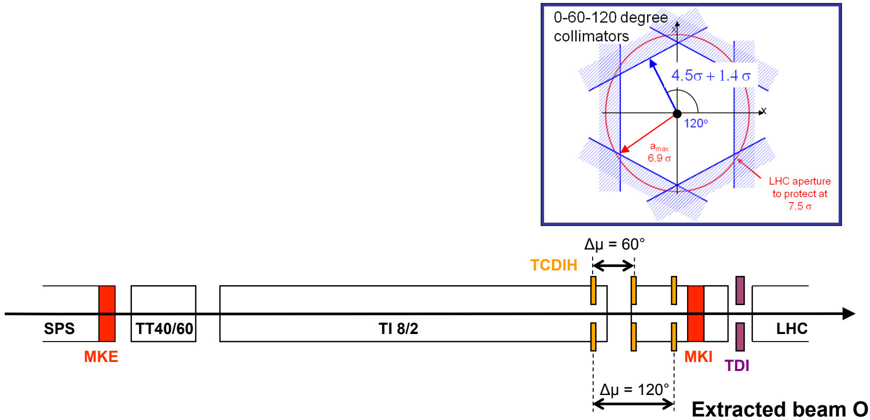


*Absorber blocks inspected after impact of HIRADMAT test #6: survived and re-installed*

Comment: small emittance (high beam brightness) can be just as much a concern as the total intensity for thermo-mechanical stresses during beam impact

# Transfer protection: e.g. SPS-to-LHC

- SPS to LHC transfer lines have dedicated transfer line collimators (TCDIH and V) in case of fast failures to protect LHC aperture:



**Extracted beam ON nominal trajectory**

---

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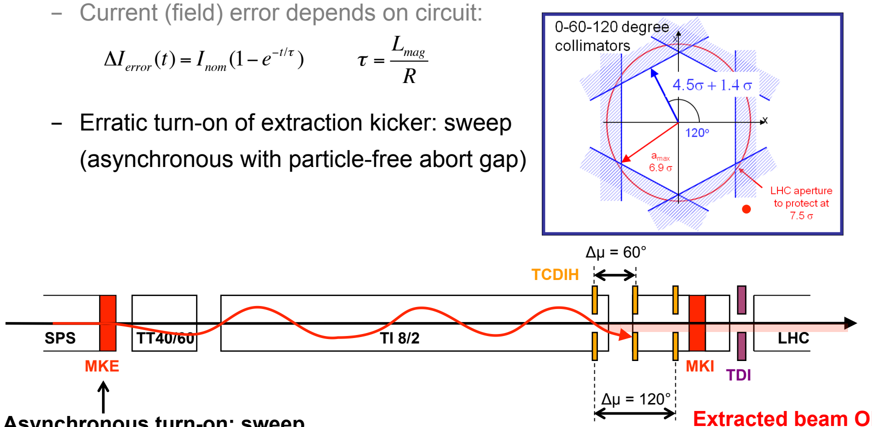
  - Magnet power supply trips at time *t* after the last extraction interlock check: beam steered onto collimator
  - Current (field) error depends on circuit:

$$\Delta I_{error}(t) = I_{nom}(1-e^{-t/\tau}) \qquad \tau = \frac{L_{mag}}{R}$$

  - Erratic turn-on of extraction kicker: sweep (asynchronous with particle-free abort gap)



**Asynchronous turn-on: sweep**

**Extracted beam OFF nominal trajectory**

---

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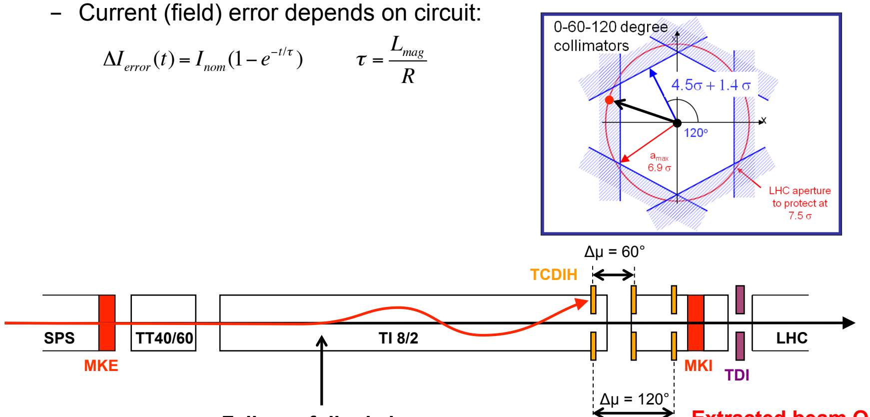

**Failure of dipole here**

**Extracted beam OFF nominal trajectory**

---

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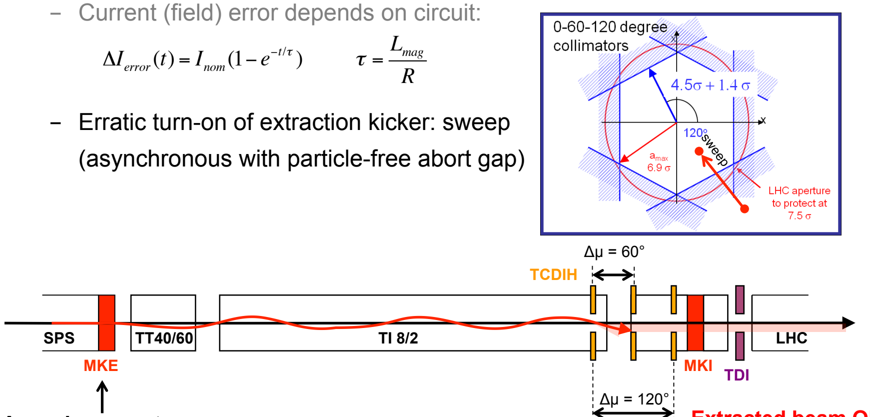

**Asynchronous turn-on: sweep**

**Extracted beam OFF nominal trajectory**

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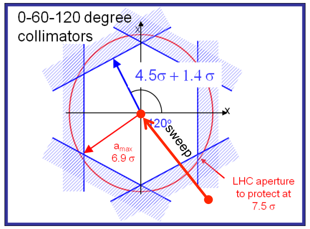

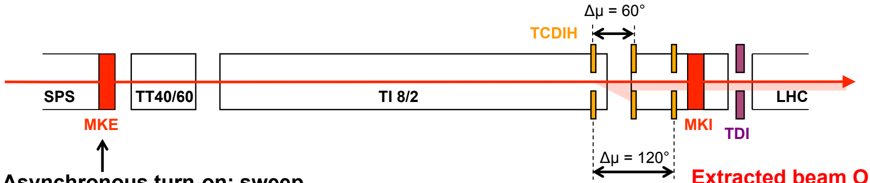

**Asynchronous turn-on: sweep**

**Extracted beam ON nominal trajectory**

---

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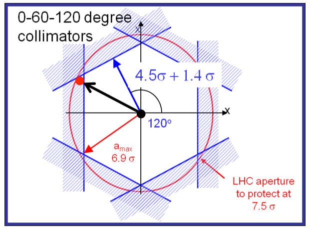
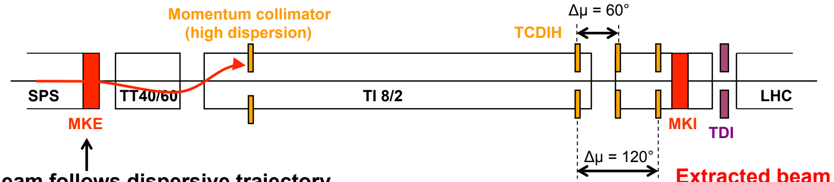
  - Erratic turn-on of extraction kicker: sweep (asynchronous with particle-free abort gap)
  - Flash-over (short-circuit) in kicker
  - Momentum mismatch at extraction:





**Beam follows dispersive trajectory**

**Extracted beam ON nominal trajectory**

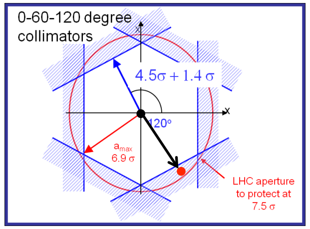
---

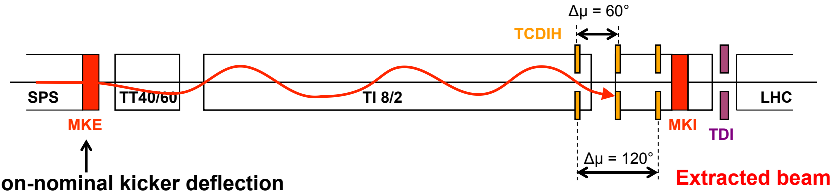
# Transfer protection: e.g. SPS-to-LHC

- SPS to LHC transfer lines have dedicated transfer line collimators (TCDIH and V) in case of fast failures to protect LHC aperture:
  - Magnet power supply trips at time *t* after the last extraction interlock check: beam steered onto collimator
  - Current (field) error depends on circuit:

$$\Delta I_{error}(t) = I_{nom}(1-e^{-t/\tau}) \qquad \tau = \frac{L_{mag}}{R}$$

  - Erratic turn-on of extraction kicker: sweep (asynchronous with particle-free abort gap)
  - Flash-over (short-circuit) in kicker:





**Non-nominal kicker deflection**

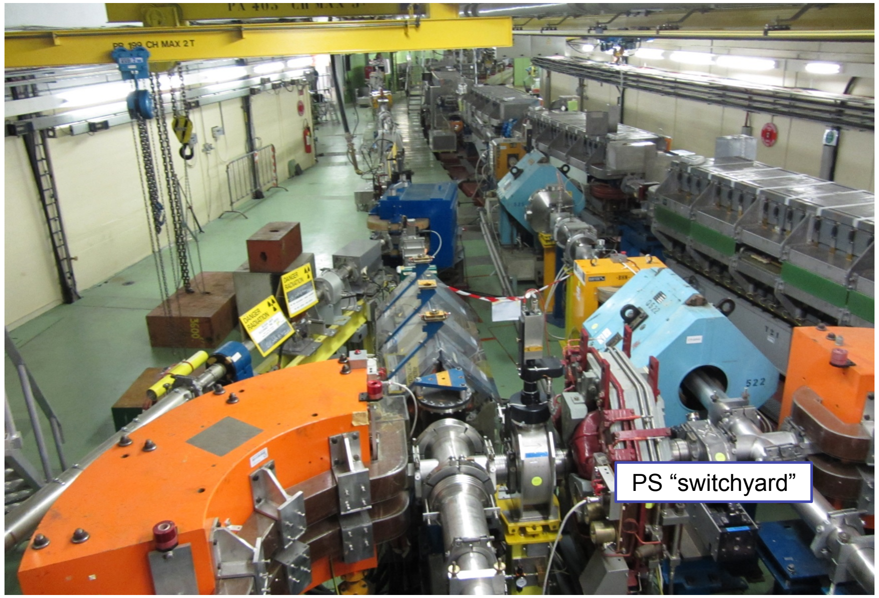
**Extracted beam ON nominal trajectory**

---

# Beam transfer lines

LHC beam dump extraction line (TD62/68)

# Beam transfer lines



# Functional requirements

Transfer lines transport beams between accelerators (extraction of one to injection of the next) and on to experimental targets and beam dumps

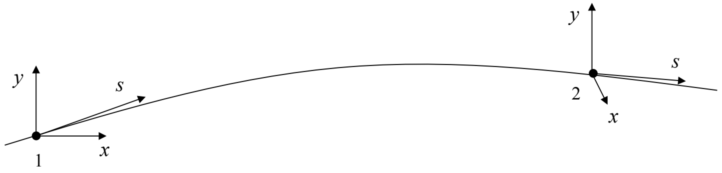
- Requirements:
  - Geometric link between machines/experiment
  - Match optics between machines/experiment
  - Preserve emittance
  - Change particles' charge state (stripping foils)
  - Measure beam parameters (measurement lines)
  - Protect downstream machine/experiment

# Beam transfer lines

- Beam transfer lines
  - Functional requirements
  - Distinctions between transfer lines and circular machines
  - Linking machines/experiments together
  - Emittance blow-up from mismatch
  - Measurement of beam parameters (measurement lines)

# General transport

Beam transport: moving from $s_1$ to $s_2$ through $n$ elements, each with transfer matrix $M_i$



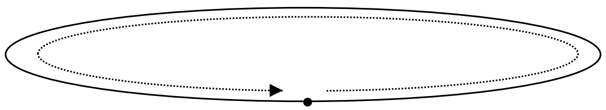
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\rightarrow 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} \qquad \mathbf{M}_{1\rightarrow 2} = \prod_{i=1}^{n} \mathbf{M}_n$$

The transfer matrix ($M_i$) can be expressed using the Twiss formalism:

$$\mathbf{M}_{1\rightarrow 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1 - \alpha_2)\cos\Delta\mu - (1 + \alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

# Circular Machine

Circumference = L



One turn:

$\Delta\mu = 2\pi Q$

$$\mathbf{M}_{1\to2} = \mathbf{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -1/\beta\left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $D_1 = D_2$
- This condition *uniquely* determines α(*s*), β(*s*), μ(*s*), D(*s*) around the whole ring
  - i.e. a single matched ellipse exists for each given location, *s*

---

# Transfer line

One pass: $\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$ $\begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$

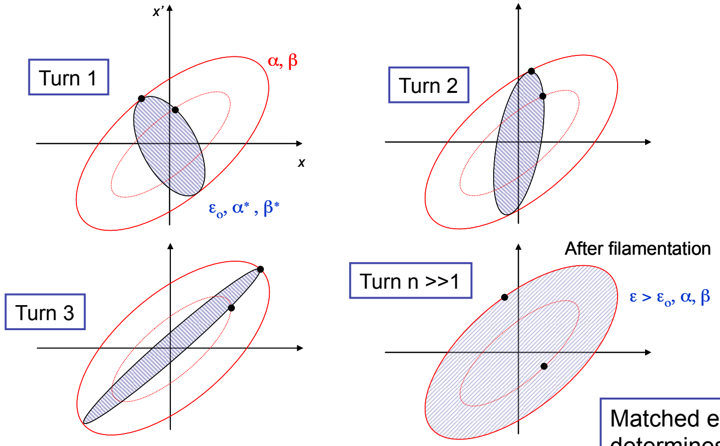
$$\mathbf{M}_{1\to2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[\left(\alpha_1-\alpha_2\right)\cos\Delta\mu - \left(1+\alpha_1\alpha_2\right)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2\sin\Delta\mu\right) \end{bmatrix}$$

- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, α(s) β(s) are functions of $\alpha_1$ and $\beta_1$

---

# Circular Machine

- At a location with matched ellipse (α, β) a mismatched injected beam ($\alpha^*$, $\beta^*$) with emittance $\varepsilon_0$, generates (via filamentation) a larger ellipse with the matched α, β, but larger emittance: $\varepsilon > \varepsilon_0$



Matched ellipse determines beam shape

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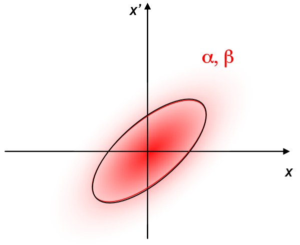
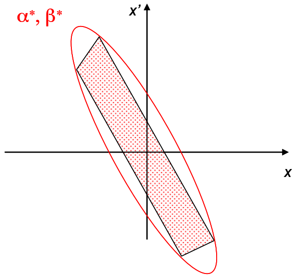
# Transfer line

- Initial α, β are defined for a transfer line by the beam shape at the entrance



Gaussian beam

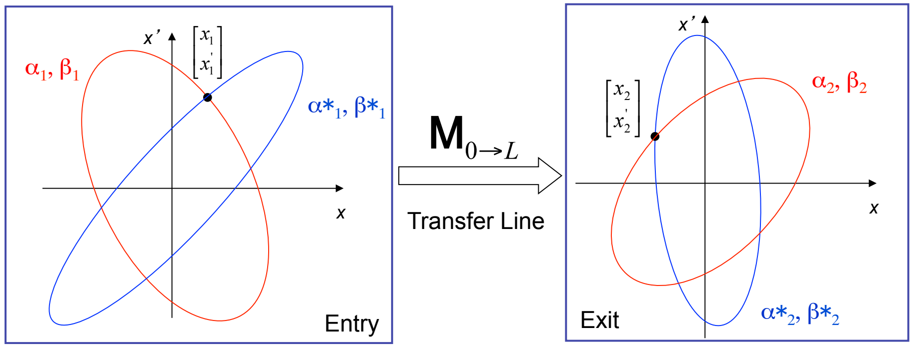


Non-Gaussian beam
(e.g. slow extracted)

- Propagation of this beam ellipse depends on the line
- A transfer line optics is different for different input beams:
  - Synchrotrons are often multi-purpose, accelerating different beams but extracting through a common line transfer line: optics must switch to match the input and output conditions for each beam type

# Transfer line

- On a single pass of a finite transfer line there is no regular motion from entrance to exit
  - Periodicity is not enforced: it's actually a design choice
  - Infinite number of possible starting ellipses are transported to an infinite number of final ellipses



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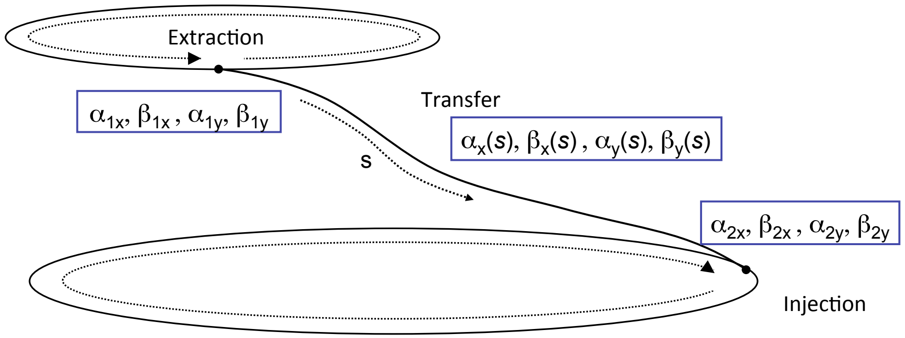
# Linking Machines



The Twiss parameters can be propagated when the transfer matrix **M** is known

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

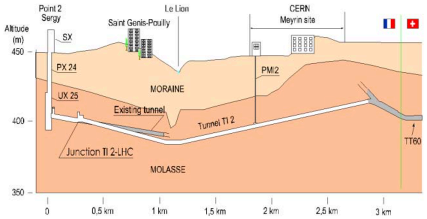
$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

---

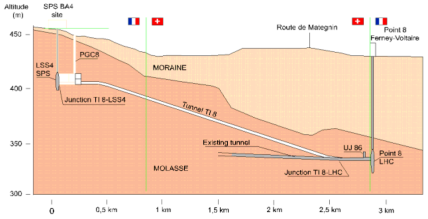
# Linking Machines

- Beams have to be transported from extraction of one machine to injection of the next machine:
  - Trajectory must be matched in all 6 geometric degrees of freedom (x,y,z,θ,Φ,ψ)

- Other important constraints can include:
  - Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology or other obstacles, etc.



SPS to LHC transfer tunnel TI2 (Beam 1)



SPS to LHC transfer tunnel TI8 (Beam 2)

An example of how geology can influence transfer line design

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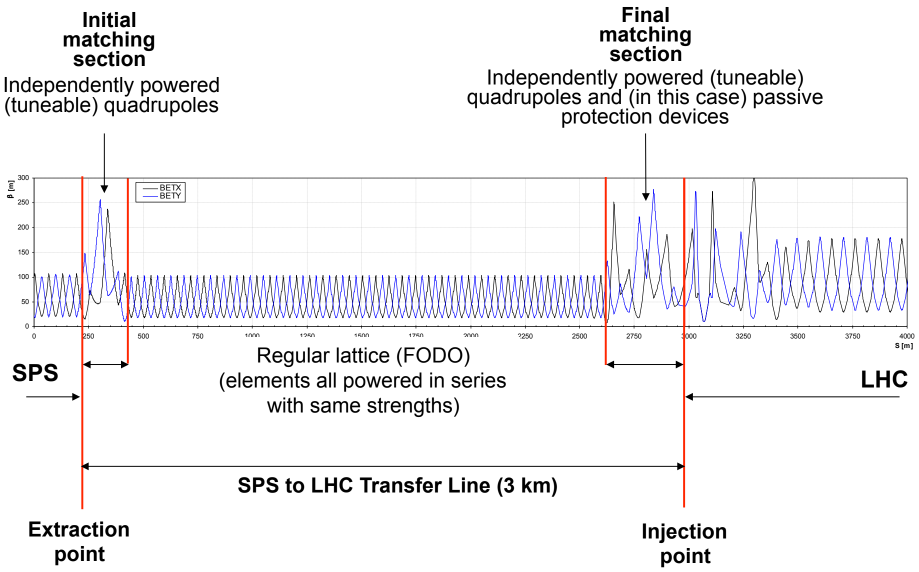
# Linking Machines

- Linking the optics is a complicated process:
  - Parameters at start of line have to be propagated to matched parameters at the end of the line (injection to another machine, fixed target etc. )
  - Need to "match" 8 variables ($\alpha_x$, $\beta_x$, $D_x$, $D'_x$ and $\alpha_y$, $\beta_y$, $D_y$, $D'_y$)
  - Matching done with number of independently power ("matching") quadrupoles
  - Maximum β and D values are imposed by magnetic apertures
  - Other constraints exist:
    - Phase conditions for collimators
    - Insertions for special equipment like stripping foils
    - Low beam energy (β<<1) re-bunching cavities might be necessary, i.e. RF gymnastics in the transfer line

- Matching with computer codes and relying on mixture of theory, experience, intuition, trial and error.

# Optics Matching



**Initial matching section**

Independently powered (tuneable) quadrupoles

**Final matching section**

Independently powered (tuneable) quadrupoles and (in this case) passive protection devices

**SPS**

Regular lattice (FODO) (elements all powered in series with same strengths)

**LHC**

**SPS to LHC Transfer Line (3 km)**

**Extraction point**

**Injection point**

---

# Blow-up from betatron mismatch

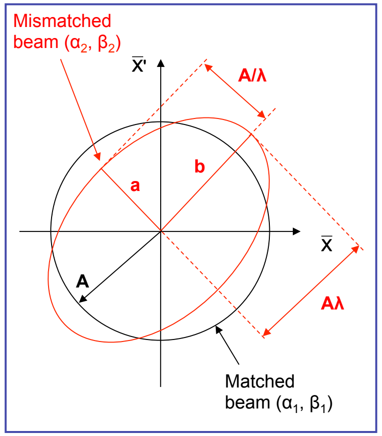
- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch
- Filamentation will produce an emittance increase
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse
- The emittance after filamentation:

$$\varepsilon_{diluted} = \frac{\varepsilon_{matched}}{2}\left(\lambda^2 + \frac{1}{\lambda^2}\right) \quad \text{where} \quad \lambda = \sqrt{b/a}$$

- Writing $\lambda$ as a function of the matched and mismatched Twiss parameters is an exercise in geometry:

$$\varepsilon_{diluted} = \frac{1}{2}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)\varepsilon_{matched}$$

See appendix for derivation



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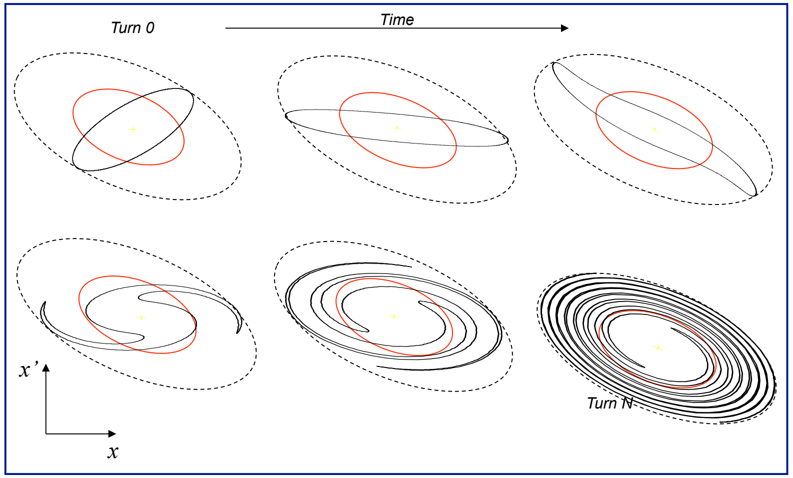
# Optical Mismatch at Injection

- Filamentation fills larger ellipse with same shape as matched ellipse



- Dispersion mismatch at injection will also cause emittance blow-up

---

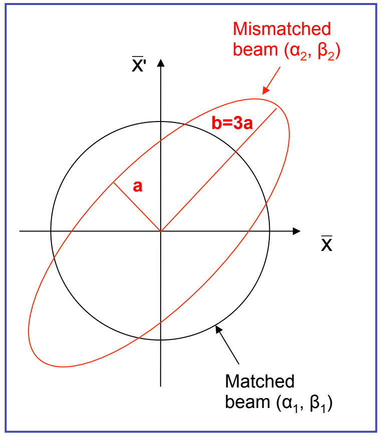
# Blow-up from betatron mismatch

- A numerical example…
- Consider $b = 3a$ for the mismatched ellipse:

$$\lambda = \sqrt{b/a} = \sqrt{3}$$

$$\varepsilon_{diluted} = \frac{\varepsilon_{matched}}{2}\left(\lambda^2 + \frac{1}{\lambda^2}\right)$$

$$= 1.67\varepsilon_{matched}$$



# Blow-up from dispersion mismatch

- Dispersion mismatch will also introduce emittance blow-up through filamentation much like optical mismatch
- Introducing normalised dispersion: $D_n = \frac{D}{\sqrt{\beta}}$ $\quad D'_n = \frac{\alpha}{\sqrt{\beta}} D + \sqrt{\beta} D'$
- With a momentum error of $\delta = \frac{\Delta p}{p}$ the mismatch is:

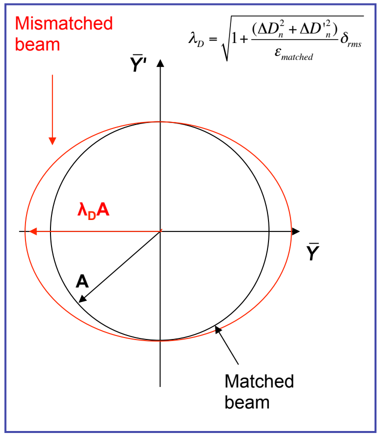
$$\bar{X} = \bar{X} + \Delta D_n \delta \qquad \bar{X}' = \bar{X}' + \Delta D'_n \delta$$

- Rotating the reference frame to a convenient reference (see plot):

$$\bar{Y} = \bar{Y} + \sqrt{\Delta D_n^2 + \Delta D'^2_n}\,\delta \qquad \bar{Y}' = \bar{Y}'$$

- And averaging over a distribution of particles, one can write the emittance blow-up as:

$$\varepsilon_{diluted} = \varepsilon_{matched} + \frac{\Delta D_n^2 + \Delta D'^2_n}{2} \delta_{rms}^2$$



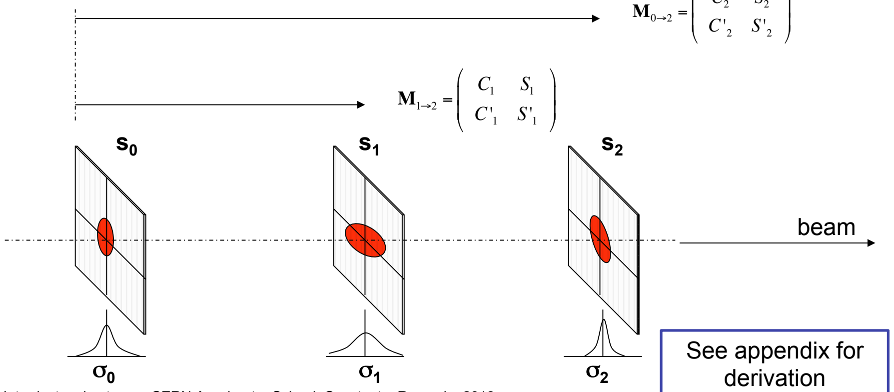
$$\lambda_D = \sqrt{1 + \frac{(\Delta D_n^2 + \Delta D'^2_n)}{\varepsilon_{matched}} \delta_{rms}}$$

# Optics measurement with 3 screens

- Assume 3 screens in a dispersion free region and that the emittance is constant along the line:

$$\varepsilon = \frac{\sigma_0^2}{\beta_0} = \frac{\sigma_1^2}{\beta_1} = \frac{\sigma_2^2}{\beta_2}$$

- Measurements of σ at $s_1$, $s_2$, $s_3$ plus knowledge of the two transfer matrices $M_{12}$ and $M_{13}$ allows determination of ε, α and β



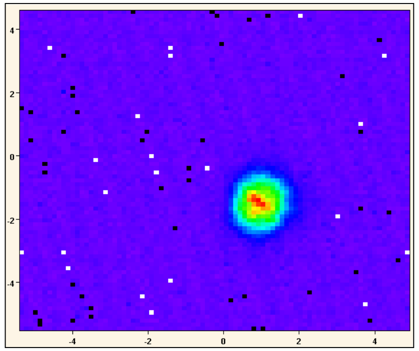
$$\mathbf{M}_{0\to 2} = \begin{pmatrix} C_2 & S_2 \\ C'_2 & S'_2 \end{pmatrix}$$

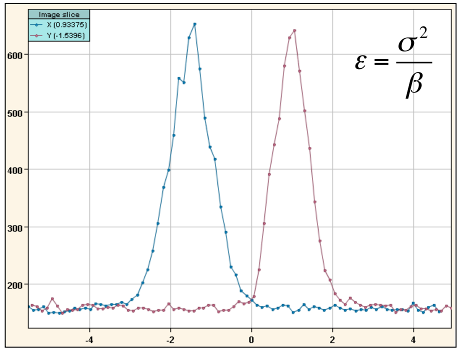
$$\mathbf{M}_{1\to 2} = \begin{pmatrix} C_1 & S_1 \\ C'_1 & S'_1 \end{pmatrix}$$

See appendix for derivation

# Optics measurement with screens

- A profile monitor is needed to measure the beam size
  - e.g. beam screen (luminescent) provides 2D density profile of the beam
- Profile fit gives transverse beam size: σ
- If optics (Twiss parameters) are known, ε can be calculated from a single screen:





$$\varepsilon = \frac{\sigma^2}{\beta}$$

# Summary

- Depending on the injection/extraction concept we chose a dedicated combination of septa (spatial separation of fields) and kickers (temporal separation of fields)
- Transfer lines present interesting challenges and differences from circular machines:
  - No periodic condition mean optics is defined by transfer line element strengths and by initial beam ellipse
  - Matching is subject to many constraints
  - Emittance blow-up is an important consideration, and arises from several sources: mis-steering, mismatch (betatron and dispersion)
  - Measurement of beam parameters is important for ensuring beams are well matched between machines and/or experiments

**Thank you for your attention**

# Bibliography for Septa

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- M.J. Barnes, B. Balhan, J. Borburgh, T. Fowler, B. Goddard, W.J.M. Weterings, A. Ueda, “**Development of an Eddy Current Septum for LINAC4**”, EPAC 2008.
- J. Borburgh, B. Balhan, T. Fowler, M. Hourican, W.J.M. Weterings, “**Septa and Distributor Developments for H- Injection into the Booster from Linac**4”, EPAC 2008.
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- J.M. Cravero & J.P. Royer, “**The New Pulsed Power Converter for the Septum Magnet in the PS Straight Section 42**”, CERN PS/PO/ Note 97-03, 1997.
- J.P. Royer, “**High Current with Precision Flat-Top Capacitor Discharge Power Converters for Pulsed Septum Magnets**”, CERN/PS 95-13 (PO), 1995.

# Example parameters for kickers at CERN

| **Kicker** Location | Beam momentum (GeV/c) | # Magnets | Gap Height [$V_{ap}$] (mm) | Current (kA) | Impedance (Ω) | Rise Time (ns) | Total Deflection (mrad) |
|---|---|---|---|---|---|---|---|
| CTF3 | 0.2 | 4 | 40 | 0.056 | 50 | ~4 | 1.2 |
| PS Inj. | 2.14 | 4 | 53 | 1.52 | 26.3 | 42 | 4.2 |
| SPS Inj. | 13/26 | 16 | 54 to 61 | 1.47/1.96 | 16.67/12.5 | 115/200 | 3.92 |
| SPS Ext. (MKE4) | 450 | 5 | 32 to 35 | 2.56 | 10 | 1100 | 0.48 |
| LHC Inj. | 450 | 4 | 54 | 5.12 | 5 | 900 | 0.82 |
| LHC Abort | 450 to 7000 | 15 | 73 | 1.3 to 18.5 | 1.5 (not T-line) | 2700 | 0.275 |

# Bibliography for Kickers

- M.J. Barnes, L. Ducimetiére, T. Fowler, V. Senaj, L. Sermeus, “**Injection and extraction magnets: kicker magnets**”, CERN Accelerator School CAS 2009: Specialised Course on Magnets, Bruges, 16-25 June 2009, arXiv:1103.1583 [physics.acc-ph].
- D. Fiander, K.D. Metzmacher, P.D. Pearce, “**Kickers and Septa at the PS complex**, CERN”, Prepared for KAON PDS Magnet Design Workshop, Vancouver, Canada, 3-5 Oct 1988, pp71-79.
- M.J. Barnes, G.D. Wait, I.M. Wilson, “**Comparison of Field Quality in Lumped Inductance versus Transmission Line Kicker Magnets**”, EPAC 1994, pp2547-2549.
- G. Kotzian, M. Barnes, L. Ducimetière, B. Goddard, W. Höfle, “**Emittance Growth at LHC Injection from SPS and LHC**”, LHC Project Report 1116.
- J. N. Weaver et al., “**Design, Analysis and Measurement of Very Fast Kicker Magnets at SLAC**,” Proc of 1989 PAC, Chicago, pp. 411–413.
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- W. Zhang, J. Sandberg, J. Tuozzolo, R. Cassel, L. Ducimetière, C. Jensen, M.J. Barnes, G.D. Wait, J. Wang, “**An Overview of High Voltage Dielectric Material for Travelling Wave Kicker Magnet Application**”, proc. of 25th International Power Modulator Conference and High Voltage Workshop, California, June 30-July 3, 2002, pp674-678.
- J. Bonthond, J.H. Dieperink, L. Ducimetikrre, U. Jansson, E. Vossenberg, “**Dual Branch High Voltage Pulse Generator for the Beam Extraction of the Large Hadron Collider**”, 2002 Power Modulator Symposium, Holloywood, USA, 30 June-3 July 2002, pp114-117.

# Example parameters for septa at CERN

| **Septum** Location | Beam momentum (GeV/c) | Gap Height (mm) | Max. Current (kA) | B (T) | Deflection (mrad) | Septum thickness (mm) |
|---|---|---|---|---|---|---|
| LEIR/AD/CTF (13 systems) | Various | 25 to 55 | 1 DC to 40 pulsed | 0.5 to 1.6 | up to 130 | 1.7 - 19.2 |
| PS Booster (6 systems) | 1.4 | 25 to 60 | 28 pulsed | 0.1 to 0.6 | up to 80 | 1 – 15 |
| PS complex (8 systems) | 26 | 20 to 60 | 2.5 DC to 33 pulsed | 0.2 to 1.2 | up to 55 | 3 - 11.2 |
| SPS Ext. | 450 | 20 | 24 | 1.5 | 2.25 | 4.2 - 17.2 |

# Blow-up from betatron mismatch

- General betatron motion:

$$x_2 = \sqrt{a_2\beta_2}\,\sin(\varphi+\varphi_o), \qquad x'_2 = \sqrt{a_2/\beta_2}\left[\cos(\varphi+\varphi_o) - \alpha_2\sin(\varphi+\varphi_o)\right]$$

- Applying the normalisation transformation for the matched beam…

$$\begin{bmatrix} \mathbf{X}_2 \\ \mathbf{X'}_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

…an ellipse is obtained in normalised phase space:

$$A^2 = \mathbf{X}_2^2 \underbrace{\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right]}_{\gamma_{new}} + \mathbf{X'}_2^2 \underbrace{\frac{\beta_2}{\beta_1}}_{\beta_{new}} - 2\mathbf{X}_2\mathbf{X'}_2 \underbrace{\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]}_{\alpha_{new}}$$

# Blow-up from betatron mismatch

- We can evaluate the square of the distance of a particle from the origin as:

$$\mathbf{A}_{new}^2 = \overline{\mathbf{X}}_{new}^2 + \overline{\mathbf{X}}'^2_{new} = \lambda^2 \cdot \mathbf{A}_0^2 \sin^2(\phi+\phi_1) + \frac{1}{\lambda^2}\mathbf{A}_0^2\cos^2(\phi+\phi_1)$$

- The new emittance is the average for all particles with positions Ai over all phases:

$$\varepsilon_{diluted} = \frac{1}{2}\left\langle \mathbf{A}_{new}^2 \right\rangle = \frac{1}{2}\left(\lambda^2\left\langle \mathbf{A}_0^2 \sin^2(\varphi+\varphi_1)\right\rangle + \frac{1}{\lambda^2}\left\langle \mathbf{A}_0^2\cos^2(\varphi+\varphi_1)\right\rangle\right)$$

$$= \frac{1}{2}\left\langle \mathbf{A}_0^2\right\rangle\left(\lambda^2\left\langle \cancelto{1/2}{\sin^2(\varphi+\varphi_1)}\right\rangle + \frac{1}{\lambda^2}\left\langle \cancelto{1/2}{\cos^2(\varphi+\varphi_1)}\right\rangle\right) = \frac{1}{2}\varepsilon_0\left(\lambda^2 + \frac{1}{\lambda^2}\right)$$

- If we're feeling diligent, we can substitute back for λ:

$$\varepsilon_{diluted} = \frac{1}{2}\varepsilon_{matched}\left(\lambda^2 + \frac{1}{\lambda^2}\right) = H\varepsilon_{matched} = \frac{1}{2}\varepsilon_{matched}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

where subscript 1 refers to the matched and 2 refers to mismatched cases

# Blow-up from betatron mismatch

- From general ellipse properties one can write:

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \quad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right) \quad \text{where} \quad H = \frac{1}{2}\left(\gamma_{new} + \beta_{new}\right)$$

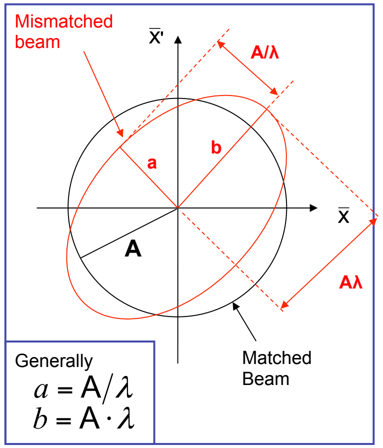
Giving:

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right),$$

$$\frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

- The co-ordinates of the mismatched beam can be expressed:

$$\mathbf{X}_{new} = \lambda \cdot \mathbf{A}\sin(\phi+\phi_1), \qquad \mathbf{X'}_{new} = \frac{1}{\lambda}\mathbf{A}\cos(\phi+\phi_1)$$



# Optics measurement with 3 screens

- Remember how we propagate Twiss parameters from $s_0$ to $s_1$:

$$\begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} C_1^2 & -2C_1S_1 & S_1^2 \\ -C_1C_1' & C_0S_0' + S_0C_0' & -S_1S_1' \\ C_1'^2 & -2C_1'S_1' & S_1'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

- Giving us three simultaneous equations and three unknowns $\varepsilon_0$, $\alpha_0$ and $\beta_0$:

$$\left.\begin{aligned} \beta_0 &= C_0^2\cdot\beta_0 - 2C_0S_0\cdot\alpha_0 + S_0^2\cdot\gamma_0 \\ \beta_1 &= C_1^2\cdot\beta_0 - 2C_1S_1\cdot\alpha_0 + S_1^2\cdot\gamma_0 \\ \beta_2 &= C_2^2\cdot\beta_0 - 2C_2S_2\cdot\alpha_0 + S_2^2\cdot\gamma_0 \end{aligned}\right| \times \boldsymbol{\varepsilon} \rightarrow \boxed{\begin{aligned} \sigma_0^2 &= \beta_0\varepsilon \\ \sigma_1^2 &= C_1^2\cdot\beta_0\varepsilon - 2C_1S_1\cdot\alpha_0\varepsilon + S_1^2\cdot\frac{(1+\alpha_0^2)}{\beta_0}\varepsilon \\ \sigma_2^2 &= C_2^2\cdot\beta_0\varepsilon - 2C_2S_2\cdot\alpha_0\varepsilon + S_2^2\cdot\frac{(1+\alpha_0^2)}{\beta_0}\varepsilon \end{aligned}}$$

- After a bit of algebra… we find:

$$\alpha_0 = -\frac{\beta_0}{2}W \qquad W = \frac{(\sigma_2/\sigma_0)^2/S_2^2 - (\sigma_1/\sigma_0)^2/S_1^2 - (C_2/S_2)^2 + (C_1/S_1)^2}{(C_1/S_1) - (C_2/S_2)}$$

# Optics measurement with 3 screens

- Some (more) algebra with the above equations and we can finally express the beta function at the first screen:

$$\beta_0 = 1 \Big/ \left| \sqrt{(\sigma_2/\sigma_0)^2 \big/ S_2^2 - (C_2/S_2)^2 + W(C_2/S_2)^2 - W^2/4} \right|$$

- And therefore also the emittance and the divergence of the beta function:

$$\varepsilon = \frac{\sigma_0^2}{\beta_0} \qquad \alpha_0 = \frac{\beta_0}{2} W$$

- Other methods of emittance measurement:
  - Extension of the above method to multiple screens: tomography
  - Quad scan: same as above but use one screen and change $M_{\text{quad}\rightarrow\text{screen}}$
  - Direct measurements (lower intensity/energy beams):
    - slit-grid or pepper-pot, laser “wire” for H- beams