FFA Accelerators
‘Fixed Field Alternating Gradient’

CERN Introductory Accelerator School
Constanta, Romania, September 2018

Dr. Suzie Sheehy
Royal Society University Research Fellow
John Adams Institute for Accelerator Science
University of Oxford

Many thanks to Dr. S. Machida for his advice and previous lecture materials
‘Fixed Field Alternating Gradient’ Accelerators

- Are FFAs like a synchrotron or cyclotron?
  - EMMA non-scaling FFA
- Fixed field magnets
- Beam dynamics
- Scaling FFAs
- Advanced FFA types and optics
Motivation

• Many challenges for future accelerators:

- **High power**
  - Neutrons, muons, ADS

- **Reliable**
  - Medical, ADS

- **Flexible**
  - Is industry limited by existing technology?

- **Rapid acceleration**
  - Muon beams, Unstable nuclei

- **Cheap**
  - Hadron accelerators aren’t known for being cheap
Is an FFA like a synchrotron? (1)

“Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field … which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations” - Marcus Oliphant, 1943

Image courtesy of ISIS, STFC
Is an FFA like a synchrotron? (2)

\[ \sin\left(\frac{\theta}{2}\right) = \frac{B(t)L}{2(B(t)\rho)} \]

\[ \theta \approx \frac{B(t)L}{\rho(t)/q} \]

What happens if I don’t ramp the B field with E?
Is an FFA like a synchrotron? (3)

\[ B_y = gx \]

Do we also ramp the quadrupoles in a synchrotron?

\[ k = \frac{g}{p/q} \]

‘normalised gradient’ of quad

\[ \frac{1}{f} = \frac{L(dB(t)/dx)}{p(t)/q} \]
The ‘EMMA’ accelerator

42 Quadrupole doublets
10-20 MeV e-
Demonstrates ‘non-scaling’ FFA

‘Electron Model for Many Applications’ = EMMA
Built and commissioned at STFC Daresbury Laboratory, UK
EMMA doesn’t ramp the B field with time

‘Fixed Field Alternating Gradient’ = FFA

Quadrupole with radial offset creates bending component

Note: this is just like a ‘combined function’ magnet

Figure 2: Orbits in a quadrupole doublet cell.

M. Craddock, PAC’07

It doesn’t ramp up the magnetic field with energy
Fixed-field magnets have advantages

• Simple power supplies and no synchronisation issues

• You can accelerate very quickly (as fast as your RF allows…)

  • in EMMA and in muon FFAs this is ~10 turns

• Higher repetition rate, so higher average current.
Is an FFA like a cyclotron? (1)

It has fixed field magnets too

The particles spiral outward as they gain energy
Is an FFA like a cyclotron? (2)

**Weak focusing**

Simultaneous radial and axial focusing: **Weak focusing**

\[ 0 \leq n \approx -\frac{\partial B_z}{\partial x} \leq 1 \]

slightly decreasing field

**Horizontal focusing** \( n < 1 \) means:
- \( 0 < n < 1 \) \( B_z \) can slightly decrease
- \( n < 0 \) \( B_z \) can increase as much as wanted

**Vertical focusing** \( n > 0 \) means:
- \( B_z \) should decrease with the radius

*F. Chautard*
Is an FFA like a cyclotron? (3)

You may have heard of ‘flutter’ in an AVF cyclotron
An FFA has:
• Flutter so large that the field reverses sign between ‘hills’ and ‘valleys’.
• Also: FFA has a field gradient with radius

In the AVF cyclotron the weak focusing is still important, but in the FFA the dynamics is controlled by the strong focusing
The circular fixed-field accelerator family

AG

Focusing

Weak focusing

Classical Cyclotron

Isochronous Cyclotrons

Fixed Freq.

Synchro-cyclotrons

Variable RF

RF modulation

FFAs
But that’s not the whole story…

• So an FFA is like a synchrotron but with fixed-field magnets

• OR like a cyclotron with a field gradient and strong focusing, (and variable RF frequency**)

• But that’s not all there is to it…

**FFAs do not always have variable RF frequency…
## Circular Accelerators

<table>
<thead>
<tr>
<th></th>
<th>Cyclotron</th>
<th>Synchrotron</th>
<th>FFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolution time</td>
<td>Constant</td>
<td>Variable</td>
<td>Variable (except relativistic)</td>
</tr>
<tr>
<td>Orbit radius</td>
<td>Variable</td>
<td>Constant</td>
<td>Variable</td>
</tr>
<tr>
<td>Transverse focusing</td>
<td>Variable</td>
<td>Constant</td>
<td>Variable</td>
</tr>
</tbody>
</table>
What does variable focusing mean?

- In a synchrotron the tune is fixed away from resonance lines.
- But in an FFA, the betatron tunes can vary…
Resonance crossing

\[ n \nu_x + m \nu_y = 0, 1, 2 \ldots \]

There are many resonance lines in tune space.

Normally, particles would be lost on resonance, but if the resonance is weak and the crossing is fast the beam can survive.
Results from EMMA

Orbit and tune shift with momentum

No beam ‘blowup’ despite resonance crossing

Can you have an FFA with fixed RF frequency?

- Suppose we choose rf frequency that is synchronized with revolution frequency at the center.

In the first half of a cycle, a particle lags behind the rf. At the center momentum, a particle is synchronized with rf. In the second half, a particle lags again.

If the total time lag is less than a half of rf cycle, a beam has net energy gain.

Figure 2: Orbits in a quadrupole doublet cell.
If the RF voltage is sufficient, we can accelerate over the whole energy range.

Similar to acceleration in a cyclotron but with imperfect isochronicity.

This is called ‘serpentine’ acceleration and was demonstrated in EMMA.

But that’s not the whole story…

• Electrons & muons are easy to accelerate quickly, but for hadrons it’s harder…

• If resonance crossing could be harmful for hadron FFAs, what can we do to fix it?

• In a synchrotron, off-momentum tune variations = chromaticity

• Can we have stable tunes in an FFA?
Scaling FFA

• In fact, the first FFAs had constant tunes and were designed not to cross resonances, we call them ‘scaling’ FFAs

The orbits are made ‘similar’

\[
\frac{\partial}{\partial p} \left( \frac{\rho}{\rho_0} \right)_{\theta=\text{const.}} = 0
\]

\[
\rho_0 \quad \text{Average bending radius}
\]

\[
\rho \quad \text{Local bending radius}
\]

\[
\theta \quad \text{Generalised azimuth}
\]

The ‘field index’ is constant

\[
\frac{\partial k}{\partial p} \bigg|_{\theta=\text{const.}} = 0
\]

\[
k = \frac{r}{B} \left( \frac{\partial B}{\partial r} \right)
\]

\[
B_y(r)
\]
Scaling FFA

- If the field profile is of this form, the ‘cardinal conditions’ are satisfied.
- We call this type of FFA a ‘Scaling’ type.
- Alternating magnets have opposite bending fields

\[ B_y = B_0 \left( \frac{r}{r_0} \right)^k F(\theta) \]

Note that this field profile does NOT satisfy isochronicity (see M. Seidel’s cyclotron lecture)

\[ \omega = \frac{eB}{m\gamma} \neq \text{const.} \]
The FFA is not so new…

1956

Chandrasekhar
Bohr
Scaling FFA types

\[ B(r, \theta) = B_0 \left( \frac{r}{r_0} \right)^k F(\vartheta) \]

\[ F(\vartheta) = F\left( \theta - \tan \zeta \cdot \ln \frac{r}{r_0} \right) \]

Spiral sector type

Spiral angle gives strong edge focusing.

\[ \Delta p_z = \frac{e}{\gamma} \int_{\gamma x = 0} \left( -v_x B_z \right) dx = -eB_0 \tan \zeta \cdot z \]

Recent Scaling FFAs

- In the late 90’s and in 2000’s, the FFA idea was re-awakened in Japan,
- Particular focus on hadron FFAs of scaling type

Proof of Principle machine finished in 1999 at KEK, demonstrated 1kHz rep. rate

3-stage FFA for ADSR studies:
- 2.5 MeV spiral (ion beta) FFA with induction cores
- 25 MeV radial (booster) FFA with RF
- 150 MeV radial (main) FFA with RF
Technology for scaling FFAs

Magnetic Alloy (MA) Cavity

- Large aperture
- High shunt impedance
- Low Q - can cover large range of frequencies.

Image credit: A. Takagi

Image credit: Y. Mori,
Aside: Injection/extraction

• How do we inject/extract beams without a time dependent field?
• Well, pulsed kickers/septum can still be used.
• Can also exploit the orbit movement with acceleration.
# Circular Accelerators

<table>
<thead>
<tr>
<th></th>
<th>Cyclotron</th>
<th>Synchrotron</th>
<th>Non-scaling FFA</th>
<th>Scaling FFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolution time</td>
<td>Constant</td>
<td>Variable (except relativistic)</td>
<td>Variable (small)</td>
<td>Variable</td>
</tr>
<tr>
<td>Orbit radius</td>
<td>Variable</td>
<td>Constant</td>
<td>Variable (small)</td>
<td>Variable</td>
</tr>
<tr>
<td>Transverse focusing</td>
<td>Variable</td>
<td>Constant</td>
<td>Variable</td>
<td>Constant</td>
</tr>
</tbody>
</table>
A quick summary…

• ‘Scaling’ type is a very specific type of FFA. Anything else is the ‘non-scaling’ type.

• EMMA is a linear non-scaling FFA, which again is quite specific.

• …Are there any other possibilities?
Advanced FFA optics (1)

“There are other variations of these designs which preserve betatron oscillation stability, hold $\nu_x$ and $\nu_y$ constant, but do not retain the property of similar of equilibrium orbits.”

“The magnet edges of focusing and defocusing sectors can be made non-radial, and the fields in positive- and negative- field magnets made different functions of radius”


Tune-stable non-scaling FFA designs have been developed

\[
B_z = B_{z0} \left( \frac{r_0 + r}{r_0} \right)^k = B_{z0} \left( 1 + \sum_{n=1}^{k} \frac{k(k-1)\ldots(k-n+1)}{n!} \frac{r^n}{r_0^n} \right)
\]

Rectangular magnets, Simplified field profile Higher stability region (S. Machida, S. Sheehy)
Advanced FFA optics (2)

Radial designs with edge profiles
(C. Johnstone)

Vertical orbit excursion FFA
(S. Brooks)
Current status of FFA designs

- A whole spectrum of designs have emerged in the last 5-10 years

Potential applications include:
- Accelerator Driven Subcritical Reactor
- Boron Neutron Capture Therapy
- Proton/carbon therapy
- Accelerator-based Neutron Source
- Emittance/Energy Recovery with Internal Target (ERIT)
  - e-RHIC injector
  - Muon or neutrino factory source
  - + many more…
Summary

• FFAs are just a generalisation of synchrotrons or cyclotrons
• Two main types ‘scaling’ and ‘non-scaling’
  • Scaling: specific optics and orbit requirements put a strict requirement on the field profile (zero-chromaticity)
  • Non-scaling: removes these restrictions, very general type (chromatic)
• FFAs may be suitable for many future applications

• In my view, the next big challenge is demonstrating high power operation
Reading List

- Proceedings of the FFA (formerly called FFAG) workshops

Notes on FFAs from CAS schools: