Linear Accelerators

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LINAC APPLICATIONS

- Injectors for synchrotrons
- Medical applications: radiotherapy
- Industrial applications
  - Nuclear waste treatment and controlled fission for energy production (ADS)
  - Spallation sources for neutron production
  - Material testing for fusion nuclear reactors
- National security
- Material treatment
- Ion implantation
- Material/food sterilization

~10^4 LINACs operating around the world
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

- **Particle source**
- **Accelerating structures**
- **Focusing elements:** quadrupoles and solenoids
- **Longitudinal dynamics of accelerated particles**
- **Transverse dynamics of accelerated particles**

**1st PART OF THE LECTURE**

**2nd PART OF THE LECTURE**

**LINAC COMPONENTS AND TECHNOLOGY**
The overall LINAC has to be designed to obtain the desired beam parameters in term of:
- output energy/energy spread
- beam current (charge)
- long and transverse beam dimensions/divergence (emittance)

Having, in general, constraints in term of:
- space
- cost
- power consumption
- available power sources
...
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

- Longitudinal dynamics of accelerated particles
- Transverse dynamics of accelerated particles
- Particle source
- Accelerating structures
- Focusing elements: quadrupoles and solenoids
- Accelerated beam
The basic equation that describes the acceleration/bending/focusing processes is the Lorentz Force. Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

\[
\frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]

- \( \vec{p} = \text{momentum} \)
- \( m = \text{mass} \)
- \( \vec{v} = \text{velocity} \)
- \( q = \text{charge} \)

**ACCELERATION**
To accelerate, we need a force in the direction of motion.

- 2\textsuperscript{nd} term always perpendicular to motion => no energy gain

**BENDING AND FOCUSING**

**LORENTZ FORCE: ACCELERATION AND FOCUSING**

**Longitudinal Dynamics**

**Transverse Dynamics**
The first historical linear particle accelerator was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. Electrons emitted by the heated cathode were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced X-rays.

Particle energies are typically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt: 1 eV=1.6x10^{-19} J

\[
\frac{d\vec{p}}{dt} = q\vec{E} \quad \Rightarrow \quad \Delta E = q\Delta V
\]

\[\vec{p} = \text{momentum}\]
\[q = \text{charge}\]
\[E = \text{energy}\]
PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

Single particle

- rest mass $m_0$
- rest energy $E_0 (= m_0 c^2)$
- total energy $E$
- relativistic mass $m$
- velocity $v$
- momentum $p (= m v)$
- Kinetic energy $W = E - E_0$

**Relativistic factor**

- $\beta = v/c \ (<1)$
- $\gamma = E/E_0 \ (\geq 1)$

- $E^2 = E_0^2 + p^2 c^2$

**Relativistic factor**

- $\beta = \sqrt{1 - 1/\gamma^2}$
- $\gamma = 1/\sqrt{1 - \beta^2}$

- $W = (\gamma - 1)m_0 c^2 \approx \frac{1}{2} m_0 v^2 \text{ if } \beta \ll 1$

$\Rightarrow$ **Light particles** (as electrons) are practically fully relativistic ($\beta \approx 1$, $\gamma \gg 1$) at relatively low energy and reach a constant velocity ($\sim c$). The acceleration process occurs at constant particle velocity.

$\Rightarrow$ **Heavy particles** (protons and ions) are typically weakly relativistic and reach a constant velocity only at very high energy. The velocity changes a lot during acceleration process.

$\Rightarrow$ This implies important differences in the technical characteristics of the accelerating structures. In particular for protons and ions we need different types of accelerating structures, optimized for different velocities and/or the accelerating structure has to vary its geometry to take into account the velocity variation.
To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a dielectric belt transporting positive charges to an isolated electrode hosting an ion source. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.

**LIMITS OF ELECTROSTATIC ACCELERATORS**

DC voltage as large as ~10 MV can be obtained (E~10 MeV). The main limit in the achievable voltage is the breakdown due to insulation problems.

**APPLICATIONS OF DC ACCELERATORS**

DC particle accelerators are in operation worldwide, typically at \( V<15\text{MV} \) (\( E_{\text{max}}=15\text{MeV} \)), \( I<100\text{mA} \). They are used for:

- material analysis
- X-ray production,
- ion implantation for semiconductors
- first stage of acceleration (particle sources)

750 kV Cockcroft-Walton Linac2 injector at CERN from 1978 to 1992
RF ACCELERATORS: WIDERÖE “DRIFT TUBE LINAC” (DTL)
(protons and ions)

Basic idea: the particles are accelerated by the electric field in the gap between electrodes connected alternatively to the poles of an AC generator. This original idea of Ising (1924) was implemented by Wideroe (1927) who applied a sine-wave voltage to a sequence of drift tubes. The particles do not experience any force while travelling inside the tubes (equipotential regions) and are accelerated across the gaps. This kind of structure is called Drift Tube LINAC (DTL).

⇒ If the length of the tubes increases with the particle velocity during the acceleration such that the time of flight is kept constant and equal to half of the RF period, the particles are subject to a synchronous accelerating voltage and experience an energy gain of $\Delta E = q\Delta V$ at each gap crossing.

⇒ In principle a single RF generator can be used to indefinitely accelerate a beam, avoiding the breakdown limitation affecting the electrostatic accelerators.

⇒ The Wideroe LINAC is the first RF LINAC.
ACCELERATION: ENERGY GAIN

We consider the acceleration between two electrodes in DC.

\[ E^2 = E_0^2 + p^2 c^2 \Rightarrow 2EdE = 2pdp c^2 \Rightarrow dE = \frac{mc^2}{E} dp \Rightarrow dE = v dp \]

\[ \frac{dp}{dt} = qE_z \Rightarrow v \frac{dp}{dz} = qE_z \Rightarrow \frac{dE}{dz} = qE_z \] (and also \( \frac{dW}{dz} = qE_z \)) \hspace{1cm} W = E - E_0

\[ \Rightarrow \Delta E = \int_{\text{gap}} \frac{dE}{dz} dz = \int_{\text{gap}} qE_z dz \Rightarrow \Delta E = q \Delta V \]

rate of energy gain per unit length

energy gain per electrode
RF ACCELERATION: BUNCHEDED BEAM

We consider now the acceleration between two electrodes fed by an RF generator.

\[ \Delta V = V_{RF} \cos(\omega_{RF}t) \quad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}} \]

\[ \Rightarrow \Delta E = q\hat{V}_{acc} \cos(\omega_{RF}t_{inj}) \]

Only these particles are accelerated.

These particles are not accelerated and basically are lost during the acceleration process.

**DC acceleration**

**RF acceleration**

_Bunched beam_ (in order to be synchronous with the external AC field, particles have to be gathered in non-uniform temporal structure)
We consider now the acceleration between two electrodes fed by an RF generator.

\[
\Delta V = V_{RF} \cos(\omega_{RF} t) \quad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}
\]

\[
V_{RF} = \int_{\text{gap}} E_{RF}(z) dz 
\]

\[
E_z(z, t) = E_{RF}(z) \cos(\omega_{RF} t)
\]

\[
E_z(z, t)_{\text{seen by particle}} = E_{RF}(z) \cos[\omega_{RF} (t + t_{inj})] = E_{RF}(z) \cos(\omega_{RF} t + \phi_{inj})
\]

\[
\phi_{inj} = \omega_{RF} t_{inj}
\]

Hyp. of symmetric accelerating field

\[
\Delta E = q \int_{\text{gap}} E_z(z, t)_{\text{seen by particle}} dz = \int_{-L/2}^{+L/2} E_{RF}(z) \cos\left(\omega_{RF} \frac{z}{v} + \phi_{inj}\right) dz
\]

\[
\hat{V}_{acc} = \int_{\text{gap}} E_{RF}(z) dz
\]

\[
V_{RF} = \int_{\text{gap}} E_{RF}(z) dz
\]

\[
\int E_{RF}(z) dz
\]

\[
\cos(\phi_{inj})
\]

\[
\text{Peak gap voltage}
\]

\[
\text{Transit time factor}
\]

\[
\hat{E}_{acc} = \frac{\hat{V}_{acc}}{L} \quad \text{Average accelerating field in the gap}
\]

\[
E_{acc} = \frac{V_{acc}}{L} \quad \text{Average accelerating field seen by the particle}
\]
If now we consider a DTL structure with an injected particle at an energy $E_{in}$, we have that at each gap the maximum energy gain is $\Delta E_n = qV_{acc}$ and the particle increase its velocity accordingly to the previous relativistic formulae.

\[
E_n = E_{in} + nqV_{acc}
\]

\[
v_n = c\beta_n = c\sqrt{1 - \frac{1}{\gamma_n^2}} = c\sqrt{1 - \left(\frac{E_o}{E_n}\right)^2}
\]

\[
\Delta V = V_{RF} \cos(\omega_{RF} t)
\]

$\Rightarrow$ In order to be synchronous with the accelerating field at each gap the length of the n-th drift tube has to be $L_n$:

\[
t_n = \frac{L_n}{v_n} = \frac{T_{RF}}{2} \Rightarrow L_n = \frac{1}{2} v_n T_{RF} = \frac{1}{2} \beta_n c T_{RF}
\]

\[
L_n = \frac{1}{2} \beta_n \lambda_{RF}
\]

$\Rightarrow$ The energy gain per unit length (i.e. the average accelerating gradient) is given by:

\[
\frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = \frac{2qV_{acc}}{\lambda_{RF} \beta_n}
\]
ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important consequences of the previous obtained formulae:

\[ L_n = \frac{1}{2} \beta_n \lambda_{RF} \]

The condition \( L_n << \lambda_{RF} \) (necessary to model the tube as an equipotential region) requires \( \beta << 1 \). \( \Rightarrow \) The Wideröe technique can not be applied to relativistic particles.

\[ \frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = qE_{acc} = \frac{2qV_{acc}}{\lambda_{RF} \beta_n} \]

Moreover when particles get high velocities the drift spaces get longer and one looses on the efficiency. The average accelerating gradient \( (E_{RF} \text{ [V/m]}) \) increase pushes towards small \( \lambda_{RF} \) (high frequencies).

\( \Rightarrow \) The solution consists of enclosing the system in a cavity which resonant frequency matches the RF generator frequency.

\( \Rightarrow \) Each cavity can be independently powered from the RF generator.

High frequency high power sources became available after the 2nd world war pushed by military technology needs (such as radar). However, the concept of equipotential DT can not be applied at small \( \lambda_{RF} \) and the power lost by radiation is proportional to the RF frequency.

As a consequence we must consider accelerating structures different from drift tubes.
High frequency RF accelerating fields are confined in cavities.

The cavities are metallic closed volumes where the e.m fields have a particular spatial configuration (resonant modes) whose components, including the accelerating field $E_z$, oscillate at some specific frequencies $f_{RF}$ (resonant frequency) characteristic of the mode.

The modes are excited by RF generators that are coupled to the cavities through waveguides, coaxial cables, etc...

The resonant modes are called Standing Wave (SW) modes (spatial fixed configuration, oscillating in time).

The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) by solving the Maxwell equations with the proper boundary conditions.
Alvarez's structure can be described as a special DTL in which the electrodes are part of a resonant macrostructure.

⇒ The DTL operates in 0 mode for protons and ions in the range $\beta = 0.05-0.5$ ($f_{RF} = 50-400$ MHz) 1-100 MeV;

⇒ The beam is inside the "drift tubes" when the electric field is decelerating. The electric field is concentrated between gaps;

⇒ The drift tubes are suspended by stems;

⇒ Quadrupole (for transverse focusing) can fit inside the drift tubes.

⇒ In order to be synchronous with the accelerating field at each gap the length of the n-th drift tube $L_n$ has to be:

$$L_n = \beta_n \lambda_{RF}$$
CERN LINAC 2 tank 1:  
200 MHz 7 m x 3 tanks, 1 m diameter, final energy 50 MeV.

CERN LINAC 4: 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes,  
Energy: 3 MeV to 50 MeV, $\beta=0.08$ to 0.31 → cell length from 68mm to 264mm.
When the β of the particles increases (>0.5) one has to use **higher RF frequencies** (>400-500 MHz) to increase the accelerating gradient per unit length.

⇒ **the DTL structures became less efficient** (effective accelerating voltage per unit length for a given RF power);

**Real cylindrical cavity**
(TM$_{010}$-like mode because of the shape and presence of beam tubes)

Cylindrical single or multiple cavities working on the **TM$_{010}$-like** mode are used.

For a **pure cylindrical structure** (also called pillbox cavity) the first accelerating mode (i.e. with non zero longitudinal electric field on axis) is the **TM$_{010}$ mode**. It has a well known analytical solution from Maxwell equation.

$$f_{\text{res}} = \frac{c}{2.405a}$$

$$E_z = AJ_0\left(\frac{2.405}{a}\right)\cos(\omega_{RF}t)$$

$$H_\theta = A\frac{1}{Z_0}J_1\left(\frac{2.405}{a}\right)\sin(\omega_{RF}t)$$
ACCELERATING VOLTAGE (\(V_{\text{acc}}\))

DISSIPATED POWER (\(P_{\text{diss}}\))

STORED ENERGY (\(W\))

**SHUNT IMPEDANCE**

The shunt impedance is the parameter that qualifies the **efficiency of an accelerating mode**. The higher is its value, the larger is the obtainable accelerating voltage for a given power. Traditionally, it is the quantity to optimize in order to **maximize the accelerating field for a given dissipated power**:

\[
R = \frac{\hat{V}_{\text{acc}}^2}{P_{\text{diss}}} \quad [\Omega]
\]

**SHUNT IMPEDANCE PER UNIT LENGTH**

\[
r = \frac{\left(\frac{\hat{V}_{\text{acc}}}{L}\right)^2}{P_{\text{diss}}/L} = \frac{\hat{E}_{\text{acc}}^2}{P_{\text{diss}}/L} \quad [\Omega/m]
\]

NC cavity R~1M\(\Omega\)

SC cavity R~1T\(\Omega\)

**QUALITY FACTOR**

\[
Q = \omega_{RF} \frac{W}{P_{\text{diss}}}
\]

Example:

R~1M\(\Omega\)

\(P_{\text{diss}}\)=1 MW

\(V_{\text{acc}}\)=1 MV

For a cavity working at 1 GHz with a structure length of 10 cm we have an average accelerating field of 10 MV/m
MULTI-CELL SW CAVITIES
(electrons or protons and ions at high energy)

- In a multi-cell structure there is one RF input coupler. As a consequence the total number of RF sources is reduced, with a simplification of the layout and reduction of the costs;

- The shunt impedance is n time the impedance of a single cavity

- They are more complicated to fabricate than single cell cavities;

- The fields of adjacent cells couple through the cell irises and/or through properly designed coupling slots.
**MULTI-CELL SW CAVITIES: \( \pi \) MODE STRUCTURES**  
(*electrons or protons and ions at high energy*)

- The N-cell structure behaves like a system composed by **N coupled oscillators** with **N coupled multi-cell resonant modes**.

- The modes are characterized by a cell-to-cell phase advance given by:
  \[
  \Delta \phi_n = \frac{n\pi}{N - 1} \quad n = 0, 1, \ldots, N - 1
  \]

- The multi cell mode generally used for acceleration is the **\( \pi \), \( \pi/2 \) and 0 mode** (DTL as example operate in the 0 mode).

- In this case as done for the DTL structures the cell length has to be chosen in order to synchronize the accelerating field with the particle traveling into the structure at a certain velocity.

\[
\Rightarrow \text{For ions and protons the cell length has to be increased and the linac will be made of a sequence of different accelerating structures matched to the ion/proton velocity.}
\]

\[
\Rightarrow \text{For electron, } \beta=1, \ d=\lambda_{RF}/2 \text{ and the linac will be made of an injector followed by a series of identical accelerating structures, with cells all the same length.}
\]
MODE STRUCTURES: EXAMPLES

LINAC 4 (CERN) PIMS (PI Mode Structure) for protons: \( f_{RF} = 352 \) MHz, \( \beta > 0.4 \)

Each module has 7 identical cells

European XFEL (Desy): electrons

800 accelerating cavities
1.3 GHz / 23.6 MV/m

All identical \( \beta = 1 \)
Superconducting cavities

Cryomodule housing: 8 cavities, quadrupole and BPM
MULTI-CELL SW CAVITIES: $\pi/2$ MODE STRUCTURES
(electrons or protons and ions at high energy)

⇒ It is possible to demonstrate that over a certain number of cavities (>10) working on the $\pi$ mode, the overlap between adjacent modes can be a problem (as example the field uniformity due to machining errors is difficult to tune).

⇒ The criticality of a working mode depend on the frequency separation between the working mode and the adjacent mode.

⇒ the $\pi/2$ mode from this point of view is the most stable mode. For this mode it is possible to demonstrate that the accelerating field is zero every two cells. For this reason the empty cells are put of axis and coupling slots are opened from the accelerating cells to the empty cells.

⇒ this allow to increase the number of cells to >20-30 without problems.

\[ f_{RF} = 800 - 3000 \text{ MHz for proton (} \beta = 0.5-1) \text{ and electrons} \]
Spallation Neutron Source Coupled Cavity Linac (protons)

4 modules, each containing 12 accelerator segments CCL and 11 bridge couplers. The CCL section is a RF Linac, operating at 805 MHz that accelerates the beam from 87 to 186 MeV and has a physical installed length of slightly over 55 meters.
TRAVELLING WAVE (TW) STRUCTURES (electrons)

⇒ To accelerate charged particles, the electromagnetic field must have an electric field along the direction of propagation of the particle.

⇒ The field has to be synchronous with the particle velocity.

⇒ Up to now we have analyzed the cases standing standing wave (SW) structures in which the field has basically a given profile and oscillate in time (as example in DTL or resonant cavities operating on the TM$_{010}$-like).

\[ E_z(z, t) = E_{RF}(z) \cos(\omega_{RF} t) \]

⇒ There is another possibility to accelerate particles: using a travelling wave (TW) structure in which the RF wave is co-propagating with the beam with a phase velocity equal to the beam velocity.

⇒ Typically these structures are used for electrons because in this case the phase velocity can be constant all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low $\beta$ particle that changes its velocity during acceleration.
TW CAVITIES: CIRCULAR WAVEGUIDE AND DISPERSION CURVE

*electrons*

In TW structures an e.m. wave with $E_z \neq 0$ travel together with the beam in a special guide in which the phase velocity of the wave matches the particle velocity ($v$). In this case the beam absorbs energy from the wave and it is continuously accelerated.

As example if we consider a simple circular waveguide the first propagating mode with $E_z \neq 0$ is the TM$_{01}$ mode. Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this constant cross section waveguide will never be synchronous with a particle beam since the phase velocity is always larger than the speed of light $c$.

$$E_z^n_{TM_{01}} = E_0(r) \cos(\omega_{RF} t - k^* z)$$

$$v_{ph} = \frac{\omega_{RF}}{k^*} > c$$

$$\omega = 2\pi f \quad (f = \text{RF generator frequency})$$
In order to slow-down the wave phase velocity, iris-loaded periodic structure have to be used.

**CIRCULAR WAVEGUIDE**

MODE TM\(_{01}\)  

\[
E_z \big|_{TM_{01}} = E_0(r) \cos(\omega_{RF} t - k^* z)
\]

IRIS LOADED STRUCTURE

MODE TM\(_{01}\)-like  

Periodic in z of period D  

\[
E_z \big|_{TM_{01}\text{-like}} = \hat{E}_{acc}(r, z) \cos(\omega_{RF} t - k^* z)
\]

\[
\frac{\omega}{k} = c
\]

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⇒ The field in this kind of structures is that of a special wave travelling within a spatial periodic profile.

⇒ The structure can be designed to have the phase velocity equal to the speed of the particles.

⇒ This allows acceleration over large distances (few meters, hundred of cells) with just an input coupler and a relatively simple geometry.
APPENDIX: TW CONSTANT GRADIENT STRUCTURES

In a TW structure, the RF power enters into the cavity through an input coupler, flows (travels) through the cavity in the same direction as the beam and an output coupler at the end of the structure is connected to a matched power load.

If there is no beam, the input power reduced by the cavity losses goes to the power load where it is dissipated.

In the presence of a large beam current, however, a fraction of the TW power is transferred to the beam.

In a purely periodic structure, made by a sequence of identical cells (also called “constant impedance structure”), the RF power flux and the intensity of the accelerating field decay exponentially along the structure:

\[
\hat{E}_{acc}(z) = E_0 e^{-\alpha z}
\]

It is possible to demonstrate that, in order to keep the accelerating field constant along the structure, the iris apertures have to decrease along the structure.
LINAC TECHNOLOGY
ACCELERATING CAVITY TECHNOLOGY

The structures are powered by RF generators (like klystrons).

The cavities (and the related LINAC technology) can be of different material:

- **copper** for normal conducting (NC, both SW than TW) cavities;
- **Niobium** for superconducting cavities (SC, SW);

We can choose between NC or the SC technology depending on the required performances in term of:

- **accelerating gradient** (MV/m);
- **RF pulse length** (how many bunches we can contemporary accelerate);
- **Duty cycle**: pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- **Average beam current**.

Dissipated power into the cavity walls is related to the surface currents

\[ P_{diss} = \int_{\text{cavity wall}} \frac{1}{2} R_s H_{tan}^2 dS \]

Between copper and Niobium there is a factor \(10^5-10^6\)
The most widely used NC metal for RF structures is **OFHC copper** (Oxygen free high conductivity) for several reasons:

1. Easy to machine (good achievable roughness at the few nm level)
2. Easy to braze/weld
3. Easy to find at relatively low cost
4. Very good electrical (and thermal) conductivity
5. Low SEY (multipacting phenomena)
6. Good performances at high accelerating gradient

**SC: NIOBIUM**

The most common material for SC cavities is Nb because:

1. Nb has a relatively **high transition temperature** \((T_c=9.25 \text{ K})\).
2. SC can be destroyed by magnetic field greater than a critical field \(H_c \Rightarrow\) Pure Nb has a **relatively high critical magnetic field** \(H_c=170-180 \text{ mT}\).
3. It is chemically inert
4. It can be **machined and deep-drawn**
5. It is available as bulk and sheet material in any size, fabricated by forging and rolling....

**APPENDIX: NC AND SC MATERIALS**

- **Higher dissipation**
- **Pulsed** operation
- **Higher peak accelerating gradient** (up to 50-100 MV/m)
- **Standard cleaning** procedures for the cavity fabrication
- **Cooling** of dissipated power with pipes

- **lower dissipation**
- **Allow continuous operation**
- **lower peak accelerating gradient** (max 30-40 MV/m)
- **Special cleaning procedures** for the cavity fabrication
- They need a **cryostat** to reach the SC temperature of few K
The “beam structure” in a LINAC is directly related to the “RF structure”. There are basically two possible types of operations:

- CW (continuous wave) ⇒ allow, in principle, to operate with a continuous beam
- PULSED OPERATION ⇒ there are RF pulses at a certain repetition rate (Duty Cycle (DC)=pulsed width/period)

⇒ Because of the very low power dissipation and low RF power required to achieve a certain accelerating voltage the SC structures allow operation at very high Duty Cycle (DC) up to a CW operation with high gradient (>20 MV/m).
⇒ On the other hand NC structures can operate in pulsed mode at very low DC with higher peak field (TW structures can >50-80 MV/m peak field).
⇒ NC structures can also operate in CW but at very low gradient because of the dissipated power.
The cells and couplers are fabricated with milling machines and lathes starting from OFHC forged or laminated copper with precisions that can be of the order of few um and surface roughness <50 nm. The cells are then piled up and brazed together in vacuum or hydrogen furnace using different alloys at different temperatures (700-1000 C) and/or in different steps.
APPENDIX: FABRICATION PROCESS SC SW STRUCTURES

**Nb** is available as **bulk and sheet material** in any size, fabricated by forging and rolling. **High Purity Nb** is made by **electron beam melting** under good vacuum. The most common fabrication techniques for the cavities are to **deep draw or spin half-cells**. **Alternative techniques** are: hydroforming, spinning an entire cavity out of single sheet or tube and Nb sputtering.

After forming the parts are **electron beam welded** together.

**CAVITY TREATMENT**

The cavity treatment after the welding is quite complicated and require several steps between:

- buffered **chemical polishing (BCP)**, **electropolishing** and etching to remove surface damaged layers of the order of 100 \( \mu m \)
- **rinsed with ultraclean water** also at high pressure (100 bar)
- **Thermal treatments** up to >1000 C to diffuse H\(_2\) out of the material increasing the Nb purity (RRR)
- **high-temperature treatment** with Ti getter (post-purification)
- RF tuning

**WHY**

- Clean welding
- RRR enhancement
- Remove contamination and damage layer
- Get rid of hydrogen
- Remove diffusion layer (O, C, N)
- e.g., remove S particles due to EP
- Get rid of dust particles
- Ancillaries: antennas, couplers, vacuum ports...
- Decrease high field losses (Q-drop)
- Get rid of "re-contamination"?
- Cavity's performance
- Decrease field emission

**Forming**
**EB Welding**
**Ti purification**
**Chemical etching 100-200 \( \mu m \)**
**Annealing 800°C, 2h (or 600°C, 10h)**
**Chemical etching 5-20 \( \mu m \)**
**High pressure rinsing (HPR)**
**Assembling**
**Baking, 120°C, 48h**
**Post processing**
**Test RF**
**He processing, HPP**
EXAMPLES: EUROPEAN XFEL

Nominal Energy | GeV | 17.5
Beam pulse length | ms | 0.60
Repetition rate | Hz | 10
Max. # of bunches per pulse | nS | 2700
Min. bunch spacing | ns | 220
Bunch charge | nC | 1
Bunch length, $\sigma_z$ | $\mu$m | < 20
Emittance (slice) at undulator | $\mu$rad | < 1.4
Energy spread (slice) at undulator | MeV | 1

101 cryomodules in total

RF system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)

Cryomodule housing: 8 cavities, quadrupole and BPM

800 accelerating cavities
1.3 GHz / 23.6 MV/m
EXAMPLE: SWISSFEL LINAC (PSI)

Courtesy T. Garvey
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

- **Particle source**
- **Accelerating structures**
- **Focusing elements: quadrupoles and solenoids**
- **Longitudinal dynamics of accelerated particles**
- **Transverse dynamics of accelerated particles**

**LINAC BEAM DYNAMICS**

**LINAC COMPONENTS AND TECHNOLOGY**
ELECTRON SOURCES: RF PHOTO-GUNS

RF guns are used in the first stage of electron beam generation in FEL and acceleration.

- Multi cell: typically 2-3 cells
- SW $\pi$ mode cavities
- operate in the range of 60-120 MV/m cathode peak accelerating field with up to 10 MW input power.
- Typically in L-band- S-band (1-3 GHz) at 10-100 Hz.
- Single or multi bunch (L-band)
- Different type of cathodes (copper,...)

The electrons are emitted on the **cathode** through a laser that hit the surface. They are then accelerated trough the electric field that has a longitudinal component on axis $TM_{010}$. 
RF PHOTO-GUNS: EXAMPLES

**LCLS**
- Frequency = 2,856 MHz
- Gradient = 120 MV/m
- Exit energy = 6 MeV
- Copper photocathode
- RF pulse length ~2 µs
- Bunch repetition rate = 120 Hz
- Norm. rms emittance
  - 0.4 mm·mrad at 250 pC

**PITZ L-band Gun**
- Frequency = 1,300 MHz
- Gradient = up to 60 MV/m
- Exit energy = 6.5 MeV
- Rep. rate 10 Hz
- Cs₂Te photocathode
- RF pulse length ~1 ms
- 800 bunches per macropulse
- Normalized rms emittance
  - 1 nC 0.70 mm·mrad
  - 0.1 nC 0.21 mm·mrad

Solenoids field are used to compensate the space charge effects in low energy guns. The configuration is shown in the picture.
**Basic principle:** create a plasma and optimize its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

**LINAC BEAM DYNAMICS**

- **Longitudinal dynamics of accelerated particles**
- **Transverse dynamics of accelerated particles**

**LINAC COMPONENTS AND TECHNOLOGY**

- Particle source
- Accelerating structures
- Focusing elements: quadrupoles and solenoids
- Accelerated beam
LORENTZ FORCE: ACCELERATION AND FOCUSING

Particles are accelerated through electric field and are bended and focalized through magnetic field. The basic equation that describe the acceleration/bending /focusing processes is the Lorentz Force.

\[ \frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

- \( \vec{p} = \text{momentum} \)
- \( m = \text{mass} \)
- \( \vec{v} = \text{velocity} \)
- \( q = \text{charge} \)

ACCELERATION

To accelerate, we need a force in the direction of motion.

BENDING AND FOCUSING

2\textsuperscript{nd} term always perpendicular to motion => no energy gain.

Longitudinal Dynamics

Transverse Dynamics
**MAGNETIC QUADRUPOLE**

Quadrupoles are used to **focalize the beam in the transverse plane**. It is a **4 poles magnet**:

⇒ **B=0** in the center of the quadrupole

⇒ The **B intensity increases linearly** with the off-axis displacement.

⇒ If the quadrupole is **focusing in one plane is defocusing in the other plane**

\[
\begin{align*}
B_x &= G \cdot y \\
B_y &= G \cdot x \\
F_y &= qvG \cdot y \\
F_x &= -qvG \cdot x
\end{align*}
\]

\[
G = \text{quadrupole gradient} \left[ \frac{T}{m} \right]
\]

**Electromagnetic quadrupoles** G <50-100 T/m

\[
\frac{F_B}{F_E} = v \Rightarrow \begin{cases} 
F_B(1T) = F_E \left(300 \frac{MV}{m} \right) @ \beta = 1 \\
F_B(1T) = F_E \left(3 \frac{MV}{m} \right) @ \beta = 0.01
\end{cases}
\]
Also solenoids can be used for focalization of beams (in particular electron beams).

Particles that enter into a solenoidal field with a transverse component of the velocity (divergence) start to **spiralize describing circular trajectories**. Useful for beam emittance compensation at low energies.
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.

**Longitudinal dynamics of accelerated particles**

**Transverse dynamics of accelerated particles**

**Particle source**

**Accelerating structures**

**Accelerated beam**

**Focusing elements: quadrupoles and solenoids**
Let us consider a SW linac structure made by accelerating gaps (like in DTL) or cavities.

In each gap we have an accelerating field oscillating in time and an integrated accelerating voltage \( V_{\text{acc}} \) still oscillating in time than can be expressed as:

\[
V_{\text{acc}} = \hat{V}_{\text{acc}} \cos(\omega_{RF} t + \theta)
\]

Let’s assume that the “perfect” synchronism condition is fulfilled for a phase \( \phi_s \) (called synchronous phase). This means that a particle (called synchronous particle) entering in a gap with a phase \( \phi_s = \omega_{RF} t_s \) with respect to the RF voltage receive a energy gain (and a consequent change in velocity) that allow entering in the subsequent gap with the same phase \( \phi_s \) and so on.

For this particle the energy gain in each gap is:

\[
\Delta E = q\hat{V}_{\text{acc}} \cos(\phi_s + \theta) = qV_{\text{acc}_s}
\]

Obviously both \( \phi_s \) and \( \phi_s^* \) are synchronous phases.
Let us consider now the first synchronous phase $\phi_s$ (on the positive slope of the RF voltage). If we consider another particle “near” to the synchronous one that arrives later in the gap ($t_1 > t_s$, $\phi_1 > \phi_s$), it will see an higher voltage, it will gain an higher energy and an higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially compensating its initial delay.

Similarly if we consider another particle “near” to the synchronous one that arrives before in the gap ($t_1 < t_s$, $\phi_1 < \phi_s$), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.

On the contrary if we consider now the synchronous particle at phase $\phi_s^*$ and another particle “near” to the synchronous one that arrives later or before in the gap, it will receive an energy gain that will increase further its distance form the synchronous one

The choice of the synchronous phase in the positive slope of the RF voltage provides longitudinal focusing of the beam: phase stability principle.

The synchronous phase on the negative slope of the RF voltage is, on the contrary, unstable.

Relying on particle velocity variations, longitudinal focusing does not work for fully relativistic beams (electrons). In this case acceleration “on crest” is more convenient.
ENERGY-PHASE EQUATIONS (1/2)
(protons and ions or electrons at extremely low energy)

In order to study the **longitudinal dynamics in a LINAC**, the following variables are used, which describe the generic particle **phase** (time of arrival) and **energy** with respect to the synchronous particle:

- Arrival time (phase) of a **generic particle** at a certain gap (or cavity)\( \varphi = \phi - \phi_s = \omega_{RF}(t - t_s) \)
- Arrival time (phase) of the **synchronous particle** at a certain gap (or cavity)
- Energy of a **generic particle** at a certain position along the linac\( W = E - E_s \)
- Energy of the **synchronous particle** at a certain position along the linac

The **energy gain per cell** (one gap + tube in case of a DTL) of a generic particle and of a synchronous particle are (we put \( \theta = 0 \) in the generic expression of the accelerating voltage just for simplicity):

\[
\begin{align*}
\Delta E_s &= q\hat{V}_{acc} \cos \phi_s \\
\Delta E &= q\hat{V}_{acc} \cos \phi = q\hat{V}_{acc} \cos(\phi_s + \phi)
\end{align*}
\]

subtracting

\[ \Delta w = \Delta E - \Delta E_s = q\hat{V}_{acc} \left[ \cos(\phi_s + \varphi) - \cos \phi_s \right] \]

Dividing by the accelerating cell length \( \Delta L \) and assuming that:

\[ \frac{\hat{V}_{acc}}{\Delta L} = \hat{E}_{acc} \]

Average accelerating field over the cell (or accelerating gradient)

\[ \frac{\Delta w}{\Delta L} = q\hat{E}_{acc} \left[ \cos(\phi_s + \varphi) - \cos \phi_s \right] \]

Approximating

\[ \frac{\Delta w}{\Delta L} \approx \frac{dw}{dz} = q\hat{E}_{acc} \left[ \cos(\phi_s + \varphi) - \cos \phi_s \right] \]
On the other hand we have that the **phase variation per cell** of a generic particle and of a synchronous particle are:

\[
\begin{align*}
\Delta \phi_s &= \omega_{RF} \Delta t_s \\
\Delta \phi &= \omega_{RF} \Delta t
\end{align*}
\]

$\Delta t$ is basically the time of flight between two accelerating cells.

$v, v_s$ are the average particles velocities.

\[\Delta \varphi = \omega_{RF} (\Delta t - \Delta t_s)\]

Dividing by the accelerating cell length $\Delta L$

\[
\frac{\Delta \varphi}{\Delta L} = \omega_{RF}\left( \frac{\Delta t}{\Delta L} - \frac{\Delta t_s}{\Delta L} \right) = \omega_{RF}\left( \frac{1}{v} - \frac{1}{v_s} \right) \approx \frac{\omega_{RF}}{cE_0 \beta_s^3 \gamma_s^3} \, w
\]

This system of coupled (non linear) differential equations **describe the motion of a non synchronous particles** in the longitudinal plane with respect to the synchronous one.

\[
\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0 \beta_s^3 \gamma_s^3} \, w
\]

\[
\frac{dw}{dz} = q\hat{E}_{acc} [\cos(\phi_s + \varphi) - \cos \phi_s]
\]

**MAT**

\[
\omega_{RF}\left( \frac{1}{v} - \frac{1}{v_s} \right) = \omega_{RF}\left( \frac{v_s - v}{vv_s} \right) \quad \geq \quad -\frac{\omega_{RF}}{v_s^2} \Delta v = -\frac{\omega_{RF}}{c} \frac{\Delta \beta}{\beta_s^2} \quad \text{remembering that} \quad \beta = \sqrt{1-1/\gamma^2} \Rightarrow \beta d\beta = d\gamma/\gamma^3 \Rightarrow -\frac{\omega_{RF}}{c} \frac{\Delta \beta}{\beta_s^2} \equiv -\frac{\omega_{RF}}{c} \frac{\Delta \gamma}{\beta_s^3 \gamma_3} = -\frac{\omega_{RF}}{c} \frac{\Delta E}{E_0 \beta_s \gamma_s^3}
\]
SMALL AMPLITUDE ENERGY-PHASE OSCILLATIONS
(protons and ions or electrons at extremely low energy)

\[
\frac{dw}{dz} = qE_{\text{acc}}[\cos(\phi_s + \varphi) - \cos\phi_s]
\]

Assuming small oscillations around the synchronous particle that allow to approximate \(\cos(\phi_s + \varphi) - \cos\phi_s \approx \varphi \sin\phi_s\)

Deriving both terms with respect to \(z\) and assuming an adiabatic acceleration process i.e. a particle energy and speed variations that allow to consider

\[
\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w
\]

\[
\frac{d^2\varphi}{dz^2} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3} \frac{dw}{dz}
\]

\[
\frac{d^2\varphi}{dz^2} + q\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}E_{\text{acc}} \sin(-\phi_s) \varphi = 0
\]

harmonic oscillator equation

\[\Omega_s^2 > 0 \implies \sin(-\phi_s) > 0\]

\[V_{\text{acc}} > 0 \implies \cos\phi_s > 0\]

\[\Rightarrow \frac{\pi}{2} < \phi_s < 0\]

\[\Omega_s^2 \approx \frac{\omega_{RF}E_{\text{acc}}}{cE_0\beta_s^3\gamma_s^3} \varphi = 0\]

if we accelerate on the rising part of the positive RF wave we have a longitudinal force keeping the beam bunched around the synchronous phase.

\[\phi = \hat{\phi}\cos(\Omega_s z)\]

\[w = \hat{w}\sin(\Omega_s z)\]
ENdGY-PHASE OSCILLATIONS IN PHASE SPACE
(protons and ions or electrons at extremely low energy)

The energy-phase oscillations can be drawn in the longitudinal phase space:

\[
\begin{align*}
\varphi &= \hat{\varphi} \cos(\Omega_s z) \\
\hat{w} &= \hat{w} \sin(\Omega_s z)
\end{align*}
\]

⇒ The trajectory of a generic particle in the longitudinal phase space is an ellipse.

⇒ The maximum energy deviation is reached at \( \varphi=0 \) while the maximum phase excursion corresponds to \( w=0 \).

⇒ The bunch occupies an area in the longitudinal phase space called longitudinal emittance and the projections of the bunch in the energy and phase planes give the energy spread and the bunch length.
APPENDIX: LARGE OSCILLATIONS AND SEPARATRIX

To study the longitudinal dynamics at large oscillations, we have to consider the non linear system of differential equations without approximations. By neglecting particle energy and speed variations along the LINAC (adiabatic acceleration) it is possible to easily obtain the following relation between $w$ and $\phi$ that is the Hamiltonian of the system related to the total particle energy:

$$\frac{1}{2} \left( \frac{\omega_{RF}}{cE_0\beta^3_s\gamma^3_s} \right)^2 w^2 + \frac{\omega_{RF} q\hat{E}_{acc}}{cE_0\beta^3_s\gamma^3_s} \left[ \sin(\phi_s + \phi) - \varphi \cos \phi_s - \sin(\phi_s) \right] = \text{const} = H$$

For each $H$ we have different trajectories in the longitudinal phase space.

The oscillations are stable within a region bounded by a special curve called separatrix: its equation is:

$$\frac{1}{2} \frac{\omega_{RF}}{cE_0\beta^3_s\gamma^3_s} w^2 + \frac{q\hat{E}_{acc}}{cE_0\beta^3_s\gamma^3_s} \left[ \sin(\phi_s + \phi) - (2\varphi_s + \varphi) \cos \phi_s + \sin(\phi_s) \right] = 0$$

The region inside the separatrix is called RF bucket. The dimensions of the bucket shrinks to zero if $\phi_s=0$.

Trajectories outside the RF buckets are unstable.

We can define the RF acceptance as the maximum extension in phase and energy that we can accept in an accelerator:

$$\Delta \phi_{MAX} \approx 3\phi_s$$

$$\Delta w_{MAX} = \pm 2 \left[ \frac{q c E_0 \beta^3_s \gamma^3_s \hat{E}_{acc} (\phi_s \cos \phi_s - \sin \phi_s)}{\omega_{RF}} \right]^{1/2}$$
To study the longitudinal dynamics at large oscillations, we have to consider the non-linear system of differential equations without approximations. By neglecting particle energy and speed variations along the LINAC we obtain:

\[
\frac{d^2 \varphi}{dz^2} = -\frac{\omega_{RF} q \hat{E}_{acc}}{c E_0 \beta_s^3 \gamma_s^3} \left[ \cos(\phi_s + \varphi) - \cos \phi_s \right] = F
\]

The restoring force \( F \) can not be considered purely elastic anymore and may be derived from a potential function according to the usual definition:

\[
U = -\int_0^\varphi F d\varphi' = \frac{\omega_{RF} q \hat{E}_{acc}}{c E_0 \beta_s^3 \gamma_s^3} \left[ \sin(\phi_s + \varphi) - \varphi \cos \phi_s - \sin(\phi_s) \right]
\]

With few simple passages we obtain an “energy conservation”–like law:

\[
\frac{d}{dz} \left[ \left( \frac{d\varphi}{dz} \right)^2 \right] = 2 \frac{d\varphi}{dz} \frac{d^2\varphi}{dz^2} = 2 \frac{d\varphi}{dz} \left( -\frac{dU}{d\varphi} \right) = -2 \frac{d}{dz} U \Rightarrow \frac{d}{dz} \left[ \left( \frac{d\varphi}{dz} \right)^2 + 2U \right] = 0 \Rightarrow \frac{1}{2} \left( \frac{d\varphi}{dz} \right)^2 + U = \text{const}
\]

\[
\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{c E_0 \beta_s^3 \gamma_s^3} w
\]

\[
\frac{1}{2} \left( \frac{\omega_{RF}}{c E_0 \beta_s^3 \gamma_s^3} \right)^2 w^2 + \frac{\omega_{RF} q \hat{E}_{acc}}{c E_0 \beta_s^3 \gamma_s^3} \left[ \sin(\phi_s + \varphi) - \varphi \cos \phi_s - \sin(\phi_s) \right] = \text{const} = H
\]
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS

From previous formulae it is clear that there is no motion in the longitudinal phase plane for ultrarelativistic particles ($\gamma >> 1$).

It is interesting to analyze what happen if we inject an electron beam produced by a cathode (at low energy) directly in a TW structure (with $v_{ph} = c$) and the conditions that allow to capture the beam (this is equivalent to consider instead of a TW structure a SW designed to accelerate ultrarelativistic particles at $v = c$).

Particles enter the structure with velocity $v < c$ and, initially, they are not synchronous with the accelerating field and there is a so-called slippage.

After a certain distance they can reach enough energy (and velocity) to become synchronous with the accelerating wave. This means that they are captured by the accelerator and from this point they are stably accelerated.

⇒ This is the case of electrons whose velocity is always close to speed of light $c$ even at low energies.

⇒ Accelerating structures are designed to provide an accelerating field synchronous with particles moving at $v = c$, like TW structures with phase velocity equal to $c$. 

If this does not happen (the energy increase is not enough to reach the velocity of the wave) they are lost.
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS:
PHASE SPLIPPAGE

The accelerating field of a TW structure can be expressed by

\[ E_{acc} = \hat{E}_{acc} \cos(O_{RF} t - k z) \phi(z,t) \]

The equation of motion of a particle with a position \( z \) at time \( t \) accelerated by the TW is then

\[ \frac{d}{dt}(mv) = q\hat{E}_{acc} \cos(\phi(z,t)) \Rightarrow m_0 c \frac{d}{dt}(\gamma \beta) = m_0 \gamma^3 \frac{d\beta}{dt} = q\hat{E}_{acc} \cos(\phi) \]

It is useful to find which is the relation between \( \beta \) and \( \phi \)

\[ \sin \phi_{fin} = \sin \phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q\hat{E}_{acc}} \left( \frac{1 - \beta_{in}}{\sqrt{1 + \beta_{in}} - \sqrt{1 + \beta_{fin}}} \right) \]

Suppose that the particle reach asymptotically the value \( \beta_{fin}=1 \) we have:

\[ \sin \phi_{fin} > \sin \phi_{in} \Rightarrow \phi_{fin} > \phi_{in} \]
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE ACCELERATING FIELD

For a given injection energy ($\beta_{in}$) and phase ($\phi_{in}$) we can find which is the accelerating field ($E_{acc}$) that is necessary to have the completely relativistic beam at phase fin (that is necessary to capture the beam at phase fin)

Example:
$E_{in} = 50$ keV, (kinetic energy), $\phi_{in} = -\pi/2$,
$\phi_{\infty} = 0 \Rightarrow \gamma_{in} \approx 1.1; \beta_{in} \approx 0.41$
$f_{RF} = 2856$ MHz $\Rightarrow \lambda_{RF} \approx 10.5$ cm

We obtain $E_{acc} \approx 20$ MV/m;

The minimum value of the electric field ($E_{acc}$) that allow to capture a beam. Obviously this correspond to an injection phase $\phi_{in} = -\pi/2$ and $\phi_{\infty} = \pi/2$.

Example: For the previous case we obtain: $E_{acc\_MIN} \approx 10$ MV/m;
During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by velocity modulation (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).
In order to increase the capture efficiency of a traveling wave section, pre-bunchers are often used. They are SW cavities aimed at pre-forming particle bunches gathering particles continuously emitted by a source.

Once the capture condition $E_{RF} > E_{RF,MIN}$ is fulfilled the fundamental equation of previous slide sets the ranges of the injection phases $\phi_{in}$ actually accepted. Particles whose injection phases are within this range can be captured the other are lost.

**BUNCHER AND CAPTURE SECTIONS**

( electrons )

Thermionic Gun

Buncher (SW Cavity)

Capture section (SW or TW)

Continuous beam with velocity modulation

Bunched beam

Bunched and captured beam

$\Rightarrow$ **Bunching** is obtained by modulating the energy (and therefore the velocity) of a continuous beam using the longitudinal E-field of a SW cavity. After a certain drift space the velocity modulation is converted in a density charge modulation. The density modulation depletes the regions corresponding to injection phase values incompatible with the capture process.

$\Rightarrow$ A TW accelerating structure (capture section) is placed at an optimal distance from the pre-buncher, to capture a large fraction of the charge and accelerate it till relativistic energies. The amount of charge lost is drastically reduced, while the capture section provide also further beam bunching.
APPENDIX: PHASE SPLIPPAGE CALCULATIONS

The accelerating field of a TW structure can be expressed by

\[ E_{acc} = \hat{E}_{acc} \cos(\omega_{RF} t - kz) \phi(z, t) \]

The equation of motion of a particle with a position \( z \) at time \( t \) accelerated by the TW is then

\[ \frac{d}{dt}(mv) = q\hat{E}_{acc} \cos(\phi(z, t)) \Rightarrow m_0c \frac{d}{dt}(\gamma \beta) = m_0c\gamma^3 \frac{d\beta}{dt} = q\hat{E}_{acc} \cos \phi \]

This is the phase of the TW wave seen by the particle at a certain time \( t \) and position \( z \).

The phase motion during acceleration is then

\[ \frac{d\phi}{dt} = \omega_{RF} - k \frac{dz}{dt} = \omega_{RF} - \frac{\omega_{RF}}{c} \frac{dz}{dt} \Rightarrow \frac{d\phi}{dt} = \omega_{RF} (1 - \beta) > 0 \]

The phase of the wave "seen" by the particle always increase because \( \beta < 1 \).

It is useful to find which is the relation between \( \beta \) and \( \phi \)

\[ m_0c\gamma^3 \frac{d\beta}{dt} = m_0c\gamma^3 \frac{d\beta}{d\phi} \frac{d\phi}{dt} = \frac{1}{(1 + \beta)\sqrt{1 - \beta^2}} \frac{d\beta}{d\phi} = \frac{q\hat{E}_{acc}}{\omega_{RF}m_0c} \cos \phi \Rightarrow \frac{1}{(1 + \beta)\sqrt{1 - \beta^2}}d\beta = \frac{q\hat{E}_{acc}}{\omega_{RF}m_0c} \cos \phi d\phi \]

We can integrate both sides between the initial and the final condition to find the dependence of phase on velocity during the acceleration process (using, as example, the new variable \( \beta = \cos \alpha \))

\[ \sin \phi_{fin} = \sin \phi_{in} + \frac{2\pi m_0c^2}{\lambda_{RF} q\hat{E}_{acc}} \left( \frac{1 - \beta_{in}}{\sqrt{1 + \beta_{in}}} - \frac{1 - \beta_{fin}}{\sqrt{1 + \beta_{fin}}} \right) \]
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.**

**LINAC BEAM DYNAMICS**

- Longitudinal dynamics of accelerated particles
- Transverse dynamics of accelerated particles

**LINAC COMPONENTS AND TECHNOLOGY**

- Particle source
- Accelerating structures
- Focusing elements: quadrupoles and solenoids
- Accelerated beam
The RF fields act on the transverse beam dynamics because of the transverse components of the E and B field.

\[ E_z(z, t) = E_{RF}(z) \cos(\omega_{RF} t + \phi) \]

⇒ According to Maxwell equations the divergence of the field is zero and this implies that in traversing one accelerating gap there is a focusing/defocusing term:

\[ \nabla \cdot E = 0 \]
\[ \nabla \times B = \frac{1}{c^2} \dot{E} \]

\[ F_r = q (E_r - vB_\theta) = -q \frac{r}{2} \left( \frac{\partial E_z}{\partial z} \right) - \frac{\beta}{c} \frac{\partial E_z}{\partial t} \]

\[ F_r|_E = -q \frac{r}{2} \frac{\partial E_{RF}(z)}{\partial z} \cos \left( \omega_{RF} \frac{z}{\beta c} + \phi \right) \]

\[ F_r|_B = q \frac{r}{2} \frac{\beta}{c} E_{RF}(z) \sin \left( \omega_{RF} \frac{z}{\beta c} + \phi \right) \]

-\( f_{RF} = 350 \text{ MHz} \)
-\( \beta = 0.1 \)
-\( L = 3 \text{ cm} \)
RF DEFOCUSING/FOCUSBING

From previous formulae it is possible to calculate the transverse momentum increase due to the RF transverse forces. Assuming that the velocity and position changes over the gap are small we obtain to the first order:

\[
\Delta p_r = \int_{-L/2}^{+L/2} F_r \frac{dz}{\beta c} = -\frac{\pi q \hat{E}_{acc} L \sin \phi}{c \gamma^2 \beta^2 \lambda_{RF}}
\]

⇒ transverse defocusing scales as \( \sim 1/\gamma^2 \) and disappears at relativistic regime (electrons)

⇒ At relativistic regime (electrons), moreover, we have, in general, \( \phi=0 \) for maximum acceleration and this completely cancel the defocusing effect

⇒ Also in the non relativistic regime for a correct evaluation of the defocusing effect we have to:

⇒ take into account the velocity change across the accelerating gap

⇒ the transverse beam dimensions changes across the gap (with a general reduction of the transverse beam dimensions due to the focusing in the first part)

Both effects give a reduction of the defocusing force
RF FOCUSING IN ELECTRON LINACS

- RF defocusing is negligible in electron linacs.
- There is a second order effect due to the non-synchronous harmonics of the accelerating field that give a focusing effect. These harmonics generate a ponderomotive force i.e. a force in an inhomogeneous oscillating electromagnetic field.

NON-SYNCHRONOUS RF HARMONICS: SIMPLE CASE OF SW STRUCTURE

\[ F_r = -q \frac{r}{2} \left( \frac{\partial E_z}{\partial z} + \beta \frac{\partial E_z}{\partial t} \right) \]

The Lorentz force is linear with the particle displacement.

Transverse equation of motion

\[ \ddot{r} = \frac{1}{\gamma m_0} \sum_n a_n \cos(n \omega t) \]

RF harmonics

Average focusing force

\[ \overline{F_r} = -r \left( q \hat{E}_{acc} \right)^2 \gamma m_0 c^2 \eta(\phi) \]

With accelerating gradients of few tens of MV/m can easily reach the level of MV/m²
COLLECTIVE EFFECTS: SPACE CHARGE AND WAKEFIELDS

Collective effects are all effects related to the number of particles and they can play a crucial role in the longitudinal and transverse beam dynamics.

**SPACE CHARGE**

- Effect of Coulomb repulsion between particles (space charge).
- These effects cannot be neglected especially at low energy and at high current because the space charge forces scale as $1/\gamma^2$ and with the current $I$.

EXAMPLE: Uniform and infinite cylinder of charge moving along $z$

$$\vec{F}_{SC} = q \frac{I}{2\pi \varepsilon_0 R_b^2 \beta c \gamma^2} r_q \hat{r}$$

In this particular case it is linear but in general it is a non-linear force.

**WAKEFIELDS**

The other effects are due to the wakefield. The passage of bunches through accelerating structures excites electromagnetic field. This field can have longitudinal and transverse components and, interacting with subsequent bunches (long range wakefield), can affect the longitudinal and the transverse beam dynamics. In particular the transverse wakefields, can drive an instability along the train called multibunch beam break up (BBU).

Several approaches are used to absorb these fields from the structures like loops, couplers, waveguides, beam pipe absorbers.
MAGNETIC FOCUSING AND CONTROL OF THE TRANSVERSE DYNAMICS

⇒ Defocusing RF forces, space charge or the natural divergence (emittance) of the beam need to be compensated and controlled by focusing forces.

⇒ Quadrupoles are focusing in one plane and defocusing on the other. A global focalization is provided by alternating quadrupoles with opposite signs.

⇒ In a linac one alternates accelerating sections with focusing sections.

⇒ The type of magnetic configuration and magnets type/distance depend on the type of particles/energies/beam parameters we want to achieve.

This is provided by quadrupoles along the beam line. At low energies also solenoids can be used.
Due to the alternating quadrupole focusing system each particle perform transverse oscillations along the LINAC.

The equation of motion in the transverse plane is of the type:

\[ \frac{d^2 x}{ds^2} + \left[ \kappa^2(s) + k_{RF}^2(s) \right] x - F_{SC} = 0 \]

Focusing period = length after which the structure is repeated (usually as N\(\beta\lambda\)).

The single particle trajectory is a pseudo-sinusoid described by the equation:

\[ x(s) = \sqrt{\varepsilon_0 \beta(s)} \cos \left[ \int_s^D \frac{ds}{\beta(s)} + \phi_0 \right] \]

The final transverse beam dimensions \((\sigma_{x,y}(s))\) varies along the linac and are contained within an envelope.

\[ \sigma = \int_D \frac{ds}{\beta(s)} \approx \frac{D}{\langle \beta \rangle} \]

Transverse phase advance per period. For stability should be \(0 < \sigma < \pi\).
In case of "smooth approximation" of the LINAC (we consider an average effect of the quadrupoles and RF) we obtain a simple harmonic motion along s of the type ($\beta$ is constant):

$$x(s) = \sqrt{\varepsilon} \sqrt{1/K_0} \cos (K_0 s + \phi_0)$$

If we consider also the SC contribution in the simple caso of an ellipsoidal beam (linear space charges) we obtain:

$$K_0 = \sqrt{\left(\frac{qGl}{2m_0 c \gamma \beta}\right)^2 - \frac{\pi q \hat{E}_{acc} \sin(-\phi)}{m_0 c^2 \lambda_{RF} (\gamma \beta)^3}} - \frac{3Z_0 q l \lambda_{RF} (1 - f)}{8\pi m_0 c^2 \beta^2 \gamma^3 r_x r_y r_z}$$

For ultrarelativistic electrons RF defocusing and space charge disappear and the external focusing is required to control the emittance and to stabilize the beam against instabilities.
GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (1/2)

⇒ Beam dynamics dominated by space charge and RF defocusing forces
⇒ Focusing is usually provided by quadrupoles
⇒ Phase advance per period ($\sigma$) should be, in general, in the range 30-80 deg, this means that, at low energy, we need a strong focusing term (short quadrupole distance and high quadrupole gradient) to compensate for the defocusing, but the limited space ($\beta \lambda$) limits the achievable G and beam current
⇒ As $\beta$ increases, the distance between focusing elements can increase ($\beta \lambda$ in the DTL goes from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV), and can be increased to 4-10$\beta \lambda$ at higher energy (>40 MeV).
⇒ A linac is made of a sequence of structures, matched to the beam velocity, and where the length of the focusing period increases with energy. As $\beta$ increases, longitudinal phase error between cells of identical length becomes small and we can have short sequences of identical cells (lower construction costs).
⇒ Keep sufficient safety margin between beam radius and aperture

Transverse (x) r.m.s. beam envelope along Linac4

![Graph showing beam envelope along Linac4 with different beta values and structures.]
GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (2/2)

- **Space charge only at low energy and/or high peak current**: below 10-20 MeV (injector) the beam dynamics optimization has to include emittance compensation schemes with, typically solenoids;
- **At higher energies no space charge and no RF defocusing effects** occur but we have **RF focusing due to the ponderomotive force**: focusing periods up to several meters
- **Optics design** has to take into account **longitudinal and transverse wakefields** (due to the higher frequencies used for acceleration) that can cause energy spread increase, head tail oscillations, multi-bunch instabilities
- **Longitudinal bunch compressors** schemes based on magnets and chicanes have to take into account, for short bunches, the interaction between the beam and the emitted synchrotron radiation (**Coherent Synchrotron Radiation effects**)
- **All these effects are important** especially in LINACs for **FEL that requires extremely good beam qualities**

![Graphs showing beam dynamics and wakefields effects](image)
At low proton (or ion) energies ($\beta \sim 0.01$), space charge defocusing is high and quadrupole focusing is not very effective. Moreover, cell length becomes small and conventional accelerating structures (DTL) are very inefficient. At this energy, it is used a (relatively) new structure, the Radio Frequency Quadrupole (1970).

These structures allow to simultaneously provide:

**Transverse Focusing**

**Acceleration**

**Bunching of the beam**

Electrodes

Courtesy M. Vretenar
RFQ: PROPERTIES

1-Focusing
The resonating mode of the cavity (between the four electrodes) is a focusing mode: Quadrupole mode (TE\(_{210}\)). The alternating voltage on the electrodes produces an alternating focusing channel with the period of the RF (electric focusing does not depend on the velocity and is ideal at low \(\beta\)).

2-Acceleration
The vanes have a longitudinal modulation with period = \(\beta\lambda_{RF}\). This creates a longitudinal component of the electric field that accelerates the beam (the modulation corresponds exactly to a series of RF gaps).

3-Bunching
The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient. One can start at -90° phase (linac) with some bunching cells, progressively bunch the beam (adiabatic bunching channel), and only in the last cells switch on the acceleration.

The RFQ is the only linear accelerator that can accept a low energy continuous beam.
RFQ: EXAMPLES

The 1st 4-vane RFQ, Los Alamos
1980: 100 KeV - 650 KeV, 30 mA, 425 MHz

The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle

TRASCO @ INFN Legnaro
Energy In: 80 keV
Energy Out: 5 MeV
Frequency 352.2 MHz
Proton Current (CW) 30 mA
### APPENDIX: THE CHOICE OF THE FREQUENCY

<table>
<thead>
<tr>
<th>Structure dimensions</th>
<th>Scales with $1/f$</th>
</tr>
</thead>
</table>
| Shunt impedance (efficiency) per unit length $r$ | **NC structures** $r$ increases and this push to adopt higher frequencies $\propto f^{1/2}$  
**SC structures** the power losses increases with $f^2$ and, as a consequence, $r$ scales with $1/f$ this push to adopt lower frequencies |

| Power sources | At very high frequencies (>10 GHz) power sources are commercially not available or expensive |

| Mechanical realization | Cavity fabrication at very high frequency requires higher precision but, on the other hand, at low frequencies one needs more material and larger machines/brazing oven |

| Bunch length | **short bunches** are easier with higher $f$ (FEL) |
| RF defocusing (ion linacs) | Increases with frequency ($\propto f$) |

| Cell length ($\beta \lambda_{RF}$) | $1/f$ |
| Wakefields | more critical at high frequency ($w_{||} \propto f^2$, $w_{\perp} \propto f^3$) |

$\Rightarrow$ **Higher frequencies** are economically convenient (shorter, less RF power, higher gradients possible) but the limitation comes from **mechanical precision** (tight tolerances are expensive!) and **beam dynamics** for ion linacs.

$\Rightarrow$ **Electron** linacs tend to use higher frequencies (1-12 GHz) than ion linacs.  
**SW SC**: 500 MHz-1500 MHz  
**TW NC**: 3 GHz-12 GHz

$\Rightarrow$ **Proton** linacs use lower frequencies (100-800 MHz), increasing with energy (ex.: 350–700 MHz): compromise between focusing, cost and size.  
**Heavy ion** linacs tend to use low frequencies (30-200 MHz),
THE CHOICE OF THE ACCELERATING STRUCTURE

In general the choice of the accelerating structure depends on:

- **Particle type**: mass, charge, energy
- **Beam current**
- **Duty cycle** (pulsed, CW)
- **Frequency**
- **Cost** of fabrication and of operation

Moreover a given accelerating structure has also a curve of efficiency (shunt impedance) with respect to the particle energies and the choice of one structure with respect to another one depends also on this.

As example a very general scheme is given in the Table (absolutely not exhaustive).

<table>
<thead>
<tr>
<th>Cavity Type</th>
<th>β Range</th>
<th>Frequency</th>
<th>Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFQ</td>
<td>0.01– 0.1</td>
<td>40-500 MHz</td>
<td>Protons, Ions</td>
</tr>
<tr>
<td>DTL</td>
<td>0.05 – 0.5</td>
<td>100-400 MHz</td>
<td>Protons, Ions</td>
</tr>
<tr>
<td>SCL</td>
<td>0.5 – 1</td>
<td>600 MHz-3 GHz</td>
<td>Protons, Electrons</td>
</tr>
<tr>
<td>SC Elliptical</td>
<td>&gt; 0.5-0.7</td>
<td>350 MHz-3 GHz</td>
<td>Protons, Electrons</td>
</tr>
<tr>
<td>TW</td>
<td>1</td>
<td>3-12 GHz</td>
<td>Electrons</td>
</tr>
</tbody>
</table>
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