RF Cavity Design

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CERN Accelerator School
Accelerator Physics (Intermediate level)
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Overview

• DC versus RF
  – Basic equations: Lorentz & Maxwell, RF breakdown
• Some theory: from waveguide to pillbox
  – rectangular waveguide, waveguide dispersion, group and phase velocity, standing waves ... waveguide resonators, round waveguides, Pillbox cavity
• Accelerating gap
  – Principle, ferrite cavity, drift tube linac
• Characterizing a cavity
  – Accelerating voltage, transit time factor
  – Resonance frequency, shunt impedance,
  – Beam loading, loss factor, RF to beam efficiency,
  – Transverse effects, Panofsky-Wenzel, higher order modes, PS 80 MHz cavity (magnetic coupling)
• More examples of cavities
  – PEP II, LEP cavities, PS 40 MHz cavity (electric coupling),
• RF Power sources
• Many gaps
  – Why?
  – Example: side coupled linac, LIBO
• Travelling wave structures
  – Brillouin diagram, iris loaded structure, waveguide coupling
• Superconducting Accelerating Structures
DC VERSUS RF

DC accelerator

RF accelerator

DC versus RF
Lorentz force

A charged particle moving with velocity \( \vec{v} = \frac{\vec{p}}{m \gamma} \) through an electromagnetic field experiences a force

\[
\frac{d\vec{p}}{dt} = q (\vec{E} + \vec{v} \times \vec{B})
\]

The total energy of this particle is \( W = \sqrt{(mc^2)^2 + (pc)^2} = \gamma mc^2 \), the kinetic energy is \( W_{\text{kin}} = mc^2 (\gamma - 1) \).

The role of acceleration is to increase the particle energy!

Change of \( W \) by differentiation:

\[
W dW = c^2 \vec{p} \cdot d\vec{p} = q c^2 \vec{p} \cdot (\vec{E} + \vec{v} \times \vec{B}) \, dt = q c^2 \vec{p} \cdot \vec{E} \, dt
\]

\[
dW = q \vec{v} \cdot \vec{E} \, dt
\]

Note: **Only the electric field can change the particle energy!**

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Maxwell’s equations (in vacuum)

\[
\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = \mu_0 \vec{J} \quad \nabla \cdot \vec{B} = 0
\]

\[
\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = \mu_0 \varepsilon_0 \rho
\]

why not DC?

1) **DC (\( \frac{\partial}{\partial t} \equiv 0 \)):** \( \nabla \times \vec{E} = 0 \) which is solved by \( \vec{E} = -\nabla \Phi \)

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

2) Circular machine: DC acceleration impossible since \( \oint \vec{E} \cdot d\vec{s} = 0 \)

With time-varying fields:

\[
\nabla \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \quad \oint \vec{E} \cdot d\vec{s} = -\oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}
\]

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Maxwell’s equation in vacuum (contd.)

\[ \nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0 \]
\[ \nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = 0 \]

curl of 3rd and \( \frac{\partial}{\partial t} \) of 1st equation:

\[ \nabla \times \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0 \]

vector identity:

\[ \nabla \times \nabla \times \vec{E} = \nabla \nabla \cdot \vec{E} - \Delta \vec{E} \]

with 4th equation:

\[ \Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0 \]

i.e. Laplace in 4 dimensions.

Another reason for RF: breakdown limit

surface field, in vacuum, Cu surface, room temperature

Wang & Loew, SLAC-PUB-7684, 1997

Approximate limit for CLIC parameters (12 GHz, 140 ns, breakdown rate: \(10^{-7} \text{ m}^{-1}\)); 260 MV/m

Kilpatrick 1957, \( \sqrt[4.25]{f} = E_c \cdot e^{\frac{E_c}{24.67}} \)

\( f \) in GHz, \( E_c \) in MV/m

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Homogeneous plane wave

\[ \vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r}) \]
\[ \vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r}) \]

\[ \vec{k} \cdot \vec{r} = \frac{\omega}{c} (\cos(\varphi)z + \sin(\varphi)x) \]

Wave vector \( \vec{k} \):
the direction of \( \vec{k} \) is the direction of propagation,
the length of \( \vec{k} \) is the phase shift per unit length.
\( \vec{k} \) behaves like a vector.

\[ k_\perp = \frac{\omega}{c} \]
\[ k = \frac{\omega}{c} \]
\[ k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_z}{\omega}\right)^2} \]
Wave length, phase velocity

- The components of \( \mathbf{k} \) are related to the wavelength in the direction of that component as \( \lambda_z = \frac{2\pi}{k_z} \) etc., to the phase velocity as \( v_{\phi z} = \frac{\omega}{k_z} = f \lambda_z \).

\[
\begin{align*}
k_{\perp} &= \frac{\omega \varepsilon}{c} \\
\end{align*}
\]

Superposition of 2 homogeneous plane waves

Metallic walls may be inserted where \( E_y \equiv 0 \) without perturbing the fields.

Note the standing wave in \( x \)-direction!

This way one gets a hollow rectangular waveguide.
Rectangular waveguide

Fundamental (TE$_{10}$ or H$_{10}$) mode in a standard rectangular waveguide.

**Example:** “S-band”: 2.6 GHz ... 3.95 GHz.

Waveguide type WR284 (2.84” wide), dimensions: 72.14 mm x 34.04 mm.

Operated at $f = 3$ GHz.

Waveguide dispersion

What happens with different waveguide dimensions (different width $a$)?

1:

- $a = 52$ mm,
- $f/f_c = 1.04$

2:

- $a = 72.14$ mm,
- $f/f_c = 1.44$

3:

- $a = 144.3$ mm,
- $f/f_c = 2.88$
Phase velocity

The phase velocity is the speed with which the crest or a zero-crossing travels in z-direction.

Note on the three animations on the right that, at constant f, it is $\propto \lambda_2$.

Note that at $f = f_c$, $v_{p,z} = \infty$.

With $f \to \infty$, $v_{p,z} \to c$!

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Note on the three animations on the right that, at constant $f$, it is $\propto \lambda_2$.

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With $f \to \infty$, $v_{p,z} \to c$!

Summary waveguide dispersion and phase velocity:

In a general cylindrical waveguide:

$$\gamma = j \sqrt{\left(\frac{\omega}{c}\right)^2 - k_\perp^2}$$

$$Z_0 = \frac{j \omega \mu}{\gamma} \text{ for TE}, \quad Z_0 = \frac{\gamma}{j \omega \varepsilon} \text{ for TM}$$

$$k_z = \text{Im}\{\gamma\} = \frac{2\pi}{\lambda_g}$$

e.g.: TE_{10} wave in rectangular waveguide:

$$\gamma = j \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_0 = \frac{j \omega \mu}{\gamma}$$

$$\lambda_{cutoff} = 2a$$

In a hollow waveguide: phase velocity $> c$, group velocity $< c$. 

In a hollow waveguide: phase velocity $> c$, group velocity $< c$. 

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Rectangular waveguide modes

Radial waves

Also radial waves may be interpreted as superposition of plane waves. The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.

\[ E_z \propto H_n^{(2)}(k_r r) \cos(n \varphi) \quad E_z \propto H_n^{(1)}(k_r r) \cos(n \varphi) \quad E_z \propto J_n(k_r r) \cos(n \varphi) \]
Round waveguide

\[ f/f_c = 1.44 \]

\( f_c \) for TE11 - fundamental, TM01 - axial field, TE01 - low loss.

Circular waveguide modes

- \( \text{TE}_{11} \)
- \( \text{TE}_{21} \)
- \( \text{TE}_{31} \)
- \( \text{TE}_{01} \)
- \( \text{TM}_{01} \)
- \( \text{TM}_{11} \)

plot: \( E \)-field
General waveguide equations:
Transverse wave equation (membrane equation):
\[ \Delta T + \left( \frac{\omega_c}{c} \right)^2 T = 0 \]

**TE (or H) modes**
boundary condition:
\[ \vec{n} \cdot \nabla T = 0 \]

longitudinal wave equations (transmission line equations):
\[ \frac{dU(z)}{dz} + \gamma Z_0 I(z) = 0 \]
\[ \frac{dI(z)}{dz} + \frac{\gamma}{Z_0} U(z) = 0 \]

propagation constant:
\[ \gamma = \frac{j \omega \mu}{c} \sqrt{1 - \left( \frac{\omega_c}{\omega} \right)^2} \]

characteristic impedance:
\[ Z_0 = \frac{j \omega \mu}{\gamma} \quad \quad \quad Z_0 = \frac{\gamma}{j \omega \varepsilon} \]

ortho-normal eigenvectors:
\[ \vec{e} = \vec{u}_z \times \nabla T \quad \quad \quad \quad \vec{e} = -\nabla T \]

transverse fields:
\[ \vec{E} = U(z) \vec{e} \quad \quad \quad \vec{H} = I(z) \vec{u}_z \times \vec{e} \]

longitudinal field:
\[ H_z = \left( \frac{\omega_c}{c} \right)^2 \frac{T U(z)}{j \omega \mu} \quad \quad \quad E_z = \left( \frac{\omega_c}{c} \right)^2 \frac{T I(z)}{j \omega \varepsilon} \]

Rectangular waveguide: transverse eigenfunctions

**TE (H) modes**:
\[ T^{(H)}_{mn} = \frac{1}{\pi} \sqrt{\frac{ab \varepsilon_m \varepsilon_n}{(mb)^2 + (na)^2}} \cos \left( \frac{m \pi}{a} x \right) \cos \left( \frac{n \pi}{b} y \right) \]

**TM (E) modes**:
\[ T^{(E)}_{mn} = \frac{2}{\pi} \sqrt{\frac{ab}{(mb)^2 + (na)^2}} \sin \left( \frac{m \pi}{a} x \right) \sin \left( \frac{n \pi}{b} y \right) \]

Round waveguide: transverse eigenfunctions

**TE (H) modes**:
\[ T^{(H)}_{mn} = \sqrt{\frac{\varepsilon_m}{\pi \chi_{mn}} \frac{J_m \left( \frac{\rho}{a} \right)}{J_{m-1}(\chi_{mn})}} \left\{ \begin{array}{ll} \cos(m \varphi) & \text{for } \rho/a < 1 \\ \sin(m \varphi) & \text{for } \rho/a = 1 \end{array} \right\} \]

**TM (E) modes**:
\[ T^{(E)}_{mn} = \sqrt{\frac{\varepsilon_m}{\pi \chi_{mn}} \frac{J_m \left( \frac{\rho}{a} \right)}{J_{m-1}(\chi_{mn})}} \left\{ \begin{array}{ll} \sin(m \varphi) & \text{for } \rho/a < 1 \\ \cos(m \varphi) & \text{for } \rho/a = 1 \end{array} \right\} \]

where
\[ \varepsilon_i = \begin{cases} 1 & \text{for } i = 0 \\ 2 & \text{for } i \neq 0 \end{cases} \]
Waveguide perturbed by notches

Reflections from notches lead to a superimposed standing wave pattern. "Trapped mode"

Signal flow chart

Short-circuited waveguide

TM$_{010}$ (no axial dependence)  TM$_{011}$  TM$_{012}$
Single WG mode between two shorts

![Signal flow chart](image)

Eigenvalue equation for field amplitude $a$:

$$a = e^{-jk_z \ell} a$$

Non-vanishing solutions exist for $2k_z \ell = 2\pi m$:

With $k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(\frac{c}{2\ell} m\right)^2$

Simple pillbox

![TM010-mode](image)

electric field (purely axial)

magnetic field (purely azimuthal)
Pillbox cavity field (w/o beam tube)

\[ T(\rho, \varphi) = \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01} J_1\left(\frac{\chi_{01}}{a}\right)} \]

\[ \chi_{01} = 2.40483... \]

The only non-vanishing field components:

\[ E_z = \frac{1}{j\omega \varepsilon_0} \frac{\chi_{01}}{a} \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)} \]

\[ B_\varphi = \mu_0 \sqrt{\frac{1}{\pi}} \frac{J_1\left(\frac{\chi_{01}}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)} \]

\[ \omega_0|_{\text{pillbox}} = \frac{\chi_{01}}{a} \frac{c}{\varepsilon_0} \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \Omega \]

\[ Q|_{\text{pillbox}} = \frac{\sqrt{2 \alpha \eta \sigma \chi_{01}}}{2\left(1 + \frac{a}{h}\right)} \]

\[ \frac{R}{Q|_{\text{pillbox}}} = \frac{4\eta}{\chi_{01}^2 \pi J_1^2\left(\chi_{01}\right)} \frac{\sin^2\left(\frac{\chi_{01}}{2} h\right)}{h/a} \]

Pillbox with beam pipe

One needs a hole for the beam pipe – circular waveguide below cutoff

TM₀₁₀-mode (only 1/4 shown)
A more practical pillbox cavity

Round of sharp edges (to reduce field enhancement!)

TM_{010}-mode (only 1/4 shown)

ACCELERATING GAP
Accelerating gap

- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on ground potential.
- Use \( \int \vec{E} \cdot d\vec{s} = -\int \frac{d\vec{B}}{dt} \cdot d\vec{A} \)
- The “shield” imposes a
  - upper limit of the voltage pulse duration or
  - equivalently –
  - a lower limit to the usable frequency.
- The limit can be extended with a material which acts as “open circuit”!
- Materials typically used:
  - ferrites (depending on \( f \)-range)
  - magnetic alloys (MA) like Metglas®, Finemet®, Vitrovac®…
- resonantly driven with RF (ferrite loaded cavities) – or with pulses
  (induction cell)

Ferrite cavity

CERN PS Booster, '98
0.6 – 1.8 MHz
< 10 kV gap
NiZn ferrites
Gap of PS cavity (prototype)

Drift Tube Linac (DTL) – how it works

For slow particles!
E.g. protons @ few MeV

The drift tube lengths can easily be adapted.

electric field

colour coding
1.000e+00
9.000e-01
8.000e-01
7.000e-01
6.000e-01
5.000e-01
4.000e-01
3.000e-01
2.000e-01
1.000e-01
0.000e+00
Drift tube linac – practical implementations

CHARACTERIZING A CAVITY
Acceleration voltage & $R$-upon-$Q$

I define $V_{acc} = \int E_z e^{j\omega z/\beta c} dz$. The exponential factor accounts for the variation of the field while particles with velocity $\beta c$ are traversing the gap (see next page).

With this definition, $V_{acc}$ is generally complex – this becomes important with more than one gap. For the time being we are only interested in $|V_{acc}|$.

**Attention, different definitions are used!**

The square of the acceleration voltage is proportional to the stored energy $W$.
The proportionality constant defines the quantity called $R$-upon-$Q$:

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2 \omega_0 W}$$

**Attention, also here different definitions are used!**

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Transit time factor

The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see.

$$TT = \frac{|V_{acc}|}{\left|\int E_z dz\right|} = \frac{\left|\int E_z e^{j\omega z/\beta c} dz\right|}{\left|\int E_z dz\right|}$$

The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length) $h$ is:

$$TT = \sin\left(\frac{\chi_0 h}{2a}\right) \sqrt{\left(\frac{\chi_0 h}{2a}\right)}$$

Field rotates by 360° during particle passage.
Shunt impedance

The square of the acceleration voltage is proportional to the power loss $P_{\text{loss}}$. The proportionality constant defines the quantity “shunt impedance”

$$R = \frac{|V_{\text{acc}}|^2}{2P_{\text{loss}}}$$

Attention, also here different definitions are used!

Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.

Cavity resonator - equivalent circuit

Simplification: single mode

$$L = \frac{R}{Q\omega_0}$$
$$C = \frac{Q}{(R\omega_0)}$$

$\beta$: coupling factor

$R$: Shunt impedance

$\sqrt{\frac{L}{C}}$: $R$-upon-$Q$
Reentrant cavity

Nose cones increase transit time factor, round outer shape minimizes losses.

Example: KEK photon factory 500 MHz
- R probably as good as it gets -

<table>
<thead>
<tr>
<th></th>
<th>this cavity</th>
<th>optimized pillbox</th>
</tr>
</thead>
<tbody>
<tr>
<td>R/Q:</td>
<td>111 Ω</td>
<td>107.5 Ω</td>
</tr>
<tr>
<td>Q:</td>
<td>44270</td>
<td>41630</td>
</tr>
<tr>
<td>R:</td>
<td>4.9 MΩ</td>
<td>4.47 MΩ</td>
</tr>
</tbody>
</table>
Loss factor

\[ k_{loss} = \frac{\omega_0 R}{2Q} = \frac{|V_{gap}|^2}{4W} = \frac{1}{2C} \]

Energy deposited by a single charge \( q \):

\[ k_{loss} q^2 \]

Voltage induced by a single charge \( q \):

\[ \frac{V_{\text{gap}}}{2k_{loss}q} \]

Impedance seen by the beam:

\[ L = R/(Q\omega_0) \]
\[ C = Q/(R\omega_0) \]

Summary: relations \( V_{\text{gap}}, W, P_{\text{loss}} \)

\[ \frac{R}{Q} = \frac{|V_{\text{gap}}|^2}{2\omega_0 W} \]

\[ k_{loss} = \frac{\omega_0 R}{2Q} = \frac{|V_{\text{gap}}|^2}{4W} \]

Energy stored inside the cavity:

\[ W \]

Shunt impedance:

\[ R_{\text{shunt}} = \frac{|V_{\text{gap}}|^2}{2P_{\text{loss}}} \]

Power lost in the cavity walls:

\[ P_{\text{loss}} \]

Q factor:

\[ Q = \frac{\omega_0 W}{P_{\text{loss}}} \]
Beam loading – RF to beam efficiency

- The beam current “loads” the generator, in the equivalent circuit this appears as a resistance in parallel to the shunt impedance.

- If the generator is matched to the unloaded cavity, beam loading will cause the accelerating voltage to decrease.

- The power absorbed by the beam is $P = \frac{1}{2} \text{Re}\{V_{\text{gap}} I^* B\}$, the power loss $P = \frac{|V_{\text{gap}}|^2}{2R}$.

- For high efficiency, beam loading shall be high.

- The RF to beam efficiency is $\eta = \frac{I_B}{1 + \frac{|V_{\text{gap}}|}{R|I_B|}}$.

Characterizing cavities

- Resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

- Transit time factor

  - field varies while particle is traversing the gap

- Shunt impedance

  - gap voltage – power relation $|V_{\text{gap}}|^2 = 2R_{\text{shunt}}P_{\text{loss}}$

- $Q$ factor $\omega_0 W = QP_{\text{loss}}$

- $R/Q$

  - independent of losses – only geometry! $R = \frac{Q}{2} \omega_0 W = \sqrt{\frac{L}{C}}$

- Loss factor $k_{\text{loss}} = \frac{\omega_0}{2} R = \frac{|V_{\text{gap}}|^2}{4W}$
Example Pillbox:

\[ \omega_0|_{\text{pillbox}} = \frac{\chi_{01} c}{a} \]
\[ Q|_{\text{pillbox}} = \frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1 + \frac{a}{h}\right)} \]
\[ R|_{\text{pillbox}} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01} h}{2a}\right)}{h/a} \]

\[ \chi_{01} = 2.4048 \]
\[ \eta = \frac{\mu_0}{\varepsilon_0} = 377 \, \Omega \]
\[ \sigma_{\text{Cu}} = 5.8 \cdot 10^7 \text{S/m} \]

Higher order modes

\[ n_1 \quad n_2 \quad n_3 \]

\[ R_1, Q_1, \omega_1 \quad R_2, Q_2, \omega_2 \quad R_3, Q_3, \omega_3 \]

\[ I_B \]
Panofsky-Wenzel theorem

For particles moving virtually at \( v=c \), the integrated transverse force (kick) can be determined from the transverse variation of the integrated longitudinal force!

\[
j \frac{\omega}{c} \vec{F}_\perp = \nabla_\perp F_\parallel
\]

Pure TE modes: No net transverse force!

Transverse modes are characterized by
• the transverse impedance in \( \omega \)-domain
• the transverse loss factor (kick factor) in \( t \)-domain!

CERN/PS 80 MHz cavity (for LHC)

inductive (loop) coupling, low self-inductance

Higher order modes
Example shown: 80 MHz cavity PS for LHC.
Color-coded:

\[ |E| \]
Higher order modes (measured spectrum)

Without dampers

With dampers

MORE EXAMPLES OF CAVITIES
PS 19 MHz cavity (prototype, photo: 1966)

Examples of cavities

PEP II cavity
476 MHz, single cell,
1 MV gap with 150 kW,
strong HOM damping,

LEP normal-conducting Cu RF cavities,
350 MHz, 5 cell standing wave + spherical cavity
for energy storage, 3 MV
CERN/PS 40 MHz cavity (for LHC)

example for capacitive coupling

coupling C

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RF POWER SOURCES
RF Power sources

> 200 MHz: Klystrons

Thales TH1801, Multi-Beam Klystron (MBK), 1.3 GHz, 117 kV. Achieved:
48 dB gain, 10 MW peak, 150 kW average, $\eta = 65\%$

< 1000 MHz: grid tubes

48 dB: $\frac{\text{output power}}{\text{input power}} = 10^{4.8}$

pictures from http://www.thales-electrondevices.com

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Example of a tetrode amplifier (80 MHz, CERN/PS)

400 kW, with fast RF feedback

- 18 Ω coaxial output (towards cavity)
- 22 kV DC anode voltage feed-through with λ/4 stub
- Tetrode cooling water feed-throughs

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MANY GAPS

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What do you gain with many gaps?

- The $R/Q$ of a single gap cavity is limited to some 100 $\Omega$.

  Now consider to distribute the available power to $n$ identical cavities: each will receive $P/n$, thus produce an accelerating voltage of $\sqrt{2RP/n}$.

  The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of $nR$.

  \[
  V_{\text{acc}} = n\sqrt{\frac{2R}{n}P} = \sqrt{2(nR)P}
  \]

Standing wave multicell cavity

- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).

- Coupled cavity accelerating structure (side coupled)

  - The phase relation between gaps is important!
Example of Side Coupled Structure

LIBO (= Linac Booster)

A 3 GHz Side Coupled Structure to accelerate protons out of cyclotrons from 62 MeV to 200 MeV

Medical application: treatment of tumours (proton therapy)

Prototype of Module 1 built at CERN (2000)

Collaboration
CERN/INFN/TERA Foundation

LIBO prototype

This Picture made it to the title page of CERN Courier vol. 41 No. 1 (Jan./Feb. 2001)
TRAVELLING WAVE STRUCTURES

Brillouin diagram
Travelling wave structure

\[ \omega = \frac{\beta}{c} \]

speed of light line,
\( \omega = \beta / c \)

synchronous
Iris loaded waveguide

30 GHz structure

11.4 GHz structure (NLC)

Disc loaded structure with strong HOM damping
“choke mode cavity”

Dimensions in mm
Power coupling with waveguides

3 GHz Accelerating structure (CTF3)
Examples (CLIC structures @ 11.4, 12 and 30 GHz)

“T18” reached 105 MV/m!

“HDS” – novel fabrication technique

SUPERCONDUCTING ACCELERATING STRUCTURES
LEP Superconducting cavities

SUPERCONDUCTING CAVITY WITH ITS CRYOSTAT

10.2 MV/ per cavity

LHC SC RF, 4 cavity module, 400 MHz
ILC high gradient SC structures at 1.3 GHz

25 -35 MV/m

Small $\beta$ superconducting cavities (example RIA, Argonne)

115 MHz split-ring cavity, 172.5 MHz $\beta = 0.19$ "lollipop" cavity

345 MHz $\beta = 0.4$ spoke cavity

57.5 MHz cavities:

$\beta = 0.06$ QWR (quarter wave resonator)

$\beta = 0.03$ fork cavity

$\beta = 0.021$ fork cavity

pictures from Shepard et al.: "Superconducting accelerating structures for a multi-beam driver linac for RIA", Linac 2000, Monterey