Introduction to „Transverse Beam Dynamics“

Bernhard Holzer

The Ideal World

I.) Magnetic Fields and Particle Trajectories
yes ... yes ... there is NO talk without it ...

The Higgs

ATLAS event display: Higgs => two electrons & two muons
Luminosity

Example: Luminosity run at LHC

\[ \beta_{x,y} = 0.55 \, m \]
\[ \epsilon_{x,y} = 5 \times 10^{-10} \, \text{rad m} \]
\[ \sigma_{x,y} = 17 \, \mu m \]
\[ f_0 = 11.245 \, kHz \]
\[ n_b = 2808 \]
\[ I_p = 584 \, mA \]

\[ L = \frac{1}{4\pi e^2 f_0 n_b} \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y} \]

\[ L = 1.0 \times 10^{34} \, \frac{1}{cm^2 \cdot s} \]
The Tune …
…is the number of these transverse oscillations per turn and corresponds to the „Eigenfrequency“ or sound of the particle oscillations. As in any oscillating system (e.g. pendulum) we have to avoid resonance conditions between the eigenfrequency of the system (= particle) and any external frequency that might act on the beam. Most prominent external frequency is the revolution frequency itself !! -> avoid integer tunes.

The Beta function shows the overall effect of all focusing fields; it has a certain value (m) that depends on the actual position in the ring.

The beam emittance describes the quality of the particle ensemble. It measure the area in phase space and can be considered like the temperature of a gas. The lower the emittance the better the beam quality. Together with the beta function it defines the beam dimension.

The lattice cell is the special magnet arrangement of the principle building block in an accelerator. Moist appropriate for high energy accelerators is the FoDo.

The Higgs particle is very small, $10^{-36}$ cm$^2$, and so it is difficult to produce.
Mini-Beta-Insertions in phase space

A mini-β insertion is always a kind of special symmetric drift space.

\( \Rightarrow \) greetings from Liouville

the smaller the beam size
the larger the beam divergence
The LHC Insertions

**ATLAS R1**

**Inner Triplet**

- Q1
- Q2
- Q3
- D1 (1.38 T)

**Separation/Recombination**

- TAN (3.8 T)
- D2
- Q4
- Q5
- Q6
- Q7

**Matching Quadrupoles**

- Matching Quadrupoles

**Warm**

- 1.9 K

**1.9 K**

**4.5 K**

**4.5 K**

**4.5 K**

**1.9 K**

**mini β optics**

**LHC Error Analysis**

- MAD-X 3.00.03
- 03/12/08 10:35:00

**β_x, β_y**

**s (m) [10^(-8)]**
13.) **Liouville during Acceleration**

\[ \varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) x'^2(s) \]

*Beam Emittance* corresponds to the area covered in the \( x, x' \) Phase Space Ellipse

*Liouville*: Area in phase space is constant.

**But so sorry ... \( \varepsilon \neq \text{const} \) !**

*Classical Mechanics:*

*phase space* = diagram of the two canonical variables

*position* & *momentum*

\[ x \quad \quad \quad p_x \]
According to Hamiltonian mechanics:
phase space diagram relates the variables $q$ and $p$

**Liouville's Theorem:**
\[
\int p \, dq = \text{const}
\]
\[
\int p_x \, dx = \text{const}
\]

_for convenience (i.e. *because we are lazy bones*) we use_
in accelerator theory:

\[
\begin{align*}
x' &= \frac{dx}{ds} = \frac{dx}{dt} \frac{dt}{ds} = \frac{\beta_x}{\beta} = \frac{p_x}{p} \\
\int x' \, dx &= \int \frac{p_x}{p} \, dx \\
&\propto \frac{\text{const}}{m_0 c \cdot \gamma \beta}
\end{align*}
\]

\[\Rightarrow \varepsilon = \int x' \, dx \propto \frac{1}{\beta \gamma}\]

_the beam emittance shrinks during acceleration \( \varepsilon \sim 1 / \gamma \)_

\[
\gamma = \sqrt{1 - \frac{v^2}{c^2}}
\]
\[
\beta_x = \frac{v_x}{c}
\]
Nota bene:

1.) A proton machine ... or an electron linac ... needs the highest aperture at injection energy !!! as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

$$\sigma = \sqrt{\varepsilon \beta}$$

2.) At lowest energy the machine will have the major aperture problems, here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy: A Mini Beta concept will only be adequate at flat top.

LHC injection optics at 450 GeV

LHC mini beta optics at 7000 GeV
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$
flat top energy: 920 GeV $\gamma = 980$

emittance $\varepsilon (40\text{GeV}) = 1.2 \times 10^{-7}$
$\varepsilon (920\text{GeV}) = 5.1 \times 10^{-9}$

7 $\sigma$ beam envelope at $E = 40$ GeV

... and at $E = 920$ GeV
The „not so ideal world“

14.) The „Δp / p ≠ 0“ Problem

ideal accelerator: all particles will see the same accelerating voltage.
⇒ Δp / p = 0

„nearly ideal“ accelerator: Cockroft Walton or van de Graaf

Δp / p ≈ 10⁻⁵

Vivitron, Straßbourg, inner structure of the acc. section

MP Tandem van de Graaf Accelerator at MPI for Nucl. Phys. Heidelberg
RF Acceleration

Energy Gain per „Gap“:

\[ W = n \cdot q \cdot U_0 \cdot \sin(\omega_{RF} t) \]

*RF Acceleration*: multiple application of the same acceleration voltage; brilliant idea to gain higher energies

* 1928, Wideroe

\[ W = n \cdot q \cdot U_0 \cdot \sin(\omega_{RF} t) \]

\[ n \] number of gaps between the drift tubes
\[ q \] charge of the particle
\[ U_0 \] Peak voltage of the RF System
\[ \Psi_s \] synchronous phase of the particle

500 MHz cavities in an electron storage ring
RF Acceleration-Problem: panta rhei !!!
(Heraklit: 540-480 v. Chr.)

just a stupid (and nearly wrong) example)

\[ \nu = 400 \, MHz \]
\[ c = \lambda \nu \]
\[ \lambda = 75 \, cm \]

\[ \sin(90^\circ) = 1 \]
\[ \sin(84^\circ) = 0.994 \]

\[ \frac{\Delta U}{U} = 6.0 \times 10^{-3} \]

typical momentum spread of an electron bunch:

\[ \frac{\Delta p}{p} = 1.0 \times 10^{-3} \]
Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???
Sure there are !!!

font colors due to pedagogical reasons
15.) Dispersion and Chromaticity: Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: prop. to magn. field & prop. zu $1/p$

**dipole magnet**

$$\alpha = \frac{\int B \, dl}{p/e}$$

**focusing lens**

$$k = \frac{g}{p/e}$$

$$x_D(s) = D(s) \frac{\Delta p}{p}$$

*particle having ... to high energy to low energy ideal energy*
Dispersion

Example: homogeneous dipole field

Matrix formalism:

\[
x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}
\]

\[
x(s) = C(s) \cdot x_0 + S(s) \cdot x'_0 + D(s) \cdot \frac{\Delta p}{p}
\]
or expressed as $3 \times 3$ matrix

\[
\begin{pmatrix}
    x \\
    x'
\end{pmatrix}
= \begin{pmatrix}
    C & S & D \\
    C' & S' & D'
\end{pmatrix}
\begin{pmatrix}
    x' \\
    \Delta p/p
\end{pmatrix}
\]

Example

\[
x_{\beta} = 1 \ldots 2 \text{ mm} \\
D(s) \approx 1 \ldots 2 \text{ m} \\
\frac{\Delta p}{p} \approx 1 \cdot 10^{-3}
\]

Amplitude of Orbit oscillation

contribution due to Dispersion $\approx$ beam size

Dispersion must vanish at the collision point

Calculate $D, D':$ ... takes a couple of sunny Sunday evenings!

\[
D(s) = S(s) \int_{s_0}^{S_1} \frac{1}{\rho} C(\bar{s}) d\bar{s} - C(s) \int_{s_0}^{S_1} \frac{1}{\rho} S(\bar{s}) d\bar{s}
\]

(proof see appendix)
Dispersion is visible

\[ x_D = D(s) \frac{\Delta p}{p} \]

Attention: at the Interaction Points we require \( D = D' = 0 \)

HERA Standard Orbit

HERA Dispersion Orbit

dedicated energy change of the stored beam

\( \rightarrow \) closed orbit is moved to a dispersions trajectory
**Periodic Dispersion:**

„*Sawtooth Effect*“ at LEP (CERN)

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*In the straight sections they are accelerated by the rf cavities so much that they ‟overshoot“ and reach nearly the outer side of the vacuum chamber.*

*In the arc the electron beam loses so much energy in each octant that the particle are running more and more on a dispersion trajectory.*
16.) Chromaticity:

A Quadrupole Error for $\Delta p/p \neq 0$

Influence of external fields on the beam: prop. to magn. field & prop. zu 1/p

Remember the normalisation of the external fields:

focusing lens

$$k = \frac{g}{\frac{p}{e}}$$

A particle that has a higher momentum feels a weaker quadrupole gradient and has a lower tune.

$$\Delta Q = -\frac{1}{4\pi \frac{\Delta p}{p_0}} k_0 \beta(s) ds$$

definition of chromaticity:

$$\Delta Q = Q' \frac{\Delta p}{p}$$
... what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself!!

Q' is a number indicating the size of the tune spot in the working diagram,
Q' is always created if the beam is focussed
→ it is determined by the focusing strength k of all quadrupoles

\[ \Delta Q = - \frac{1}{4\pi} \frac{\Delta p}{p_0} k_0 \beta(s) ds \]

k = quadrupole strength
β = betafunction indicates the beam size … and even more the sensitivity of
the beam to external fields

Example: LHC

\[ Q' = 250 \]
\[ \Delta p/p = +/- 0.2 \times 10^{-3} \]
\[ \Delta Q = 0.256 \ldots 0.36 \]

→ Some particles get very close to resonances and are lost
in other words: the tune is not a point it is a pancake
Tune signal for a nearly uncompensated cromaticity \((Q' \approx 20)\)

**Ideal situation: cromaticity well corrected,**
\((Q' \approx 1)\)
Tune and Resonances

\[ m*Q_x + n*Q_y + l*Q_z = \text{integer} \]

Tune diagram up to 3rd order

... and up to 7th order

Homework for the operateurs:
find a nice place for the tune
where against all probability
the beam will survive
Chromaticity Correction:

We need a magnetic field that focuses stronger those individual particles that have larger momentum and focuses weaker those with lower momentum. ... but that does not exist.

The way the trick goes:
1.) sort the particle trajectories according to their energy
   we use the dispersion to do the job

2.) introduce magnetic fields that increase stronger than linear
   with the distance $\Delta x$ to the centre

3.) calculate these fields (sextupoles) in a way that the lack of focusing strength is exactly compensated.
Correction of $Q'$:

Need: additional quadrupole strength for each momentum deviation $\Delta p/p$

1.) sort the particles according to their momentum

\[ x_D(s) = D(s) \frac{\Delta p}{p} \]

... using the dispersion function

2.) apply a magnetic field that rises quadratically with $x$ (sextupole field)

\[
B_x = \tilde{g} xy \\
B_y = \frac{1}{2} g (x^2 - y^2)
\]

\[
\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g} x \quad \text{linear amplitude dependent}\]

„gradient“:
**Correction of $Q'$:**

$k_1$ normalised quadrupole strength

$k_2$ normalised sextupole strength

\[ k_1(\text{sext}) = \frac{\bar{g}x}{p/e} = k_2 \ast x \]

\[ = k_2 \ast D \frac{\Delta p}{p} \]

**Sextupole Magnets:**

Combined effect of „natural chromaticity“ and Sextupole Magnets:

\[ Q' = -\frac{1}{4 \pi} \left\{ \int k_1(s)\beta(s)ds + \int k_2 \ast D(s)\beta(s)ds \right\} \]

You only should not forget to correct $Q'$ in both planes ... and take into account the contribution from quadrupoles of both polarities.
Einstellung am Speicherring:
Sextupolströme so variieren, dass $\xi \approx +1 \ldots +2$
A word of caution: keep non-linear terms in your storage ring low.

\[ B_y + iB_x = B_{\text{ref}} \sum_{n=1}^{\infty} (b_n + ia_n) \left( \frac{x + iy}{r_0} \right)^{n-1} \]

"effective magnetic length"

\[ B \cdot l_{\text{eff}} = \int_0^{l_{\text{mag}}} B ds \]
Clearly there is another problem … … if it were easy everybody could do it

Again: the phase space ellipse

for each turn write down – at a given position „s“ in the ring – the single particle amplitude \( x \) and the angle \( x' \)… and plot it.

\[
\begin{pmatrix}
  x \\
  x'
\end{pmatrix}_{s1} = M_{\text{turn}} \cdot \begin{pmatrix}
  x \\
  x'
\end{pmatrix}_{s0}
\]

A beam of 4 particles – each having a slightly different emittance:
Installation of a weak (!!!) sextupole magnet

The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equations; instead: Computer simulation "particle tracking"
Effect of a strong (!!!) Sextupole ...

→ Catastrophy!

„dynamic aperture“
Any CAS yellow report is worth reading!!

1.) Edmund Wilson: *Introd. to Particle Accelerators*  
*Oxford Press, 2001*

2.) Klaus Wille: *Physics of Particle Accelerators and Synchrotron Radiation Facilicties, Teubner, Stuttgart 1992*

3.) Peter Schmüser: *Basic Course on Accelerator Optics, CERN Acc. School: 5th general acc. phys. course CERN 94-01*


5.) Herni Bruck: *Accelerateurs Circulaires des Particules, presse Universitaires de France, Paris 1966 (english / francais)*


7.) Frank Hinterberger: *Physik der Teilchenbeschleuniger, Springer Verlag 1997*

8.) Mathew Sands: *The Physics of e+ e- Storage Rings, SLAC report 121, 1970*

9.) D. Edwards, M. Syphers: *An Introduction to the Physics of Particle Accelerators, SSC Lab 1990*
Luminosity…
…describes the performance of a collider to hit the „target“ (i.e. the other particles) and so to produce „hits“.

The Mini-Beta scheme …
… focusses strongly the beams to achieve smallest possible beam sizes at the IP. The obtained small beta function at the IP is called $\beta^*$. Don’t forget the cat.

A proton beam shrinks during acceleration, we call it unfortunately „adiabatic shrinking“. Nota bene: An electron beam in a ring is growing with energy !!

Dispersion …
… is the particle orbit for a given momentum difference.

Chromaticity …
… is a focusing problem. Different momenta lead to different tunes $\rightarrow$ attention … resonances !!

Sextupoles …
have non-linear fields and are used to compensate chromaticity.

Strong non-linear fields can lead to particle losses (dynamic aperture)