Beam-Beam Interactions

Tatiana Pieloni (BE-ABP-ICE)

Thanks to W. Herr
Hadron Circular Colliders

\[ E^* \approx 2 \times E \]

\[ N_{\text{event/s}} = L \cdot \sigma_{\text{event}} \]

\[ L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{\text{rev}} \]

Bunch intensity: \( N_p = 1.15 - 1.65 \times 10^{11} \text{ ppb} \)

Transverse Beam size: \( \sigma_{x,y} = 16 - 30 \text{ \(\mu\)m} \)

Number of bunches \( 1370 - 2808 \)

Revolution frequency \( 11 \text{ kHz} \)
When do we have beam-beam effects?

- They occur when two beams get closer and collide.

- Two types:
  - High energy collisions between two particles *(wanted)*
  - Distortions of beam by electromagnetic forces *(unwanted)*

- Unfortunately: usually both go together...
  - 0.001% (or less) of particles collide
  - 99.999% (or more) of particles are distorted
Beam-beam effects: overview

- Circular Colliders: interaction occurs at every turn
  - Many effects and problems
  - Try to understand some of them

- Overview of effects (single particle and multi-particle effects)
- Qualitative and physical picture of effects
- Observations from the LHC
- And CAS Proceedings
References:


...much more on the LHC Beam-beam webpage:
http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/
Beams EM potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

Force on itself (space charge) and opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted

A beam acts on particles like an electromagnetic lens, but...
Beam-beam Mathematics

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

\[ \Delta U = -\frac{1}{\epsilon_0} \rho(x, y, z) \]

\[ U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi \epsilon_0} \int \int \int \frac{\rho(x_0, y_0, z_0) dx_0 dy_0 dz_0}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} \]

\[ \vec{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z) \]

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]

Making some assumptions we can simplify the problem and derive analytical formula for the force...

Derive potential from Poisson equation for charges with distribution \( \rho \)

Solution of Poisson equation

Then compute the fields

From Lorentz force one calculates the force acting on test particle with charge \( q \)
Round Gaussian distributions:

Gaussian distribution for charges:
Round beams:
Very relativistic, Force has only radial component:

\[ F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

\[ \Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) \, dt \]

\[ \Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

\[ \sigma_x = \sigma_y = \sigma \]
\[ \beta \approx 1 \quad r^2 = x^2 + y^2 \]

Beam-beam Force

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

Only radial component in relativistic case

How does this force looks like?
$F_r(r) = \pm \frac{ne^2(1 + \beta_{rel}^2)}{2\pi\varepsilon_0} \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$
Why do we care?

Pushing for luminosity means stronger beam-beam effects

\[ \mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \]

\[ F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

Physics fill lasts for many hours 10h – 24h

Strongest non-linearity in a collider YOU CANNOT AVOID!

Two main questions:
What happens to a single particle?
What happens to the whole beam?
Beam-Beam Force: single particle...

Lattice defocusing quadrupole

Beam-beam force

\[ F = -k \cdot r \]

For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!
Can we quantify the beam-beam strength?

Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force

For small amplitudes: linear force

\[ F \propto -\xi \cdot r \]

The slope of the force gives you the beam-beam parameter \( \xi \)

\[ \Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

\[ \Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[ 1 - \left( 1 - \frac{r^2}{2\sigma^2} + \ldots \right) \right] \]
Colliders:

For round beams:

\[ \xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} \]

For non-round beams:

\[ \xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi \gamma \sigma_{x,y}(\sigma_x + \sigma_y)} \]

**Examples:**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LHC nominal</th>
<th>LHC 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity (N_{p,e}/\text{bunch})</td>
<td>(1.15 \times 10^{11})</td>
<td>(1.6 \times 10^{11})</td>
</tr>
<tr>
<td>Energy GeV</td>
<td>7000</td>
<td>4000</td>
</tr>
<tr>
<td>Beam emittance</td>
<td>3.75 (\mu\text{mrad})</td>
<td>2.2-2.5 (\mu\text{mrad})</td>
</tr>
<tr>
<td>Crossing angle ((\mu\text{rad}))</td>
<td>285</td>
<td>290</td>
</tr>
<tr>
<td>(\beta_{x,y}^*) (m)</td>
<td>1.25-0.05</td>
<td>0.60-0.60</td>
</tr>
<tr>
<td>Luminosity</td>
<td>(1 \times 10^{34})</td>
<td>7.6 (\times 10^{33})</td>
</tr>
<tr>
<td>(\xi_{\sigma_{bb}})</td>
<td>0.0034</td>
<td>0.006</td>
</tr>
</tbody>
</table>
Linear Tune shift

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

\[ F \propto -\xi \cdot r \]

Focal length:

\[ \frac{1}{f} = \frac{\Delta x'}{x} = \frac{Nr_0}{\gamma\sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*} \]

Beam-beam matrix:

\[
\begin{pmatrix}
1 & 0 \\
-\frac{\xi \cdot 4\pi}{\beta^*} & 1
\end{pmatrix}
\]

Perturbed one turn matrix with perturbed tune \( \Delta Q \) and beta function at the IP \( \beta^* \):

\[
\begin{pmatrix}
\cos(2\pi(Q + \Delta Q)) & \beta^* \sin(2\pi(Q + \Delta Q)) \\
-\frac{1}{\beta^*} \sin(2\pi(Q + \Delta Q)) & \cos(2\pi(Q + \Delta Q))
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & 0 \\
-\frac{1}{2f} & 1
\end{pmatrix} \cdot \begin{pmatrix}
cos(2\pi Q) & \beta_0^* \sin(2\pi Q) \\
-\frac{1}{\beta_0^*} \sin(2\pi Q) & \cos(2\pi Q)
\end{pmatrix} \cdot \begin{pmatrix}
1 & 0 \\
-\frac{1}{2f} & 1
\end{pmatrix}
\]
Linear tune

Solving the one turn matrix one can derive the tune shift $\Delta Q$ and the perturbed beta function at the IP $\beta^*$:

**Tune is changed**

$$\cos(2\pi(Q + \Delta Q)) = \cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi \xi}{\beta^*} \sin(2\pi Q)$$

**$\beta$-function is changed:**

$$\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi(Q + \Delta Q))}$$

...how do they change?
Tune dependence of tune shift and dynamic beta

Tune shift as a function of tune

Larger $\xi$ $\rightarrow$ Strongest variation with $Q$
Head-on and Long-range interactions

\[ L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev} \]

Other beam passing in the center force: **HEAD-ON** beam-beam interaction

Other beam passing at an offset of the force: **LONG-RANGE** beam-beam interaction
Multiple bunch Complications

MANY INTERACTIONS

\[ \mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \]

Num. of bunches \( n_b = 2808 \)

For 25ns case 124 BBIs per turn: 4 HO and 120 LR

3.7 m
LHC, KEKB... colliders

- Crossing angle operation
- High number of bunches in train structures

<table>
<thead>
<tr>
<th></th>
<th>SppS</th>
<th>Tevatron</th>
<th>RHIC</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Bunches</td>
<td>6</td>
<td>36</td>
<td>109</td>
<td>2808</td>
</tr>
<tr>
<td>LR interactions</td>
<td>9</td>
<td>70</td>
<td>0</td>
<td>120/40</td>
</tr>
<tr>
<td>Head-on interactions</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

72 bunches
A beam is a collection of particles

Beam-beam force

Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude (detuning with amplitude or tune spread)
A beam will experience all the force range

Beam-beam force

Second beam passing in the center
**HEAD-ON** beam-beam interaction

Beam-beam force

Second beam displaced offset
**LONG-RANGE** beam-beam interaction

Different particles will see different force
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude.

\[ \Delta Q \propto \frac{\delta F}{\delta r} \]

\[ \Delta Q_{quad} = const \]

\[ \Delta Q_{bb} \approx const \]

For small amplitude test particle linear tune shift

\[ \lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} = \xi \]
Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread

\[ \Delta Q_{quad} = \text{const} \]
\[ \Delta Q(x) = \frac{N \sigma_0 \beta}{4 \pi \gamma \sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp - (\frac{x}{2})^2 I_0(\frac{x}{2})^2 - 1) \]

Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude

And in the other plane?
THE SAME DERIVATION
same tune spread

FOOTPRINT
2-D mapping of the detuning with amplitude of particles
And for long-range interactions?

Second beam centered at $d$ (i.e. $6\sigma$)
- Small amplitude particles positive tune shifts
- Large amplitude can go to negative tune shifts

Long range tune shift scaling for distances

$$d > 6\sigma$$

$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$
Long-range footprints

Separation in vertical plane!
And in horizontal plane?
The test particle is centered with the opposite beam
tune spread more like for head-on at large amplitudes

The picture is more complicated now the LARGE amplitude particles see the second beam and have larger tune shift
Beam-beam tune shift and spread

Footprints depend on:
• number of interactions
• Type (Head-on and long-range)
• Plane of interaction

When long-range effects become important footprint wings appear and alternating crossing important

Aim to reduce the area as much as possible!

Passive compensation of tune shift Ref[7]
Complications

PACMAN and SUPER PACMAN bunches

72 bunches

Pacman:

miss long range BBI

Super Pacman:

miss head-on BBI

IP2 and IP8 depending on filling scheme

Different bunch families: Pacman and Super Pacman
LHC Complications: filling schemes

72 bunches

PS extraction kicker

SPS extraction kicker

Abort Gap

Nominal Bunches

Pacman Bunche: different number of long-range interactions
Pacman and Super-pacman

...operationally it is even more complicated!
...intensities, emittances...
Particle Losses

Dynamic Aperture: area in amplitude space with stable motion
Stable area of particles depends on beam intensity and crossing angle

\[ F_{bb} \propto N_p \]

Stable Area (\( \sigma \))

\[ d_{tr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}} \]

10-12 \( \sigma \) separation

Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, \( \beta^* \), intensity) is the result of careful study of different effects!

Ref [6]
DO we see the effects of LR in the LHC?

Small crossing angle = small separation
If separation of long range too small particles become unstable and get lost

Particle losses follow number of Long range interactions
Nominal LHC will have twice the number of interactions
Long-range BB and Orbit Effects

Long Range Beam-beam interactions lead to orbit effects

Long range kick

\[ \Delta x' (x + d, y, r) = -\frac{2N r_0}{\gamma} \frac{(x + d)}{r^2} [1 - \exp \left( -\frac{r^2}{2\sigma^2} \right)] \]

For well separated beams \( d \gg \sigma \)

The force has an amplitude independent contribution: ORBIT KICK

\[ \Delta x' = \frac{\text{const}}{d} \left[ 1 - \frac{x}{d} + O \left( \frac{x^2}{d^2} \right) + \ldots \right] \]

Orbit can be corrected but we should remember PACMAN effects
LHC orbit effects

Orbit effects different due to Pacman effects and the many long-range add up giving a non negligible effect

\[ L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}} \]

Ref [7]
Long range orbit effect

Long range interactions leads to orbit offsets at the experiment a direct consequence is deterioration of the luminosity

Measurement of the vertex centroid by ATLAS

Calculations for nominal LHC

Effect is already visible with reduced number of interactions

Ref [7]
Long range orbit effect observations:

Vertical oscillation starts when one beam is ejected and dumped
Coherent dipolar beam-beam modes

Coherent beam-beam effects arise from the forces which an exciting bunch exerts on a whole test bunch during collision.

We study the collective behaviour of all particles of a bunch.

Coherent motion requires an organized behaviour of all particles of the bunch.

**Coherent beam-beam force**

- Beam distributions $\Psi_1$ and $\Psi_2$ mutually changed by interaction.
- Interaction depends on distributions.
  - Beam 1 $\Psi_1$ solution depends on beam 2 $\Psi_2$.
  - Beam 2 $\Psi_2$ solution depends on beam 1 $\Psi_1$.
- Need a self-consistent solution.
Coherent beam-beam effects

• Whole bunch sees a kick as an entity (coherent kick)
• Coherent kick seen by full bunch different from single particle kick
• Requires integration of individual kick over particle distribution

\[ \Delta r' = - \frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[ 1 - e^{-\frac{r^2}{4\sigma^2}} \right] \]

• Coherent kick of separated beams can excite coherent dipolar oscillations
• All bunches couple because each bunch “sees” many opposing bunches (LR): many coherent modes possible!
Coherent effects
Self-consistent treatment needed

Perturbative methods
static source of distortion: example magnet

Self-consistent method
source of distortion changes as a result of the distortion

For a complete understanding of BB effect a self-consistent treatment should be used
Simple case: one bunch per beam

Coherent mode: two bunches are "locked" in a coherent oscillation
0-mode is stable (mode with NO tune shift)
π-mode can become unstable (mode with largest tune shift)
Incoherent tune spread is the Landau damping region any mode with frequency laying in this range should not develop
• $\pi$-mode has frequency out of tune spread ($Y$) so it is not damped!
Coherent modes at RHIC

Tune spectra before collision and in collision two modes visible

Courtesy W. Fischer (BNL)
Head-on beam-beam coherent mode: LHC

BBQ Signals

\[ \Delta Q = 0.034 \]

\[ \xi \approx 0.016 \]
Beam-beam coherent modes and Landau Damping

Pacman effect on coherent modes
Single bunch diagnostic so important
Different Tunes

Tune split breaks symmetry and coherent modes disappear

Analytical calculations in Reference [8]
Different tunes or intensities

RHIC running with mirrored tune for years to break coherent oscillations

LHC has used a tune split to suppress coherent BB modes
2010 Physics Run
Different bunch intensities

For two bunches colliding head-on in one IP the coherent mode disappears if intensity ratio between bunches is 55% Reference[9]

We assumed:
• equal emittances
• equal tunes
• NO PACMAN effects (bunches will have different tunes)

For coherent modes the key is to break the simmetry in your coupled system...(tunes, intensities, collision patterns...)
And Long range interactions?

- Each bunch will have different number of modes and tune spectra
- No Landau damping of long-range coherent modes

Single bunch diagnostic can make the difference
Beam-beam compensations:

**Head-on**

- Linear e-lens, suppress shift
- Non-linear e-lens, suppress tune spread

- Past experience: at Tevatron linear and non-linear e-lenses, also hollow...
- Present: test for half compensation at RHIC with non-linear e-lens
Beam-beam compensations: long-range

Beam-beam wire compensation

\[ \xi_{x,y} = \frac{N r_0 B_{x,y}}{2 \pi \gamma \sigma^2} \]

- Past experience: at RHIC several tests till 2009...
- Present: simulation studies on-going for possible use in HL-LHC...
...not covered here...

- Linear colliders special issues
- Asymmetric beams effects
- Coasting beams
- Beamstrahlung
- Synchrobetatron coupling
- Beam-beam experiments
- Beam-beam and impedance
- ...
Thank You!