Diagnostics II

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Introduction
Beam Charge / Intensity
Beam Position
Summary

$\Big\}$ Diagnostics I

Introduction
Transverse Beam Emittance
Longitudinal Beam Emittance
Summary

$\Big\}$ Diagnostics II

Introduction

Reminder:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \underbrace{\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_{fi}}_{R} \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

transformation of the phase space coordinates (x,x') of a single particle (from i→f) given in terms of the transport matrix, R

Equivalently, and complementarily, the Twiss parameters (α, β, and γ) obey

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1+2R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix}_{fi} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$

The elements of the transfer matrix R are given generally by

$$\mathbf{R}_{fi} = \begin{pmatrix} \sqrt{\frac{\beta_f}{\beta_i}}(\cos\phi_{fi} + \alpha_i \sin\phi_{fi}) & \sqrt{\beta_f\beta_i}\ \sin\phi_{fi} \\ -\frac{1+\alpha_f\alpha_i}{\sqrt{\beta_f\beta_i}}\sin\phi_{fi} + \frac{\alpha_i-\alpha_f}{\sqrt{\beta_f\beta_i}}\cos\phi_{fi} & \sqrt{\frac{\beta_i}{\beta_f}}(\cos\phi_{fi} - \alpha_f \sin\phi_{fi}) \end{pmatrix}$$

or if the initial and final observations points are the same, by the one-turn-map:

$$\mathbf{R}_{\mathrm{otm}} = \begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ -\gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{pmatrix}$$

where μ is the 1-turn phase advance: $\mu = 2\pi Q$

A third equivalent approach involves the beam matrix defined as

$$\Sigma^x_{\text{beam}} = \epsilon_x \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

in terms of Twiss parameters

$$= \begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle x x' \rangle - \langle x \rangle \langle x' \rangle \\ \langle x' x \rangle - \langle x' \rangle \langle x \rangle & \langle x'^2 \rangle - \langle x' \rangle^2 \end{pmatrix}$$

in terms of the moments of the beam distribution

note $\epsilon_x = \sqrt{\det \Sigma^x_{\text{beam}}}$

Here <x> and <x^2> are the first and second moments of the beam distribution:

$$\langle x \rangle = \frac{\int\limits_0^\infty x f(x)\,\mathrm{d}x}{\int\limits_0^\infty f(x)\,\mathrm{d}x} \qquad \langle x^2 \rangle = \frac{\int\limits_0^\infty x^2 f(x)\,\mathrm{d}x}{\int\limits_0^\infty f(x)\,\mathrm{d}x}$$

where f(x) is the beam intensity distribution

The transformation of the initial beam matrix $\Sigma_{\text{beam},0}$ to the desired observation point is

$$\Sigma_{\text{beam}} = R \Sigma_{\text{beam},0} R^t$$

where R is again the transfer matrix

Neglecting the mean of the distribution (disregarding the static position offset of the core of the beam; i.e. <x>=0) :

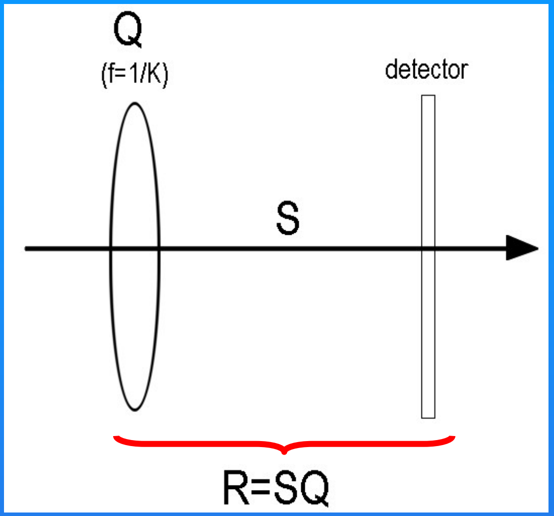
$$\Sigma^x_{\text{beam}} = \begin{pmatrix} \langle x^2 \rangle & \langle x x' \rangle \\ \langle x x' \rangle & \langle x'^2 \rangle \end{pmatrix}$$

and the root-mean-square (rms) of the distribution is

$$\sigma_x = \langle x^2 \rangle^{\frac{1}{2}}$$

Measurement of the Transverse Beam Emittance

<u>Method I:</u> quadrupole scan



Principle: with a well-centered beam, measure the beam size as a function of the quadrupole field strength

Here
Q is the transfer matrix of the quadrupole
R is the transfer matrix between the quadrupole and the beam size detector

With $Q = \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix}$ then $R = \begin{pmatrix} S_{11} + KS_{12} & S_{12} \\ S_{21} + KS_{22} & S_{22} \end{pmatrix}$ with $\Sigma_{\mathrm{beam}} = R\Sigma_{\mathrm{beam},0}R^t$

The (11)-element of the beam transfer matrix is found after algebra to be:

$$\begin{aligned} \Sigma_{11}(=\langle x^2\rangle) = {} & ({S_{11}}^2 \Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + {S_{12}}^2\Sigma_{22_0}) \\ & + (2S_{11}S_{12}\Sigma_{11_0} + 2{S_{12}}^2\Sigma_{12_0})K + {S_{12}}^2\Sigma_{11_0}K^2 \end{aligned}$$

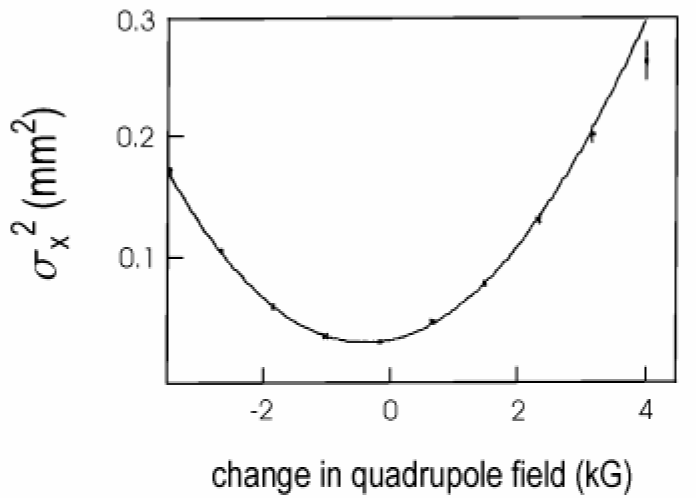
which is quadratic in the field strength, K

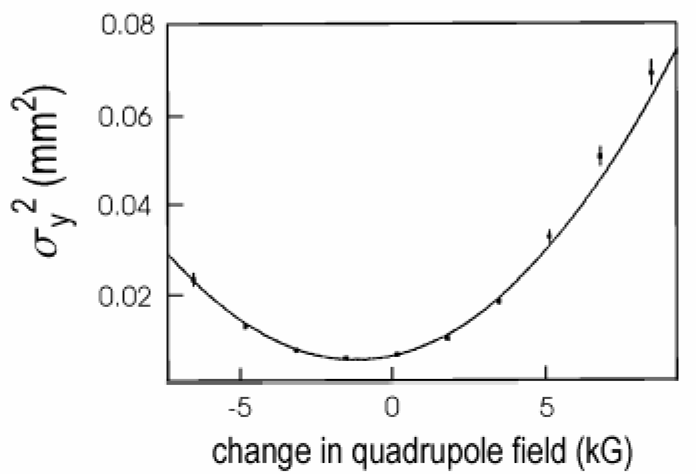
Measurement: measure beam size versus quadrupole field strength

recall:

$$\Sigma_{11}(=\langle x^2\rangle) = ({S_{11}}^2\Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + {S_{12}}^2\Sigma_{22_0}) + (2S_{11}S_{12}\Sigma_{11_0} + 2{S_{12}}^2\Sigma_{12_0})K + {S_{12}}^2\Sigma_{11_0}K^2$$

data: (SLC transfer line)





fitting function (parabolic):

$$\begin{aligned}\Sigma_{11} &= A(K-B)^2 + C \\ &= AK^2 - 2ABK + (C + AB^2)\end{aligned}$$

equating terms (drop subscripts 'o'),

$$A = S_{12}^2\Sigma_{11}\,,$$

$$-2AB = 2S_{11}S_{12}\Sigma_{11} + 2S_{12}^2\Sigma_{12}\,,$$

$$C + AB^2 = {S_{11}}^2\Sigma_{11} + 2S_{11}S_{12}\Sigma_{12} + {S_{12}}^2\Sigma_{22}$$

solving for the beam matrix elements:

$$\Sigma_{11} = A/{S_{12}}^2\,,$$

$$\Sigma_{12} = -\frac{A}{{S_{12}}^2}\left(B + \frac{S_{11}}{S_{12}}\right) \quad (= \Sigma_{21})$$

$$\Sigma_{22} = \frac{1}{{S_{12}}^2}\left[(AB^2 + C) + 2AB\left(\frac{S_{11}}{S_{12}}\right) + A\left(\frac{S_{11}}{S_{12}}\right)^2\right]$$

The emittance is given from the determinant of the beam matrix:

$$\epsilon_x = \sqrt{\det\ \Sigma^x_{\text{beam}}}$$

$$\begin{aligned}\det\ \Sigma^x_{\text{beam}} &= \Sigma_{11}\Sigma_{22} - {\Sigma_{12}}^2 \\ &= AC/S^4_{12}\,,\end{aligned}$$

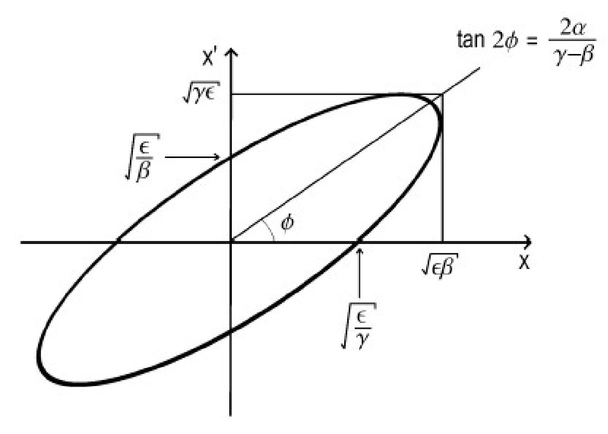
$$\Rightarrow \quad \epsilon_x = \sqrt{AC}/S^2_{12}$$

With these 3 fit parameters (A,B, and C), the 3 Twiss parameters are also known:

$$\beta_x = \frac{\Sigma_{11}}{\epsilon} = \sqrt{\frac{A}{C}}\,,$$

$$\alpha_x = -\frac{\Sigma_{12}}{\epsilon} = \sqrt{\frac{A}{C}}\left(B + \frac{S_{11}}{S_{12}}\right)\,,$$

$$\gamma_x = \frac{{S_{12}}^2}{\sqrt{AC}}\left[(AB^2 + C) + 2AB\left(\frac{S_{11}}{S_{12}}\right) + A\left(\frac{S_{11}}{S_{12}}\right)^2\right]$$



as a useful check, the beam-ellipse parameters should satisfy $(\beta_x\gamma_x - 1) = \alpha^2$

Method II: fixed optics, measure beam size using multiple measurement devices

Recall: the matrix used to transport the Twiss parameters:

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1+2R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix}_{fi} \begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_i$$

with fixed optics and multiple measurements of σ at different locations:

$$\underbrace{\begin{pmatrix} ({\sigma_x}^{(1)})^2 \\ ({\sigma_x}^{(2)})^2 \\ ({\sigma_x}^{(3)})^2 \\ \dots \\ ({\sigma_x}^{n})^2 \end{pmatrix}}_{\Sigma_x} = \underbrace{\begin{pmatrix} ({R_{11}}^{(1)})^2 & 2R_{11}^{(1)}R_{12}^{(1)} & ({R_{12}}^{(1)})^2 \\ ({R_{11}}^{(2)})^2 & 2R_{11}^{(2)}R_{12}^{(2)} & ({R_{12}}^{(2)})^2 \\ ({R_{11}}^{(3)})^2 & 2R_{11}^{(3)}R_{12}^{(3)} & ({R_{12}}^{(3)})^2 \\ \dots & & \\ ({R_{11}}^{(n)})^2 & R_{11}^{(n)}R_{12}^{(n)} & ({R_{12}}^{(n)})^2 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \beta(s_0)\epsilon \\ -\alpha(s_0)\epsilon \\ \gamma(s_0)\epsilon \end{pmatrix}}_{o}$$

simplify notation:

$$\mathbf{\Sigma_x} = \mathbf{B}\cdot\mathbf{o}$$

goal is to determine the vector o by minimizing the sum (least squares fit):

$$\chi^2 = \sum_{l=1}^{n} \frac{1}{\sigma^2_{\Sigma_x^{(l)}}} \left(\Sigma_x^{(l)} - \sum_{i=1}^{3} B_{li} o_i \right)^2$$

with the symmetric n×n covariance matrix, $\mathbf{T} = (\hat{\mathbf{B}}^t \cdot \hat{\mathbf{B}})^{-1}$

➡ the least-squares solution is $\mathbf{o} = \mathbf{T}\cdot\hat{\mathbf{B}}^t\cdot\hat{\Sigma}_x$

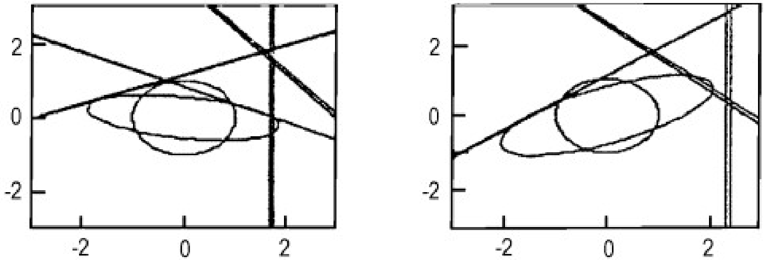
(the 'hats' show weighting: $\hat{B}_{li} = \dfrac{B_{li}}{\sigma_{\Sigma_x^{(l)}}}$ $\hat{\Sigma}_x^{(l)} = \dfrac{\Sigma_x^{(l)}}{\sigma_{\Sigma_x^{(l)}}}$)

once the components of o are known,

$$\epsilon = \sqrt{o_1 o_3 - o_2^2}\,,$$
$$\beta = o_1/\epsilon\,,\ \text{and}$$
$$\alpha = -o_2/\epsilon\,.$$

graphical representation of results:

LI01 x-plane elec			LI01 y-plane elec	
20.88 +/- 0.99	(20.00)	$\gamma\epsilon$ (10^{-5} m-r)	32.29 +/- 2.67	(20.00)
39.00 +/- 1.02	(20.00)	$B_{mag}\gamma\epsilon$ (10^{-5} m-r)	54.23 +/- 1.32	(20.00)
1.87 +/- 0.11	(1.00)	B_{mag}	1.68 +/- 0.17	(1.00)
28.01 +/- 1.93	(8.28)	β (m)	15.90 +/- 2.27	(6.00)
-3.05 +/- 0.23	(-1.03)	α	2.06 +/- 0.39	(1.13)
1463.90 +/- 29.29	(843.17)	σ_1 (μm)	1893.29 +/- 33.89	(718.04)
1439.99 +/- 28.80	(620.33)	σ_2 (μm)	1514.39 +/- 30.29	(818.45)
888.05 +/- 13.75	(915.43)	σ_3 (μm)	2052.81 +/- 41.06	(545.70)
778.71 +/- 15.57	(730.80)	σ_4 (μm)	632.62 +/- 12.65	(731.27)
3.68 +/- 0.24		intensity (10^{10} ppb)	3.43 +/- 0.36	
307.4		χ^2 / dof	578.7	



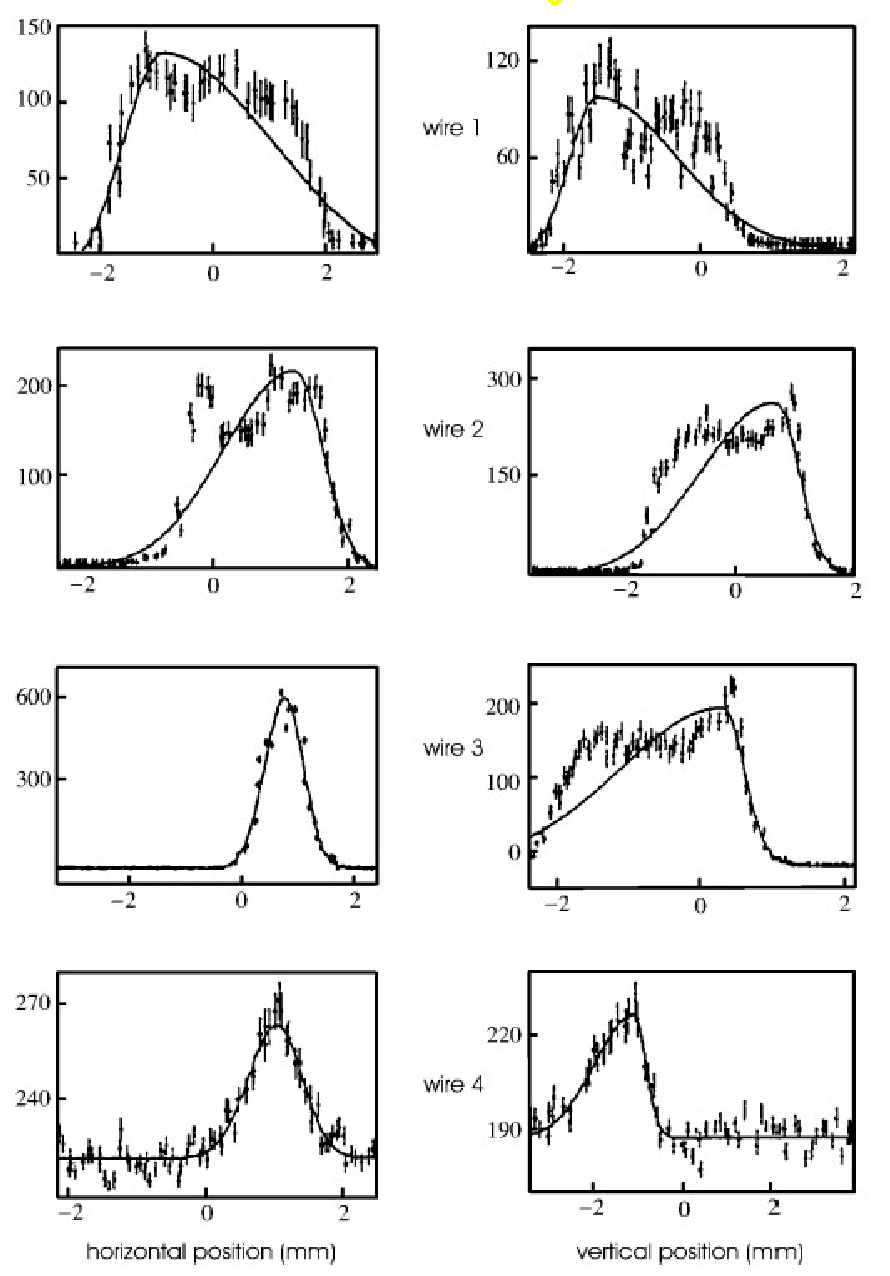
note: coordinate axes are so normalized (design phase ellipse is a circle):

$$\left(\frac{x}{\sqrt{\beta_x}}, \frac{\alpha_x x + \beta_x x'}{\sqrt{\beta_x}} \right)$$

lines show phase space coverage of wires:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{ref point}} = R^{-1} \begin{pmatrix} \sigma_{x,w} \\ x'_w \end{pmatrix}$$

data: from the SLC injector linac



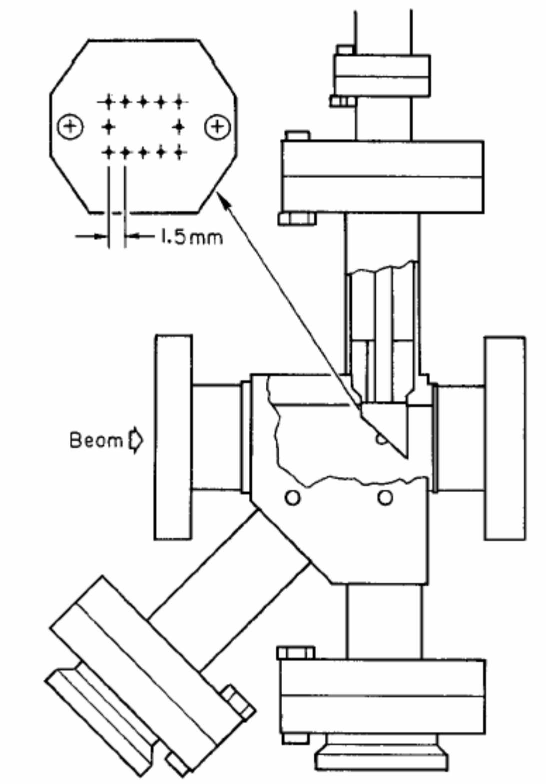
With methods I & II, the beam sizes may be measured using e.g. screens or wires

Transverse Beam Emittance – Screens

principle: intercepting screen (eg. Al_2O_3Cr possibly with phosphorescent coating) inserted into beam path (usually 45 deg); image viewed by camera → direct observation of x-y (η = 0) or y-δ ($\eta \neq$ 0) distribution

R. Jung et al, "Single Pass Optical Profile Monitoring" (DIPAC 2003)

fluorescence – light emitted (t~10 ns) as excited atoms decay to ground state
phosphorescence – light continues to emit (~ μs) after exciting mechanism has ceased (i.e. oscilloscope "afterglow")
luminescence – combination of both processes



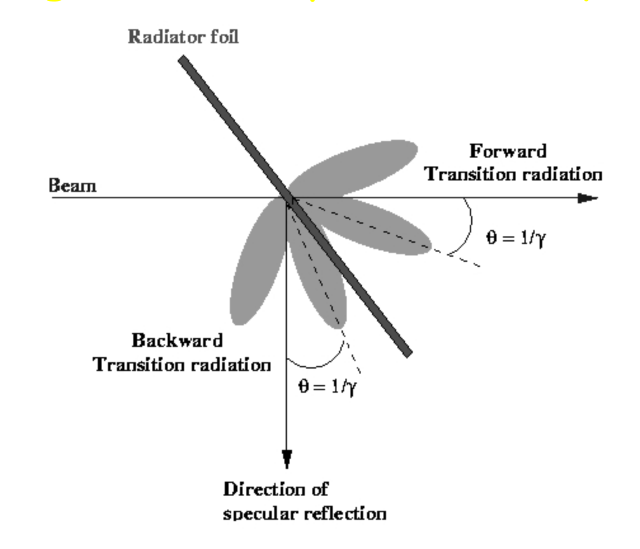
(courtesy P. Tenenbaum, 2003)

emittance measurement
image is digitized, projected, fitted with Gaussian
calibration: often grid lines directly etched onto screen or calibration holes drilled, either with known spacing

issues
spacial resolution (20-30 μm) given by phosphor grain size and phosphor transparency
temporal resolution – given by decay time
radiation hardness of screen and camera
dynamic range (saturation of screen)

Transverse Beam Emittance – Transition Radiation (1)

principle: when a charged particle crosses between two materials of different dielectric constant (e.g. between vacuum and a conductor), transition radiation is generated, temporal resolution is ~ 1 ps → useful for high bunch repetition frequencies



forward and backward transition radiation with foil at 45 degrees allowing for simple vacuum chamber geometry (courtesy K. Honkavaara, 2003)

foil: Al, Be, Si, Si + Al coating, for example

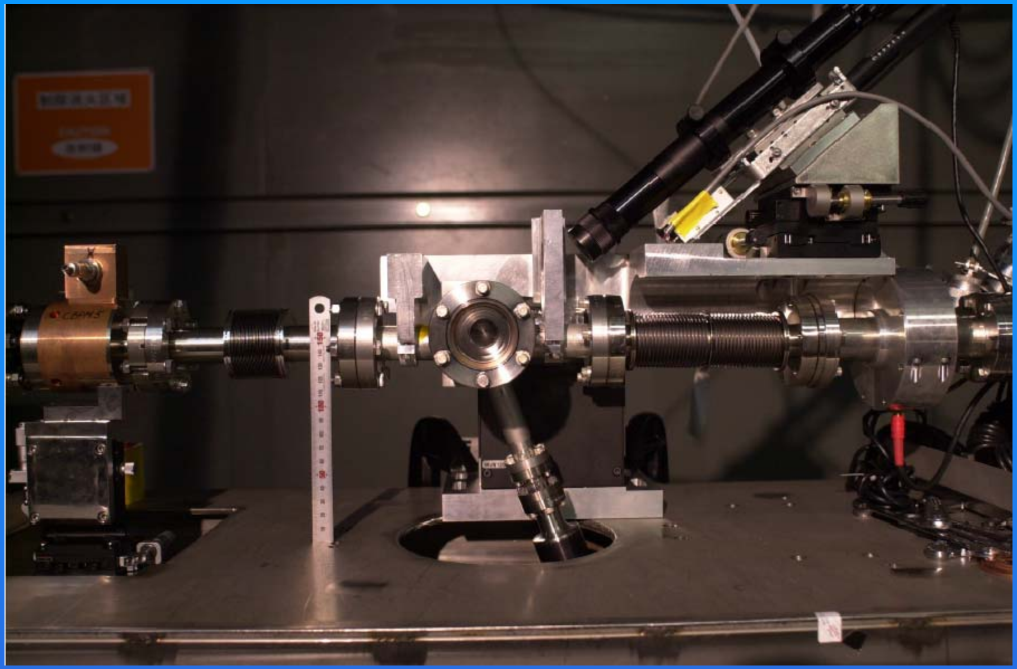
emittance measurement (as for screens):

- digitized and fitted image
- calibration: also with etched lines of known spacing

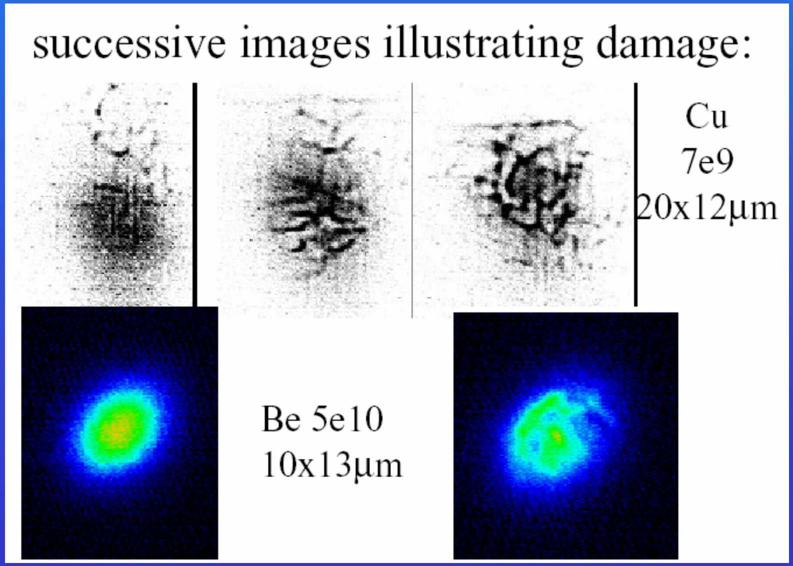
issues:

- spatial resolution (<5 μm) as introduced by the optics
- damage to radiator with small spot sizes
- (geometrical depth of field effect when imaging backward TR)

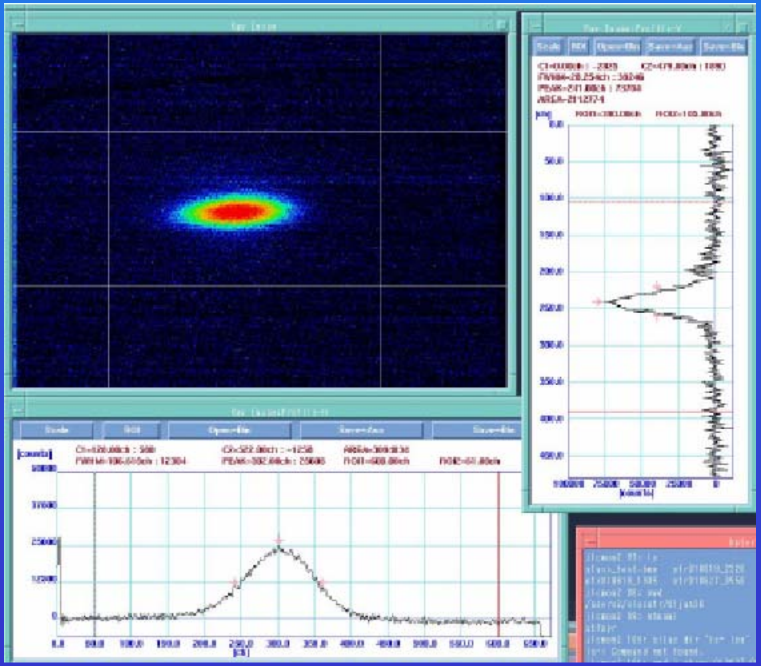
Transverse Beam Emittance – Transition Radiation (2)



the SLAC-built OTR as installed in the extraction line of the ATF



beam spot as measured with the OTR at the ATF at KEK

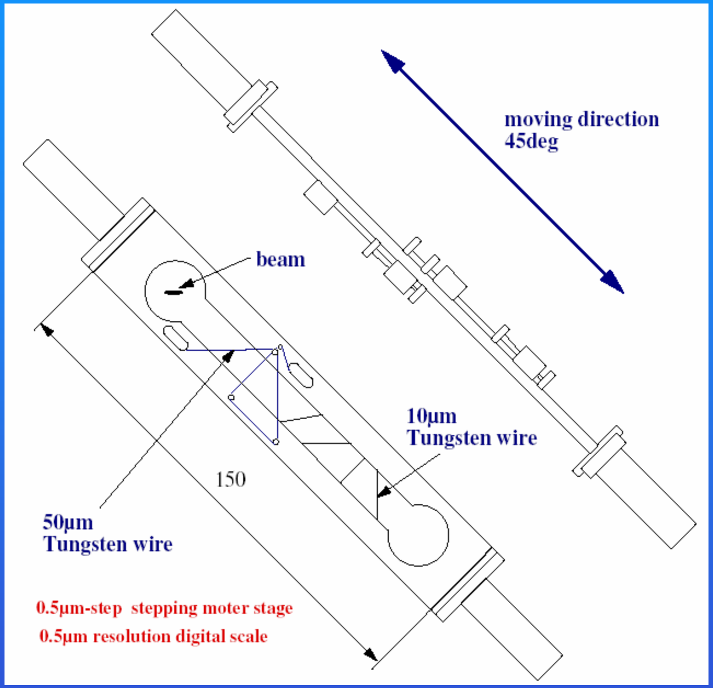


(all figures courtesy M. Ross, 2003)

Transverse Beam Emittance – Wire Scanners (1)

principle: precision stage with precision encoder propels shaft with wire support wires (e.g. C, Be, or W) scanned across beam (or beam across wire) interaction of beam with wire detected, for example with PMT

wire mount used at the ATF (at KEK) with thin W wires and 5 μm precision stepper-motors and encoders (courtesy H. Hayano, 2003)



wire velocity: depends on desired interpoint spacing and on the bunch repetition frequency

detection of beam with wire:

1. change in voltage on wire induced by secondary emission
2. hard Bremsstrahlung – forward directed γs which are separated from beam via an applied magnetic field and converted to e+/e- in the vacuum chamber wall and detected with a Cerenkov counter or PMT (after conversion to γs in front end of detector)
3. via detection at 90 deg (δ-rays)
4. using PMTs to detect scattering and electromagnetic showers

(5. via change in tension of wire for beam-tail measurements)

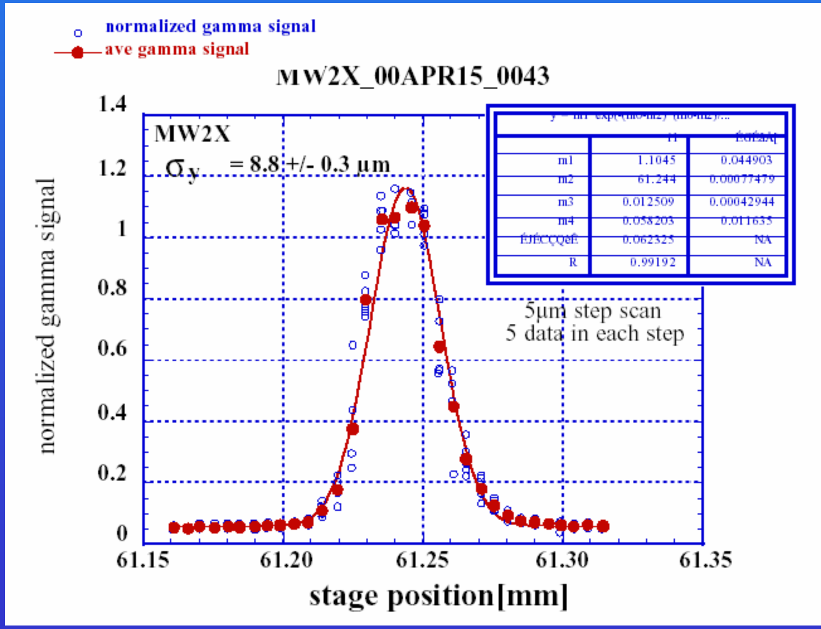
emittance measurement: as for screens (quad scan or 3-monitor method)

Transverse Beam Emittance – Wire Scanners (2)

issues:

- different beam bunch for each data point
- no information on x-y coupling with 1 wire (need 3 wires at common location)
- dynamic range: saturation of detectors (PMTs)
- single-pulse beam heating
- wire thickness (adds in quadrature with beam size)
- higher-order modes

(left) wire scanner chamber installed in the ATF (KEK) extraction line and (right) example wire scan (courtesy H. Hayano, 2003)

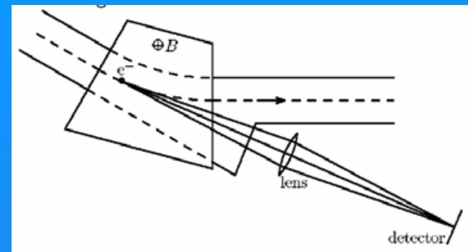


Transverse Beam Emittance – Synchrotron Radiation (1)

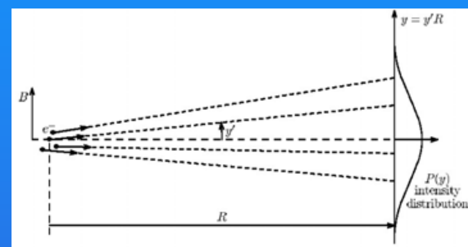
principle: charged particles, when accelerated, emit synchrotron radiation

measurements:

imaging → beam cross section



direct observation →angular spread

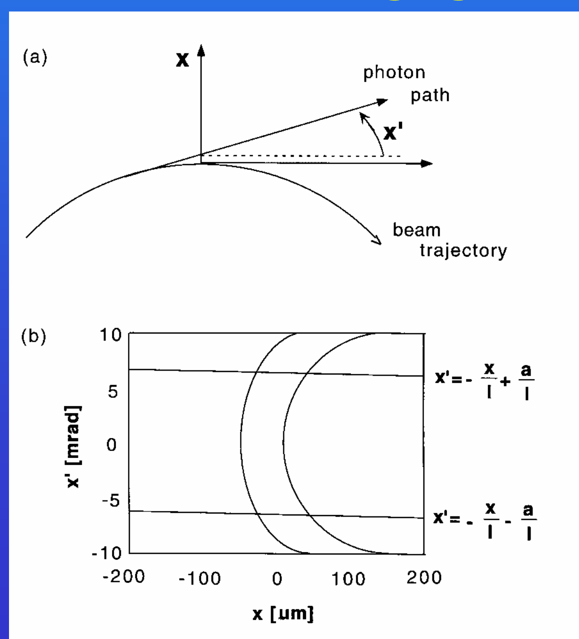


A. Hofmann, from this lecture series, 2003

depth of field effect in direct imaging (ref. A. Sabersky)

phase space coordinates of the photon beam

photon beam phase space at distance l from source (for a 100 μm beam at emission point)



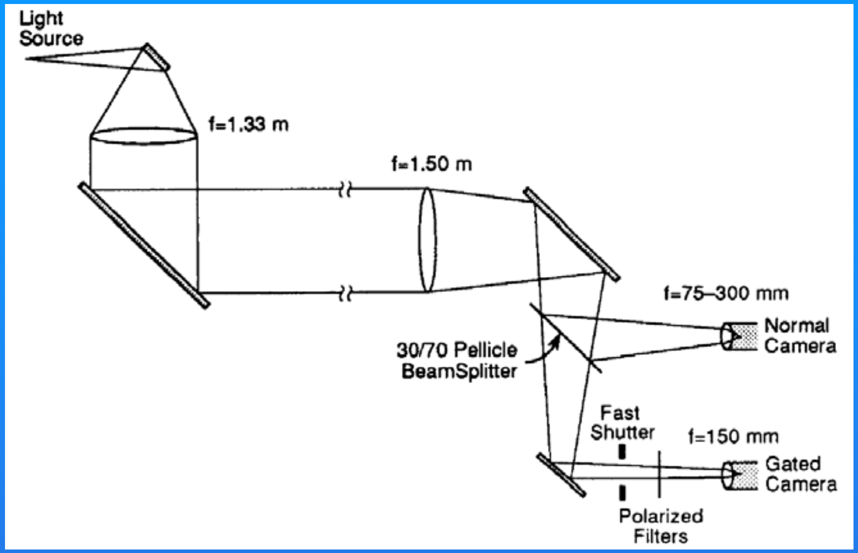
a = half-width of and
l = distance to defining aperture

measured beam parameters correspond to a projection of this phase space onto the horizontal axes

$$\bar{x} = \frac{\int I(x) x dx}{\int I(x) dx}$$

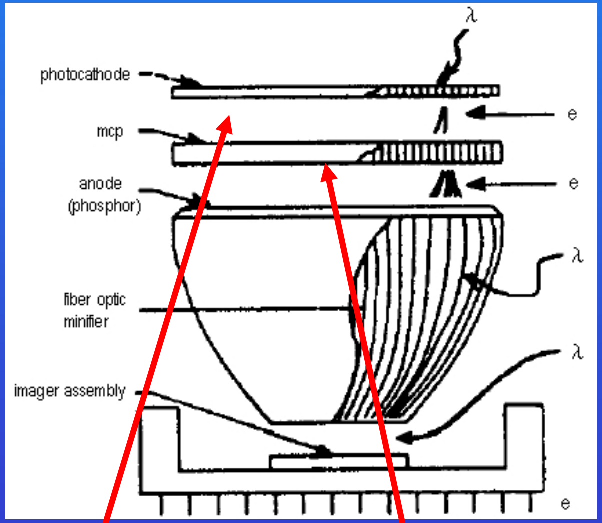
$$\sigma_{r,x}{}^2 = \frac{\int I(x)(x-\bar{x})^2 dx}{\int I(x) dx}$$

Transverse Beam Emittance – Synchrotron Radiation (2)

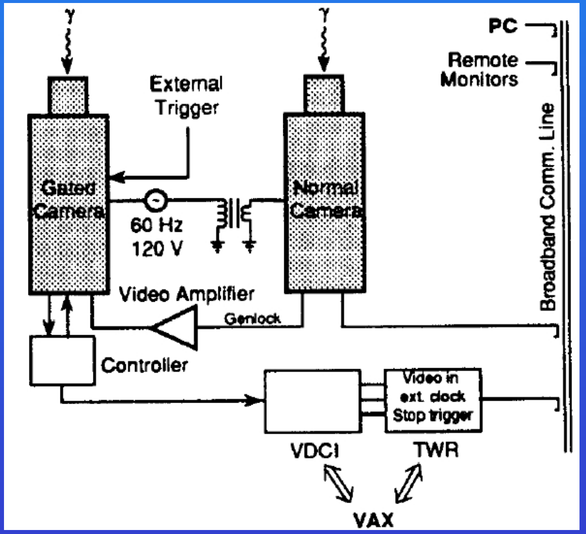


optics used at the SLC (damping rings) for imaging SR emitted in a dipole magnet (not to scale)

intensifier of the gated camera used for fast (turn by turn) imaging of the radiation (from Xybion Corp.)



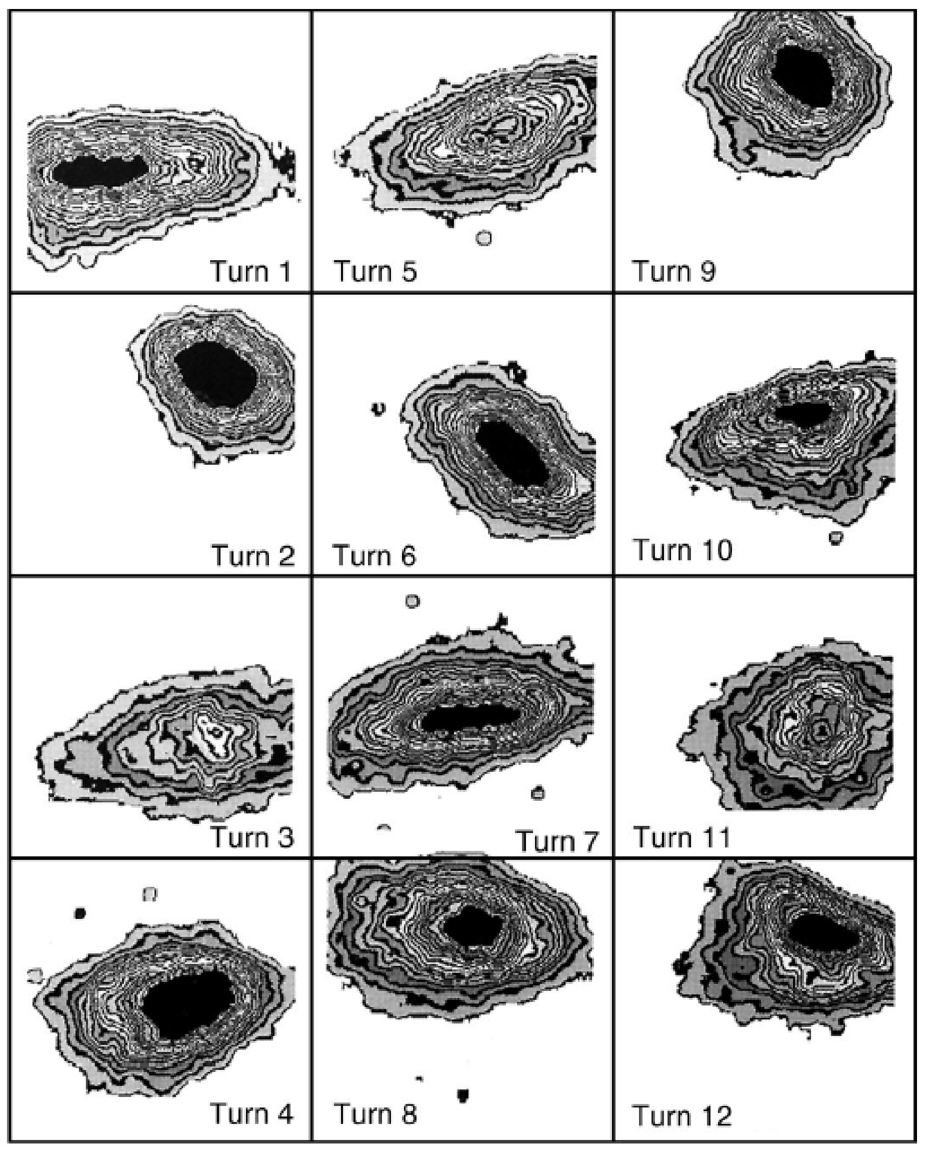
gated camera synchro-nization using a standard TV camera



key feature: gated high bias between photocathode and MCP

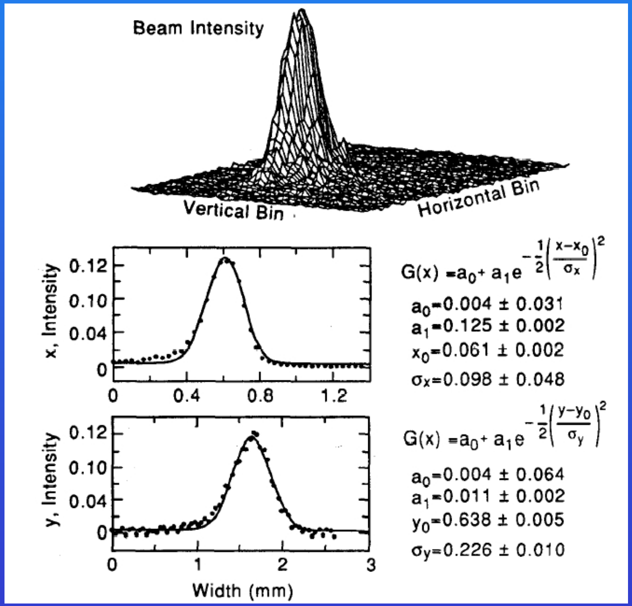
(scrolling from emi removed using a line-locked TV camera for composite synchronization)

Transverse Beam Emittance – Synchrotron Radiation (3)



(left) raw data showing turn-by-turn images at injection (average over 8 pulses)

(bottom) digitized image in 3-D and projections together with Gaussian fits (bottom)



originally, these studies aimed at measuring the transverse damping times, but were then extended to measure emittance mismatch and emittance of the injected beams …

Transverse Beam Emittance – Synchrotron Radiation (4)

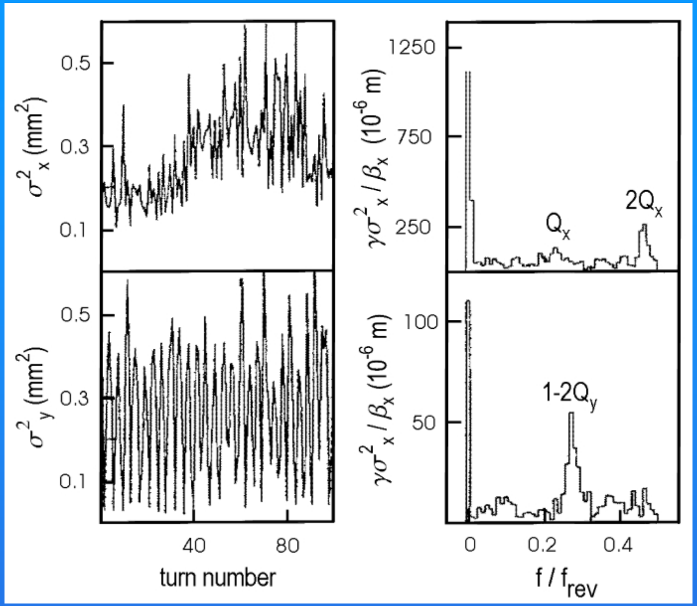
<u>injection matching</u> – using the definition of the mismatch parameter (M. Sands)

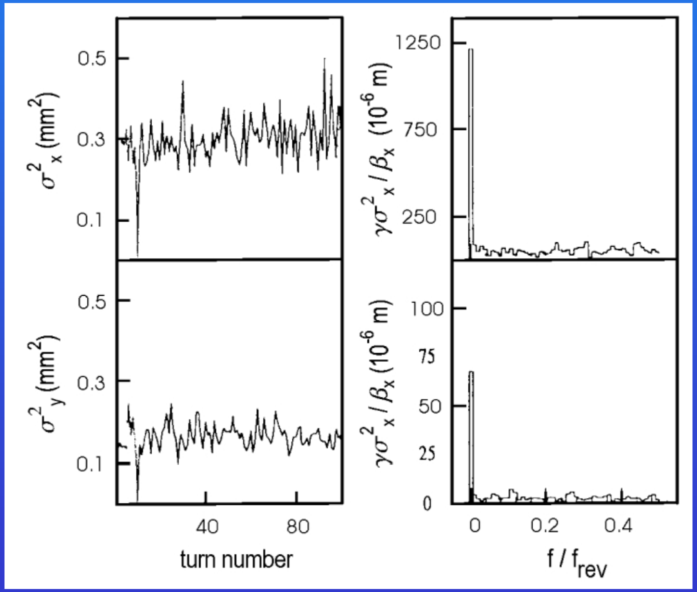
$A_1 = B\varepsilon$ = amplitude of DC peak
$A_2 = \mathrm{sqrt}\,(B^2-1)$ = amplitude of peak at 2Q

with $\rho = A_1/A_2$ ➡ $B = 1/\mathrm{sqrt}(1-\rho^{-2})$

<u>beam emittance</u>

➡ $\varepsilon = A_1/(A_2^2+1)$





(left) turn-by-turn beam size measured before (top) and after (bottom) injection matching; (right) spectrum of beam shape oscillations (FT of $\sigma_x^2(t)$)

M. Minty and F. Zimmermann "Measurement and Control of Charged Particle Beams", Springer Verlag (2003)

Transverse Beam Emittance – Laser Wire Scanners

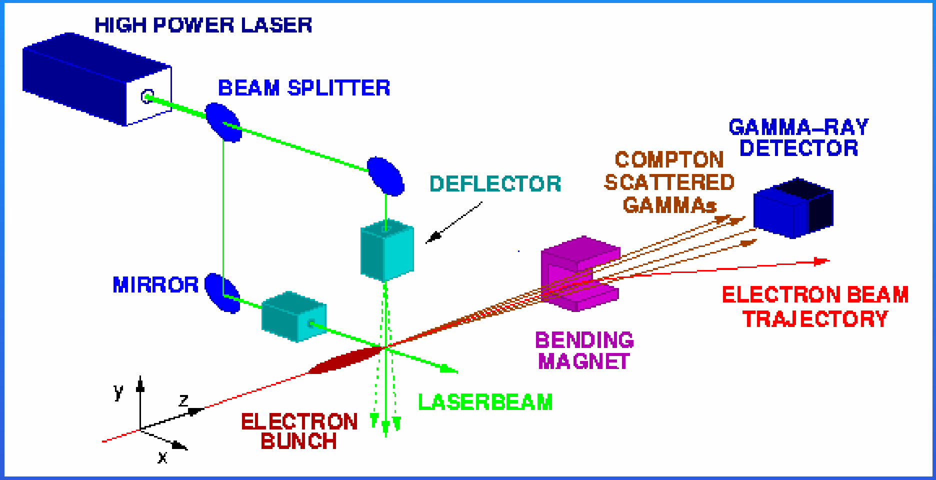
principle: laser wire provides a non-invasive and non-destructable target wire scanned across beam (or beam across wire)

constituents: laser, optical transport line, interaction region and optics, detectors

beam size measurements: forward scattered Compton γs or lower-energy electrons after deflection by a magnetic field

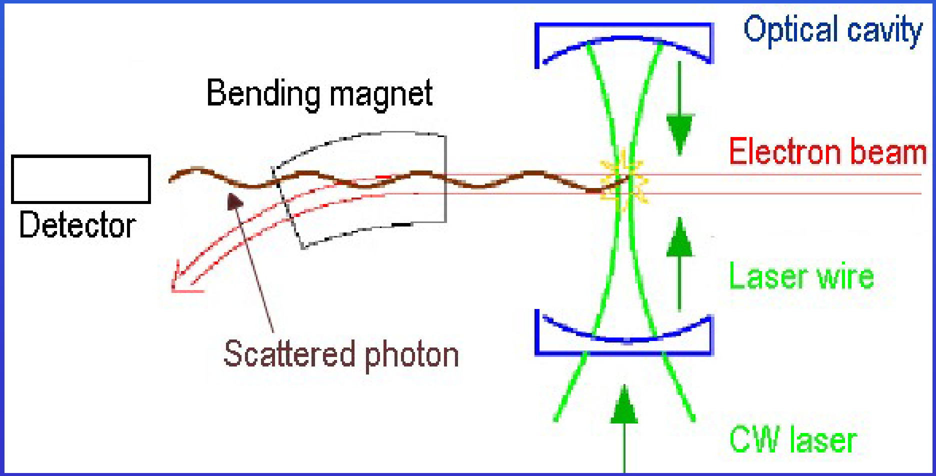
schematic of the laser wire system planned for use at PETRA and for the third generation synchrotron light source PETRA 3 (courtesy S. Schreiber, 2003)

high power pulsed laser

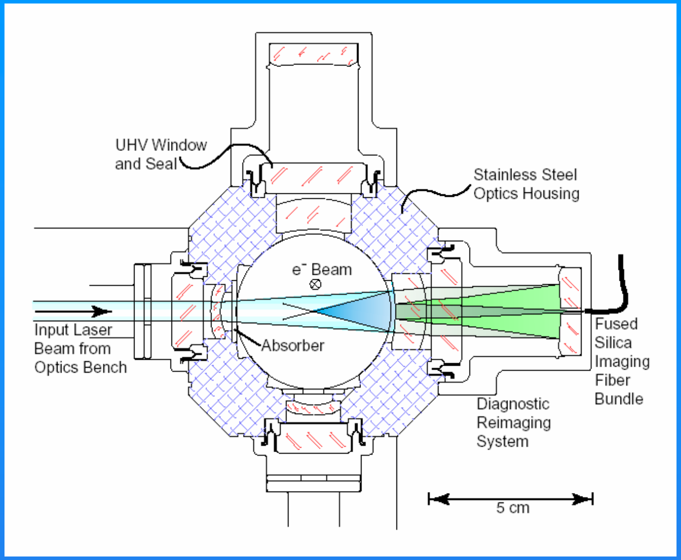


overview of the laser wire system at the ATF (courtesy H. Sakai, 2003)

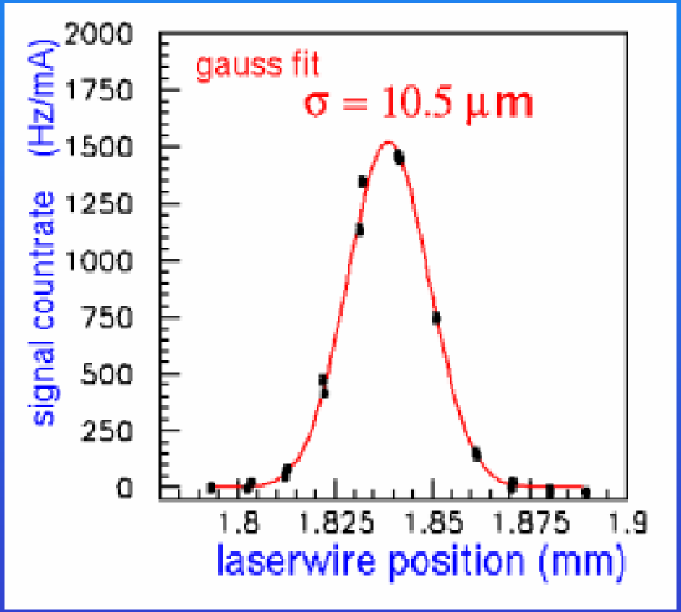
optical cavity pumped by CW laser (mirror reflectivity ~99+%)



laser-electron interaction point from the pioneering experiment at the SLC final focus (courtesy M. Ross, 2003)



beam size measured at the laser wire experiment of the ATF (courtesy H. Sakai, 2003)



$$\sigma_y = \sqrt{{\sigma_{obs}}^2 - (\frac{w_0}{2})^2}$$

$$\beta_y \epsilon_y = (\sigma_y)^2 - (\eta_y \frac{\sigma_p}{p})^2$$

(as with normal wires, the wire size must be taken into account)

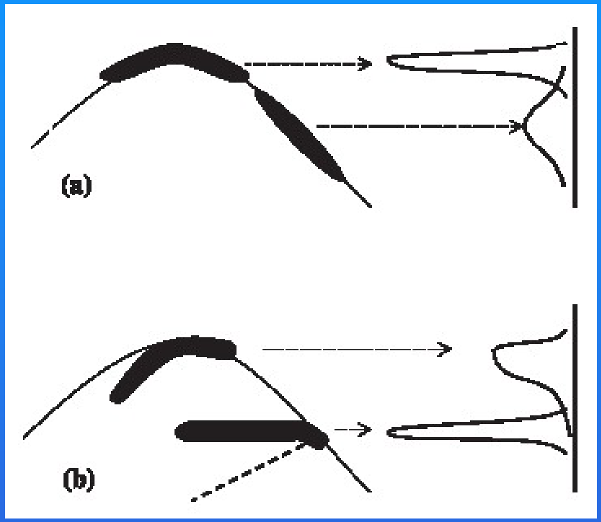
(here w_0 is the 2σ wire thickness)

<u>issues:</u> waist of laser < beam size (in practice, waist size $\sim\lambda$)
background and background subtraction
depth of focus large so that sensitivity of σ_y on x_e is minimized
synchronization (for pulsed lasers)

Beam Energy Spread

In circular accelerators, the beam energy spread is usually very small (~10^{-4})

In linear accelerators, the energy spread is determined to a large extent by the length of the bunch and its overlap with the sinusoidal accelerating voltage



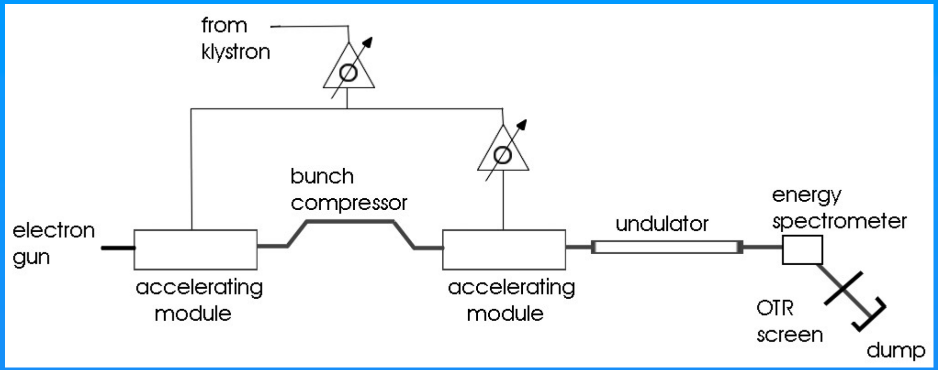
effective energy gain (left) and energy spread (right) for low (a) and high (b) current bunches illustrating optimum phasing of the rf structures for minimum energy spread

Principle: the beam size as measured, e.g. with a screen or wire, is the convolution of the natural beam size $\sigma_{\beta,\,x,y}$ and the energy spread $\delta=\Delta E/E$:

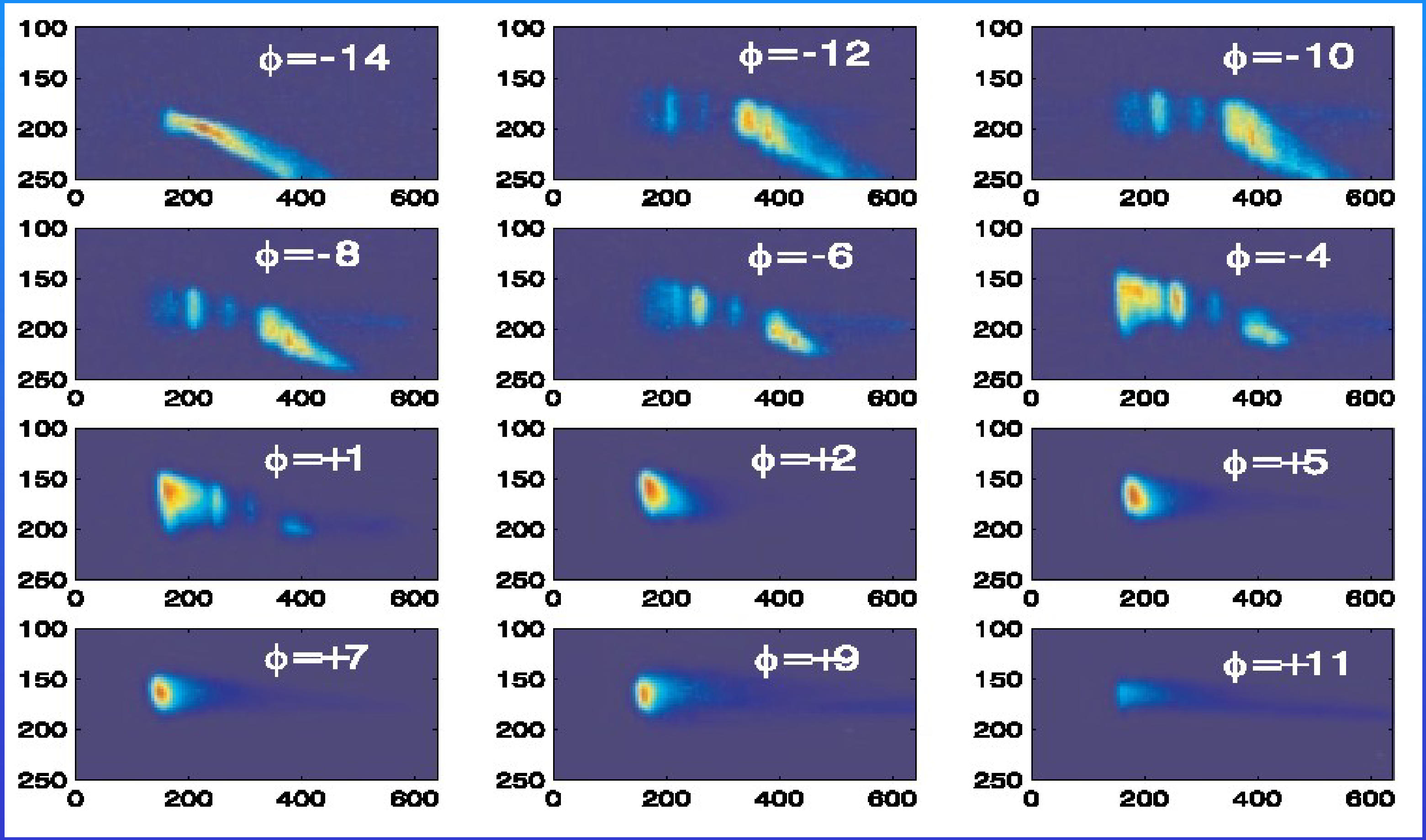
$$\sigma = \mathrm{sqrt}\ (\sigma_\beta^2 + [\eta\delta]^2)$$

where η is the dispersion function

By proper selection of location (for a screen /wire), where η is large, the beam energy spread δ can be directly measured



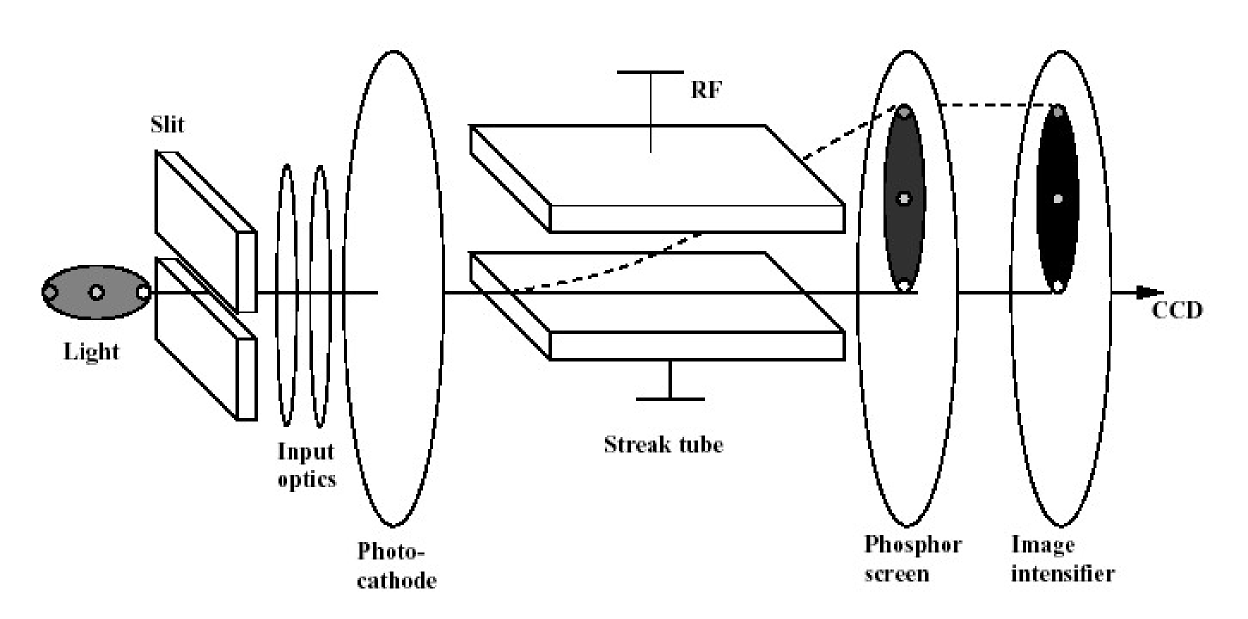
sketch of the layout of TTF I



Single-bunch OTR images from TTF I obtained in a region of high dispersion (courtesy F. Stulle, 2003)

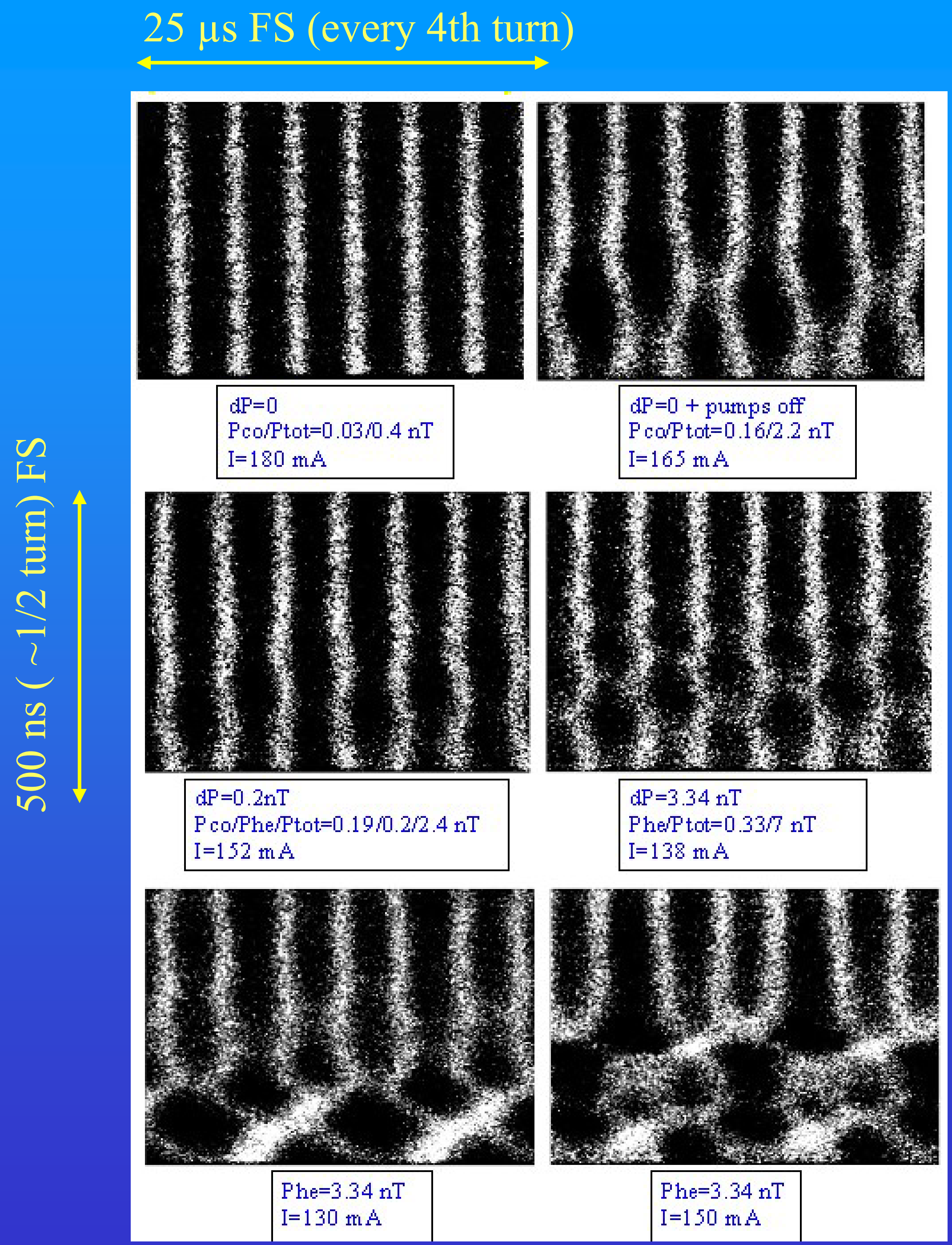
Bunch Length – Streak Cameras

Principle: photons (generated e.g. by SR, OTR, or from an FEL) are converted to e-, which are accelerated and deflected using a time-synchronized, ramped HV electric field; e- signal is amplified with an MCP, converted to γs (via a phosphor screen) and detected using an imager (e.g. CCD array), which converts the light into a voltage



Principle of a streak camera (from M. Geitz, "Bunch Length Measurements", DIPAC 99)

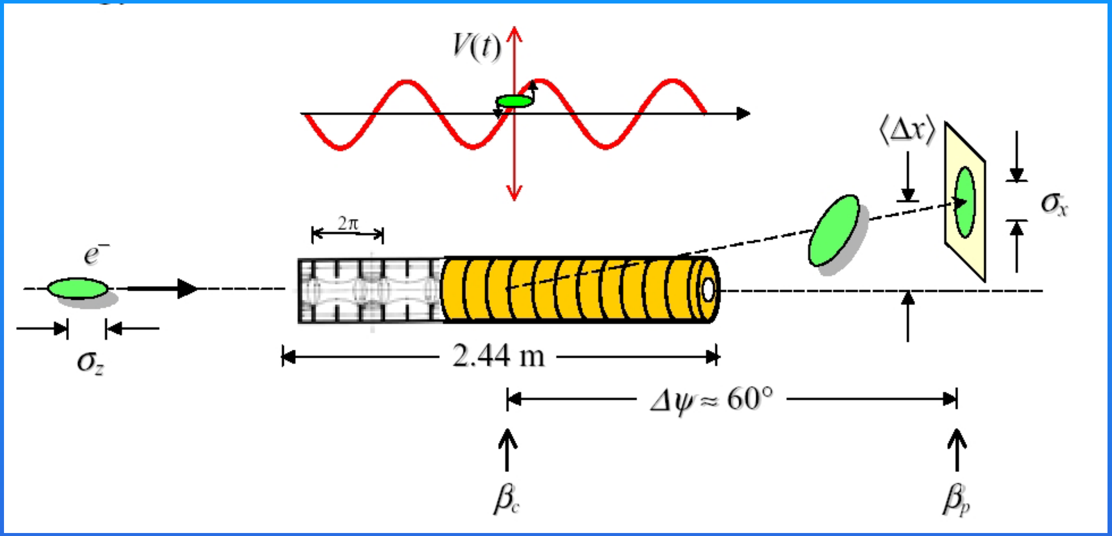
Issues: energy spread of e- from the photocathode (time dispersion)
chromatic effects ($dt/dE_{\gamma}(\lambda)$) in windows
space charge effects following the photocathode



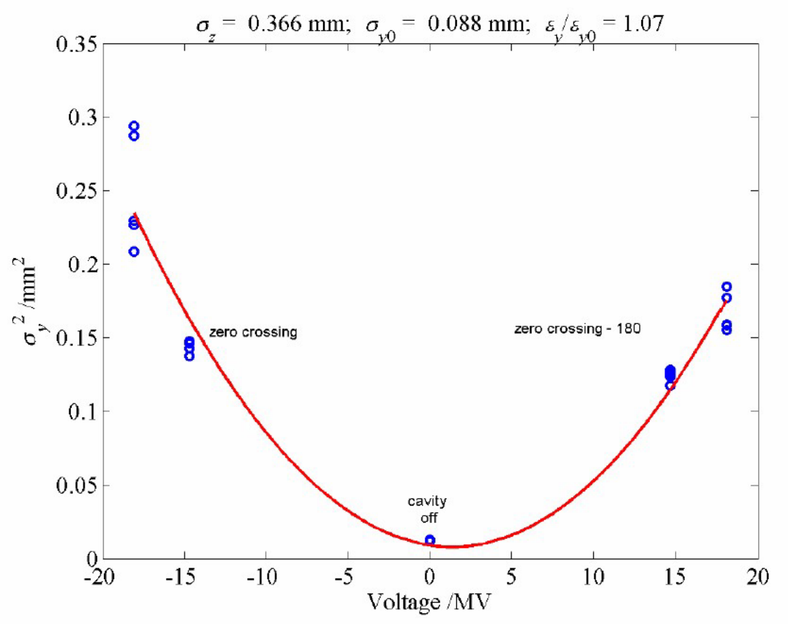
Streak camera images from the Pohang light source evidencing beam oscillations arising from the fast-ion instability (courtesy M. Kwon, 2000)

Bunch Length – Transverse Mode Cavities

Principle: use transverse mode deflecting cavity to "sweep"/kick the beam, which is then detected using standard profile monitors



Principle of the TM_{11} transverse mode deflecting cavity



(figures courtesy R. Akre, 2003)

Summary

We reviewed multiple, equivalent methods for describing the transport of beam parameters between 2 points

Two methods for measuring the transverse beam emittance were presented:

- the quadrupole scan – optics are varied, single measurement location
- the fixed optics method with at least three independent beam size measurements

Methods for measuring the beam size were reviewed including

- screens
- transition radiation
- (conventional) wire scanners
- (direct imaging of) synchrotron radiation
- laser wires

The presented method for measuring the bunch energy spread used hardware as for the transverse beam size measurements, but required situation at a location where the dispersion is nonzero

Bunch length measurements using streak cameras and transverse mode cavities were briefly reviewed.