Diagnostics I
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Summary

Diagnostics I

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Introduction

Accelerator performance depends critically on the ability to carefully measure and control the properties of the accelerated particle beams.

In fact, it is not uncommon, that beam diagnostics are modified or added after an accelerator has been commissioned.

This reflects in part the increasingly difficult demands for high beam currents, smaller beam emittances, and the tighter tolerances placed on these parameters (e.g. position stability) in modern accelerators.

A good understanding of diagnostics (in present and future accelerators) is therefore essential for achieving the required performance.

A beam diagnostic consists of:

- the measurement device
- associated electronics and processing hardware
- high-level applications

Reference: “Beam Diagnostics and Applications”, A. Hofmann (BIW 98)
Detection of charged particle beams - beam detectors:

\( i_w \) is a current source with infinite output impedance, \( i_w \) will flow through any impedance placed in its path.

Many "classical" beam detectors consist of a modification of the walls through which the currents will flow.

**Sensitivity of beam detectors:**

- **Beam charge:**
  \[ S(\omega) = \frac{V(\omega)}{I_w(\omega)} \] (in \( \Omega \))
  = ratio of signal size developed to the wall current \( I_w(\omega) \)

- **Beam position:**
  \[ S(\omega) = \frac{V(\omega)}{D(\omega)} \] (in \( \Omega /m \))
  = ratio of signal size developed / dipole mode of the distribution, given by \( D(\omega) = I_w(\omega) z \), where \( z = x \) (horizontal) or \( z = y \) (vertical)
Beam Charge – the Faraday Cup

Principle: beam deposits (usually) all energy into the cup (invasive) charge converted to a corresponding current voltage across resistor proportional to instantaneous current absorbed

In practice:
termination usually into 50 Ω; positive bias to cup to retain e- produced by secondary emission; bandwidth-limited (~1 GHz) due to capacitance to ground

cylindrically symmetric blocks of lead (~35 rad lengths) carbon and iron (for suppression of em showers generated by the lead)
bias voltage (~many 100 Volts) for suppression of secondary electrons

cross-sectional view of the FC of the KEKB injector linac (courtesy T. Suwada, 2003)
Beam Intensity - Toroids (1)

Consider a magnetic ring surrounding the beam, from Ampere’s law:

\[ \oint B \cdot dl = \mu I \]

if \( r_0 \) (ring radius) >> thickness of the toroid,

\[ B = \frac{\mu i_b}{2\pi r_0} \]

Add an N-turn coil - an emf is induced which acts to oppose B:

\[ \epsilon = \frac{d\phi}{dt} \quad \text{where} \quad \phi = \int \vec{B} \cdot d\vec{a} \\
= \frac{\mu A}{2\pi r_0} \frac{di_b}{dt} \]

Load the circuit with an impedance; from Lenz’s law, \( i_R = i_b/N \):

Principle: the combination of core, coil, and R produce a current transformer such that \( i_R \) (the current through the resistor) is a scaled replica of \( i_b \). This can be viewed across R as a voltage.
Beam Intensity - Toroids (2)

sensitivity:

\[ S = \frac{R}{\sqrt{1 + \left(\frac{\omega}{\omega_L}\right)^2}} \]

\[ \omega_L = \frac{R}{L} \]

with \( R_h = \) reluctance of magnetic path

\[ R_h = \frac{l}{\mu A} \text{ [H}^{-1}] \]

\[ L = \frac{N^2 \mu_r \mu_0 A}{l} \]

cutoff frequency, \( \omega_L \), is small if \( L \sim N^2 \) is large

detected voltage:

\[ V(t) = \frac{i_b R}{N} e^{-\left(\frac{R}{L}\right) t} \]

if \( N \) is large, the voltage detected is small

\( \left\{ \begin{array}{c} \text{trade-off} \\ \text{between} \\ \text{bandwidth} \\ \text{and signal} \\ \text{amplitude} \end{array} \right\} \)
Beam Intensity - Toroids (3)

schematic of the toroidal transformer for the TESLA Test facility (courtesy, M. Jablonka, 2003)

(one of many) current transformers available from Bergoz Precision Instruments (courtesy J. Bergoz, 2003)

A iron
B Mu-metal } shielding
C copper
D “Supermalloy” (distributed by BF1 Electronique, France) with $\mu \sim 8 \times 10^4$
E electron shield
F ceramic gap

(based on design of K. Unser for the LEP bunch-by-bunch monitor at CERN)
linacs: resolution of $3 \times 10^6$
storage rings: resolution of 10 nA rms

details: www.bergoz.com
Beam Intensity – Toroids (4)

recent developments of toroids for TTF II (DESY)

2 iron halves

50 Ω output impedance

calibration windings

(25 ns, 100 mV / dvsn)

(ferrite rings (for suppression of high frequency resonance))

Beam Intensity - BPM Sum signals

Remarks:
1) as we will see, higher-order nonlinearities must occasionally be taken into account
2) in circular e+/- accelerators, assembly is often tilted by 45 degrees
Beam Position - Wall Gap Monitor (1)

principle:
remove a portion of the vacuum chamber and replace it with some resistive material of impedance $Z$

detection of voltage across the impedance gives a direct measurement of beam current since $V = i_w(t) Z = -i_b(t) Z$

(add high-inductance metal shield
add ferrite to increase L
add ceramic breaks
add resistors (across which $V$ is to be measured)

alternate topology - one of the resistors has been replaced by the inner conductor of a coaxial line

(susceptible to em pickup and to ground loops)
Beam Position - WGM (2)
sensitivity:

Circuit model using parallel RLC circuit:

\[
\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C
\]

High frequency response is determined by \( C \):

\[
|Z(\omega \to \infty)| = \frac{R}{\sqrt{1 + (\frac{\omega}{\omega_C})^2}} \quad (\omega_C = 1/RC)
\]

Low frequency response determined by \( L \):

\[
|Z(\omega \to 0)| = \frac{R}{\sqrt{1 + (\frac{\omega L}{\omega})^2}} \quad (\omega_L = R/L)
\]

Intermediate regime: \( R/L < \omega < 1/RC \) - for high bandwidth, \( L \) should be large and \( C \) should be small

Remark: this simplified model does not take into account the fact that the shield may act as a resonant cavity.
Beam Position – Capacitive Monitors (1)
(capacitive monitors offer better noise immunity since not only the wall current, but also PS and/or vacuum pump returns and leakage current, for example, may flow directly through the resistance of the WGM)

principle: vacuum chamber and electrode act as a capacitor of capacitance, $C_e$, so the voltage generated on the electrode is

$$V = \frac{Q}{C_e} \quad \text{with} \quad Q = i_w t = \frac{i_w L}{c}$$

where $L$ is the electrode length and $c = 3 \times 10^8 \text{ m/s}$

long versus short bunches:

since the capacitance $C_e$ scales with electrode length $L$, for a fixed $L$, the output signal is determined by the input impedance $R$ and the bunch length $\sigma$

for $\omega << \omega_c$

(bunch long compared to electrode length $\sigma > L$)
the electrode becomes fully charged during bunch passage
signal output is differentiated
signal usually coupled out using coax attached to electrode

for $\omega >> \omega_c$

output voltage rises rapidly and is followed by extended negative tail (since dc component of signal is zero)
induced voltage usually detected directly through a high impedance amplifier
Beam Position – Capacitive Monitors (2)

**Position Information:**
replace cylinder by curved electrodes (usually 2 or 4) symmetrically placed with azimuth +/-/ψ (usually small to avoid reflections between the edges and the output coupling)

**Example – Capacitive Split Plate:**

\[ \sigma = \frac{1}{2\pi a} \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{r_0}{r} \right)^2 \cos n(\phi - \phi_0) \right] \]

**Surface Charge Density \( \sigma \) due to a unit line charge collinear to electrodes at \((r_0, \phi_0)\):**

\[ I_R = \int_{-\psi}^{\psi} \sigma(r d\phi) \]
\[ = \frac{2\psi}{2\pi} \left[ 2\psi + 2\frac{r_0}{a} \sin \psi + \frac{x_0^2 - y_0^2}{a^2} \sin 2\psi + ... \right] \]

\[ I_L = \int_{-\psi}^{\psi} \sigma(r d\phi) \]
\[ = \frac{2\psi}{2\pi} \left[ 2\psi - 2\frac{x_0}{a} \sin \psi + \frac{x_0^2 - y_0^2}{a^2} \sin 2\psi + ... \right] \]

**Voltage Across Impedance \( R \):**

\[ V = \frac{(I_R - I_L)R}{2i_\omega R} \]
\[ = \frac{\pi a}{(\sin \psi)x_0} + ... \]

**Sensitivity:**

\[ S = \frac{V}{i_\omega x_0} = \frac{2R}{\pi a} \sin \psi + ... \]

The voltage and sensitivity are large if the azimuthal coverage is large or the radius \( a \) is small; e.g. \( \psi = 30 \) deg, \( R = 50 \Omega \), \( a = 2.5 \text{ cm} \) \( \rightarrow S = 2 \Omega /\text{mm} \).
Beam Position – Capacitive Monitors (3)

example – capacitive split cylinder:

charge in each detector half is found by integrating the surface charge density:

\[ Q_i = \frac{\lambda}{2} [L \pm \frac{r_0}{2\pi} \sin \phi_0 \tan \theta] \]

\[ C_e = \frac{C}{2} \]

\[ C = \frac{L}{Z_0 c} \]

\[ \Delta x = r_0 \cos \phi_0 \]

\[ V = \frac{Q_l - Q_r}{C_e} = \frac{Z_0 \tan \theta}{2\pi L} (-i_{\mu}) \Delta x \]

\[ S = \frac{Z_0 \tan \theta}{2\pi L} \]

The capacitive split cylinder is a linear detector; there are no geometry-dependent higher order contributions to the position sensitivity. \( S \) is maximal for \( \theta = \pi/4 \)
Beam Position – Button Monitors

Buttons are used frequently in synchrotron light sources as a variant of the capacitive monitor (2), however terminated into a characteristic impedance (usually by a coax cable with impedance $50\,\Omega$). The response obtained must take into account the signal propagation (like for transmission line detectors, next slide).

button electrode for use between the undulators of the TTF II SASE FEL (courtesy D. Noelle and M. Wendt, 2003)

cross-sectional view of the button BPM assembly used in the DORIS synchrotron light facility

design reflects geometrical constraints imposed by vacuum chamber geometry

note: monitor has inherent nonlinearities (courtesy F. Peters, 2003)
Beam Position – Stripline / Transmission Line Detectors (1)

principle: electrode (spanning some azimuth $\psi$) acts as an inner conductor of a coaxial line; shield acts as the grounded outer conductor $\rightarrow$ signal propagation must be carefully considered

- unterminated transmission line
- transmission line terminated (rhs) to a matched impedance

reminder: characteristic impedance $Z_0$ terminated in a resistor $R$

$$\rho = \text{reflection coefficient} = \frac{R-Z_0}{R+Z_0} = \begin{cases} 0 & \text{if } R=Z_0 \\ -1 & \text{if } R=0 \\ >0 & \text{if } R>Z_0 \\ <0 & \text{if } R<Z_0 \end{cases}$$

$$\Gamma = 1-\rho = \text{transmission coefficient}$$
Beam Position – Stripline / Transmission Line Detectors (2)

The voltage appearing across each resistor is evaluated by analyzing the current flow in each gap:

\[ V_{\text{R}_1,g_1} = \frac{i_w}{2} \left[ 1 + \left( \frac{R_2 - Z_0}{R_2 + Z_0} \right) e^{-2j\omega\Delta t} \right] R_1 \]

\[ V_{\text{R}_1,g_2} = -\frac{i_w}{2} e^{-j\omega\Delta t} \left[ 1 - \left( \frac{R_1 - Z_0}{R_1 + Z_0} \right) e^{-j\omega\Delta t} \right] R_1 \]

**initial reflection**

**beam delay**

**transmission**
Beam Position - Stripline / Transmission Line Detectors (3)

Similarly, voltage at $R_2$:

$$V_{R_2,g_1} = \frac{i_{\omega}}{2} e^{-j\omega \Delta t} \left[ 1 - \left( \frac{R_2 - Z_0}{R_2 + Z_0} \right) R_2 \right]$$

Signal delay

$$V_{R_2,g_2} = -\frac{i_{\omega}}{2} e^{-j\omega \Delta t} \left[ 1 + \left( \frac{R_1 - Z_0}{R_1 + Z_0} e^{-2j\omega \Delta t} \right) R_2 \right]$$

Transmission

On each resistor:

$$V_{R_1} = V_{R_1,g_1} + V_{R_1,g_2}$$

$$V_{R_3} = V_{R_1,g_1} + V_{R_1,g_2}$$

Voltage at each gap:

$$\Delta t = \frac{L}{c}$$

Special cases:

(i) $R_1 = Z_0$, $R_2 = 0$ (terminated to ground)

$$V_{R_1} = \frac{i_{\omega}}{2} \left( 1 - e^{-\frac{2i\omega t}{c}} \right) R_1$$

$$V_{R_2} = 0$$

(ii) $R_1 = R_2 = Z_L$ (matched line)

$$V_{R_1} = \frac{i_{\omega}}{2} \left( 1 - e^{-\frac{2i\omega L}{c}} \right) Z_L$$

$$V_{R_2} = 0$$

(iii) $R_1 = R_2 \neq Z_L$ then solution as in (ii) to second order in $\rho$
again, \[ V_{R_1} = \frac{i_w}{2} \left( 1 - e^{-\frac{2\pi L}{\lambda}} \right) R_1 \]

sensitivity \[ |S| = \left| \frac{V}{i_w} \right| = R_1 \left| \sin^2 \omega \Delta t \right| \]

signal peaks at \[ \omega \Delta t = \frac{2\pi L}{\lambda} = \frac{\pi}{2} \rightarrow L = \frac{\lambda}{4} \]

spacing between zeros \[ \omega \Delta t = 0 \rightarrow L = \frac{\lambda}{2} \]

the LEUTL at Argonne shorted S-band quarter-wave four-plate stripline BPM (courtesy R.M. Lill, 2003)
specially designed to enhance port isolation (using a short tantalum ribbon to connect the stripline to the molybdenum feedthrough connector) and to reduce reflections

L=28 mm (electrical length \sim 7\% longer than theoretical quarter-wavelength), \( Z_0=50 \, \Omega \)
Beam Position - Cavity BPMs (1)

principle: excitation of discrete modes (depending on bunch charge, position, and spectrum) in a resonant structure; detection of dipole mode signal proportional to bunch charge, \( q \times \) transverse displacement, \( \delta x \)

theoretical treatment: based on solving Maxwell's equations for a cylindrical waveguide with perpendicular plates on two ends

motivation: high sensitivity (signal amplitude / \( \mu \)m displacement)

accuracy of absolute position, LCLS design report

dipole mode cavity BPM consists of (usually) a cylindrically symmetric cavity, which is excited by an off-axis beam:

\[ TM_{010}, "common mode" (\propto I) \]
\[ TM_{110}, dipole mode of interest \]

amplitude detected at position of antenna contains contributions from both modes \( \rightarrow \) signal processing

reference:
"Cavity BPMs", R. Lorentz (BIW, Stanford, 1998)
Beam Position – Cavity BPMs (2)

\[ V_{110}^{\text{out}}(\delta x) = V_{110}^{\text{in}}(\delta x) \frac{R}{Q} \sqrt{\frac{50\Omega}{Q_L}} \sqrt{1 - \frac{Q_L}{Q_0}} \]

\[ V_{110}^{\text{in}} \approx \delta x \cdot q \frac{T_t^2}{r^3} \cdot 0.2474 \]

\[ T_t = \frac{\sin \eta}{\eta} \quad \text{with} \quad \eta = \frac{\pi l}{\lambda_{mn0}} \]

\[ T_{tr}, (R/Q) \quad \text{transit time factor} \]
\[ Q_0, Q_L \quad \text{geometrical property of cavity} \]
\[ \text{unloaded and loaded } Q\text{-factors} \]
\[ L \quad \text{cavity length} \]
\[ r \quad \text{cavity radius} \]
\[ \lambda_{mn0} \quad \text{wavelength of mode of interest} \]
\[ \delta x \quad \text{transverse displacement} \]

\[ \text{for the TTF cavity BPM:} \]
\[ r = 115.2 \text{mm} \]
\[ L = 52 \text{ mm} \]
\[ \rightarrow V_{110}^{\text{out}} \approx 115 \text{ mV/mm for } 1 \text{ nC} \]

Pioneering experiments: 3 C-band cavity “RF” BPMs in series at the FFTB (SLAC)
\[ \rightarrow 25 \text{ nm position resolution at } 1 \text{ nC bunch charge} \]

(courtesy, T. Shintake, 2003)
Beam Position – “Reentrant Cavity BPMs”

principle: detection of the evanescent field of the cavity fundamental mode (those waves with exponential attenuation below the cut-off frequency):

excite cavity at frequency $f_0$ with respect to cavity resonant frequency $f_r$ while Q-factor decreases by $\sqrt{f_0/f_r}$, the attenuation constant of evanescent fields below $\sim 1/2$ the cut-off frequency is practically constant $\Rightarrow$ maintain high signal amplitude

from R. Bossart, “High Precision BPM Using a Re-Entrant Coaxial Cavity”, LINAC94

using URMEL, the equivalent circuit for impedance model was developed

schematic of the reentrant cavity BPM used successfully at TTF I and planned for use at TTF II (courtesy C. Magne, 2003)
Summary

Detection of the wall current $I_w$ allows for measurements of the beam intensity and position.

The detector sensitivities are given by

$$S(\omega) = \frac{V(\omega)}{I_w(\omega)}$$

for the beam charge and intensity

$$S(\omega) = \frac{V(\omega)}{D(\omega)}$$

with

$$D(\omega) = I_w(\omega)x$$

$$D(\omega) = I_w(\omega)y$$

for the horizontal position

for the vertical position

We reviewed basic beam diagnostics for measuring:
- the beam charge - using Faraday cups
- the beam intensity - using toroidal transformers and BPM sum signals
- the beam position
  - using wall gap monitors
  - using capacitive monitors (including buttons)
  - using stripline / transmission line detectors
  - using resonant cavities and re-entrant cavities

We note that the equivalent circuit models presented were often simplistic. In practice these may be tailored given direct measurement or using computer models. Impedances in the electronics used to process the signals must also be taken into account as they often limit the bandwidth of the measurement. Nonetheless, the fundamental design features of the detectors presented were discussed (including variations in the designs) highlighting the importance of detector geometries and impedance matching as required for high sensitivity.