Part III
Technology of Insertion Devices

Pascal ELLEAUME
European Synchrotron Radiation Facility, Grenoble
The main issue in the magnetic design of a planar undulator or wiggler is to produce a sinusoidal field with a high peak field $B$ and the shortest period $\lambda_0$ within a given aperture (gap).

Three type of technologies can be used:
- Permanent magnets (NdFeB, Sm$_2$Co$_{17}$)
- Room temperature electromagnets (iron and coils)
- Superconducting electromagnets (superconducting coils with or without iron)
Current Equivalent of a Magnetised Material

Air coil with Surface Current Density $[\text{A/m}] \approx \frac{B_r}{\mu_0}$

Periodic array of magnets

\[ B_T \approx \frac{2B_r[T]}{\mu_0} \]

or Current Density \[ [A/m^2] \approx \frac{4B_r[T]}{\mu_0 \lambda_0} \]

Example: \( B_r = 1 \, T \), \( \lambda_0 = 20 \, \text{mm} \) \Rightarrow \n
Equiv. Current Density=160A/mm²!!

Permanent Magnet Undulator

Hybrid

Pole (Steel)

Pure Permanent Magnet

(NdFeB, Sm$_2$Co$_{17}$,...)

<table>
<thead>
<tr>
<th>Material</th>
<th>$B_r$ [T]</th>
<th>$\mu_{r,\parallel}$</th>
<th>$\mu_{r,\perp}$</th>
<th>$H_{ci}$ [kA/m]</th>
<th>$10^{-2}$/°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>SmCO$_5$</td>
<td>0.9–1.01</td>
<td>1.05</td>
<td></td>
<td>1500–2400</td>
<td>–0.04</td>
</tr>
<tr>
<td>Sm$<em>2$CO$</em>{17}$</td>
<td>1.04–1.12</td>
<td>1.05–1.08</td>
<td></td>
<td>800–2000</td>
<td>–0.03</td>
</tr>
<tr>
<td>NdFeB</td>
<td>1.0–1.4</td>
<td>1.04–1.06</td>
<td>1.15–1.17</td>
<td>1000–3000</td>
<td>–0.10</td>
</tr>
</tbody>
</table>
Magnetic Field of a Pure Permanent Magnet Undulator
(Halbach Formula)

Assume relative permeability of magnet = 1 with remanent field $B_r$, then the exact field computation gives:

$$B_n = 2B_r \frac{\sin(n \frac{\pi}{4})}{n \frac{\pi}{4}} \exp(-n\pi \frac{\text{gap}}{\lambda_0})(1 - \exp(2n\pi \frac{h}{\lambda_0})) \cos(2n\pi \frac{s}{\lambda_0})$$

if $h > \frac{\lambda_0}{2} \Rightarrow 1 - \exp(2n\pi \frac{h}{\lambda_0}) \sim 1$

$\Rightarrow B_n = B_r \frac{\text{gap}}{\lambda_0}$

$\Rightarrow n = 1$ dominates

$$B_z(s) \approx 1.8 \ B_r \ \exp(-\pi \frac{\text{gap}}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0})$$

<table>
<thead>
<tr>
<th>$b_1$</th>
<th>$0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_3$</td>
<td>$0.30$</td>
</tr>
<tr>
<td>$b_5$</td>
<td>$-0.18$</td>
</tr>
<tr>
<td>$b_7$</td>
<td>$-0.13$</td>
</tr>
</tbody>
</table>

Field from Pure Permanent Magnet vs Hybrid

Magnet Volume = 2 N \lambda^3
L x Magnet = 2 \lambda

First Harmonic
••••• Peak Field

Hybrid
(Vanadium Permendur)

Pure Permanent Magnet

Numerical Computation of Magnetic Field

- **No Iron** (perm. magnet & coil):
  - Integration of Biot and Savart Law
    \[ \mathbf{B} = \mu_0 \int I \frac{d\mathbf{l} \times \hat{u}}{r^2} \]
  - Simple Numerical Methods based on the current sheet or surface charge model. The total field is the linear sum of the field produced by each block. Particularly simple and efficient for parallelepipedic shapes.

- **With Iron** (perm. magnet & coil & iron): Best solved with numerical methods
  - Finite Element Method
    - Used dominantly for Dipole/Quadrupole/Sextupole … Magnets
    - 2D: POISSON (Public Domain)
      - from http://laacg1.lanl.gov/laacg/services/possup.html
    - 3D: Commercial Codes (TOSCA, FLUX3D, ANSYS,…)
  - Volume Integral Method: Radia
    - Particularly adapted to undulators and Wigglers
    - Compute field and field integral in 3D
    - Public Domain http://www.esrf.fr/machine/groups/insertion_devices/Codes/software.html
Design Process

User Requirements:
- Photon Energy Range
- Linear/Circular Polarization
- Divergence, Power

Pre-Design:
- Choice of Technology
  - Wiggler/Undulator
  - Period, Field, Length

Detailed Design:
- Central Period
  - End design

Field Measurement and Shimming

Install in the Ring

Radiation Computation:
- With Ideal Field
- Photon Energy Range
- Brilliance, Flux

Measurements of Radiation

Machine Constraints:
- Minimum Gap
- Electron Energy

Measure effect on the e-beam

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Magnetic Forces

Force between upper and lower magnetic arrays:

\[
Force = \frac{B^2 WL}{4\mu_0}
\]

<table>
<thead>
<tr>
<th></th>
<th>B [T]</th>
<th>W [mm]</th>
<th>L [m]</th>
<th>F [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undulator</td>
<td>0.8</td>
<td>40</td>
<td>1.6</td>
<td>8.1</td>
</tr>
<tr>
<td>Wiggler</td>
<td>1.5</td>
<td>120</td>
<td>1.6</td>
<td>85.9</td>
</tr>
</tbody>
</table>

Force on each magnet can be large:
⇒ rigid holding structures
⇒ special assembly tools
ESRF Undulators

Magnetic Force: 1-10 Tons
Gap Resolution: < 1 µm
Parallellism < 20 µm

Undulators are Fundamentally **Small Gap Devices**

- Like any accelerator magnet, the smaller the magnetic gap the less volume of magnetic material required to reach a specific field geometry.
- The lower the gap the higher the energy of the harmonics in the undulator emission.

\[ \lambda_n = \frac{\lambda_0}{2n\gamma^2} \left(1 + \frac{K^2}{2}\right) \]

*with*

\[ K = 0.0934 B_0[T] \lambda_0[mm] \]

\[ B_0 \sim 1.8 B_r \exp(-\pi \frac{\text{gap}}{\lambda_0}) \]
Application: Build a pure permanent magnet undulator with NdFeB Magnets ($B_r = 1.2$ T)

Undulator with $K=1$

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.72</td>
<td>15</td>
<td>15.2</td>
<td>6.0</td>
</tr>
<tr>
<td>10</td>
<td>0.49</td>
<td>22</td>
<td>10.3</td>
<td>7.3</td>
</tr>
<tr>
<td>15</td>
<td>0.38</td>
<td>28</td>
<td>8.2</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Flexible Chambers

<table>
<thead>
<tr>
<th>Ring</th>
<th>( \lambda_o ) (mm)</th>
<th>L (m)</th>
<th>( g_{\text{vac}} ) (mm)</th>
<th>( g_{\text{mag}} ) (mm)</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX</td>
<td>24</td>
<td>0.83</td>
<td>6.0</td>
<td>7.6</td>
<td>1.7</td>
</tr>
<tr>
<td>NSLS</td>
<td>16</td>
<td>0.32</td>
<td>3.8</td>
<td>7.5</td>
<td>0.7</td>
</tr>
<tr>
<td>ESRF</td>
<td>26</td>
<td>0.80</td>
<td>6.0</td>
<td>9.0</td>
<td>1.7</td>
</tr>
</tbody>
</table>

In Vacuum Undulators

- Developed at NSLS, Spring-8, ESRF
- Required by many new light sources (SLS, CLS, LBL, Diamond, Soleil, ..)
- Open the gap during injection if needed
- Allow a minimum magnetic gap of 3 to 6 mm

ESRF In-vacuum Undulator

Figure 5.26 3D view of an ESRF in-vacuum undulator.

Electro-Magnet Undulator

-Limited by the electrical power requirement and associated cooling of the coils:
-Current Densities < 10-15 A/mm²
-Only interesting for long periods

Delta Superconducting Wiggler

- High field: up to 10 T => Shift the spectrum to higher energies
- Sophisticated engineering & high cost
SRS Superconducting Wiggler
Local Field Measuring Bench

Optimized for fast longitudinal field scanning:
- Optical & Laser Encoder
- 3-axis Hall probe sensor
- On-the-fly scanning 2000pts/m
- Measuring length 2-10 m
- Essential for phase shimming

Field Integral Measuring Bench

Either:
- Rotating multiturn coil
- Moving stretched wire

- Measure Horiz & Vertical single and double field integrals
- Absolute accuracy < 10 Gcm
- Essential for multipole shimming
Magnetic Field Errors of Permanent Magnet Insertion Devices:

- Originate from:
  - Non uniform magnetization of the magnet blocks (poles).
  - Dimensional and Positional errors of the poles and magnet blocks.
  - Interaction with environmental magnetic field

- Need to purchase highly uniformly magnetized blocs and
  - perform a systematic characterization
  - Perform a pairing of the blocks to cancel field integrals
  - but still insufficient.

- Type of Field Errors
  - Multipole Field Errors (Normal and skew dipole, quadrupole, sextupole,…).
  - Phase errors which reduce the emission on the high harmonic numbers
Undulator Shimming

• **Mechanical**: Moving permanent magnet or iron pole vertically or horizontally

• **Magnetic**: Add thin iron piece at the surface of the blocks
  – More precise and local
  – Field reduction
Magnetic shims

Field Integral and Multipole Shimming

Horizontal Deflection
Quadrupole
Sextupole …

Vertical Deflection
Skew Quadrupole
Skew sextupole …

Each pulse interfere constructively if $T_p = T$ for all $p$ and for a wavelength so that $\lambda = 2T/n$ where $n$ is an integer (harmonic number). Real undulators have small field errors which result in fluctuations of $T_p$. These are also called phase errors.
Assuming: \( T_p \) identically independently distributed for every \( p \) with:

\[
\langle T_p \rangle = T, \quad \langle (T_p - T)^2 \rangle = \sigma_T
\]

Then, it is a consequence of the Fourier Transform that:

\[
\frac{d\Phi_n}{d\theta_x d\theta_z} \frac{d\lambda}{\lambda} (0, 0, \lambda_n) = \frac{d\Phi_n}{d\theta_x d\theta_z} \frac{d\lambda}{\lambda} (0, 0, \lambda_n)_{\text{ideal}} e^{-(n\pi \frac{\sigma_T}{T})^2}
\]

independently of the number of period \( N \).

The effect is usually characterised by a rms phase error \( \sigma \) expressed in degrees:

\[
\sigma[\text{deg}] = \frac{1}{180} \frac{\sigma_T}{T}
\]

On-axis angular flux, flux and brilliance are multiplied by \( e^{-(n\pi \frac{\sigma_T}{T})^2} \)

<table>
<thead>
<tr>
<th>Phase error [deg]</th>
<th>6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic #</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>0.76</td>
<td>0.99</td>
</tr>
<tr>
<td>9</td>
<td>0.41</td>
<td>0.98</td>
</tr>
<tr>
<td>13</td>
<td>0.16</td>
<td>0.95</td>
</tr>
</tbody>
</table>

A Practical Example of Phase Shimming of an ESRF Undulator:
(period 35 mm, N=46 periods, Gap=11 mm)

Measured Vertical Field

Calculated Trajectory @ 6 GeV

Calculated on-axis single electron emission spectrum

Remarks on Phase Errors

- Small phase errors may have a large impact on the undulator spectrum in particular on the high harmonic numbers. The associated magnetic field errors can be detected on the field plot where they appear as period and peak field fluctuations. Some of them (generating internal angles) may also be visible from the wandering of trajectory.

- Emittance and energy spread induce a broadening of the peak and may mask a part of the spectral flux lost due to phase errors. Nevertheless, in most cases, even with large emittance and energy spread, low phase error undulators perform much better on the high harmonics.

- They are important for long undulators or undulators intended to be used on a high harmonic number

- They are usually not important in undulators used on the fundamental of the spectrum such as in Free Electron Lasers