CERN Accelerator School

Bruges, Belgium June 16-25, 2009

MAGNETS

Solenoids

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A few things that I know about solenoids…

- Flux tubes
- Formulas
- Homogeneity
- Bitter Magnet
- Solenoid lense
- CMS
- Iseult
Maxwell equations : think Flux

- The phenomenons are perfectly known.
- All the day we obey to Maxwell equations to create magnetic field.
- The representation of flux tubes is a very powerful method.
- Flux is going from North to south
**DISCUSSION 3.6: “Double-Pancake” vs. “Layer-Wound”**

Of the two magnet winding techniques, one is commonly known as “double-pancake” or simply “pancake,” and the other as “layer-wound.” A double-pancake coil is generally wound with flat conductor, e.g., tape, and sometimes with “large” square- or rectangular-cross-sectioned conductor, e.g., CIC. Each is wound with a

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**Fig. 3.23** Pictorial view of a double-pancake coil, with the top and bottom pancakes separated axially for clarity. The pancakes in this drawing are wound with a tape conductor. Points A and B indicate the ends of a continuous conductor, with Point C marking the approximate midpoint.
Maxwell’s equations for magneto-statics

- Maxwell’s equations have been solved by different methods.
- The precision request is in the range of $10^{-6}$
- Sophisticated computer codes have been developed on a large scale due for RMN & MRI industry
- These codes need 3D analytical representation of the field and “double precision”
The Ring Coil

Center field of a ring coil of radius $a$

$B_0 = \mu_0 \frac{I}{2a}$

$\mu_0 = 4\pi \cdot 10^{-7}$

$I \quad B \quad 2a$

1A 1mT 1.25mm
200A 0.8mT 300mm
Ring Coil

Field Distribution along the axis

\[ B_\phi(z) = B_0 \sin^3 \theta \]

\[ z = a \]
\[ \theta = \frac{\pi}{4} \]

\[ B_\phi(a) \approx 0.35 B_0 \]
Field distribution of a ring coil along axis

Equivalent length: $2a$

Inflexion at $a/2$

Long distance effect
Ring Coil

Equivalence with Permanent Magnet

Flux is going from North to South outside of the magnet
Laplace’s theorem: the 3 fingers of the right hand

\[ \mathbf{F} = \mathbf{B} i \mathbf{L} \]
« Hoop stress »

**Forces sur les conducteurs**

\[ \mathbf{dF} = I \mathbf{dl} \wedge \mathbf{B} \]

cas de la spire circulaire
Hoop stress in the ring coil (manuscript of Pr Guy Aubert)

The radial force is trying to extend the radius of the ring coil and it is equilibrated by the tension force. The approximation is valid for a turn on itself and slightly pessimistic in solid coils

Stress = J*B*R
(MPa) = (A/mm²)*Tesla*m
100 MPa = 100A/mm²*2T*0.5m
Field of a thin solenoid of finite length

\[ B_0 = \mu_0 n i \cos \alpha_0 \]
\[ (L \to \infty, B_0 \to \mu_0 n i) \]

À l'intérieur \[ B_A = \frac{\mu_0 n i (\cos \alpha_A + \cos \beta_A)}{2} \]
Plotting the flux lines of a thick air core solenoid

Flux tubes step 1
Plotting the flux lines of an air core solenoid
Flux tubes step2
Plotting the flux lines of an air core solenoid

Flux tubes step3

Please remember: Think FLUX
Plotting the flux lines of an air core solenoid

- In the middle, field is homogeneous, flux lines are parallel.
- In the ends, field is roughly half, flux tubes should be twice larger.
- Near the axis, flux tubes are going very far to close on themselves.
- Close to the coil, flux is turning, creating a longitudinal compression.
- Near the median plane, part of the flux is returning inside the coil around a zero field point.
- The inner part of the coil is suffering an expanding effort.
- The outer part of the coil is suffering a radial compression.
- The total force is an expanding effort.
Thick solenoid

Fig. 3.6. Computer plot of the field in a simple solenoid showing, on the left hand, magnetic lines of force and, on the right hand contours of constant field intensity $|B|$ relative to the central field $B_0$ (C. W. Trowbridge, Rutherford Laboratory, private communication).
La compréhension des forces est primordiale
Thick solenoid

- Field formulas for thick solenoids are complicated
- Peak field
- Lehmann Point
- Internal magnetic forces
- Magnetostatic pressure is equivalent to stored energy density: \( B^2 / 2 \mu_0 \)

<table>
<thead>
<tr>
<th>Field</th>
<th>Magnetic pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6 Tesla</td>
<td>1 MPa</td>
</tr>
<tr>
<td>4 Tesla</td>
<td>6.25 MPa</td>
</tr>
</tbody>
</table>
Discussion on homogeneity SMC magnet parameters

- 2 M Ampere turns per meter are creating around 2.57 T

If a finite solenoid of 300mm in diameter is only 2 meters long the field drop at 0.1m is already:
Ampere’s Theorem

Please remember:

<table>
<thead>
<tr>
<th>Current</th>
<th>Field</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>1mT</td>
<td>1.25 mm</td>
</tr>
</tbody>
</table>

800*2500 = 2M Ampere turns
i.e. for 500 Amps 4000 turns per meter

It applies to infinite solenoids

<table>
<thead>
<tr>
<th>Factor</th>
<th>2500</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1m</td>
</tr>
</tbody>
</table>

| 2.57 Tesla |
| Factor     |

2500
Even a long thin solenoid is not homogeneous.

- \( B_0 = \mu_0 n I \cos(\alpha_0) \)
- \( B_1 = \mu_0 n I (\cos \alpha_1 + \cos \alpha_2) \)

\[ \begin{align*}
\text{ALPHA}_0 &= 8,530 \text{ degrés} \quad B_0 = 0,9941 \text{ Tesla} \\
\text{At } z &= 0,100 \text{m} \\
\text{ALPHA}_1 &= 9,462 \text{ degrés} \quad \text{ALPHA}_2 = 7,765 \text{ degrés} \\
B_1 &= 0,98861205 \\
\text{DeltaB/B0} &= 0,005607 = 5 \times 10^{-3}
\end{align*} \]
Long solenoid with small error gap in the winding

- 0.1 mm error gap is a negative ring coil of 2000 A

\[ \Delta B = -0.0008 \text{T in the range of 500 ppm} \]

If you need a magnet with 25 ppm, it is huge
The Ring Coil

Center Field of a ring coil of radius $a$

\[ B_0 = \mu_0 \frac{I}{2a} \]

\[ \mu_0 = 4\pi \times 10^{-7} \]

$I$ \quad $B$ \quad $2a$

1A \quad 1mT \quad 1.25mm

200A \quad 0.8mT \quad 300mm
It is impossible to realise the winding of a long solenoid without a lot of distributed errors in the range of 0.1mm. The error results in a superposition of ring coil curves randomly distributed. It must be corrected by a family of longitudinal trim coils independently supplied. They will also compensate for the natural drop.
B in a point \( z \) of the axis created by a ring coil of radius \( a \) situated in \( b \) \( z=b/2 \) and with current \( I \)

\[
B_z(z) = \frac{\mu_0 I s^2 a^2}{2 \left( a^2 + \left( z - \frac{b}{2} \right)^2 \right)}
\]

For two rings

\[
B_z(z) = 0.5 \mu_0 I s^2 a^2 \left( \frac{1}{a^2 + \left( z - \frac{b}{2} \right)^2} + \frac{1}{a^2 + \left( z + \frac{b}{2} \right)^2} \right)
\]
For two ring coils

\[
\frac{d^2 B_z}{dz^2} = \frac{3 \mu_0 I s^2 a^2 (b^2 - a^2)}{\left(\frac{7}{2}\right) \left(a^2 + \frac{b^2}{4}\right)}
\]

\[
\frac{d^4 B_z}{dz^4} = \frac{45 \mu_0 I s^2 a^2 (8 a^4 - 24 a^2 b^2 + 4 b^4)}{\left(\frac{11}{2}\right) \left(a^2 + \frac{b^2}{4}\right)}
\]

\[
\frac{d^2 B_z}{dz^2} = 0 \quad \text{conduit à} \quad a = b \quad \text{(position de Helmholtz)}
\]
a = 0.350 2b = 0.700

Ordre 2: 2 Helmholtz coils (12000A)
a = 0.350  2b = 0.600

Ordre 2: 2 Helmholtz coils (12000A)
$a = 0.350 \quad 2b = 0.500$

Ordre 2: 2 Helmholtz coils (12000A)
\(a = 0.350\)  \(2b = 0.400\)

Ordre 2: 2 Helmholtz coils (12000A)

![Graph showing magnetic field distribution for 2 Helmholtz coils.](image-url)
Ordre 2: 2 Helmholtz coils (12000A)

\[ a = 0.350 \]
\[ b = 0.300 \]
Ordre 2: 2 Helmholtz coils (12000A)
a = 0.350  2b = 0.100

Ordre 2: 2 Helmholtz coils (12000A)
$a = 0.350 \quad 2b = 0.000$

Ordre 2: 2 Helmholtz coils (12000A)
Ordre 2: 2 Helmholtz coils (12000A)

1000ppm in +-65mm

\[ a = 0.350 \]
\[ 2b = 0.350 \]
Optimisation with 3 symmetric ring coils

The rings have two different currents $I_1$ and $I_2$.

We look for:

$$\frac{d^2 B_z}{dz^2} = 0$$

and

$$\frac{d^4 B_z}{dz^4} = 0$$
I1=0A I2=6377A

Ordre 4: two symmetric coils (2b=0.532) I1 & one central coil I2
I1=6000A I2=6377A

Ordre 4: two symmetric coils (2b=0.532) I1 & one central coil I2
$I_1 = 7000 \text{A}$ $I_2 = 6377 \text{A}$

Ordre 4: two symmetric coils ($2b = 0.532$) $I_1$ & one central coil $I_2$
Ordre 4: two symmetric coils (2b=0.532) I1 & one central coil I2
$I_1=9000A$ $I_2=6377A$

Ordre 4: two symetric coils ( $2b=0.532$) $I_1$ & one central coil $I_2$
$I_1 = 10000 \text{A} \quad I_2 = 6377 \text{A}$

**Ordre 4: two symmetric coils** ($2b=0.532$) $I_1$ & one central coil $I_2$
$I_1 = 11000\, A \quad I_2 = 6377\, A$

**Ordre 4: two symmetric coils (2b=0.532) $I_1$ & one central coil $I_2$**

![Graph showing magnetic field distribution](image-url)
$I_1=12000\text{A}$  $I_2=6377\text{A}$

Ordre 4: two symetric coils (2b=0.532) $I_1$ & one central coil $I_2$
I1 = 13000A  I2 = 6377A

Ordre 4: two symetric coils (2b=0.532) I1 & one central coil I2
$I_1=14000\text{A}$ $I_2=6377\text{A}$

Ordre 4: two symetric coils ( $2b=0.532$ ) $I_1$ & one central coil $I_2$
I1=15000A  I2=6377A

Ordre 4: two symmetric coils (2b=0.532) I1 & one central coil I2

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I1=12000A  I2=6377A

Ordre 4: two symmetric coils (2b=0.532) I1 & one central coil I2

1000 ppm in +-115mm
Ordre 2: 2 Helmholtz coils (12000A)
How to increase the field

My Magnets (Part 2 of 3 Parts)
—Passage from Francis Bitter’s Magnets: The Education of a Physicist

…By using a variable-size wire in the construction of a coil, I found it possible to increase the magnetic field at the center by a factor of just 1.52 over the best design for a coil with a uniform winding. Therefore, by going into all kinds of practical complications, one could improve the performance of coils only one and a half times or a little more. However, this now was settled; there was no use worrying about it any more. In the end these calculations did show me a practical way of improving the performance of coils by an appreciable amount.
Bitter Magnet

Bitter’s design employed a conductor in the form of a stack of annular plates, each with a slit and separated by a thin sheet of insulation except over a sector. The slit allows the bare sector to pressure-contact the next plate’s bare sector, enabling the current to commutate from one plate to the next in a quasi-helical path as it flows from one end of the stack to the other. Each “Bitter” plate is punched with hundreds of cooling holes. To generate a high field, tens of thousands of amperes of current are pushed through the electrically resistive stack, consuming megawatts of electrical power, which heat the stack. This heat is removed by water forced through the cooling holes at high velocity, \( \sim 20 \text{ m/s} \). A silhouette of two nested “Florida-Bitter” plates, developed in the 1990s at the National High Magnetic Field Laboratory (NHMFL), is shown in Fig. 3.18 [3.9]. A radial slit in each plate is clearly visible. Also note that each water passage hole is not circular as in Bitter’s plates; the elongated—in the direction of current—shape was first developed by Weggel at M.I.T. in the 1970s. The outer plate of the set here is 148 mm in diameter; plate sizes have been more than 400 mm in diameter. The sixteen large holes are for axially clamping the plates with tie rods. A key feature of the Bitter magnet construction is that it is modular, consisting of many similar plates. Plate thickness, mechanical properties, and electrical properties can be tailored to the axial position to optimize magnet performance.
NHMFL Bitter Magnet

Fig. 3.18 Silhouette of two nested “Florida-Bitter” plates, with an outer plate of 140 mm in diameter, in “water” magnets at the National High Magnetic Field Laboratory [3.9].

Courtesy of Pr Y. Iwasa
NHMFL 45 T Hybrid Magnet

Figure 3.22 shows a cross-sectional view of the “water” magnet, SCM, and some auxiliary components of the 45-T hybrid magnet at the NHMFL [3.31]. The water magnet has four nested coils; it generates a center field of 31 T at 24 MW. The SCM, consisting of three coils, A, B, and C, operated at 1.8 K, initially generated 14 T but now operates at 11 T [3.30]; the water magnet has been redesigned to contribute 34 T at 30 MW. The system includes a superfluid helium supply cryostat, to which the SCM cryostat is connected by a pipe, shown truncated at the right, middle of the figure.
Fig. 3.22  Cross sectional view of the 45-T hybrid magnet at NHMFL [3.31].
Solenoid Lens for Ion Beam source & LEBT
Coil parameters

<table>
<thead>
<tr>
<th></th>
<th>1A</th>
<th>1mT</th>
<th>1.25mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>160000At</td>
<td>0,800T</td>
<td>Factor 200</td>
</tr>
</tbody>
</table>

The magnetic circuit is saturated
\( J_{\text{eng}} = 10 \text{ A/mm}^2 \)
An electrical Power in the range of 15kW is necessary
Field level computed by TOSCA

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Field curve of a magnetic lens
Steering coils are installed inside the solenoid to save space.
View of the coils: due to high power, the connections are space consuming.
In accelerators solenoids are used
For focusing in the low energy sections:
Electron guns
CLIC-CTF3 Probe beam LINAC

Structure 3 GHz nue

Structure 3 GHz avec solénoïdes

Califes: vue upstream

Vue downstream
Let's stay at CERN: CMS (Compact Muon Solenoid)

Design compact
Basé sur un solenoïde SC
6 m de diamètre
13 m de long
Champ élevé 4T
+ culasse en fer doux
Du Virtuel au Réel : 1998-2006

Champ central : 4 T
Courant nominal : 20 kA
Energie stockée : 2,6 GJ
Masse froide
Longueur : 12,5 m
Diamètre interne : 6 m
Poids : 220 t
A few challenges of the CMS Magnet CMS

- Magnetic pressure is 6.4 MPa
- Conductor 20 kA reinforced mechanically by aluminum alloy
- Stress=
  
- Winding in 5 modules, each of 4 layers chacun. The winding is practised in an outer mandrel

- Magnetic pressure 6.25 MPa
- Attraction force between modules: 6.25 MPa*20m²=12000 tons

- Stores energy 11.6 kJ/kg in the cold mass
CMS conductor

High purity aluminum for stabilisation: 99.998%

Câble Supraconductor (32 strands)

Aluminum Alloy For mechanical reinforcement: 6082 T5

Electron beam welding
Manufacturing of modules (06/04)

Polymérisation CB-1
Finition CB0
Winding CB+1
outer cylinder CB+2
Rings between the modules are taking the axial attraction force or 12000 tons which is just magnetic pressure (6MPa) cross the section (20 m²)
Août 05: insertion of cold mass in the vacuum vessel
Efforts on the cold mass

- Axial stress along Z axis
- Circumferential stress
- Von Mises stress

Contrainte de Von Mises mesurée à 4T : 138 MPa
En accord complet avec les calculs de 1998.
MRI limits

Magnet 1.5T (GE) SHFJ/CEA
Magnet 3.0T (Bruker) SHFJ

Iseult 11.7 T
Magnet 9.4 T GE 600 mm (USA)

Increase spatial and time resolution

1 mm 1s
0.1 mm 0.1s

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MRI Magnet architecture
Winding

Main Coil

compensation coil
Main parameters of the ISEULT Magnet

- **B0 / Warm Bore**: 11.75 T / 900 mm
- **Field stability**: 0.05 ppm/hr
- **Field Homogeneity**
  - cm DSV: < 0.5 ppm over 22 cm DSV
- **Stray field (5 G line)**
  - 9.6 m axial, 5.1 m radial
- **Stored Energy**: 330 MJ
- **Inductance**: 301 H
- **Winding Current Density**: 28.2 A/mm²
- **Temperature**: 1.8 K
- **Current**: 1487 A
- **Conductor size**: 9.2 mm x 4.6 mm
- **Conductor weight**: 60 t of NbTi
Winding pack design

For magnetic shielding anti coils are used with:

\[ B_1 S_1 = -B_0 S_0 \]
Rôle du blindage actif (2/2)
Winding pack design

- Original Double Pancake design
  - The objective is to design a magnet theoretically **intrinsically** homogeneous

\[
B_z (r, \theta, \varphi) = B_0 + \sum_{n=1}^{\infty} r^n \left[ Z_n P_n (\cos \theta) + \sum_{m=1}^{n} \left( \frac{\cos m \varphi}{X_n^m} + \frac{\sin m \varphi}{Y_n^m} \right) W_n^m P_n^m (\cos \theta) \right]
\]

- Design cross-check several times
Assembly of the double pancakes
Summary and Conclusion

I hope I have given you a flavour the magic world of solenoids

James Clerk MAXWELL

Do Coils please

« Think Flux »
Field Analysis of Solenoidal Coil

\[ H_z(r, \theta, \varphi) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} 2^n (n+m+1) P_n^m(\cos \varphi) (A_n^m \cos m\varphi + B_n^m \sin m\varphi) \]

with \( \mu = \cos \theta \)

If \( \frac{\partial \phi}{\partial \psi} = 0 \)

\[ H_z(z) = \sum_{n=0}^{\infty} z^n (n+1) A_n^0 \]