Manufacturing and calibration of search coils

Part II – Calibration

Marco Buzio, CERN

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Introduction
What does it mean to calibrate a coil?

Main assumption: **2D field** (i.e. thin lens approximation, \( L_{\text{magnet}} << \lambda_{\text{betatron}} \))

[see also: lecture by A. Wolski]

\[
\Phi(\theta) = \Re \left( \sum_{n=1}^{\infty} \frac{\kappa_n}{r_{\text{ref}}^{n-1}} C_n e^{i n \theta} \right)
\]

\[
\{ C_n = B_n + iA_n \}
\]

**time**

\[
\Delta \Phi_{\text{coil}}
\]

**coil position**

\[
\{ \kappa_n \}
\]

**Measurement(s)**

\[
\text{FFT coefficients}
\]

Result analysis

\[
C_n = \frac{2r_{\text{ref}}^{n-1} \Psi_{n+1}}{N} \frac{\kappa_n}{\kappa_n}, \ n = 1 \ldots \frac{N}{2}
\]

Azimuthal sampling over \( N \) points + inverse FFT

[L. Bottura, LHC-XMT-ES-0001]

**Calibration**

Finding the \( \{ \kappa_n \} \) needed to infer the \( \{ C_n \} \) from \( \Phi_{\text{coil}} \) (within a certain accuracy)
Further assumptions:

- Perfect rectangular geometry
- Infinitesimally thin winding cross-section

\[ \kappa_n = \frac{N_T L}{n} (z_2^n - z_1^n) = \frac{N_T L}{n} \left( R_2^n e^{i\frac{\alpha}{2}} - R_1^n e^{-i\frac{\alpha}{2}} \right) e^{i\phi_0} \]

\text{Finite winding size correction}

if square cross-section \( \leq 1 \text{ mm}^2 \), \( R_0 \geq 10 \text{ mm} \), \( n \leq 6 \) \( \Rightarrow \) no correction for a dipole, correction \( \ll 10^{-4} \) for a quadrupole, \( \ll 10^{-3} \) for higher harmonics

The correction can be either ignored or computed from the nominal geometry (no special calibration necessary)

\[ [L. \text{ Deniau, CERN LHC-MTA IN 98-026, Coils calibration correction factors for rectangular windings}] \]
### Harmonic coil coefficients: typical cases

<table>
<thead>
<tr>
<th>n</th>
<th>radial coil ( \varphi_0=0 )</th>
<th>tangential coil ( \varphi_0=\pi/2, w=2R_0\sin\alpha/2 )</th>
<th>tangential coil ( \varphi_0=\pi/2, \alpha\approx0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_1 )</td>
<td>( N_T Lw )</td>
<td>( N_T Lw )</td>
<td>( N_T Lw )</td>
</tr>
<tr>
<td>( \kappa_2 )</td>
<td>( \frac{1}{2} N_T LwR_0 )</td>
<td>( i \cos \frac{\alpha}{2} N_T LwR_0 )</td>
<td>( iN_T LwR_0 )</td>
</tr>
<tr>
<td>( \kappa_3 )</td>
<td>( N_T Lw \left( \frac{w^2}{12} + R_0^2 \right) )</td>
<td>( -\frac{1}{3} (1 + 2 \cos \alpha) N_T LwR_0^2 )</td>
<td>( -N_T LwR_0^2 )</td>
</tr>
<tr>
<td>( \kappa_4 )</td>
<td>( N_T LwR_0 \left( \frac{w^2}{4} + R_0^2 \right) )</td>
<td>( -i \cos \alpha \cos \frac{\alpha}{2} N_T LwR_0^3 )</td>
<td>( -iN_T LwR_0^3 )</td>
</tr>
<tr>
<td>( \kappa_5 )</td>
<td>( N_T Lw \left( \frac{w^4}{80} + \frac{w^2 R_0^2}{2} + R_0^4 \right) )</td>
<td>( \frac{1}{5} (1 + 2 \cos \alpha + 2 \cos 2\alpha) N_T LwR_0^4 )</td>
<td>( N_T LwR_0^4 )</td>
</tr>
<tr>
<td>( \kappa_6 )</td>
<td>( N_T LwR_0 \left( \frac{w^4}{16} + \frac{5}{6} w^2 R_0^2 + R_0^4 \right) )</td>
<td>( \frac{i}{3} N_T \cos \frac{\alpha}{2} (4 \cos^2 \alpha - 1) LwR_0^5 )</td>
<td>( iN_T LwR_0^5 )</td>
</tr>
</tbody>
</table>

- All coefficients can be calculated from coil length \( L \), width \( w \) and radius \( R_0 \)
- All coefficients proportional to total coil area \( A_c=N_T Lw \)
- All coefficients increase like radius \( R_0^{n-1} \)

**NB:** calibration coefficients can be used at any field level – **inherently linear sensor**

However: S/N at calibration gets better at high field
Calibration dataflow

Geometry measurements

Coil geometry nominal = \( N_T, L, w, R_0, \varphi_0 \) + imperfections

Magnetic measurements

compute from definition:

\[
\kappa_n = \frac{N_T L}{n} \left( R_2^n e^{i \alpha \frac{n}{2}} + R_1^n e^{-i \alpha \frac{n}{2}} \right) e^{i \varphi_0}
\]

\[
A_c = \kappa_1, \quad w_{eff} = \frac{\kappa_1}{N_T L}, \quad R_0 = \frac{2\kappa_2}{A_c}
\]

• **Geometry measurement**: practically only \( L \) is accurate enough (10^{-3} \sim 10^{-4}). Moreover: if coil completely within or outside the magnet, precise determination of \( L \) not necessary.
• **Geometry may be the only option** when no suitable reference magnet is available (e.g. large or curved coils ...)
• **Magnetic measurement**: best option when possible (all non-idealities automatically included in the result)
  Reference dipole and quadrupole are the norm; sextupole sometimes used
• **Computation of \( \kappa_n \) from geometry**: done for the orders where no reference comparison is available.
  In this case, accuracy is estimated from calculations, repeatability, cross-checks with different instruments

\[
\kappa_n = \frac{2 r_{ref}^{n-1}}{N} \frac{\Psi_{n+1}}{C_n}
\]
Finite Length Effects
3D coil calibration

\[ \Phi = N_T \int_0^L w(s)B(s)ds = B \overline{w}_{\text{eff}}L = \frac{B\overline{d\ell}}{A_{\text{eff}}} \]

- The flux corresponding to a given coil position can be obtained in various ways (flipping or rotating the coil, pulsing the field from zero)
- \( L \) can be considered as known from mechanical measurements
- General case: unless \( B(s) \) or \( w(s) \) are constant and can be taken out of the integral sign, the flux cannot be obtained from average width and average field:
  \[ \frac{\frac{1}{L} \int_0^L w(s)B(s)ds}{\frac{1}{L} \int_0^L B(s)ds \frac{1}{L} \int_0^L w(s)ds} \neq 1 \]
- Define: effective width (\( N_T \) gets lumped in for convenience) = average of width weighted with the field

Considerations made here for a dipole field hold true for other components as well.
Adapting the coil to the magnet

$$\Phi = \overline{w_{\text{eff}}} B d\ell = \overline{B A_{\text{eff}}}$$

**error made if** $B(s)_{\text{calibration}} \neq B(s)_{\text{measurement}}$

most favourable case: $B=\text{const.}$  

$$\text{error} = \frac{\delta w \delta L}{w_0 L}$$

$L_c << L_M$

**local measurement**

if we can assume $B=\text{constant}$ (during both calibration and measurement), then the calibrated value $A_{\text{eff}}$ can be used in any magnet to obtain the average field.

Usage:
- local field quality (diagnostic tool for SC coils)
- integral measurement by scanning

scanning required, $w(z)$ unimportant, $w_{\text{avg}}$ sufficient

$L_c > L_M$

**integral measurement**

$w_{\text{eff}}$ depends on both $w(s)$ and $B(s)$ – accurate absolute measurements cannot be done even if $w(s)$ is known perfectly

Usage:
- precise and efficient determination of the field integral for relative measurements
- absolute calibration may be done with another instrument (stretched wire) – to be repeated if $B(s)$ changes
Rounded ends: BdL can be measured precisely only if B is constant or linear over the end regions.

\[
\Phi = N_T \int_{-w/2}^{0} 2\sqrt{\frac{w^2}{4} - s^2} B(s)ds + N_T \int_{0}^{L} B(s)ds + N_T \int_{L}^{w/2} 2\sqrt{\frac{w^2}{4} - (s-L)^2} B(s)ds
\]

<table>
<thead>
<tr>
<th>End shape</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangular (default)</td>
<td>racetrack</td>
</tr>
<tr>
<td>▪ precise definition of coil length</td>
<td></td>
</tr>
<tr>
<td>▪ best practical solution to reconstruct field integral from multiple longitudinal measurements (eg. gap between coils in long SC dipoles must be a multiple of twist pitch)</td>
<td></td>
</tr>
<tr>
<td>▪ sharp corners spell wire problems: risk of breakage, bulge out, bond does not tak</td>
<td></td>
</tr>
<tr>
<td>▪ well-defined geometry facilitates winding</td>
<td></td>
</tr>
<tr>
<td>▪ calculations more complicated</td>
<td></td>
</tr>
<tr>
<td>▪ should be used only if the ends fall entirely within B=0 or B=const. region</td>
<td></td>
</tr>
</tbody>
</table>
Integral field of SC magnets

- Superconducting magnets made with Rutherford cable develop a periodical field modulation, due to non-uniform interstrand current distribution, with period $\lambda = \text{twist pitch length}$ and relative amplitude of order $10^{-3}$
- To measure the integral correctly, coil length and gap must be integer multiples of $\lambda$
- To correctly fill in the gaps by interpolation/extrapolations, gaps must occur in the flat (central) field region

P. Pugnat, INVESTIGATION OF THE PERIODIC MAGNETIC FIELD MODULATION INSIDE APERTURES OF LHC SUPERCONDUCTING DIPOLE MODELS, EPAC 2000
Quality Control
Quality control

Quality control = **acceptance test** to verify broad conformity with specifications **not** necessarily a precise quantitative measurement (go/no go) done routinely **after manufacture** and **during operation** (periodical or on-call checks)

Routine controls carried out at CERN:

**Visual Inspection**

- Regular winding geometry
- Epoxy polymerized uniformly, no air bubbles, cracks or swellings

**Geometrical Controls**

- Mechanical measurement of **length, width** of winding form/finished coil. Derived quantities: total coil area, winding thickness (including the epoxy bonding)
- **Microphotogrammetry** gives a lot of information, but it is destructive. Carried out occasionally to measure cross-section parameters to be used for the calculation of finite cross-section effects.
- **Radiographic** photogrammetry: might be useful (non-destructive), but is costly
• Geometry of winding form and finished coils checked systematically by mechanical means
• Coil fixed to precision support with calibrated pins
• A sliding spring-loaded feeler is used to take horizontal and vertical micrometric measurements at the bottom of the groove
• Measurements from both sides are averaged to cancel out systematic errors
• Mechanical area is computed at the center line of the measured winding thickness and compared with magnetic calibration
The gauge can turn by 90° to take horizontal measurements. The wedge-spaced feeler is used to reach the bottom of the 0.5 mm wide groove.
Example: mechanical survey of dipole coils for LHC

Coil width = 8.56 ± 0.07 mm

Winding width = 0.63 ± 0.06 mm
(variations in wire diameter, bonding thickness, ..)

Core width = 7.96 ± 0.05 mm
Example – comparison of mechanical and magnetic calibration

- Obtained area ~1% larger than specified on average
- Magnetic area 0.25% larger than geometric on average
- Spread (including outliers) is about 1% of the nominal value
- Even excluding outliers, about 0.7% spread of the difference between geometrical and magnetic values is ⇒ this kind of mechanical measurements cannot be used as a reliable predictor of magnetic area.
Apply a local magnetic field to obtain an indication of the width variations
Relative measurement – absolute width may be obtained from average w calibrated in a reference dipole
Potentially preferable to mechanical measurements – takes into account all non-idealities
Flux signal can be obtained in different ways (flipping the coil in a constant field, translating the coil at constant velocity, applying a pulsed or AC field)

\[
\Phi = N_T \int_0^L w(s)B(s)ds \approx N_T \int_{s-\Delta/2}^{s+\Delta/2} w(s)B(s)ds \approx N_T w(\bar{s}) \left( \int_{s-\Delta/2}^{s+\Delta/2} B(s)ds \right)
\]

\[
w(\bar{s}) = \frac{\Phi}{N_T B d \ell_{\text{ref}}}
\]
A possible implementation of the idea: a simple permanent magnet-based dipole

- Leakage flux must be pushed as far away as possible (better if perpendicularly to the coil)
- Additional iron screens may improve the gradient dB/ds at the boundaries
**Electrical Controls**

- Continuity test
- Resistance measurement: difficult to detect single-turn shorts (e.g., 1 inter-turn short on a 400-turn coil = 0.25% R drop equivalent to 0.5 °C !!)

\[ R_c = \gamma_R (\ell_c, w_c, \varnothing_w) \rho N_T \]

\[ \frac{1}{R_c} \frac{\partial R_c}{\partial N_T} = \frac{1}{N_T} \]

- Inductance measurement: no material property dependence, 2×sensitivity (actually better than that, as the shorted turns add a mutual inductance in phase opposition).

\[ L_c = \gamma_L (\ell_c, w_c, \varnothing_w) \mu_0 N_T^2 \]

\[ \Rightarrow \quad \frac{1}{L_c} \frac{\partial L_c}{\partial N_T} = \frac{2}{N_T} \]

- Measurement of the time constant of the inductive discharge \((\tau^2=L/R)\) may be useful

Example of R variability in a batch of equal coils
Polarity checker for coil assemblies

- up to ~20 connector assemblies in series – very easy to get inversions
- check phase of V induced in the coil vs. source
Example: calibration bench for long shaft at CERN (SM18)

- Calibration bench conceived for quick on-site magnetic and electrical verification of 15-m long coil shafts for LHC cryomagnets during the series test phase
- 0.5 T horizontal field over 1.5 m, very good uniformity (b3<2 units), travelling magnet
- Calibration of field direction for long shafts + verification of strength (coil area);
- Realistic coil voltages for electrical checks
Example of application: intermittent open circuit during the rotation due to failing connection – a nightmare to find otherwise!
Metrological Aspects
Josephson effect: \( V = n f \Phi_0 \)

Quantum hall effect: \( R = n \Phi_0 / e \)
**Calibration: main issues**

### Possible requirements

- **Absolute calibration**
  the “real” value of the magnetic field (according to some objective reference standard) is required to compare the result with an external source
  Examples:
  - field of a spectrometer (must provide absolute beam energy values)
  - matching the energy between chained accelerators

- **Relative calibration**
  the value of the magnetic field in comparison to a local reference fulfils the purpose of the measurement
  Examples:
  - integrated strength of the main magnets of a synchrotron (they must be equal to guarantee the same bending radius/focusing when they are powered in series)
  - field mapping to compute homogeneity/harmonics

**relative measurements are both easier to do and more precise**
**differential measurements allow the cancellation of a number of error sources**
**null methods allow to work in the most linear range of the instrumentation**

In both cases: open-loop (geometry-based) calibration does not well enough for harmonic coils ⇒ recourse to appropriate reference magnets is necessary

Also: applications where few % calibration is sufficient (e.g. measurement in pulsed magnets, which are self-calibrated - relative)
Reference standards of any physical quantity must fulfill three basic requirements:

1. Realize the quantity in a useful range
2. $\frac{\partial}{\partial t} = 0$ (long term stability)
3. $\frac{\partial}{\partial x} = 0$ (transportability)

The instrument to be calibrated is checked directly or indirectly.

An instrument calibrated with a reference standard is used to characterize a user field source, which can then be used as a reference.

Coil + instrument = the reference

In-situ calibration: using symmetries, the instrument and the magnet are calibrated simultaneously.

In practice: cross check of different instruments, confidence comes from redundancy.
Magnetic Field Reference

Flux Density \([B] = [M]^1 [L]^0[T]^{-2}[I]^{-1}\) 1 Tesla = 1 N/Am = 1 Wb/m²

No official primary standard. The established secondary standard is the proton gyromagnetic ratio \(\gamma_0 = 42.5759\) MHz/T, at the basis of the NMR measurements (e.g. Metrolab teslameter). In practice, one has to realize their own reference by taking a dipole magnet and mapping it. Main limitations: absolute accuracy 5 ppm; tracking rate 1%/s; homogeneity 13 ppm/mm.
Magnetic Flux Reference

Magnetic Flux \[ \Phi = [M]^1 [L]^2 [T]^{-2} [I]^{-1} \]  
1 Weber = 1 Vs = 1 HA = 1 Tm2 = 1 J/A

No official primary standard. Secondary standards of flux and (equivalently) mutual inductance employed routinely as calibrators (e.g. Hibbert’s standards, Campbell coils, or more modern calculable sources). The flux quantum is logically a candidate to replace the old standards, but for the moment it is impractical to use it directly.

NB: realizations of the Vs can be obtained with high precision, and are used to calibrate acquisition electronics; since they do not represent physically a magnetic flux, however, they cannot be used to calibrate directly a coil.

Historical Note: Hibbert’s flux standard

- Narrow annular gap in a permanent-magnet based iron circuit
- Coil can slide between mechanical stops on a rod into and out of the field
- A few % reproducibility with artificial ageing to improve long-term stability

Helmoltz Coils

- Two circular coils of radius R separated by a distance R
- Zero sextupole at the center – 6% \( \Delta B/B_0 \) at \( z=R/2 \)
- Excellent accessibility of the homogeneous field volume
- Can be used in reverse mode as a search coil (e.g. for measurement of small permanent magnets) – it will be sensitive mainly to the dipole component only
- Square arrangement of conductors carried small penalty
- Calculable system – suitable for use as a reference

\[
B_0 = \frac{3\sqrt{4}}{5} \frac{\mu_0 nI}{R} = 0.715 \frac{\mu_0 nI}{R}
\]

S. R. Trout, "Use of Helmholtz Coils for Magnetic Measurements", IEEE Trans Magnetics, V.24, No.4, Jul 88

http://www.laboratorio.elettrofisico.com
Double Helmoltz Coil System (CERN)

- Two perpendicular Helmoltz coils can create combinations of normal and skew dipole + quadrupole field
- 2D geometry → optimal spacing between coils is $1/\sqrt{3}$ of the distance between conductors
- 2400×120×240 mm coils, 30×30 mm cross section, 150×
- 64 turns of 4 mm² air-cooled Cu conductor, nominal current $I=20$ A ⇒ max. modulus field = 0.76 T
- Force on each conductor = 6.5 kN → needs a strong structure
- Must be kept far from metallic masses
Other Calculable Systems

- Many variations on the Helmoltz theme with different coil numbers, arrangement and optimization criteria
- Braunbeck Coils: 4 coils in series used to cancel Earth’s field to better than 1 nT – used for instrument calibration inside a large volume.

3-axis Braunbeck coil system
arbitrary B vector direction

need a really electromagnetically quiet environment!
Calibration in a Dipole
Flip Coil Method

- Not a novel technique ...
- Find the total flux linked in a given position
- Relates coil area and average field
- Returning back to the original position allows estimation and correction of integrator drift
- Equivalent to harmonic (rotating) coil measurement with two sampling points 180° apart
- Works in any field: use to calibrate \((B \rightarrow A)\) is possible only if \(B\) is known precisely ...

\[-\int_0^t V_c \, dt = \Phi - (-\Phi) = 2A_c B\]

Transform an \textbf{integral w.r.t. time} into another \textbf{w.r.t. position} (same as for harmonic coil method).

\begin{center}
\textbf{guarantees insensitivity to speed and trajectory fluctuations}
\end{center}

- Field of 1 Tesla (@ 310 A), l = 2.5 m
- Permanently monitored with NMR-probe
- Probe Ø 70 mm max.
- Up to 5 coils simultaneously
# Reference Magnets vs. In Situ Calibration

<table>
<thead>
<tr>
<th></th>
<th>Reference Magnet</th>
<th>In Situ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pros</td>
<td>• best accuracy if parameters are consistently tracked (current, temperature, ...)&lt;br&gt;• best convenience – doubts can be dispelled by a quick check&lt;br&gt;• can be done offline</td>
<td>• most accurate&lt;br&gt;• not always practical (geometry reasons)&lt;br&gt;• adds time to series measurements</td>
</tr>
<tr>
<td>Cons</td>
<td>• may require transportation of the instrument</td>
<td></td>
</tr>
</tbody>
</table>
- Field mapped yearly and checked continuously with a local NMR (used for scaling)
- Monitor temperature and current during the measurements!

\[ A_C = \frac{\Delta \Phi}{2B_1} \]
Dipole calibration cross-check

- Individual coils are calibrated in a NMR-mapped reference dipole fringe fields are excluded
- LHC dipole field integral is measured in the same conditions with: 15 m coil shaft (12 coil sectors) + Single Stretched Wire (SSW)
- The SSW provides an independent calibration standard for integral fields only (precision comes from high-precision translation stages ~1 μm/10 mm. No direct comparison between SSW and NMR-mapped magnet is possible

Magnetic Field

- copper-beryllium wire stretched through the magnet
- return wire
- induced voltage V in the wire loop proportional to both magnetic field and speed of movement
- two translation stages move the wire from A-A to B-B with micrometer accuracy
- integrator to multiply the voltage V times the duration of the movement to get flux

integrated voltage (Vs) is proportional to the average field across the area spanned by the wire
Dipole calibration cross-check

Comparison between SSW and harmonic coils gives an indication of absolute accuracy
In this particular case: RMS spread = $2 \cdot 10^{-4}$

cross-checks with multiple instruments are an essential tool to get independent confirmation of calibration accuracy
• Connect coils with equal nominal area in series opposition
  \[
  \kappa_1^1 = A_1 \cos \epsilon_1 + i \sin \epsilon_2 \\
  \kappa_2^2 \approx A_2 (1 + i \epsilon_1)
  \]

  \[
  \kappa_1^{\text{diff}} = \kappa_2^2 - \kappa_1^1 \approx \Delta A + i A \Delta \epsilon
  \]

• Flip the array 180° to measure $\Delta \Phi$. Choose initial phase $\varphi_0 = \pi/2$ to be insensitive to area differences.

  \[
  \Delta \Phi = 2 \Re \left( \kappa_1^{\text{diff}} B_1 e^{i \varphi_0} \right) = 2 B_1 \left( \Delta A \cos \varphi_0 - A \Delta \epsilon \sin \varphi_0 \right)
  \]

• Compute the tilt angle difference. File and/or shim, then iterate.

  \[
  \Delta \epsilon = \frac{\Delta \Phi}{2 A B_1}
  \]

• NB: the tilt of the reference coil remains unknown (see field direction calibration later on)
Calibration in a Quadrupole
Calibration in a quadrupole

Precisely controlled horizontal displacement to calibrate the gradient

\[
B_2 = \frac{r_{\text{ref}}}{\Delta x} \frac{\Phi(0, x_0 + \Delta x) - \Phi(0, x_0)}{A_c}
\]

\[
\Phi(\vartheta) = \Re \left( \kappa_1 B_1 e^{i n \vartheta} + \kappa_2 B_2 e^{2i n \vartheta} \right) = \frac{A_c B_2}{r_{\text{ref}}} \left( x_0 \cos \vartheta + \frac{1}{2} R_0 \cos 2\vartheta \right)
\]

\[
\begin{align*}
\Phi(0) &= \frac{A_c B_2}{r_{\text{ref}}} \left( x_0 + \frac{1}{2} R_0 \right) \\
\Phi\left(\frac{\pi}{2}\right) &= \frac{A_c B_2}{r_{\text{ref}}} \left( - \frac{1}{2} R_0 \right) \\
\Phi(\pi) &= \frac{A_c B_2}{r_{\text{ref}}} \left( - x_0 + \frac{1}{2} R_0 \right)
\end{align*}
\]

\[
x_0 = r_{\text{ref}} \frac{\Phi(0) - \Phi(\pi)}{A_c B_2}
\]

\[
R_0 = 2 r_{\text{ref}} \frac{\Phi(0) + \Phi(\pi)}{A_c B_2}
\]

- Assume for simplicity a purely horizontal rotation axis offset
- It is essential that the rotation axis be the same during calibration and measurements (same ball bearings)
- Typical accuracy cross-checked via SSW about 17 units.
- No reference standard for field gradient: use a SSW validated previously with a NMR (Guy ?)
- SSW and rotating coil check: 1.7 units of difference on field integral
Calibration of field direction
Calibration of main field direction

- No standard reference (despite being the oldest magnetic measurement in the world ...)
- Main alternatives:
  - exploit symmetries (must have fixed rotating coil axis or an attached reference e.g. gravity)
  - use a reference magnet (stretched wire method) or re-calibrate in situ.
- Rotation of the reference magnet allows one to verify polarity and gain
Calibration by $180^\circ$ rotation

- Measurement affected by unknown internal offset (e.g. mechanical tilt angle of inclinometer to encoder zero)
- Turning the instrument by $180^\circ$ allows to measure the wanted angle + calibrate the offset

\[
\begin{align*}
\alpha_{\text{meas}}^1 &= \alpha + \Delta \alpha \\
\alpha_{\text{meas}}^2 &= \pi - \alpha + \Delta \alpha
\end{align*}
\Rightarrow
\begin{align*}
\alpha &= \frac{\pi}{2} + \frac{\alpha_{\text{meas}}^1 - \alpha_{\text{meas}}^2}{2} \\
\Delta \alpha &= -\frac{\pi}{2} + \frac{\alpha_{\text{meas}}^1 + \alpha_{\text{meas}}^2}{2}
\end{align*}
\]
Calibration of axis
Axis reference quadrupole

- No standard reference – ad-hoc techniques needed.
- Reference benches with quadrupole and higher multipole magnets
- Allow easy translation of the magnets for verification of polarity and gain
- Cross-check with multiple systems: rotating coils, AC coils, stretched wire
Measurement of magnetic axis

- Main axis measurement methods: harmonic feed-down from rotating coil data, morgan-coil type signal balance (AC or pulsed), stretched wire (AC or DC), Hall probe (null technique)
- Measured harmonics contain information about the null field region
- Assuming small offsets and one dominant harmonic, a linearized equation provides the center as a function of the strength of the dominant harmonic and the next lower one
- The computed $\Delta z$ is relative to the coil rotation axis → this must be transferred (mechanically or optically) to an external frame of reference (magnet fiducial references).

\[
\frac{\Delta z}{r_{\text{ref}}} \approx -\frac{1}{n+1} \frac{C_n}{C_{n+1}}
\]

For a purely normal magnet:

\[
\begin{align*}
\frac{\Delta x}{r_{\text{ref}}} & \approx -\frac{1}{n+1} \frac{B_n}{B_{n+1}} \\
\frac{\Delta y}{r_{\text{ref}}} & \approx -\frac{1}{n+1} \frac{A_n}{B_{n+1}}
\end{align*}
\]

\[
\begin{align*}
\frac{\Delta x}{r_{\text{ref}}} & \approx \Phi_1 - \Phi_3 \\
\frac{\Delta y}{r_{\text{ref}}} & \approx \frac{2A \cdot B_2}{2A \cdot B_2}
\end{align*}
\]
Example: calibration of translational offsets

- Reverse by 180 one of the reference systems (the probe in the example below)
- As $\Delta z$ is re-measured, the precise location of the vertical rotation axis is irrelevant
- Add an optical target (fixed w.r.t. the coil rotation radius) measured in an external reference $(X, Y)$

\[
\begin{align*}
\Delta z &= \Delta x + i \Delta y \\
\left\{ 
X_{o1} &= X_M + \Delta x_1 + x_0' \\
X_{o2} &= X_M + \Delta x_2 - x_0'
\right.
\end{align*}
\]
Example: calibration of linac drift tubes

- Linac4 currently in construction at CERN
- Drift tubes include permanent quads
- Must be aligned in each DFT tank to better than 0.08 mm

M. Vretenar, “Linac4 - a new linear accelerator for the CERN complex”
Example: calibration of linac drift tubes

\[(x,y) = \text{rotating coil reference frame}\]

\[(X,Y) = \text{DT reference frame}\]

- Magnetic field: \(B = (B_y + iB_x)\)
- Drift tube stem axis
- Magnetic center: \((X_{CM}, Y_{CM})\)
- Field direction: \(\psi\)
- Coil axis: \((\Delta X, \Delta Y)\)
- Angular encoder offset: \(\gamma\)
- Support tilt: \(\beta\)
- Stem perpendicularity error: \(\varepsilon\)

8 unknowns:
- Magnetic center \((X_{CM}, Y_{CM})\), field direction \(\psi\)
- Coil axis \((\Delta X, \Delta Y)\), angular encoder offset \(\gamma\), support tilt \(\beta\)
- Stem perpendicularity error \(\varepsilon\)
Drift Tube Measurement – Horizontal Flip

Position 0 - nominal

\[
\begin{align*}
X &= \Delta X + x - \alpha y \\
Y &= \Delta Y + \alpha x + y \\
\psi &= \varphi + \alpha
\end{align*}
\]

Position 1 – flip around X

\[
\begin{align*}
X' &= -2\varepsilon R + X + 2\varepsilon Y \\
Y' &= -Y + 2\varepsilon X \\
\psi &= \pi - \varphi - \alpha + 2\varepsilon
\end{align*}
\]
Drift Tube Measurement – Vertical Flip

Position 2 – flip around S

\[
\begin{align*}
X'' &= 2 \beta R - X + 2 \beta Y \\
Y'' &= Y + 2 \beta X \\
\psi &= -\varphi + \alpha + 2 \beta
\end{align*}
\]

Position 3 – flip around X and S

\[
\begin{align*}
X''' &= -2(\beta + \varepsilon)R - X + 2(\beta + \varepsilon)Y \\
Y''' &= -2(\beta + \varepsilon)X - Y \\
\psi &= -\pi + \varphi + \alpha + 2(\beta + \varepsilon)
\end{align*}
\]
Drift Tube Measurement – Calculation of calibrated measurement results

Substitution of measurement results in position 0,1,2,3 and linearization of the equations:

\[
\begin{align*}
\text{Pos.0} & \quad X_{cm} = \Delta X + x_0 - \alpha y_0 \\
& \quad Y_{cm} = \Delta Y + \alpha x_0 + y_0 \\
\text{Pos.1} & \quad X_{cm} = -2\varepsilon R + (\Delta X + x_1 - \alpha y_1) + 2\varepsilon(\Delta Y + \alpha x_1 + y_1) \\
& \quad Y_{cm} = 2\varepsilon(\Delta X + x_1 - \alpha y_1) - (\Delta Y + \alpha x_1 + y_1) \\
\text{Pos.2} & \quad X_{cm} = 2\beta R - (\Delta X + x_2 - \alpha y_2) + 2\beta(\Delta Y + \alpha x_2 + y_2) \\
& \quad Y_{cm} = 2\beta(\Delta X + x_2 - \alpha y_2) + (\Delta Y + \alpha x_2 + y_2) \\
\text{Pos.3} & \quad X_{cm} = -2(\beta + \varepsilon)R - (\Delta X + x_3 - \alpha y_3) + 2(\beta + \varepsilon)(\Delta Y + \alpha x_3 + y_3) \\
& \quad Y_{cm} = -2(\beta + \varepsilon)(\Delta X + x_3 - \alpha y_3) - (\Delta Y + \alpha x_3 + y_3)
\end{align*}
\]

\[
\begin{align*}
\psi = \varphi_0 + \alpha \\
\psi = \pi - \varphi_1 - \alpha + 2\varepsilon \\
\psi = -\varphi_2 + \alpha + 2\beta \\
\psi = -\pi + \varphi_3 + \alpha + 2(\beta + \varepsilon)
\end{align*}
\]

8 unknowns, 12 equations:
- magnetic center \((X_{cm}, Y_{cm})\), field direction \(\psi\)
- coil axis \((\Delta X, \Delta Y)\), angular encoder offset \(\gamma\), support tilt \(\beta\)
- stem perpendicularity error \(\varepsilon\)

Redundant equations useful to double-check and estimate error bars!
Drift Tube Measurement – Calculation of measurement results

Field direction measurements ⇒ calculation of calibrated angular parameters

\[
\begin{pmatrix} \psi \\ \alpha \\ \varepsilon \\ \beta \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} -\varphi_0 \\ \varphi_1 - \pi \\ \varphi_2 \\ -\varphi_3 + \pi \end{pmatrix}
\]

- \( \psi \): magnetic field direction in the DT reference frame
- \( \gamma = \alpha + \beta \): offset angle between encoder Ø and support
  (this is a fixed quantity useful to check stability)
- \( \varepsilon \): \perp error of stem w.r.t. fiducial line \( T_1-T_2 \)
  (can be cross-checked with geometry survey results)

Magnetic center measurements ⇒ calculation of calibrated linear parameters

\[
\begin{align*}
X_{cm} &= \beta R + \frac{x_0 - x_2}{2} = -(\beta + 2\varepsilon)R + \frac{x_1 - x_3}{2} \\
Y_{cm} &= \frac{y_0 - y_1}{2} = \frac{y_2 - y_3}{2}
\end{align*}
\]

\[
\begin{align*}
\Delta X &= 2\beta R - \frac{x_0 + x_2}{2} \\
\Delta Y &= -\frac{y_1 + y_2}{2}
\end{align*}
\]

\[
\begin{align*}
N_C &= -R\beta + \Delta X \\
S_C &= R + \Delta Y
\end{align*}
\]

- \( (X_{CM}, Y_{CM}) \): magnetic axis position in the DT reference frame
- \( (N_C, S_C) \): coordinates of the coil rotation axis in a frame fixed to the test bench
  (another built-in parameter useful to check system stability)
Calibration of fixed coils
Calibration of fixed coils – pulsed measurements

- In principle: calibration coefficients work whatever the source of $\partial \Phi / \partial t$, so all previous considerations apply
- Field mapping techniques require normally repeated measurements in different coil positions – one has to worry about the repeatability of the magnetic field:
  - stability of power supply (10^{-4}~10^{-5})
  - accuracy of current measurements
  - reproducibility of hysteresis cycle (dependence upon history, time, temperature, ramp rates, ripple of the power supply $\rightarrow$ minor hysteresis loops)
- Magnets must be stabilized into a reproducible magnetic state by suitable pre-cycling (a few to a few dozen cycles may be needed)
- Variable-field measurements with moving coils are possible, but the analysis is much more complicated [BNL]
- The start and stop currents must be chosen in a range where the power supply is stable – no ripple (unipolar power converters may be unstable close to zero current)
- Integration lengths should be integer multiples of the period of the dominant perturbations (50 Hz mains, converter ripple ...)

$$\int_0^L B(s, I(t)) ds - \int_0^L B(s, I_0) ds = -\frac{1}{W_{\text{eff}}} \int_0^t V_c dt$$
Example: calibration of CNAO fluxmeter

- Main problem: long curved coil shape → whole coil calibration inside a reference magnet impossible.
- Worse: in-situ comparison with a stretched wire systems also impossible!
- Solution: **relative measurements w.r.t. a reference coil**
- Surface area is expected to change $\sim 10^{-3}$ due to the bending (wire is stretched outboard and compressed inboard). Area change should be systematic → coils sorted based on original areas
- Three best coils at the centre and extremities of the goos field region to minimize error. The reference chosen as the closest to the average.
- A caveat: moving the coil is difficult and likely to affect mechanical stability and calibration.

<table>
<thead>
<tr>
<th>Coiling ordering</th>
<th>Position in the support assembly (mm)</th>
<th>$N_\text{of the Coil}$</th>
<th>Surfaces (m$^2$)</th>
<th>Delta surface (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>-90</td>
<td>24</td>
<td>2.3674</td>
<td>-10.86</td>
</tr>
<tr>
<td>12</td>
<td>-75</td>
<td>18</td>
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<td>-0.01</td>
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<td>11</td>
<td>-60</td>
<td>16</td>
<td>2.4191</td>
<td>10.74</td>
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<tr>
<td>10</td>
<td>-45</td>
<td>23</td>
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<td>7.67</td>
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<tr>
<td>9</td>
<td>-30</td>
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<td>-0.01</td>
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<td>19</td>
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<td>14</td>
<td>2.3554</td>
<td>-15.86</td>
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</table>

<table>
<thead>
<tr>
<th>Coiling ordering</th>
<th>Reference support</th>
<th>$N_\text{of the Coil}$</th>
<th>Surfaces (m$^2$)</th>
<th>Delta surface (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0</td>
<td>21</td>
<td>2.3991</td>
<td>2.40</td>
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</table>

<table>
<thead>
<tr>
<th>Spare Coils</th>
<th>Surface (m$^2$)</th>
<th>Delta surface (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.3381</td>
<td>-2.16</td>
</tr>
</tbody>
</table>
Relative calibration

- Reference and calibrated coils are placed symmetrically w.r.t. midplane – assuming allowed harmonics only, they see the same field (automatic compensation of power supply instability, magnetic history, drifts ..)
- A measurement in series opposition gives therefore:

\[
\frac{\Phi_j}{\bar{W}_{eff}^j} = \frac{\Phi_{ref}^j}{\bar{W}_{eff}^{ref}} \implies \bar{W}_{eff}^j = \bar{W}_{eff}^{ref} \left(1 + \frac{\Delta \Phi_j}{\Phi_{ref}^j}\right)
\]

- Important advantage: each calibration takes into account the proper profile \(B(s)\) for each position ("in situ" concept: the reference follows the instrument)
Calibrated measurement of field homogeneity – before shimming

- Curved magnets with parallel end generate strong quadrupole component (fringe field \( \perp \) ends)
- Nominal shimming with linear + parabolic profile to offset quadrupole and sextupole

7-block shim pieces

Nominal shim profile

... why bother measuring if the simulation is good?
- Nasty surprise – effect of stainless steel shim bolts on field uniformity curves
- Solved resorting to bi-metal screws – iron half engages into the yoke, steel half remains outside
Tracking measurements

- Relative measurement of BdL w.r.t. reference magnets
- Objective: shim all main dipoles to ensure an uniform BdL (accuracy of the beam orbit, facilitates maintenance)
- Method used with long-term success in many other cases (e.g. SPS).

Fluxmeter inside the magnet to be shimmed

Reference inside the reference dipole

<table>
<thead>
<tr>
<th>I (A)</th>
<th>dB/Bref (1E-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.5</td>
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<tr>
<td>1000</td>
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<tr>
<td>1500</td>
<td>1.5</td>
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<tr>
<td>2000</td>
<td>2.0</td>
</tr>
<tr>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td></td>
</tr>
</tbody>
</table>
Final Remarks
Conclusions

- Like for a magnet, good measurements start with good mechanics
- The two biggest enemies of accuracy: time and temperature – do your best to control them
- Cross-checks with multiple systems are a good practical way to find out and eliminate systematic errors
- Carry out calibration and actual measurement changing conditions as little as possible (environment, electronics, magnet …)
- Good quality assurance practice is the hallmark of a professional work: track and record everything, who does what, when and with which equipment