Field Measurement Methods

Luca.Bottura@cern.ch

CAS on Magnets

Novotel Brugge Centrum, Bruges, Belgium
16 - 25 June, 2009
Overview

- NMR/EPR, *the golden standard*
- Fluxmeters, *the workhorse for accelerators*
  - fixed, moving, flipping, rotating coils and wires
- Hall generators and magneto-resistors, *cheap*
- Other *fun* methods
  - Fluxgate magnetometers, *sensitive*
  - SQUIDS, *quantum sensitive*
  - Atomic and SERF magnetometers, *yet more sensitive*
  - Faraday rotation, *fast*
- Concluding remarks, *and a design graph*
NMR/EPR principle

- A particle with a spin and magnetic moment in an applied field precesses at a (Larmor) frequency $f$:

$$f = \gamma B$$

- Gyromagnetic ratio
- Precession
- Applied magnetic field
- Longitudinal component $M_z$
- Magnetic moment $M$
- Rotating component $M_r$
Gyromagnetic ratio

Electron Paramagnetic Resonance (EPR), Electron Spin Resonance (ESR)

Nuclear Magnetic Resonance (NMR)

<table>
<thead>
<tr>
<th>particle</th>
<th>$\gamma$ (MHz/T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-$</td>
<td>$28.025 \times 10^3$</td>
</tr>
<tr>
<td>$^1H$</td>
<td>$42.576396(3)$</td>
</tr>
<tr>
<td>$^2H$</td>
<td>6.535</td>
</tr>
<tr>
<td>$^2He$</td>
<td>32.4326</td>
</tr>
<tr>
<td>$^13C$</td>
<td>10.71</td>
</tr>
<tr>
<td>$^14N$</td>
<td>3.08</td>
</tr>
<tr>
<td>$^19F$</td>
<td>40.08</td>
</tr>
<tr>
<td>$^{23}Na$</td>
<td>11.27</td>
</tr>
<tr>
<td>$^{27}Al$</td>
<td>11.093</td>
</tr>
<tr>
<td>$^{31}P$</td>
<td>17.25</td>
</tr>
</tbody>
</table>

1 GHz NMR magnet $\downarrow$ 23.5 T

cryogenic probes
Resonance and coherence

- A transverse RF pulse of frequency $f$ induces resonance in the precession and coherence in $M_t$. 

![Diagram of RF pulse and coherence decay](image-url)
Fig. 4. Scale drawing of an xy section of the r-f head. The spherical sample $S$ is surrounded by a receiver coil $R$, which is in turn surrounded by a transmitter coil $T$, the whole being encased in a shield. A rotably mounted paddle $P$ is used to steer the transmitter flux. Leads to the receiver coil are the coaxial leads $L_1L_2$ while the transmitter leads are $L_3$ and $L_4$. The outer shield is split to avoid 60-cycle eddy currents.
A DIY NMR magnetometer

0.1 ppm absolute accuracy achievable (0.1 Hz)
Field tracking

- tracking is slow (Hz): maximum field variation tolerated for latching $\delta B/B < 1 \% s^{-1}$
- field gradients *blur* signal: field homogeneity $\nabla B/B < 10 \ldots 100 \text{ ppm/mm}$
  - gradient coils to measure inhomogeneous fields!
Free Induction Decay

- $M_t$ decay after RF pulse (FID)
  - high accuracy for long measurement times
  - main tool for spectroscopy
    - analysis of chemicals, molecules
    - structure determination (COSY, NOSY, ...)

![Signal and spectrum diagram with $T_2^*$ time constant]
Imaging

- Magnetic Resonance Imaging (MRI)

\[ f = \gamma B \]

2000 Ig Nobel Prize winner, *Annals of Improbable Research*

I.W. Schultz, P. van Andel, I. Sabelis, E. Mooyaart

*Magnetic resonance imaging of male and female genitals during coitus and female sexual arousal*

Fluxmeter

- Magnetic flux: \[ \varphi = \int_S \mathbf{B} d\mathbf{S} \]

- Induction law: \[ V = -\frac{d\varphi}{dt} \]

\[
\varphi_{\text{end}} - \varphi_{\text{start}} = - \int_{t_{\text{start}}}^{t_{\text{end}}} V \, dt
\]

needs an \textit{integrator}...

\[
B_{\text{end}} - B_{\text{start}} = \frac{\varphi_{\text{end}} - \varphi_{\text{start}}}{\kappa}
\]

... and coil \textit{calibration}
Ann. Der Physik, 2, 209, 1853

**Inductions-Inclinatorium**

<table>
<thead>
<tr>
<th>Date</th>
<th>Observed</th>
<th>Calculated</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1805 Dec.</td>
<td>69° 29'</td>
<td>69° 36' 43''</td>
<td>- 7' 43''</td>
</tr>
<tr>
<td>1826 Sept.</td>
<td>68 29 26''</td>
<td>68 23 17</td>
<td>+ 6' 9''</td>
</tr>
<tr>
<td>1837 Juli 1</td>
<td>67 47 0</td>
<td>67 52 41</td>
<td>- 5' 41''</td>
</tr>
<tr>
<td>1837 Juli 1</td>
<td>67 53 30</td>
<td>67 52 41</td>
<td>+ 0' 49''</td>
</tr>
<tr>
<td>1841 Oct. 8</td>
<td>67 42 43</td>
<td>67 42 0</td>
<td>+ 0' 43''</td>
</tr>
<tr>
<td>1842 Juni 21</td>
<td>67 39 39</td>
<td>67 40 18</td>
<td>- 0' 39''</td>
</tr>
<tr>
<td>1850 Aug. 7</td>
<td>67 18 38</td>
<td>67 18 34</td>
<td>+ 0' 4''</td>
</tr>
</tbody>
</table>

... daß die durch Vermittlung der Induction mit dem Magnetometer an Präcision auch den durch die sorgfältigsten Beobachtungen mit den besten bisherigen Inclinatorien gewonnenen Resultaten nicht nachstehen;

... that the determination of the inclination through the evaluation of the induction with the magnetometer are not worse than the results obtained so far with the best inclinometers;

magnetic inclination in Göttingen, also measured by:
- Gauss
- Humboldt
- Forbes

few arcmin change observed over 50 years
Analog integrator (Miller)

\[ V_{out} = -\frac{1}{RC} \int_{-\infty}^{t} V_{in} dt \]

shielding and control:
- leakage current
- ground current
- temperature coefficient
- dielectric absorption

simple, inexpensive, effective
accuracy limited by analog electronics
Digital integrator

\[
\int_{t_{\text{start}}}^{t_{\text{end}}} f dt = K_{VFC} \int_{t_{\text{start}}}^{t_{\text{end}}} V_{in} dt
\]

digital output, no cumulative error, 10...100 ppm accuracy!

VFC linearity and stability, counter resolution \((\Delta t/(4 f_{\text{ref}}))\) ppm
Numerical integrator

digital output, powerful numerical integration possible
precise time required, *dead*-times, may need 2 DVM’s
Fast Digital Integrator

- \(-V_{\text{max}} \ldots +V_{\text{max}}\)
- \(0 \ldots 2^{N\text{bit}}\)

\(-V_{\text{max}} \ldots +V_{\text{max}}\)\hfill \text{ADC}\hfill f_{\text{ref}}\hfill \text{CLK} \hfill \Delta \phi \hfill \text{C-PCI/PXI BUS}

C-PCI/PXI BUS

Digital Signal Processor

Field Programmable Gate Array

FPGA

DSP

ADC

\(f_{\text{ref}}\)

\(\text{CLK}\)

\(-V_{\text{max}}\ldots +V_{\text{max}}\)

\(V_{\text{in}}\)

\(R_{\text{coil}}\)

instrumentation amplifier \((Z_{\text{input}})\)

Analog to Digital Converter

faster integrals (100 kHz), improved resolution (1 ppm)
Point coils – the Fluxball


measure average field in a small volume (point-like)
can be approximated by co-axial solenoids of proper R/H
Line and area coils

integrated field over long lengths relevant for accelerators

Courtesy of J. Billan, CERN
Harmonic coils

Gradient coils
\( dB_z/dx \)

Morgan coil \((B_4)\)

- Measure field gradients or higher order terms (bucking)
- High resolution through compensation of background field signal

Fluxmeter zoo

**Fixed coil measurements**
- Static coils \((d\mathbf{S}/dt=0)\), the field change \((dB/dt)\) induces the voltage
- Provides only a relative measurement \((B_{\text{end}}-B_{\text{start}})\)
- The voltage offsets cannot be distinguished from physical signal

**Moving coil measurements**
- Steady field \((dB/dt=0)\), the coil movement \((d\mathbf{S}/dt)\) induces the voltage
- Requires well controlled mechanics, simple movements
- Provides an absolute measurement if:
  - initial \(B_{\text{start}}=0\) moving from far away into the magnet (zero-gauss chamber): moving coil
  - using symmetries \(B_{\text{end}}=-B_{\text{start}}\): flip coil, rotating coil
- A voltage offsets can be distinguished from physical signal

\[
V = -\frac{d\varphi}{dt} \quad \varphi = \int_{S} B d\mathbf{S}
\]

*Most flexible method* in all its many variants, although...

“...This type of magnetometer is obsolete.”

http://en.wikipedia.org/wiki/Magnetometer
A rotating coil ???

... no, actually this is a *fixed coil* with 800 turns and $\approx 250 \text{ m}^2$ surface that has been used to verify e.m. coupling of LEP and SPS.
Rotating *snakes* @ CERN

16 m

0.1 μT, 0.05 mrad resolution, 100 ppm accuracy
Complex formalism

- SC magnets for accelerators
  - 2-D field (slender magnet), with components only in $x$ and $y$ and no component along $z$
  - Ignore $z$ and define the complex plane $s = x + iy$

- Complex field function:
  $$B = By + iB_x$$

- $B$ is analytic in $s$ and can be expanded in Taylor series (the series converges) inside a current-free disk:
  $$By + iB_x = \sum_{n=1}^{\infty} C_n \left( \frac{s}{R_{ref}} \right)^{n-1}$$
  $$C_n = B_n + iA_n$$
**Multipoles**

- Complex multipole coefficients:
  
  \[ C_n = B_n + iA_n \]

\[ B_y + iB_x = \sum_{n=1}^{\infty} C_n \left( \frac{s}{R_{\text{ref}}} \right)^{n-1} \]

- **n=1**
  - \( B_1 \neq 0 \), normal dipole
  - \( A_1 \neq 0 \), skew dipole

- **n=2**
  - \( B_2 \neq 0 \), normal quadrupole
  - \( A_2 \neq 0 \), skew quadrupole

\[ \text{Diagram of magnetic fields for different n values} \]
Rotating coil in normal dipole

maxima and minima located at 0 and $\pi$

$\cos(\theta)$ waveform
maxima and minima located at $\pi/2$ and $3/2 \pi$

flux de-phased by $-\pi/2$ with respect to normal dipole

$\sin(\theta)$ waveform
Rotating in normal quadrupole

maxima and minima located at 0, \( \pi/2 \), \( \pi \) and \( 3/2 \pi \)

flux variation twice faster than in a dipole

\( \cos(2 \theta) \) waveform
maxima and minima located at $1/4 \pi$, $3/4 \pi$, $5/4 \pi$ and $7/4 \pi$

flux de-phased by $-\pi/4$ with respect to normal quadrupole

$\sin(2 \theta)$ waveform
Rotating in normal sextupole

\[
\cos(3 \theta)
\]

waveform ...
Fourier analysis

\[ n=1, B_1 \neq 0 \]

\[ n=1, A_1 \neq 0 \]

\[ n=2, B_2 \neq 0 \]

and so on…

...by induction: \[ C_n = \frac{\tilde{\phi}_n}{\kappa_n} \]

Fourier transformed flux

coil calibration
Hall generator principle

thin slab of semiconducting material ($A^{III}B^V$):
- InAs
- InSb
- GaAs

DC current

Lorentz force $F = v \times B$

transverse DC (Hall) voltage

$B \cos(\theta)$
SOMETIMES during the last University year...

iron foil

current lead

voltage tap

magnet
Hall coefficient

- Hall voltage: \( V_H = G R_H IB \cos(\theta) \)
  - \( R_H(B) \): material dependent Hall coefficient
    - high mobility, low conductivity to have high \( R_H \)
      - metals (low mobility)
      - alloys (high resistivity causes heating)
    - compromise choice: semiconductors
  - temperature dependence 100 to 1000 ppm/°C

- \( G(B) \): geometry factor
  - equipotential lines deform under \( v \times B \)

- Optimal design to compensate \( R_H \) vs \( G \)
  - Cruciform design achieves 1 % linearity over wide \( B \) range
  - better definition of magnetic center

100 ppm accuracy feasible
Hall magnetometer

- a Hall sensor is a 4-terminals device
  - do NOT connect outputs in series !!!

- DC measurement can resolve 1...10 µT
- AC (lock-in) can resolve 0.1 µT
  - \( I = I_0 \sin(2\pi ft), f = 10 \text{ Hz} \ldots 1 \text{ kHz} \)
Quantum Hall effect

- Shubnikov-de Haas effect
  - oscillation in $R_H$ periodic function of $1/B$

4.2 K

$\Delta R_H/R_H$

magnetic field

B > 2 T

periodic oscillation of $\Delta R_H/R_H$

effect as much as 1 % on calibration coefficient
Planar Hall effect

\[ V_{\text{planar}} = V_{HP} B^2 \cos(2\phi) \]

\( V_{\text{planar}} \) is important when mapping 3-D fields
XXXI. The Bakerian Lecture.—On the Electro-dynamic Qualities of Metals*.  
By Professor William Thomson, M.A., F.R.S.

Received February 28.—Read February 28, 1856.
Magnetoresistors

two-terminal device, simple, inexpensive, modest sensitivity, non-linear, T effects (2500 ppm/°C)
bias field, compensated bridges, giant-magnetoresistance

Fluxgate (*Peaking Strip*)

![Diagram of fluxgate sensor with excitation, bias, and detection coils]

- $+B_{\text{excitation}}$
- $-B_{\text{excitation}}$
Fluxgate applications

- simple and inexpensive, lightweight device
- highly directional
- high sensitivity (tens of pT !)
- modest accuracy (1000 ppm)
- typical applications
  - navigation
  - geology, ores, oil fields
  - hunting submarines
  - finding mines
  - mapping of interplanetary magnetic field
A special fluxgate: the DCCT

best known device for high current (relevant for SC magnets)
1 ppm possible at 10 kA (10 mA)
SQUID principles – 1

- Change $\delta$ of phase of wave-function for paired electrons along $\Gamma$ depends on magnetic flux $\varphi$:
  \[ \delta = 2\pi \frac{\varphi}{\varphi_0} \]
  quantum fluxoid (2 x 10^{-15} Wb)

- Flux quantization: $\delta = 2\pi n$
  \[ \varphi = n\varphi_0 \]
the wave-function can tunnel through a normal-conducting (Josephson) junction

the maximum supercurrent depends on $\delta$:

$$I = I_c \sin(\delta)$$
The SQUID

- maximum supercurrent:
  \[ I = I_c \left[ \sin(\delta_1) + \sin(\delta_2) \right] \]

- phase relation:
  \[ \delta + \delta_1 - \delta_2 = 2\pi \frac{\varphi}{\varphi_0} \]

- SQUID critical current
  \[ I = 2I_c \left| \cos \left( \pi \frac{\varphi}{\varphi_0} \right) \right| \]
SQUID operation

- SQUID voltage periodic in $\varphi_0$
- flux change $\Delta \varphi$ from $\Delta V$, compute $\Delta B$ using $\kappa$

SQUID $I_c$

SQUID $V$

bias current

sensitivity few pT for bare SQUID, 1 fT with input transformer
range limited by FB, accuracy limited by calibration of surface
cortical activation of the primary and secondary somatosensory cortex during left median nerve simulation measured by MEG and fMRI...
The frontier of magnetometry

- Alkali atoms (Rb, Cs) have unpaired electrons whose spin precesses with magnetic field (as protons)
- The interaction with field can be detected as follows:
  - An alkali metal vapor is prepared in a cell
  - A first laser (the pump) aligns the spins to create coherence (as the RF pulse in NMR techniques)
  - A second laser (the probe) detects resonance, which can be seen, e.g., as a shift in an interference pattern
- Excellent device for miniaturization

Sandia National Laboratories
A digression on spectra

- The *gross* structure of the spectral line of atoms (energy level), are split in:
  - A *fine* structure (due to interaction of magnetic moment of electron spin and orbital angular momentum)
  - A *hyperfine* structure, due to interaction of nucleus magnetic moment with internal magnetic/electric fields in the atom

The hyperfine structure further splits in a magnetic field (Zeeman effect) proportionally to \( B \)

\[ \Delta \nu \approx \mu gB \]

- **87Rb D1 line**
  - \( 5^2S_{1/2} \) to \( 5^2P_{1/2} \)
  - Zeeman
    - \( F=2 \)
      - \( m=2 \)
      - \( m=1 \)
      - \( m=0 \)
      - \( m=-1 \)
      - \( m=-2 \)
  - \( 794 \text{ nm} \) (377 THz)
  - \( 6.83 \text{ GHz} \)

P. Zeeman
Atomic magnetometer DIY

- Set a laser on a given absorption line (e.g. D1 of $^{87}$Rb, 794 nm)
- AM the laser with a VCO to create two sidebands corresponding to the hyperfine structure (central $f = 3.42$ GHz)
- Modulate the VCO (by ± 3 MHz) to detect the resonances from the split hyperfine levels
- Compute the difference of frequency between two resonances, which is proportional to the magnetic field:
  
  \[ h\Delta v \approx \mu g B \]

NIST

sensitivity in the range of tens of fT
range limited to 1 mT at most, accuracy not established so far
Spin Exchange collisions preserve total momentum but cause single spins to change, scrambling the hyperfine structure.

The precession of atoms in the vapor loses quickly coherence, the resolution of the resonance is limited.

At low B and high gas density, collisions happen quicker than the precession time of the atoms. On average the hyperfine states are stable, the Spin Exchange is Relaxation Free.

The resonance is measured with improved resolution!
Premium sensitivity!

- Best quoted sensitivity is 200 aT/√Hz!
  - 0.2 x 10^{-15} T at 1 Hz

- A new world of possibilities
  - Magnetoencelography,
  - Magnetocardiography of in fetal hearts,
  - Earth field NMR ...

Example: NMR signal from water on chip

a Tricorder ?!?
850 T pulsed field at LANL

imploding flux compression generator (VNIIEF design)

material properties at high magnetic fields (8 to 10 MG)

a shot ...
850 T shot at LANL

Implosion and flux compression

how do you measure this ???
Faraday effect

- rotation of light polarization in a medium in a magnetic field

Rotation angle

$\theta = \nu B L$

Length

Field strength

Verdet constant

Material

\begin{align*}
\text{fused quartz} & : 4 \\
\text{flint glass} & : 110 \\
\text{benzene} & : 9
\end{align*}

Fast, e.m. compatible at high electric fields

Modest accuracy (1 %)
Field imaging

- Light polarization can be used to image the field:
  - Faraday rotation
  - Ferrofluid cell

Center of a RHIC quadrupole at 75 T/m gradient during cold testing

Direct magnetic center measurement (image treatment) needs higher field than e.m. methods, limited accuracy
Performance summary
Conclusions

- the art of magnetic measurements cannot be made into a science
  - at least I have given up...

- methods and instruments exist, don’t try to make them yourself, use them if you can
  - as for *italian cuisine* let Mom do it...
  - ...or buy a good cooking book before you start

- where can I find out more?
  - CAS on MM, MT, PAC, EPAC
  - (NIM) uNclear Instruments and Methods