Field Measurement Methods

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CAS on Magnets

Novotel Brugge Centrum, Bruges, Belgium

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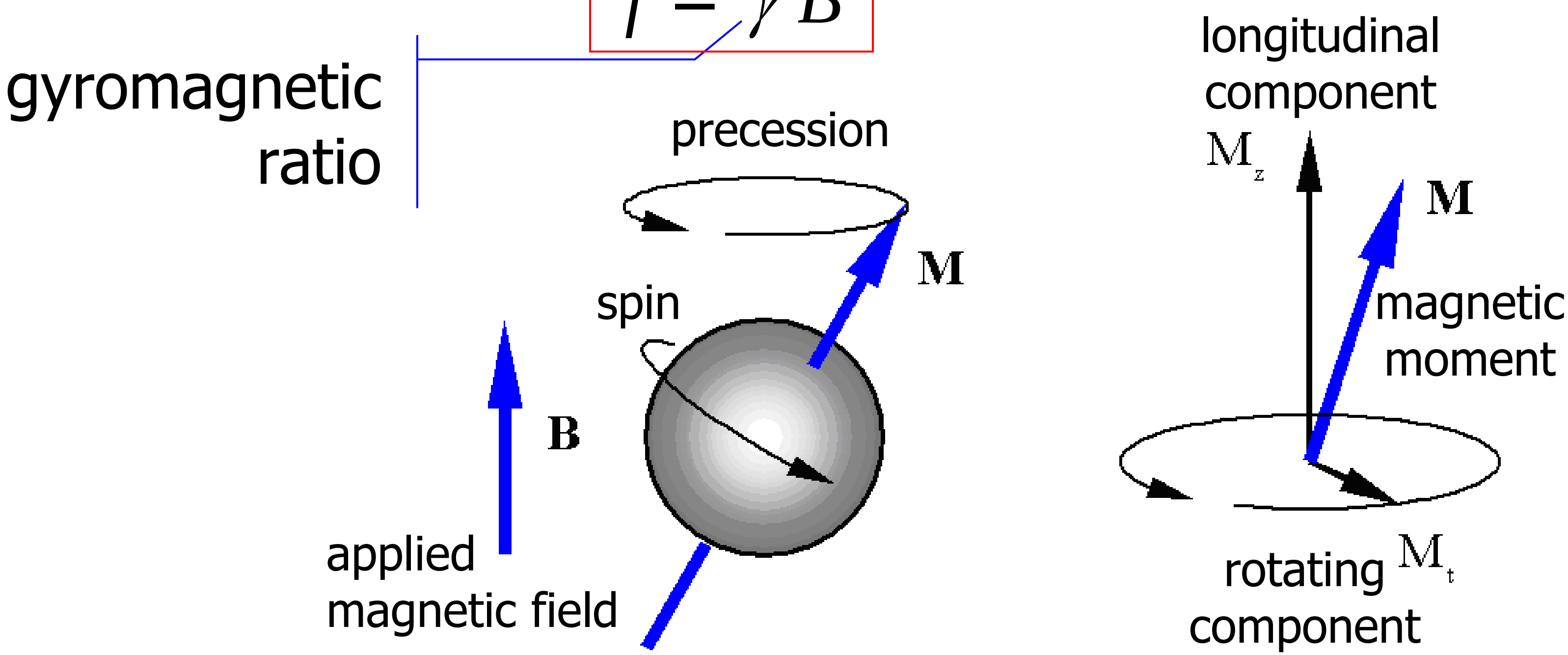
Overview

- NMR/EPR, *the golden standard*
- Fluxmeters, *the workhorse for accelerators*
  - *fixed, moving, flipping, rotating* coils and wires
- Hall generators and magneto-resistors, *cheap*
- Other *fun* methods
  - Fluxgate magnetometers, *sensitive*
  - SQUIDS, *quantum sensitive*
  - Atomic and SERF magnetometers, *yet more sensitive*
  - Faraday rotation, *fast*
- Concluding remarks, *and a design graph*

NMR/EPR principle

- a particle with a spin and magnetic moment in an applied field precesses at a (Larmor) frequency f:

$$f = \gamma B$$



Gyromagnetic ratio

Electron Paramagnetic Resonance (EPR), Electron Spin Resonance (ESR)

Nuclear Magnetic Resonance (NMR)

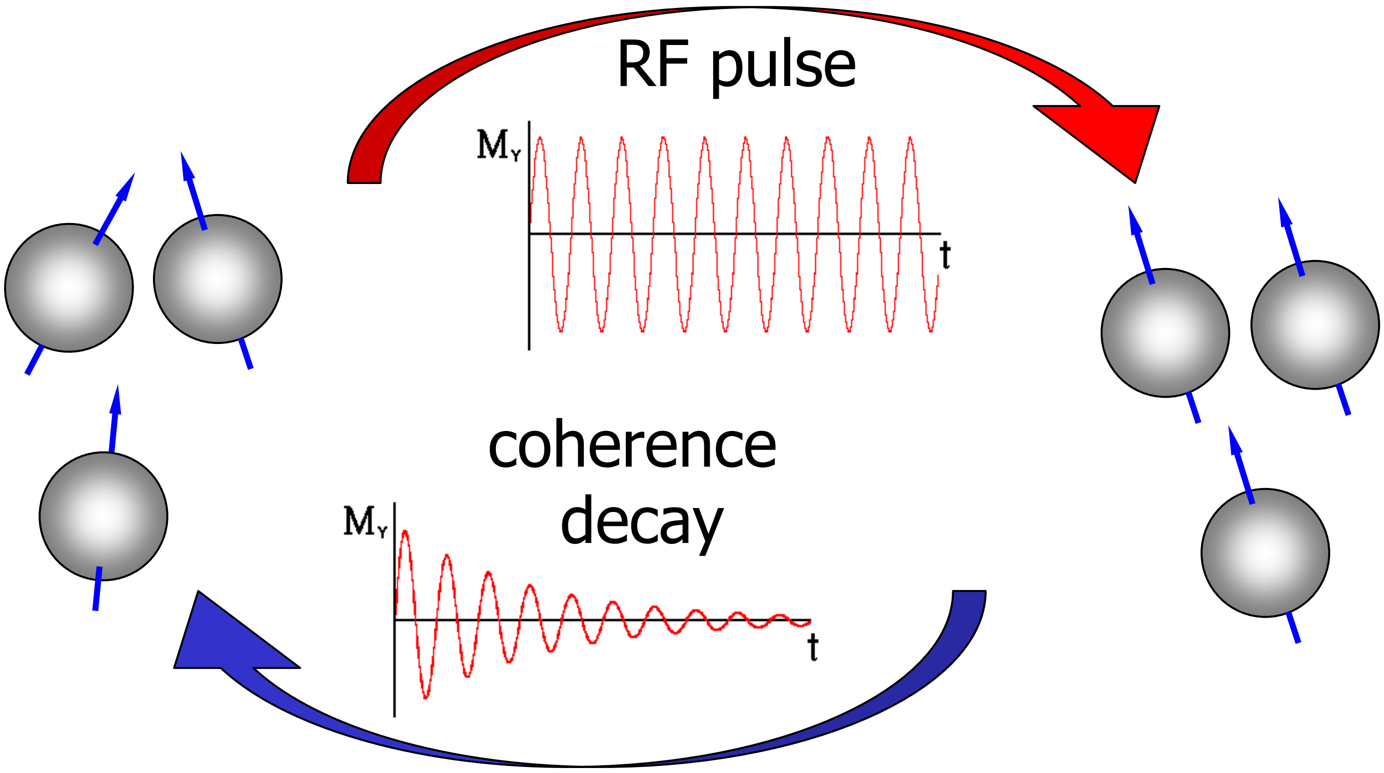
particle	γ (MHz/T)
e^-	28.025×10^3
^{1}H	42.576396(3)
^{2}H	6.535
^{2}He	32.4326
^{13}C	10.71
^{14}N	3.08
^{19}F	40.08
^{23}Na	11.27
^{27}Al	11.093
^{31}P	17.25

1 GHz NMR magnet $\Downarrow$ 23.5 T

cryogenic probes

Resonance and coherence

- a transverse RF pulse of frequency f induces resonance in the precession and coherence in M_t

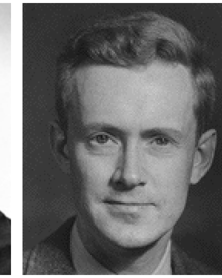


Phys. Rev., **70**, 460, **1946**

Nuclear Induction

F. Bloch, W. W. Hansen, and Martin Packard
Stanford University, Stanford University, California
January 29, 1946





Rabi, 1938 Bloch, Purcell, 1946

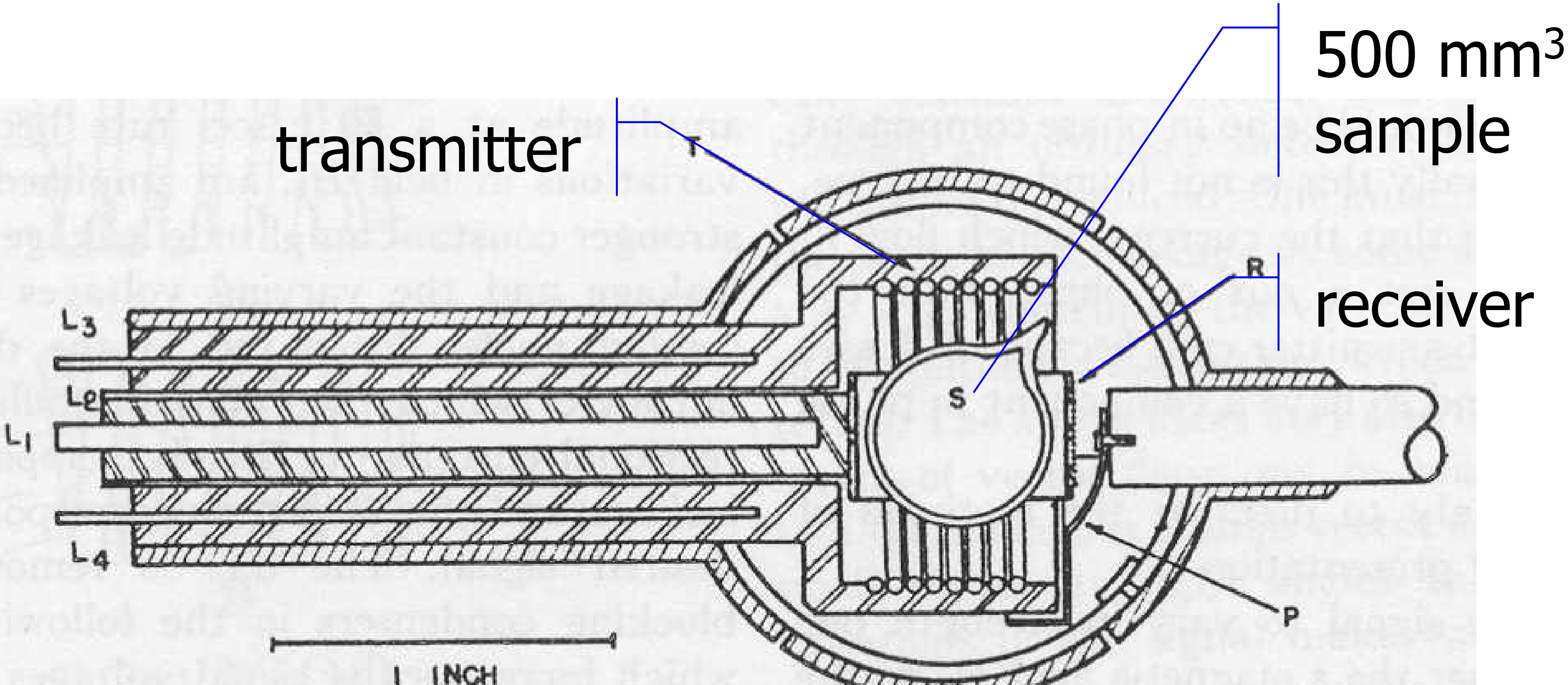
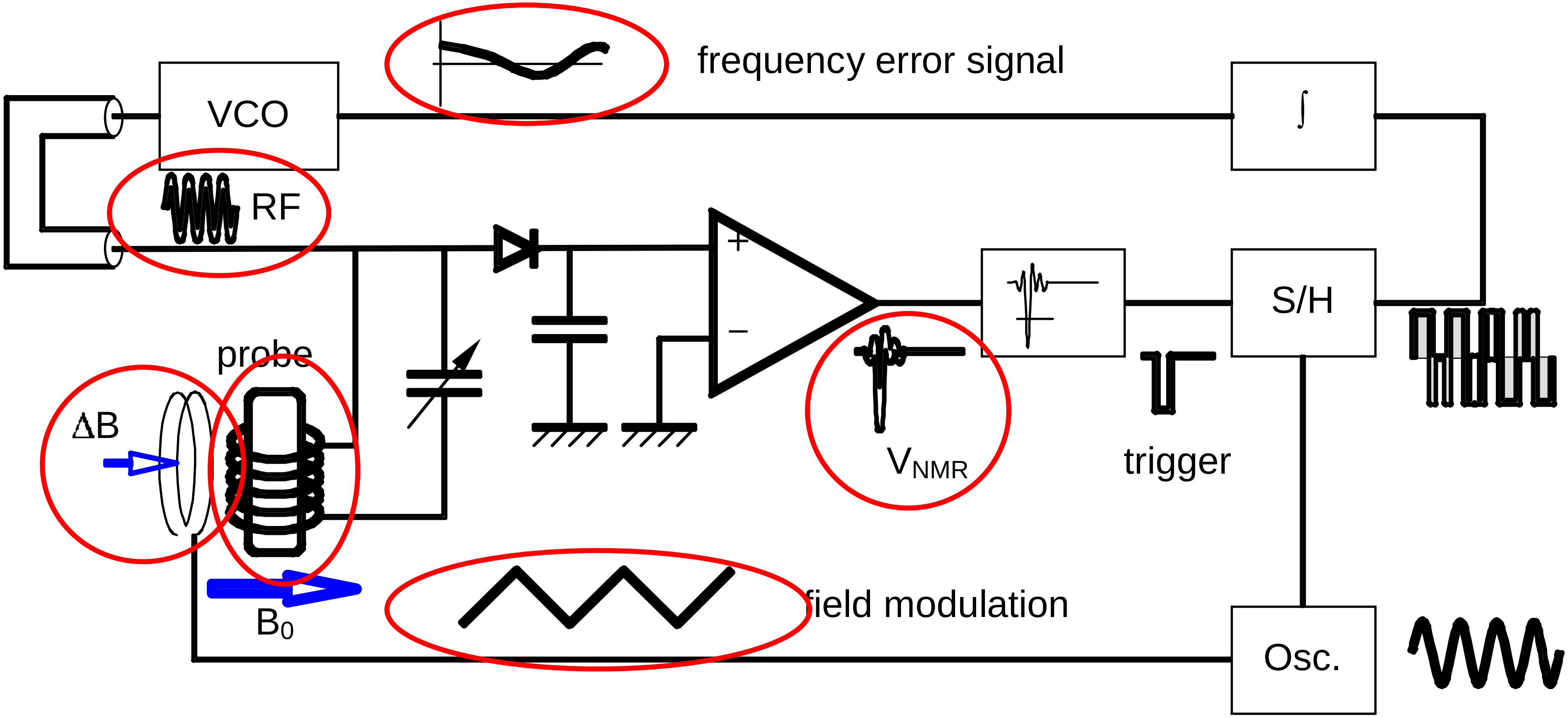


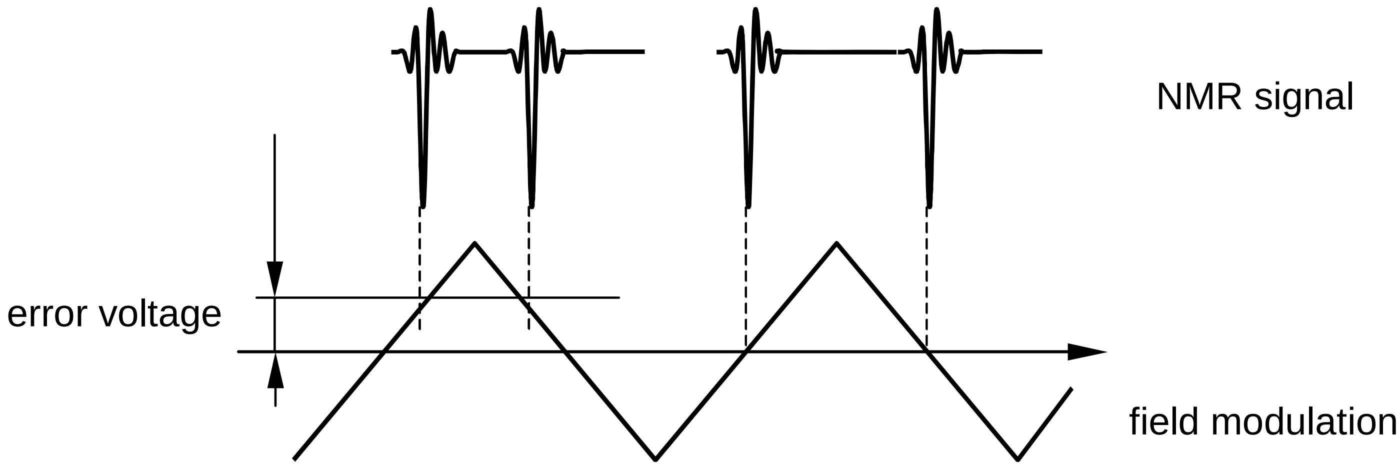
Fig. 4. Scale drawing of an *xy* section of the r-f head. The spherical sample *S* is surrounded by a receiver coil *R*, which is in turn surrounded by a transmitter coil *T*, the whole being encased in a shield. A rotably mounted paddle *P* is used to steer the transmitter flux. Leads to the receiver coil are the coaxial leads L_1L_2 while the transmitter leads are L_3 and L_4. The outer shield is split to avoid 60-cycle eddy currents.

A *DIY* NMR magnetometer



0.1 ppm absolute accuracy achievable (0.1 Hz)

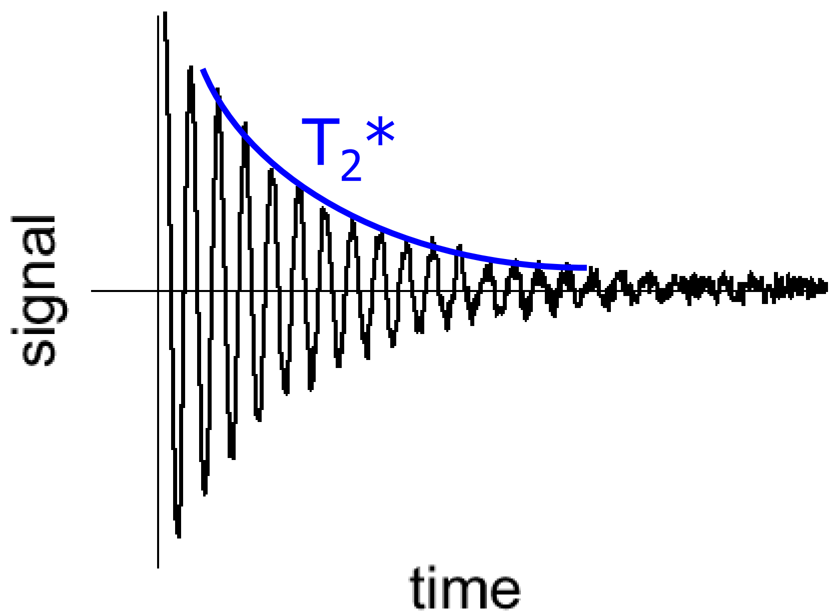
Field tracking

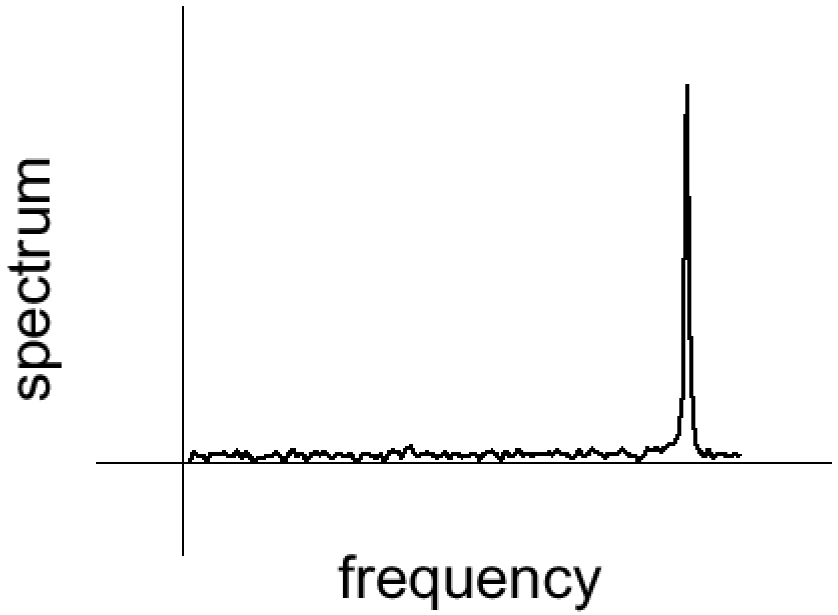


- tracking is slow (Hz): maximum field variation tolerated for latching $\delta B/B < 1\ \%\ s^{-1}$
- field gradients *blur* signal: field homogeneity $\nabla B/B < 10 \ldots 100$ ppm/mm
  - gradient coils to measure inhomogeneous fields !

Free Induction Decay

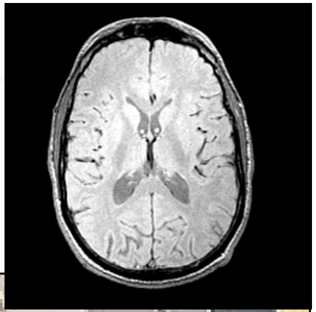
- M_t decay after RF pulse (FID)
  - high accuracy for long measurement times
  - main tool for spectroscopy
    - analysis of chemicals, molecules
    - structure determination (COSY, NOSY, …)



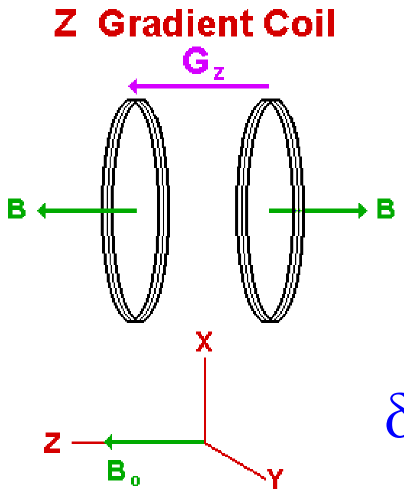


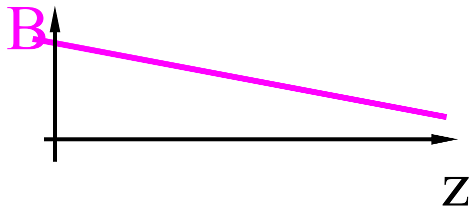
Imaging

P. Lauterbur, 1973

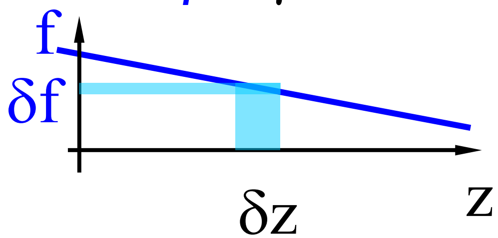


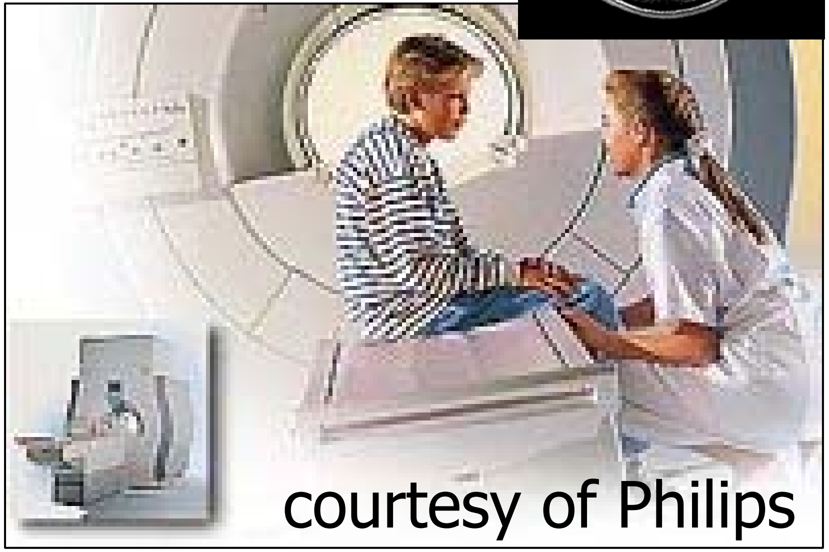
- Magnetic Resonance Imaging (MRI)





$$f = \gamma B$$

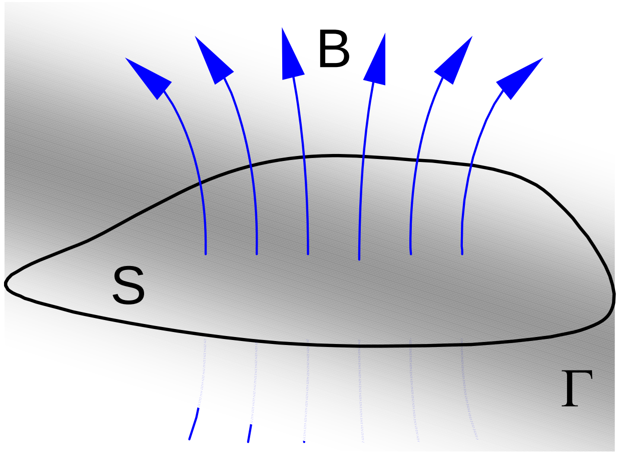




2000 Ig Nobel Prize winner, Annals of Improbable Research
I.W. Schultz, P. van Andel, I. Sabelis, E. Mooyaart
Magnetic resonance imaging of male and female genitals during coitus and female sexual arousal
British Medical Journal, **319**, 1596-1600, 1999.

Fluxmeter

- Magnetic flux: $\varphi = \int_S \mathbf{B} d\mathbf{S}$
- Induction law: $V = -\frac{d\varphi}{dt}$



$$\varphi_{end} - \varphi_{start} = -\int_{t_{start}}^{t_{end}} V\, dt$$

needs an *integrator*...

$$B_{end} - B_{start} = \frac{\varphi_{end} - \varphi_{start}}{\kappa}$$

... and coil calibration

Ann. Der Physik, 2, 209, **1853**

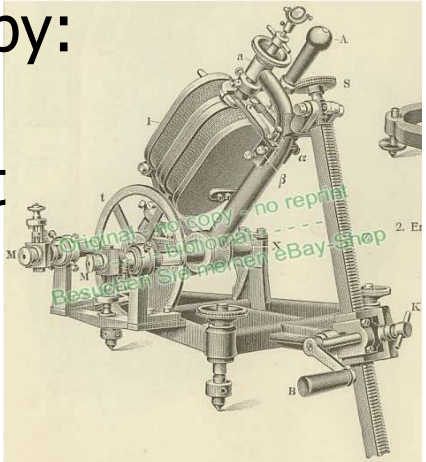
II. *Ueber die Anwendung der magnetischen Induction auf Messung der Inclination mit dem Magnetometer; von Wilh. Weber.*

(Mitgetheilt vom Hrn. Verf. im Auszuge aus d. Abhandl. d. K. Gesellsch. d. Wiss. zu Göttingen Bd. V.)

Inductions-Inclinatorium

	Beobachtet.	Berechnet.	Unterschied.
1805 Dec.	69° 29′	69° 36′ 43″	− 7′ 43″
1826 Sept.	68 29 26″	68 23 17	+ 6′ 9″
1837 Juli 1	67 47 0	67 52 41	− 5′ 41″
» »	67 53 30	67 52 41	+ 0′ 49″
1841 Oct. 8	67 42 43	67 42 0	+ 0′ 43″
1842 Juni 21	67 39 39	67 40 18	− 0′ 39″
1852 Aug. 7	67 18 38	67 18 34	+ 0′ 4″

magnetic inclination in Göttingen, also measured by:
- Gauss
- Humboldt
- Forbes



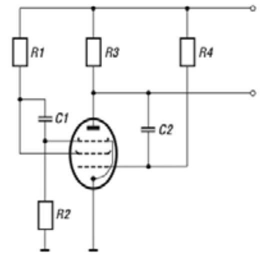
1. Induktions-Inklinatorium nach Wild (von Edelmann, München).

… daß die durch Vermittlung der Induction mit dem Magnetometer an Präcision auch den durch die sorgfältigsten Beobachtungen mit den besten bisherigen Inclinatorien gewonnenen Resultaten nicht nachstehen;

few arcmin change observed over 50 years

… that the determination of the inclination through the evaluation of the induction with the magnetometer are not worse than the results obtained so far with the best inclinometers;

Analog integrator (Miller)

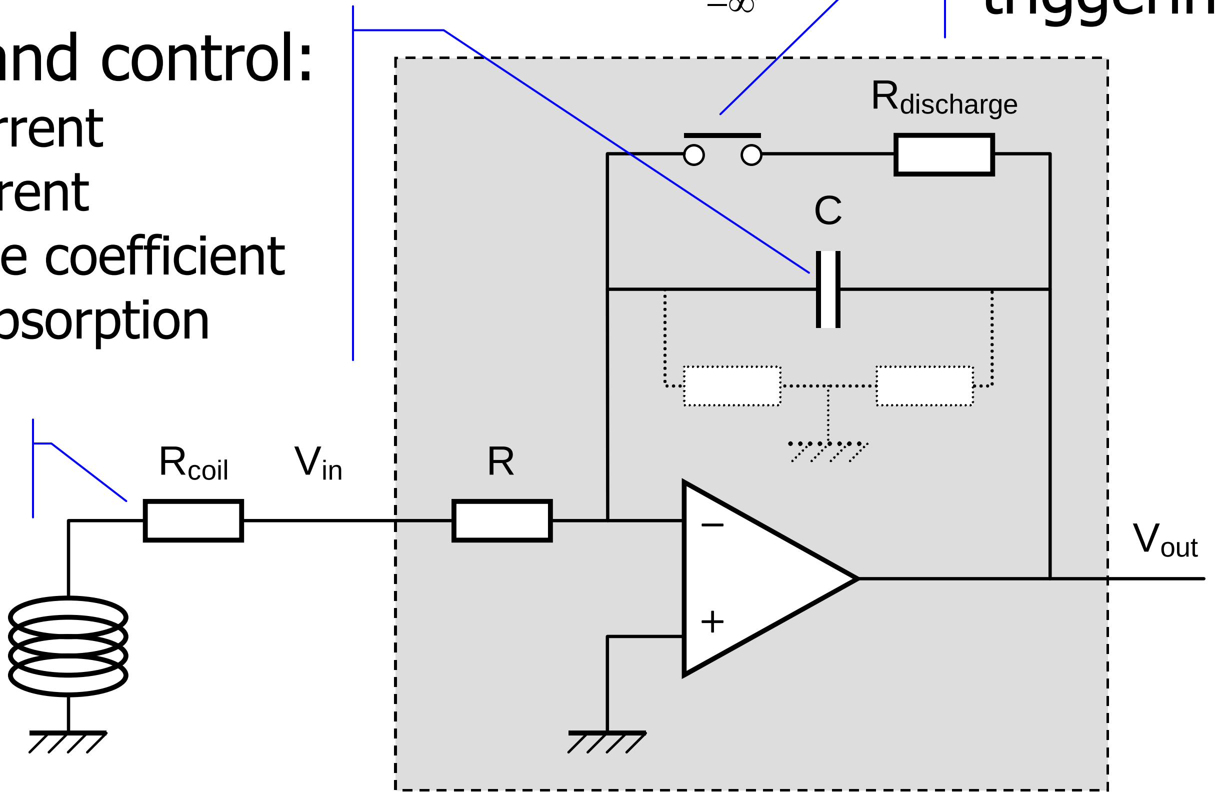


$$V_{out} = -\frac{1}{RC}\int_{-\infty}^{t} V_{in}\,dt$$

triggering

shielding and control:
- leakage current
- ground current
- temperature coefficient
- dielectric absorption

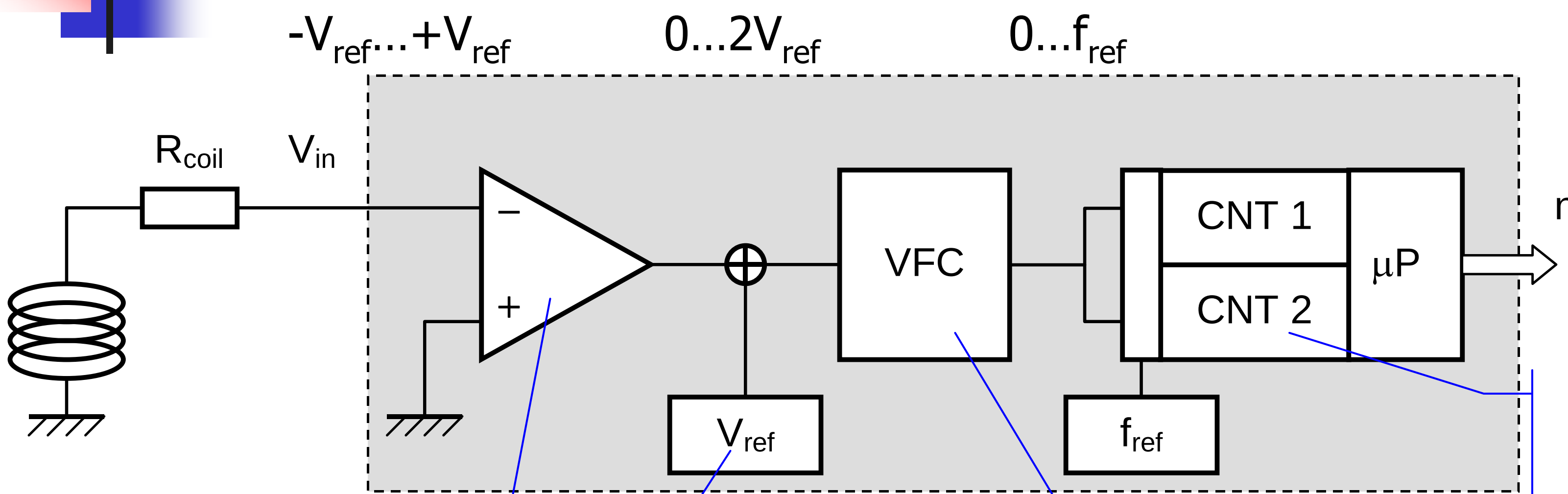
$R+R_{coil}$



simple, inexpensive, effective

accuracy limited by analog electronics

Digital integrator



parallel counters

instrumentation amplifier (Z_{input})

Voltage to Frequency Converter

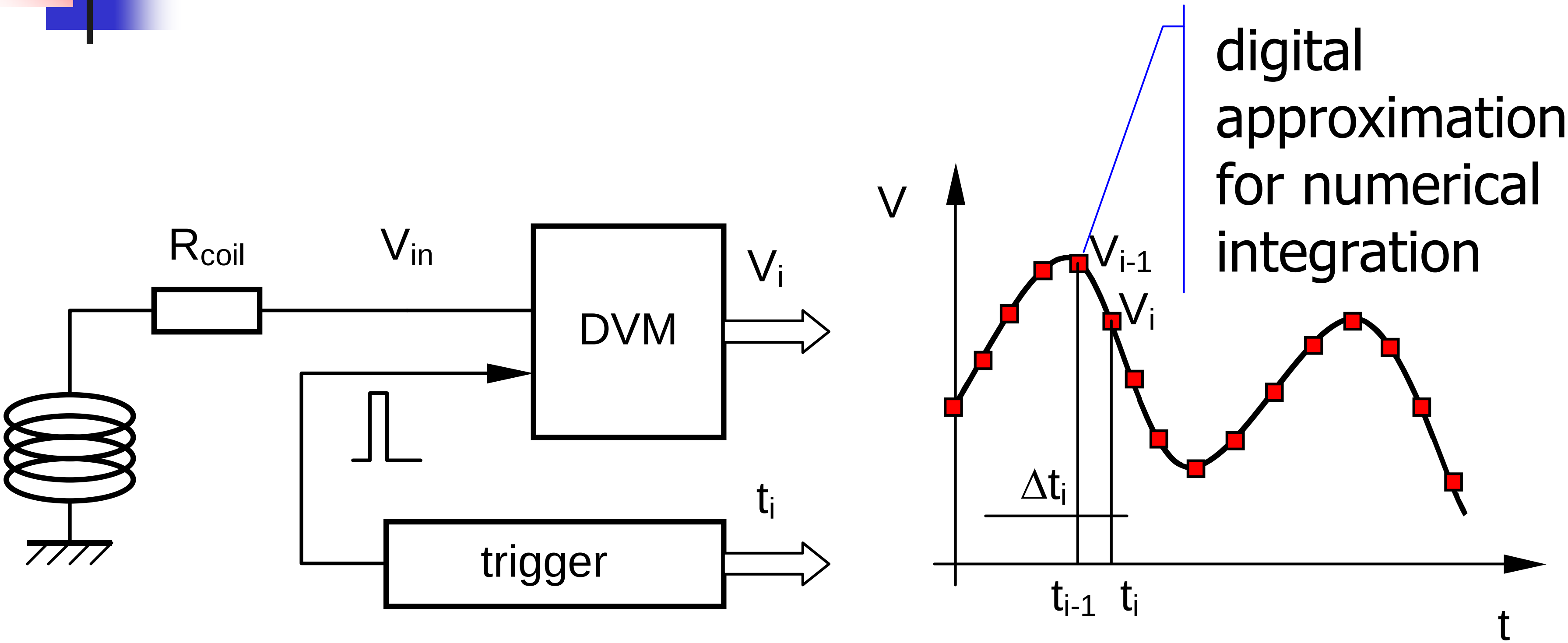
voltage offset

$$n = \int_{t_{start}}^{t_{end}} f\,dt = K_{VFC} \int_{t_{start}}^{t_{end}} V_{in}\,dt$$

digital output, no cumulative error, 10…100 ppm accuracy !

VFC linearity and stability, counter resolution ($\Delta t/(4\ f_{ref})$ ppm)

Numerical integrator

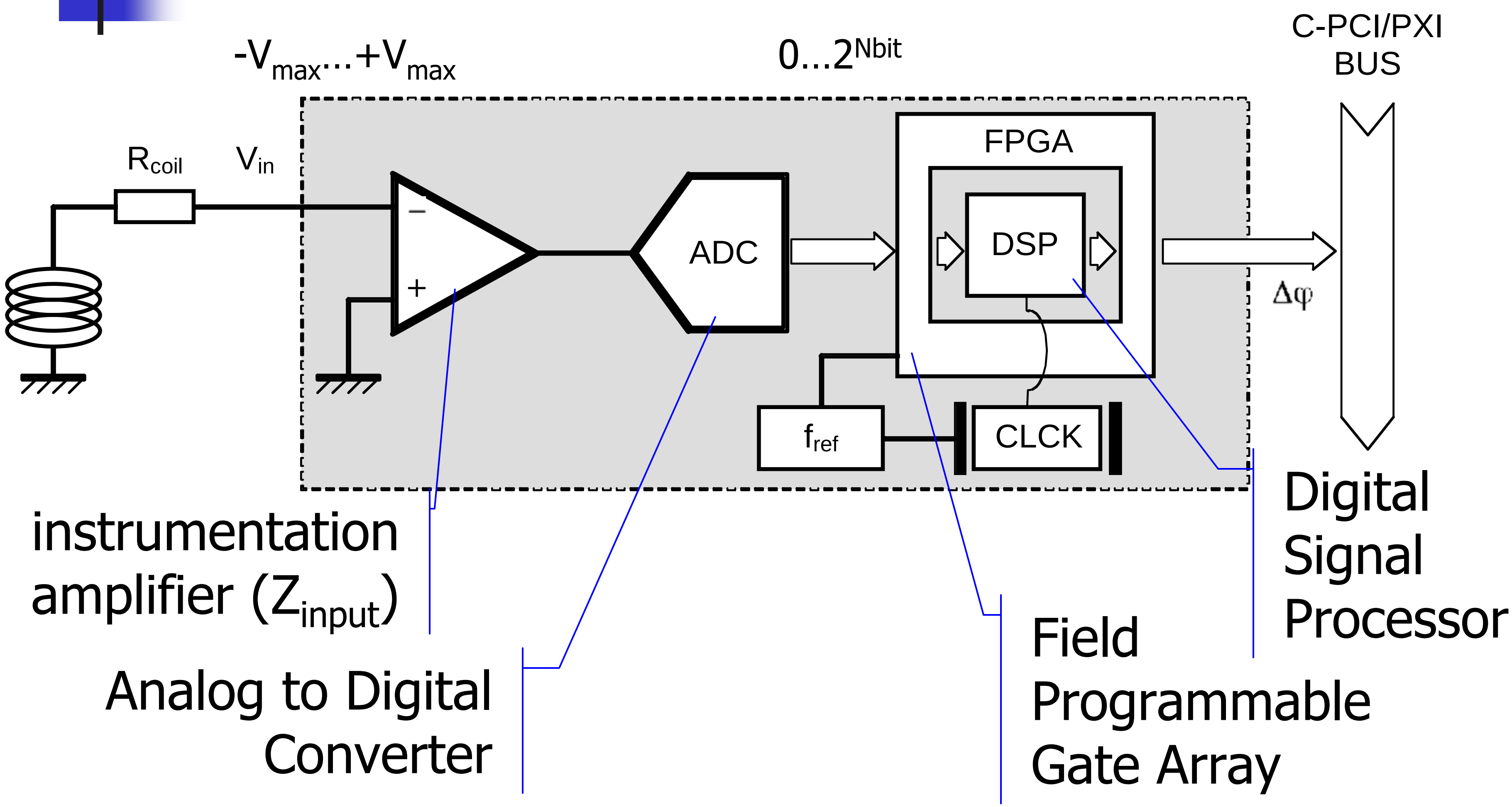


digital output, powerful numerical integration possible

precise time required, *dead*-times, may need 2 DVM’s

European Organization for Nuclear Research

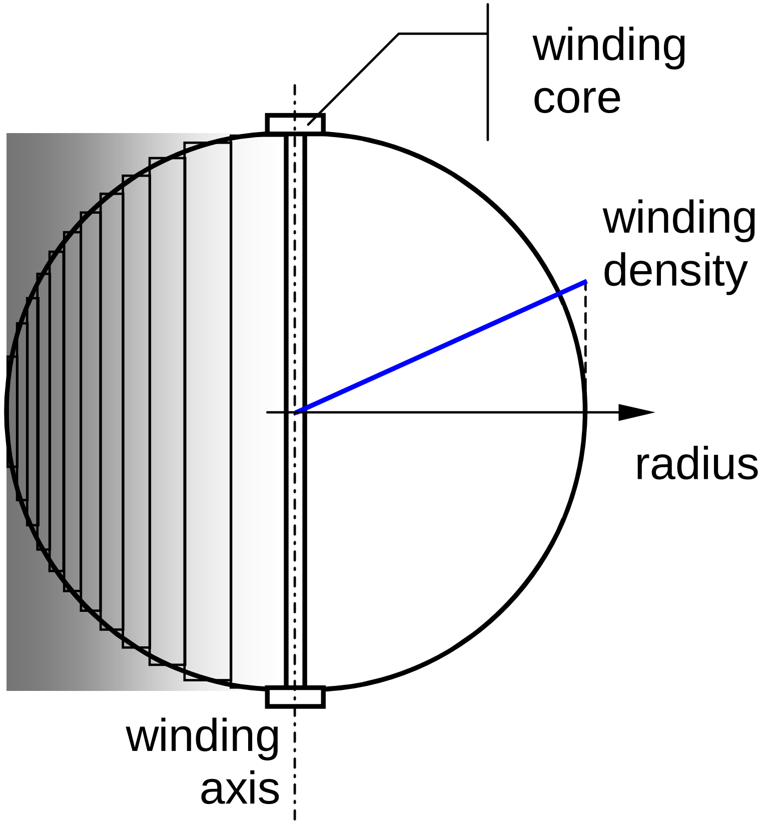
Fast Digital Integrator

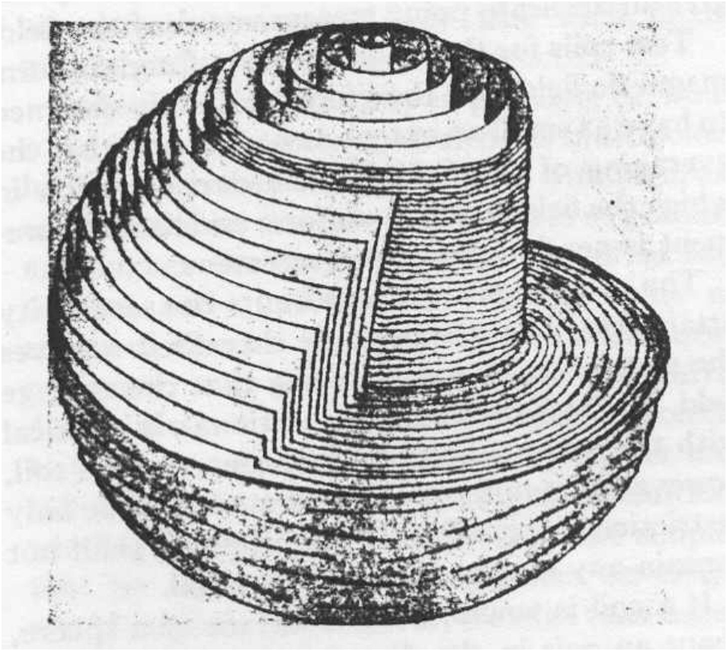


faster integrals (100 kHz), improved resolution (1 ppm)

Point coils – the Fluxball

W.F. Brown, J.H. Sweer,
Rev. Sci. Instr., 16, 276, **1945**

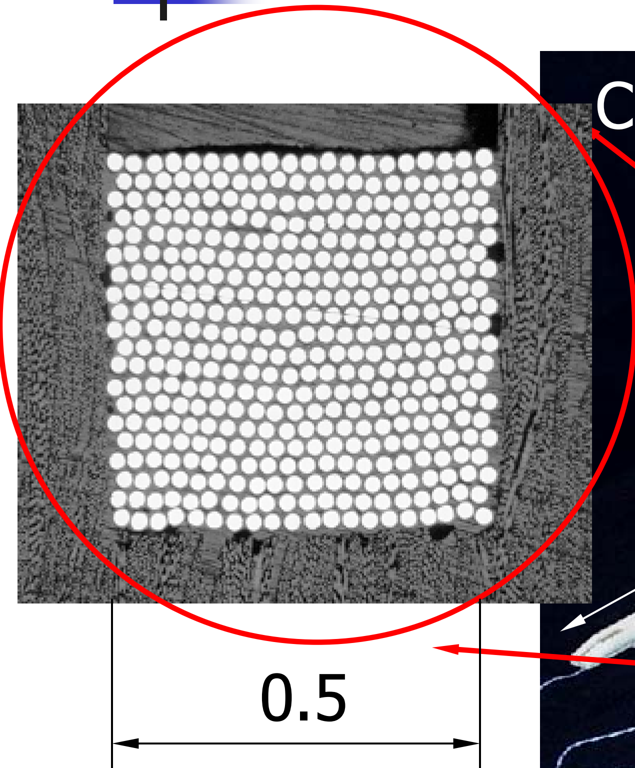


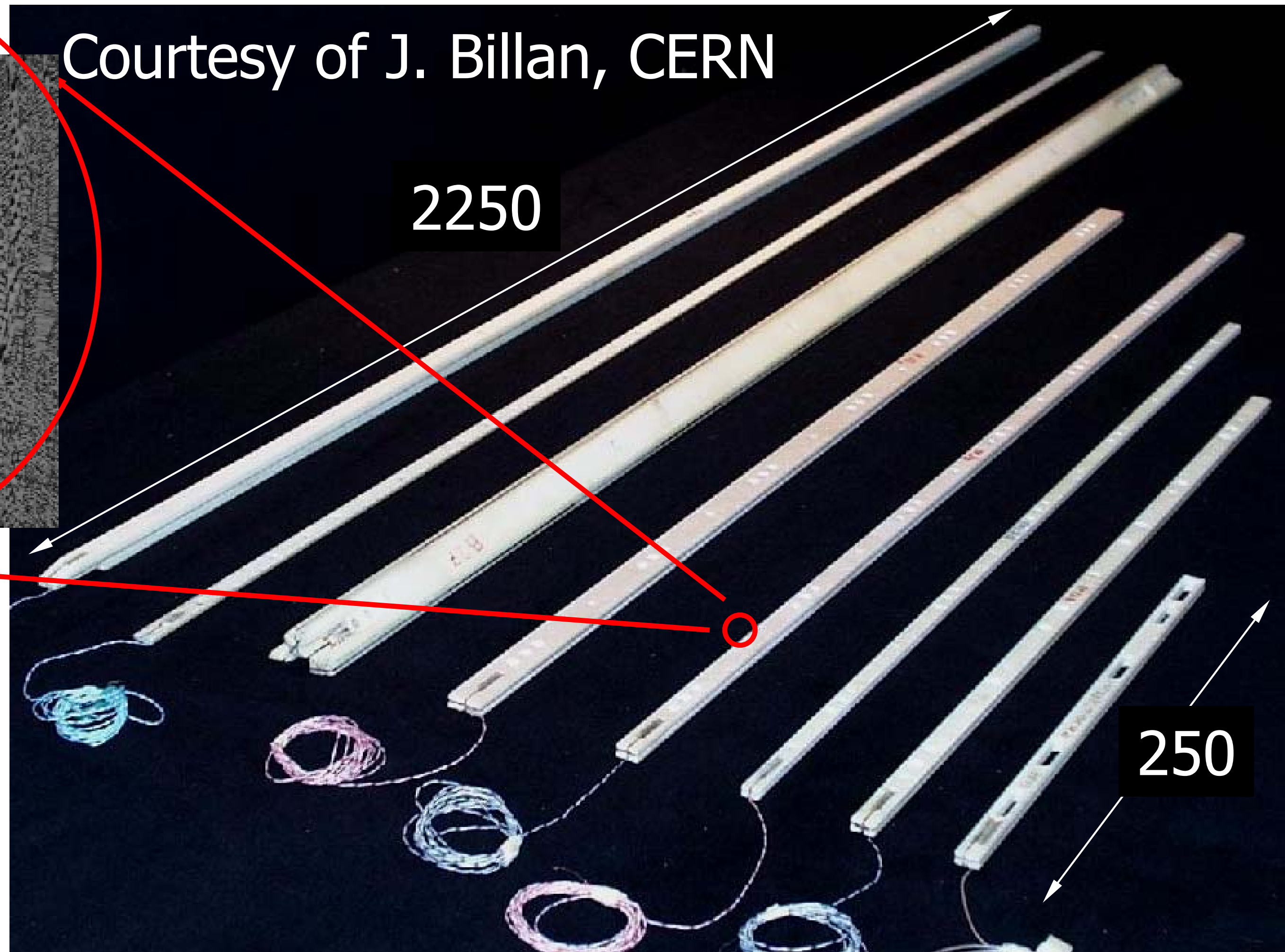


measure average field in a small volume (point-like)

can be approximated by co-axial solenoids of proper R/H

Line and *area* coils

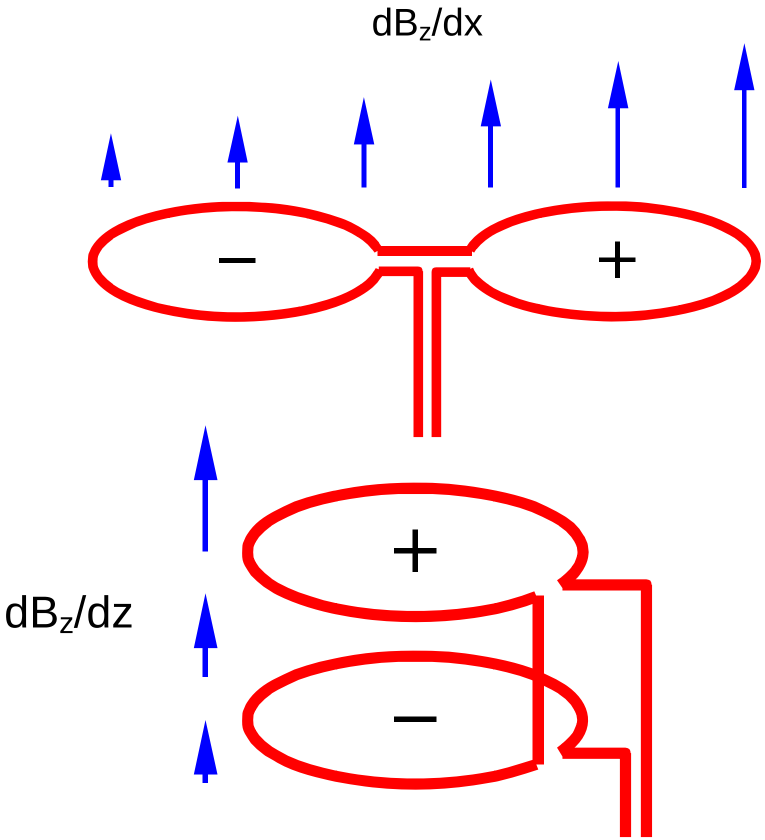




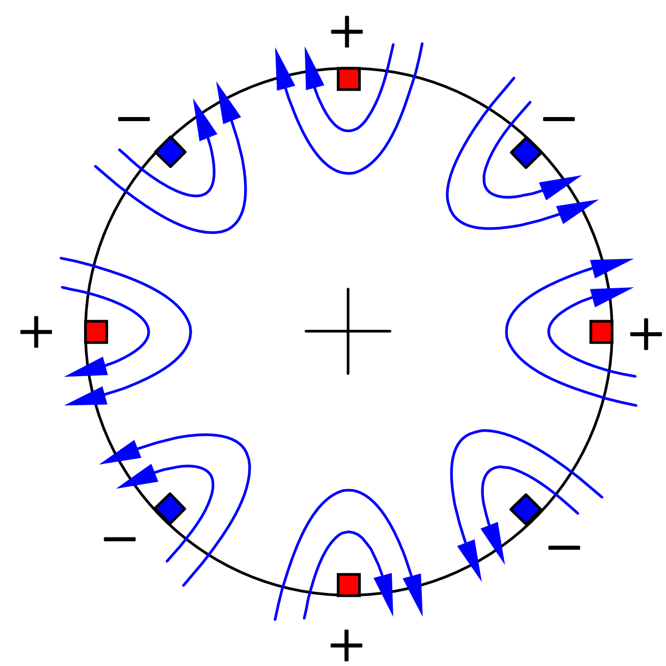
integrated field over long lengths relevant for accelerators

Harmonic coils

Gradient coils



Morgan coil (B_4)

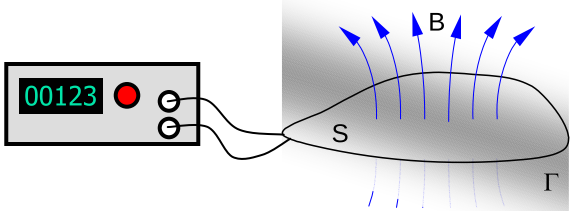


G.M. Morgan, Proc. MT-4, 787-790, **1972**

measure field gradients or higher order terms (bucking)

high resolution through compensation of background field signal

$$V = -\frac{d\varphi}{dt} \qquad \varphi = \int_S \mathbf{B}d\mathbf{S}$$



Fluxmeter zoo

- Fixed coil measurements
  - Static coils (d**S**/dt=0), the field change (d**B**/dt) induces the voltage
  - Provides only a relative measurement (B_{end}-B_{start})
  - The voltage offsets cannot be distinguished from physical signal

Most flexible method in all its many variants, although…

"…This type of magnetometer is obsolete."

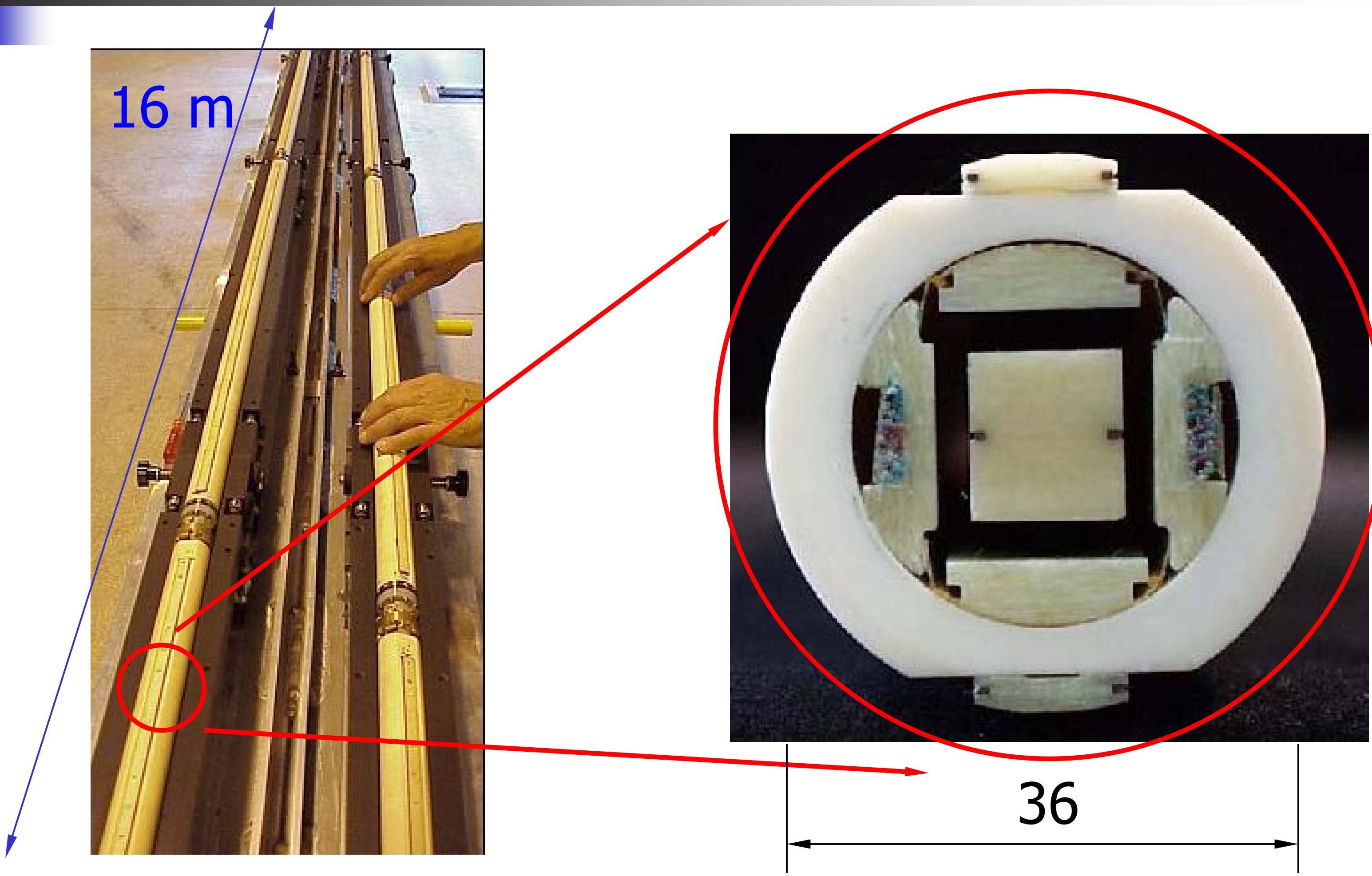
http://en.wikipedia.org/wiki/Magnetometer

- Moving coil measurements
  - Steady field (d**B**/dt=0), the coil movement (d**S**/dt) induces the voltage
  - Requires well controlled mechanics, simple movements
  - Provides and absolute measurement if:
    - initial B_{start}=0 moving from *far away* into the magnet (zero-gauss chamber): moving coil
    - using symmetries B_{end}=-B_{start}: flip coil, rotating coil
  - A voltage offsets can be distinguished from physical signal

A rotating coil ???

… no, actually this is a *fixed coil* with 800 turns and ≈ 250 m^2 surface that has been used to verify e.m. coupling of LEP and SPS

Rotating *snakes* @ CERN



0.1 μT, 0.05 mrad resolution, 100 ppm accuracy

Complex formalism

- SC magnets for accelerators
  - 2-D field (slender magnet), with components only in *x* and *y* and no component along *z*
  - Ignore *z* and define the complex plane **s** = *x* + *i y*
- Complex field function:

$$\mathbf{B} = B_y + iB_x$$

- **B** is analytic in **s** and can be expanded in Taylor series (the series converges) inside a current-free disk:

$$B_y + iB_x = \sum_{n=1}^{\infty} \mathbf{C}_n \left(\frac{\mathbf{s}}{R_{ref}} \right)^{n-1} \qquad \mathbf{C}_n = B_n + iA_n$$

$$B_y + iB_x = \sum_{n=1}^{\infty} \mathbf{C}_n \left(\frac{\mathbf{s}}{R_{ref}} \right)^{n-1}$$

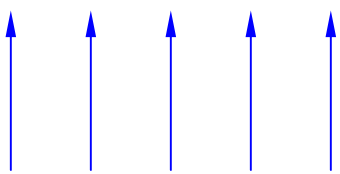
Multipoles

- complex multipole coefficients:

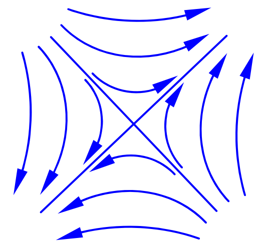
$$\mathbf{C}_n = B_n + iA_n$$

n=1 n=2 n=3

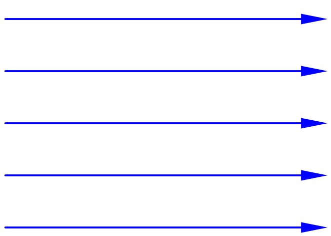
$B_1 \neq 0$, normal dipole



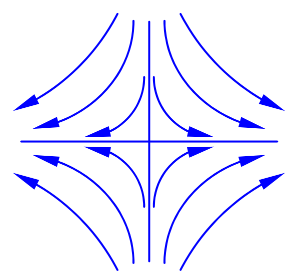
$B_2 \neq 0$, normal quadrupole



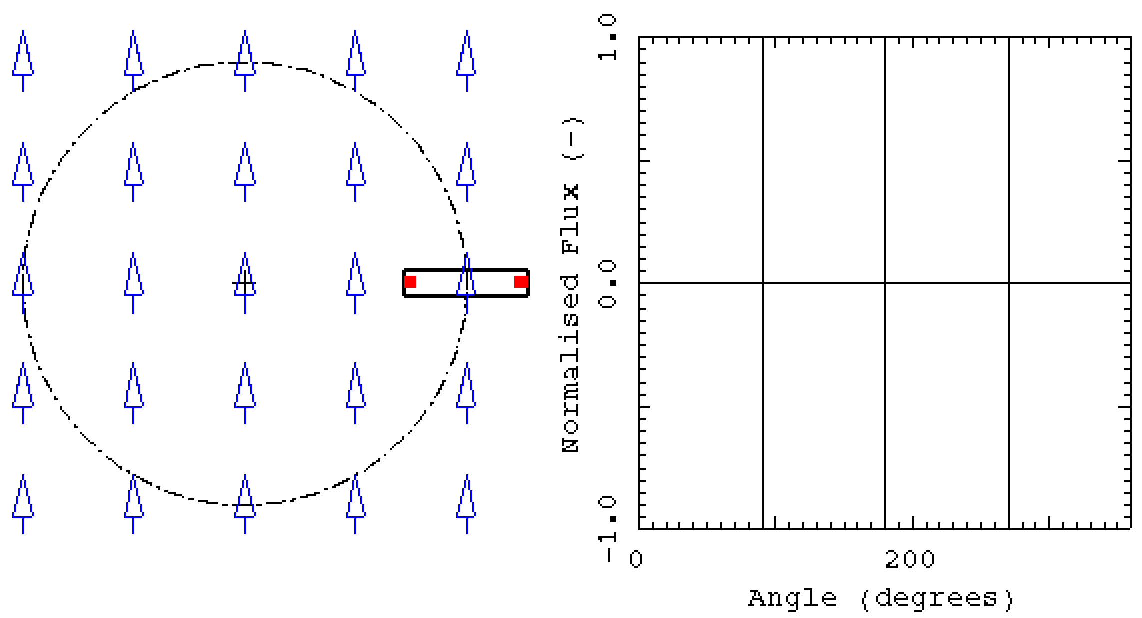
$A_1 \neq 0$, skew dipole



$A_2 \neq 0$, skew quadrupole



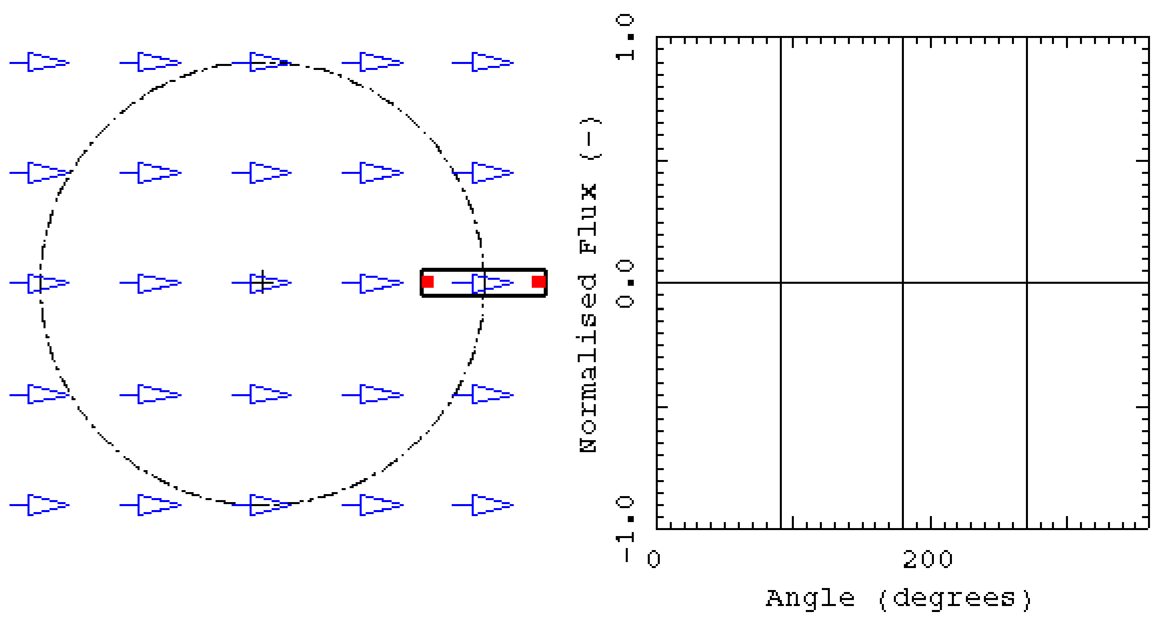
Rotating coil in normal dipole



maxima and minima located at 0 and π

$\cos(\theta)$ waveform

Rotating coil in skew dipole

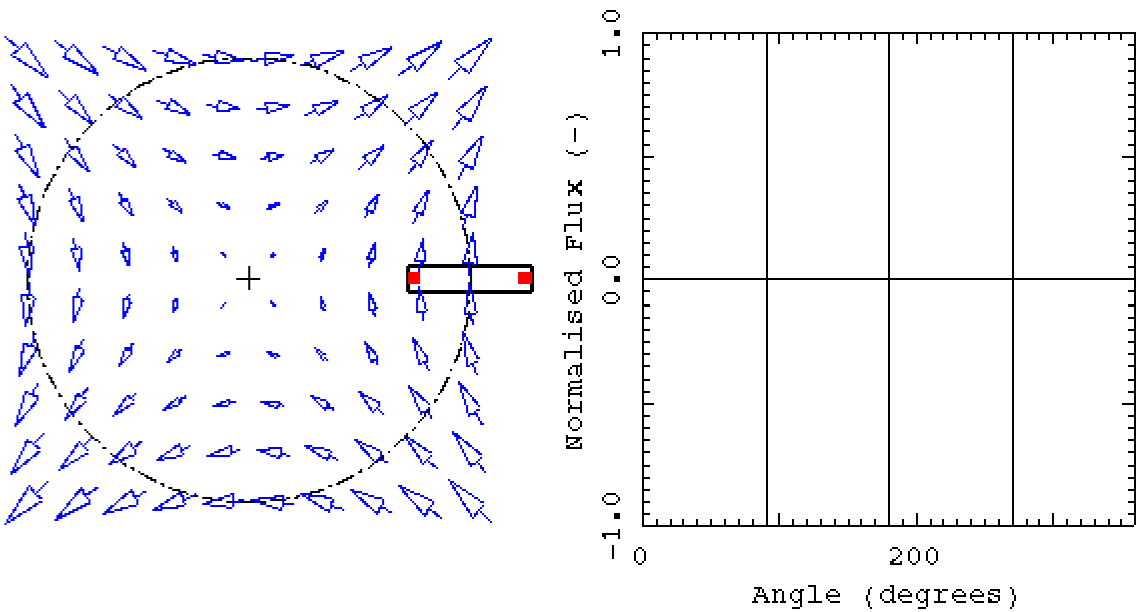


maxima and minima located at $\pi/2$ and $3/2\ \pi$

flux de-phased by $-\pi/2$ with respect to normal dipole

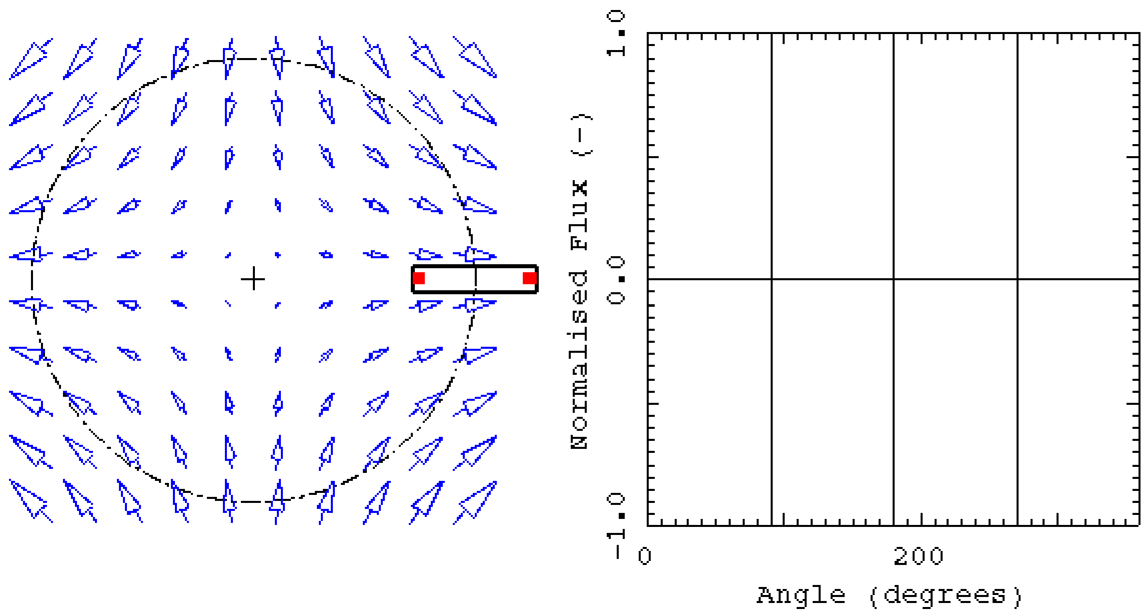
$\sin(\theta)$ waveform

Rotating in normal quadrupole



maxima and minima located at 0, $\pi/2$, π and $3/2\ \pi$
flux variation twice faster than in a dipole
cos(2 θ) waveform

Rotating in skew quadrupole

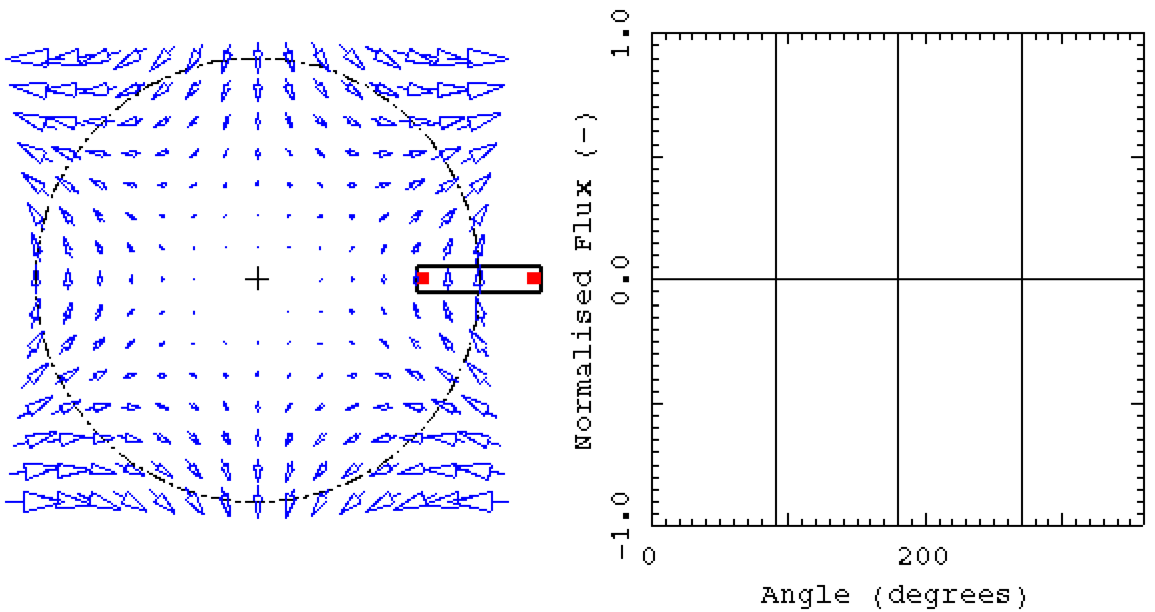


maxima and minima located at 1/4 π, 3/4 π, 5/4 π and 7/4 π

flux de-phased by –π/4 with respect to normal quadrupole

sin(2 θ) waveform

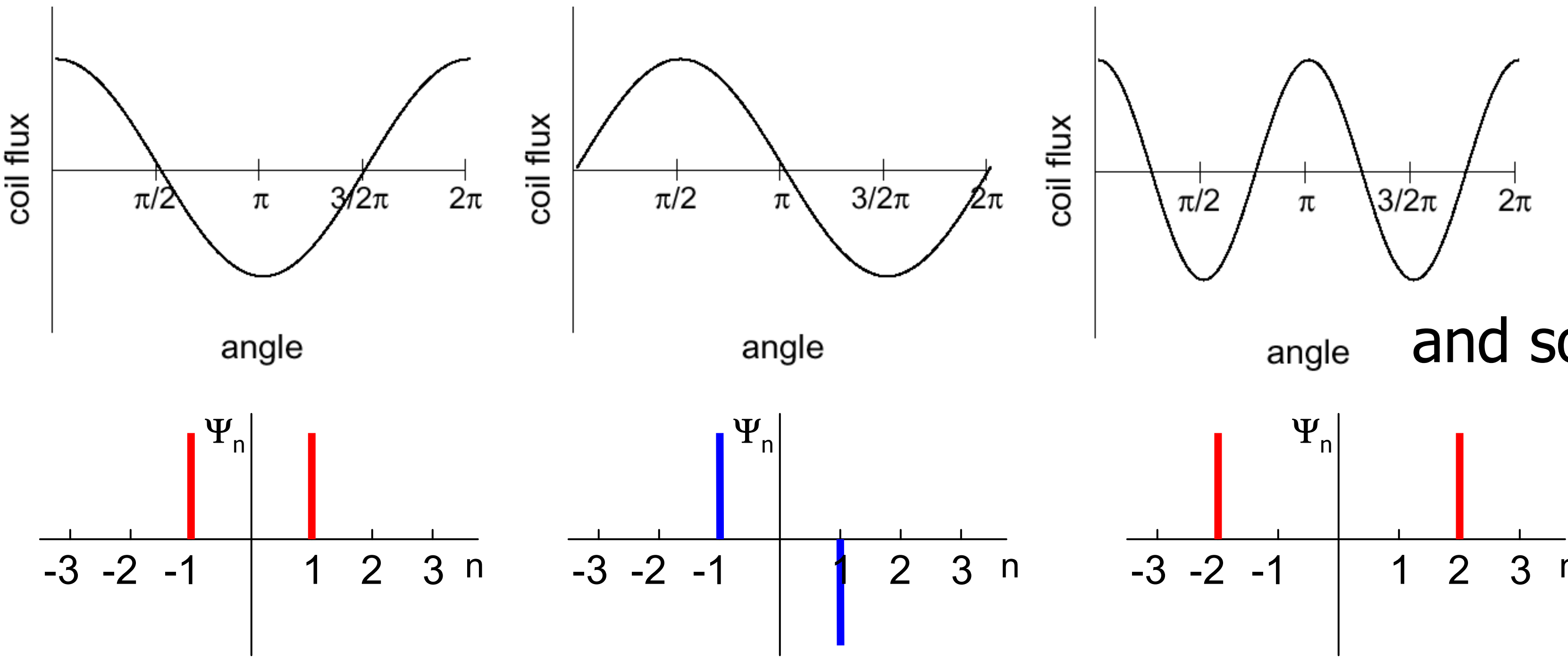
Rotating in normal sextupole



cos(3 θ) waveform …

Fourier analysis

n=1, $B_1 \neq 0$ | n=1, $A_1 \neq 0$ | n=2, $B_2 \neq 0$



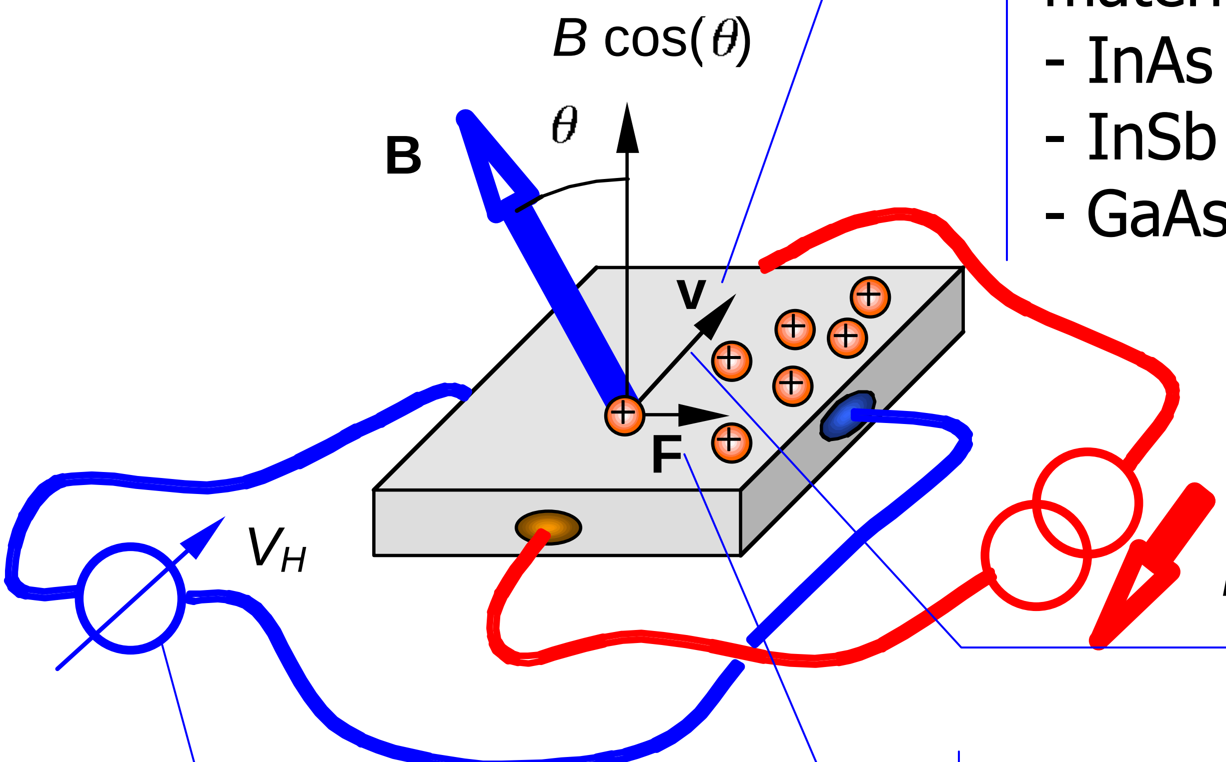
and so on...

...by induction: $$\mathbf{C}_n = \frac{\tilde{\varphi}_n}{\kappa_n}$$

Fourier transformed flux

coil calibration

Hall generator principle



thin slab of semiconducting material ($A^{III}B^{V}$):
- InAs
- InSb
- GaAs

DC current

Lorentz force F = v x B

transverse DC (Hall) voltage

Amer. J. Math., 2, 287, **1879**

On a New Action of the Magnet on Electric Currents.

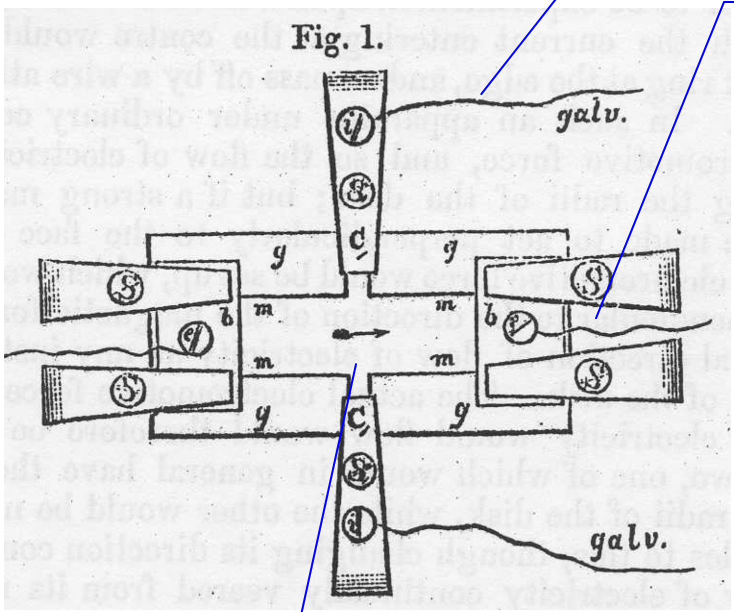
By E. H. Hall, *Fellow of the Johns Hopkins University.*

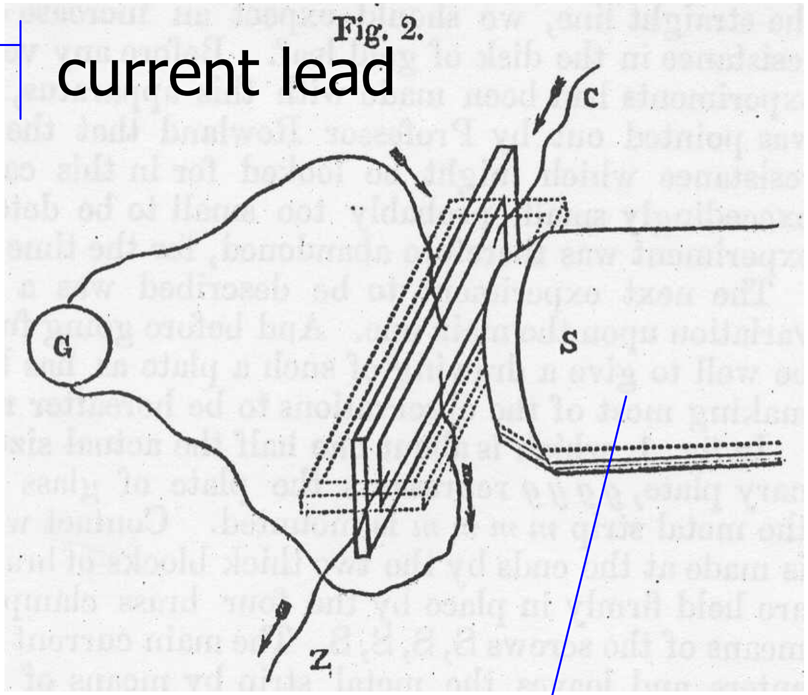
Sometime during the last University year, while I was reading Maxwell's Electricity and Magnetism in connection with Professor Rowland's

Sometime during the last University year…

voltage tap

current lead



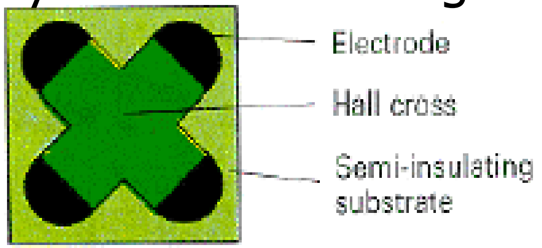


iron foil

magnet

Hall coefficient

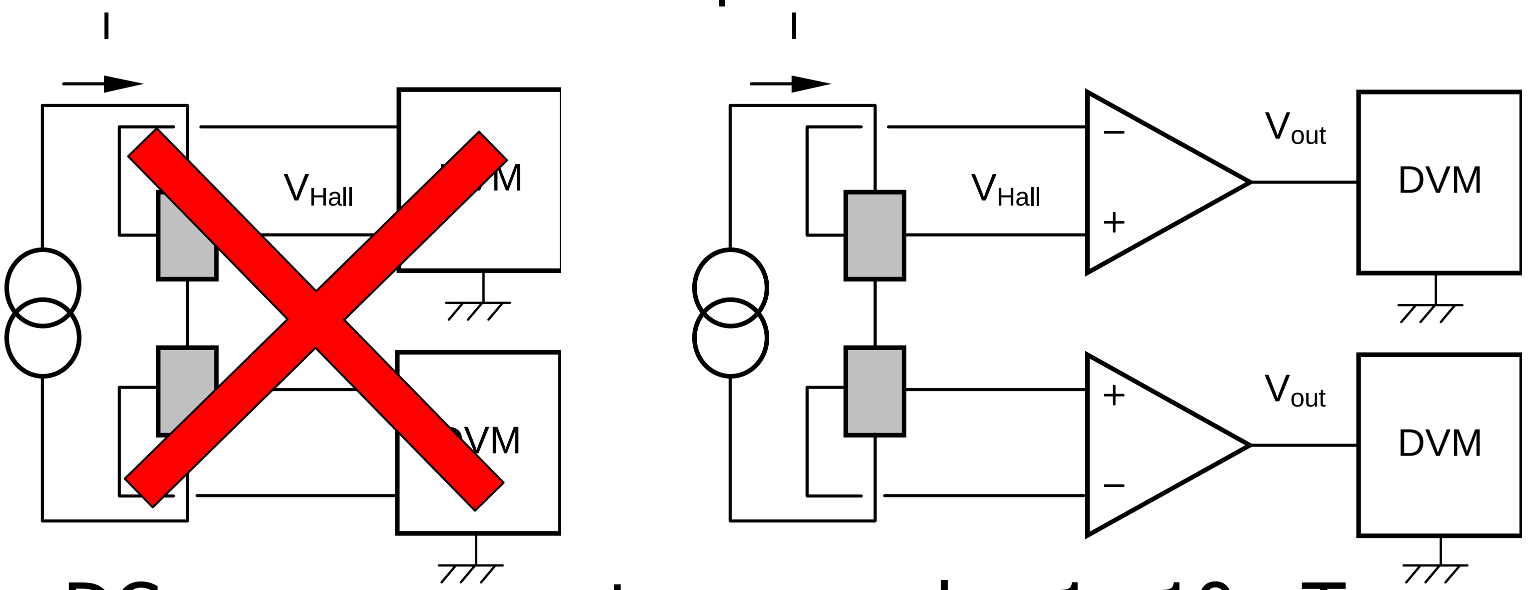
- Hall voltage: $V_H = G R_H I B \cos(\theta)$
  - R_H(B): material dependent Hall coefficient
    - high mobility, low conductivity to have high R_H
      - ~~metals~~ (low mobility)
      - ~~alloys~~ (high resistivity causes heating)
      - compromise choice: semiconductors
    - temperature dependence 100 to 1000 ppm/ºC
  - G (B): geometry factor
    - equipotential lines deform under v x B
  - Optimal design to compensate R_H vs G
    - Cruciform design achieves 1 % linearity over wide B range
    - better definition of magnetic center



100 ppm accuracy feasible

Hall magnetometer

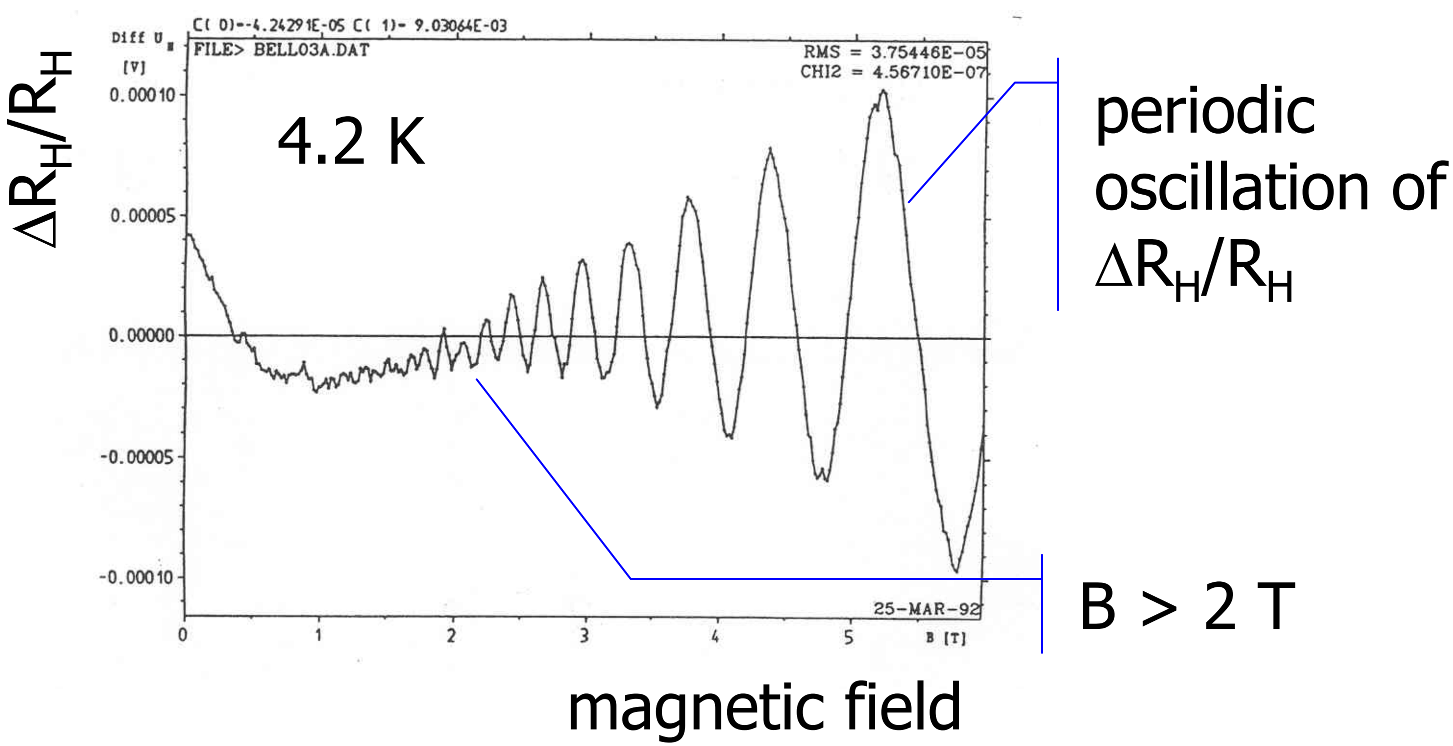
- a Hall sensor is a 4-terminals device
  - do NOT connect outputs in series !!!



- DC measurement can resolve 1…10 μT
- AC (lock-in) can resolve 0.1 μT
  - $I = I_0 \sin(2\pi ft)$, f = 10 Hz … 1 kHz

Quantum Hall effect

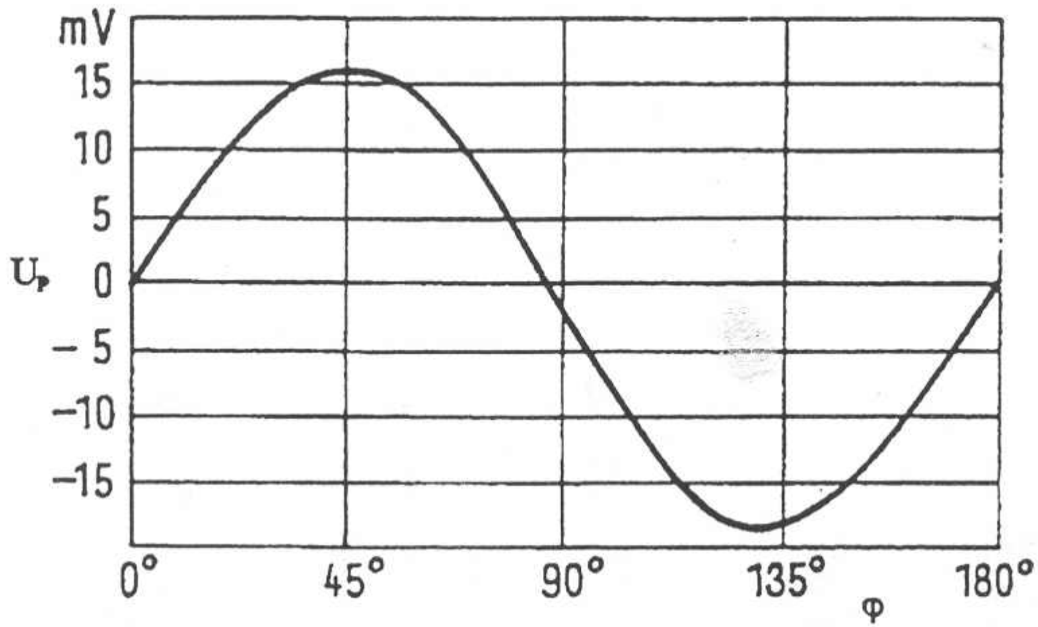
- Shubnikov-de Haas effect
  - oscillation in R_H periodic function of 1/B

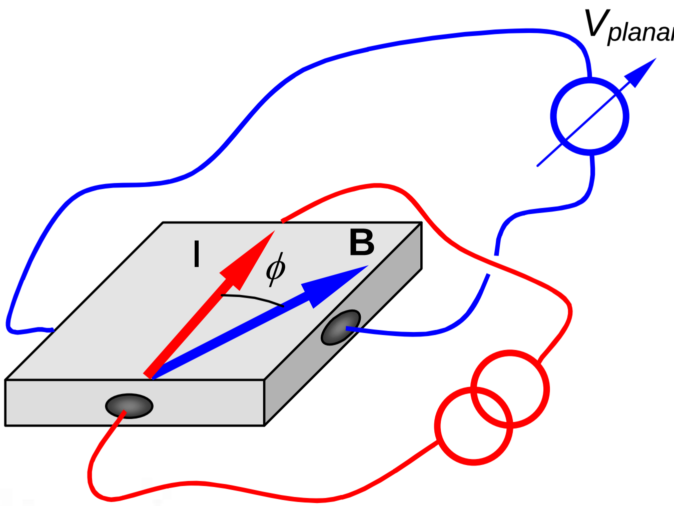


effect as much as 1 % on calibration coefficient

Planar Hall effect

$$V_{planar} = V_{HP} B^2 \cos(2\varphi)$$



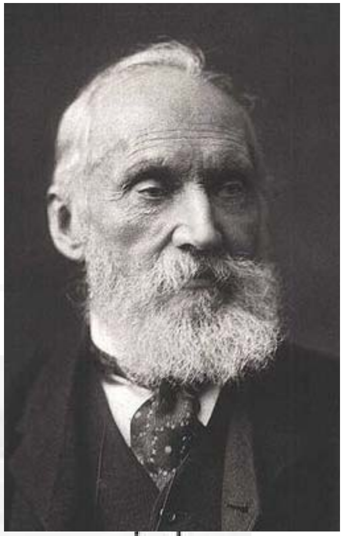


V_{planar} is important when mapping 3-D fields

Phil. Trans.,**146**, 736, **1856**

XXXI. The Bakerian Lecture.—*On the Electro-dynamic Qualities of Metals**.
By Professor William Thomson, *M.A., F.R.S.*

Received February 28,—Read February 28, 1856.



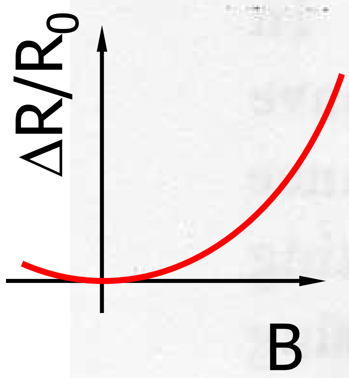
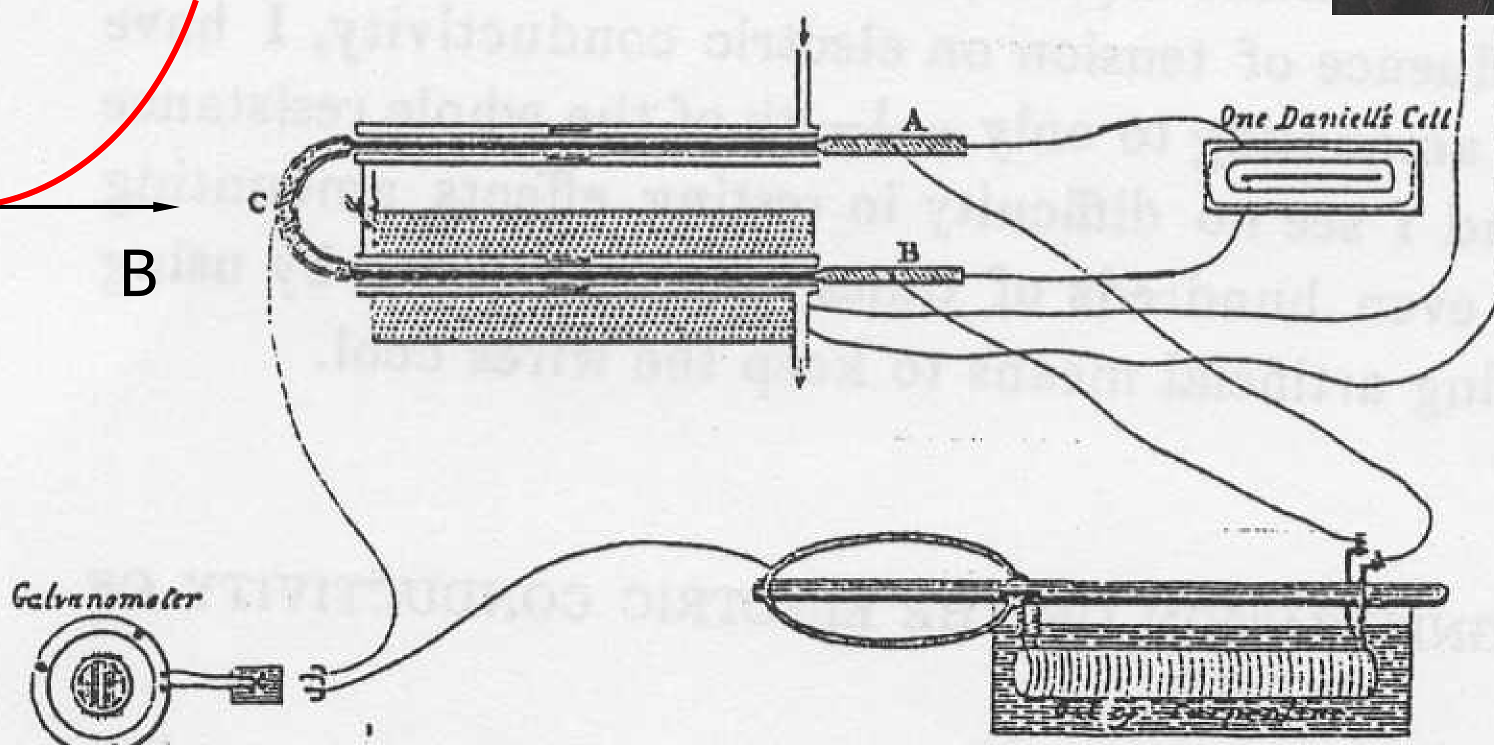
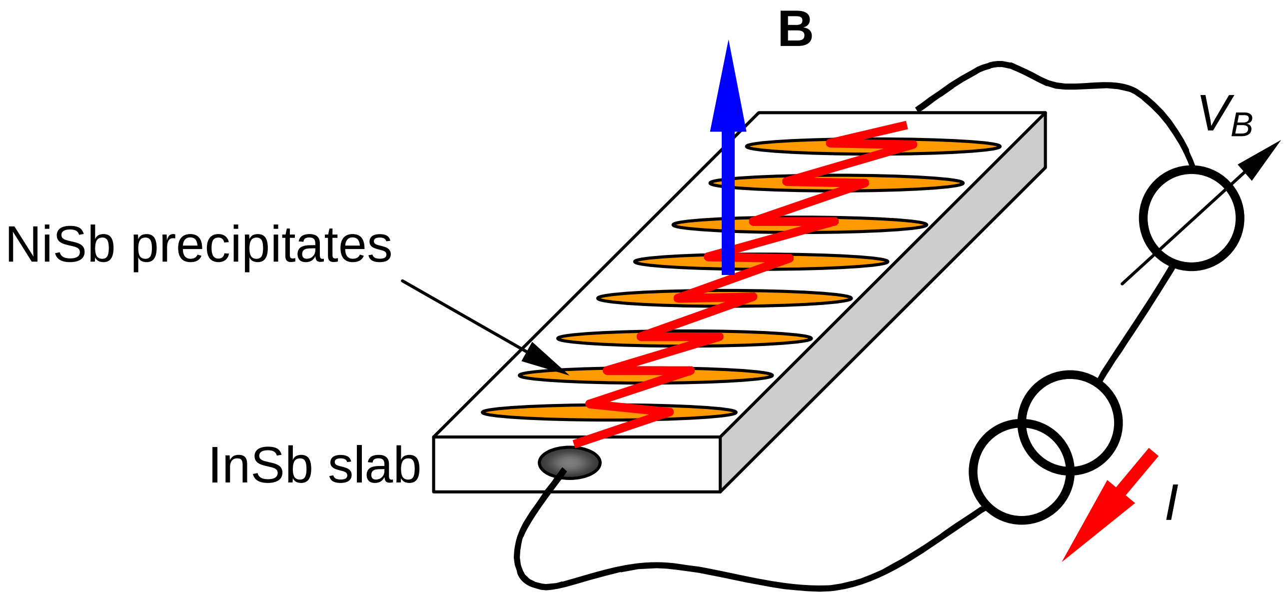


Fig. 41.



Magnetoresistors



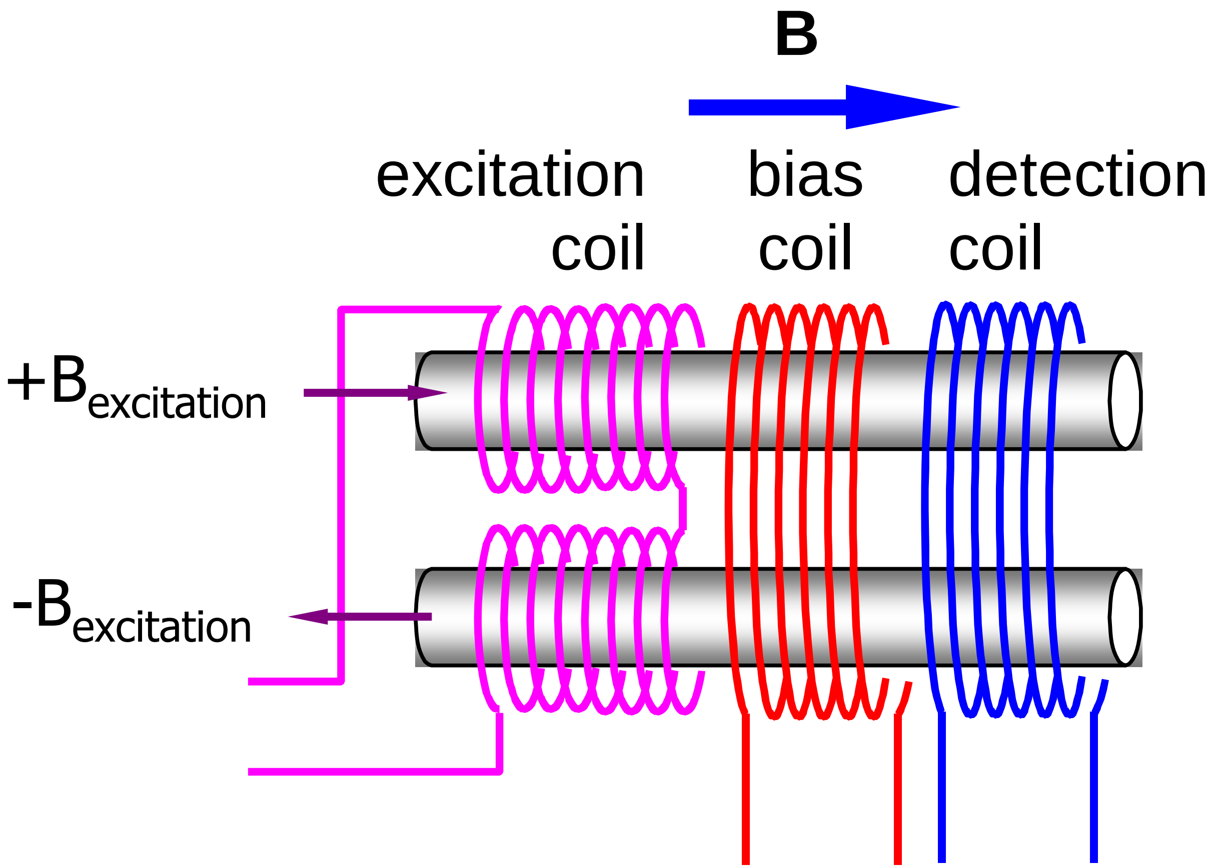
two-terminal device, simple, inexpensive

modest sensitivity, non-linear, T effects (2500 ppm/ºC)

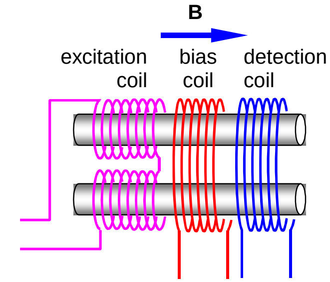
bias field, compensated bridges, giant-magnetoresistance

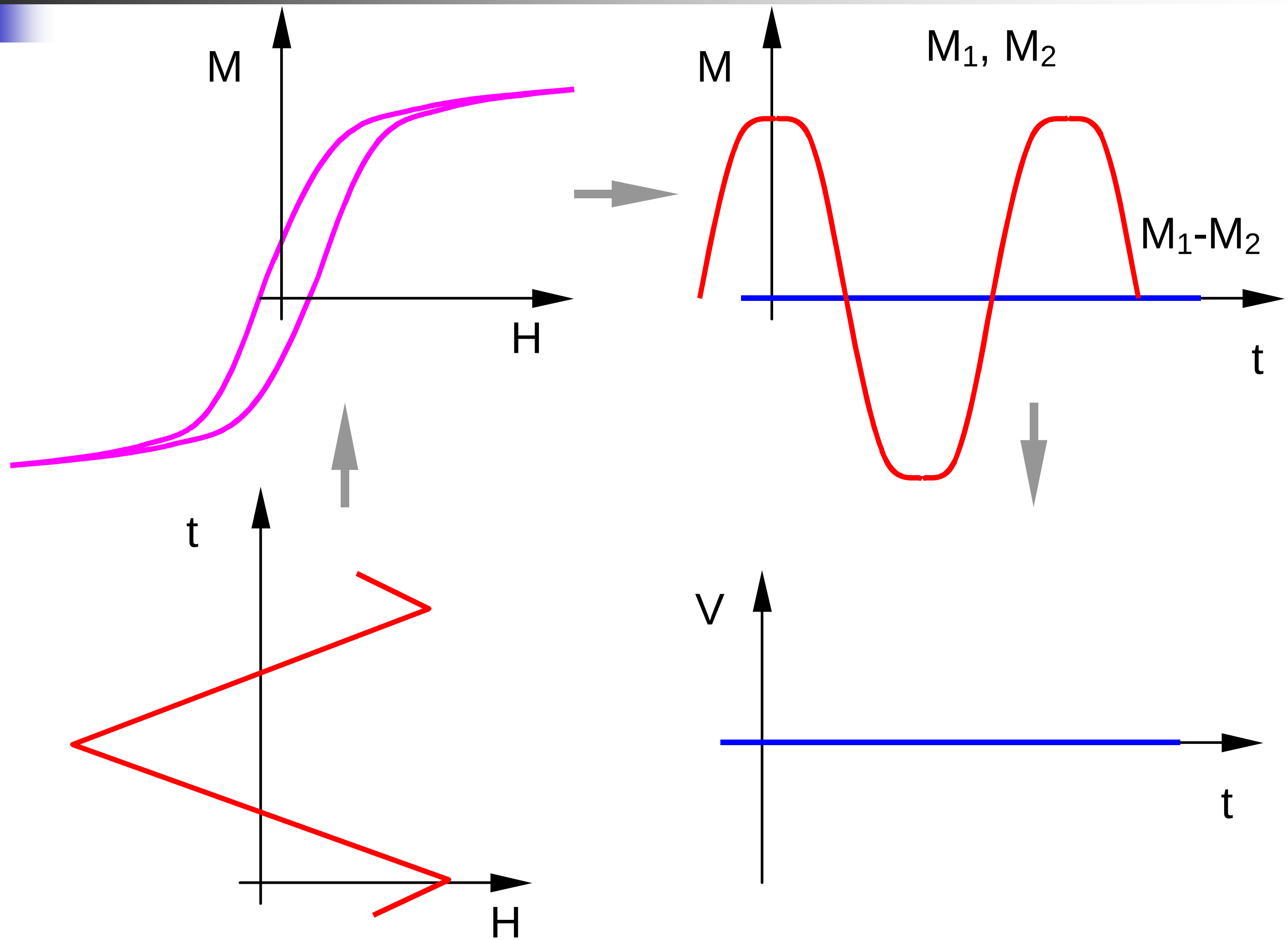
J. M. Kelly, *Magnetic Field Measurements with Peaking Strips*, Rev. Sci. Inst., **22**, 256, **1930**

Fluxgate (*Peaking Strip*)

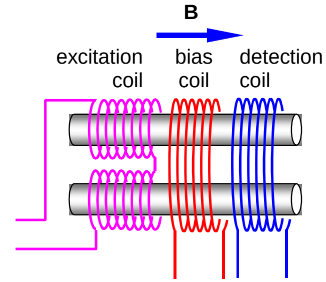


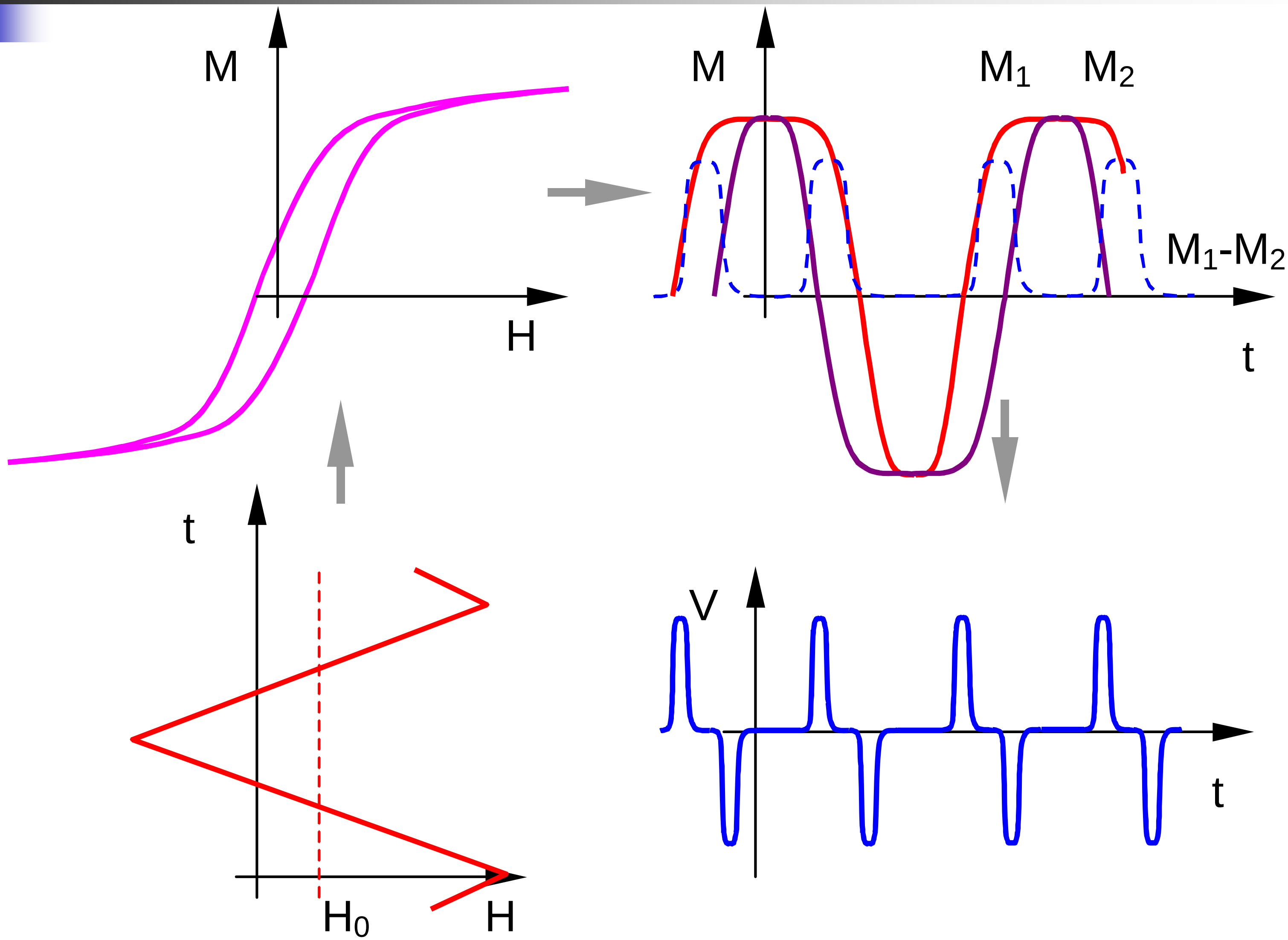
Fluxgate principle - 1





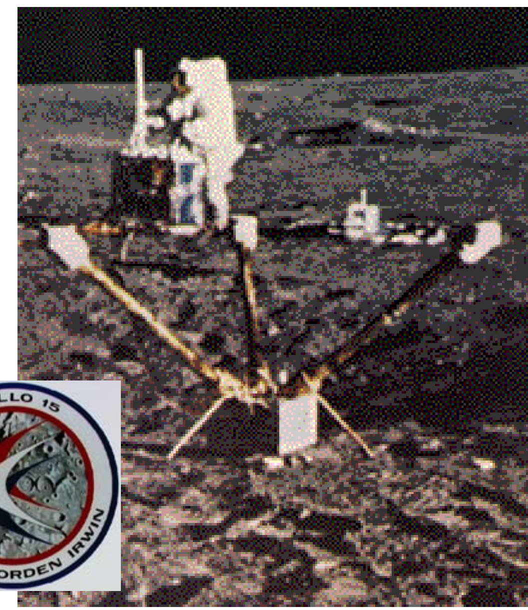
Fluxgate principle - 2



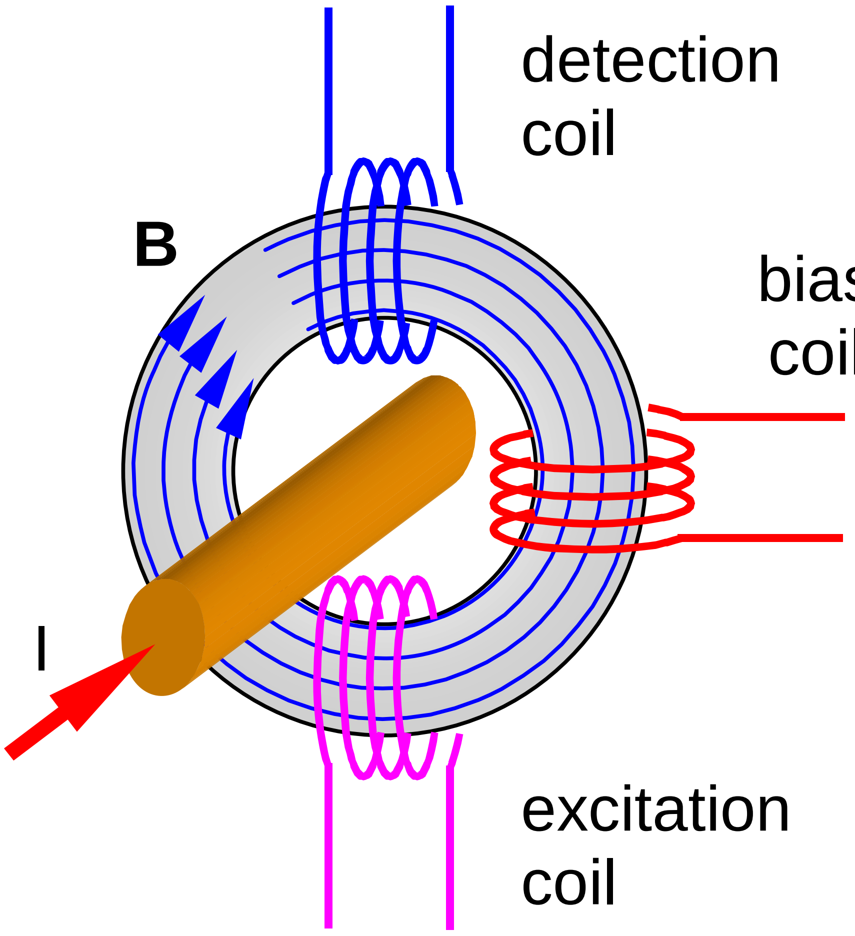


Fluxgate applications

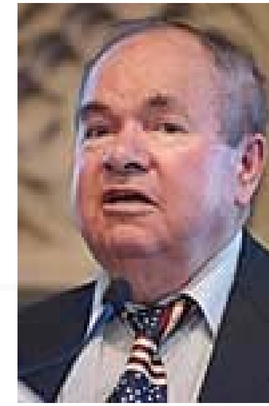
- simple and inexpensive, lightweight device
- highly directional
- high sensitivity (tens of pT !)
- modest accuracy (1000 ppm)
- typical applications
  - navigation
  - geology, ores, oil fields
  - hunting submarines
  - finding mines
  - mapping of interplanetary magnetic field



A special fluxgate: the DCCT

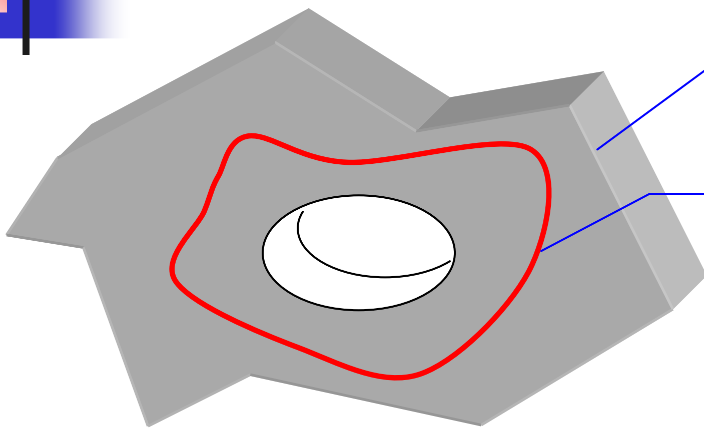


best known device for high current (relevant for SC magnets)
1 ppm possible at 10 kA (10 mA)



A. Abrikosov

SQUID principles – 1



- change δ of phase of wave-function for paired electrons along Γ depends on magnetic flux φ:

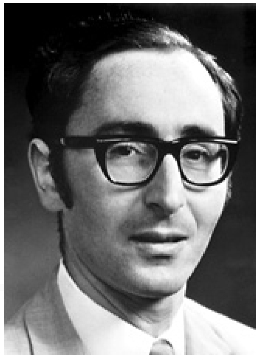
$$\delta = 2\pi \frac{\varphi}{\varphi_0}$$

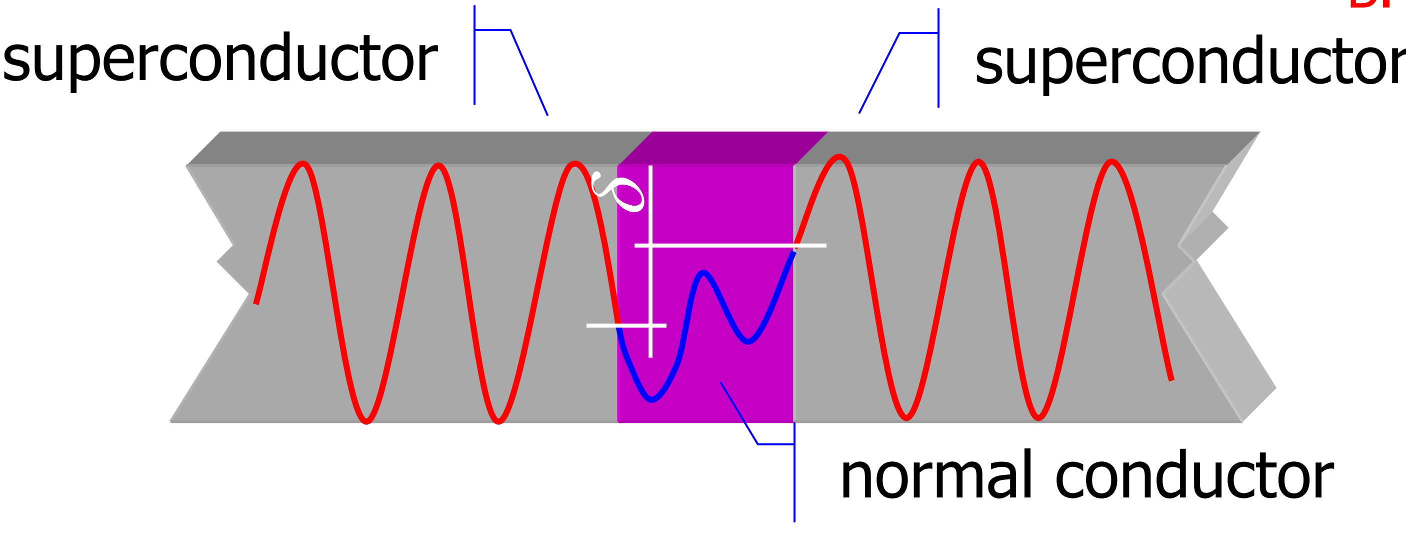
quantum fluxoid (2 x 10^{-15} Wb)

- flux quantization: $\delta = 2\pi n$

$$\varphi = n\varphi_0$$

SQUID principles – 2


B. Josephson



- the wave-function can tunnel through a normal-conducting (Josephson) junction
- the maximum supercurrent depends on δ:

$$I = I_c \sin(\delta)$$

The SQUID

- maximum supercurrent:

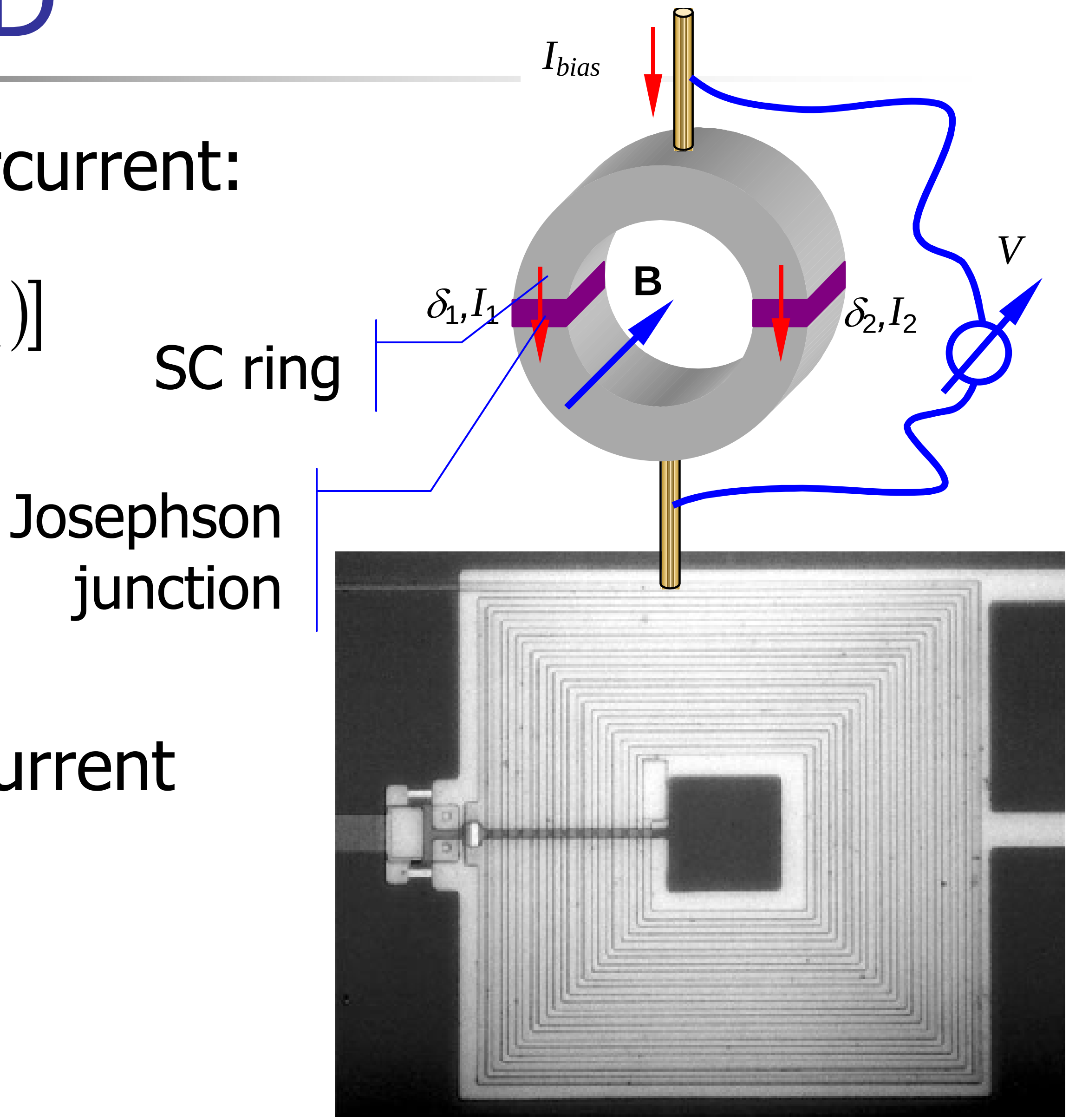
$$I = I_c\left[\sin(\delta_1) + \sin(\delta_2)\right]$$

- phase relation:

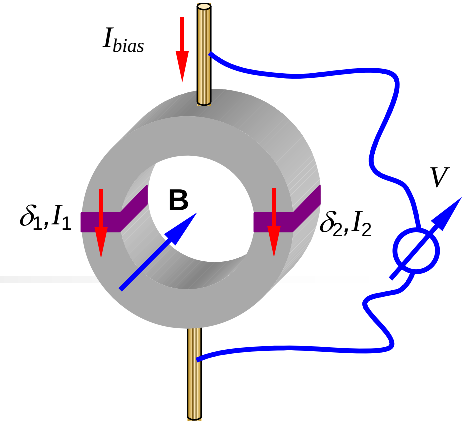
$$\delta + \delta_1 - \delta_2 = 2\pi\frac{\varphi}{\varphi_0}$$

- SQUID critical current

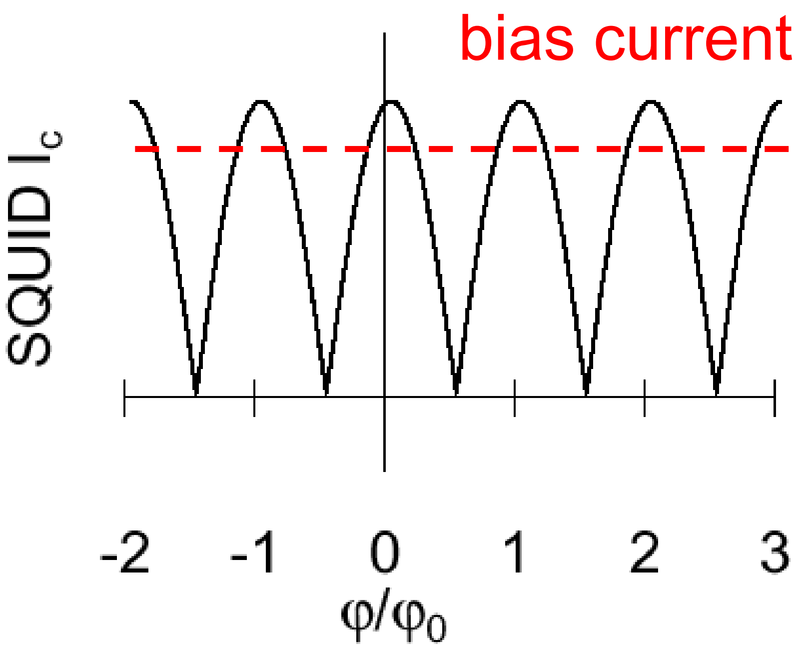
$$I = 2I_c\left|\cos\left(\pi\frac{\varphi}{\varphi_0}\right)\right|$$

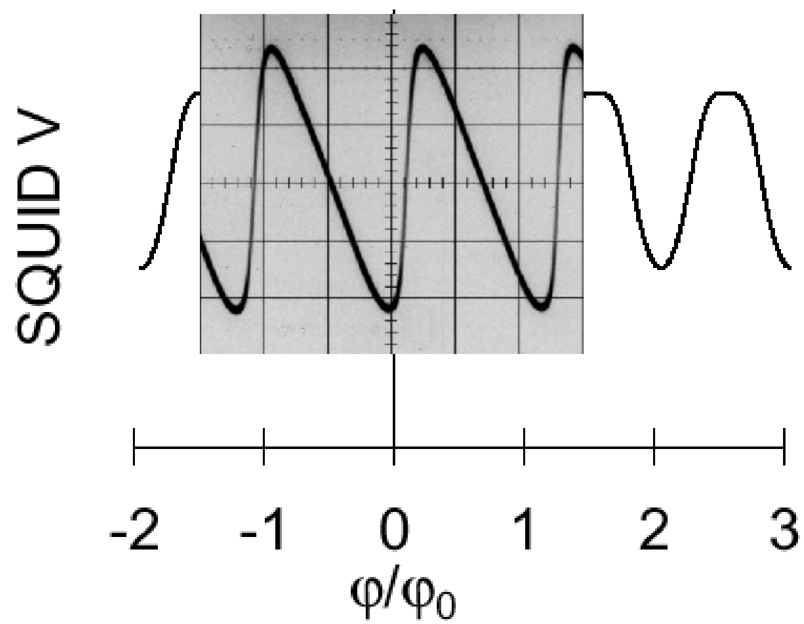


SQUID operation



- SQUID voltage periodic in φ_0
- flux change $\Delta\varphi$ from ΔV, compute ΔB using κ

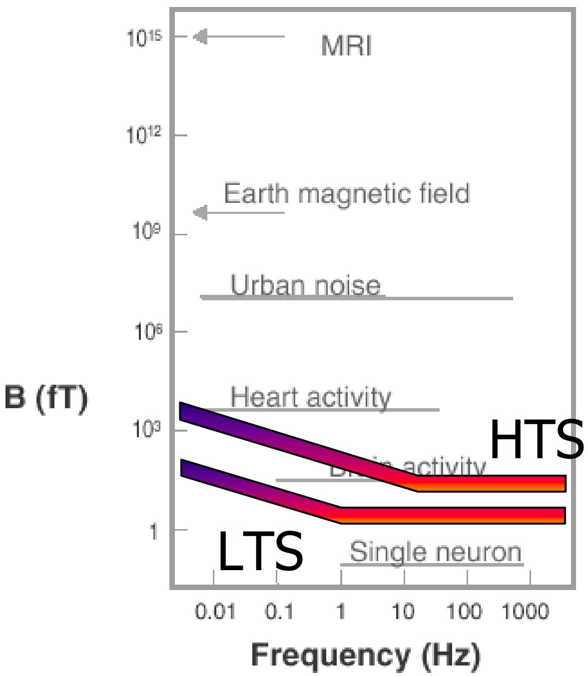




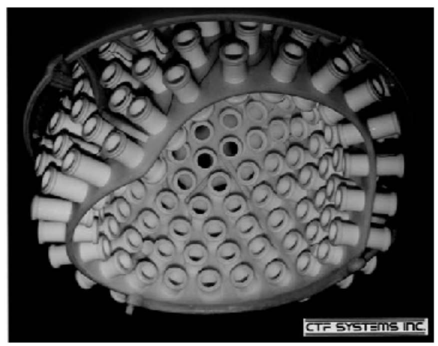
sensitivity few pT for bare SQUID, 1 fT with input transformer

range limited by FB, accuracy limited by calibration of surface

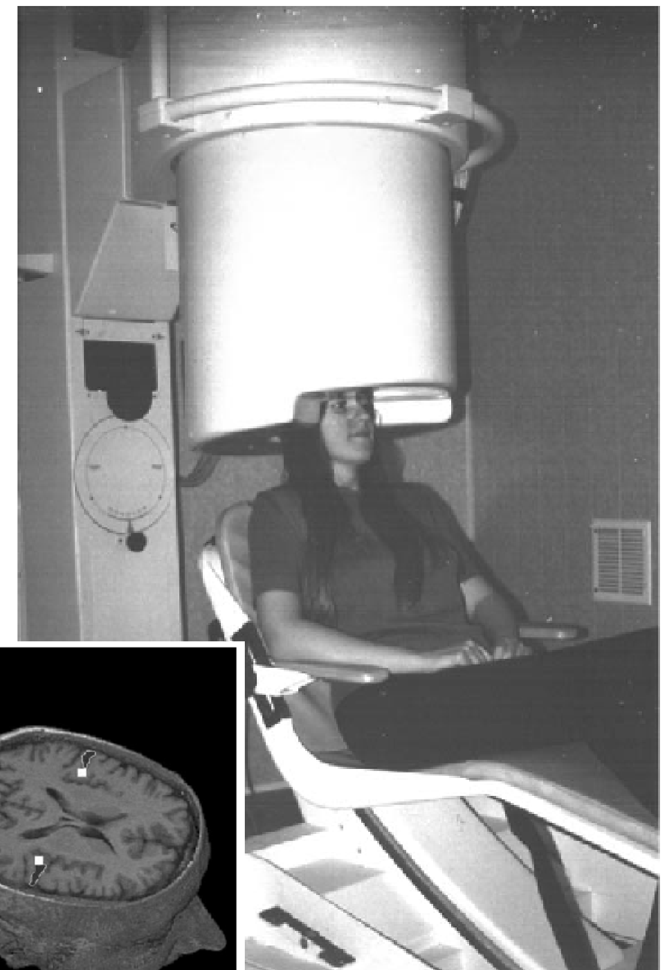
SQUID Magnetometry

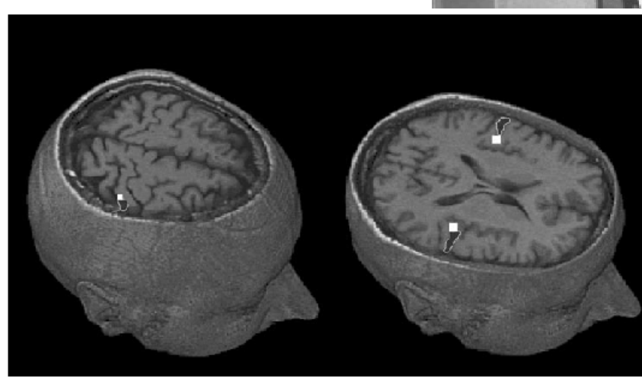


151 SQUID channels helmet



magnetoencefalogram

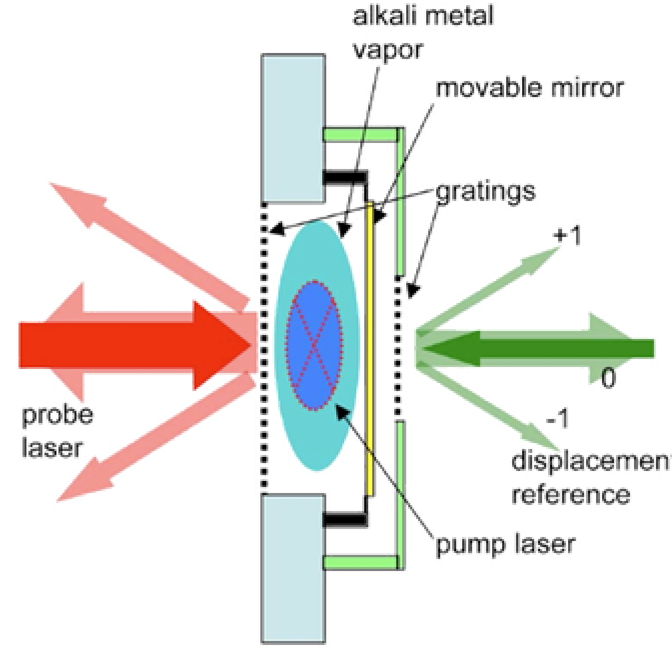


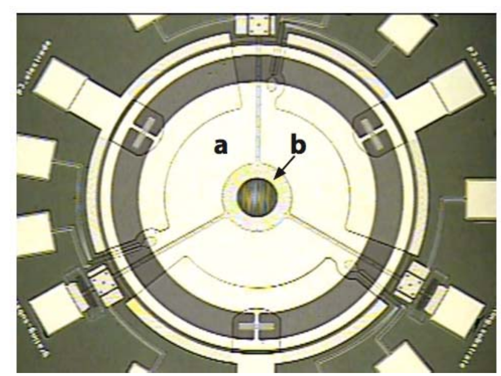


cortical activation of the primary and secondary somatosensory cortex during left median nerve simulation measured by MEG and fMRI…

The frontier of magnetometry

- Alkali atoms (Rb, Cs) have unpaired electrons whose spin precesses with magnetic field (as protons)
- The interaction with field can be detected as follows:
  - An alkali metal vapor is prepared in a cell
  - A first laser (the pump) aligns the spins to create coherence (as the RF pulse in NMR techniques)
  - A second laser (the probe) detects resonance, which can be seen, e.g., as a shift in an interference pattern
- Excellent device for miniaturization

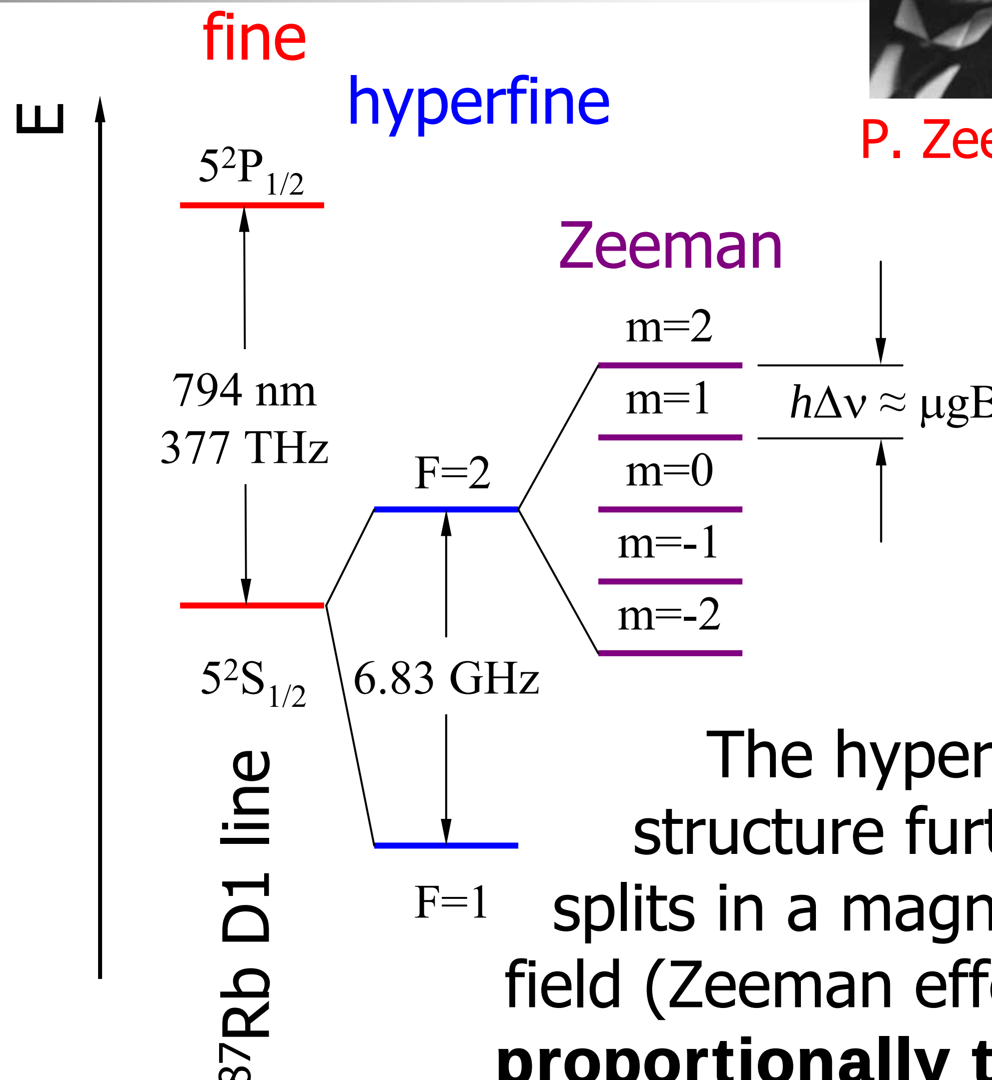


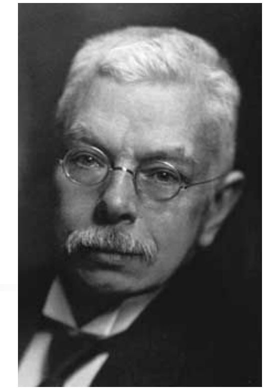


Sandia National Laboratories

A digression on spectra

- The *gross* structure of the spectral line of atoms (energy level), are split in:
  - A *fine* structure (due to interaction of magnetic moment of electron spin and orbital angular momentum)
    - A *hyperfine* structure, due to interaction of nucleus magnetic moment with internal magnetic/electric fields in the atom





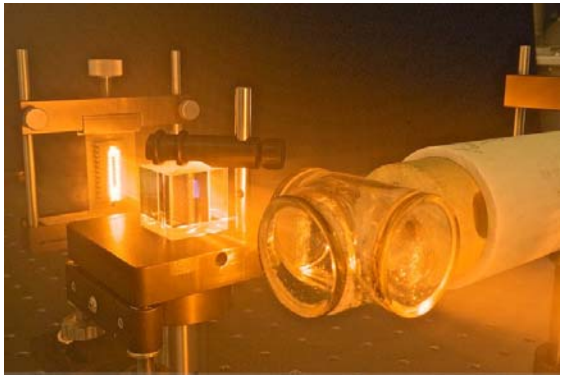
P. Zeeman

The hyperfine structure further splits in a magnetic field (Zeeman effect) **proportionally to B**

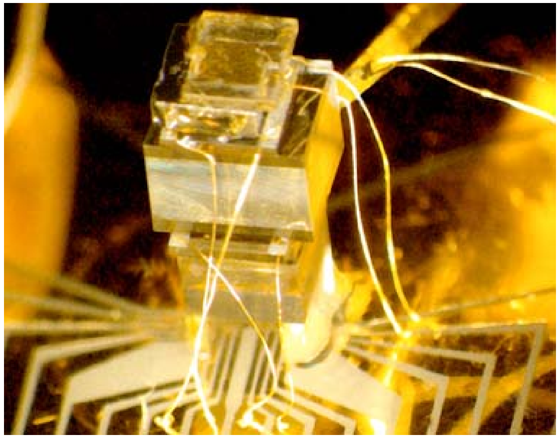
Atomic magnetometer DIY

- Set a laser on a given absorption line (e.g. D1 of ^{87}Rb, 794 nm)
- AM the laser with a VCO to create two sidebands corresponding to the hyperfine structure (central f = 3.42 GHz)
- Modulate the VCO (by ± 3 MHz) to detect the resonances from the split hyperfine levels
- Compute the difference of frequency between two resonances, which is proportional to the magnetic field:

$$h\Delta\nu \approx \mu g B$$



Princeton University



NIST

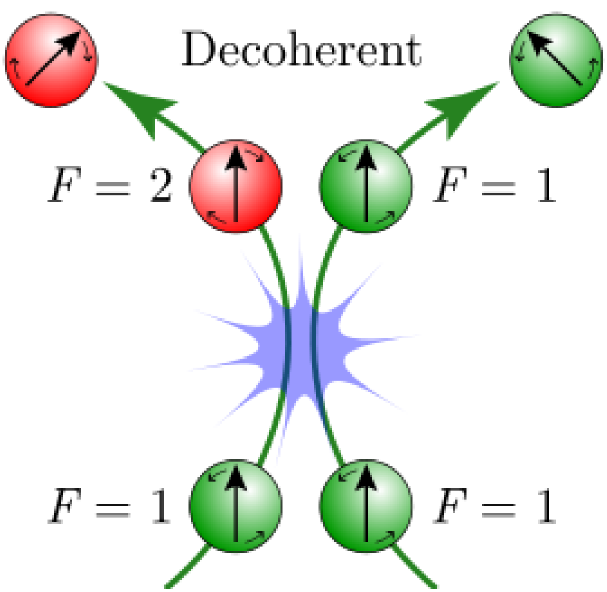
sensitivity in the range of tens of fT

range limited to 1 mT at most, accuracy not established so far

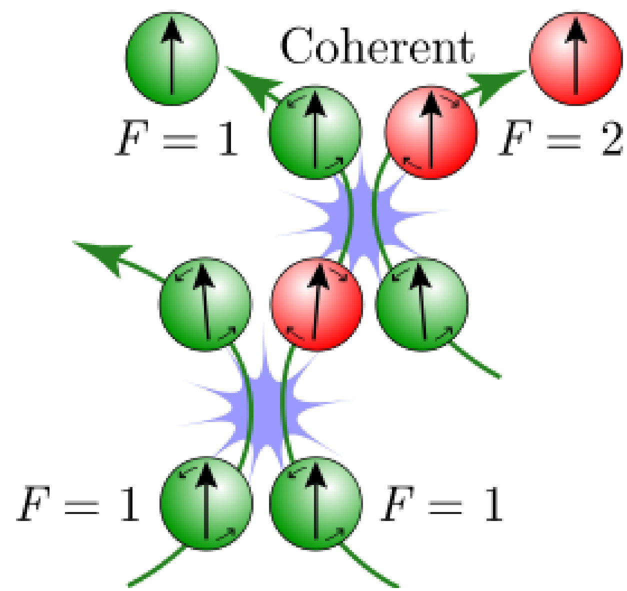
J.C. Allred, R.N Lyman, T.W. Kornack, M.V. Romalis, *High-Sensitivity Atomic Magnetometer Unaffected by Spin-Exchange Relaxation*, Phys Rev Lett **89**, 130801, **2002**

SERF atomic magnetometers

- Spin Exchange collisions preserve total momentum but cause single spins to change, scrambling the hyperfine structure
- The precession of atoms in the vapor loses quickly coherence, the resolution of the resonance is limited

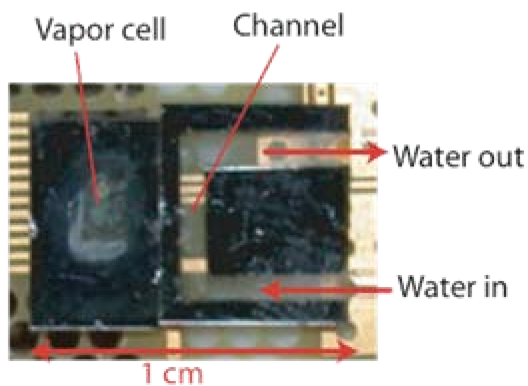


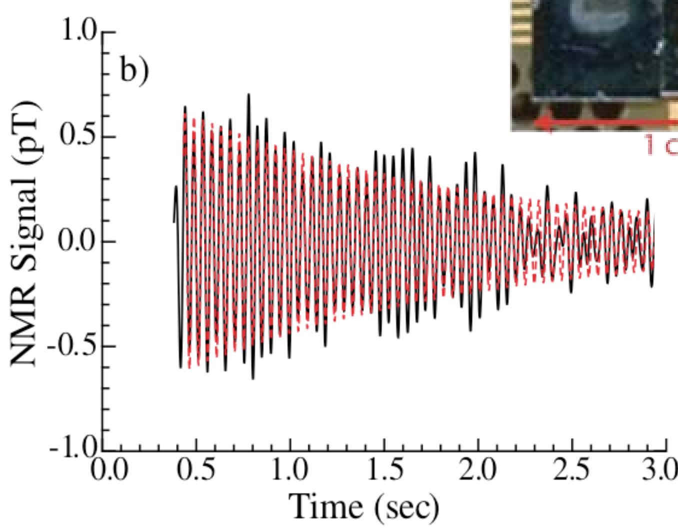
- At low B and high gas density, collisions happen quicker than the precession time of the atoms. On average the hyperfine states are stable, the **S**pin **E**xchange is **R**elaxation **F**ree
- The resonance is measured with improved resolution !



Premium sensitivity !

- Best quoted sensitivity is 200 aT/√Hz !
  - 0.2 x 10^{-15} T at 1 Hz



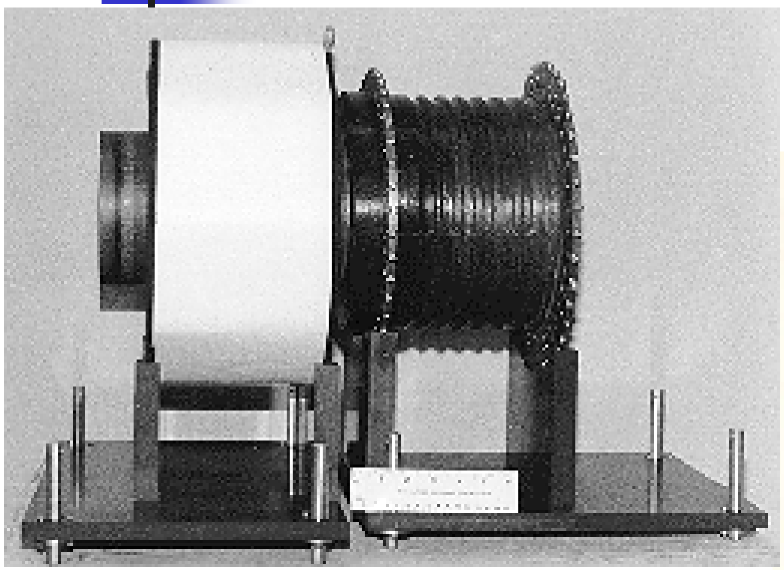


Example: NMR signal from water on chip

- A new world of possibilities
  - Magnetoencelography,
  - Magnetocardiography of in fetal hearts,
  - Earth field NMR …

a Tricorder ?!?

850 T pulsed field at LANL

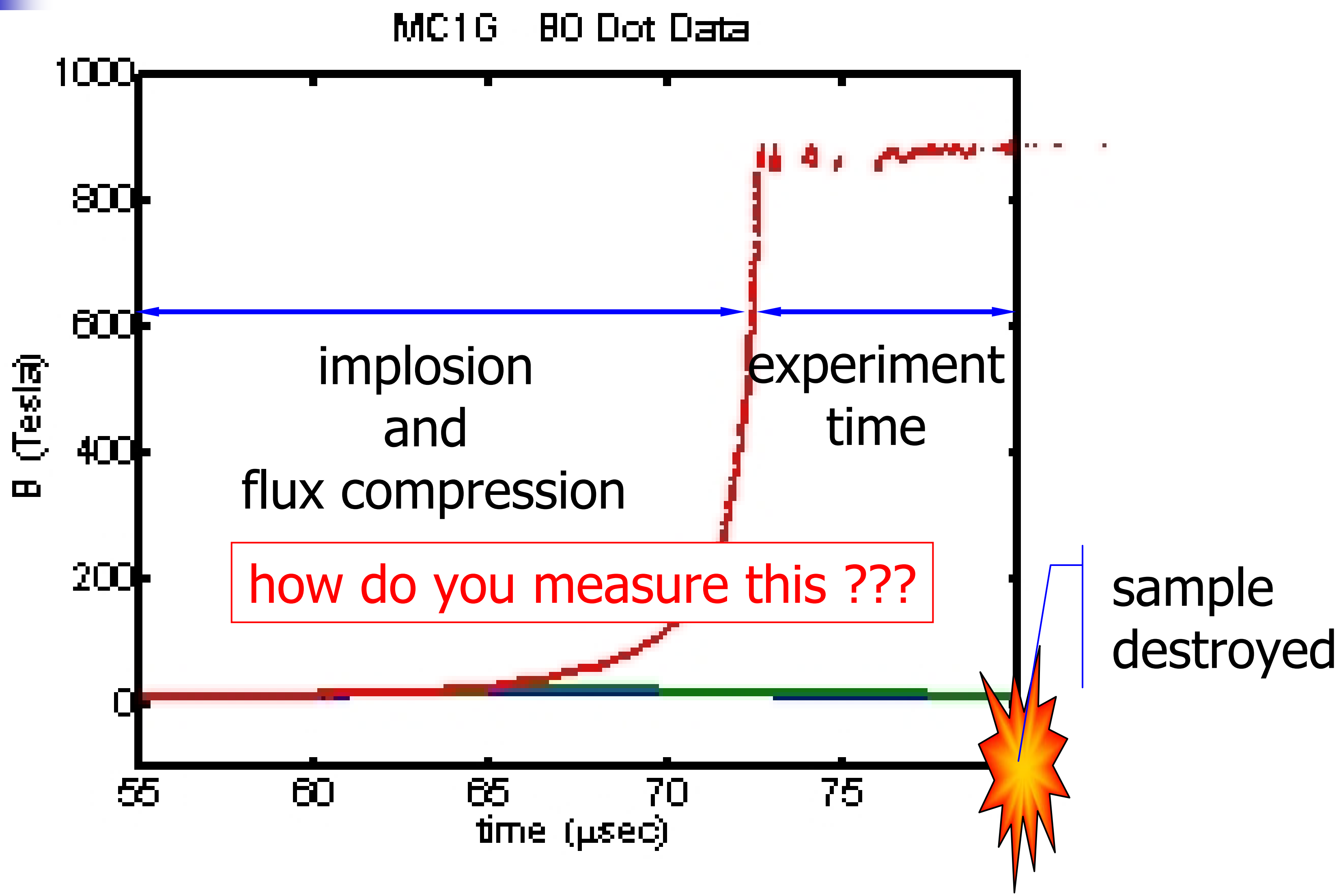


imploding flux compression generator (VNIIEF design)

material properties at high magnetic fields (8 to 10 MG)

a shot …

850 T shot at LANL

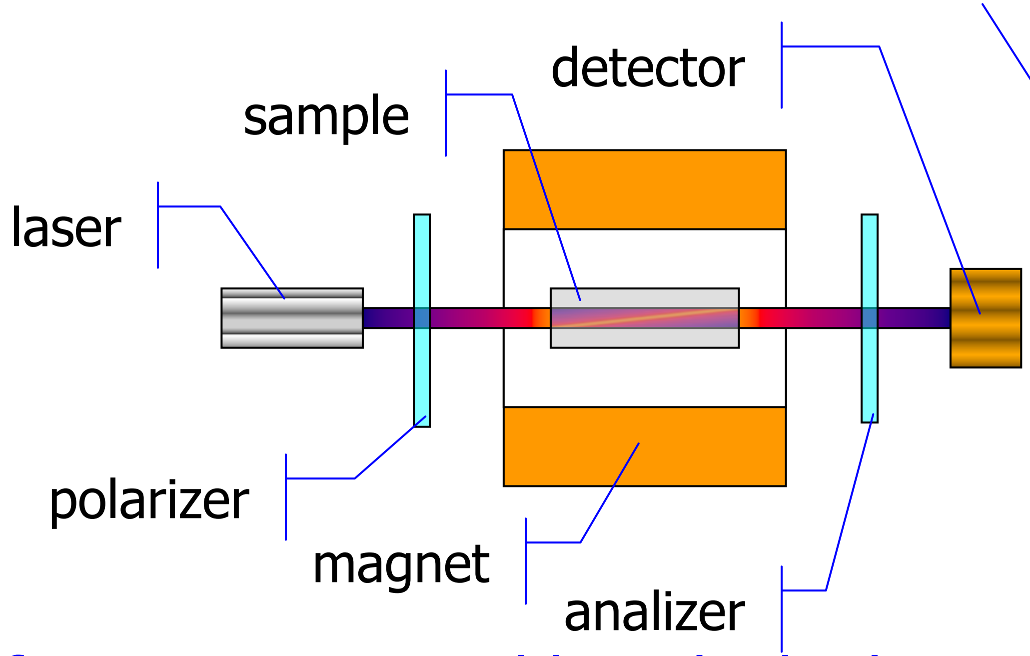


Faraday effect

- rotation of light polarization in a media in a magnetic field

$$\theta = v\, B\, L$$

rotation angle: θ; *Verdet* constant: v; field strength: B; length: L



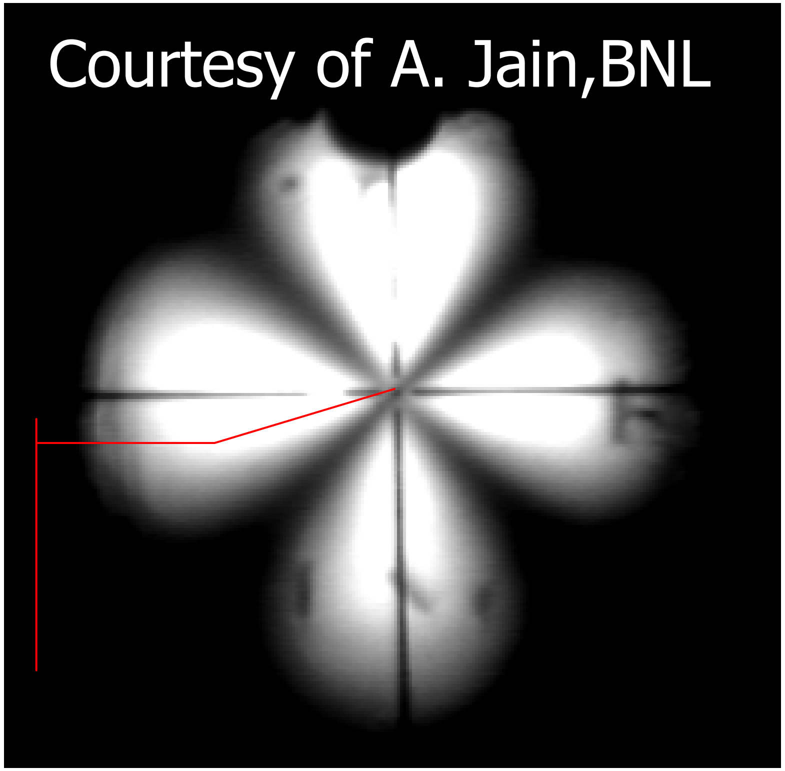
material	v (rad / T m)
fused quartz	4
flint glass	110
benzene	9

fast, e.m. compatible at high electric fields

modest accuracy (1 %)

Field imaging

- light polarization can be used to image the field:
  - Faraday rotation
  - ferrofluid cell

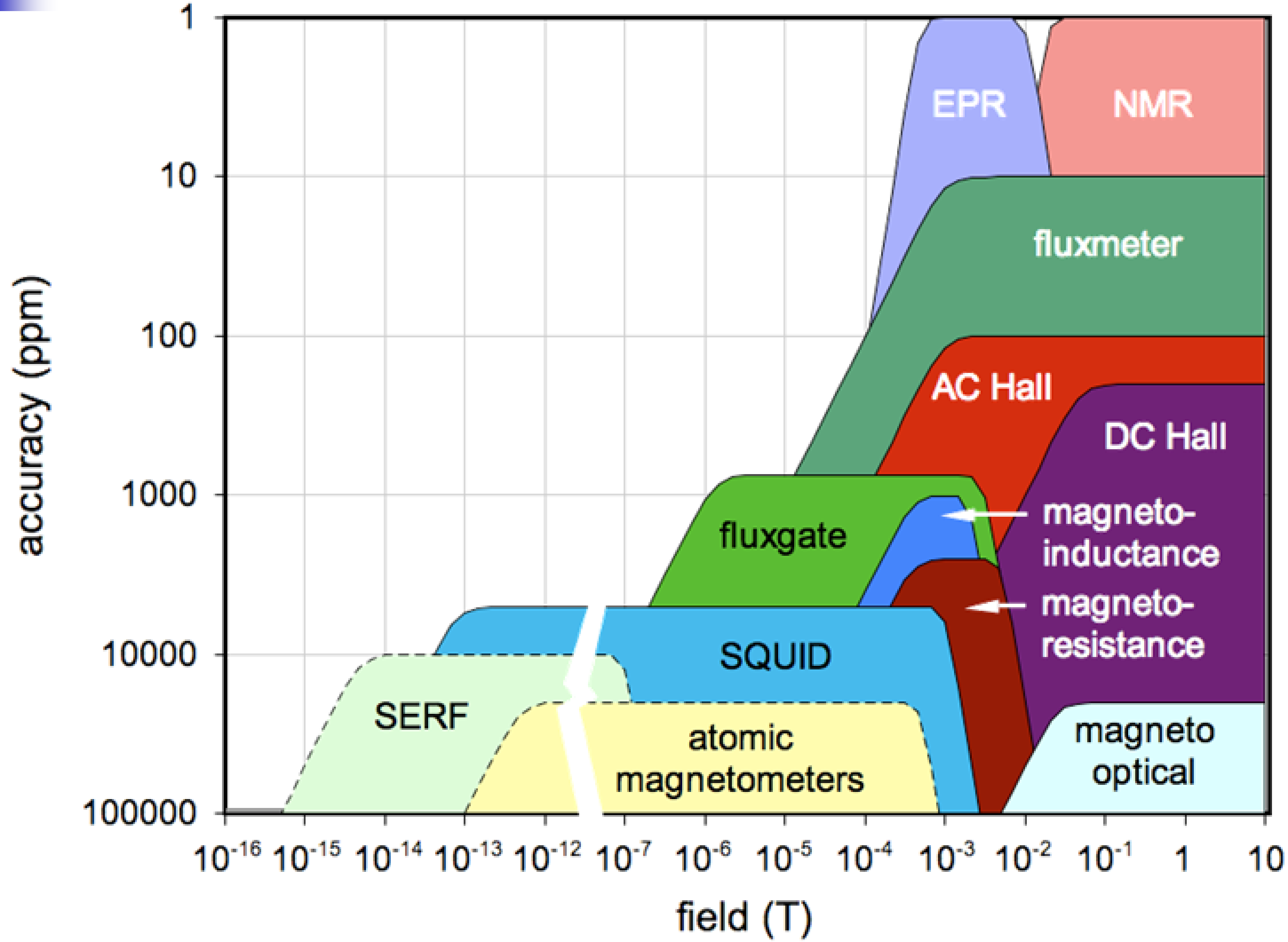


center of a RHIC quadrupole at 75 T/m gradient during cold testing

direct magnetic center measurement (image treatment)

needs higher field than e.m. methods, limited accuracy

Performance summary



Conclusions

- the *art* of magnetic measurements *cannot* be made into a science
  - at least I have given up…
- methods and instruments exist, don't try to make them yourself, use them if you can
  - as for *italian cuisine* let Mom do it…
  - …or buy a good cooking book before you start
- where can I find out more ?
  - CAS on MM, MT, PAC, EPAC
  - (NIM) uNclear Instruments and Methods