BEAM LOADING I

- DESCRIPTION OF THE COUPLED SYSTEM
  RF GENERATOR-CAVITY-BEAM
  IN TERMS OF BASIC QUANTITIES.

- THE CAVITY IMPEDANCE
  AS SEEN BY THE BEAM AND
  AS SEEN BY THE GENERATOR
  TAKING BEAMLOADING AND DETUNING INTO ACCOUNT

- SPECIAL ASPECTS FOR
  SUPERCONDUCTING CAVITIES

HELMHOLTZ GEMEINSCHAFT

Alexander Gamp

CAS on High Power Hadron Machines
Bilbao, May 27th, 2011

DESY

BEAM LOADING II

- SERVO SYSTEMS FOR RF PHASE AND AMPLITUDE CONTROL UNDER BEAMLOADING
- BEAMLOADING COMPENSATION BY MEANS OF

  DETUNING
  RF FEEDBACK
  RF FEEDFORWARD

- ROBINSON'S STABILITY CRITERION
- AN EXAMPLE OF DIGITAL RF CONTROL FOR SUPERCONDUCTING CAVITIES
- A PHASE LOOP FOR DAMPING SYNCHROTRON OSCILLATIONS

HELMHOLTZ
GEMEINSCHAFT

Alexander Gamp, DESY,
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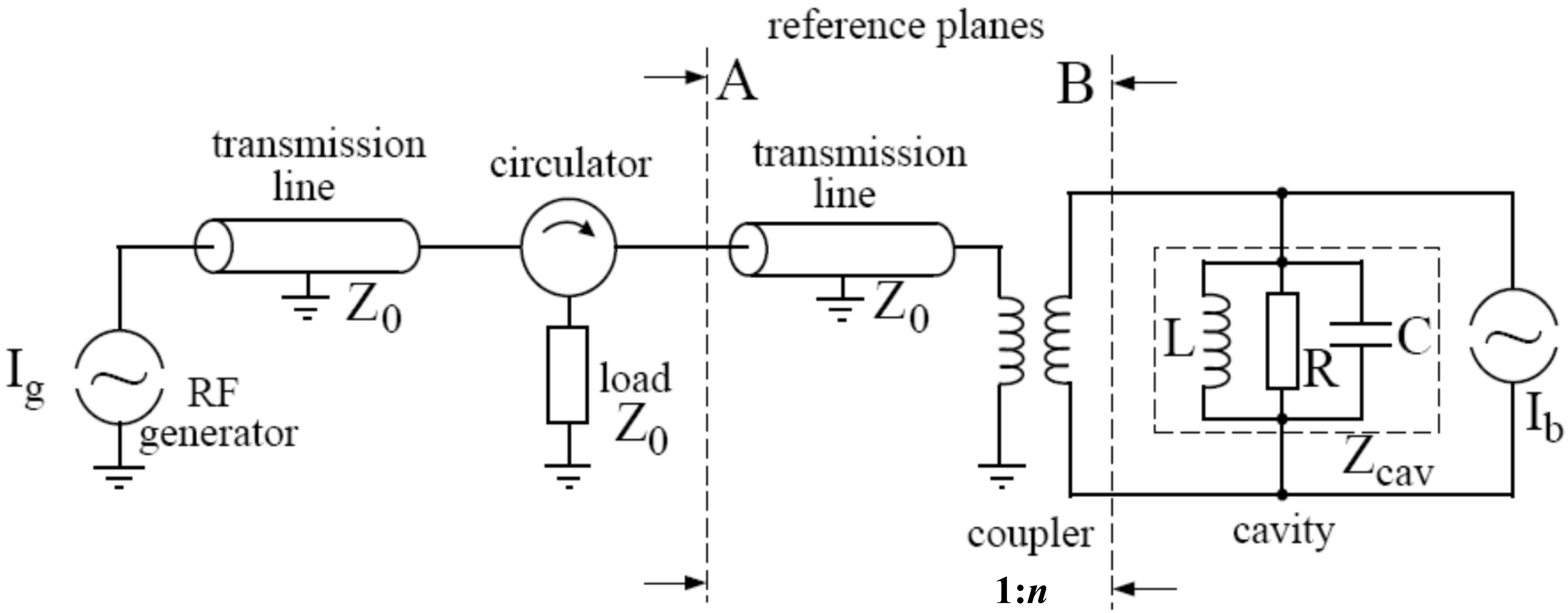
DESY

Variations of coupling different generators to a Cavity and the generator induced voltage

Optimum Generator impedance, dependence on cavity voltage without circulator, electric and magnetic coupling antennas, directional couplers, MHz-GHz, mW-MW, Zcav=f(I_B,ϕs) resulting in reflected power



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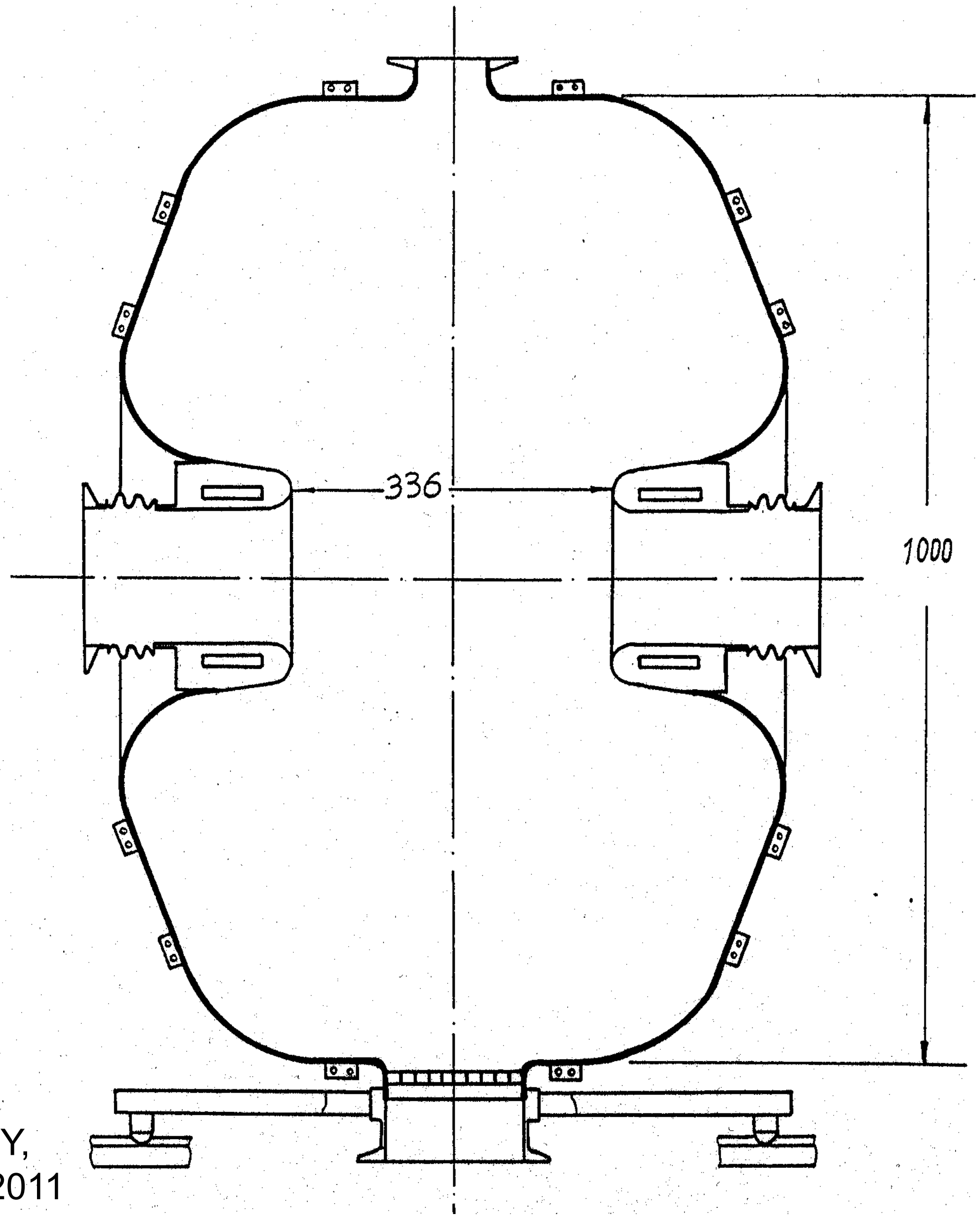
The HERA 208 MHz Cavities adapted from 200 MHz SPS Cavities



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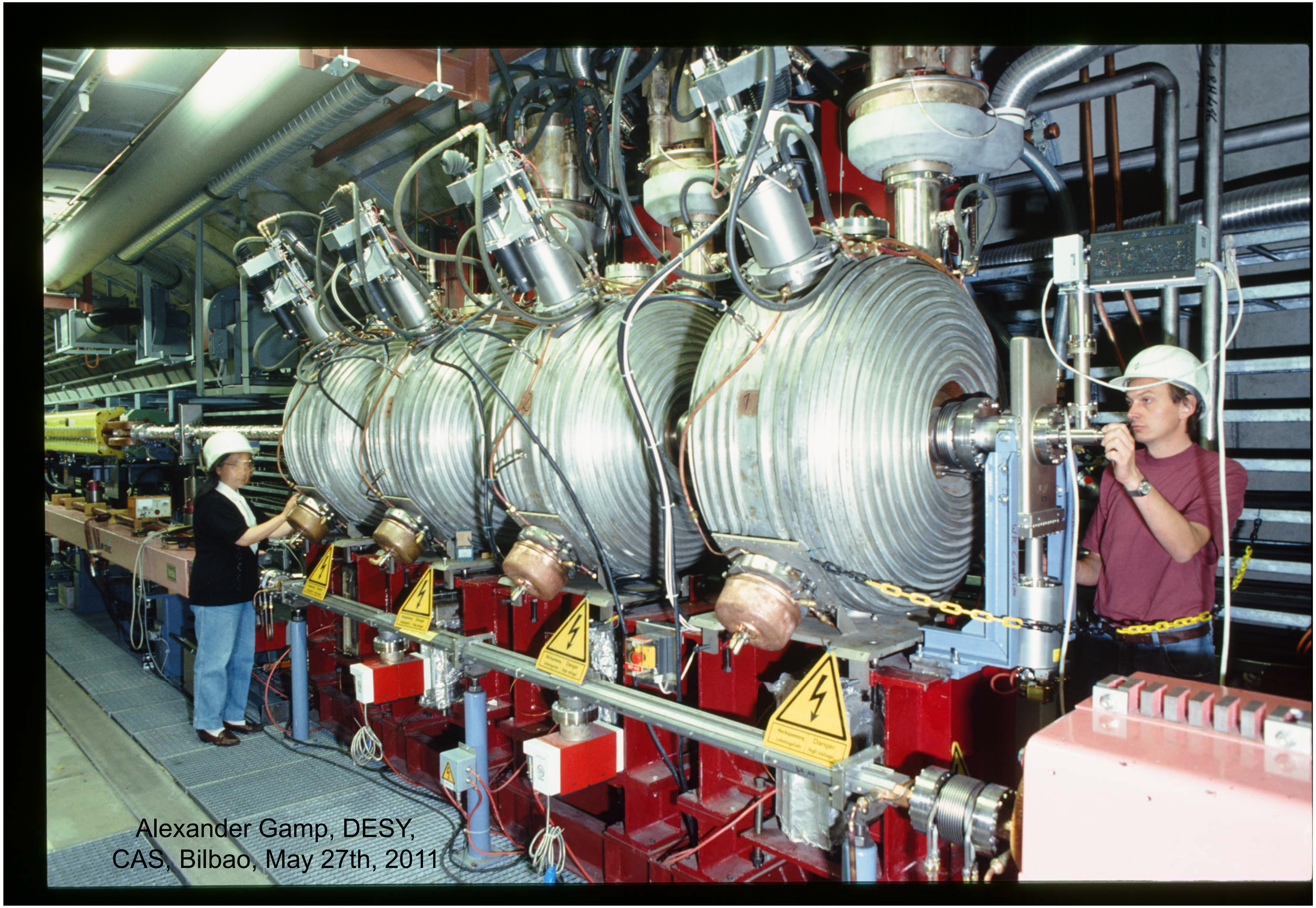
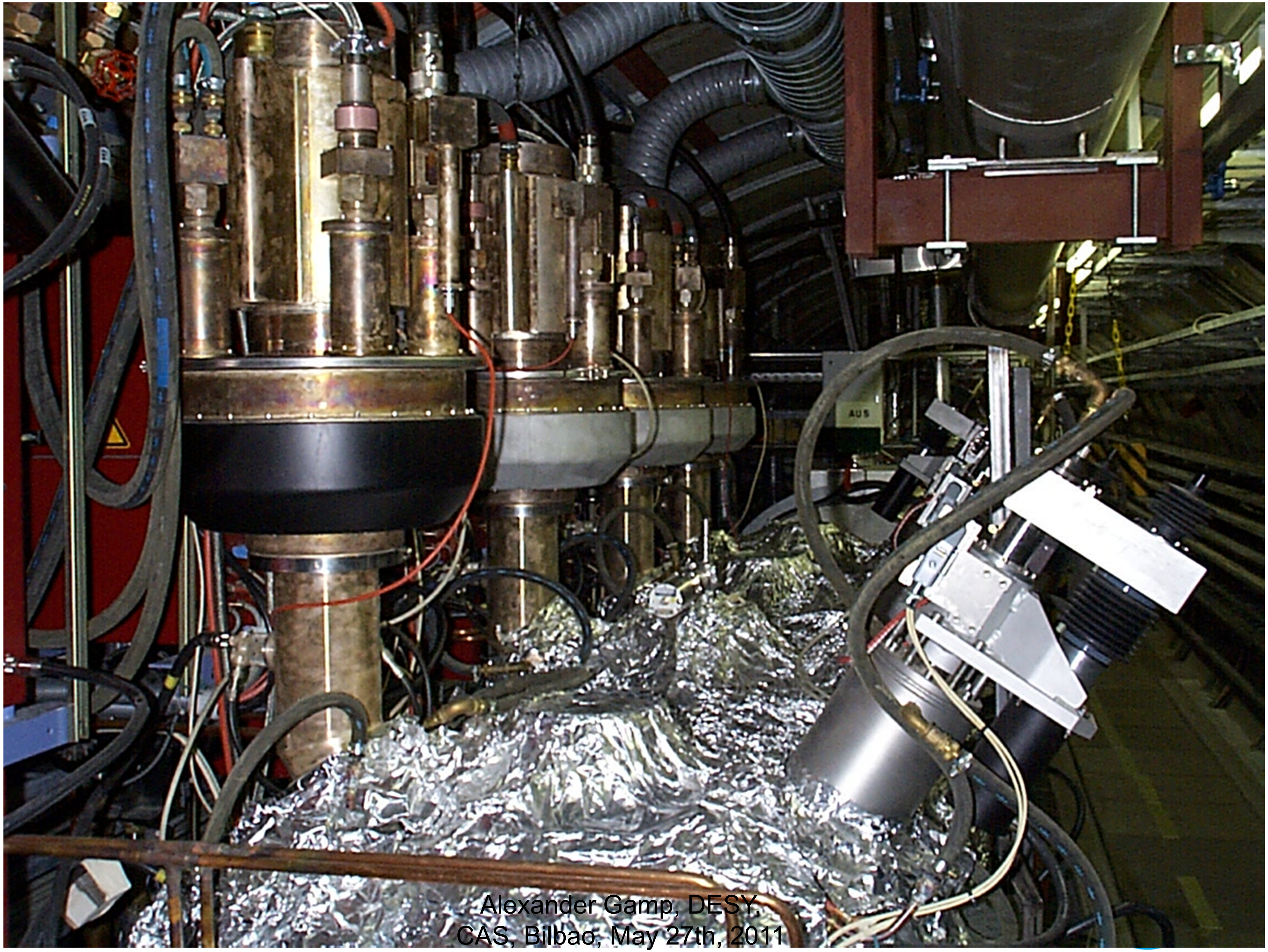
HERA Proton Cavities

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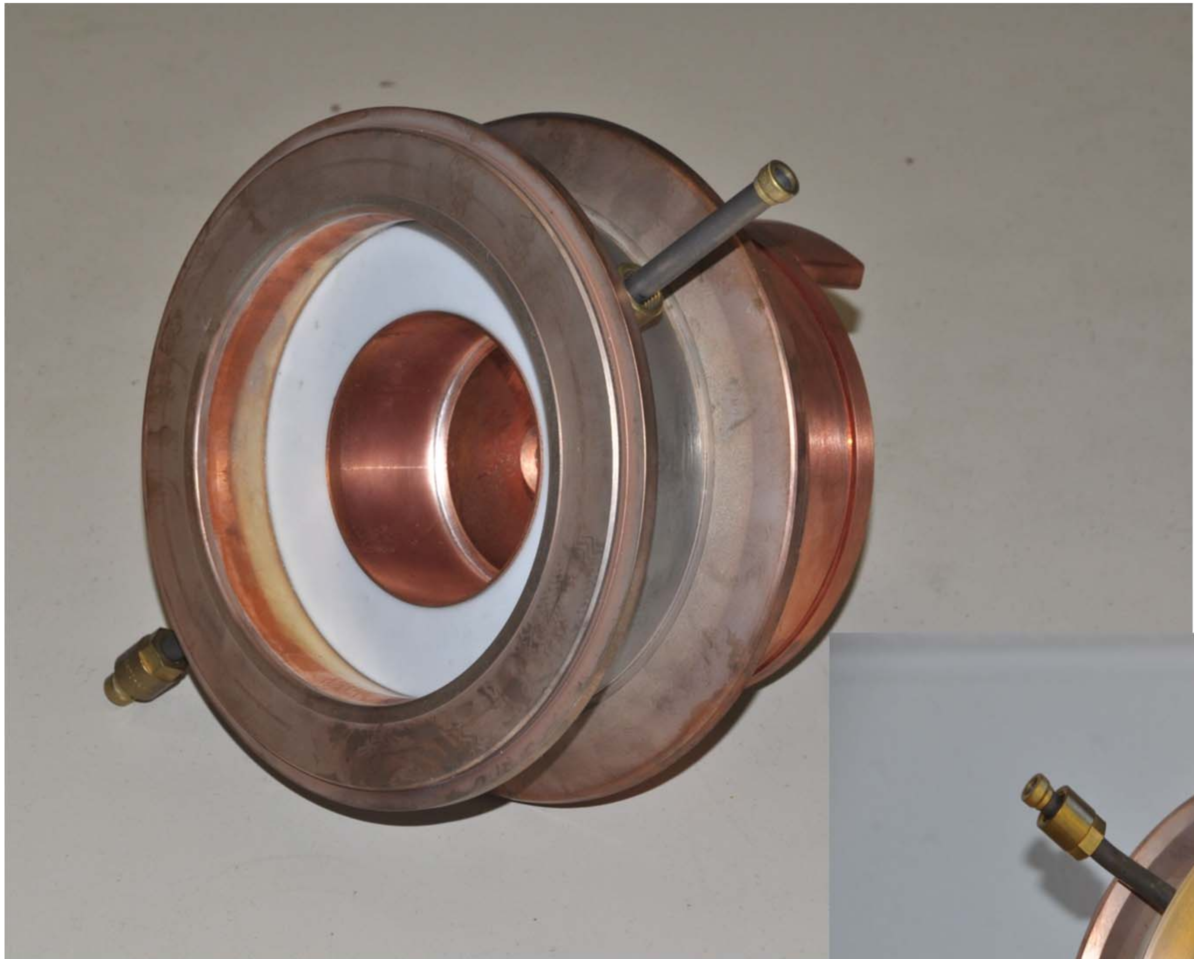
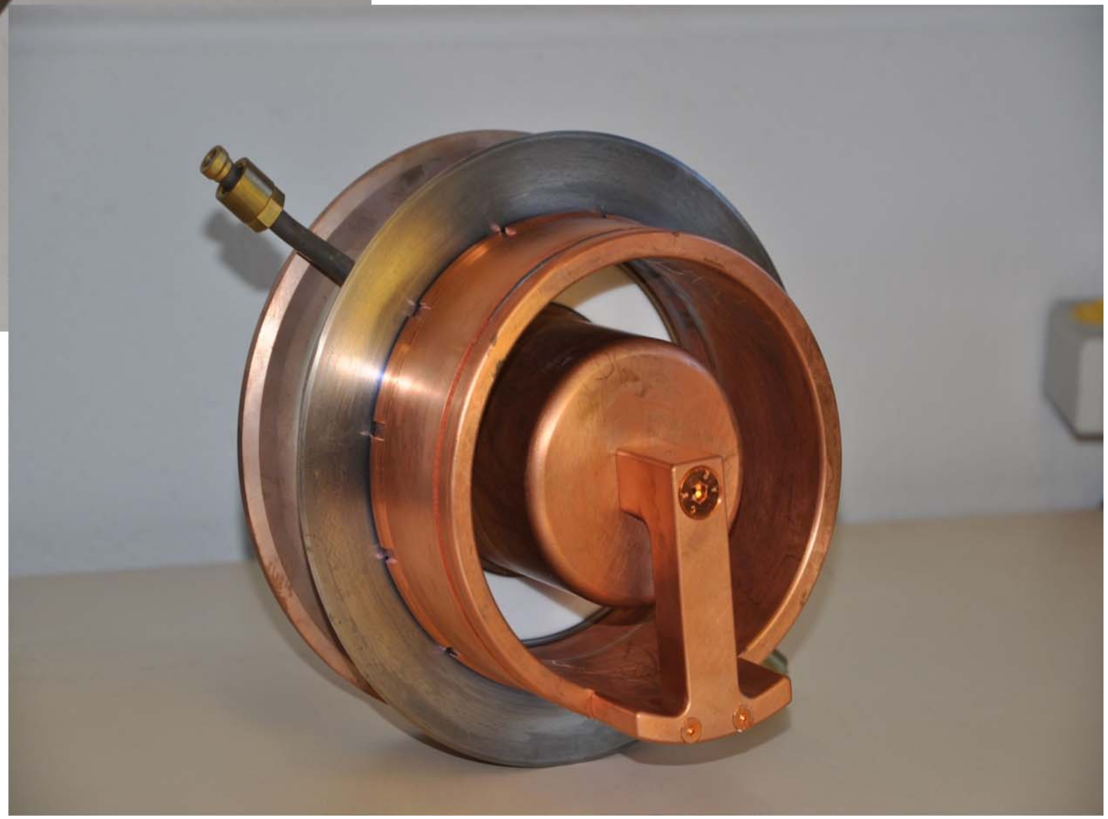
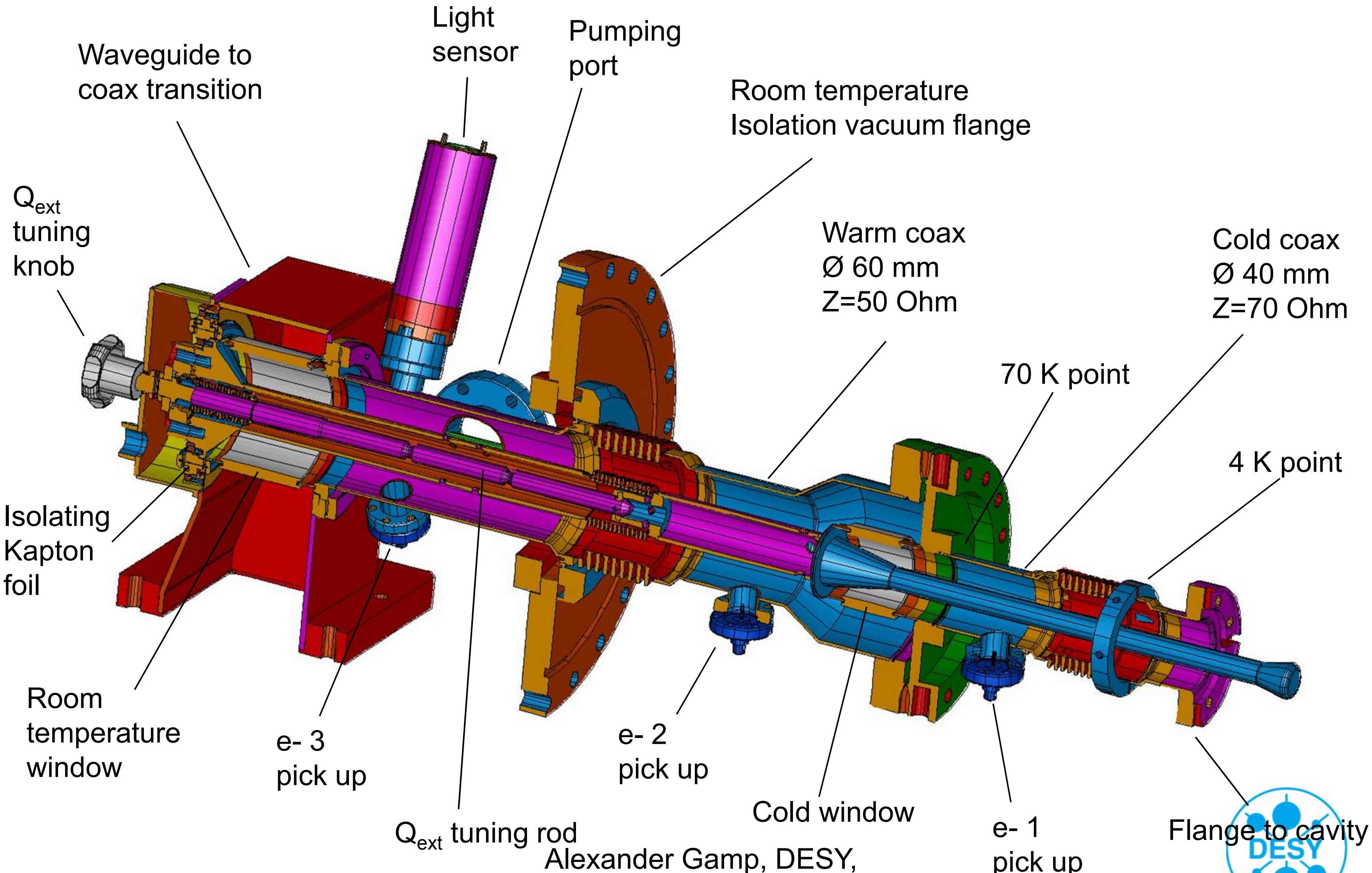


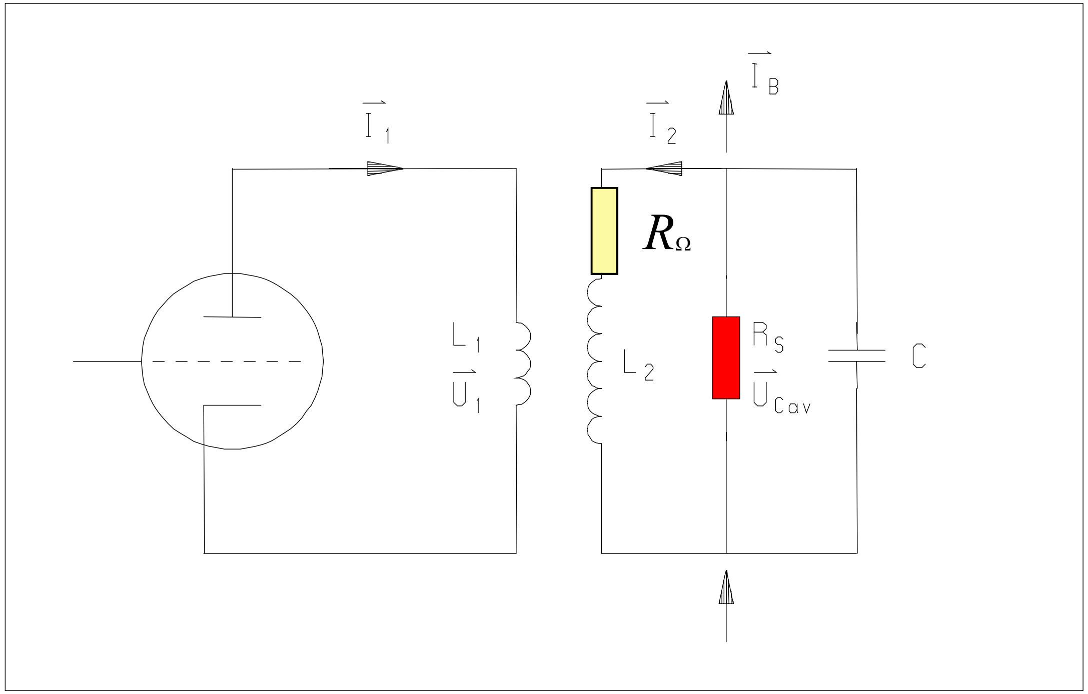


The TTF 3 Coupler Design



Equivalent circuit of a cavity[1]

Shunt Impedance: $$R_S = \hat{\vec{U}}^2_{CAV} \big/ 2P \quad = \frac{1}{R_\Omega}\frac{L_2}{C}$$

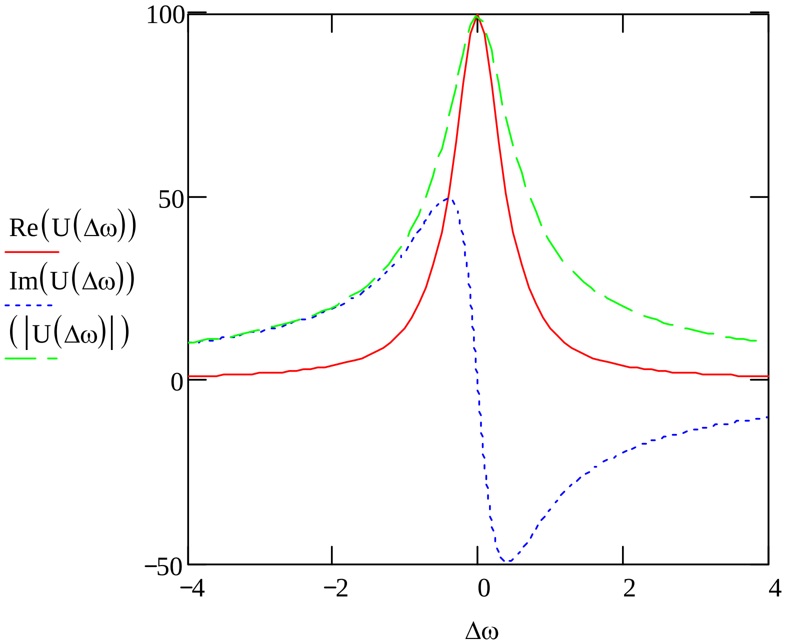


Some important definitions:

The Quality factor Q of a cavity

$$Q = 2\pi \frac{E_{tot}}{\Delta E/cycle} = \omega_{CAV} C R_S. \quad \text{Remember} \quad R_S = \frac{1}{R_\Omega}\frac{L_2}{C}$$

It also means $Q = \dfrac{\omega}{\Delta\omega}$ where $\Delta\omega$ is the FWHM Bandwidth.



$$\Gamma = \frac{1}{2CR_S} = \frac{\omega_{CAV}}{2Q}$$

The Quantity $\frac{R_S}{Q}=\sqrt{\frac{L_2}{C}}$ is a characteristic quantity of a cavity depending only on its geometry.

For a typival value of, say, 100 and Q of, say, 50000,

The shunt impedance will become $R_S = 5M\Omega$

Hence, for $\hat{\vec{U}}_{CAV} = \sqrt{2R_S\overline{P}_{Gen}} = 1MV$ a generator power of 100 kW is needed. In a „naked" cavity a beam current of 200 mA also induces 1 MV.

For a 9-cell superconducting cavity with $\frac{R_S}{Q} \approx 1000$ and $Q \approx 10^{10}$

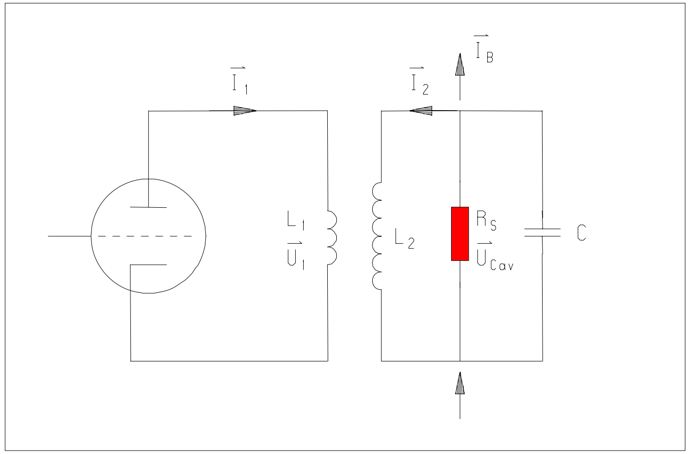
only .5 Watts are needed for 1 MV

and 31 Watts for 25 MV

THE CAVITY IMPEDANCE
AS SEEN BY THE BEAM

N IS THE TRANSFORMATION- OR STEP-UP RATIO FOR MATCHING THE GENERATOR- TO THE CAVITY IMPEDANCE

$$N^2 = R_S / R_A = L_2 / L_1$$



ON A TRANSMISSION LINE FORWARD AND REFLECTED VOLTAGES ADD:

$$\vec{U}_1 = \vec{U}_{forward} + \vec{U}_{reflected}$$

BUT CURRENTS SUBTRACT.

$$\vec{I}_1 = \vec{I}_{forward} - \vec{I}_{reflected}$$

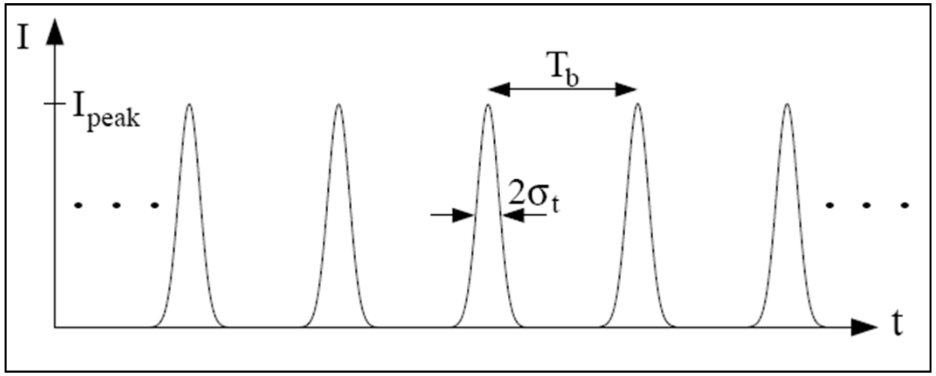
Time dependence: $\vec{U} = \hat{\vec{U}} e^{i\omega t}$

Frequency of driving force: ω

Cavity resonance frequency : $2\pi f_{CAV} = \omega_{CAV} = 1\big/\sqrt{L_2 C}.$

$\vec{I}_B(\omega)$ **Harmonic content at the frequency** ω **of the total beam current**

Relationship of average DC Beam current and its harmonic content at frequency ω **:**



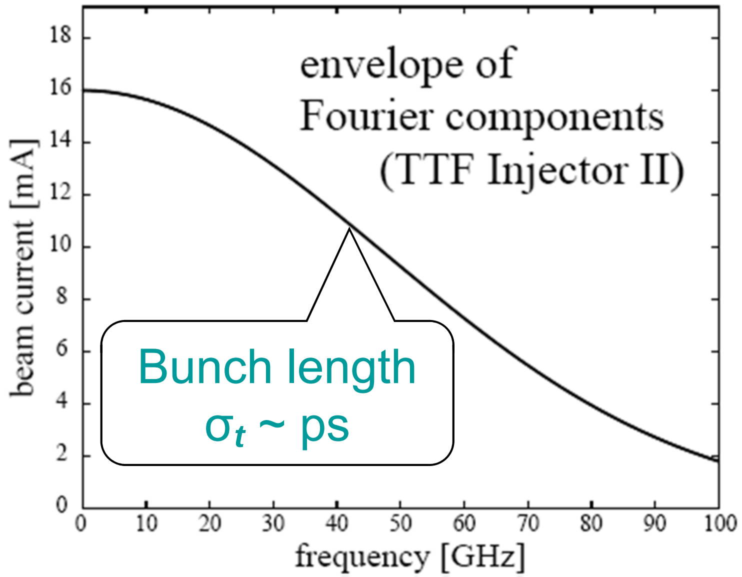
$$I(t) = \frac{Q_0}{\sqrt{2\pi}\sigma_t} \cdot e^{-\frac{t^2}{2\sigma_t^2}} =$$

$$I_{peak} \cdot e^{-\frac{t^2}{2\sigma_t^2}}$$

Fourier decomposition of bunch train:

$$I(t) = a_0/2 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_b t) + b_n \sin(n\omega_b t)\right]$$

$$\begin{cases} \omega_b = 2\pi/T_b \\ a_n = 2I_{peak}\sqrt{2\pi}\cdot\sigma_t/T_b \cdot e^{-n^2\omega_b^2\sigma_t^2/2} \quad n = 0,1,2,\cdots \\ b_n = 0 \text{ for even functions like gauss} \end{cases}$$



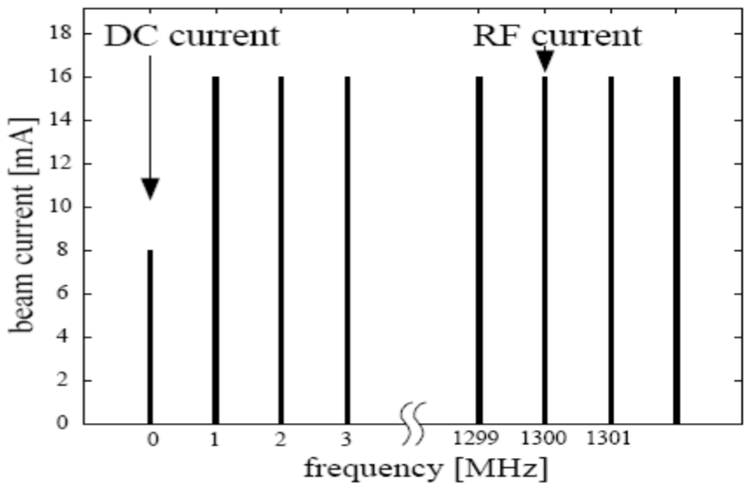
Average DC beam current:

$$I_{b0} = I_{peak}\sqrt{2\pi}\,\sigma_t/T_b = Q_0/T_b$$

Harmonic content at Frequency $n\omega_b$:

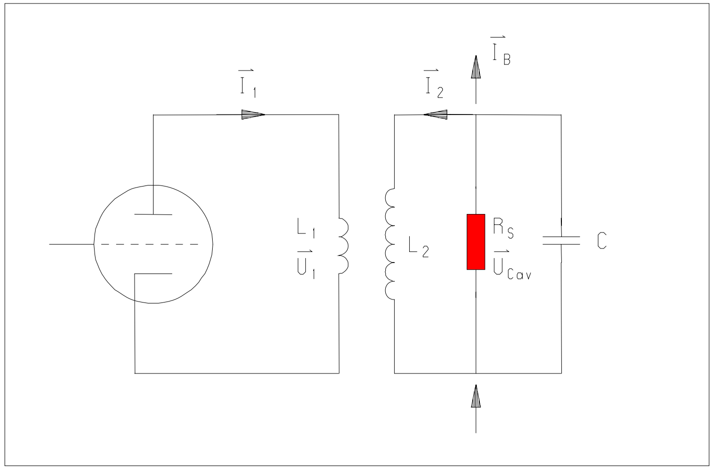
$$a_n = 2I_{b0}\cdot e^{-n^2\omega_b^2\sigma_t^2/2}$$

$$\left|\vec{I}_b(n\omega_b)\right| = a_n(1300\text{MHz}) \approx 2I_{b0}$$



At cavity resonance, i.e. $\omega = \omega_{CAV}$ there is no reflection* for $\vec{I}_B(\omega) = 0$ and $\vec{U}_1$ and $\vec{I}_1$ are identical to the generator voltage and current

(*for properly adjusted coupling ratio)



$$\vec{U}_{CAV} = N\vec{U}_1 = L_2\left(\dot{\vec{I}}_2 + \dot{\vec{I}}_1 / N\right)$$

$$\vec{I}_2 = -(\vec{I}_B + \vec{U}_{CAV} / R_S + C\dot{\vec{U}}_{CAV})$$

$$\omega_{CAV}^2 \vec{U}_{CAV} = \frac{1}{C}\left[\frac{1}{N}\dot{\vec{I}}_1 - \dot{\vec{I}}_B - \frac{1}{R_S}\dot{\vec{U}}_{CAV}\right] - \ddot{\vec{U}}_{CAV}$$

With a damping term defined as $\Gamma = \dfrac{1}{2CR_S} = \dfrac{\omega_{CAV}}{2Q}$

we obtain the equation of a damped resonant circuit with a driving force:

$$\ddot{\vec{U}}_{CAV} + 2\Gamma\dot{\vec{U}}_{CAV} + \omega_{CAV}^2\vec{U}_{CAV} = 2\Gamma R_S\left[\frac{1}{N}\dot{\vec{I}}_1 - \dot{\vec{I}}_B\right]$$

Note that $\vec{I}_B$ and $\vec{I}_1$ have opposite sign!

The beam induced voltage decelerates the beam.

Exercise: Fundamental Theorem of Beamloading[2,3]

To find the Cavity Impedance as seen by the Beam we rewrite the driving current term:

Remembering that $\vec{U}_1 = \vec{U}_{forward} + \vec{U}_{reflected}$ and $\vec{I}_1 = \vec{I}_{forward} - \vec{I}_{reflected}$ and $\vec{U}_{CAV} = N\vec{U}_1$, the generator current term $\frac{1}{\mathrm{N}}\dot{\vec{I}}_1$ becomes:

$$\frac{1}{\mathrm{N}}\dot{\vec{I}}_1 = \frac{1}{\mathrm{N}}\left[2\dot{\vec{I}}_{forward} - \frac{\dot{\vec{U}}_{CAV}}{NR_A}\right]$$

The new term with $\dot{\vec{U}}_{CAV}$ leads to a modification of the damping term:

$$\ddot{\vec{U}}_{CAV} + 2\Gamma(1+\beta)\dot{\vec{U}}_{CAV} + \omega_{CAV}^2\vec{U}_{CAV} = 2\Gamma_L R_{SL}\left[\frac{2}{N}\dot{\vec{I}}_f - \dot{\vec{I}}_B\right]$$

The loaded quantities are defined in terms of the coupling ratio β:

$$\beta = R_S \big/ \left(N^2 R_A\right), \qquad \Gamma_L = \Gamma(1+\beta),$$

$$Q_L = Q/\left(1+\beta\right) \text{ and } R_{SL} = R_S/\left(1+\beta\right)$$

Increasing β means more loading, less Q or shunt impedance, hence less cavity voltage.

$$\ddot{\vec{U}}_{CAV} + 2\Gamma\left(1+\beta\right)\dot{\vec{U}}_{CAV} + \omega_{CAV}^2 \vec{U}_{CAV} = 2\Gamma_L R_{SL}\left[\frac{2}{N}\dot{\vec{I}}_f - \dot{\vec{I}}_B\right]$$

For β=1 the shunt impedance seen by the beam is R_{SL}=0.5R_S. Why? Because Generator and Cavity impedances are in parallel!

The driving Force or Generator current is defined in terms of the forward current:

$$\dot{\vec{I}}_G = 2\dot{\vec{I}}_f \big/ N$$

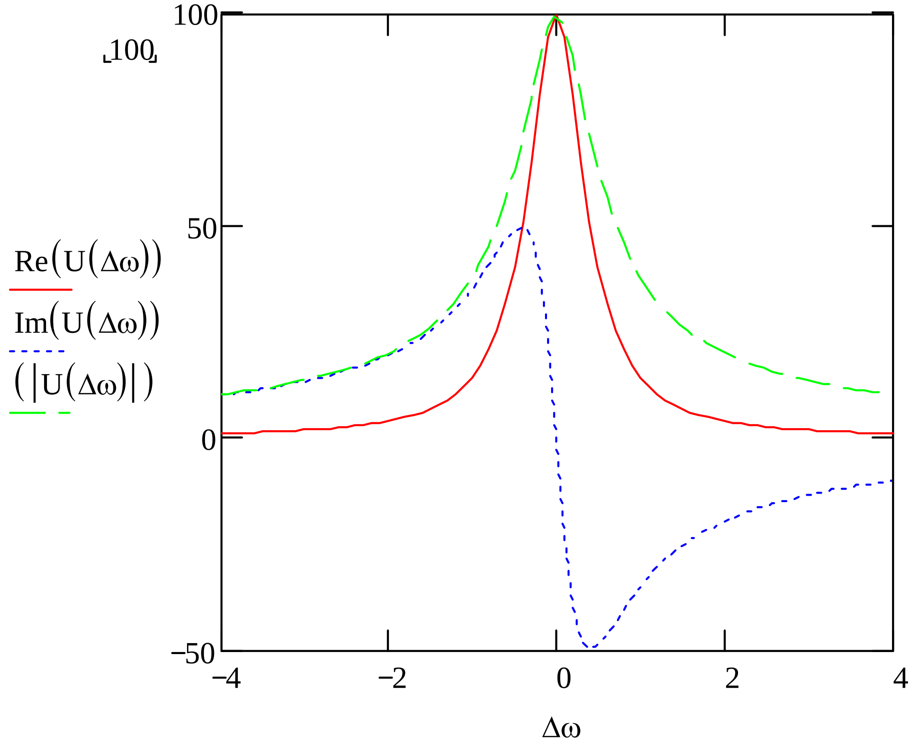
The solution can be written as [4]

$$\hat{\vec{U}}_{CAV} = \frac{i\omega}{\omega_{CAV}^2 - \omega^2 + 2i\omega\Gamma_L} 2\Gamma_L R_{SL}\left[\hat{\vec{I}}_G - \hat{\vec{I}}_B\right]$$

For small detuning, i.e. $\omega = \omega_{CAV} + \Delta\omega$

and $\Delta\omega << \omega_{CAV}$ we can approximate by

$$\hat{\vec{U}}_{CAV} \approx \frac{R_{SL}\left[\hat{\vec{I}}_G - \hat{\vec{I}}_B\right]}{1 + iQ_L 2\frac{\Delta\omega}{\omega_{CAV}}}$$



For a resonant cavity the beam induced voltage, or the beamloading, is hence given by the product of loaded shunt impedance and beam current:

$$\hat{\vec{U}}_B = -R_{SL}\hat{\vec{I}}_B$$

The ideal beam loading compensation would, therefore, minimize R_{SL} without increasing the generator power necessary to maintain the cavity voltage

For a loaded shunt impedance of 2.5 MΩ and a beam current of .2 A, we get an induced voltage of .5 MV!

To compensate this a generator current of 20 A may be needed for a typical N of, say, 100, but………possibly this results in large values of reflected power…….need of amplifier protection

The admittance which the combined system cavity and beam represents to the generator is given by [5]

$$Y = \frac{\vec{I}_1}{\vec{U}_1} = \frac{N^2}{R_S} + \frac{\vec{I}_B N^2}{\vec{U}_{CAV}} + \frac{N^2}{i\omega L_2}\left(1 - \frac{\omega^2}{\omega_{CAV}^2}\right)$$

It reduces to $Y = \frac{N^2}{R_S} = \frac{1}{R_A}$ **for a tuned cavity without beam current in the matched case where** $\beta = 1$ **.**

The beam current dependent real part necessitates a change in the coupling ratio ß for optimum coupling with beam, the beam current dependent imaginary part can be compensated by detuning.

That is much easier. Just look at a coupler and you will understand why.

FROM $\vec{U}_{ACC} = \vec{U}_{CAV} \sin\phi_s$ **AND** $\dfrac{\vec{I}_B}{\vec{U}_{CAV}} = \dfrac{|\vec{I}_B|}{|\vec{U}_{CAV}|} e^{i\left(\frac{\pi}{2}-\phi_s\right)}$

FOR $\omega=\omega_{CAV}$ ONE FINDS THE REAL PART OF Y TO BE

$$\mathrm{Re}(Y) = \frac{N^2}{R_S}\left(1 + \frac{R_S|\vec{I}_B|}{|\vec{U}_{CAV}|}\sin\phi_s\right)$$

THE CHANGE IN ADMITTANCE

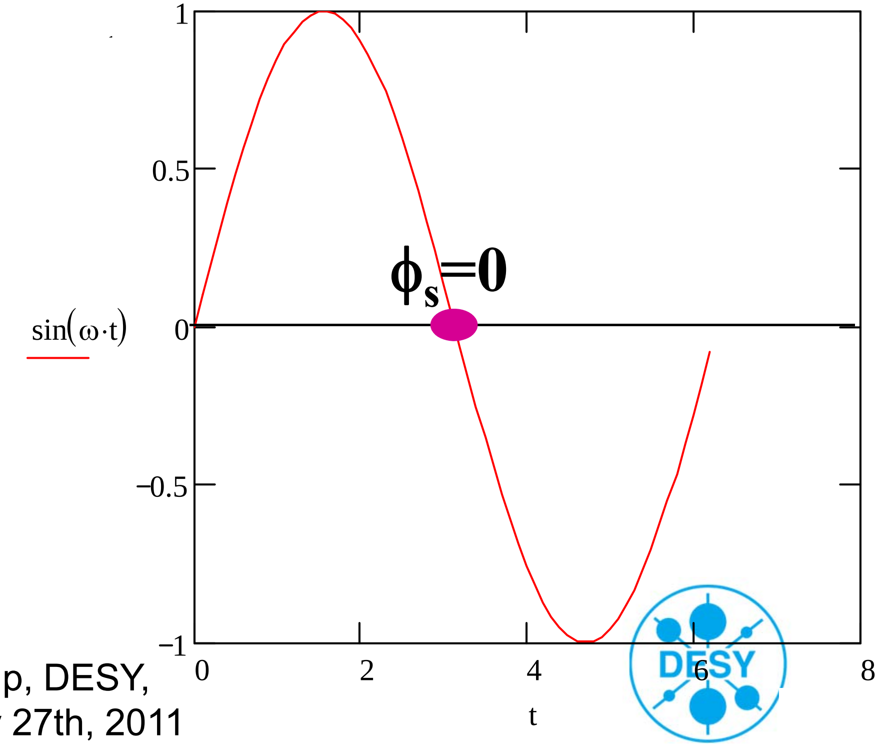
CAUSED BY THE BEAM IS

GIVEN BY THE ( ) TERM.

IT MUST BE COMPENSATED

BY A CHANGE IN THE

COUPLING FACTOR β.



NOW THERE IS OPTIMUM COUPLING FOR

$$\beta = 1 + \frac{R_S \left| \vec{I}_B \right|}{\left| U_{CAV} \right|} \sin \phi_s$$

THE COUPLING RATIO IS PROPORTIONAL TO THE RATIO OF RF POWER DELIVERED TO THE BEAM TO RF POWER DISSIPATED IN THE CAVITY WALLS.

For Rs=5 MΩ , I_B= 30 mA, i.e. I_B (ω)= 60 mA, ϕ_S = 30^0 and U_{CAV} = 1 MV ß=1.15. N.C. case for Electrons

For Rs= 10^{13} Ω, I_B= 8 mA, i.e. I_B (ω)= 16 mA, ϕ_S =90^0 and U_{CAV} = 25 MV ß=6400. S.C. case for Electrons

Then for $Q = 10^{10}$ the loaded $Q_L = Q/(1+\beta) =$

$Q/(6401) \approx 1.6 * 10^6$

This means that without beam there is a mismatch resulting in total reflection of the generator power.

$$r(\beta,\Delta\omega) = \frac{\beta - 1 - \frac{iQ2\Delta\omega}{\omega_{cav}}}{\beta + 1 + \frac{iQ2\Delta\omega}{\omega_{cav}}}$$

With the reflection coefficient (at resonance)

$$r = \frac{Z - Z_0}{Z + Z_0} = \frac{\beta - 1}{\beta + 1} = \frac{\vec{U}_{reflected}}{\vec{U}_{forward}} = \frac{6399}{6402} \approx 1$$

The VSWR becomes

$$VSWR = \frac{\vec{U}_{forward} + \vec{U}_{reflected}}{\vec{U}_{forward} - \vec{U}_{reflected}} = \frac{1+r}{1-r} \approx \infty$$

So, beamloading for superconducting cavities is even more dramatic than for normal conducting cavities!

What is the time dependence of the envelope of the reflected voltage?

The asymptotic value of the cavity voltage in terms of ß

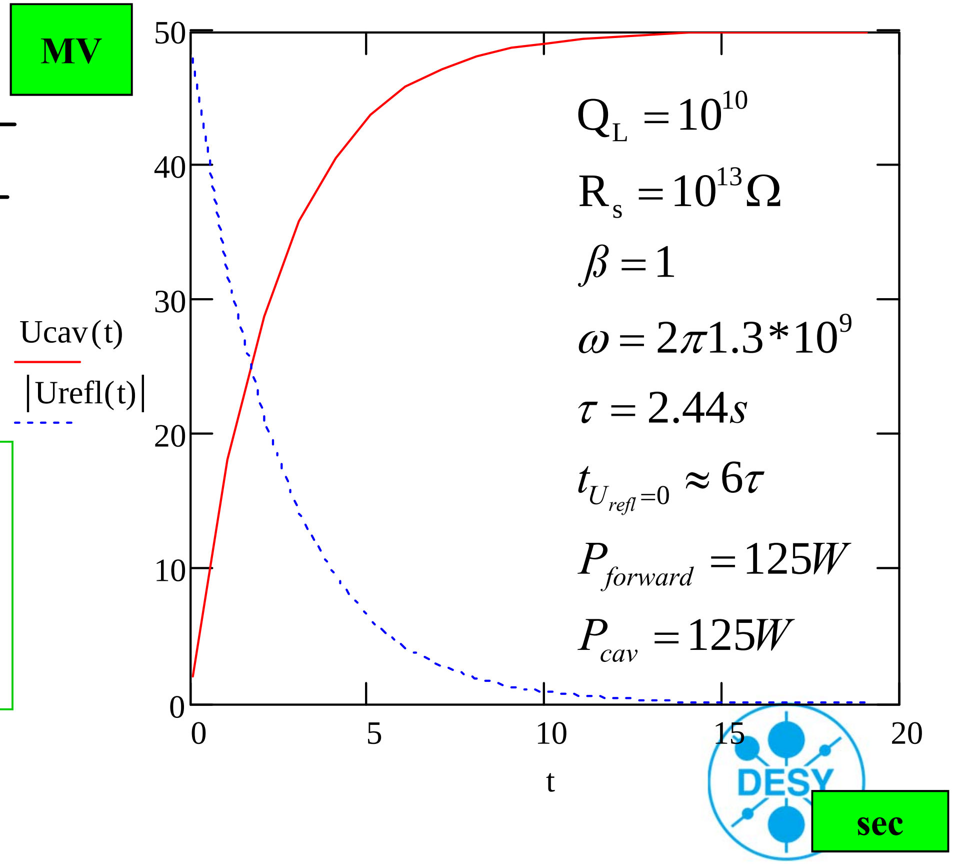
$$P_{CAV} = P_{forward} - P_{reflected} = P_{forward}(1 - r^2) = P_{forward}\frac{4\beta}{(1+\beta)^2}$$

$$r = \frac{\vec{U}_{reflected}}{\vec{U}_{forward}} \qquad r^2 = \frac{P_{reflected}}{P_{forward}}$$

$$\hat{\vec{U}}_{CAV} = \sqrt{2R_S P_{forward}\frac{4\beta}{(1+\beta)^2}}$$

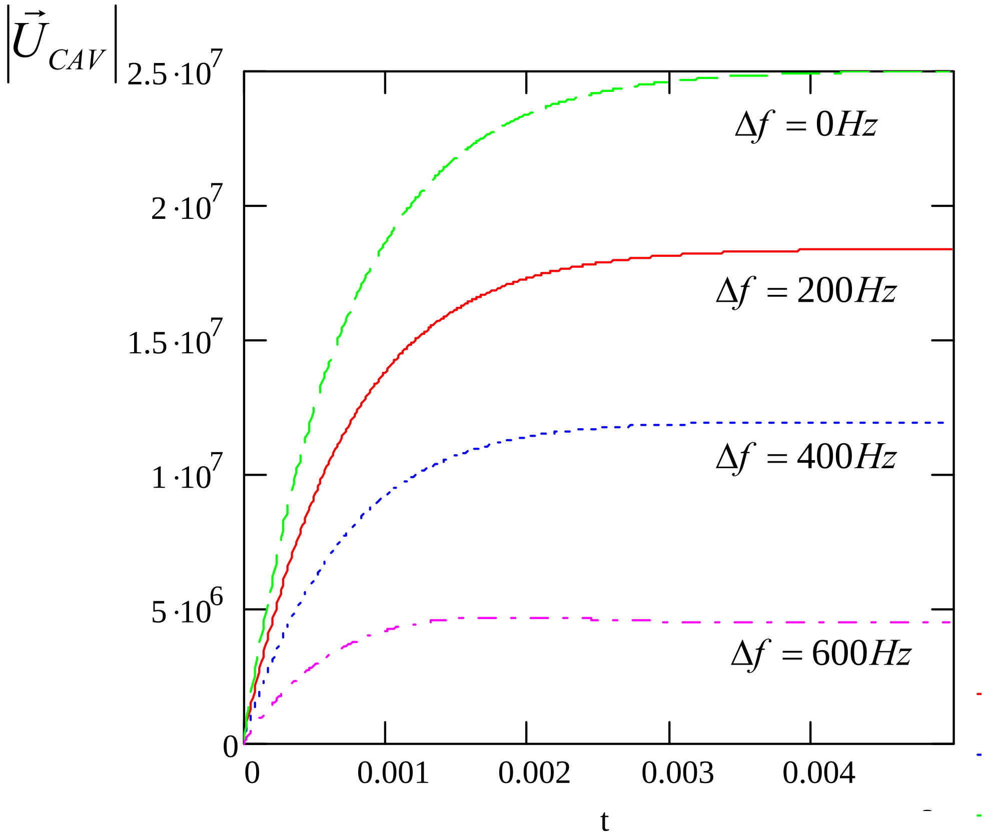
On resonance the envelope of the cavity voltage during filling is

$$\boxed{\begin{aligned}\vec{U}_{CAV}(t) &= \hat{\vec{U}}_{CAV}(1 - e^{-t/\tau}) \\ \tau &= 2Q_L/\omega\end{aligned}}$$



RF Voltage Envelope off Resonance:

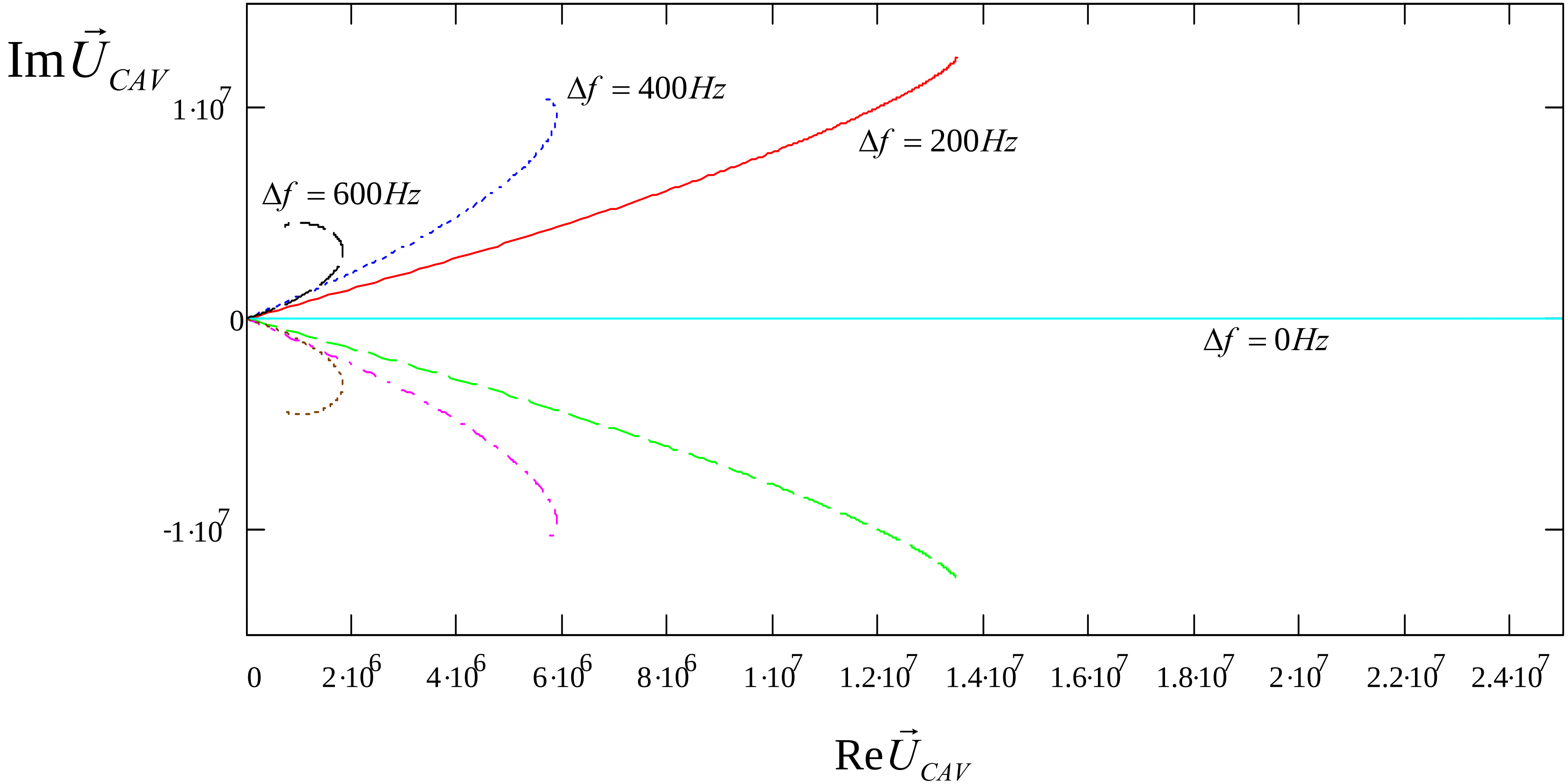
$$\boxed{\vec{U}_{CAV}(t) = \hat{\vec{U}}_{CAV}\left(1 - e^{-(1/\tau - i\Delta\omega)t}\right)}$$



$$\hat{\vec{U}}_{CAV} \approx \frac{R_{SL}\left[\hat{\vec{I}}_G - \hat{\vec{I}}_B\right]}{1 + iQ_L 2\frac{\Delta\omega}{\omega_{CAV}}}$$

$$f_{CAV} = 1.3GHz$$

$$Q_L = 3*10^6$$

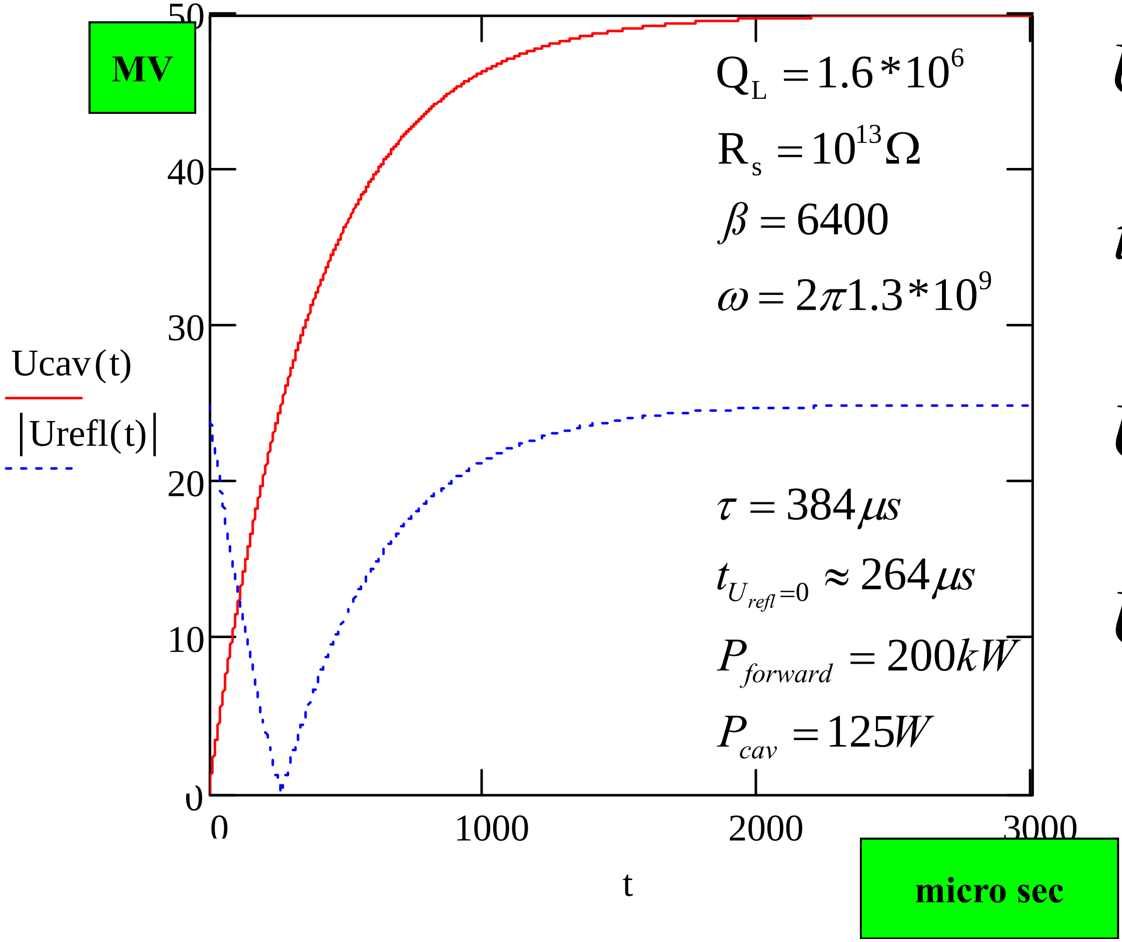


$$\vec{U}_{CAV}(t) = \frac{R_{SL}\left[\hat{\vec{I}}_G - \hat{\vec{I}}_B\right]}{1 + iQ_L 2\frac{\Delta\omega}{\omega_{CAV}}}\left(1 - e^{-(1/\tau - i\Delta\omega)t}\right)$$

$$\vec{U}_{reflected} = \vec{U}_{forward}\,\frac{\beta - 1}{\beta + 1} \qquad \vec{U}_1 = \vec{U}_{forward} + \vec{U}_{reflected} = \vec{U}_{forward}\,\frac{2\beta}{1+\beta}$$

$$\vec{U}_{CAV} = N\vec{U}_1 = N(\vec{U}_{forward} + \vec{U}_{reflected})$$

$$\vec{U}_{reflected}(t) = \frac{1}{N}\hat{\vec{U}}_{CAV}(1 - e^{-t/\tau}) - \vec{U}_{forward} = \hat{\vec{U}}_1\left[(1 - e^{-t/\tau}) - \frac{1+\beta}{2\beta}\right]$$



$$\vec{U}_{reflected}(t) = 0 \quad \text{at}$$

$$t_{U_{refl}=0} = -\tau \ln\left(1 - \frac{1+\beta}{2\beta}\right)$$

$$\vec{U}_{CAV}(t_{U_{refl}=0}) =$$

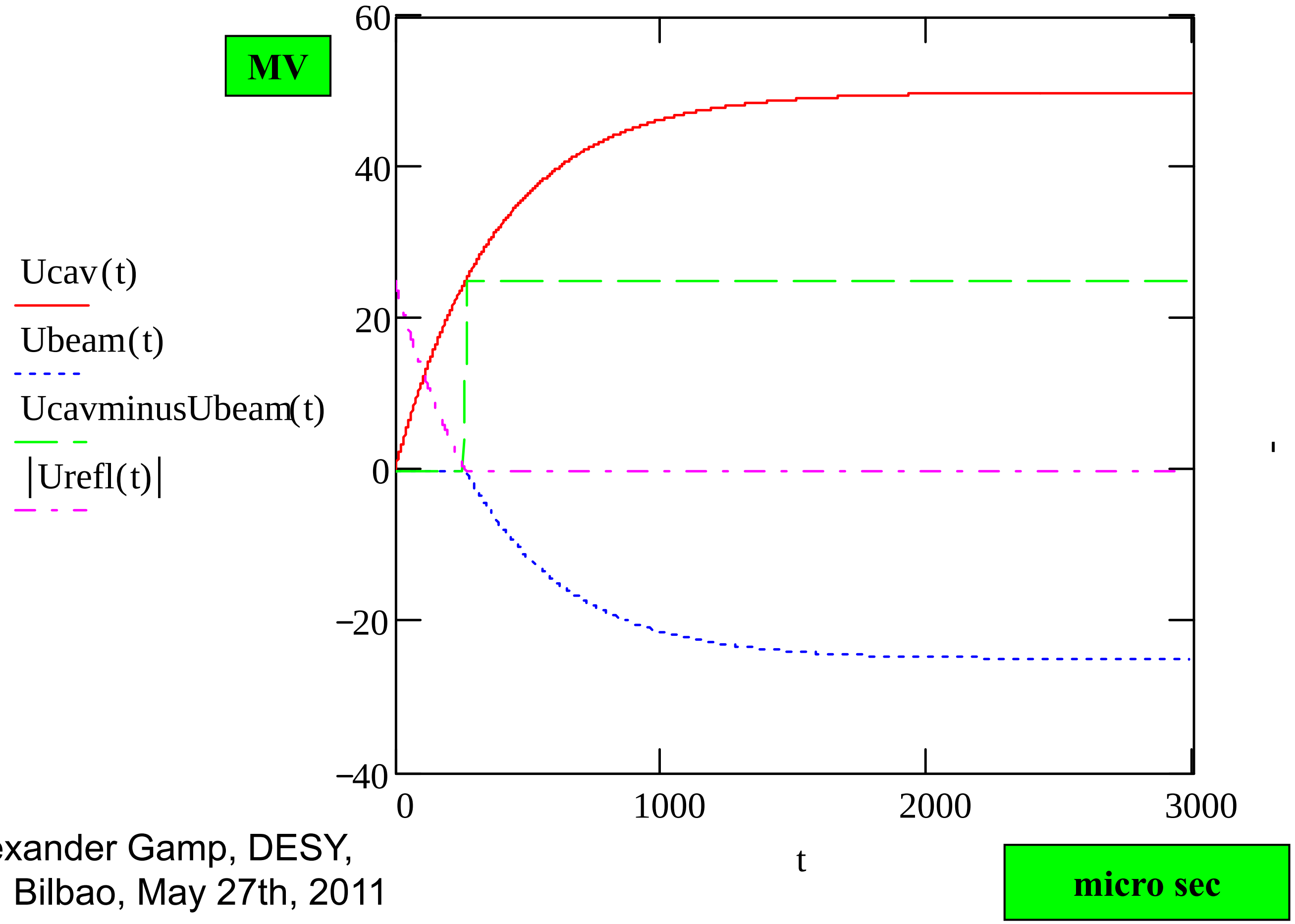
$$\hat{\vec{U}}_{CAV}\,\frac{1+\beta}{2\beta} \approx .5\hat{\vec{U}}_{CAV}$$

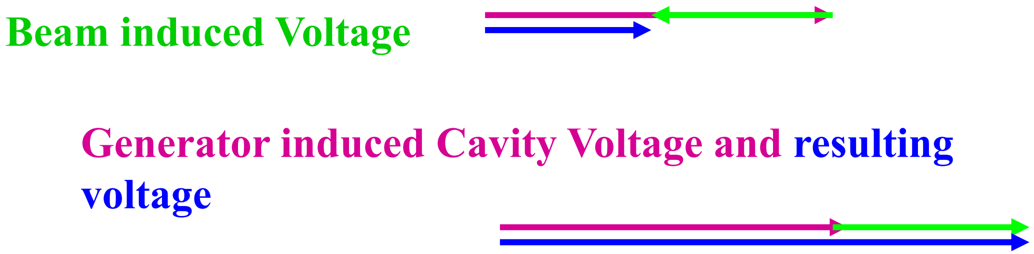
for $\beta >> 1$

Taking the beam induced voltage into account, we have

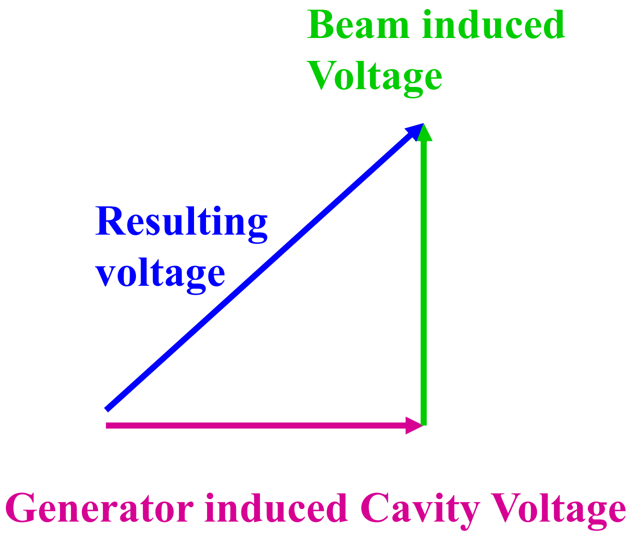
$$\hat{\vec{U}}_{CAV} = \sqrt{2R_S P_{forward}\frac{4\beta}{(1+\beta)^2}} - U_{Beam}$$

$$\vec{U}_{CAV}(t) = \hat{\vec{U}}_{CAV}(1-e^{-t/\tau}) - U_{Beam}(1-e^{-t(>t_{U_{refl}=0})/\tau})$$





Pure Real Beamloading (i.e. U_B and $U_{Gen.}$ in phase or opposite) can be compensated by adjustment of ß and Generator power. It may lead to high power dissipation in the generator.



Pure Reactive Beamloading (i.e. U_B and $U_{Gen.}$ in quadrature) can be compensated by detuning. Additional power is needed only for compensating transients, but not in steady state

The admittance which the combined system cavity and beam represents to the generator is given by

$$Y = \frac{\vec{I}_1}{\vec{U}_1} = \frac{N^2}{R_S} + \frac{\vec{I}_B N^2}{\vec{U}_{CAV}} + \frac{N^2}{i\omega L_2}\left(1 - \frac{\omega^2}{\omega_{CAV}^2}\right)$$

The beam current dependent imaginary part

$$\mathrm{Im}\left(\frac{\vec{I}_B}{\vec{U}_{CAV}}\right) + = \frac{\left|\vec{I}_B\right|}{\left|\vec{U}_{CAV}\right|}\cos\phi_s$$

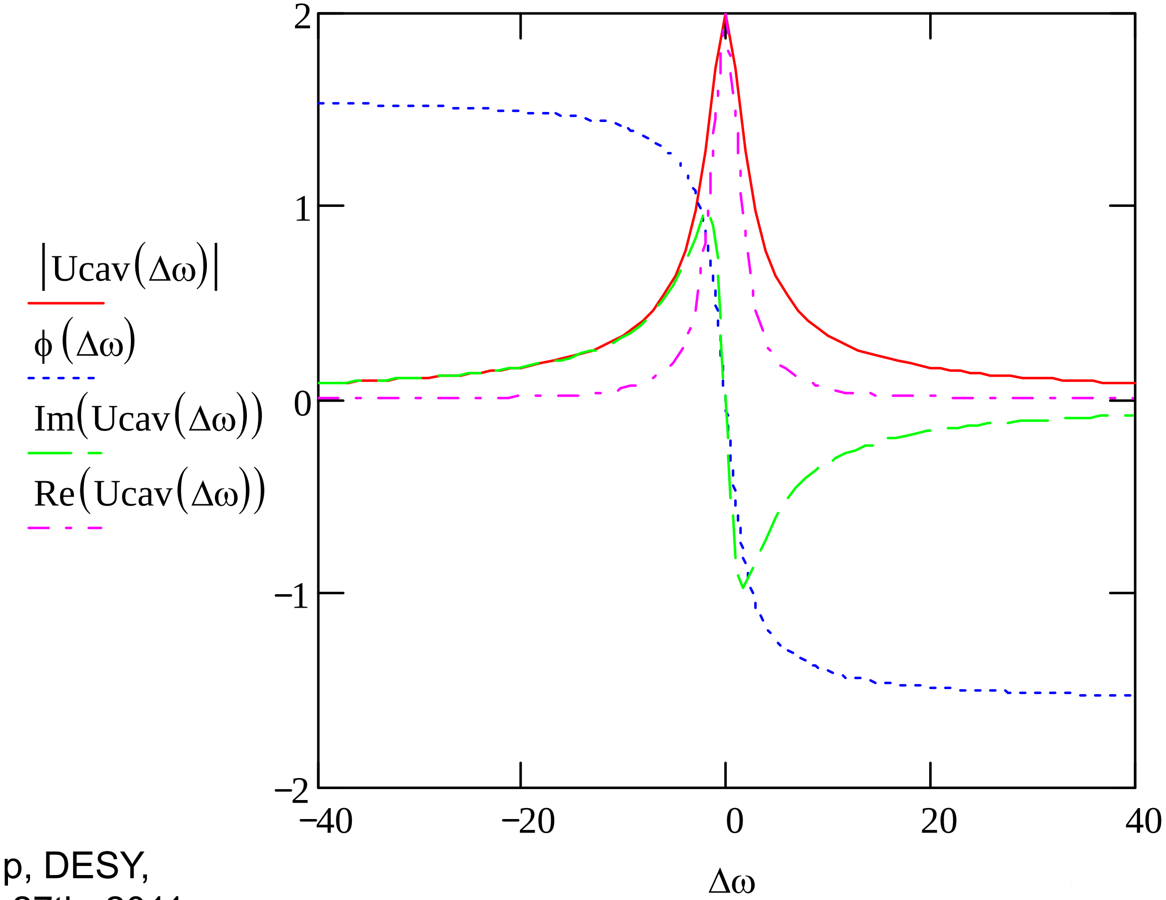
CAN BE COMPENSATED BY DETUNING THE CAVITY BY AN ANGLE WHICH IS ESSENTIALLY GIVEN BY THE RATIO OF BEAM INDUCED-TO TOTAL CAVITY VOLTAGE.

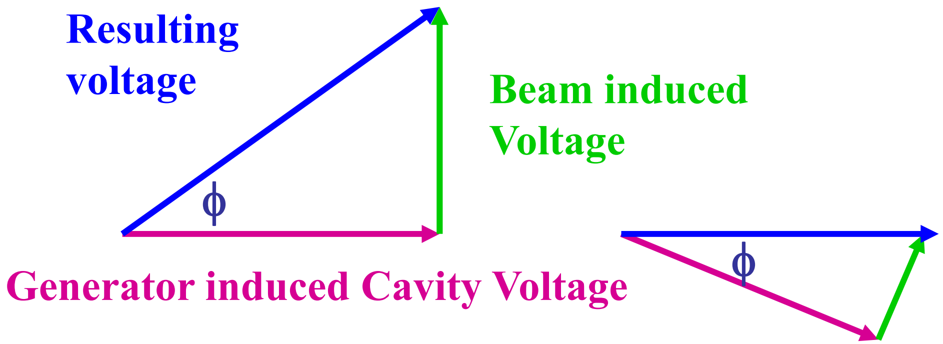
$$\omega = \omega_{CAV} + \Delta\omega$$

$$\frac{\omega}{\omega_{CAV}} = \sqrt{1 + \frac{R_S\left|\vec{I}_B\right|}{Q\left|\vec{U}_{CAV}\right|}\cos\phi_s} \approx 1 + \frac{R_S\left|\vec{I}_B\right|}{2Q\left|\vec{U}_{CAV}\right|}\cos\phi_s$$

The detuning angle is given by

$$\tan\phi = \frac{\vec{U}_{Beam}}{\vec{U}_{CAV}}\cos\phi_s = \frac{R_{SL}\left|\vec{I}_B\right|}{\left|\vec{U}_{CAV}\right|}\cos\phi_s \approx 2Q_L\frac{\Delta\omega}{\omega_{CAV}}$$





$$\hat{\vec{U}}_{CAV} \approx \frac{R_{SL}\left[\hat{\vec{I}}_G - \hat{\vec{I}}_B\right]}{1 + iQ_L 2\frac{\Delta\omega}{\omega_{CAV}}}$$

In the case of pure reactive beamloading, i.e. when U_{CAV} and U_B are in quadrature, the original cavity voltage can be restored by detuning the cavity. No additional generator power is needed in steady state, but for transient beamloading compensation sigificantly more power may be needed.

SOME REMARS ABOUT THE REFLECTED POWER WHICH MAY RESULT FROM B.L. COMPENSATION

SOLVING

$$\frac{\vec{I}_1}{\vec{U}_1} = Y = \frac{1}{R_A}\left[1 + \frac{R_s\left|\vec{I}_B\right|}{\left|U_{CAV}\right|}\sin\phi_s + i\frac{R_s\left|\vec{I}_B\right|}{\left|\vec{U}_{CAV}\right|}\cos\phi_s\right]$$

For an undetuned cavity, hence there is no B.L. compensation!

FOR $\vec{U}_r$ BY USING $\vec{U}_1 = \vec{U}_f + \vec{U}_r$ AND $\vec{I}_1 = \vec{I}_f - \vec{I}_r$ ONE FINDS FOR THE MAX. AMOUNT OF REFLECTED POWER DUE TO BEAMLOADING

$$P_{refl.} = \left|\hat{\vec{U}}_{refl}\right|^2 / 2R_A = R_s \hat{\vec{I}}_B^2 / 8$$

IT IS HALF THE POWER DELIVERED BY THE BEAM TO THE CAVITY+GENERATOR SYSTEM

EXAMPLE FOR THE AMOUNT OF REFLECTED POWER

FOR $R_S = 8M\Omega$ **AND** $\vec{I}_{Bav} = .1A$ **i.e.** $\vec{I}_B(\omega) = .2A$

$$P_{refl.} = 40kW$$

IF THE CAVITY HAS BEEN PROPERLY DETUNED FOR BEAMLOADING COMPENSATION; BUT $\beta = 1$

(that means matching without beam) THE REFLECTED

POWER BECOMES $P_{refl.} = R_s \hat{\vec{I}}_B^2 \sin^2 \phi_S / 8$

Summarizing the results of what we have learned so far we state that the beam sees the cavity shunt impedance in parallel with the transformed generator impedance. The resulting loaded impedance and Q-value are reduced by the factor $1/(1 + \beta)$.

The optimum coupling ratio between generator and cavity depends on the amount of energy taken by the beam out of the rf field.

The coupling is usually fixed and optimized for maximum beam current.

The amount of cavity detuning necessary for optimum matching, on the other hand, depends on the ratio of beam induced- to total cavity voltage.
The two latter issues depend also on the synchronous phase angle

LIMITS TO BEAMLOADING COMPENSATION BY DETUNING

We consider a hadron machine with $\beta = 1 \text{ and } \phi_s \approx 0$

Max. possible amount of detuning is limited. Instabilities arise for example when detuning comes close to the next revolution harmonic. But even before this point is reached the Robinson stability criterion may be violated.

Problems with transients due to finite tuner reaction time. **(Mechanical tuners, Ferrite tuners for large range and low Q, limiters)** $\tau_{CAV} = 2Q_L / \omega_{CAV}$ $\vec{U}_B \approx R_{SL}\vec{I}_B\left(1 - e^{-t/\tau}\right)$

After about 3τ the cavity voltage becomes

$$\left|\vec{U}_{CAV}\right| \approx R_{SL}\sqrt{\left|\vec{I}_g\right|^2 + \left|\vec{I}_B\right|^2}$$

Robinson‘s Stability Criterion [6]

Suppose there is a perturbation voltage $\vec{U}_{pertub.}(t) = \hat{\vec{U}}_{perturb.} e^{i\Omega t}$

such that $\vec{U}_{CAV}(t) = (\hat{\vec{U}}_{CAV} + \hat{\vec{U}}_{perturb.} e^{i\Omega t}) e^{i\omega_{CAV} t}$.

If Ω is close to Ω_S, a coherent synchrotron oscillation of all bunches with a damping constant D_S may be excited. This oscillation leads to two new frequency components $\pm\Omega$ in the beam current frequency ω : $\omega \pm \Omega$. These two components will induce additional rf voltages in the Cavity. Their amplitudes are unequal since $Z_{Cav}(\omega) \approx R_{SL} \Big/ (1 + iQ_L 2 \dfrac{\Delta\omega}{\omega_{CAV}})$ and hence

$$R(\omega + \Omega) \neq R(\omega - \Omega)$$

$R(\omega) = \mathrm{Re}\, Z(\omega)$

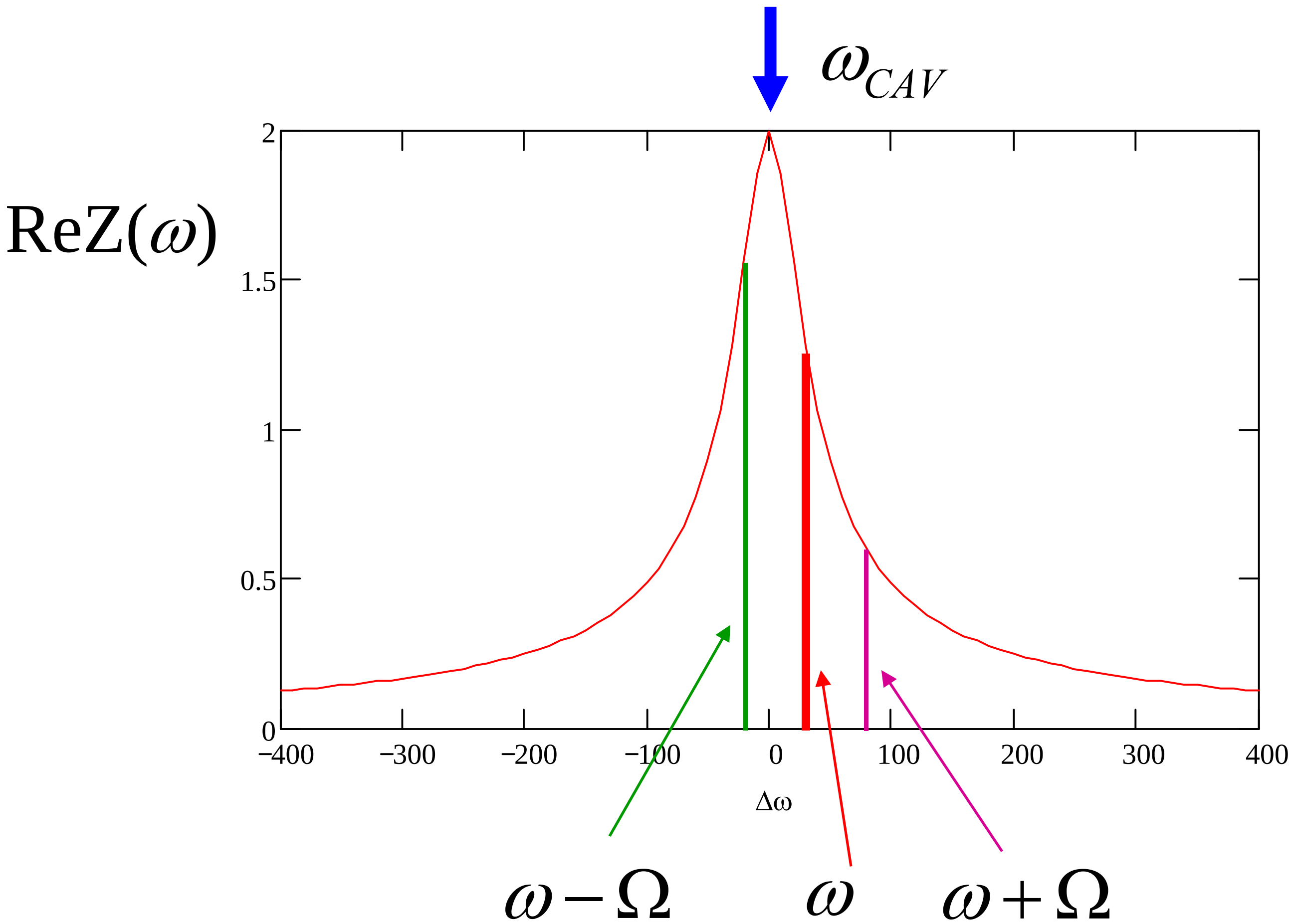
These two induced voltages act back on the beam current and when the induced voltage has the same phase and larger amplitude as the perturbation voltage the oscillation will grow and become instable.

The stability condition can be written as

$$\frac{R(\omega+\Omega)-R(\omega-\Omega)}{\vec{U}_{CAV}\sin\phi_S}\vec{I}(\omega)<4\frac{D_S}{\Omega_S}$$

$$\omega=\omega_{beam}=h\omega_{revolution}\qquad h=\mathrm{harmonicnumber}$$

This result from Piwinski [5] agrees also with the Robinson criterion. Additional cavity resonances and higher revolution harmonics may also lead to instabilities.

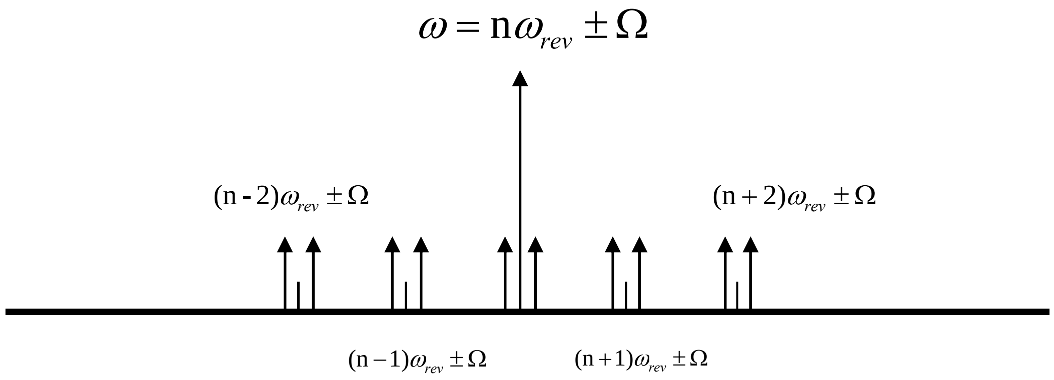


This is a stable situation, since $\omega_{CAV} < \omega$

and hence $R(\omega+\Omega) < R(\omega-\Omega)$

The situation becomes more complex when there are additional resonances or cavity modes close to other revolution harmonics of the beam current $\vec{I}_B(\omega)$ **.**

Also the spectrum can become much more complicated. Here only the fundamental synchrotron oscillation mode is drawn.



Damping of synchrotron oscillations can be achieved by

an active phase loop

an additional passive cavity with an appropriate resonance to change $R(\omega+\Omega)$ **and** $R(\omega-\Omega)$

Additional acceleration voltage with slightly diffent frequency to change the synchrotron frequency of diffrent bunches. Decoherence!

So, the possibilities of BL compensation by detuning alone are limited. In addition, transients can not be compensated by detuning. They can excite synchrotron oscillations of the beam which may result in emittance blow up and beam loss. Therefore transient beamloading compensation is a must.

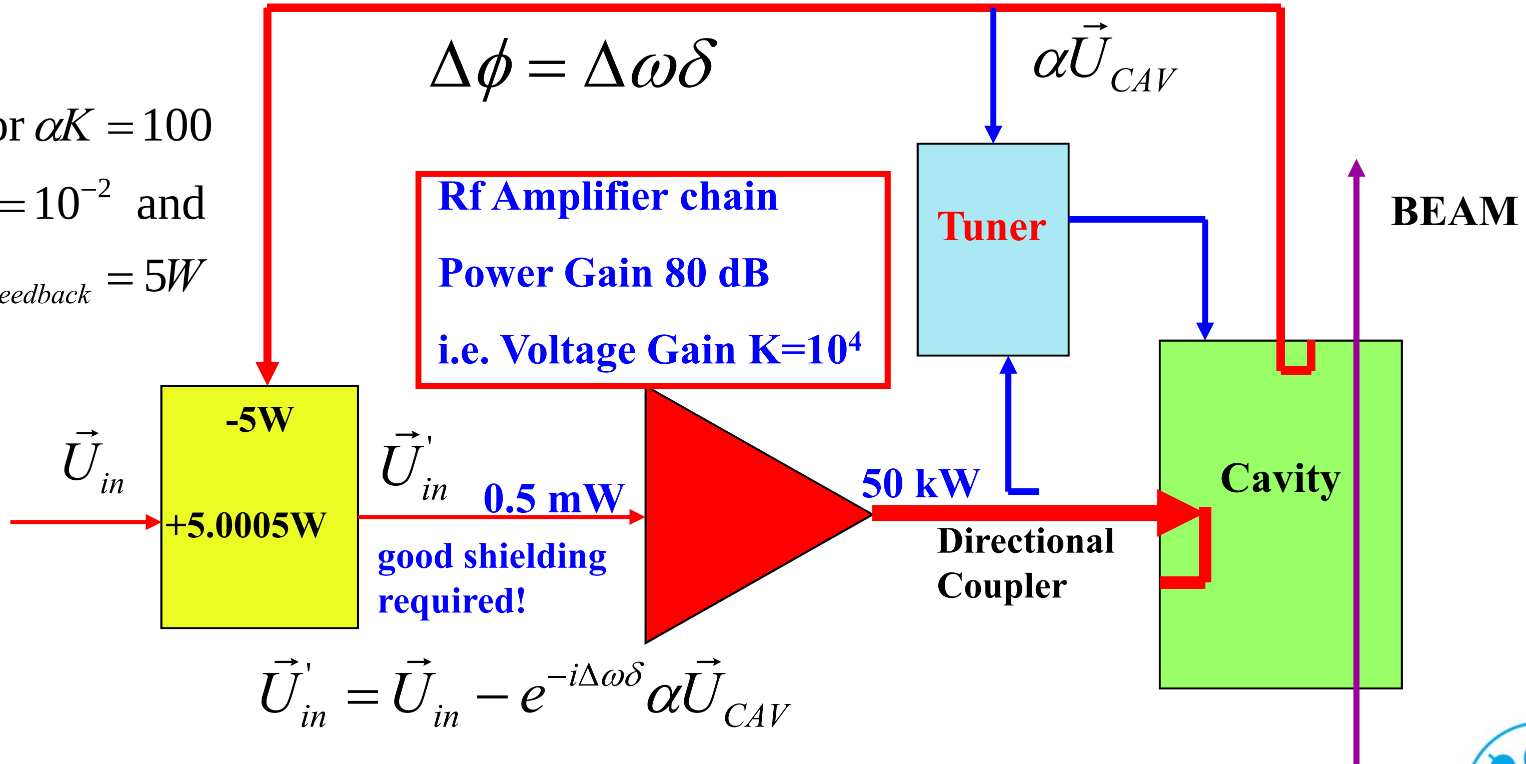
Individual phase and amplitude loops may become unstable due to the correlation of both quantities [7,8] .

$$\Delta\phi = \Delta\omega\delta$$

For $\alpha K = 100$

$\alpha = 10^{-2}$ and

$P_{Feedback} = 5W$



$$\vec{U}'_{in} = \vec{U}_{in} - e^{-i\Delta\omega\delta}\alpha\vec{U}_{CAV}$$

REDUCTION OF TRANSIENT BEAMLOADING BY FAST FEEDBACK

The total delay δ in the feedback-path is such that both signals have opposite phase at the cavity resonance frequency . For other frequencies there is a phase shift $\Delta\phi = \Delta\omega\delta$. Therefore the voltage at the amplifier input is now given by

$$\vec{U}'_{in} = \vec{U}_{in} - e^{-i\Delta\omega\delta}\alpha\vec{U}_{CAV}$$

With the voltage gain K of the amplifier we can rewrite our equ. for the cavity voltage taking feedback into account

$$\vec{U}_{CAV} \approx \frac{K\left[\vec{U}_{in} - e^{-i\Delta\omega\delta}\alpha\vec{U}_{CAV}\right] - \vec{U}_B}{1 + iQ_L 2\frac{\Delta\omega}{\omega_{CAV}}}$$

Rewriting shows the reduction of U_B due to feedback

$$\vec{U}_{CAV} \approx \frac{K\vec{U}_{in} - \vec{U}_B}{1 + iQ_L 2\frac{\Delta\omega}{\omega_{CAV}} + e^{-i\Delta\omega\delta}\alpha K}$$

We define the loop feedbackgain : $AF = \alpha K$

For $\Delta\omega = 0$ and $AF \Rightarrow\gg 1$ this reduces to

$$\vec{U}_{CAV} \approx \frac{\vec{U}_{in}}{\alpha} - \frac{\vec{U}_B}{\alpha K}$$

……RESULTS IN A REDUCTION OF U_B, NOISE VOLTAGE ETC. IN THE CAVITY BY AN AMOUNT EQUAL TO THE OPEN LOOP FEEDBACKGAIN

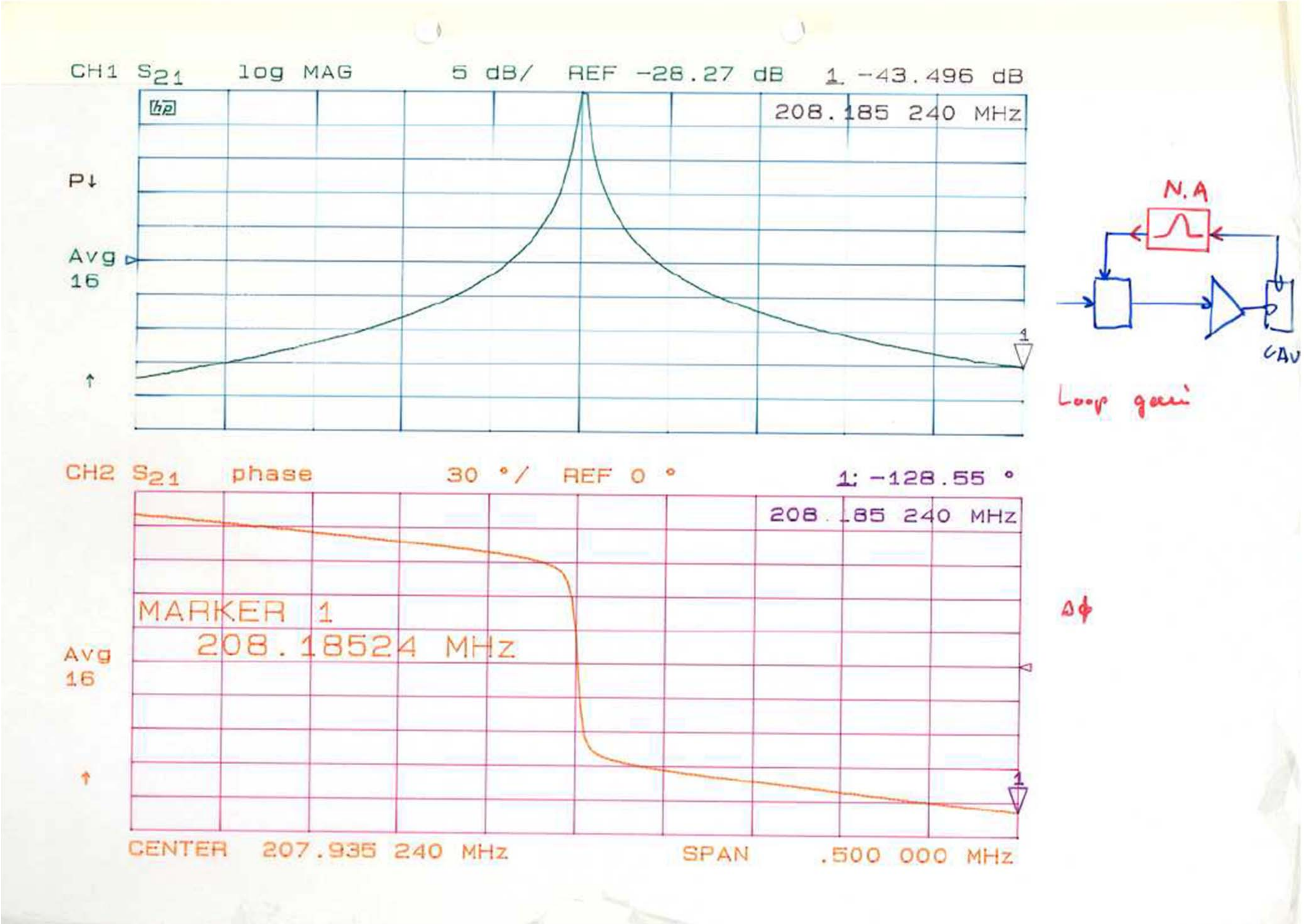
The reduction of the beam induced cavity voltage by the factor AF due to the feedback is equivalent to a similar reduction of the cavity shunt impedance as seen by the beam.

$$Z_L \approx \frac{R_{SL}}{1+iQ_L 2\frac{\Delta\omega}{\omega_{CAV}}} \rightarrow \frac{R_{SL}}{1+iQ_L 2\frac{\Delta\omega}{\omega_{CAV}} + A_F e^{-i\Delta\omega\delta}}$$

The price for this fast reduction of beam-loading is the additional amount of generator current , which is needed to almost compensate the beam current in the cavity. In terms of additional transmitter power this reads

$$P' = R_S \hat{\vec{I}}_B^2 / 8$$

Since there is no change in cavity voltage due to *P'*, this power will be reflected back to the generator, which has to have a sufficiently large plate dissipation power capability. Otherwise a circulator is needed. This critical situation of additional rf power consumption and reflection lasts, however, only until the tuner has reacted, and it may be minimized by predetuning. If there are "continuos transients" there is continously higher generator power needed!



CH1 S21 log MAG 5 dB/ REF -28.27 dB 1 -43.496 dB
208.185 240 MHz
Avg 16
CH2 S21 phase 30 °/ REF 0 ° 1: -128.55 °
208.185 240 MHz
MARKER 1
208.18524 MHz
Avg 16
CENTER 207.935 240 MHz SPAN .500 000 MHz
N.A
CAV
Loop gain
Δφ

THE MAXIMUM POSSIBLE FEEDBACKGAIN IS LIMITED BY THE SIGNAL DELAY δ AROUND THE LOOP

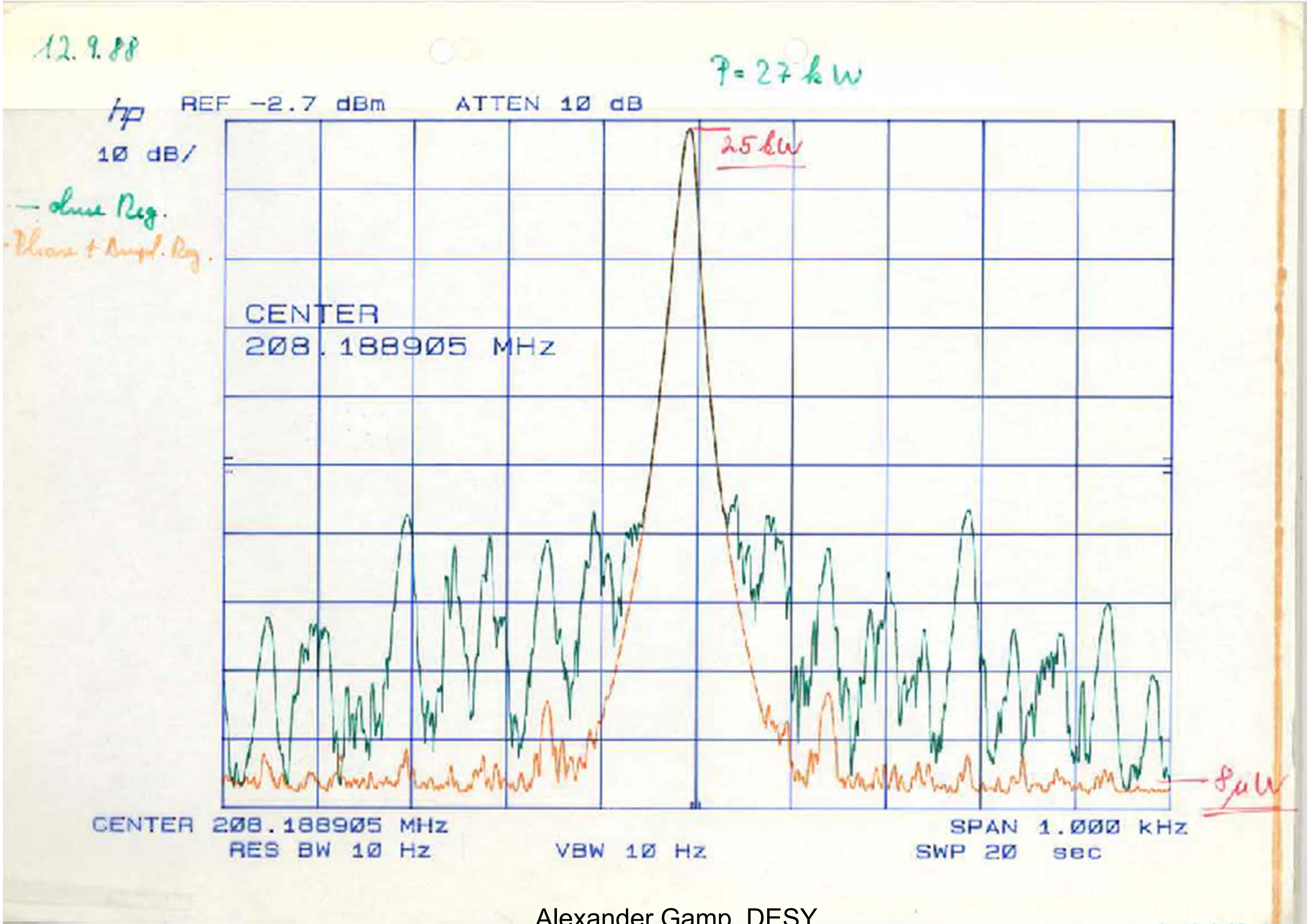
$$\left|A_F\left(\Delta\omega_{\max}\right)\right| \approx \frac{\alpha K}{1+iQ_L 2\dfrac{\Delta\omega_{\max}}{\omega_{CAV}}} \leq 1$$

$$\delta\Delta\omega_{\max} = \pm\frac{\pi}{4}$$ **THEN** $$\Delta\phi_{\max} = \pm 135^0$$

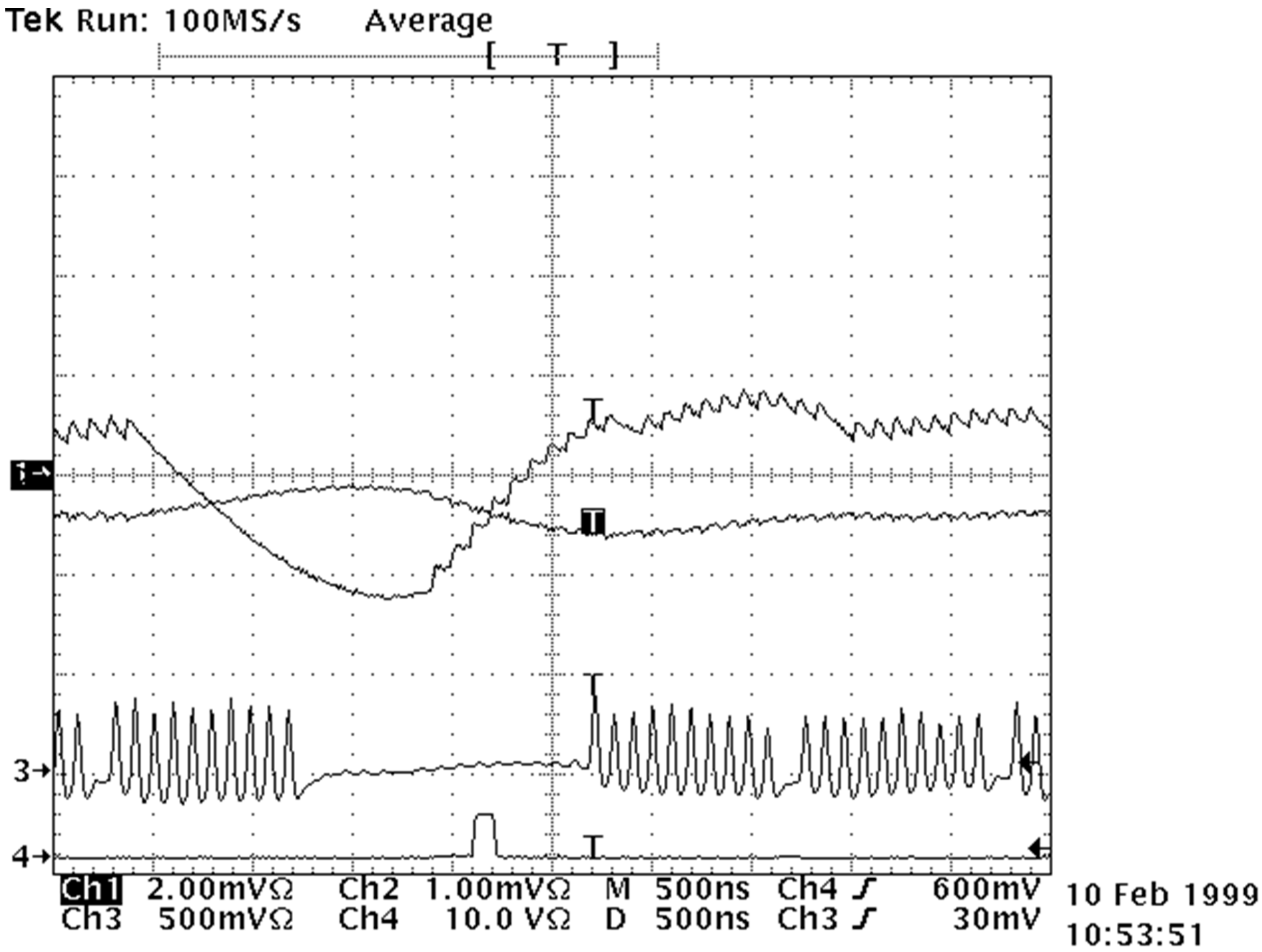
AND NYQUIST'S STABILITY CRITERION IS REACHED

$$A_F = \frac{Q_L}{4 f_{CAV}\delta}$$

A fast feedback loop of gain 100 had been realized at the HERA 208 MHz proton rf system. With a loaded cavity Q_L of ≈ 27000 the maximum tolerable delay, including all amplifier stages and cables, is δ = 330 ns. Therefore all rf amplifiers have been installed very close to the cavities in the HERA tunnel.



12.9.88
P = 27 kW
hp
REF -2.7 dBm
ATTEN 10 dB
10 dB/
25 kW
CENTER
208.188905 MHz
8 µW
CENTER 208.188905 MHz
SPAN 1.000 kHz
RES BW 10 Hz
VBW 10 Hz
SWP 20 sec



Transient behaviour of the cavity voltage under the influence of fast feedback. (Courtesy of Elmar Vogel [9])

SUMMARY FEEDBACK

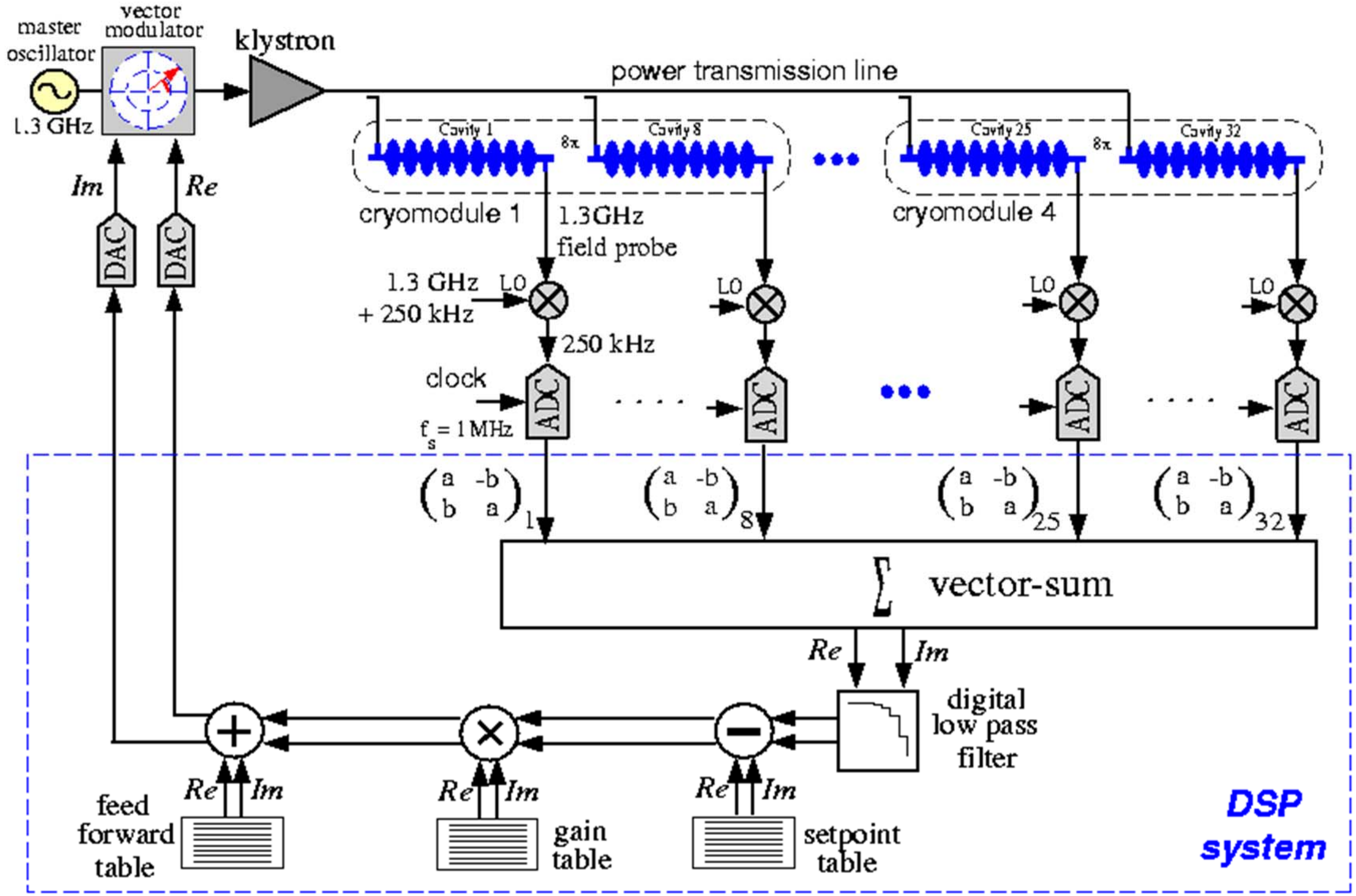
Fast feedback reduces the resonant cavity impedance as seen by an external observer (usually the beam) by the factor 1/*AF* .

It is important to realize that any noise originating from other sources than from the generator itself, especially amplitude- and phase noise from the amplifiers, will be reduced by the factor 1/*AF* because the cavity signal is directly compared to the generator signal at the amplifier input stage.

Care has to be taken that no noise be created, by diode limiters or other non-linear elements, in the path where the cavity signal is fed back to the amplifier input. This noise would be added to the cavity signal by the feedback circuit. This becomes especially important for digital F.B. systems, where the digital hardware (Downconverters, ADCs, DSPs…) is part of the F.B. loop.

Amongst the great advantages of the digital technology are very easy amplitude and phase control of each channel (analog elements are very expensive), applying easily calibration procedures and factors and…and…, but also cons like very high complexity.

FEEDBACK AND FEED-FORWARD APPLIED TO SUPERCONDUCTING CAVITIES



Courtesy S. Schilcher [10]

The RF power is essentially carried away by the beam who comes essentialy on crest of the cavity voltage. So, detuning does not help and there is a complete mismatch during cavity filling.

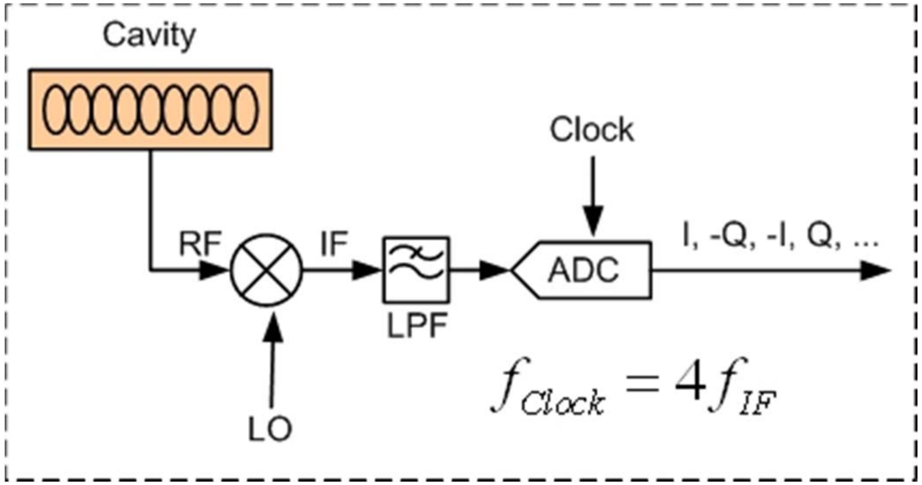
In contrast to the previous example, where all the rf power was essentially dissipated in the normally conducting cavity walls, the power needed to build up the rf cavity voltage in the superconducting cavities is only a few hundred watts.

A high efficiency 10 MW pulsed power multibeamklystron has been developed for this project.

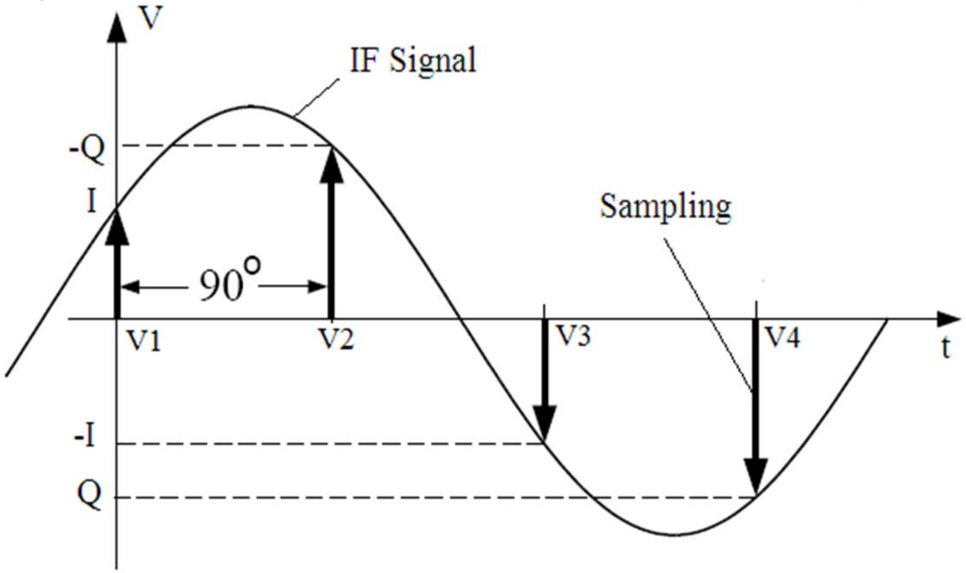
The rf seen by the beam corresponds to the vector sum of all cavity signals. It must be generated by a powerful low level RF control system involving downconversion of the cavity signals to 250 kHz intermediate frequency signals and sampling in time steps of 1 μs (for example) . This results in

IQ detection

IQ Sampling



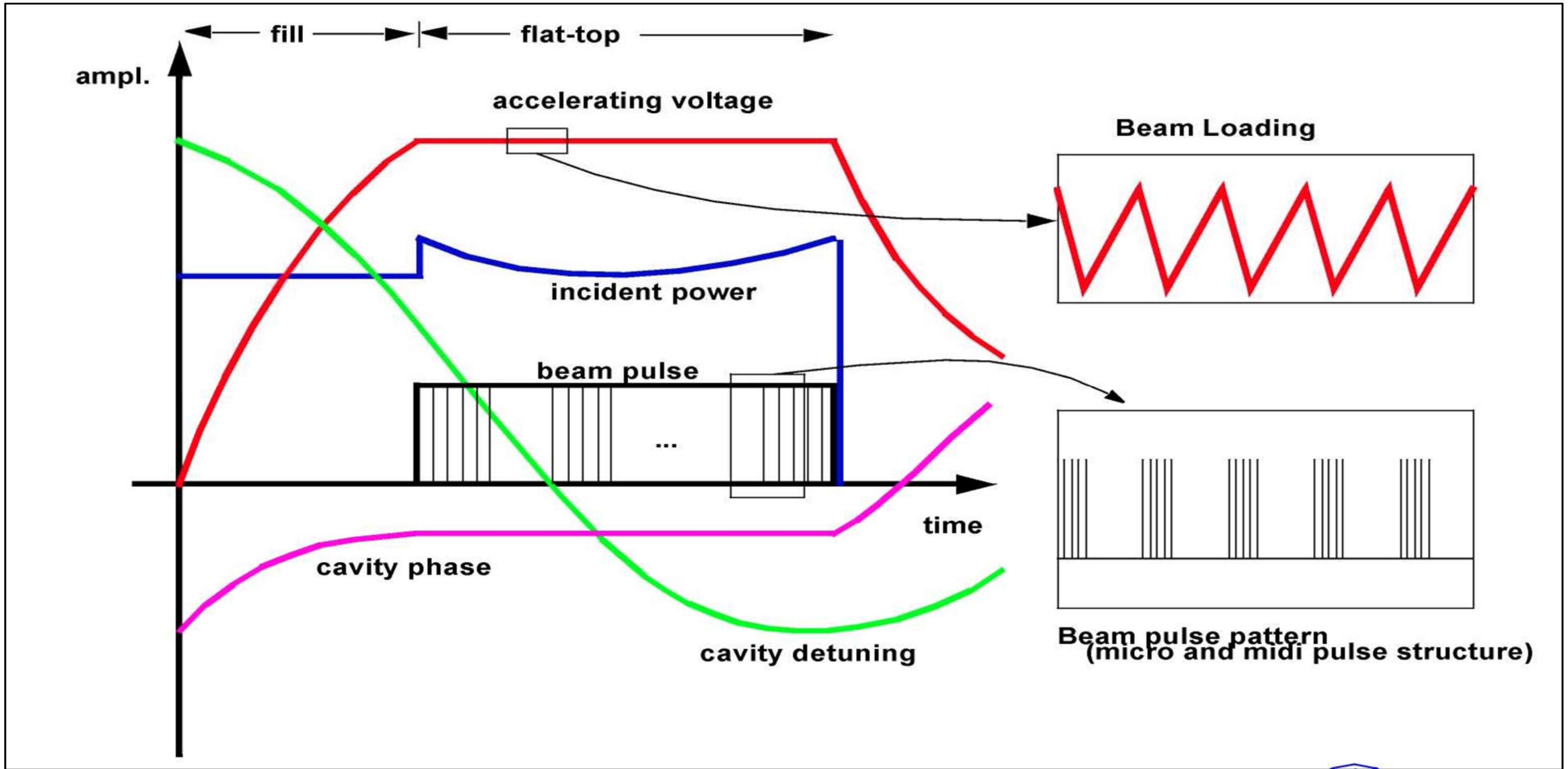
- **Digital I/Q detection**
- **IF and clock signal should be synchronized**
- **Alternating sample give I and Q components of the cavity field**



Courtesy S. Simrock [11]

In addition to feedback a (learning) feedforward correction can be added for correction of repetitive errors like a systematic decrease of beam current during the pulse due to some property of the electron source, or a systematic change of the cavity resonance frequency during the pulse. This effect exists indeed. The mechanical forces resulting from the strong pulsed rf field in the superconducting cavities cause a detuning of the order of a few hundred Hz at 25 MV/m. This effect is called Lorentz force detuning. Cavities can be pretuned to a fixed optimum value. In addition there are dynamic piezo-electric tuners which are active during the pulse.

The additional power resulting from Lorentz Force Detuning can be minimized by predetuning the cavity



Courtesy S. Simrock [11]

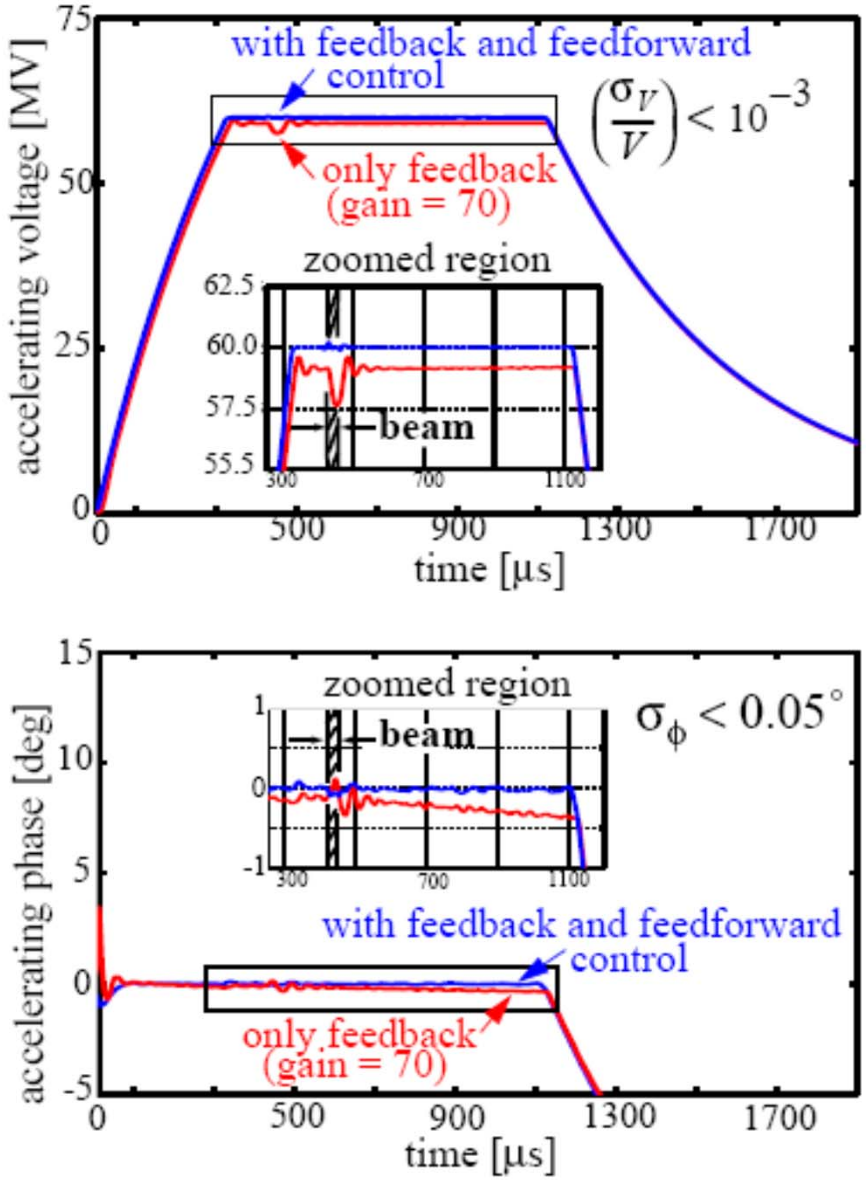


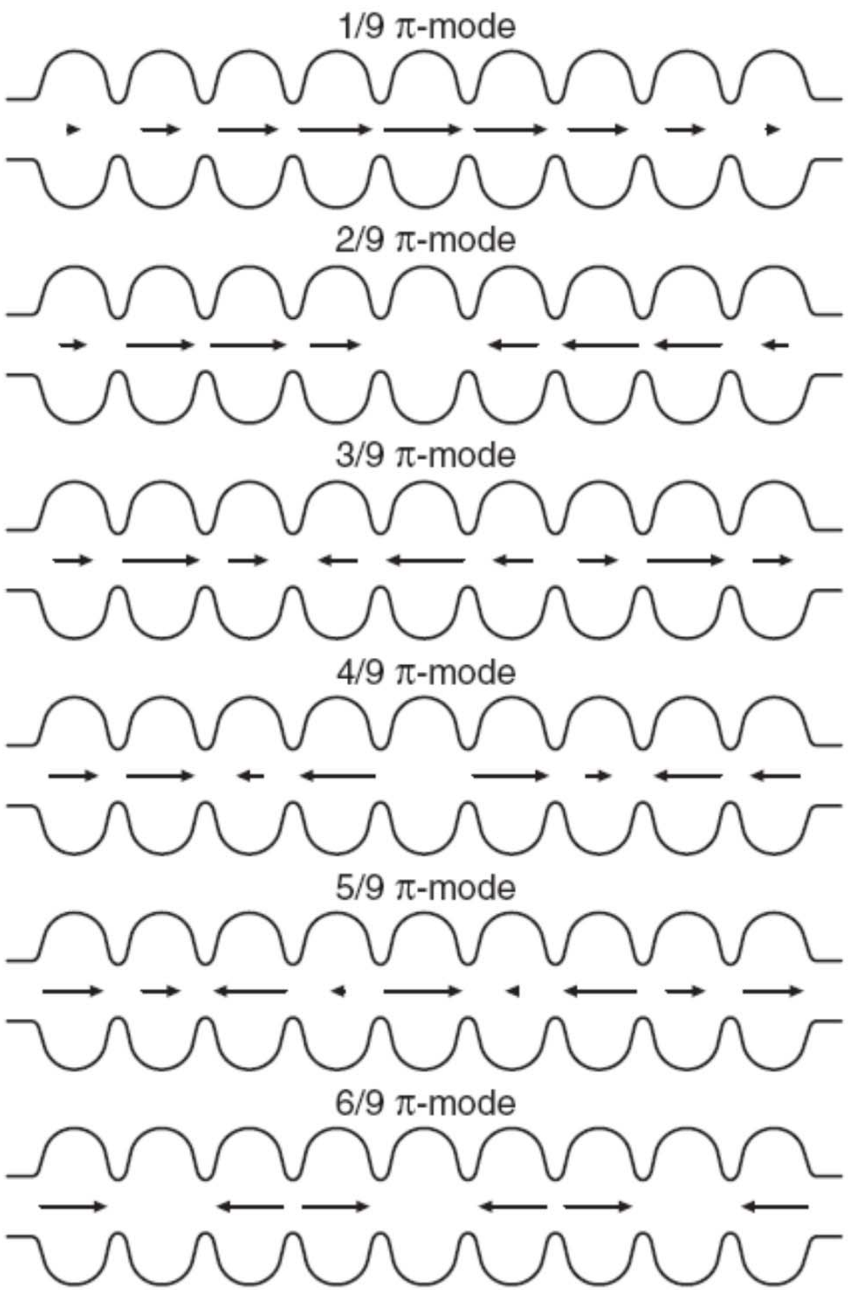
Figure 3: RF control system performance without and with adaptive feedforward.

Fig. taken from [12]

The Superconducting 9-cell FLASH and XFEL and ILC Cavities

Alexander Gamp, DESY,
CAS, Bilbao, May 27th, 2011

Phys. Rev. ST Accel. Beams **10,** 052001 (2007)



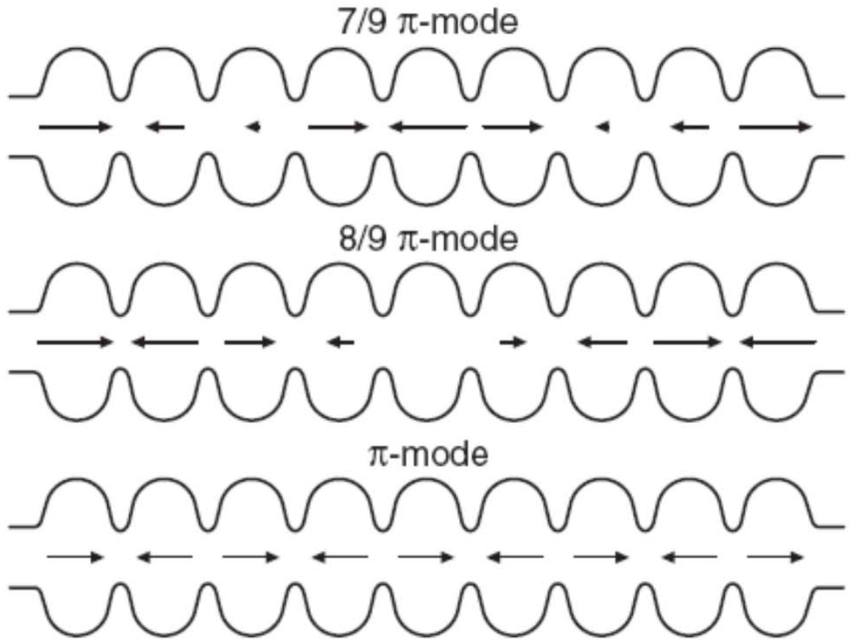


FIG. 2. The rf field amplitudes of the normal and fundamenta modes (FMs) in 9-cell cavities.

Fig. taken from E. Vogel [13] , Phys. Rev.ST Accel. Beams **10,** 052001 (2007)

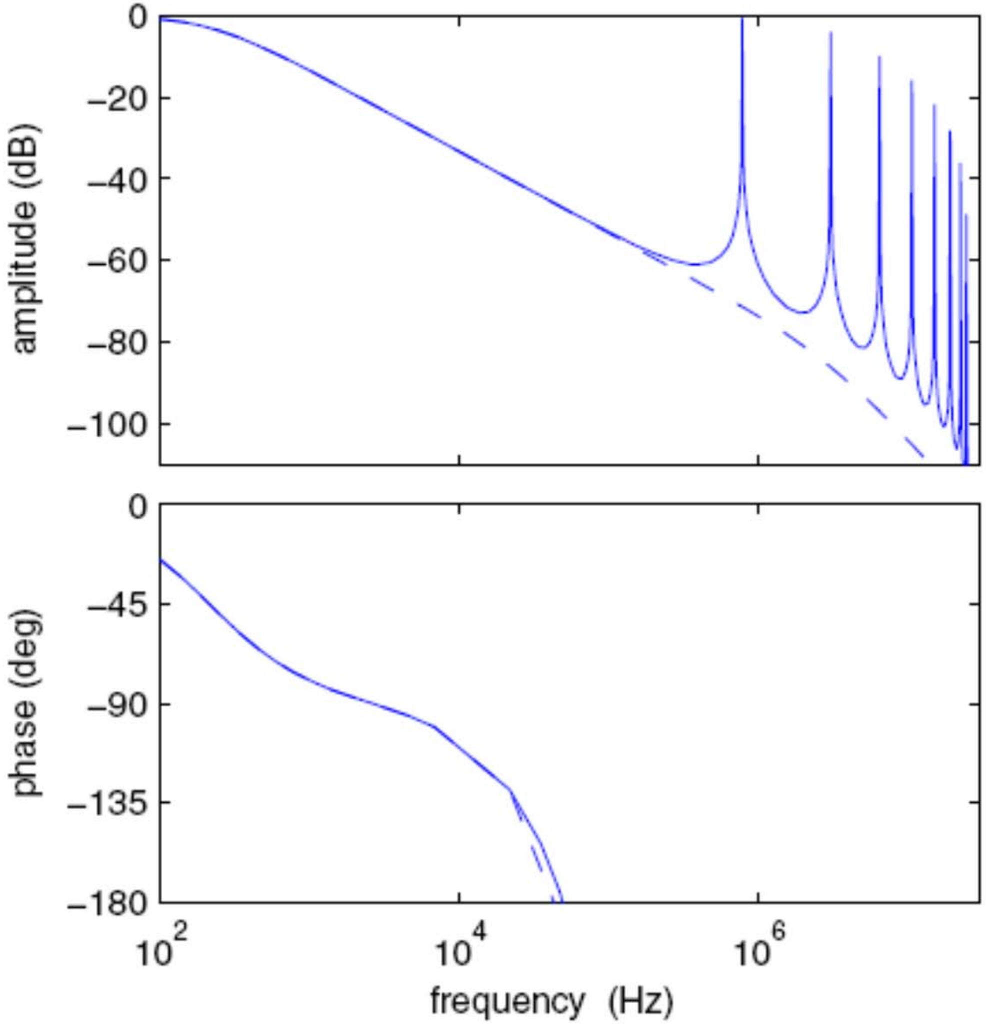


FIG. 4. (Color) Bode plot of the continuous single cavity open loop transfer function (7) with unity gain ($G = 1$) and loop delay $\tau_{\text{loop}} = 5\ \mu\text{s}$. At the dashed line all FMs with exception of the π-mode have been omitted.

Fig. taken from E. Vogel [13]

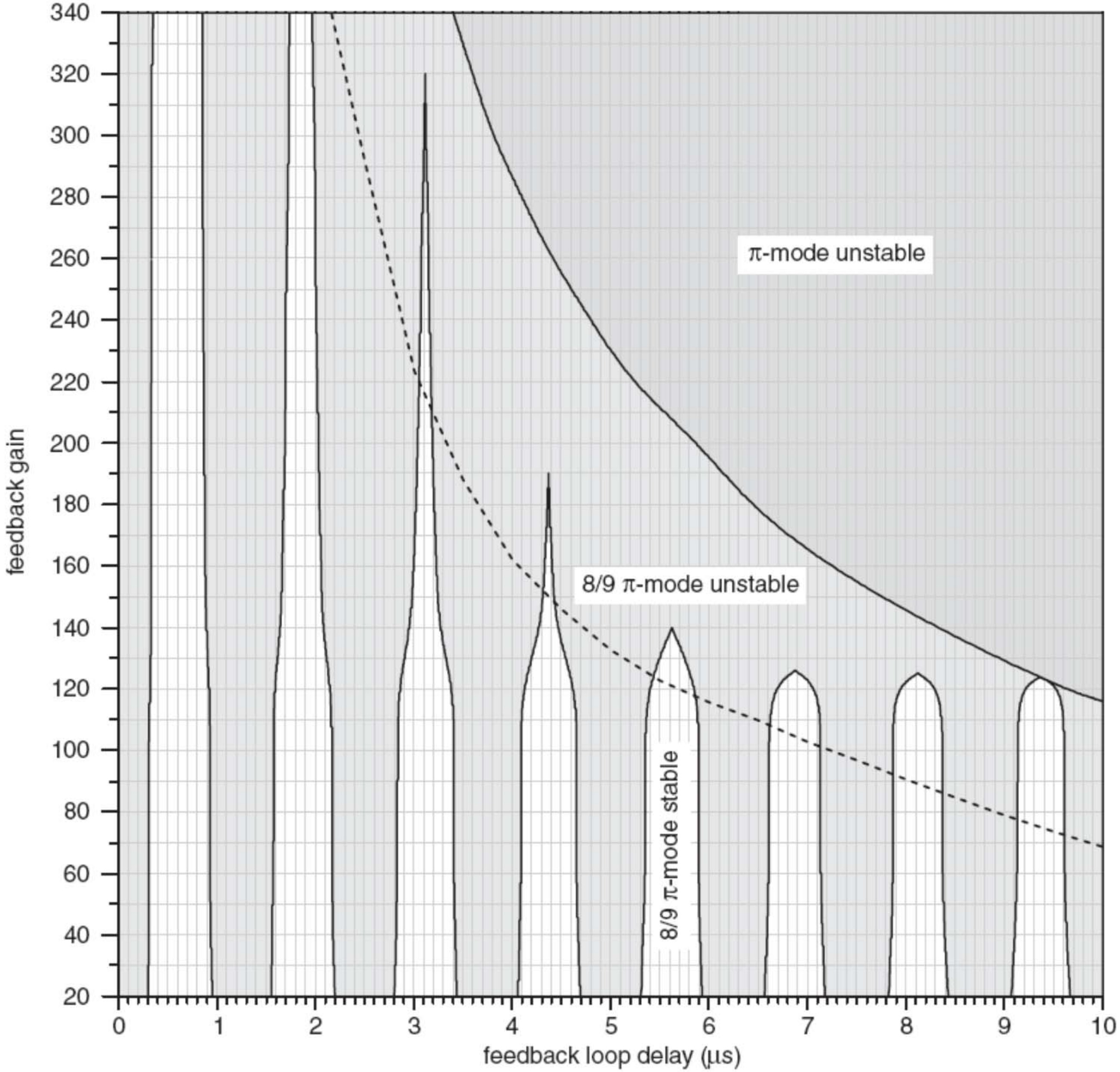
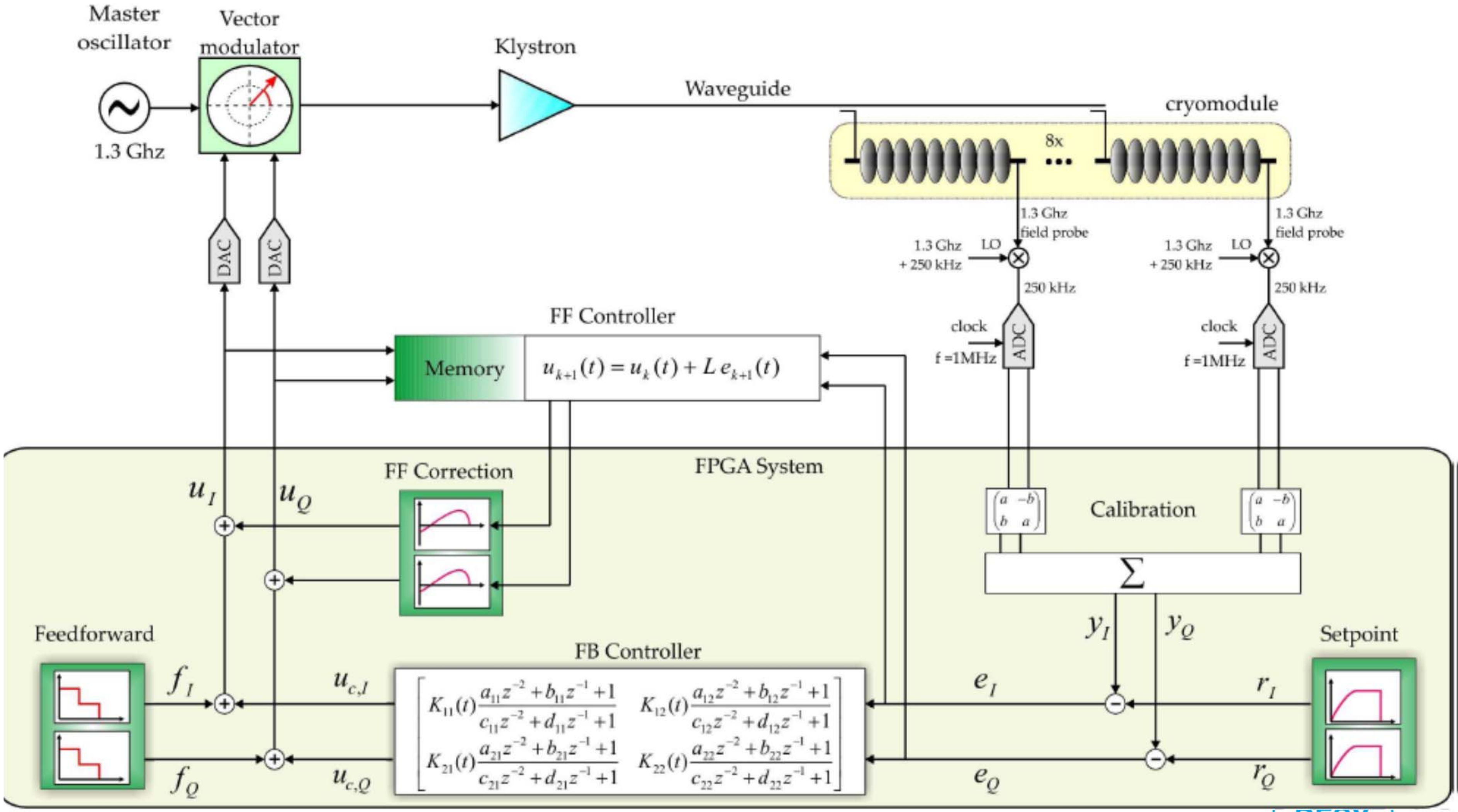


Fig. taken from E. Vogel [13]

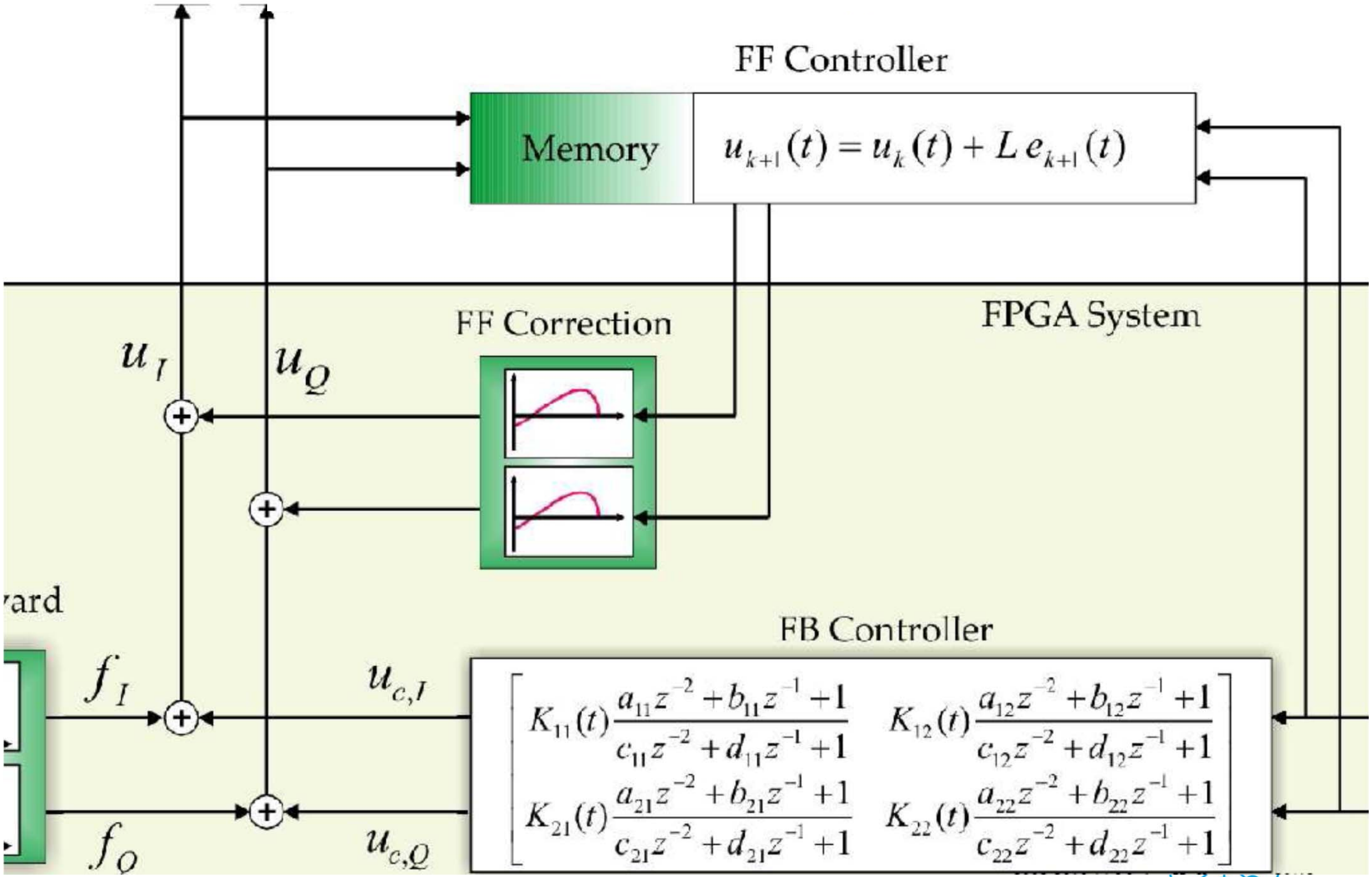
FIG. 9. Stability chart for a single cavity digital control loop operating with 1 MHz sample rate. For simplicity, 800 kHz was chosen for the $\frac{8}{9}\pi$-mode difference frequency and equal coupling for both modes ($\beta_{(8/9)\pi} = \beta_\pi$). In practice, the stable areas are more expanded, see text. A phase margin larger than 30 deg with respect to the π-mode is obtained by choosing gain values below the dashed line.

The MIMO Controller

Multiple in Multiple Out



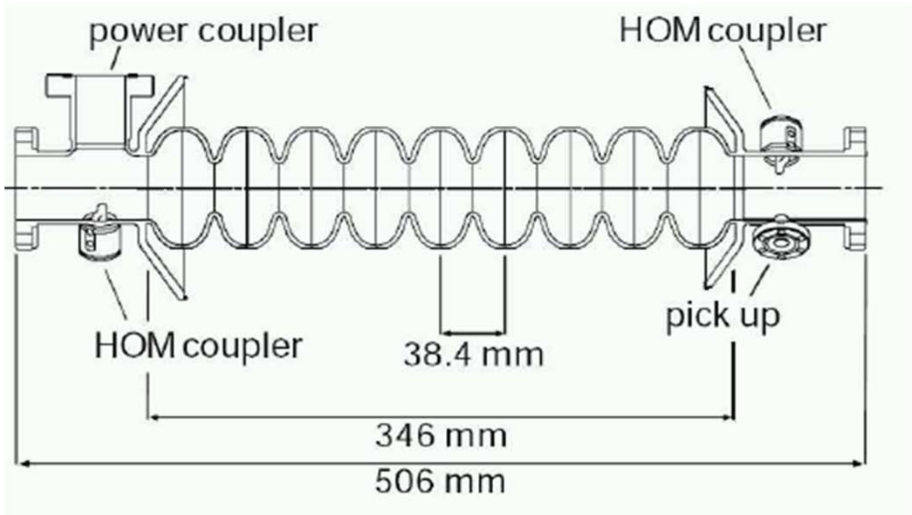
Courtesy Christian Schmidt [14] , Thesis, DESY, and TU Hamburg Harburg 2010

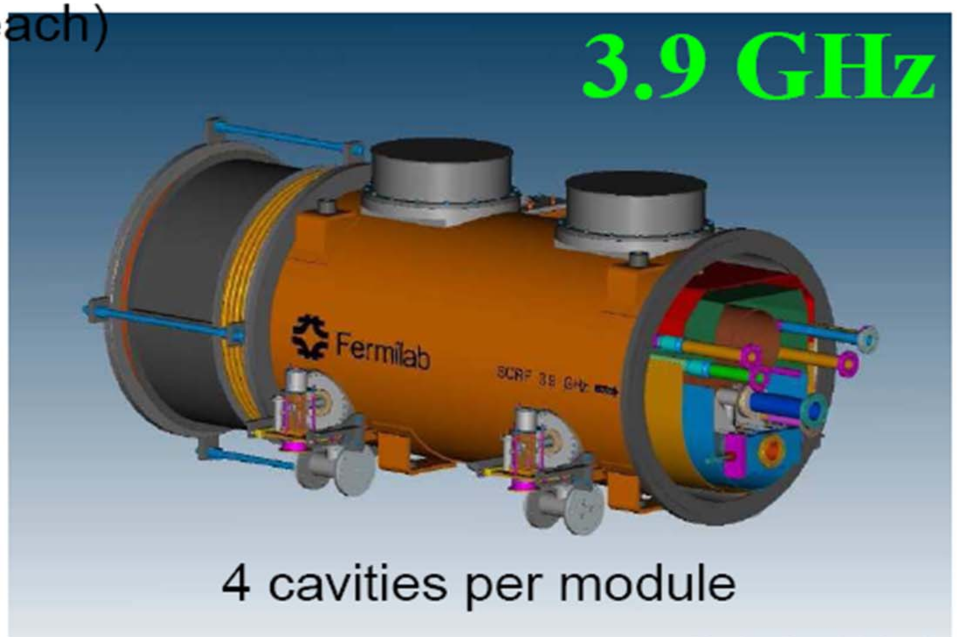


Courtesy Christian Schmidt [14] , Thesis, DESY, and TU Hamburg Harburg 2010

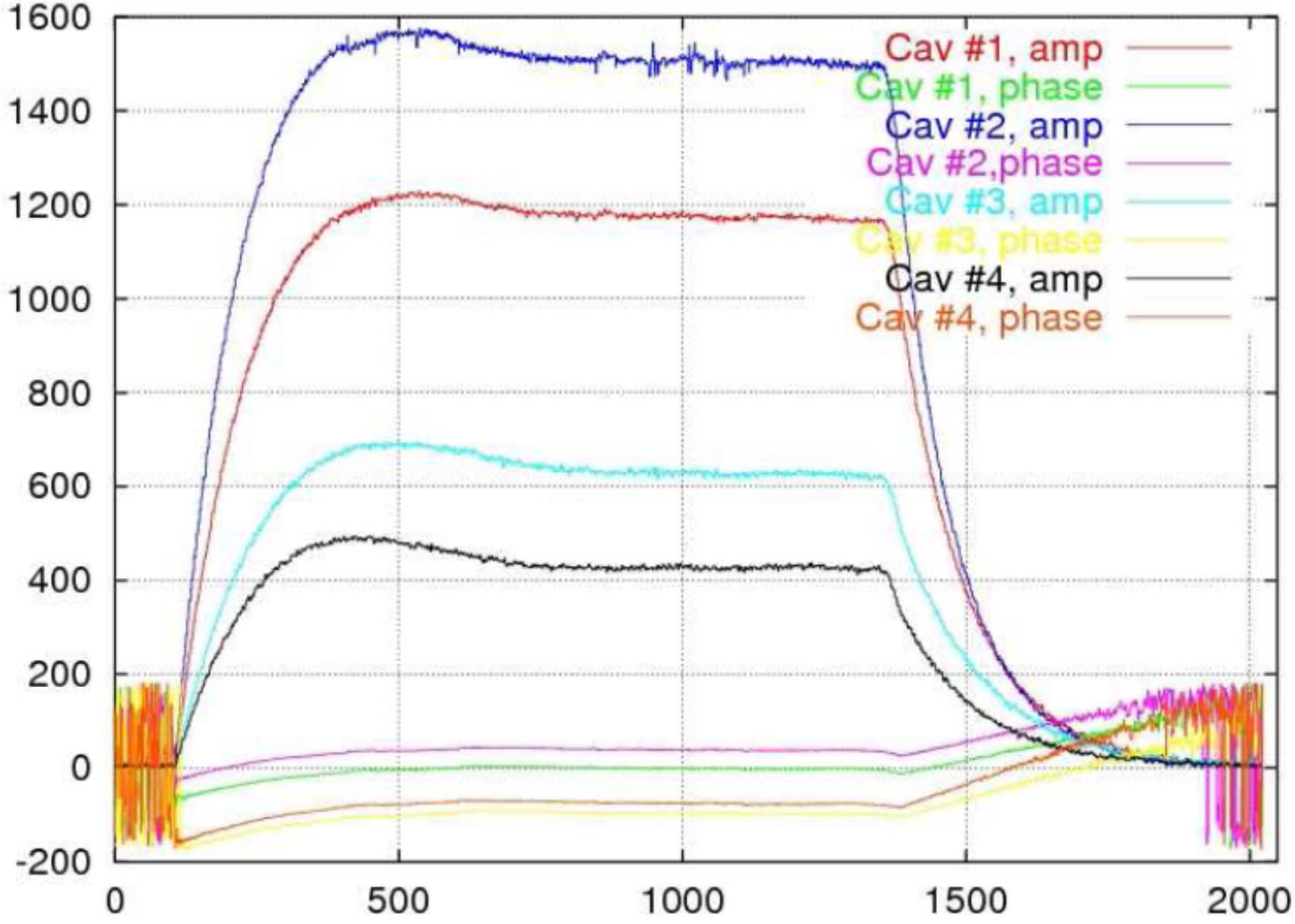
Third Harmonic System

- To improve **Bunch Compression**,
- A **peak current** of >2kA can be realized within >200 fs.
- New possibilities: pre-requiste for all **seeding** schemes.
- The System for FLASH (4 cavities)
- 2 Systems for XFEL (8 cavities each)



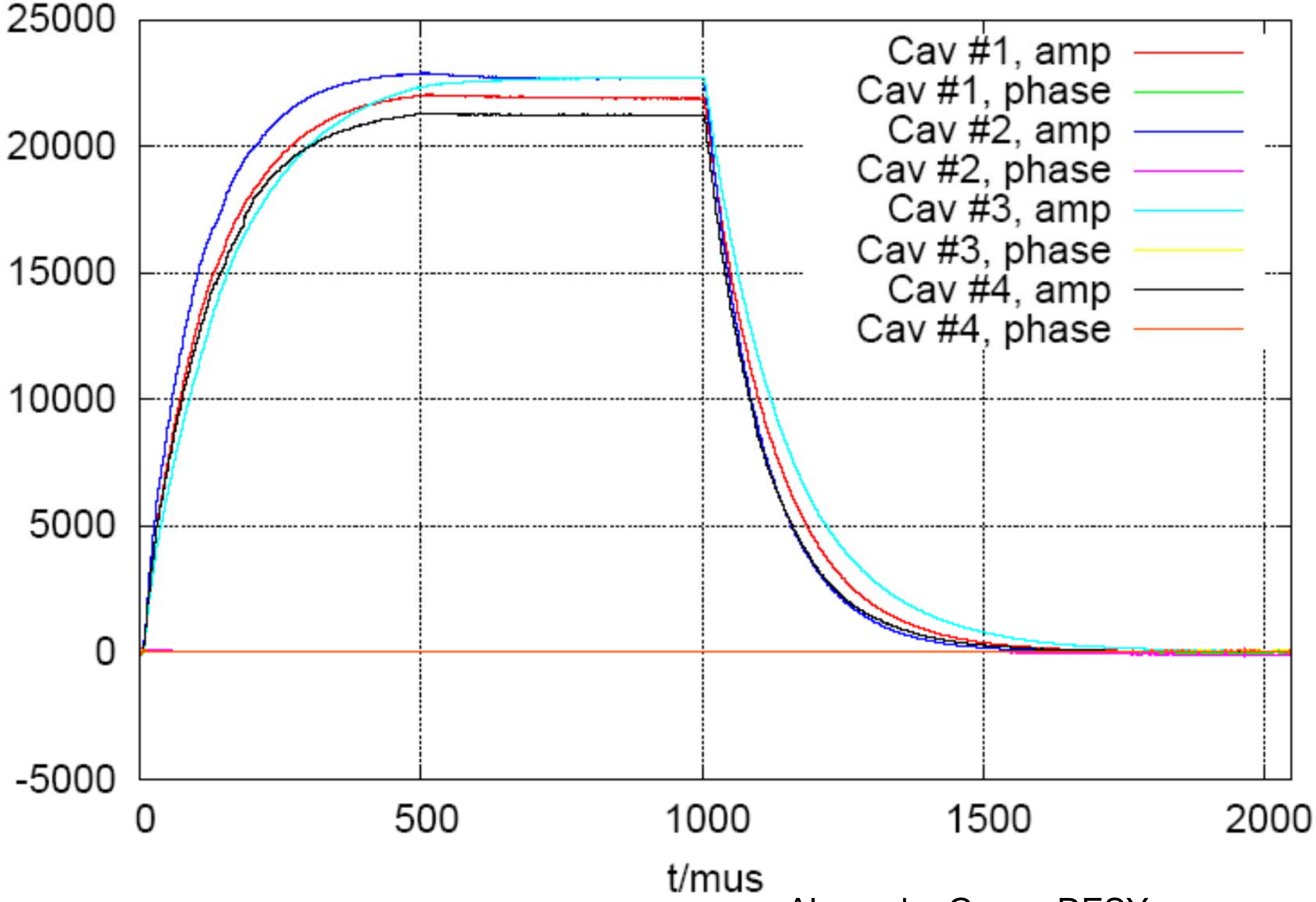


TESLA type cavities have been scaled down in size to fit the 3.9 GHz.
All auxiliaries like coupler, HOM coupler, frequency tuner, etc..., are scaled as well.
Most of this work was done by H.Edwards et al. / FNAL.



Courtesy Markus Hoffmann [15]

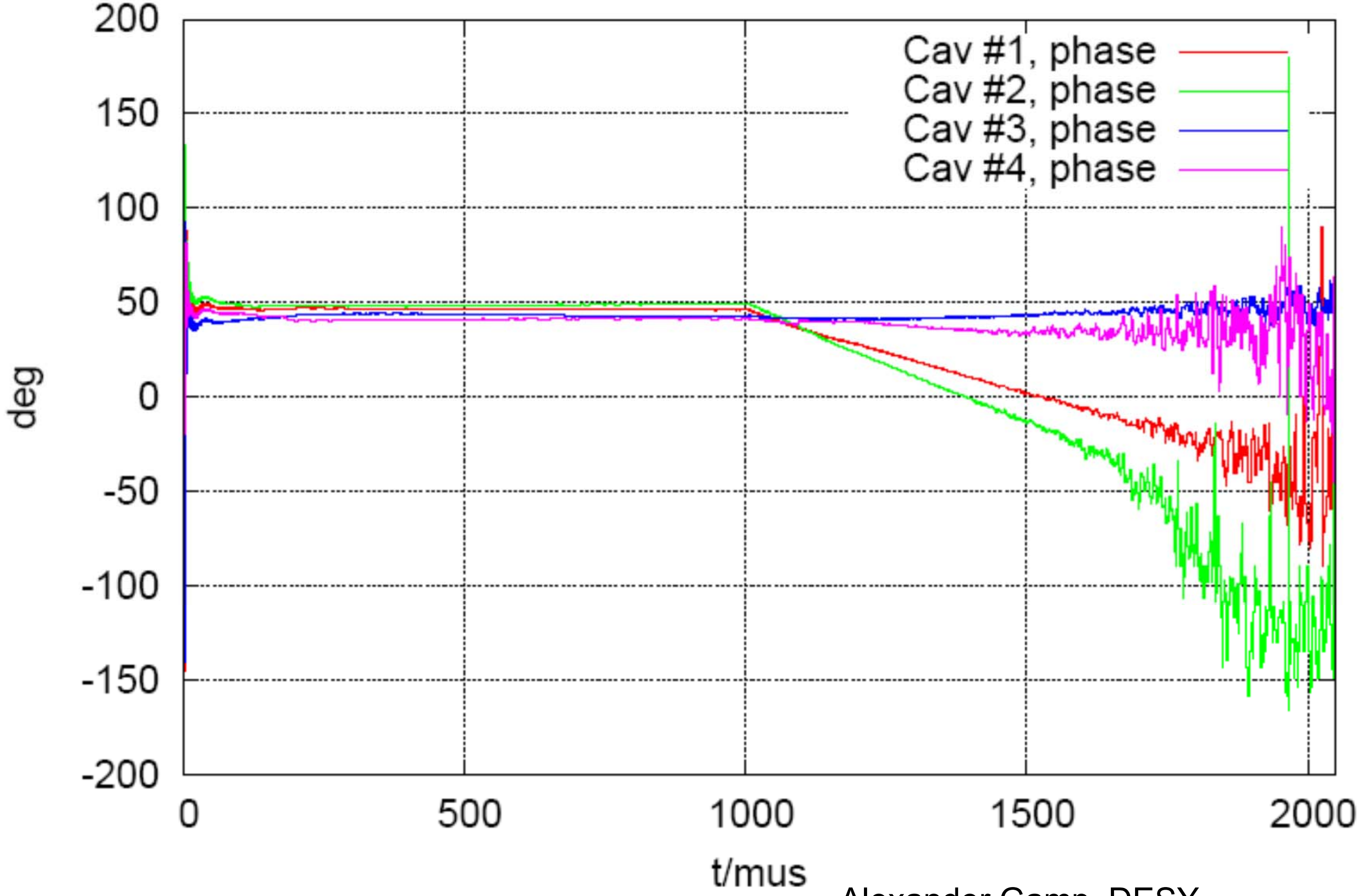
3.9 GHz Module	Required Ampl. Stability	Measured with MIMO Controller
$\Delta A / A$	10^{-4}	$1.3 * 10^{-5}$



Courtesy Markus Hoffmann [15]

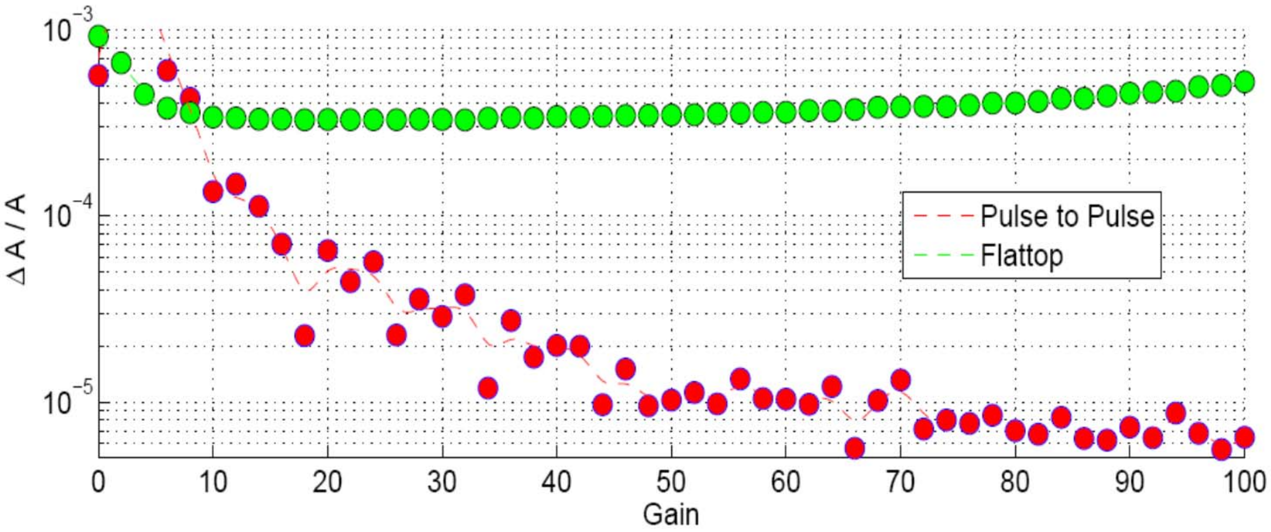
3.9 GHz Module **Required Phase Stability** **Measured with MIMO Controller**

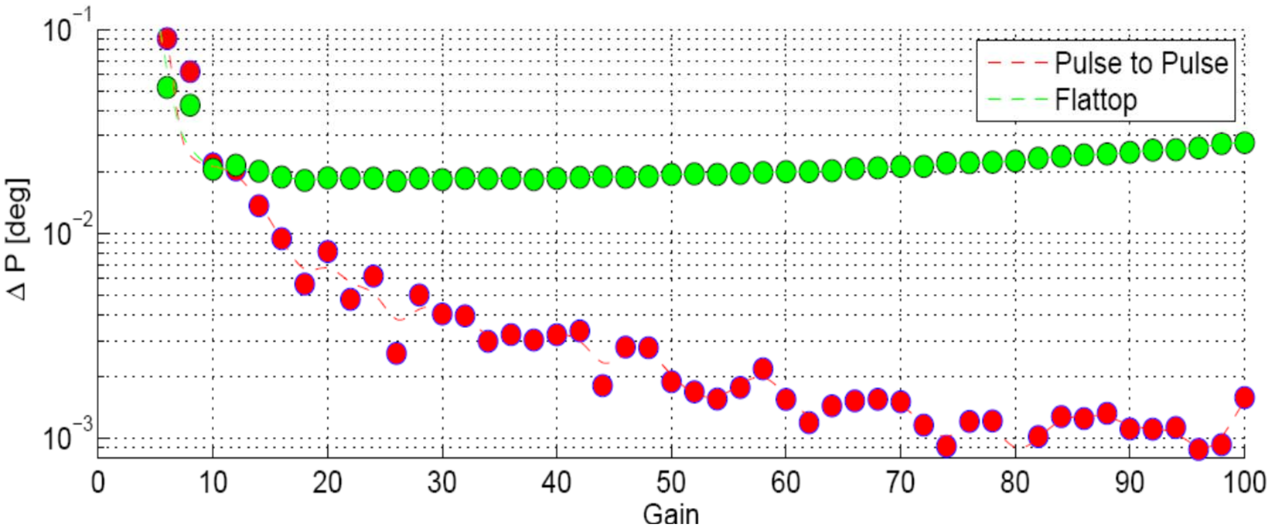
$\Delta\phi$	$.03^0$	$.003^0$



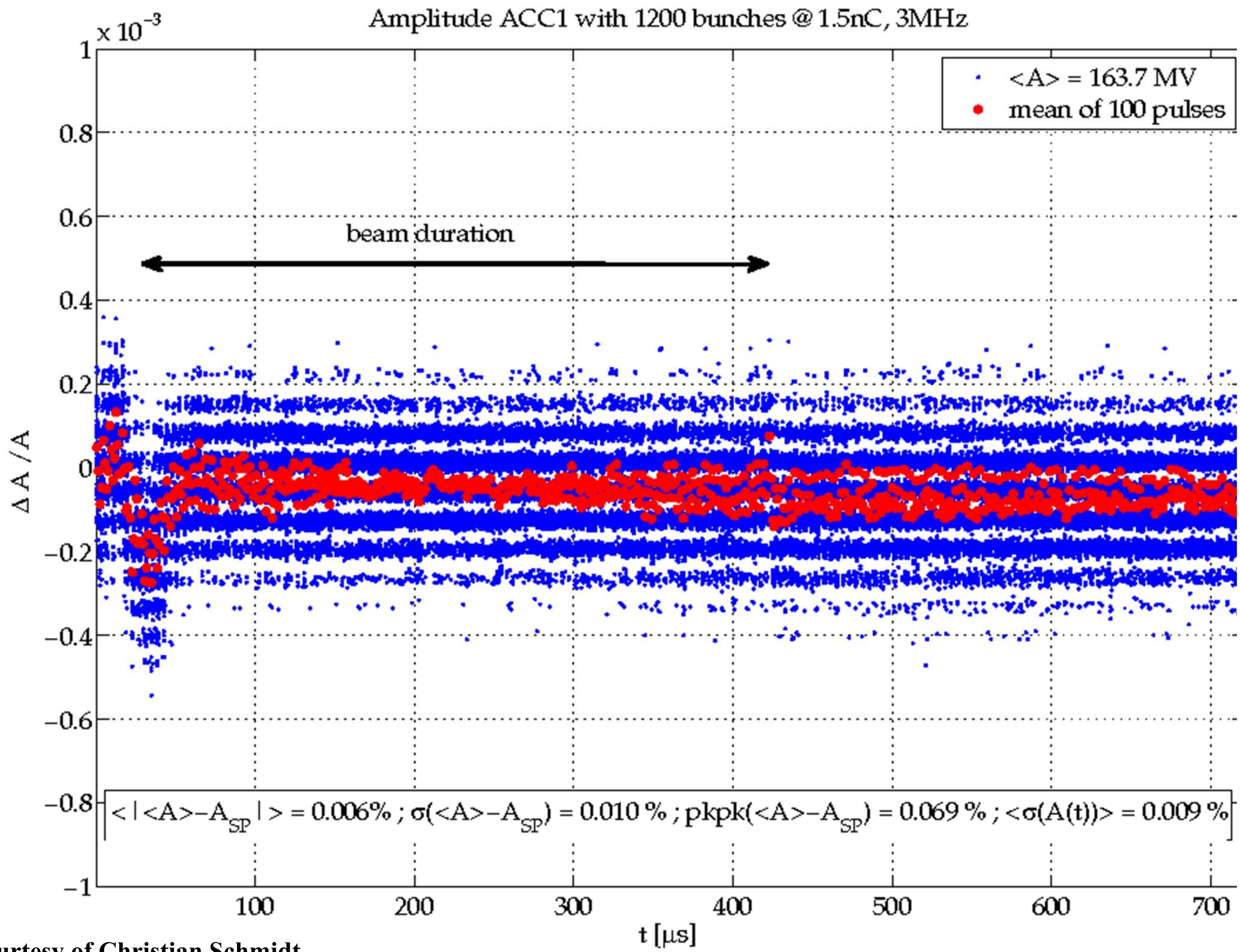
Courtesy Markus Hoffmann [15]

Improvement in Pulse to Pulse results is due to the effect of averaging over the measurement noise

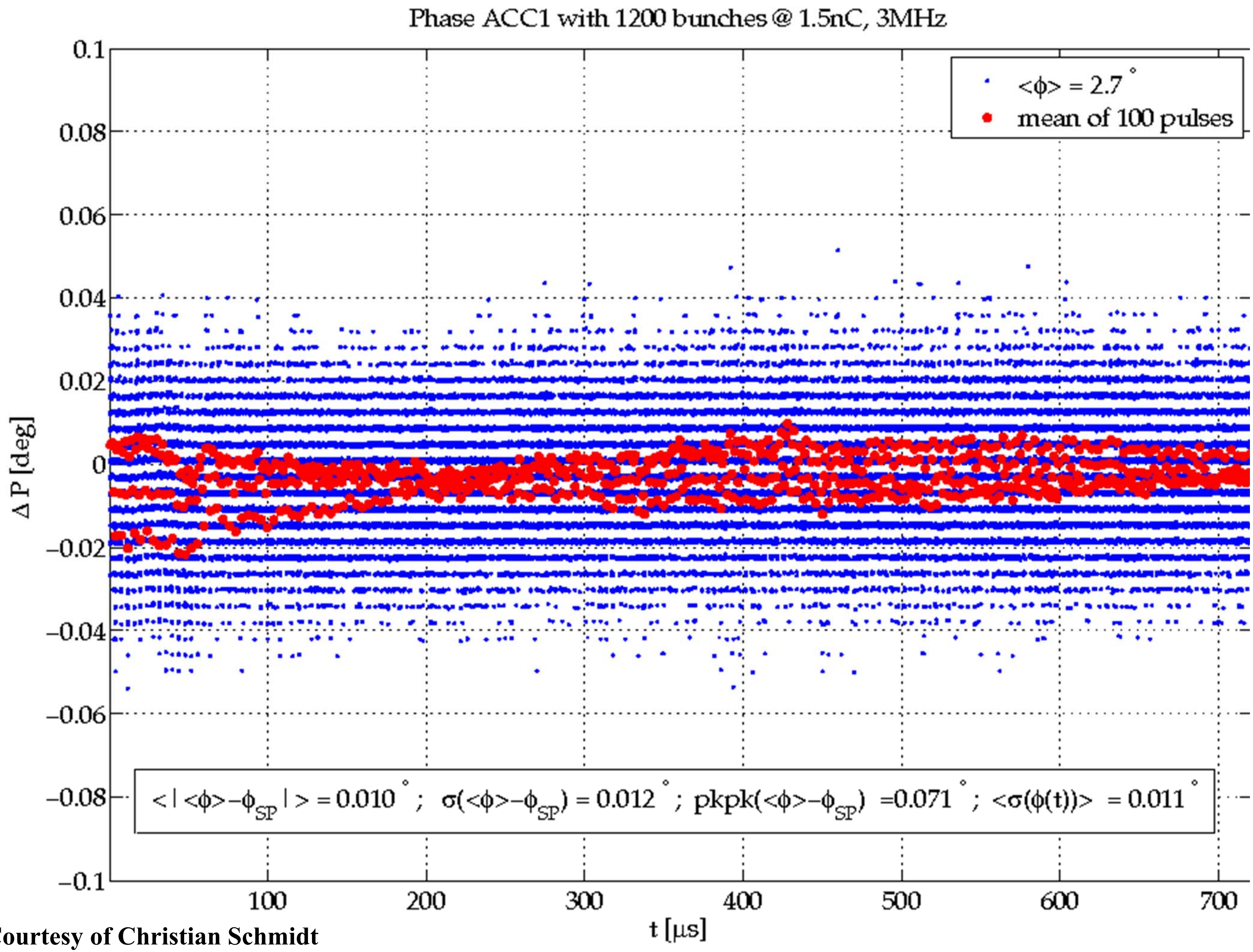




Courtesy of Christian Schmidt

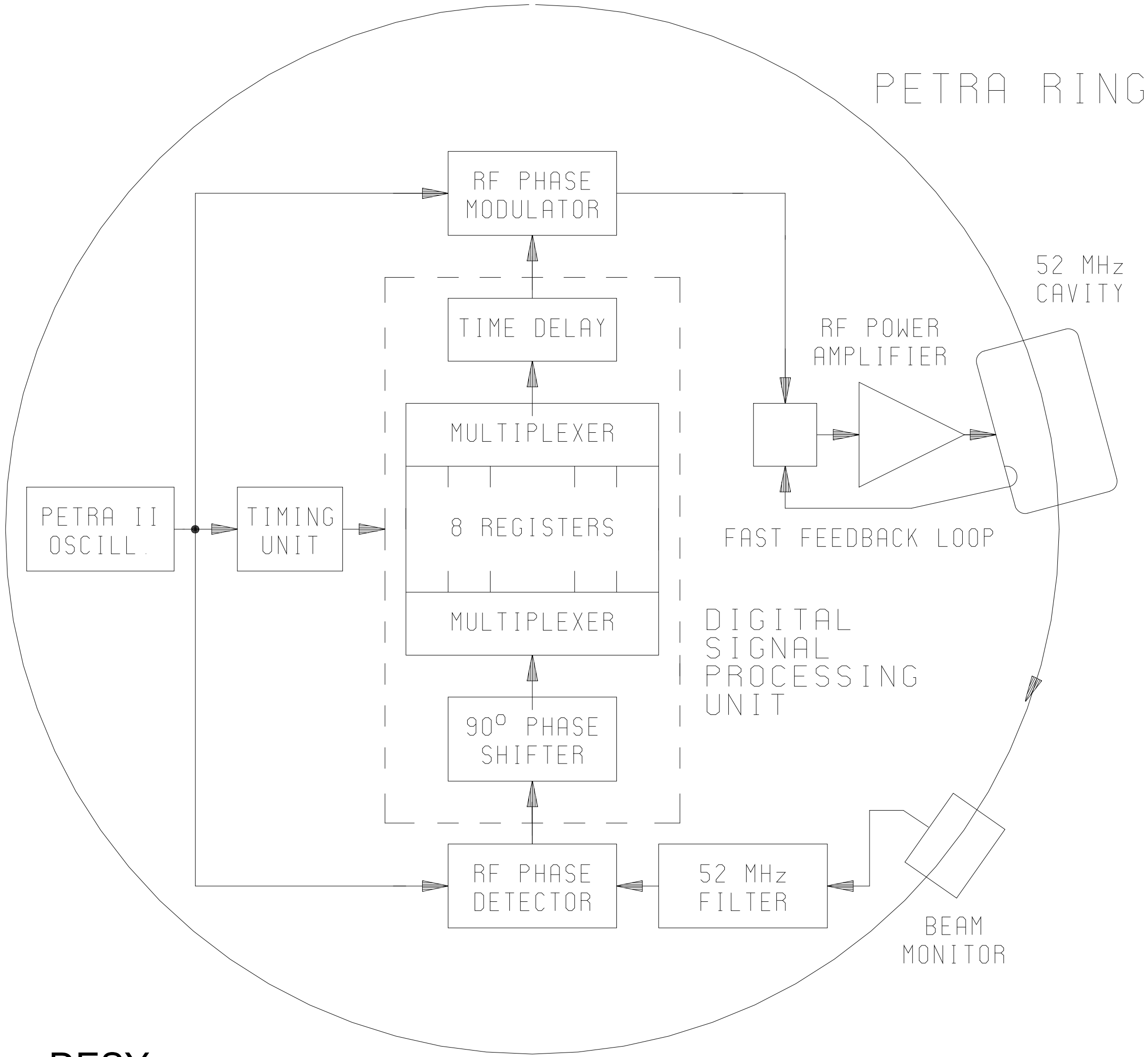


Courtesy of Christian Schmidt



Courtesy of Christian Schmidt

A PHASE LOOP FOR DAMPING SYNCHROTRON OSCILLATIONS



Alexander Gamp, DESY,
CAS, Bilbao, May 27th, 2011

A simple RC integrator of differentiator network as a 90° phase shifter is not without problems [16] .

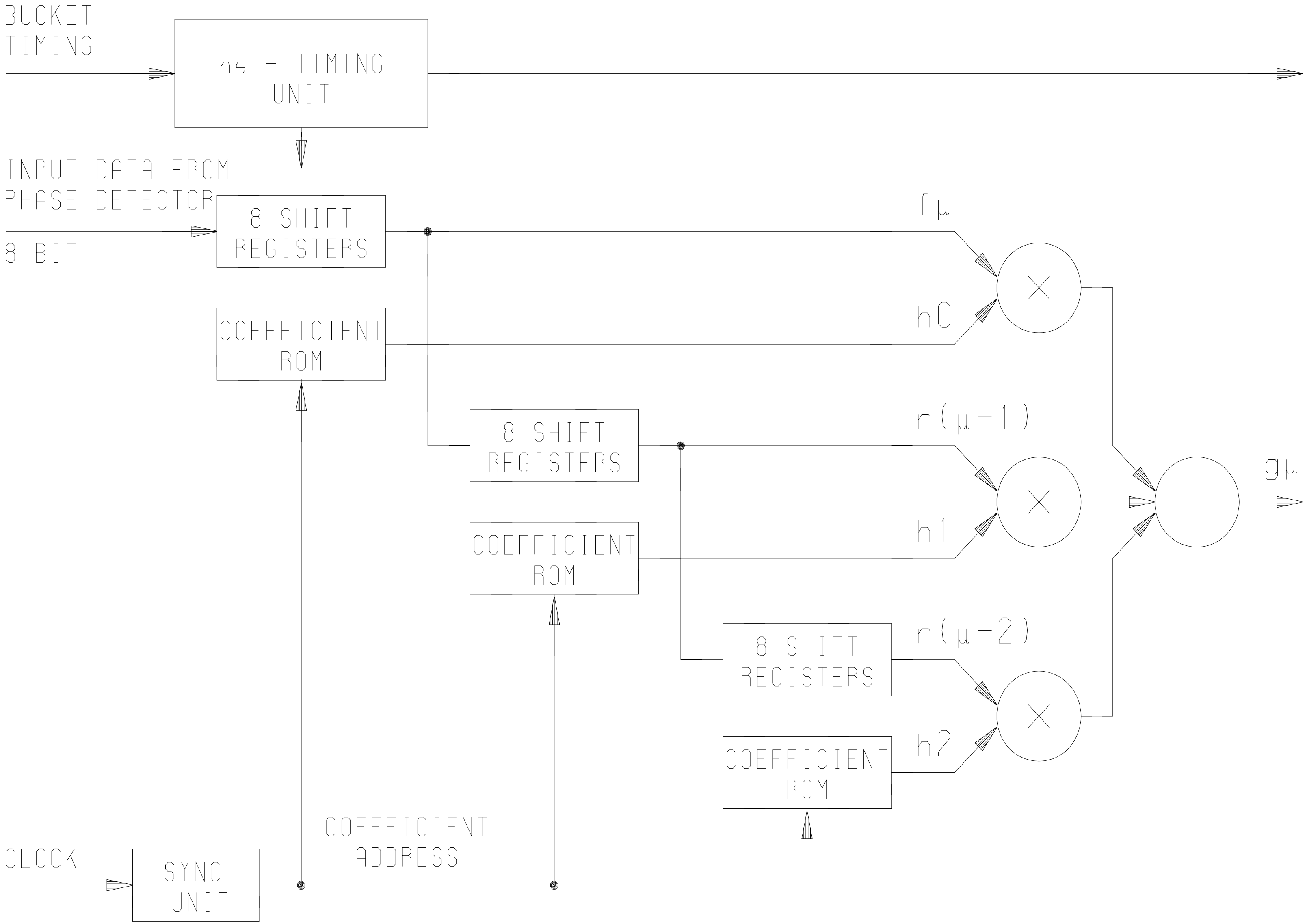
A more complex digital solution with a software controlled phase shift has been adopted. This is very attractive since during injection, acceleration and compression of the bunches the synchrotron frequency varies in the range from 200 to 350 Hz. In addition, storing and multiplexing the eight correction signals for each of the eight possible batches in PETRA II can also be realized most comfortably on the digital side. The phase shifter has been built up as a three-coefficient digital FIR (Finite-length Impulse Response) filter according to

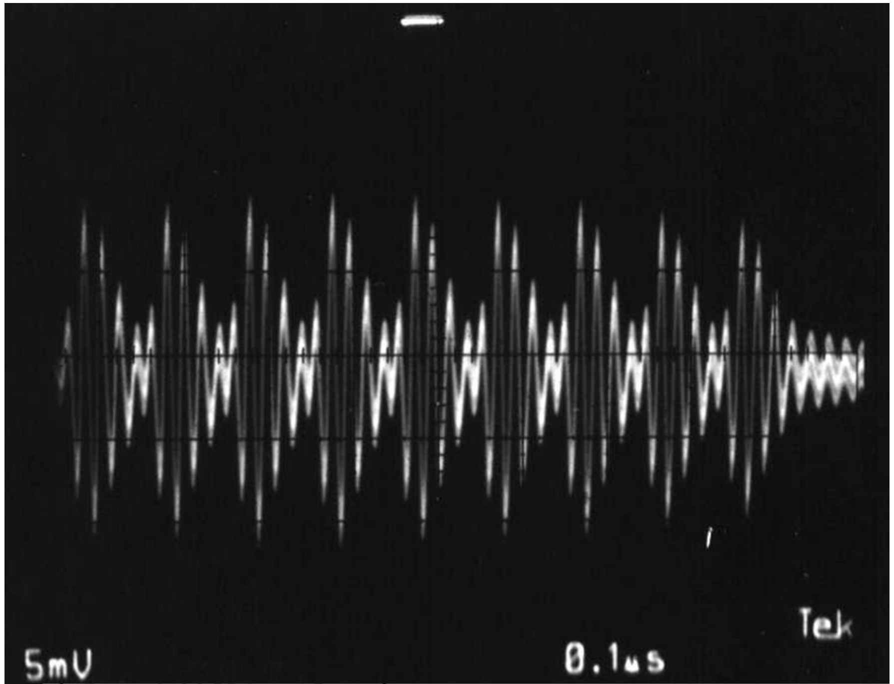
$$g_\mu = \sum_{k=0}^{2} h_k f_{\mu-k} \qquad\qquad H(\omega) = \sum_{k=0}^{2} h_k e^{-ik\omega T_S}$$

f and g are input and output data respectively. Using the coefficients

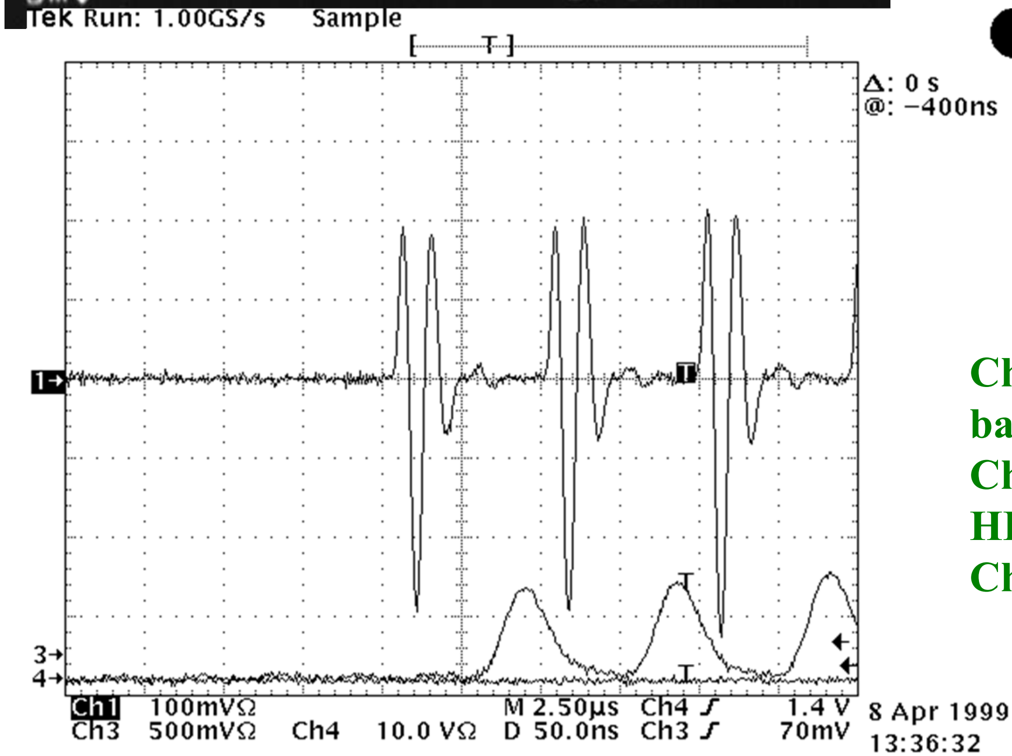
$$h_0 = \tfrac{2}{\pi}\sin\phi, \quad h_1 = \cos\phi, \quad h_2 = -\tfrac{2}{\pi}\sin\phi$$

one obtains a phase shift which, in the frequency range of interest 200 Hz $\leq f_S \leq$ 359 Hz, deviates by less than ± 0.4 from the nominal value -π/2.

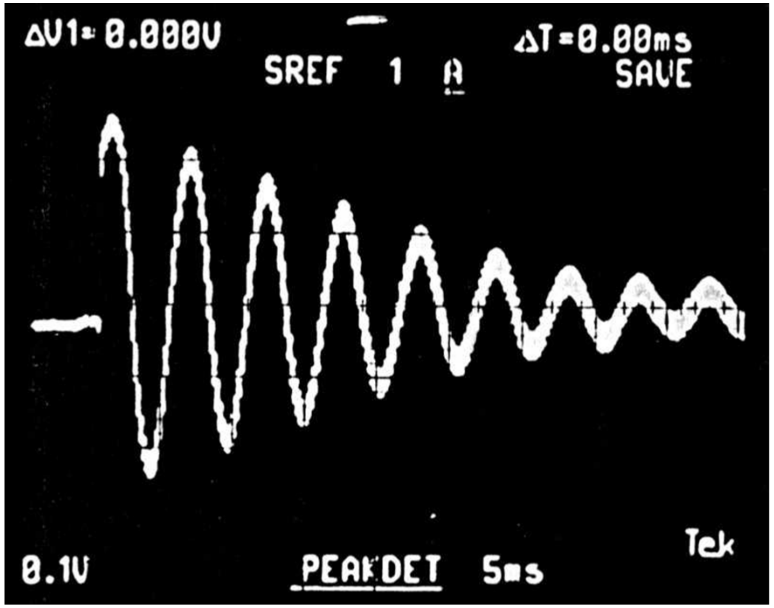

BUCKET
TIMING
ns - TIMING
UNIT
INPUT DATA FROM
PHASE DETECTOR
8 BIT
8 SHIFT
REGISTERS
fμ
COEFFICIENT
ROM
h0
8 SHIFT
REGISTERS
r(μ-1)
gμ
COEFFICIENT
ROM
h1
8 SHIFT
REGISTERS
r(μ-2)
COEFFICIENT
ROM
h2
CLOCK
SYNC.
UNIT
COEFFICIENT
ADDRESS



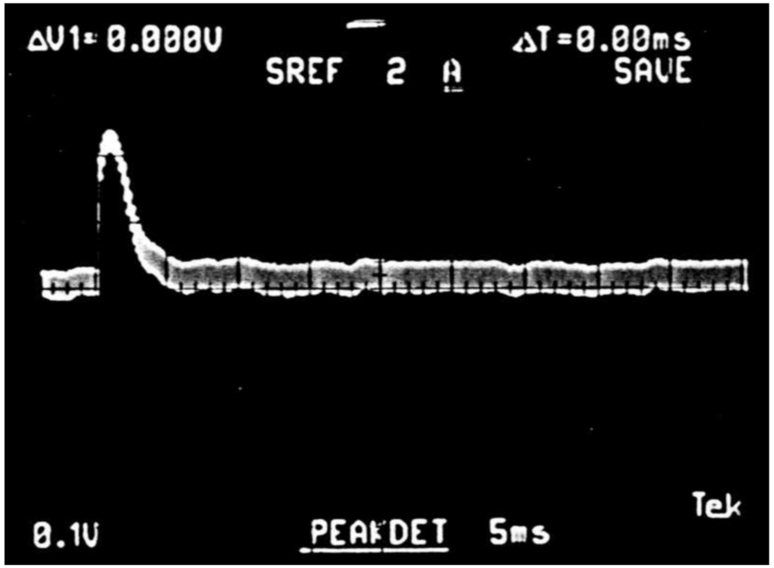
FILTERED 52 MHz COMPONENT OF THE BEAM MONITOR SIGNAL OF A BATCH OF 9 PROTON BUNCHES CIRCULATING IN PETRA: THE BUNCH SPACING IS 96 ns



Ch1: Feed-Forward Monitor Signal after band-pass filter
Ch3: Signal from Beam-Current-Monitor HERA WL
Ch4: HIT-Trigger



The synchrotron oscillation measured at the phase detector output a few ms after injection of a batch of 9 proton bunches into PETRA II. It is smeared out by Landau damping after some periods. The damping loop is not active.



Here the phase loop is active. The synchrotron oscillation is completely damped within half a synchrotron period of 5 ms.

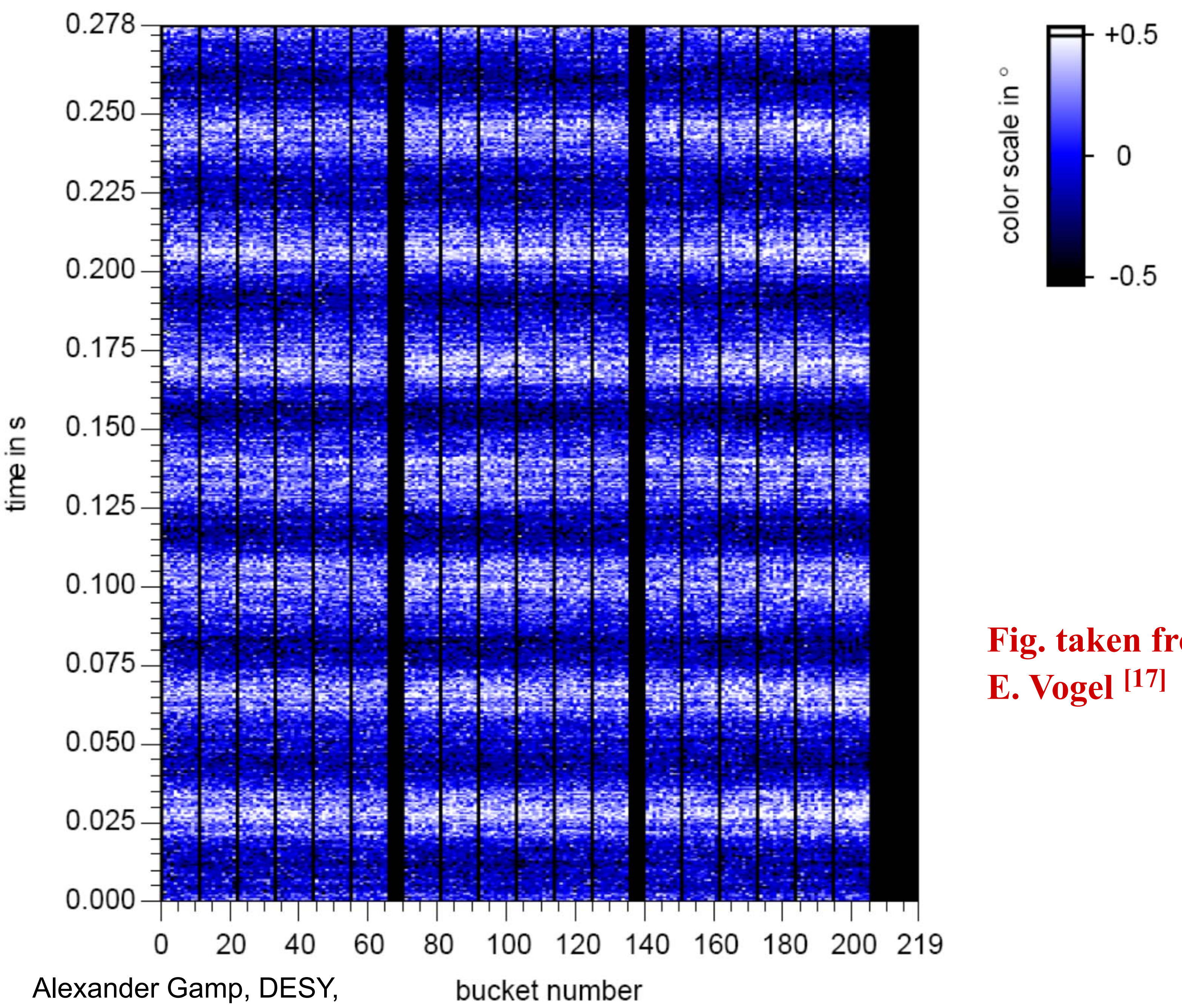


Fig. taken from E. Vogel [17]

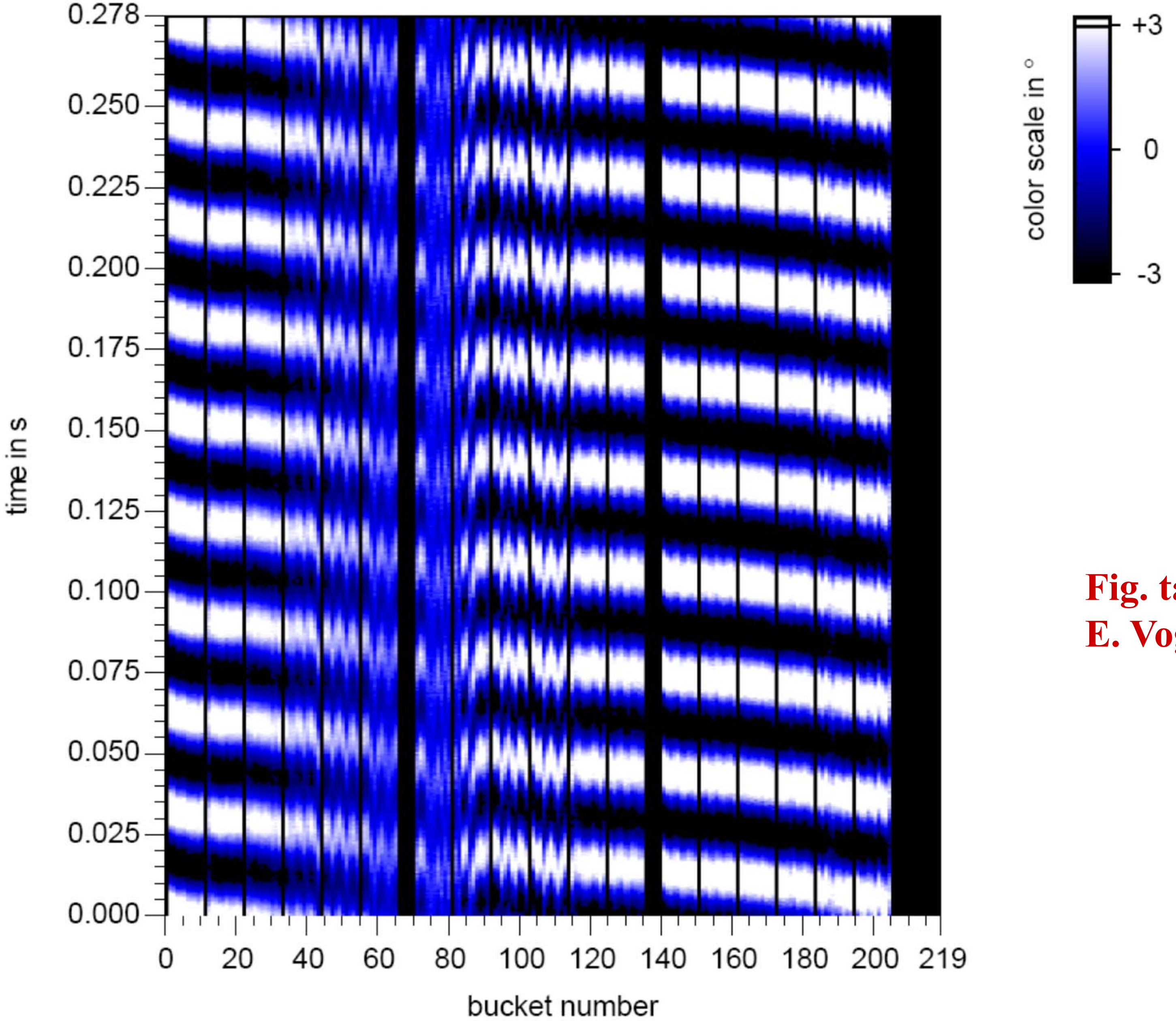


Fig. taken from E. Vogel [17]

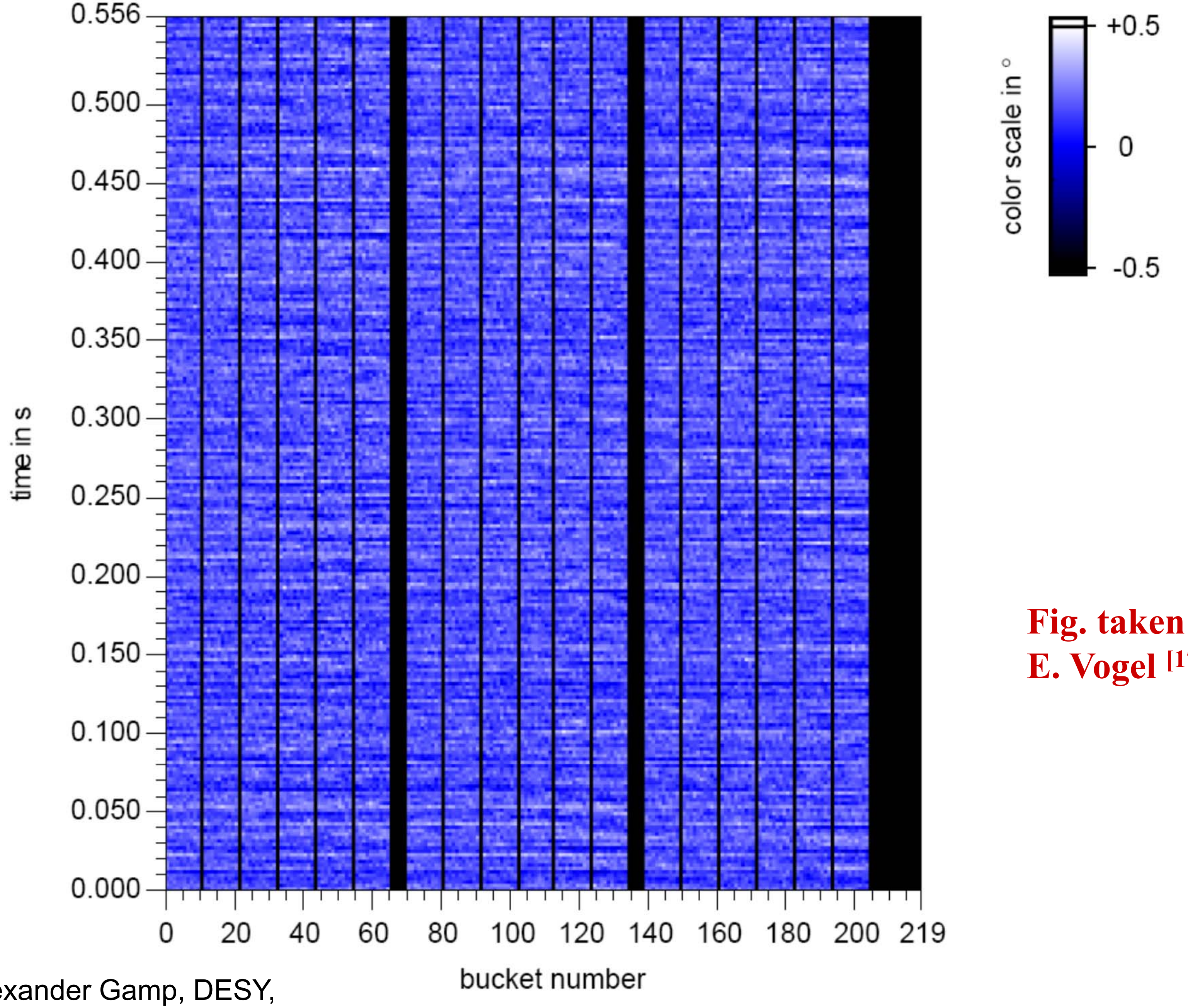


Fig. taken from E. Vogel [17]

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