Vacuum I

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CAS - Bilbao
Index

Introduction to Vacuum

Vacuum and the Beam

Flow Regimes

Creating Vacuum
Vacuum in accelerators

All beam dynamics has the purpose of controlling a charged beam particle

\[
\frac{d}{dt} m\gamma \vec{v} = e\vec{E} + ev \times \vec{B}
\]

Structure of magnets and RF structure have the purpose of creating a proper guiding (E,B) structure

However in an accelerator are present a jungle of unwanted particles which creates a damaging background for beam operation
Vacuum is considered as a Gas which is characterized by

**Macroscopic Quantities**
- Pressure
- Temperature
- Density
- Composition

**Microscopic Quantities**
- Kinetic Theory

**Aim:**
Minimize the interaction of beam with vacuum gas
# Typical numbers

<table>
<thead>
<tr>
<th></th>
<th>Particles m$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere</td>
<td>$2.5 \times 10^{25}$</td>
</tr>
<tr>
<td>Vacuum Cleaner</td>
<td>$2 \times 10^{25}$</td>
</tr>
<tr>
<td>Freeze dryer</td>
<td>$10^{22}$</td>
</tr>
<tr>
<td>Light bulb</td>
<td>$10^{20}$</td>
</tr>
<tr>
<td>Thermos flask</td>
<td>$10^{19}$</td>
</tr>
<tr>
<td>TV Tube</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>Low earth orbit (300km)</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>$H_2$ in LHC</td>
<td>$\sim 10^{14}$</td>
</tr>
<tr>
<td>SRS/Diamond</td>
<td>$10^{13}$</td>
</tr>
<tr>
<td>Surface of Moon</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>Interstellar space</td>
<td>$10^5$</td>
</tr>
</tbody>
</table>

R.J. Reid
Particle – Wall collision

The velocity after wall collision depends on the particle-wall interaction.

Momentum is propagated to the wall.
Pressure

SI Units: $P_a = \frac{N}{m^2}$

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Low vacuum</td>
<td>Atm. pressure to 1 mbar</td>
</tr>
<tr>
<td>Medium vacuum</td>
<td>1 to $10^{-3}$ mbar</td>
</tr>
<tr>
<td>High Vacuum (HV)</td>
<td>$10^{-3}$ to $10^{-8}$ mbar</td>
</tr>
<tr>
<td>Ultrahigh vacuum (UHV)</td>
<td>$10^{-8}$ to $10^{-12}$ mbar</td>
</tr>
<tr>
<td>Extreme high vacuum (XHV)</td>
<td>less than $10^{-12}$ mbar</td>
</tr>
</tbody>
</table>

other units: $1 \text{ Pa} = 10^{-2} \text{ mbar} = 7.5 \times 10^{-3} \text{ Torr} = 9.87 \times 10^{-6} \text{ atm}$
Particle – Particle interaction

In a gas large number of collisions
Most likely process P-P collision

For elastic collisions:
1) Energy conservation
2) Momentum conservation

\[
\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T
\]

\[
k_B = 1.38 \times 10^{-23} \quad [\text{JK}^{-1}]
\]
Typical numbers

Air at T= 20° C

20% O₂ → \( M_O = 8x2 \text{ g-mole} = \frac{8x2}{N_A} = 2.65 \times 10^{-23} \text{ g} \)

80% N₂ → \( M_N = 7x2 \text{ g-mole} = \frac{7x2}{N_A} = 2.32 \times 10^{-23} \text{ g} \)

Therefore

\[
< v_{N_2}^2 > = \frac{3K_B T}{M_{N_2}} = \frac{3 \times 1.38 \times 10^{-23} \times 293}{2 \times 2.32 \times 10^{-26}}
\]

\[
\sqrt{< v_{N_2}^2 >} = 511 \text{ m/s}
\]

Molecules run fast!

But the average velocity is

\[
v_a = < v > = 0.92 \sqrt{< v^2 >} = 470 \text{ m/s}
\]
When a gas is at equilibrium the distribution of the velocity follows the Maxwell-Boltzmann distribution

\[ v = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

\[ \frac{1}{N} \frac{dN}{dv} = \frac{4}{\sqrt{\pi}} \left( \frac{m}{2k_BT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2k_BT}} \]

max velocity

ave. velocity

\[ v_\alpha = \sqrt{\frac{2k_BT}{m}} \]

\[ v_a = \sqrt{\frac{8}{\pi} \frac{k_BT}{m}} \]

\[ \langle v^2 \rangle = \frac{3k_BT}{m} \]
Equation of state

An ideal gas in a container of volume V satisfies the equation of state

For a gas in equilibrium

\[ PV = nR_0T \]

SI units
- P \(\rightarrow\) Pressure [Pa]
- V \(\rightarrow\) Volume in [m\(^3\)]
- n \(\rightarrow\) moles [1]
- T \(\rightarrow\) absolute temperature [K]

- \( R_0 = 8.314 \) [Nm/(mole K)] universal constant of gas
- \( k_B = 1.38\times10^{-23} \) [JK\(^{-1}\)] Boltzmann constant
- \( N_A = 6.022\times10^{23} \) [1] Avogadro’s Number

\( R_0 = k_BN_A \)
Mean free path

free path = path of a particle between two collisions
Mean free path

\[
N(l + \Delta l) = N(l) - \frac{\sigma \Delta l A \tilde{n}}{A} N(l)
\]

\( \sigma = \pi r^2 \) \quad \text{Cross-section}
\( \tilde{n} \) \quad \text{Density of atoms}
\( A \) \quad \text{Transverse area}

Particles that did not collide

\[ \frac{dN}{N} = -\sigma \tilde{n} dl \]
Mean free path

How many particles collide between \( l \) and \( l + \Delta l \)?

\[ dN = N(l)\sigma\tilde{n}dl \]

Therefore \( dN \) particles travelled a distance \( l \) and then collided.

Probability that a particle travel \( l \) and then collide is

\[ dP = \frac{N(l)}{N_0}\sigma\tilde{n}dl \]

Mean free path

\[ \lambda = \int_0^\infty ldP(l) = \frac{1}{\sigma\tilde{n}} \]

In a gas

\[ \lambda = \frac{1}{\sqrt{2\sigma\tilde{n}}} \]
Example

Air at $T = 20^0\text{C}$ and $P = 1\text{ atm}$

From equation of state

\[ \tilde{n} = \frac{P}{k_B T} = \frac{10^5}{1.38 \times 10^{-23} \times 293} = 2.47 \times 10^{25} \text{ atom/m}^3 \]

Diameter of molecule of air $d = 3.74 \times 10^{-10} \text{ m}$ \rightarrow $\sigma = \pi d^2 = 4.39 \times 10^{-19} \text{ m}^2$

Mean free path $\lambda = 6.51 \times 10^{-8} \text{ m}$
$\text{N}_2 \text{ at } T = 20^0 \text{C}, \ d = 3.15 \times 10^{-10} \text{ m}$

<table>
<thead>
<tr>
<th>Pressure</th>
<th>$P$</th>
<th>$n$</th>
<th>$\rho$</th>
<th>$\nu$</th>
<th>$l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>atm</td>
<td>$10^5$</td>
<td>$2.5 \times 10^{25}$</td>
<td>1.16</td>
<td>$2.9 \times 10^{27}$</td>
<td>$9 \times 10^{-8}$</td>
</tr>
<tr>
<td>primary</td>
<td>1</td>
<td>$2.5 \times 10^{20}$</td>
<td>$1.16 \times 10^{-5}$</td>
<td>$2.9 \times 10^{22}$</td>
<td>$9 \times 10^{-3}$</td>
</tr>
<tr>
<td>vacuum</td>
<td>$10^{-1}$</td>
<td>$2.5 \times 10^{19}$</td>
<td>$1.16 \times 10^{-6}$</td>
<td>$2.9 \times 10^{21}$</td>
<td>$9 \times 10^{-2}$</td>
</tr>
<tr>
<td>high</td>
<td>$10^{-4}$</td>
<td>$2.5 \times 10^{16}$</td>
<td>$1.16 \times 10^{-9}$</td>
<td>$2.9 \times 10^{18}$</td>
<td>$9 \times 10^{1}$</td>
</tr>
<tr>
<td>vacuum</td>
<td>$10^{-7}$</td>
<td>$2.5 \times 10^{13}$</td>
<td>$1.16 \times 10^{-12}$</td>
<td>$2.9 \times 10^{15}$</td>
<td>$9 \times 10^{4}$</td>
</tr>
<tr>
<td>uhv</td>
<td>$10^{-10}$</td>
<td>$2.5 \times 10^{10}$</td>
<td>$1.16 \times 10^{-15}$</td>
<td>$2.9 \times 10^{12}$</td>
<td>$9 \times 10^{7}$</td>
</tr>
<tr>
<td>xhv</td>
<td>&lt;$10^{-11}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vacuum and Beam

Beam

Vacuum Gas
Beam lifetime

After a beam particle collides with a gas atom, it gets ionized and lost because of the wrong charge state with respect to the machine’s optics

Beam of particles going through a vacuum gas survives according to

\[ N(l + \Delta l) = N(l) - \frac{\sigma \Delta l A \tilde{n}}{A} N(l) \]

\[ \frac{dN}{dl} = -\sigma \tilde{n} N(l) \]

As beam particle have a velocity \( v_0 \), then

\[ \frac{dN}{dt} = -\sigma \tilde{n} v_0 N(t) \]

From the equation of state

\[ \tilde{n} = \frac{P}{k_B T} \]

\[ \frac{dN}{dt} = -\frac{\sigma P v_0}{k_B T} N(t) \]

Beam lifetime

\[ \tau = \frac{k_B T}{\sigma P v_0} \]

Beam lifetime sets the vacuum constrain
Example

It is important to know the cross-section of the interaction beam-vacuum

Of what is formed the vacuum? Does it depend on the energy?

Example LHC

H$_2$ at 7 TeV at T = 5$^0$ K

$\sigma = 9.5 \times 10^{-30}$ m$^2$

for $\tau = 100$ hours

$P = 6.7 \times 10^{-8}$ Pa

Nuclear scattering cross section at 7 TeV for different gases and the corresponding densities and equivalent pressures for a 100 hours beam lifetime

<table>
<thead>
<tr>
<th>GAS</th>
<th>Nuclear scattering cross section (cm$^2$) for a 100 hour lifetime</th>
<th>Gas density (m$^{-3}$) for a 100 hour lifetime</th>
<th>Pressure (Pa) at 5 K, for a 100 hour lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$</td>
<td>$9.5 \times 10^{-26}$</td>
<td>$9.81 \times 10^{14}$</td>
<td>$6.7 \times 10^{3}$</td>
</tr>
<tr>
<td>He</td>
<td>$1.26 \times 10^{-25}$</td>
<td>$7.41 \times 10^{14}$</td>
<td>$5.11 \times 10^{3}$</td>
</tr>
<tr>
<td>CH$_4$</td>
<td>$5.66 \times 10^{-25}$</td>
<td>$1.61 \times 10^{14}$</td>
<td>$1.11 \times 10^{3}$</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>$5.65 \times 10^{-25}$</td>
<td>$1.61 \times 10^{14}$</td>
<td>$1.11 \times 10^{3}$</td>
</tr>
<tr>
<td>CO</td>
<td>$8.54 \times 10^{-25}$</td>
<td>$1.11 \times 10^{14}$</td>
<td>$7.51 \times 10^{3}$</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>$1.32 \times 10^{-24}$</td>
<td>$7 \times 10^{13}$</td>
<td>$4.91 \times 10^{3}$</td>
</tr>
</tbody>
</table>

LHC Design Report
Electron cloud

Reflection

Y(E,\phi)

Secondary electrons

\delta(E,\phi)

Key parameters

- Synchrotron radiation
- Y(E,\phi) photoelectric yield
- \delta(E,\phi) second. electron Y
- Second. electron energy
- residual gas ionization
- Photon reflectivity
- Beam pipe shape
- Bunch intensity and spacing
- External fields (magnetic, electric, space charge)

O. Gröbner, CAS 2007
Impingement rate

For a Maxwell-Boltzmann distribution

\[ J = \frac{1}{4} \tilde{n} v_a \]

\( J \rightarrow \) units: \([\#/(s\ m^2)]\)

and

\[ v_a = \sqrt{\frac{8 k_B T}{\pi m}} \]
More on collision

mean free path

$\lambda$

$D$
Knudsen Number

\[ K_n = \frac{\lambda}{D} \]

SI units: \[1\]

\[ K_n < 0.01 \]

\[ 0.01 < K_n < 0.5 \]

\[ K_n > 0.5 \]

Flow Regimes

**Continuous Regime**

Particles collide mainly among them the fluid is continuous

**Transitional Regime**

This is a regime in between continuous and molecular

**Molecular Regime**

particles collide mainly with walls
The Throughput

Vessel with a gas at $P, V, N, T$

Number of particles

$$N = \frac{PV}{k_B T}$$

The quantity $Q = PV$ is proportional to the number of particles

$$\frac{dN}{dt} = \tilde{n} v A = \frac{P}{k_B T} v A = \frac{P}{k_B T} \frac{dV}{dt} = \frac{1}{k_B T} \frac{d}{dt} PV$$

$$\frac{dN}{dt} = \frac{1}{k_B T} \dot{Q}$$

$\dot{Q}$ is called throughput

SI units: [Pa m$^3$/s]
Conductance

In absence of adsorption and desorption processes the throughput does not change

\[ \dot{Q}_1 \text{ equal } \dot{Q}_2 \]

\[ T = \text{constant} \]

Conductance

\[ C = \frac{\dot{Q}}{P_1 - P_2} \]

SI units: [m³/s]
Composition (Series)

\[ \dot{Q} = C_1 (P_{u1} - P_{d1}) \]
\[ \dot{Q} = C_2 (P_{u2} - P_{d2}) \]

Conductance of 1+2
\[ \frac{1}{C_{1+2}} = \frac{1}{C_1} + \frac{1}{C_2} \]
Composition (Parallel)

\[ \dot{Q}_1 = C_1 (P_u - P_d) \]

\[ Q_{1+2} = Q_1 + Q_2 \]

\[ \dot{Q}_2 = C_2 (P_u - P_d) \]

Conductance of 1+2

\[ C_{1+2} = C_1 + C_2 \]

T = constant
Molecular Flow

Regime dominated by Particle – Wall interaction

\[ K_n > 0.5 \]
\[ P < 1.3 \times 10^{-3} \text{ mbar} \]
\[ D = 0.1 \text{ m} \]
Cosine-Law

1. A particle after a collision with wall will lose the memory of the initial direction
2. After a collision a particle has the same velocity $|v|$
3. The probability that particles emerge in a certain direction follow the cosine law

$$dP = \frac{1}{\pi} \cos \theta d\omega$$

$d\omega = \text{solid angle}$

$$\int dP = 1$$
In the molecular regime two fluxes in opposite direction coexist

\[
\text{particle flux} = \alpha(N_1 - N_2)
\]

\(N_1\) = particle per second through 1

\(N_2\) = particle per second through 2
Conductance of an Aperture

\[ P_u, T \quad I_u \quad I_d \quad P_d, T \]

**Gas current**

- **Downstream**
  \[ I_d = J_d A = \frac{1}{4} \tilde{n}_d v_a A \]

- **Upstream**
  \[ I_u = J_u A = \frac{1}{4} \tilde{n}_u v_a A \]

**Volumetric flow**

- **Downstream**
  \[ \frac{I_d}{\tilde{n}_d} = \frac{1}{4} v_a A \]

- **Upstream**
  \[ \frac{I_u}{\tilde{n}_u} = \frac{1}{4} v_a A \]

**Throughput**

\[ \dot{Q} = \dot{Q}_u - \dot{Q}_d = P_u \dot{V}_u - P_d \dot{V}_d = \frac{1}{4} v_a A (P_u - P_d) \]

**Conductance**

\[ C_a = A \sqrt{\frac{1}{2\pi} \frac{R_0 T}{M}} \]
Conductance of a long tube

A long tube has no end effect in the stationary state.

\[ C_L = \alpha C_a \]
\[ \alpha = \frac{4}{3} \frac{d}{L} \]

L \to 0 makes α \to \infty which is wrong:
This result is valid for L long

The dependence of 1/L is consistent with the rule of composition of conductance.
Continuum flow regime

$K_n < 0.01$  Roughly more than 100 collision among particles before wall collision

Local perturbation of $\tilde{n}, P, T$ propagate through a continuum medium

Collision among particles create the **Viscosity**

Collision with walls cancel the velocity

$K_n < 0.01$
$P > 6.5 \times 10^{-2}$ mbar
$V = 0.1$ m
Viscous/Laminar regime

Reynold Number

\[ Re = \frac{\rho v D_h}{\eta} \]

\( \rho \) = density of gas [Kg/m³]
\( v \) = average velocity [m/s]
\( D_h \) = hydraulic diameter [m]

\[ D_h = \frac{4A}{B} \]

\( A \) = cross sectional area
\( B \) = perimeter

\( \eta \) = fluid viscosity [Pa-s]

Laminar Regime \( Re < 2000 \)

Turbulent Regime \( Re > 3000 \)

Reynold Number as function of Throughput

\[ Re = 4 \frac{\dot{Q}}{B} \frac{M}{R_0 T} \frac{1}{\eta} \]

For air \((N_2)\) at \(T=20\text{C}\) \(\eta = 1.75 \times 10^{-5} \text{ Pa-s}\)

\[ Re = \frac{\dot{Q}}{B} k_b \quad \text{with} \quad k_b = 2.615 \quad \text{s/(m}^2\text{Pa)} \]

Therefore the transition to a turbulent flow takes place at the throughput of

\[ \dot{Q}_T = 24d \]

For a pipe of \(d = 25 \text{ mm}\) \(\dot{Q}_T = 600 \quad \text{mbar l/s}\)

At \(P = 1 \text{ atm}\) this threshold corresponds to \(v = 0.389 \text{ m/s}\)
Laminar Regime

1. Fluid is incompressible: true also for gas when Ma < 0.2
2. Fluid is fully developed
3. Motion is laminar $\leftrightarrow$ Re < 2000
4. Velocity at Walls is zero

Fully developed flow

entry length

velocity distribution at injection in the pipe

velocity distribution of a fully developed flow
Conductance in Laminar Regime

long tube = no end effect

\[ \dot{Q} = C(P_u - P_d) \]

Conductance:

\[ C = \frac{\pi D^4}{128 \eta L} \overline{P} \quad \text{where} \quad \overline{P} = \frac{P_u + P_d}{2} \]

The conductance now depends on the pressure!
The throughput becomes a complicated relation (derived from the Darcy-Weisbach formula)

\[ \dot{Q} = A \sqrt{\frac{R_0 T}{M}} \sqrt{\frac{D_h}{f_D L}} \sqrt{P_u^2 - P_d^2} \]

\[ f_D = \text{Darcy friction factor, dependent from the Reynold number} \]
Sources of vacuum degradation
Evaporation/Condensation

At equilibrium \( J_C = J_E \)

\[
J_E = P_E N_A \frac{1}{\sqrt{2\pi R_0 T M}}
\]

\( P_E \) = saturate vapor pressure

Example: water vapor pressure
Outgassing

The outgassing is the passage of gas from the wall of the Vessel or Pipe to the vacuum.

\[ \Theta = \text{fraction of sites occupied} \]

\[ \frac{d\Theta}{dt} = -\frac{\Theta}{\tau_d} \]

"Throughput" due to outgassing

\[ \dot{Q}_G = k_BT \frac{N_s\Theta}{\tau_d} \]

\[ N_s = A \times 3 \times 10^{15} \quad A = \text{surface in cm}^2 \]

Mean stay time

\[ \tau_d = \tau_0 e^{\frac{E_d}{RT}} \quad \tau_0 = 10^{-13} \text{ s} \]

<table>
<thead>
<tr>
<th>(E_d) [Kcal/mole]</th>
<th>Cases</th>
<th>(\tau_d) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>Helium</td>
<td>1.18 \times 10^{-13}</td>
</tr>
<tr>
<td>1.5</td>
<td>H\textsubscript{2} physisorption</td>
<td>1.3 \times 10^{-12}</td>
</tr>
<tr>
<td>3 \cdot 4</td>
<td>Ar, CO, N\textsubscript{2}, CO\textsubscript{2} physisorption</td>
<td>1.6 \times 10^{-11}</td>
</tr>
<tr>
<td>10-15</td>
<td>Weak chemisorption</td>
<td>2.6 \times 10^{-6}</td>
</tr>
<tr>
<td>20</td>
<td>H\textsubscript{2} chemisorption</td>
<td>66</td>
</tr>
<tr>
<td>25</td>
<td>(3.3 \times 10^5) (~half week)</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>CO/Ni chemisorption</td>
<td>1.6 \times 10^{9} (~50 years)</td>
</tr>
<tr>
<td>40</td>
<td>(4.3 \times 10^{16}) (~half age of earth)</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>O.W chemisorption</td>
<td>1.35 \times 10^{98} (larger than the age of universe)</td>
</tr>
</tbody>
</table>

P. Chiggiato, CAS 2007
# Leaks

**High vacuum system**

<table>
<thead>
<tr>
<th>( Q_L )</th>
<th>( \leq )</th>
<th>( mbar \ l/s )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_L )</td>
<td>( &lt; 10^{-6} )</td>
<td>mbar l/s</td>
<td>very tight</td>
</tr>
<tr>
<td>( Q_L )</td>
<td>( &lt; 10^{-5} )</td>
<td>mbar l/s</td>
<td>tight</td>
</tr>
<tr>
<td>( Q_L )</td>
<td>( &lt; 10^{-4} )</td>
<td>mbar l/s</td>
<td>leaks</td>
</tr>
</tbody>
</table>

**Small Channels**

### Diameter hole
- \( 0.01 \) mm
- \( 10^{-10} \) m

### Leaks rate
- \( 10^{-2} \) mbar-l/s
- \( 10^{-12} \) mbar-l/s

---

K. Zapfe, CAS 2007

317 years for leakage of 1 cm³
Pumps and Gas flow

Ideal Pump

particles that enter in this chamber never returns back !

Pumping speed \( S_0 \) [m\(^3\)/s]

\[
S = \frac{dV}{dt} = \frac{I}{\bar{n}}
\]

but

\[
I = \dot{J} \frac{D^2 \pi}{4}
\]

\[
S = v_a \frac{D^2 \pi}{16}
\]

Example

\( N_2 \) at \( T=20\degree C \)

Take \( D=0.1\) m

\( S_0 = 0.92 \) m\(^3\)/s
Pumps and Conductance

Pumping speed $S$ [m$^3$/s]

$$\dot{Q} = PS$$

Pumping speed of ideal pump $S_0$

$$\dot{Q}_0 = P_0S_0$$

As the throughput is preserved

$$\frac{1}{S} = \frac{1}{C} + \frac{1}{S_0}$$

Example: if the pipe is long $l=100d$

$$S_{eff} = \frac{0.922}{1 + 3l/(4d)} = 0.012 \text{ m}^3/\text{s}$$
Pumping Process -- Pump-down Time - - Ultimate Pressure

\[ \dot{Q} = \dot{Q}_E + \dot{Q}_G + \dot{Q}_L - \dot{Q}_S \]

\[ \frac{dP}{dt} V = \dot{Q}_T - PS \]
Pumping Process -- Pump-down Time - - Ultimate Pressure

Therefore

\[ P = P_u + (P_0 - P_u)e^{-\frac{S}{V}t} \]

Ultimate pressure \( \rightarrow \) equilibrium between sources of throughput and pumps throughput

\[ P_u = \frac{\dot{Q}_T}{S} \]

Pump down time \( \rightarrow \) \( \tau_{pd} = \frac{V}{S} \)
Multistage pumps

\[ P = P_u + (P_0 - P_u)e^{-\frac{S}{V}t} \]

First stage pump down

Second stage

\[ \frac{P}{P_0} \]

1

\[ P_u \]

\[ t \ [s] \]
Creating the Vacuum: Pumps

Examples of vacuum in some accelerators

Table 1: SNS Vacuum Level Requirements

<table>
<thead>
<tr>
<th>Front End</th>
<th>Vacuum Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTL</td>
<td>2x10^{-7} Torr</td>
</tr>
<tr>
<td>CTL</td>
<td>5x10^{-8} Torr</td>
</tr>
<tr>
<td>SCL</td>
<td>&lt;1x10^{-9} Torr</td>
</tr>
<tr>
<td>HEBT</td>
<td>5x10^{-8} Torr</td>
</tr>
<tr>
<td>Ring</td>
<td>1x10^{-8} Torr</td>
</tr>
<tr>
<td>RTBT</td>
<td>1x10^{-7} Torr</td>
</tr>
</tbody>
</table>

LHC $\rightarrow 10^{-10} - 10^{-11}$ mbar

(\textit{LHC Design Report})

FAIR

HEBT $\rightarrow 10^{-9}$ mbar

SIS100 $\rightarrow 10^{-12}$ mbar

A. Kraemer \textit{EPAC2006, TUPCH175}
Positive Displacement Pumps

Principle:
A volume of gas is displaced out of the Vessel

A compression of the volume V is always necessary to bring the pressure from $P_{in}$ to a value larger than $P_{out}$
Piston Pumps

Vessel

$P_i$

Minimum volume

$V_{min}$

$P_o$
Piston Pumps

Here the pressure $P'$ is smaller than $P_o$ but larger than $P_i$. 

Vessel $P_i$  

$P_o$
At this point the internal pressure becomes equal to $P_i$

$$P_i V_i = P_0 V_{min}$$
The internal pressure remains $P_i$ but the volume increase to $V_{\text{max}}$.

Effective gas volume entered into the pump equal to $V_{\text{max}} - V_i$: this is the gas which will be expelled.
At the Volume $V_c$ the internal pressure becomes equal to $P_o$ and the valve opens.

$$P_o \ V_c = P_i \ V_{\text{max}}$$

and from now on the gas entered into the pump goes out
Piston Pumps

Therefore if the pumps make $N_c$ cycles per second

$$S = N_c(V_{\text{max}} - V_i) = S_0 \left(1 - \frac{P_0}{P_i} \frac{V_{\text{min}}}{V_{\text{max}}} \right)$$

Conclusion: the pumping speed depends on the ratio of outlet/inlet pressure

When the inlet pressure is too low the pump stops pumping

![Graph showing the relationship between pump speed (S) and outlet pressure (P_o) with a limit at P_limit and a plateau at S_0.]

G. Franchetti

30/5/2011
If the gas compression/expansion is isentropic then all transformation follow the law

\[ PV^\gamma = \text{const.} \]

hence

\[ S = S_0 \left[ 1 - \left( \frac{P_0}{P_i} \right)^{1/\gamma} \frac{V_{min}}{V_{max}} \right] \]

Clearly the dependence affects the ultimate pressure even in absence of sources of throughput

\[ \frac{dP}{dt} V = -PS(P) \]

Ultimate pressure

\[ \frac{d}{dt} P = 0 \]

\[ P_u = P_0 \left( \frac{V_{min}}{V_{max}} \right)^\gamma \]
General Pump

Pumping speed of ideal pump $S_0$

$$\dot{Q}_0 = P_0 S_0$$

But in real pumps there is a backflow

Pump throughput

$$\dot{Q} = P_i S_0 - \dot{Q}_b$$
Zero Load Compression Rate

\[ \dot{Q} = P_{i0} S_0 - \dot{Q}_b \]

The backflow is found

\[ \dot{Q}_b = \frac{S_0 P_0}{K_0} \]

Zero Load compression rate

\[ K_0 = \frac{P_0}{P_{i0}} \]
Gas displaced in the moving vane

**Gas Ballast**

During the compression there can be gas component (G), which partial pressure \( P_G \) can be too high → condensation

But the maximum pressure during compression do not exceed \( P_0 \) therefore by injectioning non condensable gas during the compression rate \( P_G \) is lowered blow the condensation point
The gas enter in the cavity which expand. When the pressure in the cavity reaches the sature vapor pressure $P_s$, the water boils.

During the compression the vapor bubbles implode creating the CAVITATION.

At $T = 15^0 C \rightarrow P_s = 33$ mbar which sets the $P_{\text{limit}} = P_s$.
Dry Vacuum Pumps: Roots

pumping speed: 75 – 30000 m³/h
operating pressure: 10 – 10⁻³ mbar
Kinetic Vacuum Pumps

Molecular drag pump
Turbo molecular Pump
Diffusion Ejector pump
Molecular Drag Pumps

Based on the molecular drag effect
Volumetric flow: \[ S_0 = wh \frac{U}{2} \]

Taking into account of the backflow \[ S_i = S_0 \frac{K - K_0}{1 - K_0} \]

\( K_0 = \) zero load compression rate, \[ K = \frac{P_{outlet}}{P_{inlet}} \]
The zero load compression rate is \( K_0 = e^{S_0/C} \)

But for a long tube \( \frac{S_0}{C} = \frac{3 U L}{4 h v_a} \) if \( U \sim v_a \) \( \rightarrow \frac{S_0}{C} = \frac{3 L}{4 h} \)

Example \( L = 250 \text{ mm}, \ h = 3 \text{ mm} \ \rightarrow \frac{S_0}{C} > 10 \) and \( K_0 >> 1 \)

\[
S = S_0 \left( 1 - \frac{K}{K_0} \right)
\]

* A. Chambers, Modern Vacuum Physics, CRC, 2005
Example of Molecular drag pump

- **Pumping speed:** 7–300 l/s
- **Operating pressure:** $10^{-3} – 10^{3}$ Pa
- **Ultimate pressure:** $10^{-5}$ to $10^{-3}$ Pa
Turbo-molecular pump

Principle

General particle velocity distribution

tilted blades

Principle
In the reference frame of the rotating blades

U

$v_a - U$

General particle velocity distribution

tilted blades

$\phi$
Turbo-molecular pump

In the reference frame of the rotating blades

\[ U \]

\[ v_a - U \]

General particle velocity distribution
Turbo-molecular pump

Returning in the laboratory frame

The vacuum particle has received a momentum that pushes it down but the component of velocity tangential becomes too large
The rotational component is lost and another stage can be placed
Probability of pumping

\[ W = \frac{\dot{N}}{J_i A} \]

The maximum probability \( W_{\text{max}} \) is found when \( P_i = P_o \)

And we also find that

\[ W = W_{\text{max}} \frac{K_0 - K}{K_0 - 1} \]

\( K_0 = \) compression rate at zero load
The compression rate at zero load \( K_0 \propto g(\phi) \exp(U/v_\alpha) \)

Therefore \( K_0 \propto \exp(\sqrt{M}) \) \( \Rightarrow \) therefore different species have different pumping probability

In addition the maximum pumping probability \( W_{\text{max}} \) is

\[
W_{\text{max}} \propto U/v_\alpha \propto \sqrt{M}
\]

Therefore we find that the maximum pumping speed \( S_{\text{max}} = W_{\text{max}} J \) is independent on the gas mass

\[
\frac{S}{S_{\text{max}}} = \frac{K_0 - K}{K_0 - 1}
\]
Maximum compression as function of foreline pressure

(Pumping speed as function of the inlet pressure)

-pumping speed: 35- 25000 l/s
-ultimate pressure $10^{-8}$ to $10^{-7}$ Pa
End of Vacuum I