# High power, high intensity hadron synchrotrons

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# Contents

# Working principle of a synchrotron I

Ring accelerator with constant radius R,
variable revolution frequency $\omega_0$ and B-Field.

Motion in a constant, homogenous B-field ($\vec{B} = B_y \vec{e}_y$):

$$\dot{\vec{v}} = \frac{q}{\gamma m}\left(\vec{v} \times \vec{B}\right) \quad \Rightarrow \quad \omega_0 = \frac{v_0}{R} = \frac{qB_y}{\gamma m}$$

(revolution frequency)

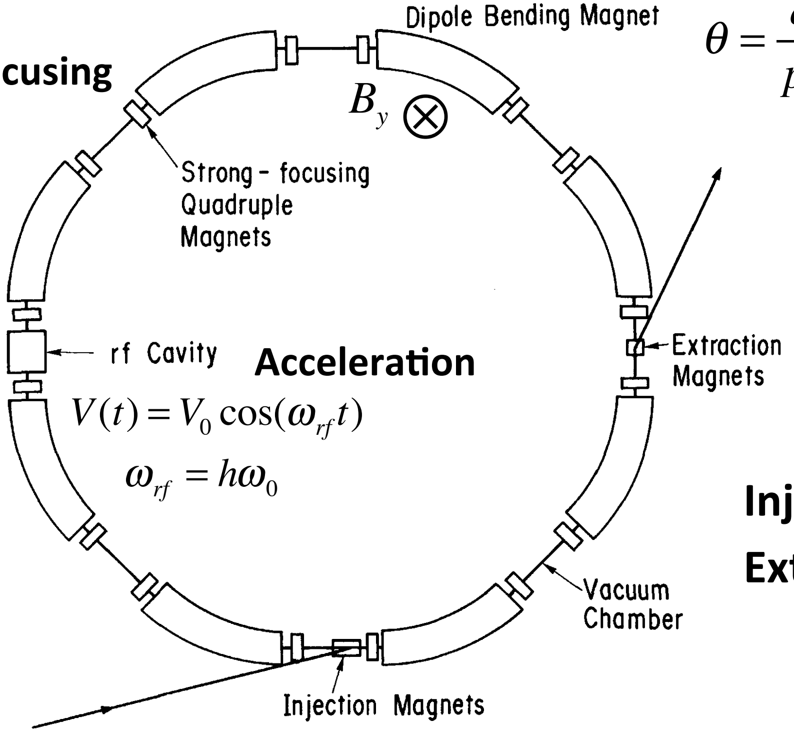
**Bending:**

$$\theta = \frac{q}{p_0} \int_{s_1}^{s_2} B_y ds \approx \frac{l}{R_0}$$

Rigidity:

$$B_y R_0 = \frac{p_0}{q}$$

**Focusing**



**Acceleration**

$$V(t) = V_0 \cos(\omega_{rf} t)$$

$$\omega_{rf} = h\omega_0$$

**Injection:** from a linac or from a ‘booster’ synchrotron.

**Extraction:** to targets or to a larger synchrotron.

**SIS-18 Synchrotron at GSI (L=216 m)**

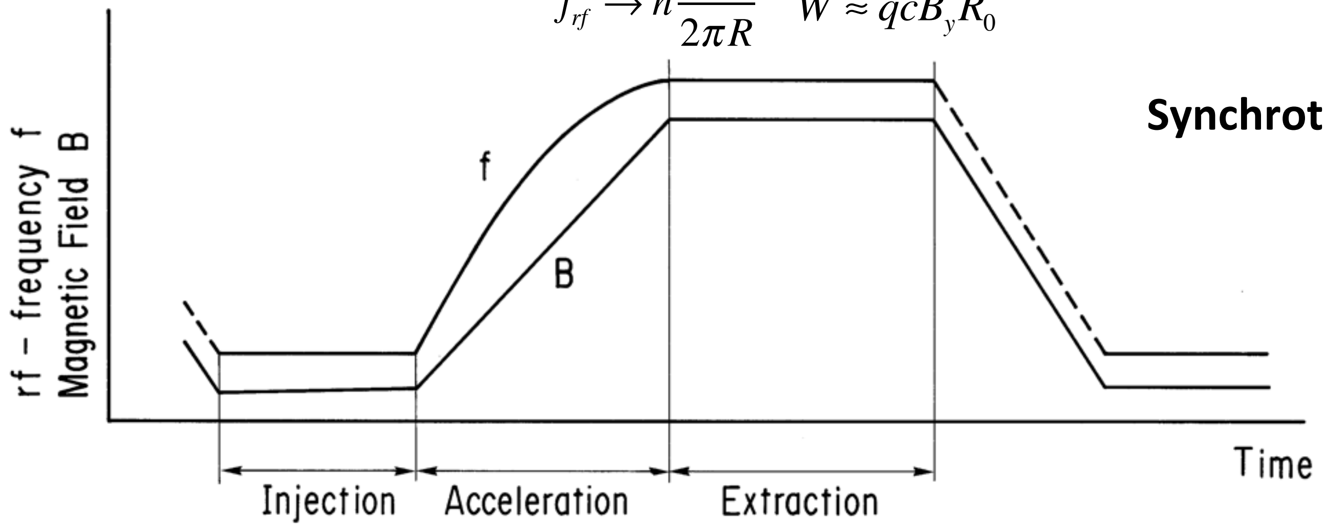
Dipole magnets (red),
quadrupole magnets (yellow)

# Working principle of a synchrotron II

Rf frequency: $f_{rf} = hf_0 = \frac{c^2 h}{2\pi}\frac{qB_y}{W}$

Particle energy: $W^2 = (pc)^2 + (mc^2)^2 = (qcBR_0)^2 + (mc^2)^2$

$f_{rf} \rightarrow h\frac{c}{2\pi R}$ $W \approx qcB_y R_0$



**Synchrotron cycle**

**Repetition rate $(T_{rep})^{-1}$:**
(time needed for one complete cycle)$^{-1}$

**Total beam energy:** $W_{tot} = NW_{kin}$

**Beam power:** $P_{beam} = \frac{W_{tot}}{T_{rep}}$

**Types of synchrotrons (a bit arbitrary):**

- slow cycling synchrotron: < 1 Hz
- fast cycling synchrotron: 1-10 Hz
- Rapid Cycling Synchrotron (RCS): > 10 Hz

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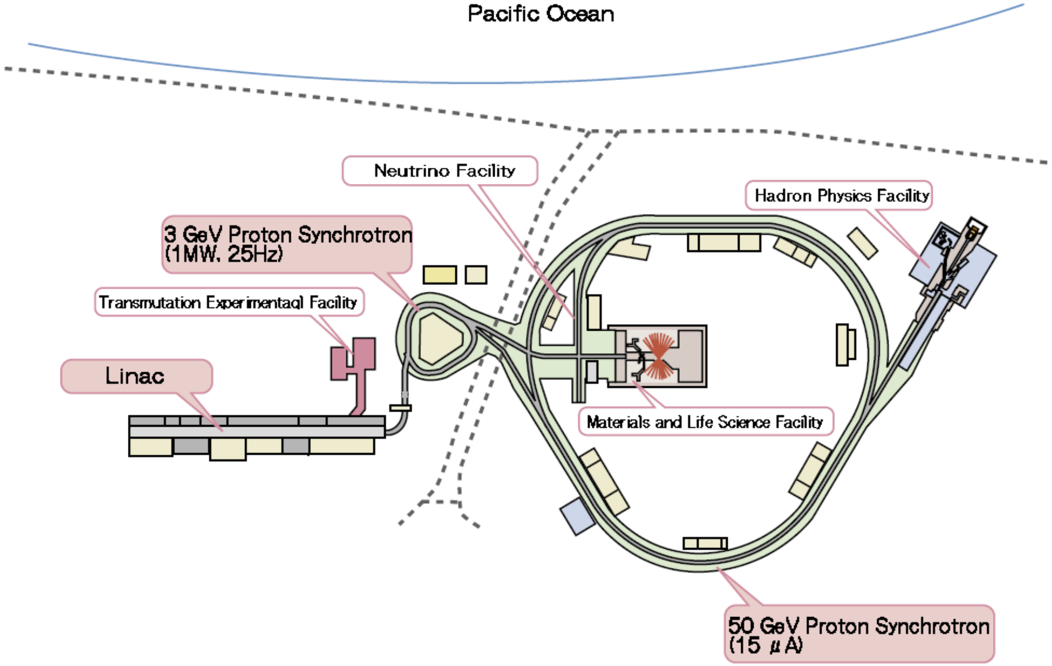
# High power, high intensity synchrotrons

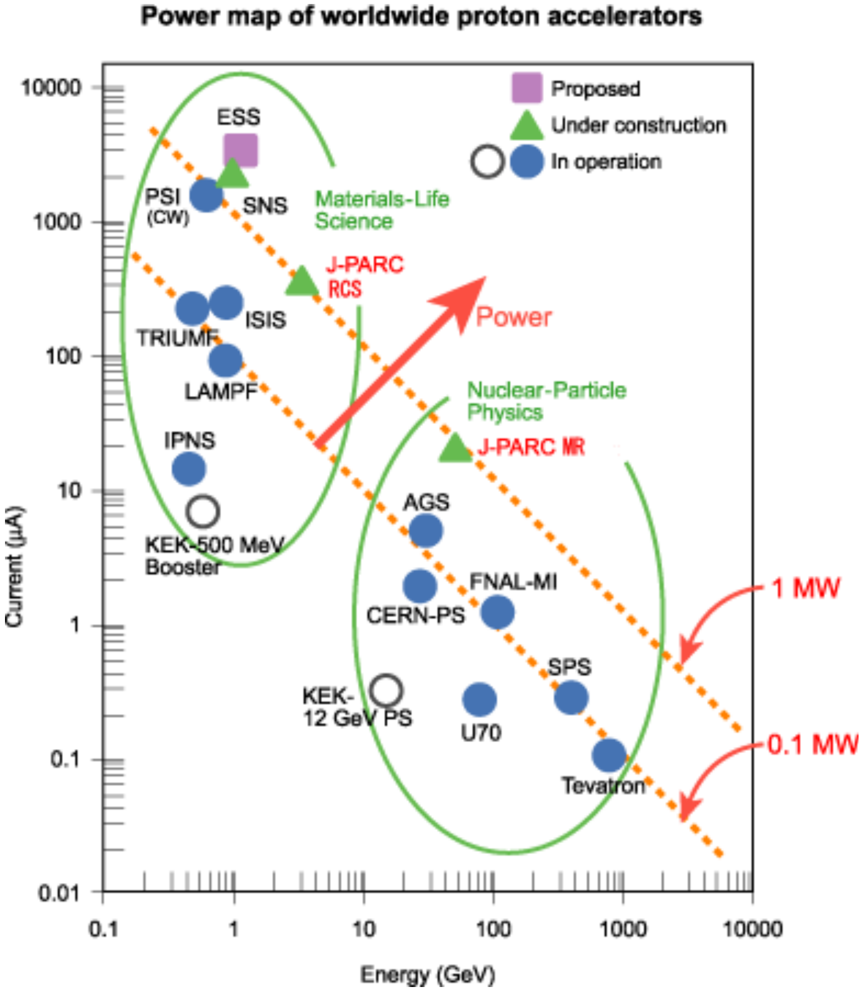
## worldwide (not complete !)

| | Energy | Radius | Rep. rate | Power | Particles /cycle | Application | Remarks |
|---|---|---|---|---|---|---|---|
| ISIS, UK | 0.8 GeV | 168 m | **50 Hz** | 0.16 MW | $3\times10^{13}$ | Neutrons, muons | RCS |
| J-PARC RCS, Japan | 3 GeV | 348 m | 25 Hz | **1 MW** (design) | $4\times10^{13}$ (design) | Injector for MR, Neutrons,... | RCS, **0.3 MW** |
| J-PARC MR, Japan | 50 GeV | 1567 m | 0.3 Hz | 0.75 MW (design) | **$4\times10^{14}$** (design) | Neutrinos, ... | |
| CERN PSB | 1.4 GeV | 157 m | 1 Hz | 1.5 kW | (4x) $2\times10^{12}$ | LHC injector chain | **4 rings** |
| CERN PS | 26 GeV | 630 m | 0.3 Hz | 25 kW | $2\times10^{13}$ | LHC injector chain | |
| AGS Booster | 1.5 GeV | 202 m | 7.5 Hz | 45 kW | $2.5\times10^{13}$ | RHIC injector chain | p-Au |
| AGS | 24 GeV | 807 m | 0.5 Hz | 130 kW | **$7\times10^{13}$** | RHIC injector chain | p-Au |
| SIS-18, GSI | 1 GeV/u | 216 m | 3 Hz | 4 kW | **$10^{10}$ Uranium** | Injector for SIS-100, RIBs | p-U |
| SIS-100, GSI | 2.7 GeV/u | 1080 m | 1 Hz | 50 kW | $5\times10^{11}$ Uranium | RIBs, pbars | p-U, sc magnets |

**RCS:** Rapid Cycling Synchrotron (> 10 Hz), **Blue**: 'Record values'

# J-PARC





J-PARC was heavily affected by the earthquake in March 2011 !
Plan for restoring the accelerator is being worked out.

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# rf cavities for synchrotrons

## variable frequency

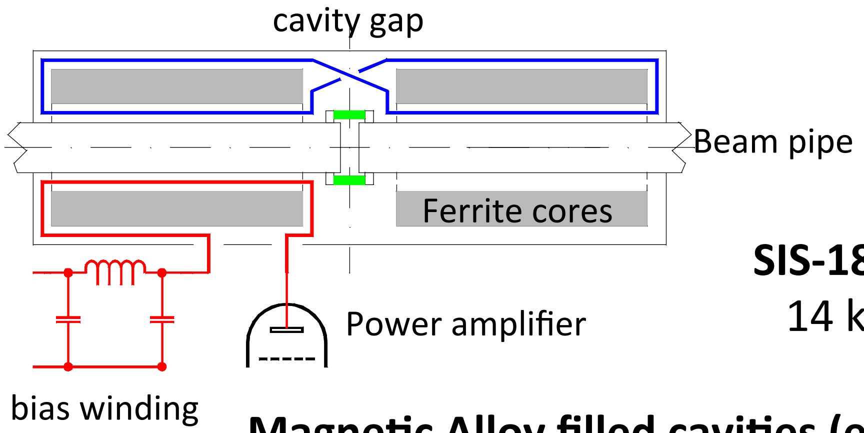
rf cavities filled with **ferrite ring cores**
(magnetic permeability $\mu(B_{bias})$)

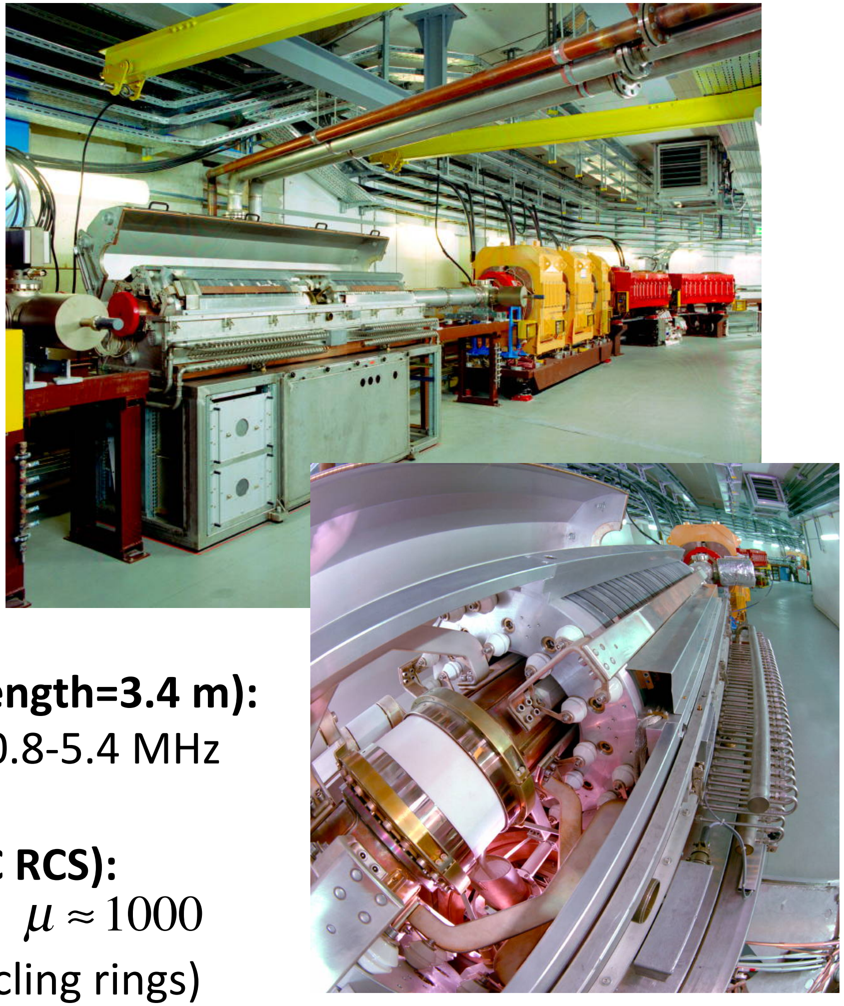
Reduced cavity eigenfrequency: $\omega_{rf}(B)=\sqrt{\frac{\mu_0}{\mu(B)}}\omega_{rf,0}$

Ferrite: $\mu \approx 100$

Cavity quality factor and max. frequency:

$Q \approx 10-50$ $f_{rf} < 10\ \mathrm{MHz}$





**SIS-18 cavity (length=3.4 m):**
14 kV, Q=10, 0.8-5.4 MHz

**Magnetic Alloy filled cavities (e.g. J-PARC RCS):**
- broadband: no tuning required $Q \lesssim 2$ $\mu \approx 1000$
- compact cavities (important for rapid cycling rings)
- larger losses

# Energy gain in a synchrotron

Synchronous particle
(enters the cavity always with the same phase $\varphi_s$):

$$(W_0)_{n+1} = (W_0)_n + qV_0 \sin\phi_s$$

Energy gain per turn: $\Delta W_0 = qV_0 \sin\phi_s$

$$W_0^2 = (p_0 c)^2 + (mc^2)^2 \quad \Rightarrow \quad \Delta W_0 = v\Delta p_0$$

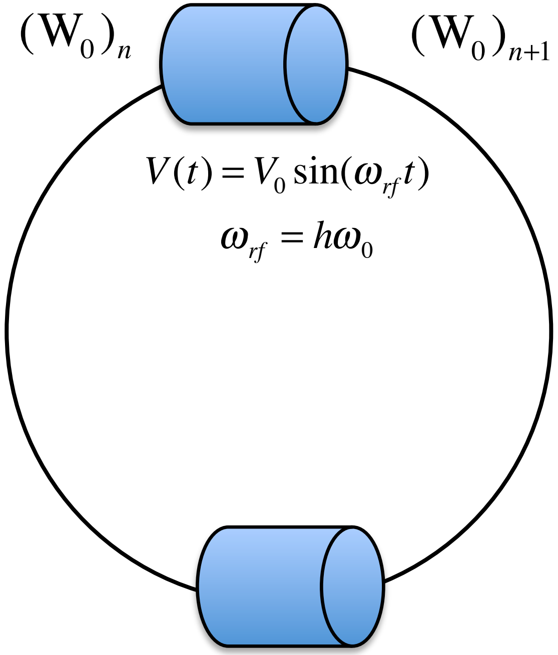
Rigidity:

$$\Delta p_0 = qR_0 \Delta B_y$$

Revolution period:

$$T_0 = \frac{2\pi R}{v_0}$$

rf cavity:



$$\Rightarrow \Delta W_0 = 2\pi R q R_0 \frac{\Delta B_y}{T_0}$$

Voltage requirement (single rf): $R_0 \dot{B}_0 = \frac{V_0}{2\pi R} \sin\phi_s$

dual rf mode: $R_0 \dot{B}_0 = \frac{V_0}{2\pi R} (\sin\phi_{s1} - \alpha \sin 2\phi_{s2})$

RCS: Installed voltage per meter counts !
Example J-PARC RCS: 400 kV/350 m ≈ 1 kV/m
-> compact Magnetic Alloy cavities !

$V(t) = \alpha V_0 \sin(2\omega_{rf} t)$
second harmonic cavity

# rf buckets

Non-synchronous particle: $\Delta W_{n+1} = \Delta W_n + qV_0(\sin\phi_n - \sin\phi_s)$ $\quad \frac{\Delta W}{T_0} \to \dot{W} \quad \Rightarrow \Delta\dot{W} = f_0 qV_0(\sin\phi - \sin\phi_s)$

$\Delta W_n = W_n - (W_0)_n$

$\Delta\phi = \phi - \phi_s$ $\quad \Delta\dot{\phi} = \dot{\phi} = \omega_{rf}\frac{\Delta T}{T_0}$ with $\frac{\Delta T}{T} = \eta\frac{\Delta p}{p}$ and $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$ $\quad \Rightarrow \quad \dot{\phi} = \frac{h\omega_0\eta}{\beta_0^2}\frac{\Delta W}{W}$

(phase difference) (frequency slip)

$$\ddot{\phi} = -\omega_s^2(\sin\phi - \sin\phi_s) \quad \Rightarrow \quad \frac{1}{2}\dot{\phi}^2 - \omega_s^2(\cos\phi + \phi\sin\phi_s) = \text{const.}$$

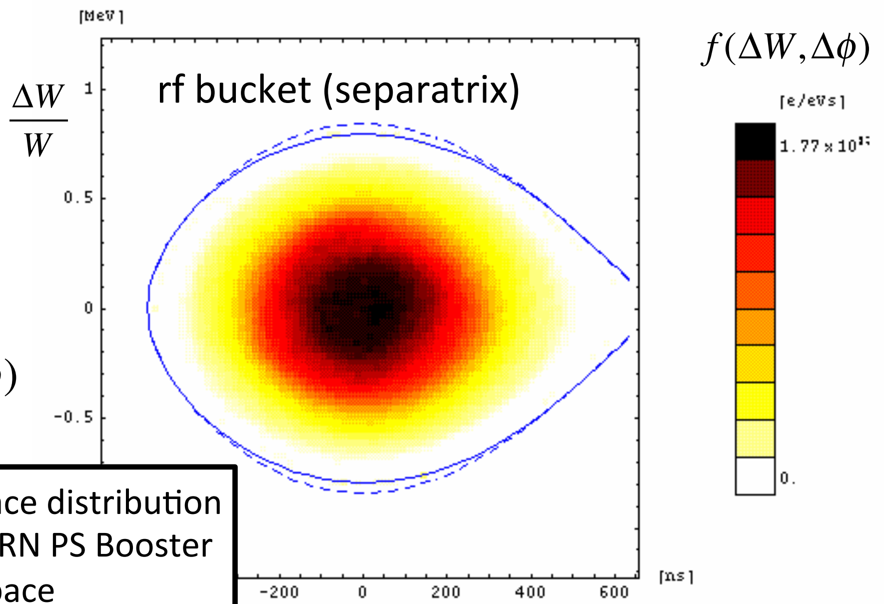
Small amplitudes: $\dot{\phi}^2 + \omega_s^2\Delta\phi^2 = \text{const.}$

Synchrotron frequency (small amplitudes):

$$\omega_s = \sqrt{\frac{qVh\cos\phi_s}{2\pi R^2 m^*}} \qquad m^* = -\frac{\gamma_0 m}{\eta}$$

Bucket and bunch area: $A_B(\phi_s) = 2\int_{\Delta\phi_1}^{\Delta\phi_2}\Delta W(\phi)d(\Delta\phi)$

$$A_B(\phi_s) \approx A_B(\phi_s = 0)\cdot\frac{1-\sin\phi_s}{1+\sin\phi_s}$$



$\tau = \frac{\Delta\phi}{\omega_{rf}}$ [ns]

Longitudinal phase space distribution after capture in the CERN PS Booster (tomographic phase space reconstruction, Hancock et al.)

**rf voltage requirement:**

Bunch area after injection determines $\varphi_s$ !

# Dual harmonic rf buckets

**Dual rf systems are employed e.g. in:**
CERN PSB, ISIS, J-PARC RCS, GSI SIS-18

Stationary ($\phi_s$=0, $\phi_{s2}$=0) rf wave form

$$V^{RF}(\phi) = V_0\left(\sin\phi - \frac{1}{2}\sin 2\phi\right) \approx \frac{1}{2}V_0\phi^3$$

Equation of motion: $\ddot{\phi} = -\frac{\omega_s^2}{2}\phi^3$

Amplitude-dependent
synchrotron frequency: $\omega_s(\hat{\phi}) \approx \omega' \hat{\phi}$

**Advantages:**

- **flattened bunches** (lower peak current)
- larger bucket area

**Complication:**

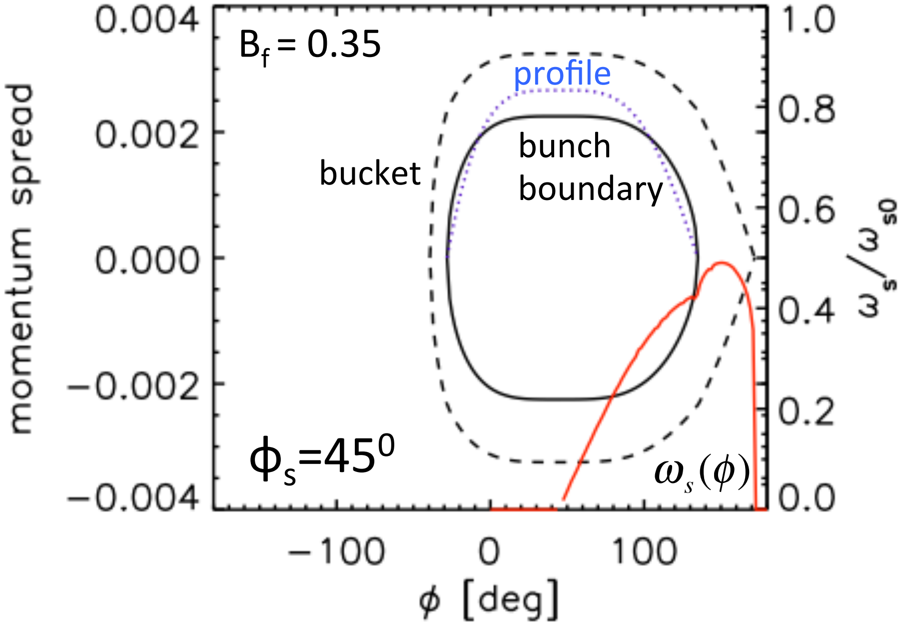
- control of the phase difference
- ‘fully nonlinear synchrotron oscillations’

Example case SIS-18:
$V_0$=40/16 kV, h=2/4 ($f_{min}$=430/860 kHz)

Non-Stationary ($\phi_s$>0, $\phi_{s2}$>0) rf wave form:

$$\frac{V(\phi)}{V_0} = \sin(\phi) - \sin(\phi_s) - \alpha\left(\sin\left[\phi_{s2} + 2(\phi - \phi_s)\right] - \sin(\phi_{s2})\right)$$

SIS-18: Dual rf bucket with flattened bunch profile



# Fast bunch compression

**For applications e.g. in nuclear physics a single, short bunch is extracted to the production target.**

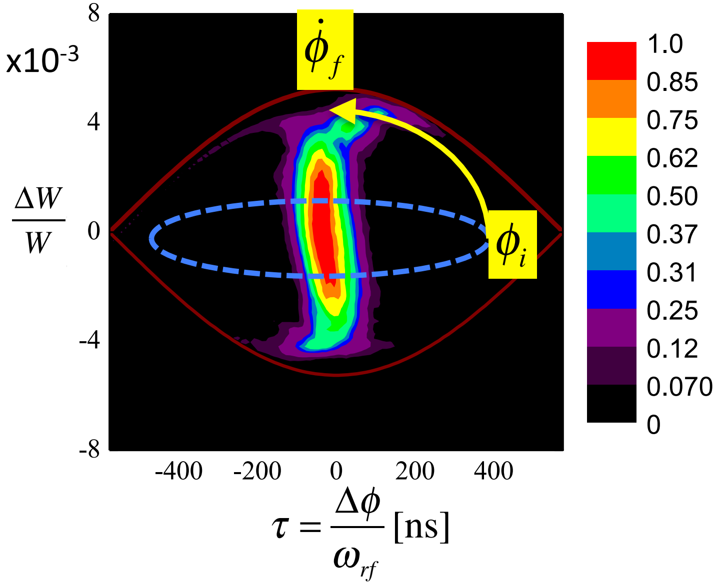
**Bunch rotation:**
Sudden switch-on of an additional rf voltage causes the bunch to rotate in the bucket.

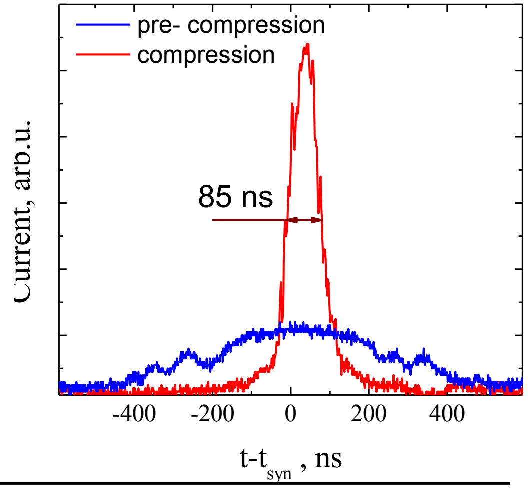
The compression takes only a quarter of a synchrotron period.

$$T_{rot}=\frac{T_s}{4}<1\text{ ms}$$

-> (broadband) rf cavity with fast rise time needed !

**Bunch rotation** in SIS-18





Stationary bucket, small amplitudes:

$$\dot\phi^2+\omega_s^2\phi^2=\text{const.}$$

$$\Rightarrow\quad \phi_f=\frac{\dot\phi_i}{\omega_s}$$

Bunch area:

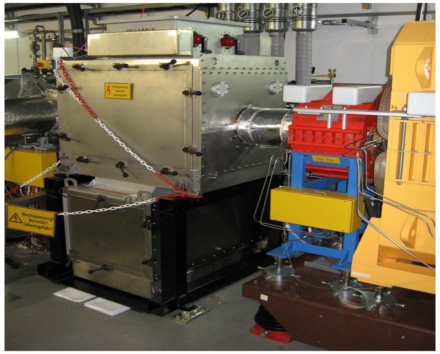
$$A_B=\phi_i\dot\phi_i=\phi_f\dot\phi_f$$

$$\Rightarrow\quad \phi_f=\frac{A_B}{\omega_s\phi_i}\propto\frac{\dot\phi_i}{\sqrt{V_0}}$$

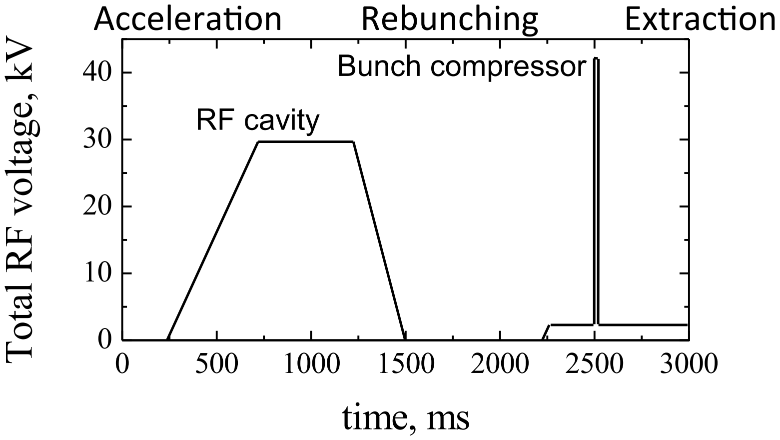
Bunch length: $\tau_f=\frac{\phi_f}{\omega_{rf}}$

Final bunch length depends on the initial momentum spread !

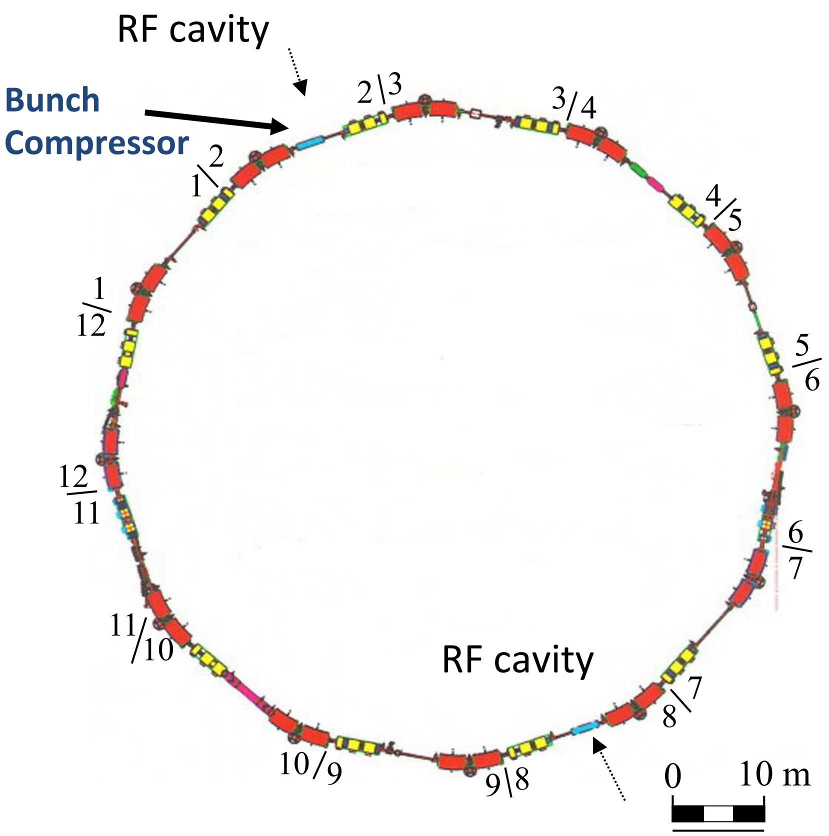
# Bunch compressor cavity



Magnetic alloy loaded cavity (length=1 m) with:
30 kV, 0.8 MHz, Q=1, 0.1 ms pulse duration.



GSI Synchrotron SIS-18



In the projected SIS-100: fast extraction of $5\text{x}10^{11}$ $U^{28+}$
in one short (50 ns) bunch -> 0.5 TW peak power !

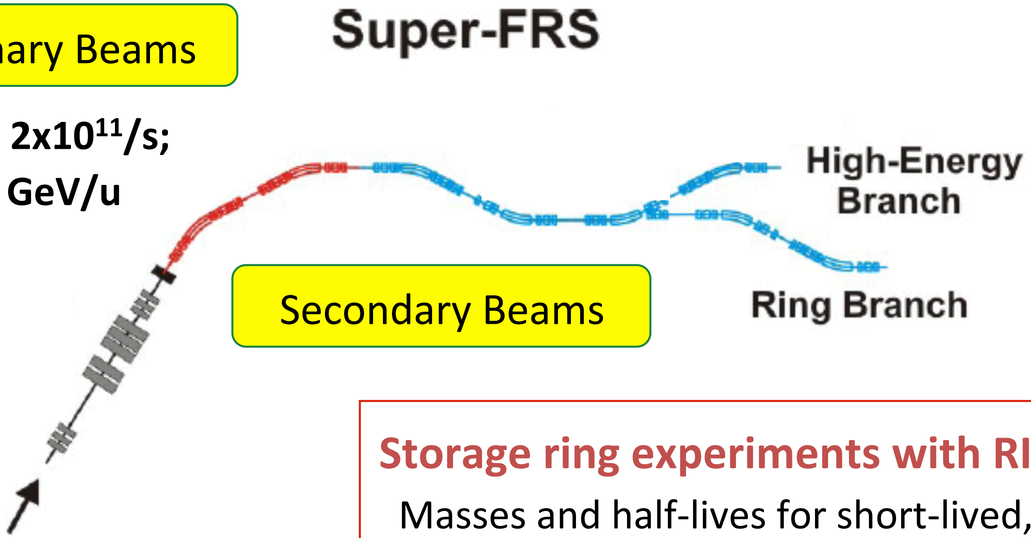
# ‘In-Flight’ production of radioactive ion beams

## Example: FAIR project at GSI

Primary Beams

$^{238}U^{28+}$; $2x10^{11}/s$;

**0.4-2.7 GeV/u**



Secondary Beams

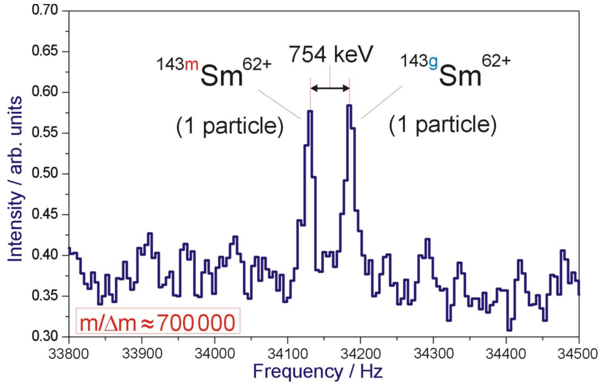
**Driver: SIS-100**

1 Hz repetition

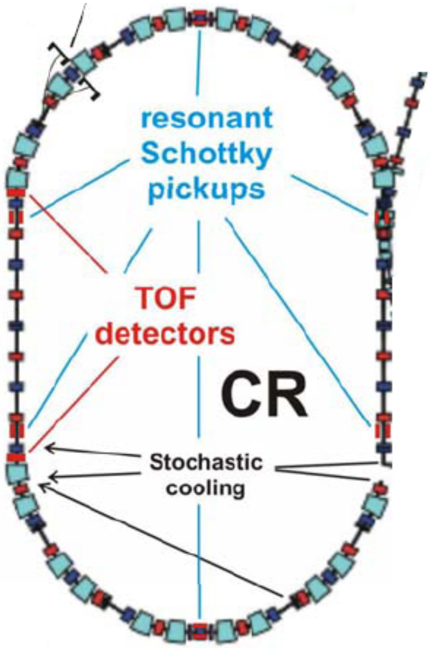
**50 ns pulse length**

**Storage ring experiments with RIBs**:

Masses and half-lives for short-lived, exotic ions



**Cooling time ≈1 s !**



**Primary heavy-ion beam intensity directly relates to the yield of exotic ions**

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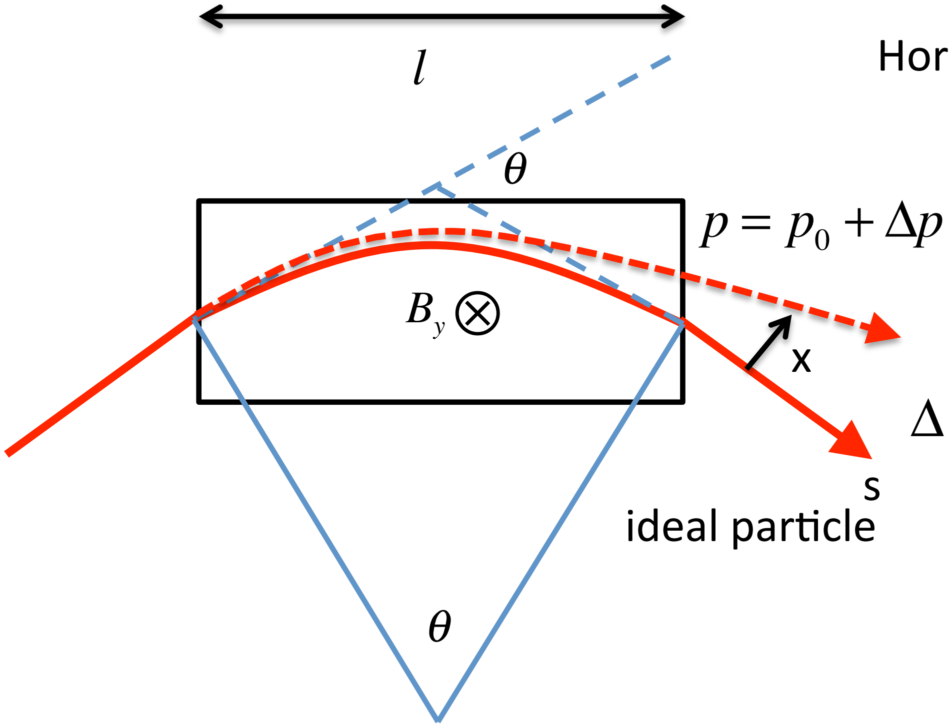
# Transverse motion in dipole magnets



$$\theta = \frac{q}{p_0}\int_{s_1}^{s_2} B_y ds \approx \frac{l}{R_0}$$

Horizontal particle offset: $x$

Divergence: $x' = \frac{dx}{ds}$

Path length: $s = \beta_0 ct$

$$\Delta\theta = \theta\frac{\Delta p}{p} \quad \Rightarrow \quad x'' = \frac{1}{R}\frac{\Delta p}{p_0}$$

$$x'' + \frac{1}{R^2}x = \frac{1}{R}\frac{\Delta p}{p_0}$$

‘weak’ focusing — inhomogeneous part

# Rapid/fast ramping dipole magnets

## Examples

**Large apertures**

SIS-18 dipoles: 20 cm x 8 cm

J-PARC RCS: 25 cm x 19 cm

**Ramping rates (Bdot):**

SIS-18 dipoles: 10 T/s

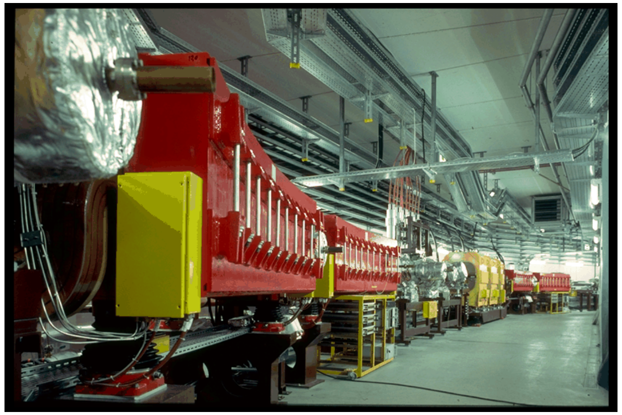
J-PARC RCS dipoles: 40 T/s

**Max. B-Field**

SIS-18: 1.9 T

J-PARC RCS: 1.1 T

Fast ramping (3 Hz) SIS-18 dipoles



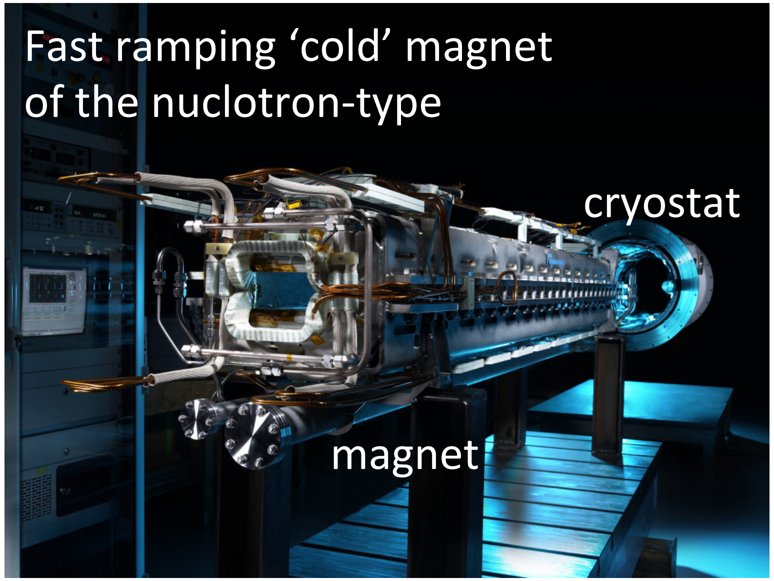
**SIS-100 superferric dipole:**

13 cm x 6 cm

Bdot = 4 T/s

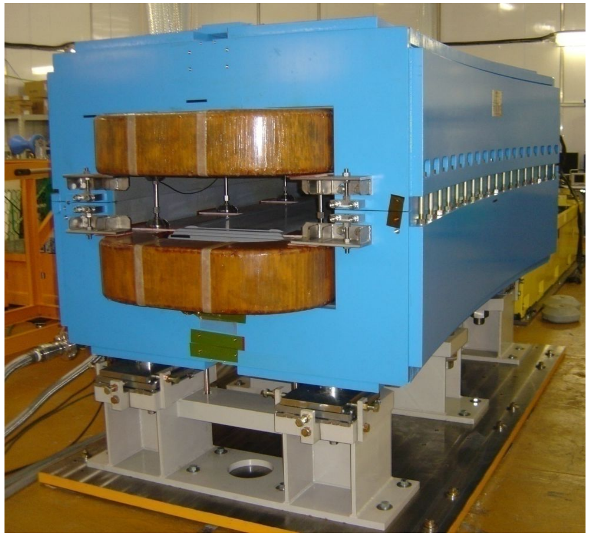
$B_{max}$= 2 T

pipe at 20 K

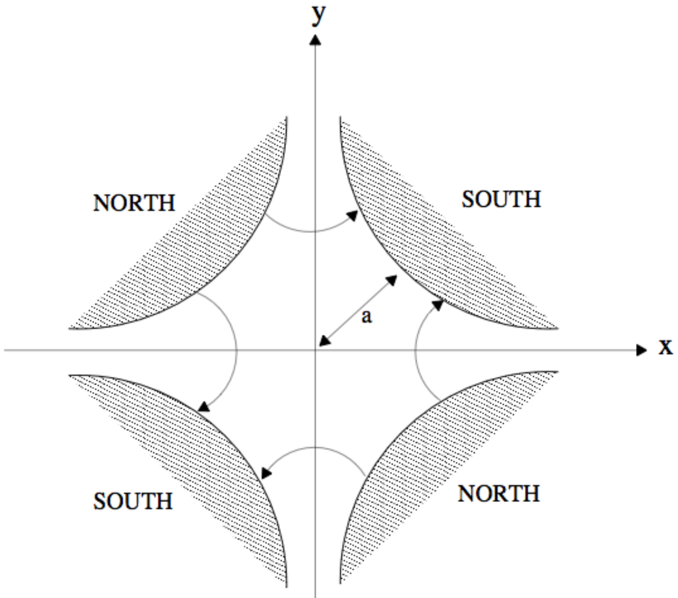


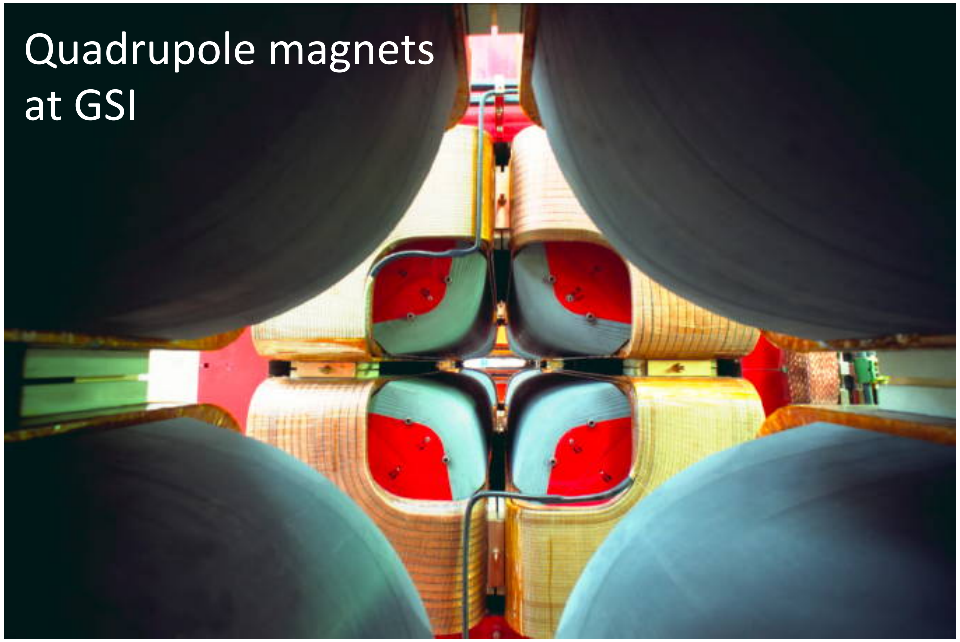
Fast ramping ‘cold’ magnet of the nuclotron-type

J-PARC RCS (25 Hz) dipole



# Quadrupole magnets and beam focusing




Quadrupole magnets at GSI

Magnetic field:

$$B_y = B_0 \frac{x}{a}, \quad B_x = B_0 \frac{y}{a}$$

Focusing gradient:

$$\kappa = \frac{q}{p_0} \frac{\partial B_x}{\partial y} = \frac{q}{p_0} \frac{\partial B_y}{\partial x}$$

Equations of motion:

$$x'' + \kappa(s) x = 0$$ (horizontal)

$$y'' - \kappa(s) x = 0$$ (vertical)

# Strong Focusing

Periodic focusing : $\kappa(s)=\kappa(s+L)$

Betatron oscillations:

$$x(s)=\sqrt{\hat{\beta}_x\epsilon_x}\cos(\psi(s)+\psi_0)$$

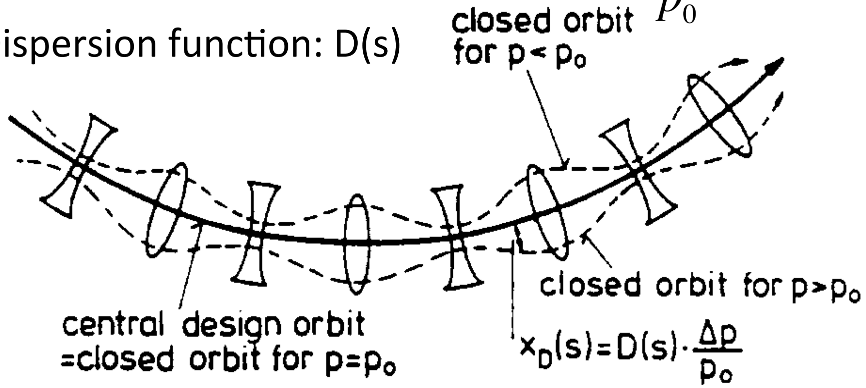
(Beam envelope)

Phase advance: $\psi(s)=\int_0^s\frac{ds}{\hat{\beta}_x(s)}.$

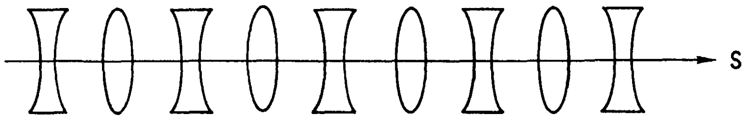
**Tune:** $Q_x=\frac{1}{2\pi}\int_0^C\frac{ds}{\hat{\beta}_x(s)}.$ Number of betatron oscillations per turn

With bends: $x(s)=x_\beta(s)+D(s)\frac{\Delta p}{p_0}$

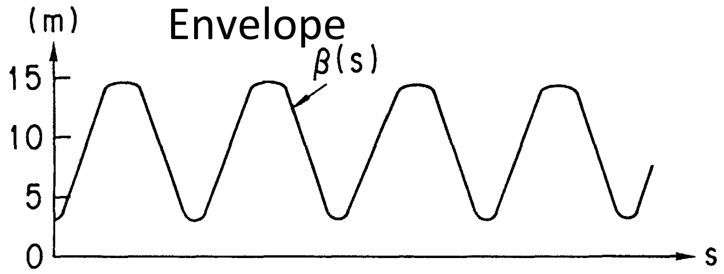
Dispersion function: D(s)



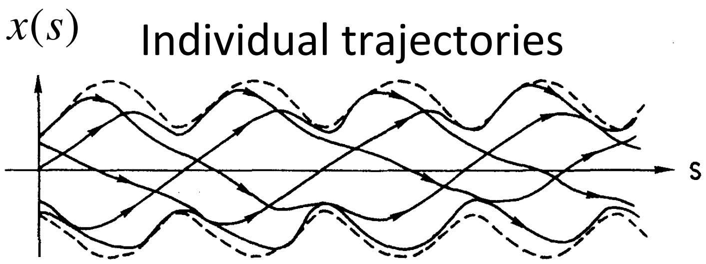
Periodic focusing (FODO)



(a)



(b)



(c)

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# Transverse resonances and 'space charge limit'

**Expansion of the B-field around the ideal pat**

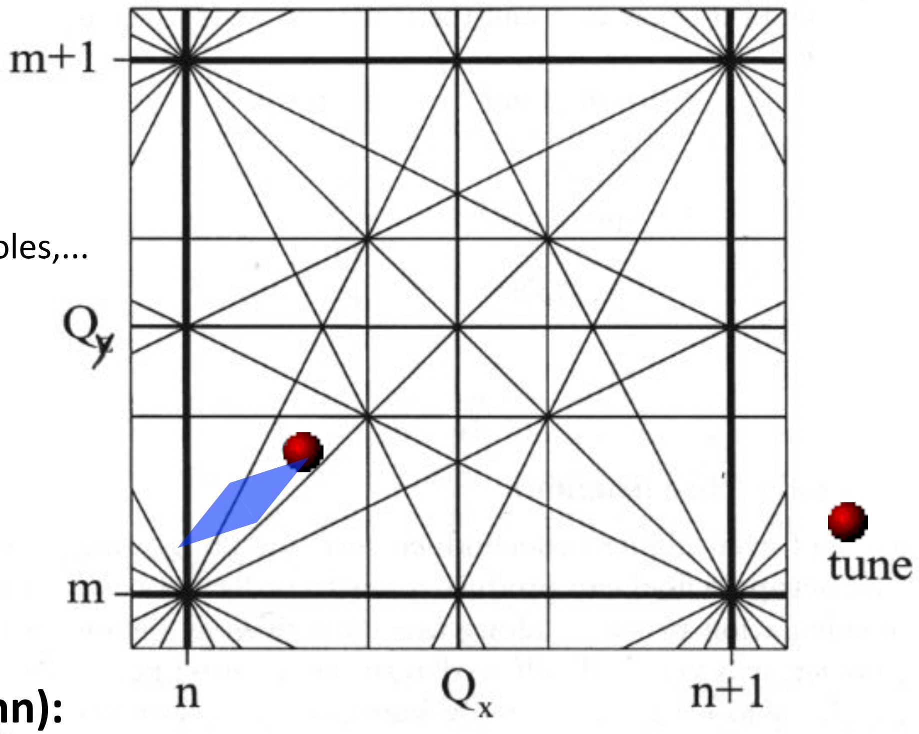
Bending Focusing

$$\frac{q}{p}B_y(x,s)=\frac{1}{R(s)}+\kappa(s)x(s)+\frac{\Delta R}{R^2}+\Delta\kappa x(s)+\mathrm{O}(x^2)$$

Dipole error Quad-error Sextupoles,...

Error resonances (m,n,p): $mQ_x+nQ_y=p$

Order: |n|+|m|=1, 2, 3, ….



space charge-
‚diamond'

**Space charge tune spread (e.g. CAS, A. Hofmann):**

$$\Delta Q_y^{sc}\propto-\frac{q^2}{m}\frac{N}{B_f}\frac{g_f}{\varepsilon_y\beta_0^2\gamma_0^3}\frac{2}{1+\sqrt{\varepsilon_y/\varepsilon_x}}$$

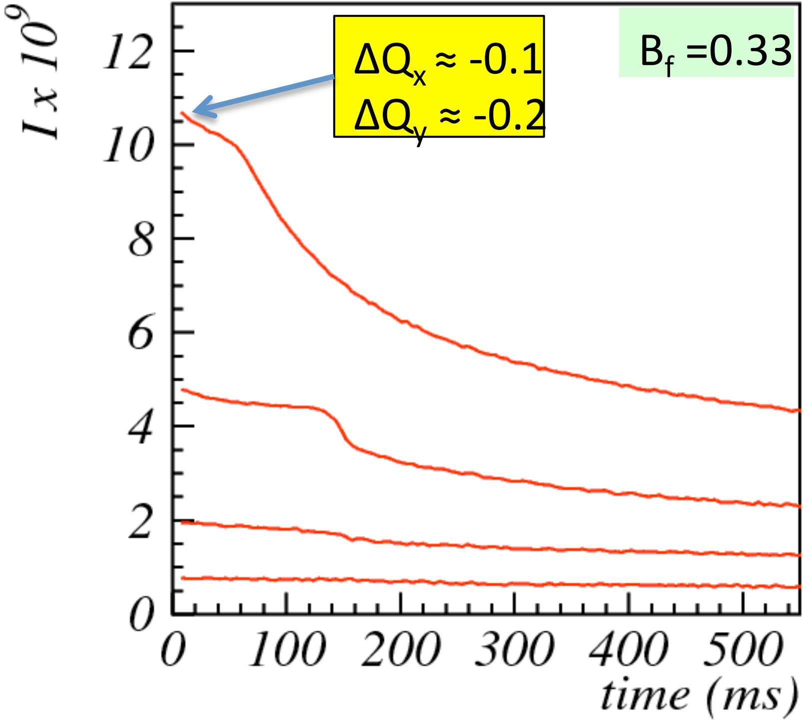
$g_f$: Transverse profile (Gauss: 2, homogenous: 1)
$B_f < 1$: bunching factor
$\varepsilon_{x,y}$: transverse emittances
**N: number of particles in the ring**
q: particle charge
m: particle mass

'Space charge limit': $|\Delta Q_y|\lesssim 0.5$
(text books)

# Space charge limit in a ‘real’ synchrotron

**Beam loss for high beam intensities**

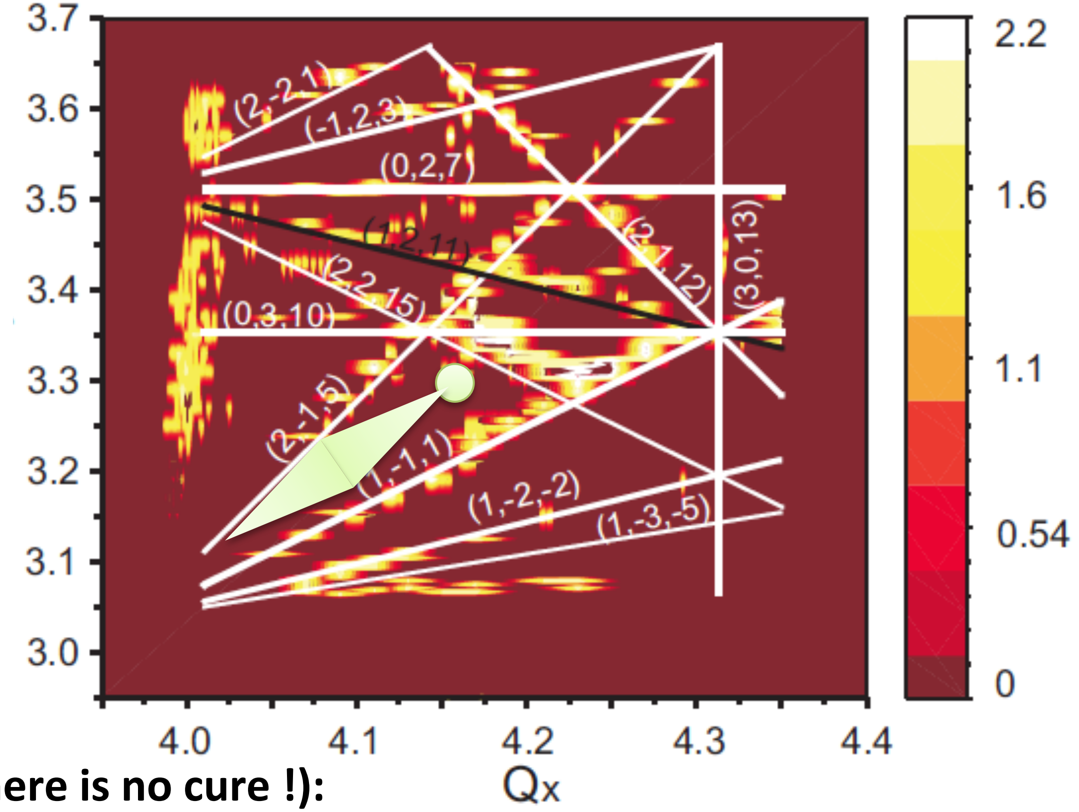
$Q_x$ = 4.17, $Q_y$ = 3.24



$\Delta Q_x \approx -0.1$
$\Delta Q_y \approx -0.2$

$B_f$ =0.33

**Resonance scan (low intensity beam )in the SIS-18.**
A. Parfenova, G. Franchetti, GSI (2011)



**Measures (there is no cure !):**

- Resonance compensation
- Flattened bunches (dual harmonic rf)

G. Franchetti, O. Chorniy, et al, Phys. Rev. ST-AB 2010

# Achieved beam intensities in the SIS-18 synchrotron

## Light vs. heavy ions

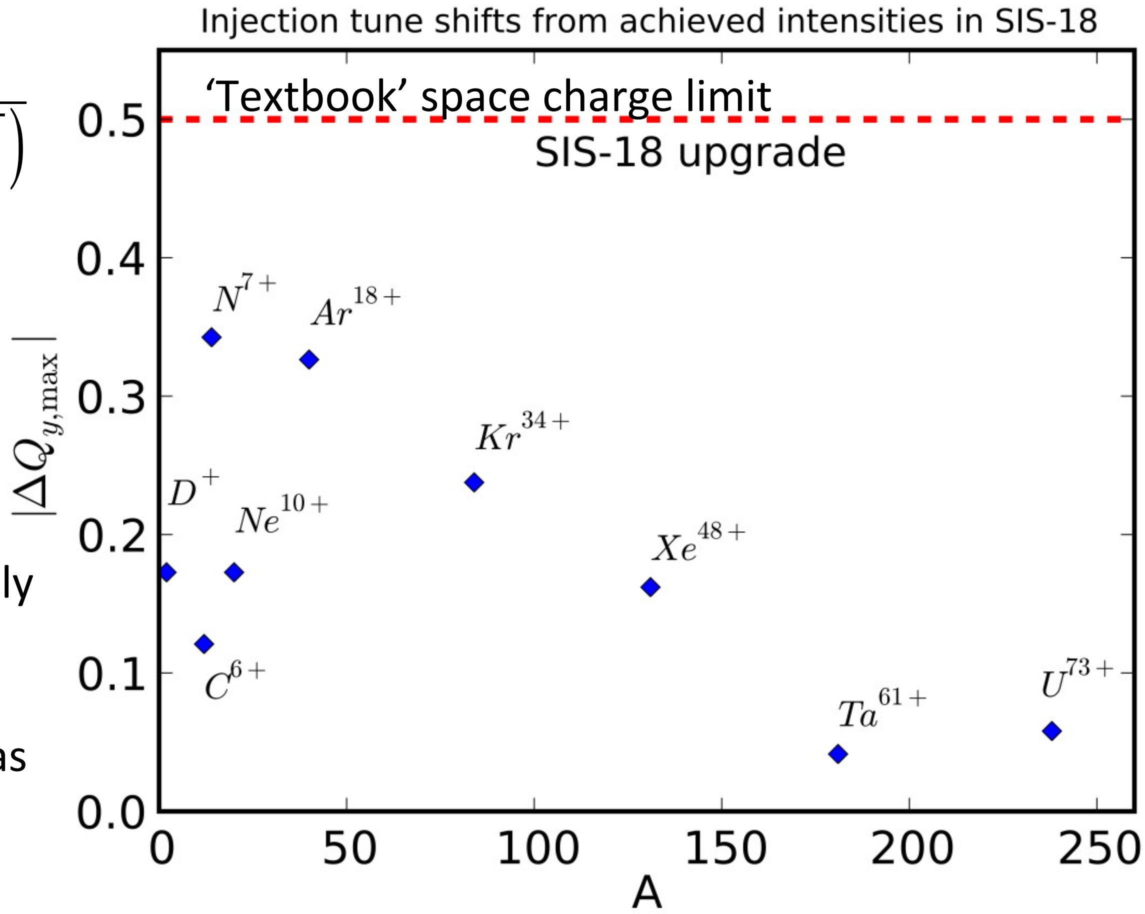
Injection energy: 11.4 MeV/u ($\beta_0$=0.155), Emittances: $\epsilon_{x,y} = 150/50$ mm mrad

$$\Delta Q_y^{sc} = -\frac{2NZ^2 g_f}{\pi A \beta_0^2 \gamma_0^3 B_f \left(\varepsilon_y + \sqrt{\varepsilon_y \varepsilon_x}\right)}$$

For protons or lighter ions the space charge limit is usually the actual intensity limiting factor.

For heavy ions there are presently other limiting factors. E.g.:

- Beam current from the source
- Beam lifetime in the residual gas



# Contents

- Working principle of synchrotrons
- High power, high intensity synchrotron facilities
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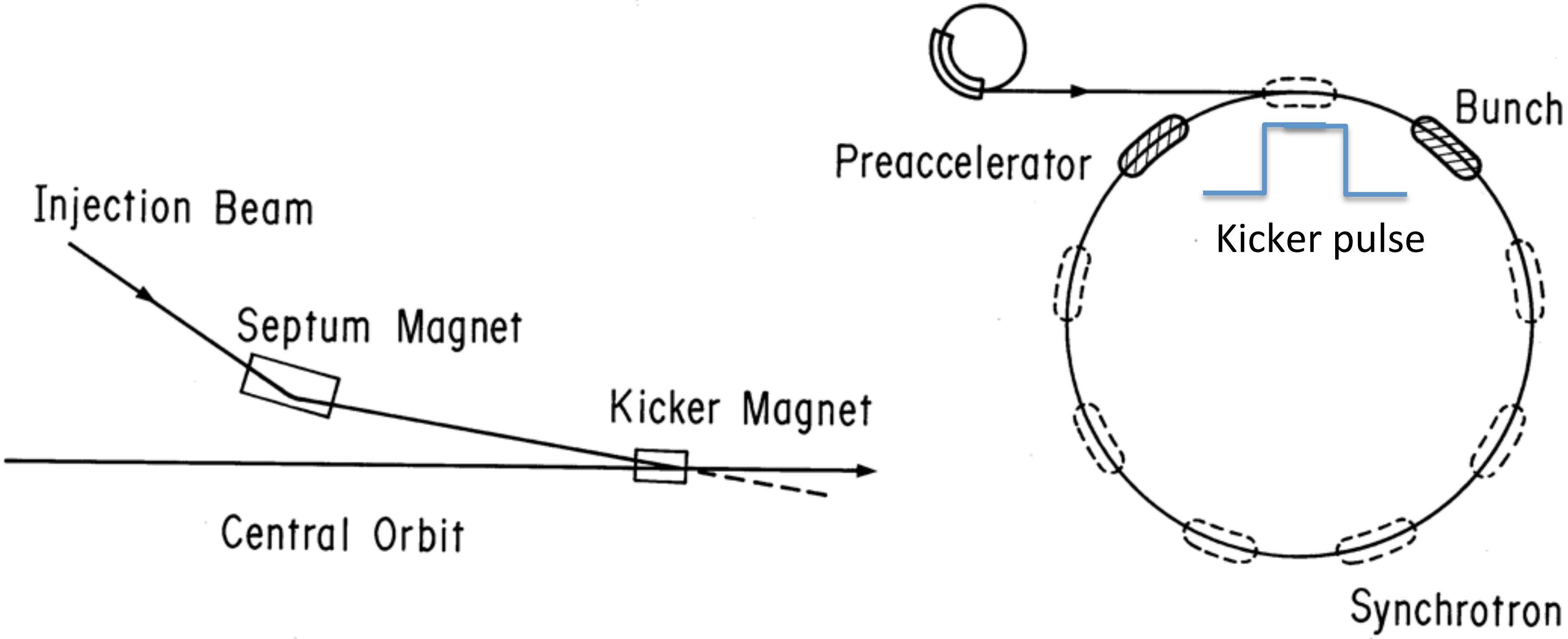
# Injection: Bunch-to-bucket

**From a smaller 'booster' synchrotron**

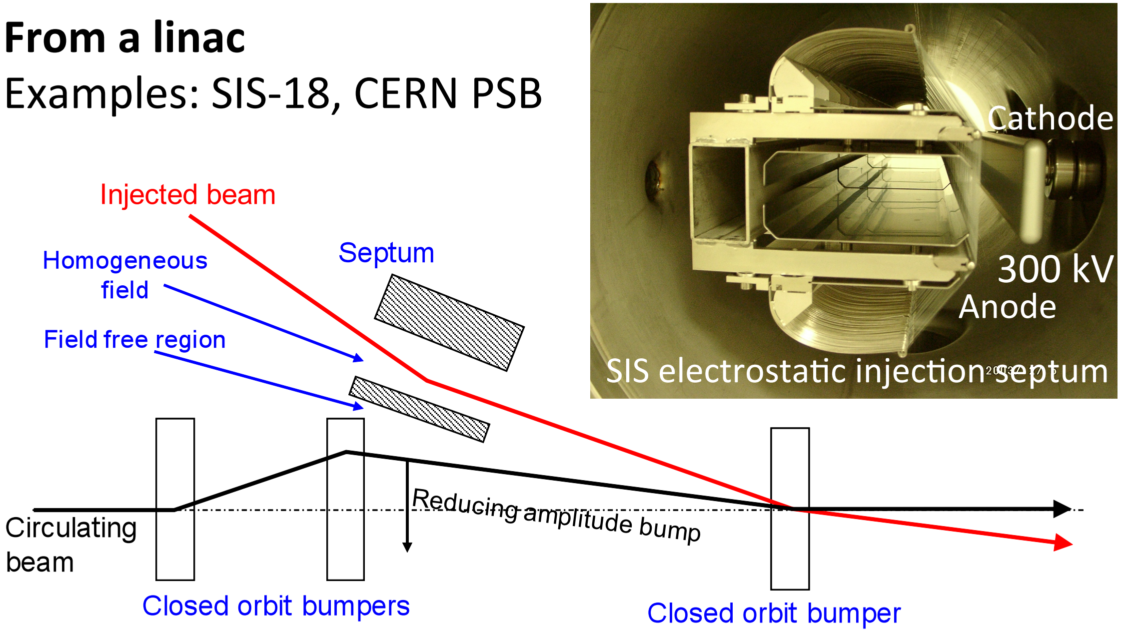


**Kicker:** fast dipole magnet with a rise time of 10-100 ns and a pulse duration of µs.

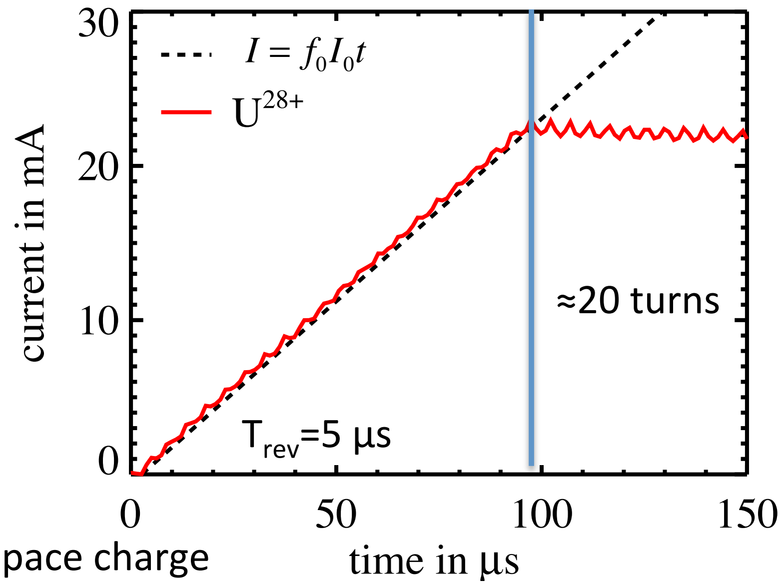
# Transverse (horizontal) multi-turn injection

**From a linac**

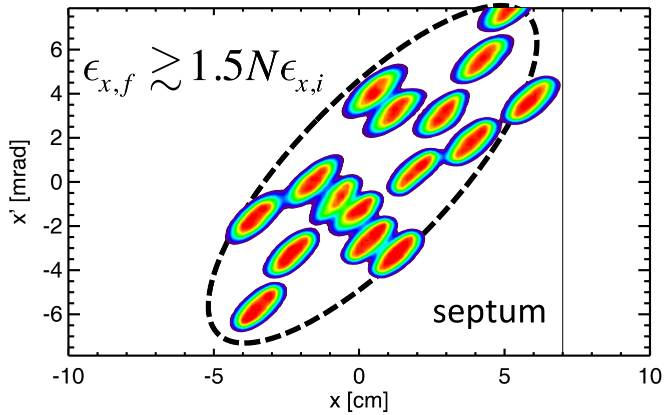
Examples: SIS-18, CERN PSB



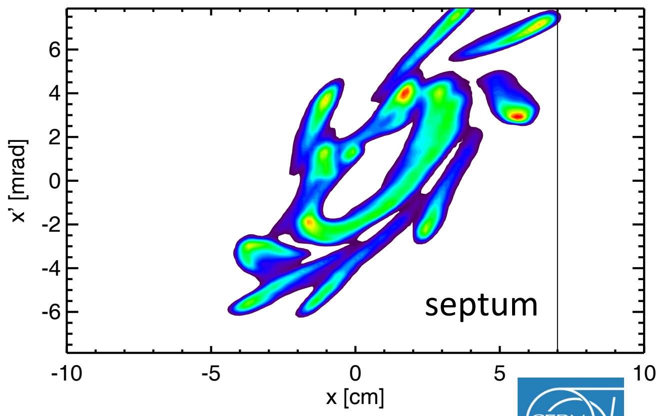
Measured MTI performance in SIS-18



Simulation: without space charge



Simulation: with space charge



**$H^-$ injection:**

Lecture by Chris Prior on Saturday

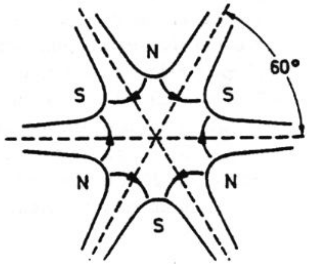
# (Slow) extraction

**Slow extraction examples:** GSI SIS-18 and SIS-100, J-PARC MR, BNL AGS

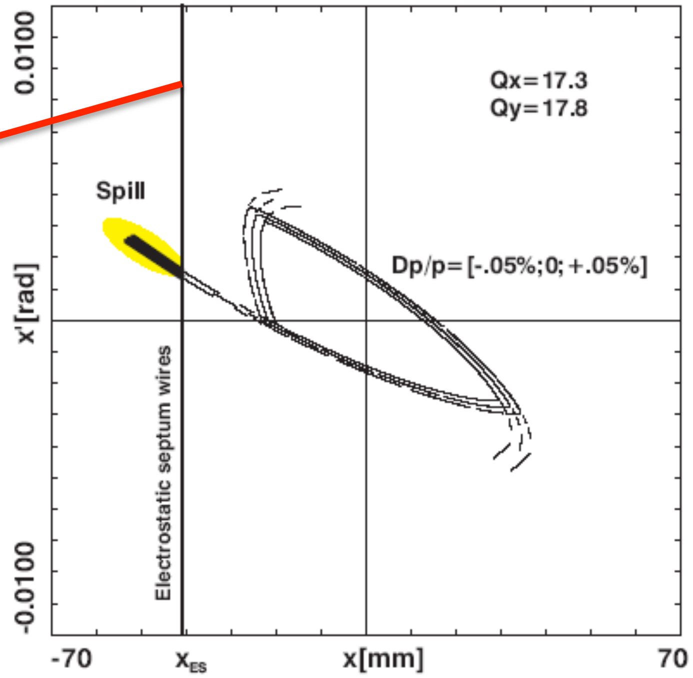
**Fast extraction:** in one turn using a kicker (e.g. after bunch compression.)

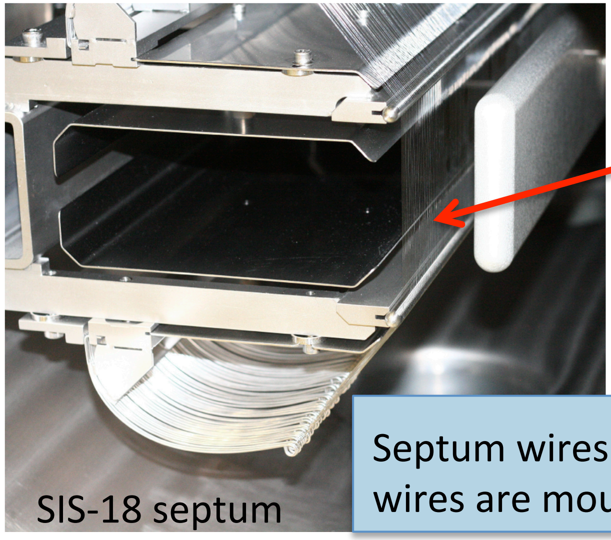
**Slow extraction:** over many turns (up to seconds !).
The horizontal tune is moved close to a third order resonance excited by sextupole magnets.
The particles on the resonance are extracted using electrostatic and magnetic septa.

Sextupole:



Separatrix (third order resonance)





SIS-18 septum

**Septum should be as thin as possible to avoid losses !**

Septum wires: Ø 0.025 mm (W-Re alloy)
wires are mounted under tension

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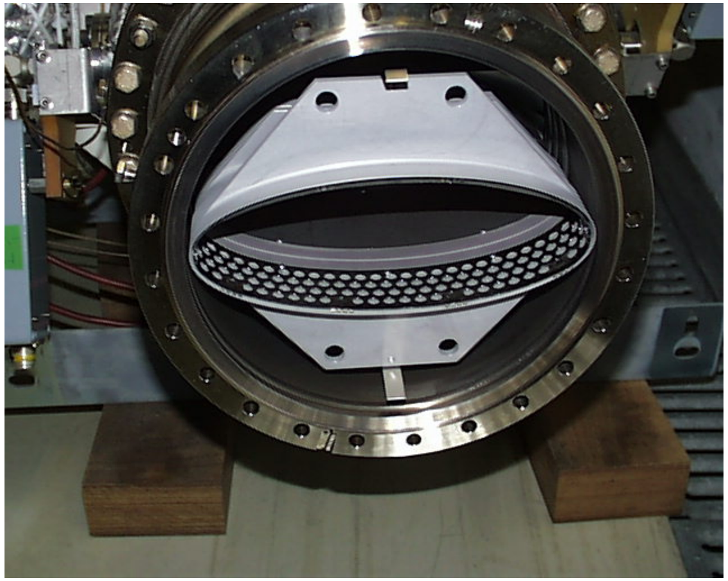
# Vacuum chamber

**Main function:**

- enclose the vacuum of $10^{-9}$ mbar (protons) or $10^{-12}$ mbar (ions)
  -> active pumping and low desorption inner surface

- shielding of the EM fields generated by the beam
  -> low beam impedance

The **beam pipe in the SIS-18 magnets** is d=0.3 mm thick (stainless steel)



**Problems:**

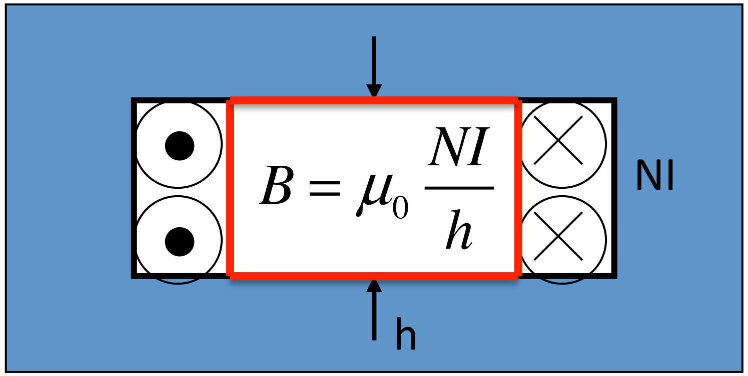
Heating of the pipe by eddy currents
-> outgassing and vacuum degradation

Magnetic fields induced by eddy currents
-> resonances and beam loss

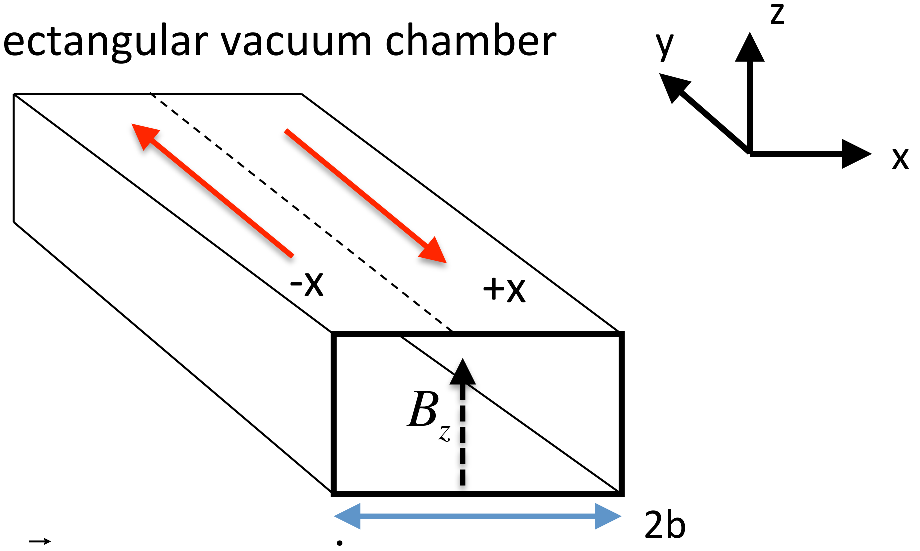
**Beam pipe is one of the most complex components in a synchrotron !**

# Eddy-currents in a rectangular beam pipe

Dipole magnet with vacuum chamber



Rectangular vacuum chamber



Faraday's law: $\oint \vec{E}\cdot d\vec{l} = -\int \dot{\vec{B}}\cdot d\vec{S} \quad \Rightarrow \quad E_y = \dot{B}_z x$

current density (conductivity σ): $j_y = \sigma E_y = -\sigma \dot{B}_z x$

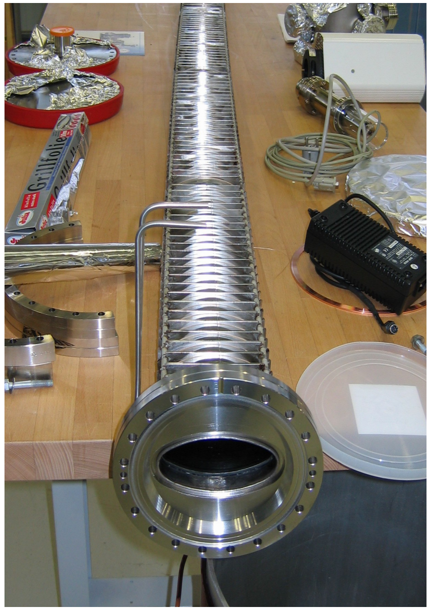
induced current between x=0 and x: $I = d\sigma \dot{B}_z x^2$ (wall thickness d)

induced field from Ampere's law: $\Delta B_z = \mu_0 \frac{d}{h} \sigma \dot{B}_z \left(\underbrace{x^2}_{\text{sextupole}} - \underbrace{b^2}_{\text{dipole}}\right)$

Power deposition / length: $\frac{P}{l} = d\sigma b^3 \dot{B}^2$

see CAS 2010
lecture by G. Moritz !

# Thin beam pipes for fast ramping synchrotrons



Thin (0.3 mm) stainless steel beam pipe
for the projected SIS-100 synchrotron

**Thin (0.2-0.3 mm) stainless steel pipes for fast ramping machines (< 5 Hz):**

- still mechanically robust (with supporting rips etc.)
- tolerable heating (< 10 W/m) and field distortion
- sufficient shielding of EM fields for frequencies larger 50 kHz
- problem: large resistive impedance !

Rf shielding: $d \gtrsim \delta_s$

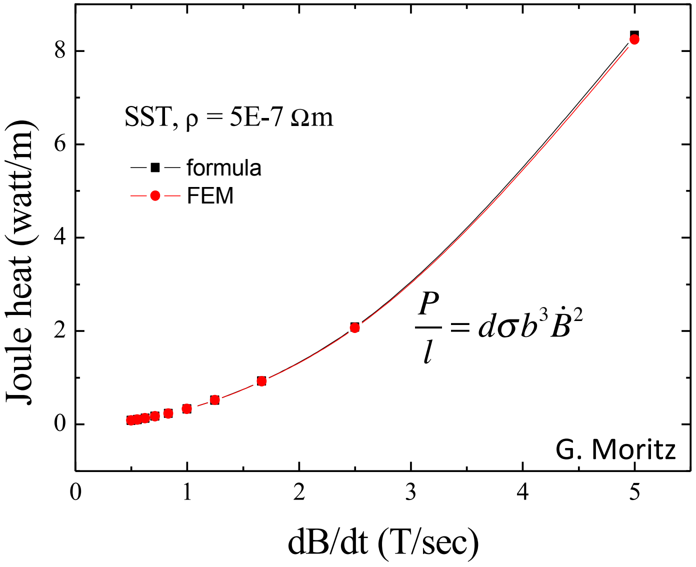
Skin depth: $\delta_s = \sqrt{\dfrac{2}{\omega \mu_0 \sigma}}$

Conductivity: $\sigma \approx 10^6\,(\Omega\mathrm{m})^{-1}$

For d=0.3 mm, $f_0$=100 kHz: $\delta_s(f_0) \approx 1.6\,\mathrm{mm}$

Transverse resistive impedance:

$$Z_\perp(\omega) = \frac{2cR}{b^3 \sigma \omega d}$$

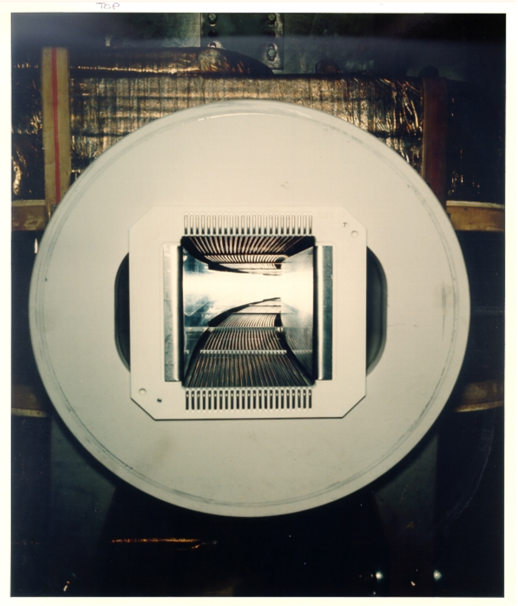


Structures behind the pipe can contribute for the lowest frequencies !

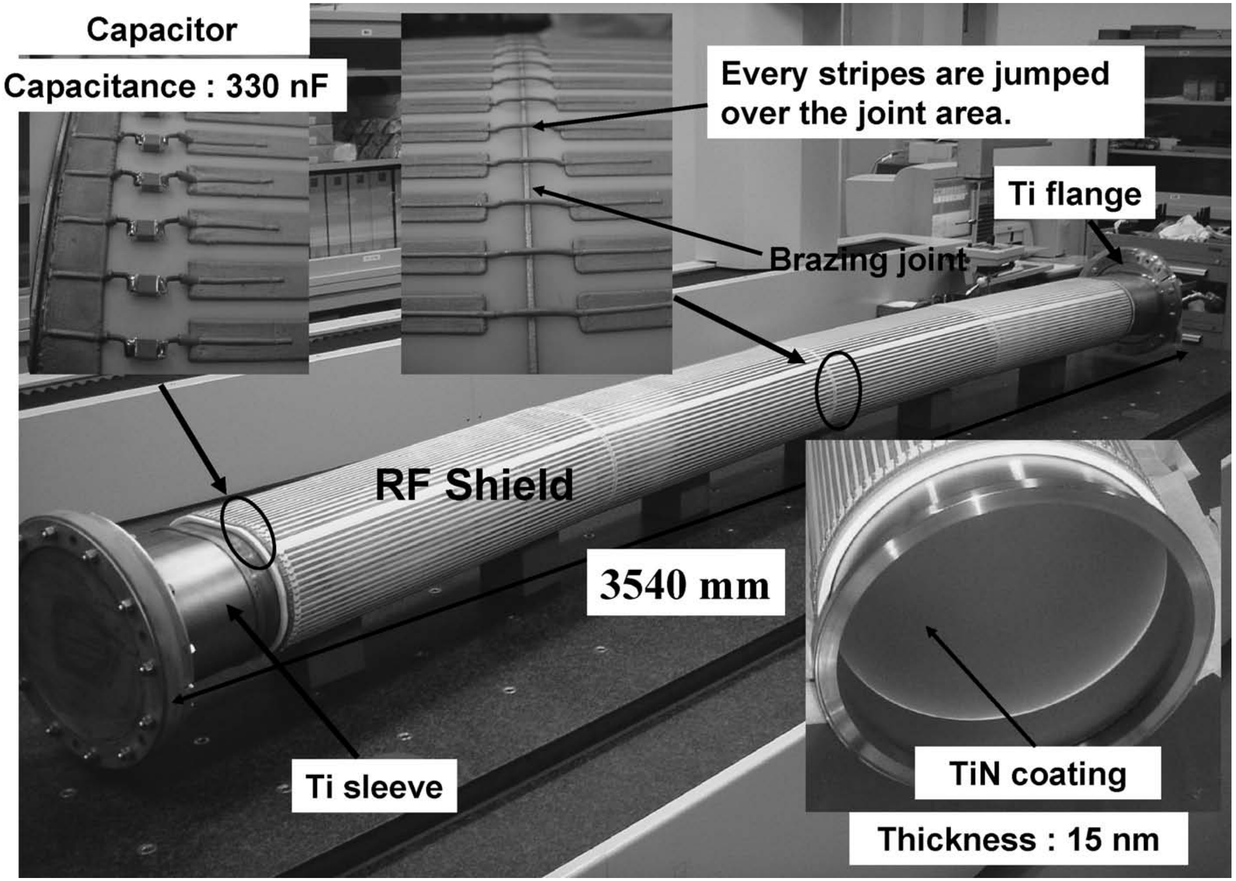
# Vacuum chambers for RCS

For rapid cycling synchrotrons (above 5-10 Hz) other solutions are required.

ISIS ceramic beam pipe with wire cage.



J-PARC RCS ceramics beam pipe with outer rf shield (copper stripes).



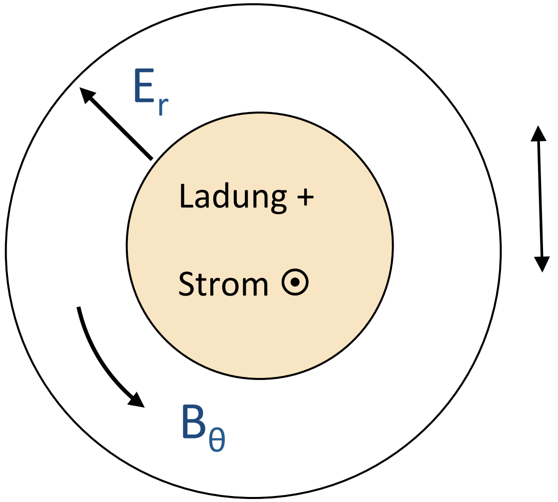
# Summary

- Synchrotrons: typically the 'working horse' in an accelerator chain.
- Average beam power up to ≈300 kW (achieved) and 1 MW (expected at J-PARC) with RCSs.
- In fast ramping synchrotrons: Large peak power per cycle due to bunch compression.
- Intensity limitations in proton synchrotrons:
  - At injection energy: Space charge tune spread and ring resonances ('hard limit').
  - At all energies: Coherent beam instabilities (not covered in this lecture)
  - At top energy: Beam loss induced activation of accelerator components
- Additional intensity limitations in heavy-ion synchrotrons:
  - Current from the ion source.
  - Efficiency of the multi-turn injection.
  - Charge changing processes with residual gas molecules.

# Additional transparencies

# Space charge tune shift

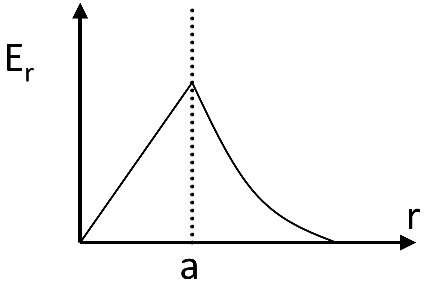
Strahl im Vakuumrohr



a: Strahlradius

Gauss'scher Satz: $\varepsilon_0 \int E_r \, dA = \int \rho \, dV$

$$\Rightarrow \quad E_r = \begin{cases} \dfrac{\rho_0 r}{2\varepsilon_0}, & r < a \\ \dfrac{\rho_0 a}{2\varepsilon_0 r}, & r \geq a \end{cases}$$



Stokes: $\int B_\theta \, ds = \mu_0 v_0 \int \rho \, dA \quad \Rightarrow \quad B_\theta = \dfrac{v_0}{c^2} E_r$

Konstante Ladungsdichte:

$$\rho(r) = \begin{cases} \rho_0(s), r \leq a \\ 0, r > a \end{cases}$$

Defokussierende Raumladungskraft auf ein Strahlteilchen:

$$F_r = q(E_r - v_0 B_\theta) = \frac{qE_r}{\gamma^2} = \frac{q\rho_0}{2\varepsilon_0 \beta_0 c \gamma_0^2} r$$

$$N = \rho_0 \pi a^2 L$$

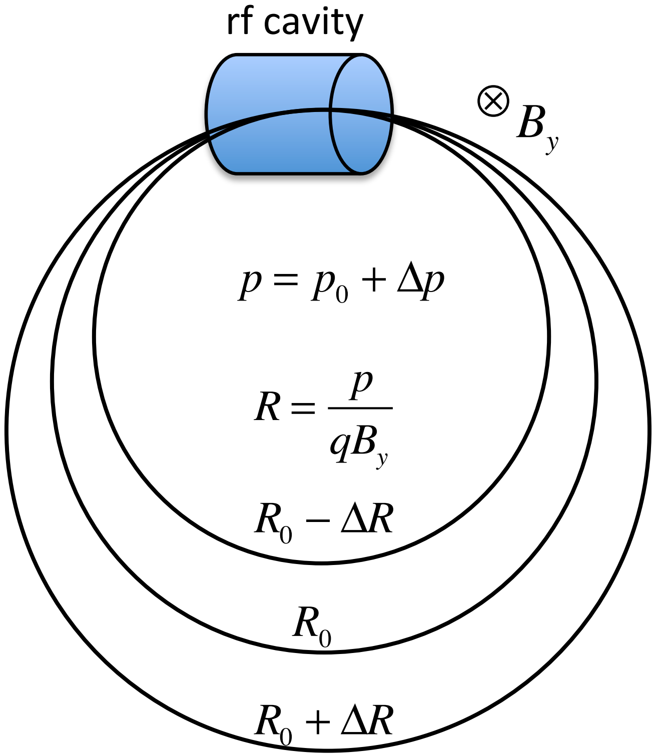
Raumladungsverschiebung des "tunes":

$$Q = Q_0 - \Delta Q^{sc} \quad \Delta Q^{sc} \propto \frac{q^2}{m} \frac{N}{a^2 \beta^2 \gamma^3}$$

Beispiel SIS-18:

$$Q_{y,0} = 3.23 \quad \Delta Q_y \leq 0.5$$

# Longitudinal motion in a circular accelerator



Revolution period: $T = \frac{2\pi R}{v}$

$$\frac{\Delta T}{T} = \frac{\Delta R}{R} - \frac{\Delta v}{v}$$

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2}\frac{\Delta p}{p}$$

$$\frac{\Delta R}{R} = \frac{1}{\gamma_t^2}\frac{\Delta p}{p}$$

Frequency slip factor: $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$

$$\boxed{\frac{\Delta T}{T} = \eta\frac{\Delta p}{p}}$$

Linac:

$$R \to \infty \quad \gamma_t \to \infty \quad \eta \to -\frac{1}{\gamma_0^2}$$

# Charge-exchange injection of H$^-$

Examples: ISIS, J-PARC RCS, CERN PSB (with Linac 4)