Paul Scherrer Institut
René Künzi
Filter Design - Passive Power Filters
CERN Accelerator School 2014, Baden, Switzerland
<table>
<thead>
<tr>
<th>Suitable Filter Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For signal filters simple RC are commonly used</strong></td>
</tr>
<tr>
<td>![Diagram of RC filter]</td>
</tr>
<tr>
<td><strong>Attenuation only 20dB/decade</strong></td>
</tr>
<tr>
<td><strong>Full current through R → Losses!</strong></td>
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<tr>
<td><strong>With a LC structure we get</strong></td>
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<td>![Diagram of LC filter]</td>
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<td><strong>40dB/decade</strong></td>
</tr>
<tr>
<td><strong>There is a high resonance</strong></td>
</tr>
<tr>
<td><strong>Series damping in order to overcome the resonance problem</strong></td>
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<tr>
<td>![Diagram of series damping]</td>
</tr>
<tr>
<td><strong>Full current through R → Losses!</strong></td>
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<tr>
<td><strong>Parallel damping in order to overcome the resonance problem</strong></td>
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<tr>
<td>![Diagram of parallel damping]</td>
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<tr>
<td><strong>Full voltage across R → Losses!</strong></td>
</tr>
<tr>
<td>Suitable Filter Structures</td>
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</tbody>
</table>
H-Bridge
H-Bridge with CM filter
2nd order filter - Transferfunction

\[ Z(s) = \frac{1}{C_1 s + \frac{1}{R_D + \frac{1}{C_{DS}}}} = \frac{R_D C_D s + 1}{C_1 R_D C_D s^2 + (C_1 + C_D) s} \]

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{Z(s)}{L_1 s + Z(s)} = \frac{R_D C_D s + 1}{L_1 C_1 R_D C_D s^3 + L_1 (C_1 + C_D) s^2 + R_D C_D s + 1} \quad (1) \]

\[ G(s) = \frac{k_1 s + 1}{k_3 s^3 + k_2 s^2 + k_1 s + 1} \]

1st order PD

3rd order PT

with
\[ k_1 = R_D C_D \]
\[ k_2 = L_1 (C_1 + C_D) \]
\[ k_3 = L_1 C_1 R_D C_D \]
3rd order PT in its normalized form:

$$G_{PT}(s) = \frac{1}{(1 + a_1 \frac{s}{\omega_0}) \cdot (1 + a_2 \frac{s}{\omega_0} + b_2 \frac{s^2}{\omega_0^2})}$$  \hspace{1cm} (2)

<table>
<thead>
<tr>
<th>Method</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butterworth</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Bessel</td>
<td>0.7560</td>
<td>0.9996</td>
<td>0.4772</td>
</tr>
<tr>
<td>Critical damping</td>
<td>0.5098</td>
<td>1.0197</td>
<td>0.2599</td>
</tr>
</tbody>
</table>
By expanding (2) and comparing the coefficients with (1) we get:

\[
G_{PT}(s) = \frac{1}{\omega_0^3} \frac{a_1 b_2}{s^3} + \frac{(a_1 a_2 + b_2)}{\omega_0^2} s^2 + \frac{(a_1 + a_2)}{\omega_0} s + 1
\]

\[
k_1 = R_D C_D = \frac{a_1 + a_2}{\omega_0} \quad \text{(3a)}
\]

\[
k_2 = L_1 (C_1 + C_D) = \frac{a_1 a_2 + b_2}{\omega_0^2} \quad \text{(3b)}
\]

\[
k_3 = L_1 C_1 R_D C_D = \frac{a_1 b_2}{\omega_0^3} \quad \text{(3c)}
\]

The 3 independent equations (3a….3c) contain 5 unknowns \((L_1, C_1, R_D, C_D\) and \(\omega_0\)). Therefore we have the choice to select 2 of them and the remaining 3 depend on that selection.
For a given frequency $\omega_B$ well in the blocking area ($\omega_B \gg \omega_0$) we can define the desired attenuation $G_B$. In the blocking area the highest order terms of both the numerator and denominator in equation (1) dominate, therefore (1) can be simplified to:

$$
G_B = \frac{R_D C_D s}{L_1 C_1 R_D C_D s^3} = \frac{a_1 + a_2}{\omega_0} s = \frac{a_1 + a_2}{a_1 b_2} \cdot \frac{\omega_0^2}{s^2} = -\frac{a_1 + a_2}{a_1 b_2} \cdot \frac{\omega_0^2}{\omega_B^2}
$$

$$(j)^2 = -1
$$

$$
\omega_0 = \omega_B \cdot \frac{|G_B| \cdot a_1 b_2}{\sqrt{a_1 + a_2}} 
$$

(4)
For cost reasons $L_1$ should be as small as possible, but a too small inductance will result in an excessive ripple current!

The DC-voltage across $C_1$ is $m \cdot V_{DC}$. When the IGBT is on, the current in $L_1$ increases and the peak-peak ripple current $\Delta I_{L1}$ can be calculated:

$$V_{L1} = L_1 \cdot \frac{di_{L1}}{dt} = V_{DC} - V_{C1} = V_{DC} \cdot (1 - m)$$

$$\Delta I_{L1} = m \cdot T \cdot \frac{di_{L1}}{dt} = m \cdot \frac{1}{f_s} \cdot \frac{V_{DC} \cdot (1 - m)}{L_1} = \frac{V_{DC} \cdot (1 - m) \cdot m}{f_s \cdot L_1}$$

$$L_1 = \frac{V_{DC} \cdot 0.25}{f_s \cdot \Delta I_{L1}} \quad (5)$$

Maximum 0.25 for $m = 0.5$
Alternative approach to determine $L_1$:

\[ I_{L1\_ripple\_pp} = \frac{v_{1\_ripple\_pp}}{2 \cdot \pi \cdot f_{ripple} \cdot L_1} \]

\[ L_1 = \frac{v_{1\_ripple\_pp}}{2 \cdot \pi \cdot f_{ripple} \cdot I_{L1\_ripple\_pp}} \]
Substitute (3a) in (3c) and we receive:

Selection: \( L_1 \) and \( \omega_0 \)

\[ C_1 = \frac{a_1 b_2}{L_1 \omega_0^2 (a_1 + a_2)} \]  \hspace{1cm} (7a)

Selection: \( C_1 \) and \( \omega_0 \)

\[ L_1 = \frac{a_1 b_2}{C_1 \omega_0^2 (a_1 + a_2)} \]  \hspace{1cm} (7b)

Selection: \( L_1 \) and \( C_1 \)

\[ \omega_0 = \sqrt{\frac{a_1 b_2}{L_1 C_1 (a_1 + a_2)}} \]  \hspace{1cm} (7c)

Solve (3b) for \( C_D \):

\[ C_D = \frac{a_1 a_2 + b_2}{L_1 \omega_0^2} - C_1 \]  \hspace{1cm} (8)

Solve (3a) for \( R_D \):

\[ R_D = \frac{a_1 + a_2}{C_D \omega_0} \]  \hspace{1cm} (9)
Design a 2\textsuperscript{nd} order filter for all three given optimization methods and compare the results.

- DC-link voltage: 200V
- DC-link current: 500A
- $\Delta I_{L1} \leq 50\text{App.}$
- $C_1$ must be $\geq 22\text{mF}$ (because of high ripple current)
Select $L_1$ to meet the ripple current requirement:

\[
L_1 = \frac{V_{1\text{-ripple}\_pp}}{2 \cdot \pi \cdot f_{\text{ripple}} \cdot I_{L1\text{-ripple}\_pp}} \quad (6)
\]

\[
L_1 = \frac{(200 \cdot 0.13)V_{pp}}{2 \cdot \pi \cdot 300s^{-1} \cdot 50App} = 300\mu H
\]

Select $C_1$:

\[
C_1 = 22mF
\]

Calculate the remaining filter elements \((7c, 8, 9)\)
Results:

<table>
<thead>
<tr>
<th></th>
<th>Butterworth</th>
<th>Bessel</th>
<th>Critical damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>1.0000</td>
<td>0.7560</td>
<td>0.5098</td>
</tr>
<tr>
<td>(a_2)</td>
<td>1.0000</td>
<td>0.9996</td>
<td>1.0197</td>
</tr>
<tr>
<td>(b_2)</td>
<td>1.0000</td>
<td>0.4772</td>
<td>0.2599</td>
</tr>
<tr>
<td>(\omega_0)</td>
<td>275 s(^{-1}) (44 Hz)</td>
<td>177 s(^{-1}) (28 Hz)</td>
<td>115 s(^{-1}) (18 Hz)</td>
</tr>
<tr>
<td>(L_1)</td>
<td>300 (\mu)H</td>
<td>300 (\mu)H</td>
<td>300 (\mu)H</td>
</tr>
<tr>
<td>(C_1)</td>
<td>22 mF</td>
<td>22 mF</td>
<td>22 mF</td>
</tr>
<tr>
<td>(C_D)</td>
<td>66 mF</td>
<td>110 mF</td>
<td>176 mF</td>
</tr>
<tr>
<td>(R_D)</td>
<td>0.11 (\Omega)</td>
<td>0.09 (\Omega)</td>
<td>0.08 (\Omega)</td>
</tr>
</tbody>
</table>

\(x3\) \(x5\) \(x8\)
2\textsuperscript{nd} order filter – Example 1

Maximum amplitude of resonance:
- Butterworth: 4.5 dB
- Bessel: 3.1 dB
- Critical damping: 2.3 dB

Frequency, for -3 dB attenuation:
- Butterworth: 74 Hz
- Bessel: 67 Hz
- Critical damping: 59 Hz

Attenuation: -28 dB @ 300 Hz
2\textsuperscript{nd} order filter – Example 2

- DC-link voltage: 120V
- $f_s = 20\text{kHz}$
- $I_{\text{Out\_max}} = 500\text{A}$
- $\Delta I_{L1} \leq 50\text{App.}$
- Attenuation: $G_B = 250$ @ $\omega_B = 2*\pi*20\text{kHz}$

Design a 2\textsuperscript{nd} order filter for all three given optimization methods and compare the results.
Select $L_1$ to meet the ripple current requirement:

$$L_1 = \frac{V_{DC} \cdot 0.25}{f_s \cdot \Delta I_{L1}} = \frac{120V \cdot 0.25}{20kHz \cdot 50A} = 30\mu H$$

(5)

Select $\omega_0$ to meet the attenuation requirement:

$$\omega_0 = \omega_B \cdot \sqrt{\frac{|G_B| \cdot a_1 b_2}{a_1 + a_2}}$$

(4)

Calculate the remaining filter elements

(7a, 8, 9)
# 2nd order filter – Example 2

## Results:

<table>
<thead>
<tr>
<th></th>
<th>Butterworth</th>
<th>Bessel</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1.0000</td>
<td>0.7560</td>
<td>0.5098</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.0000</td>
<td>0.9996</td>
<td>1.0197</td>
</tr>
<tr>
<td>$b_2$</td>
<td>1.0000</td>
<td>0.4772</td>
<td>0.2599</td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>$1.26 \times 10^5$ s$^{-1}$</td>
<td>(20kHz)</td>
<td></td>
</tr>
<tr>
<td>$G_B$</td>
<td>0.004</td>
<td>-48dB</td>
<td></td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>$5.62 \times 10^3$ s$^{-1}$ (894 Hz)</td>
<td>$3.60 \times 10^3$ s$^{-1}$ (573 Hz)</td>
<td>$2.34 \times 10^3$ s$^{-1}$ (372 Hz)</td>
</tr>
<tr>
<td>$L_1$</td>
<td>30 µH</td>
<td>30 µH</td>
<td>30 µH</td>
</tr>
<tr>
<td>$C_1$</td>
<td>528 µF</td>
<td>528 µF</td>
<td>528 µF</td>
</tr>
<tr>
<td>$C_D$</td>
<td>1'580 µF</td>
<td>2'640 µF</td>
<td>4'220 µF</td>
</tr>
<tr>
<td>$R_D$</td>
<td>0.22 Ω</td>
<td>0.18 Ω</td>
<td>0.15 Ω</td>
</tr>
</tbody>
</table>


R. Künzi      Power Filter Design     CAS 2014      10.05.2014
2\textsuperscript{nd} order filter – Example 2

Maximum amplitude of resonance:
- Butterworth: 4.5 dB
- Bessel: 3.1 dB
- Critical damping: 2.3 dB

Frequency, for -3 dB attenuation:
- Butterworth: 1.5 kHz
- Bessel: 1.4 kHz
- Critical damping: 1.2 kHz

Attenuation: -48 dB @ 20 kHz
4\textsuperscript{th} order filter - Transferfunction

\[ Z(s) = \frac{1}{C_2s + \frac{1}{R_D} + \frac{1}{C_Ds}} = \frac{R_D C_D s + 1}{C_2 R_D C_D s^2 + (C_2 + C_D)s} \]

\[ G(s) = \frac{v_3(s)}{v_1(s)} = \frac{1}{C_1s + \frac{1}{L_2s + Z(s)}} \cdot \frac{Z(s)}{L_2s + Z(s)} \]

\[ G_1(s) = \frac{v_2(s)}{v_1(s)} \]
\[ G_2(s) = \frac{v_3(s)}{v_2(s)} \]
4\textsuperscript{th} order filter - Transferfunction

\[
G(s) = \frac{R_D C_D s + 1}{L_1 L_2 C_1 C_2 R_D C_D s^5 + (C_2 + C_D) L_1 L_2 C_1 s^4 + [L_1 C_1 R_D C_D + (L_1 + L_2) C_2 R_D C_D] s^3 + \ldots}
\]

\[
\cdots + [L_1 C_1 + (L_1 + L_2)(C_2 + C_D)] s^2 + R_D C_D s + 1
\]  

(10)

\[
G(s) = \frac{k_1 s + 1}{k_5 s^5 + k_4 s^4 + k_3 s^3 + k_2 s^2 + k_1 s + 1}
\]

1\textsuperscript{st} order PD

5\textsuperscript{th} order PT

with

\[
\begin{align*}
k_1 &= R_D C_D \\
k_2 &= L_1 (C_1 + C_2 + C_D) + L_2 (C_2 + C_D) \\
k_3 &= R_D C_D (L_1 C_1 + L_2 C_2 + L_1 C_2) \\
k_4 &= L_1 L_2 C_1 (C_2 + C_D) \\
k_5 &= L_1 L_2 C_1 C_2 C_D R_D
\end{align*}
\]
$G_{PT}(s) = \frac{1}{(1 + a_1 \frac{s}{\omega_0}) \cdot (1 + a_2 \frac{s}{\omega_0} + b_2 \frac{s^2}{\omega_0^2}) \cdot (1 + a_3 \frac{s}{\omega_0} + b_3 \frac{s^2}{\omega_0^2})}$

Optimisation methods:

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$a_3$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butterworth</td>
<td>1.0000</td>
<td>1.6180</td>
<td>1.0000</td>
<td>0.6180</td>
<td>1.0000</td>
</tr>
<tr>
<td>Bessel</td>
<td>0.6656</td>
<td>1.1402</td>
<td>0.4128</td>
<td>0.6216</td>
<td>0.3245</td>
</tr>
<tr>
<td>Critical damping</td>
<td>0.3856</td>
<td>0.7712</td>
<td>0.1487</td>
<td>0.7712</td>
<td>0.1487</td>
</tr>
</tbody>
</table>
By expanding (11) and comparing the coefficients with (10) we get:

\[ G_{PT}(s) = \frac{1}{a_1 b_2 b_3 \omega_0^5 s^5 + \left( b_2 b_3 + a_1 a_2 b_3 + a_1 a_3 b_2 \right) \omega_0^4 s^4 + \left( a_2 b_3 + a_3 b_2 + a_1 b_3 + a_1 a_2 a_3 + a_1 b_2 \right) \omega_0^3 s^3 \ldots} \]

\[ \ldots \]

\[ \ldots + \frac{a_3 + b_2 + a_1 a_3 + a_1 a_2}{\omega_0^2 s^2 + \frac{a_1 + a_2 + a_3}{\omega_0} s + 1} \]

\[ k_1 = R_D C_D = \frac{a_1 + a_2 + a_3}{\omega_0} \quad (12a) \]

\[ k_2 = L_1 (C_1 + C_2 + C_D) + L_2 (C_2 + C_D) = \frac{b_3 + a_2 a_3 + b_2 + a_1 a_3 + a_1 a_2}{\omega_0^2} \quad (12b) \]

\[ k_3 = R_D C_D (L_1 C_1 + L_2 C_2 + L_1 C_2) = \frac{a_2 b_3 + a_3 b_2 + a_1 b_3 + a_1 a_2 a_3 + a_1 b_2}{\omega_0^3} \quad (12c) \]

\[ k_4 = L_1 L_2 C_1 (C_2 + C_D) = \frac{b_2 b_3 + a_1 a_2 b_3 + a_1 a_3 b_2}{\omega_0^4} \quad (12d) \]

\[ k_5 = L_1 L_2 C_1 C_2 C_D R_D = \frac{a_1 b_2 b_3}{\omega_0^5} \quad (12e) \]
4th order filter – selection of $\omega_0$ and $L_1$

The 5 independent equations (12a….12e) contain 7 unknowns ($L_1$, $C_1$, $L_2$, $C_2$, $R_D$, $C_D$ and $\omega_0$). Therefore we have the choice to select 2 of them ($\omega_0$ and $L_1$) the remaining 5 depend on that selection.

For a given frequency $\omega_B$ well in the blocking area ($\omega_B \gg \omega_0$) we can define the desired attenuation $G_B$. In the blocking area the highest order terms of both the numerator and denominator in equation (10) dominate, therefore (10) can be simplified to:

$$G_B = \frac{R_D C_D s}{L_1 L_2 C_1 C_2 R_D C_D s^5} = \frac{a_1 + a_2 + a_3}{\omega_0 s} = \frac{a_1 + a_2 + a_3}{a_1 b_2 b_3} \frac{\omega_0^4}{s^4} = \frac{a_1 + a_2 + a_3}{a_1 b_2 b_3} \frac{\omega_0^4}{\omega_B^4}$$

$$(j)^4 = +1$$

$$\omega_0 = \omega_B \cdot \left( \frac{G_B \cdot a_1 b_2 b_3}{a_1 + a_2 + a_3} \right)^{\frac{1}{4}}$$

Select $L_1$ according to ripple current requirements with (5) or (6)
By solving the equation system (12a……12e) we get:

\[
L_2 = \frac{L_1}{(k_3 k_4 - k_2 k_5)(k_1 k_2 - k_3)(k_1 k_4 - k_5)^2} - 1 \quad (14a)
\]

\[
C_2 = \frac{k_5(k_1 k_2 - k_3)}{k_1 (k_1 k_4 - k_5)(L_1 + L_2)} \quad (14b)
\]

\[
C_1 = \frac{k_5}{k_1 L_1 L_2 C_2} \quad (14c)
\]

\[
R_D = \frac{k_1 k_5}{C_2 (k_1 k_4 - k_5)} \quad (14d)
\]

\[
C_D = \frac{k_1}{R_D} \quad (14e)
\]

with

\[
k_1 = \frac{a_1 + a_2 + a_3}{\omega_0}
\]

\[
k_2 = \frac{b_3 + a_2 a_3 + b_2 + a_1 a_3 + a_1 a_2}{\omega_0^2}
\]

\[
k_3 = \frac{a_2 b_3 + a_3 b_2 + a_1 b_3 + a_1 a_2 a_3 + a_1 b_2}{\omega_0^3}
\]

\[
k_4 = \frac{b_2 b_3 + a_1 a_2 b_3 + a_1 a_3 b_2}{\omega_0^4}
\]

\[
k_5 = \frac{a_1 b_2 b_3}{\omega_0^5}
\]
Design a 4\textsuperscript{th} order filter for all three given optimization methods and compare the results.

- DC-link voltage: 120V
- \( f_s = 20\text{kHz} \)
- \( I_{\text{Out\_max}} = 500\text{A} \)
- \( \Delta I_{L1} \leq 50\text{A} \cdot \text{App.} \)
- Attenuation: \( G_B = 250 \) @ \( \omega_B = 2\pi \cdot 20\text{kHz} \)

Same premises as for example 2
Select $L_1$ to meet the ripple current requirement:

$$L_1 = \frac{V_{DC} \cdot 0.25}{f_s \cdot \Delta I_{L1}} = \frac{120V \cdot 0.25}{20kHz \cdot 50A} = 30\mu H$$

(5)

Select $\omega_0$ to meet the attenuation requirement:

$$\omega_0 = \omega_B \cdot \frac{\sqrt{G_B \cdot a_1 b_2 b_3}}{\sqrt{a_1 + a_2 + a_3}}$$

(13)

Calculate the remaining filter elements

(14a….e)
### 4th order filter – Example 3

#### Results:

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<tr>
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<th>Bessel</th>
<th>Critical damping</th>
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<td>(a_1)</td>
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<td>0.3856</td>
</tr>
<tr>
<td>(a_2)</td>
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<td>0.7712</td>
</tr>
<tr>
<td>(b_2)</td>
<td>1.0000</td>
<td>0.4128</td>
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</tr>
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<td>0.7712</td>
</tr>
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</tr>
<tr>
<td>(\omega_B)</td>
<td>1.26*10^5 s^{-1}</td>
<td>(20kHz)</td>
<td></td>
</tr>
<tr>
<td>(G_B)</td>
<td>0.004</td>
<td>(-48dB)</td>
<td></td>
</tr>
<tr>
<td>(\omega_0)</td>
<td>2.36*10^4 s^{-1}</td>
<td>(3.8 kHz)</td>
<td>1.38*10^4 s^{-1}</td>
</tr>
<tr>
<td>(L_1)</td>
<td>30 (\mu)H</td>
<td>30 (\mu)H</td>
<td>30 (\mu)H</td>
</tr>
<tr>
<td>(L_2)</td>
<td>57 (\mu)H</td>
<td>31 (\mu)H</td>
<td>17 (\mu)H</td>
</tr>
<tr>
<td>(C_1)</td>
<td>74 (\mu)F</td>
<td>90 (\mu)F</td>
<td>124 (\mu)F</td>
</tr>
<tr>
<td>(C_2)</td>
<td>7.9 (\mu)F</td>
<td>12 (\mu)F</td>
<td>16 (\mu)F</td>
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<tr>
<td>(C_D)</td>
<td>75 (\mu)F</td>
<td>168 (\mu)F</td>
<td>382 (\mu)F</td>
</tr>
<tr>
<td>(R_D)</td>
<td>1.83 (\Omega)</td>
<td>1.04 (\Omega)</td>
<td>0.62 (\Omega)</td>
</tr>
</tbody>
</table>
Maximum amplitude of resonance:
- Butterworth: 8.6 dB
- Bessel: 5.4 dB
- Critical damping: 3.8 dB

Frequency, for -3 dB attenuation:
- Butterworth: 5.5 kHz
- Bessel: 5.0 kHz
- Critical damping: 3.9 kHz

Attenuation: -48 dB @ 20 kHz
Comparison of different filter designs

**Design 1**
- 4th order
- Acc. to example 3
- Optimisation: Bessel
- \( L_1 = 30 \ \mu \text{H} \)
- \( L_2 = 31 \ \mu \text{H} \)
- \( C_1 = 90 \ \mu \text{F} \)
- \( C_2 = 12 \ \mu \text{F} \)
- \( C_D = 168 \ \mu \text{F} \)
- \( R_D = 1.04 \ \Omega \)
- \( f_0 = 2'200 \ \text{Hz} \)

**Design 2**
- 2nd order
- Acc. to example 2
- Optimisation: Bessel
- \( L_1 = 30 \ \mu \text{H} \)
- \( C_1 = 528 \ \mu \text{F} \)
- \( C_D = 2'600 \ \mu \text{F} \)
- \( R_D = 0.18 \ \Omega \)
- \( f_0 = 573 \ \text{Hz} \)

**Design 3**
- 2nd order
- Acc. to example 2 but \( L_1 = 100\mu\text{H} \)
- Optimisation: Bessel
- \( L_1 = 100 \ \mu \text{H} \)
- \( C_1 = 158 \ \mu \text{F} \)
- \( C_D = 790 \ \mu \text{F} \)
- \( R_D = 0.62 \ \Omega \)
- \( f_0 = 573 \ \text{Hz} \)

**Design 4**
- 2nd order
- Acc. to example 2 but \( L_1 = 100\mu\text{H} \) and \( G_B = 0.01 \)
- Optimisation: Bessel
- \( L_1 = 100 \ \mu \text{H} \)
  +64% \( \)
- \( C_1 = 63 \ \mu \text{F} \)
  +42% \( \)
- \( C_D = 320 \ \mu \text{F} \)
- \( R_D = 0.98 \ \Omega \)
- \( f_0 = 907 \ \text{Hz} \)
Comparison of different filter designs

Bode Plots of different filter designs
From: C To: Vout

- Design 1
- Design 2 and 3
- Design 4

Magnitude (dB)

Frequency (Hz)

Phase (deg)
Effect of a 0.5m long wire 16mm$^2$ (wiring of $C_1$ and $C_2$)

- Skin Effect
  - Skin depth in Cu @ 20kHz: 0.5mm
  - Reduces the effective cross section to 6.3 mm$^2$
  - Wire resistance @ 20kHz: 1.4m$\Omega$
- Wire inductance is approx. 0.5$\mu$H

To be avoided!
Practical Aspects – low inductive setup
The useful life time of an electrolytic capacitor depends very much on the ripple current and the ambient temperature.

<table>
<thead>
<tr>
<th>CR</th>
<th>Case dimensions</th>
<th>ESR_{typ}</th>
<th>Z_{max}</th>
<th>I_{AC,max}</th>
<th>I_{AC,R}</th>
<th>Ordering code</th>
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<td>20 °C</td>
<td>mm</td>
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<td>100 Hz</td>
<td>85 °C</td>
<td>100 Hz</td>
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<td>17.6</td>
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<tr>
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<td>17</td>
<td>47</td>
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<tr>
<td>15000</td>
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<td>22000</td>
<td>91.0 x 144.5</td>
<td>5</td>
<td>6</td>
<td>80</td>
<td>35.9</td>
<td>63.6</td>
</tr>
</tbody>
</table>

- Nominal ripple current at
  - nominal frequency (100Hz) and
  - nominal capacitor temperature (85°C).
- 17.4A in our example
Apply frequency factor:

For 10kHz a current factor of 1.35 is applicable
→ 23.5A @ 10kHz
Determine the allowed ripple current for a desired useful life and $T_a$:

For 25’000h (3 years) and $T_a = 50^\circ$C,
$\rightarrow 2.6 \times 23.5\text{A} = 61\text{A}$

For 250’000h (30 years) and $T_a = 50^\circ$C
$\rightarrow 0.85 \times 23.5\text{A} = 20\text{A}$

The useful life time dramatically decreases at higher ambient temperatures!
Thank you for your attention

Questions?

References
- U. Tietze, Ch. Schenk; Halbleiter-Schaltungs-Technik, 12. Auflage, Pages 815ff
- Epcos, Datasheet, Capacitors with screw terminals Type B43564, B43584, November 2012