

Diagnostics II

M. Minty
DESY

Introduction

Beam Charge / Intensity

Beam Position

Summary



Diagnostics I

Introduction

Transverse Beam Emittance

Summary



Diagnostics II

Introduction

Longitudinal Beam Emittance

Energy Spread

Bunch Length

Summary



Diagnostics III

Introduction

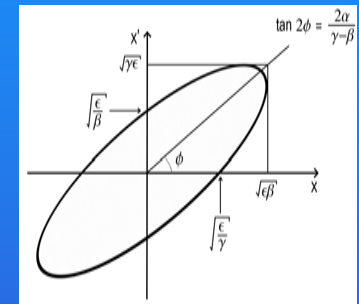
Reminder:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_{fi} \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

transformation of the phase space coordinates (x, x') of a single particle (from $i \rightarrow f$) given in terms of the "point-to-point" transport matrix, R

Equivalently, and complementarily, the Twiss parameters $(\alpha, \beta, \text{ and } \gamma)$ obey

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1 + 2R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix}_{fi} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$



The elements of the transfer matrix R are given generally by

$$\mathbf{R}_{fi} = \begin{pmatrix} \sqrt{\frac{\beta_f}{\beta_i}} (\cos \phi_{fi} + \alpha_i \sin \phi_{fi}) & \sqrt{\beta_f \beta_i} \sin \phi_{fi} \\ -\frac{1 + \alpha_f \alpha_i}{\sqrt{\beta_f \beta_i}} \sin \phi_{fi} + \frac{\alpha_i - \alpha_f}{\sqrt{\beta_f \beta_i}} \cos \phi_{fi} & \sqrt{\frac{\beta_i}{\beta_f}} (\cos \phi_{fi} - \alpha_f \sin \phi_{fi}) \end{pmatrix}$$

or if the initial and final observations points are the same, by the one-turn-map:

$$\mathbf{R}_{otm} = \begin{pmatrix} \cos \mu + \alpha \sin \mu & \beta \sin \mu \\ -\gamma \sin \mu & \cos \mu - \alpha \sin \mu \end{pmatrix}$$

where μ is the 1-turn phase advance:
 $\mu = 2\pi Q$

A third equivalent approach involves the beam matrix defined as

$$\Sigma_{\text{beam}}^x = \epsilon_x \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle xx' \rangle - \langle x \rangle \langle x' \rangle \\ \langle x'x \rangle - \langle x' \rangle \langle x \rangle & \langle x'^2 \rangle - \langle x' \rangle^2 \end{pmatrix}$$

in terms of Twiss parameters

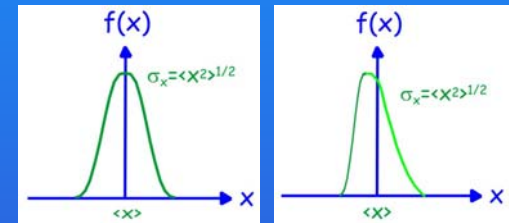
in terms of the moments of the beam distribution

$$\det \Sigma_{\text{beam}}^x = \epsilon \text{ sqrt} (\beta\gamma - \alpha^2) = \epsilon \quad \text{since} \quad \gamma \equiv (1 + \alpha^2) / \beta$$

Here $\langle x \rangle$ and $\langle x^2 \rangle$ are the first and second moments of the beam distribution:

$$\langle x \rangle = \frac{\int_0^{\infty} x f(x) dx}{\int_0^{\infty} f(x) dx}$$

$$\langle x^2 \rangle = \frac{\int_0^{\infty} x^2 f(x) dx}{\int_0^{\infty} f(x) dx}$$



where $f(x)$ is the beam intensity distribution

The transformation of the initial beam matrix $\Sigma_{\text{beam},0}$ to the desired observation point is

$$\Sigma_{\text{beam}} = R \Sigma_{\text{beam},0} R^t$$

where R is again the point-to-point transfer matrix

Neglecting the mean of the distribution (disregarding the static position offset of the core of the beam; i.e. $\langle x \rangle = 0$):

$$\Sigma_{\text{beam}}^x = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}$$

and the root-mean-square (rms) of the distribution is

$$\sigma_x = \langle x^2 \rangle^{1/2}$$

Measurement of the Transverse Beam Emittance

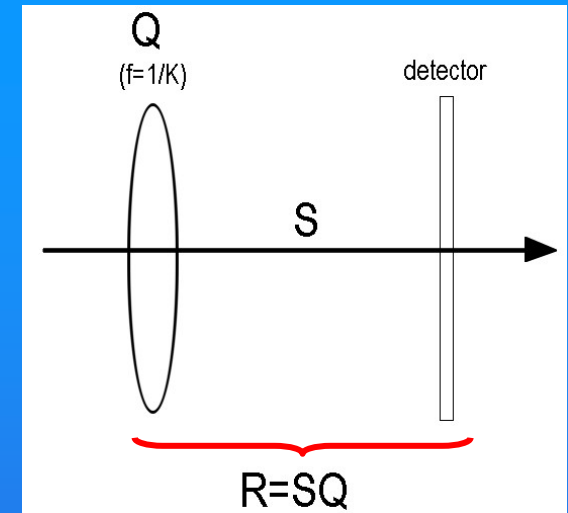
Method I: quadrupole scan

Principle: with a well-centered beam, measure the beam size as a function of the quadrupole field strength

Here

Q is the transfer matrix of the quadrupole

R is the transfer matrix between the quadrupole and the beam size detector



With $Q = \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix}$ then $R = \begin{pmatrix} S_{11} + KS_{12} & S_{12} \\ S_{21} + KS_{22} & S_{22} \end{pmatrix}$ with $\Sigma_{\text{beam}} = R\Sigma_{\text{beam},0}R^t$

The (11)-element of the beam transfer matrix is found after algebra to be:

$$\Sigma_{11}(= \langle x^2 \rangle) = (S_{11}^2 \Sigma_{11_0} + 2S_{11}S_{12} \Sigma_{12_0} + S_{12}^2 \Sigma_{22_0}) + (2S_{11}S_{12} \Sigma_{11_0} + 2S_{12}^2 \Sigma_{12_0})K + S_{12}^2 \Sigma_{11_0}K^2$$

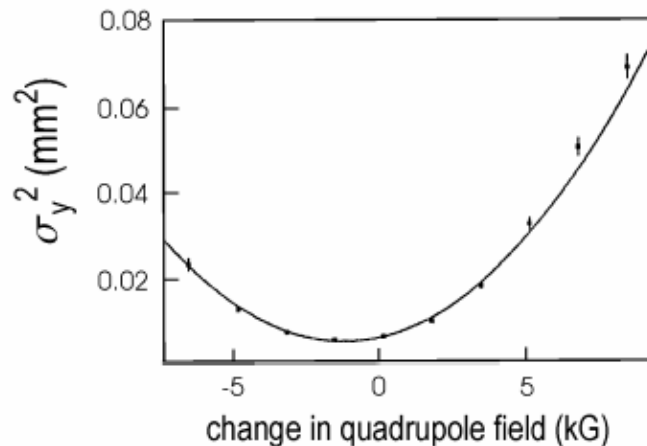
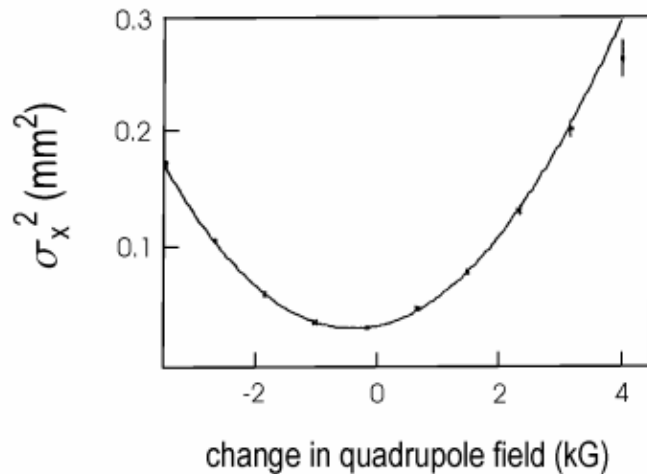
which is quadratic in the field strength, K

Measurement: measure beam size versus quadrupole field strength

again:

$$\Sigma_{11}(= \langle x^2 \rangle) = (S_{11}^2 \Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + S_{12}^2 \Sigma_{22_0}) \\ + (2S_{11}S_{12}\Sigma_{11_0} + 2S_{12}^2 \Sigma_{12_0})K + S_{12}^2 \Sigma_{11_0}K^2$$

data:



fitting function (parabolic):

$$\Sigma_{11} = A(K - B)^2 + C \\ = AK^2 - 2ABK + (C + AB^2)$$

equating terms (drop subscripts 'o'),

$$A = S_{12}^2 \Sigma_{11}, \\ -2AB = 2S_{11}S_{12}\Sigma_{11} + 2S_{12}^2 \Sigma_{12}, \\ C + AB^2 = S_{11}^2 \Sigma_{11} + 2S_{11}S_{12}\Sigma_{12} + S_{12}^2 \Sigma_{22}$$

solving for the beam matrix elements:

$$\Sigma_{11} = A/S_{12}^2, \\ \Sigma_{12} = -\frac{A}{S_{12}^2} \left(B + \frac{S_{11}}{S_{12}} \right), \\ \Sigma_{22} = \frac{1}{S_{12}^2} \left[(AB^2 + C) + 2AB \left(\frac{S_{11}}{S_{12}} \right) + A \left(\frac{S_{11}}{S_{12}} \right)^2 \right]$$

The (here, horizontal) emittance is given from the determinant of the beam matrix:

$$\epsilon_x = \sqrt{\det \Sigma_{\text{beam}}^x}$$

$$\begin{aligned} \det \Sigma_{\text{beam}}^x &= \Sigma_{11}\Sigma_{22} - \Sigma_{12}^2 \\ &= AC/S_{12}^4, \end{aligned}$$

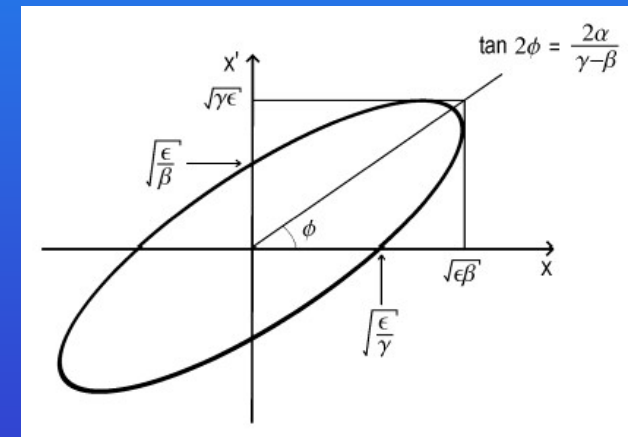
$$\rightarrow \epsilon_x = \sqrt{AC}/S_{12}^2$$

With these 3 fit parameters (A,B, and C), the 3 Twiss parameters are also known:

$$\beta_x = \frac{\Sigma_{11}}{\epsilon} = \sqrt{\frac{A}{C}},$$

$$\alpha_x = -\frac{\Sigma_{12}}{\epsilon} = \sqrt{\frac{A}{C}} \left(B + \frac{S_{11}}{S_{12}} \right),$$

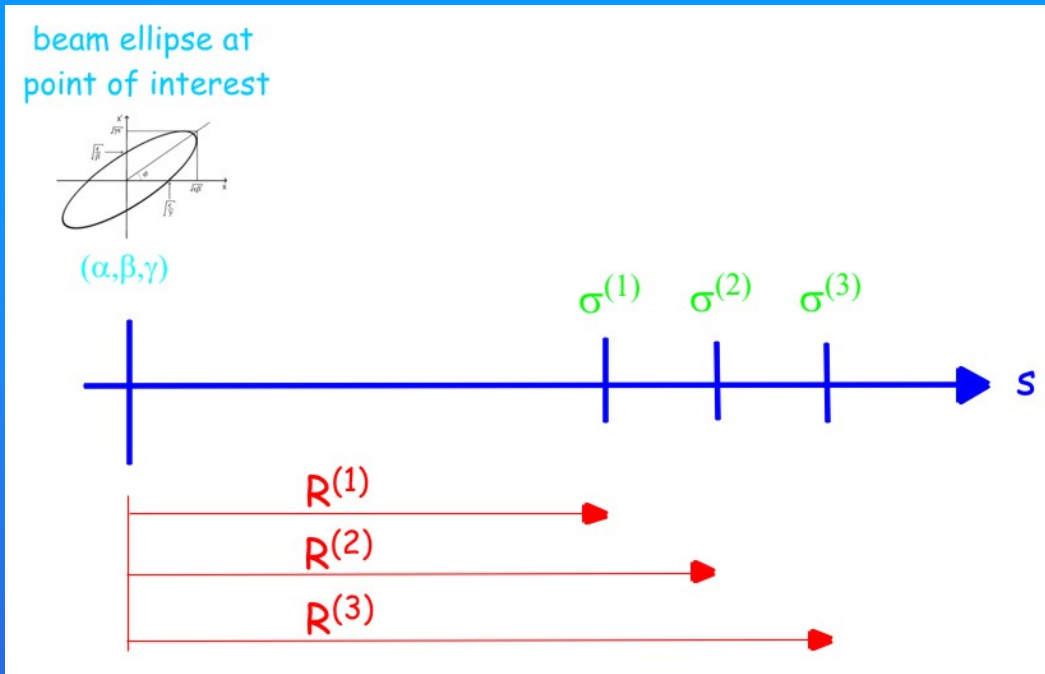
$$\gamma_x = \frac{S_{12}^2}{\sqrt{AC}} \left[(AB^2 + C) + 2AB \left(\frac{S_{11}}{S_{12}} \right) + A \left(\frac{S_{11}}{S_{12}} \right)^2 \right]$$



as a useful check, the beam-ellipse parameters should satisfy $(\beta_x\gamma_x-1)=\alpha_x^2$

Method II: fixed optics, measure beam size using multiple measurement devices

notation to be used:

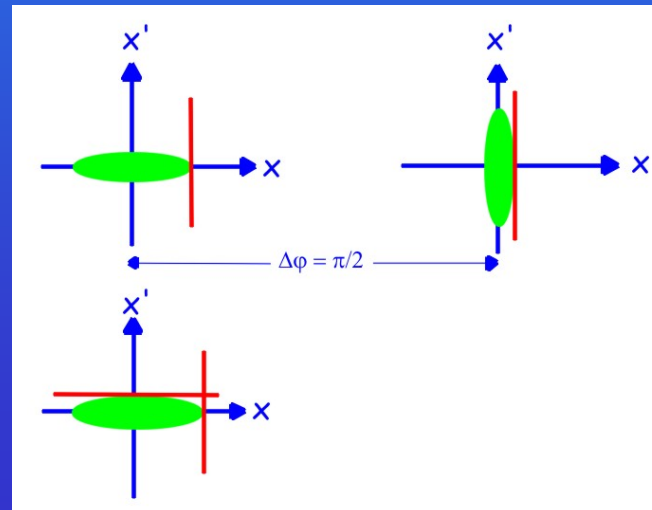


superscripts:

“measurement number”

subscripts: matrix elements

mapping of wire scanners back to reference point



(new slide)

Method II: fixed optics, measure beam size using multiple measurement devices

Recall: the matrix used to transport the Twiss parameters:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1 + 2R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix}_{fi} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$

with fixed optics and multiple measurements of σ at different locations:

subscripts: matrix elements
superscripts: "measurement number"

$$\begin{pmatrix} (\sigma_x^{(1)})^2 \\ (\sigma_x^{(2)})^2 \\ (\sigma_x^{(3)})^2 \\ \dots \\ (\sigma_x^{(n)})^2 \end{pmatrix} = \begin{pmatrix} (R_{11}^{(1)})^2 & 2R_{11}^{(1)}R_{12}^{(1)} & (R_{12}^{(1)})^2 \\ (R_{11}^{(2)})^2 & 2R_{11}^{(2)}R_{12}^{(2)} & (R_{12}^{(2)})^2 \\ (R_{11}^{(3)})^2 & 2R_{11}^{(3)}R_{12}^{(3)} & (R_{12}^{(3)})^2 \\ \dots & \dots & \dots \\ (R_{11}^{(n)})^2 & 2R_{11}^{(n)}R_{12}^{(n)} & (R_{12}^{(n)})^2 \end{pmatrix} \begin{pmatrix} \beta(s_0)\epsilon \\ -\alpha(s_0)\epsilon \\ \gamma(s_0)\epsilon \end{pmatrix}$$

simplify notation:

$$\Sigma_x = \mathbf{B} \cdot \mathbf{o}$$

Σ_x

\mathbf{B}

\mathbf{o}

goal is to determine the vector \mathbf{o} by minimizing the sum (least squares fit):

$$\chi^2 = \sum_{l=1}^n \frac{1}{\sigma_{\Sigma_x^{(l)}}^2} \left(\Sigma_x^{(l)} - \sum_{i=1}^3 B_{li} o_i \right)^2$$

with the symmetric $n \times n$ covariance matrix,

$$\mathbf{T} = (\hat{\mathbf{B}}^t \cdot \hat{\mathbf{B}})^{-1}$$

→ the least-squares solution is

$$\mathbf{o} = \mathbf{T} \cdot \hat{\mathbf{B}}^t \cdot \hat{\Sigma}_x$$

(the 'hats' show weighting:

$$\hat{B}_{li} = \frac{B_{li}}{\sigma_{\Sigma_x^{(l)}}} \quad \hat{\Sigma}_x^{(l)} = \frac{\Sigma_x^{(l)}}{\sigma_{\Sigma_x^{(l)}}})$$

once the components of σ are known,

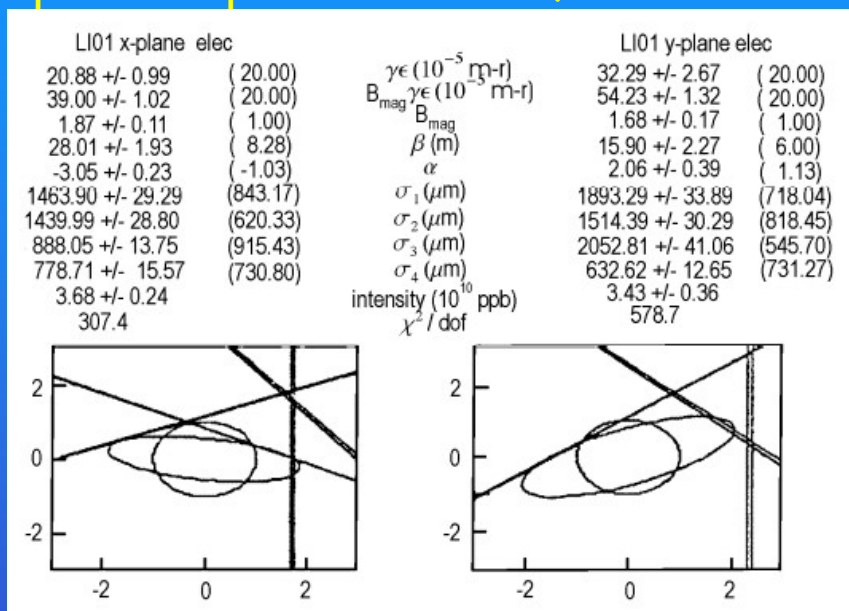
$$\epsilon = \sqrt{\sigma_1 \sigma_3 - \sigma_2^2},$$

$$\beta = \sigma_1 / \epsilon, \text{ and}$$

$$\alpha = -\sigma_2 / \epsilon.$$

data: from the SLC injector linac

graphical representation of results:

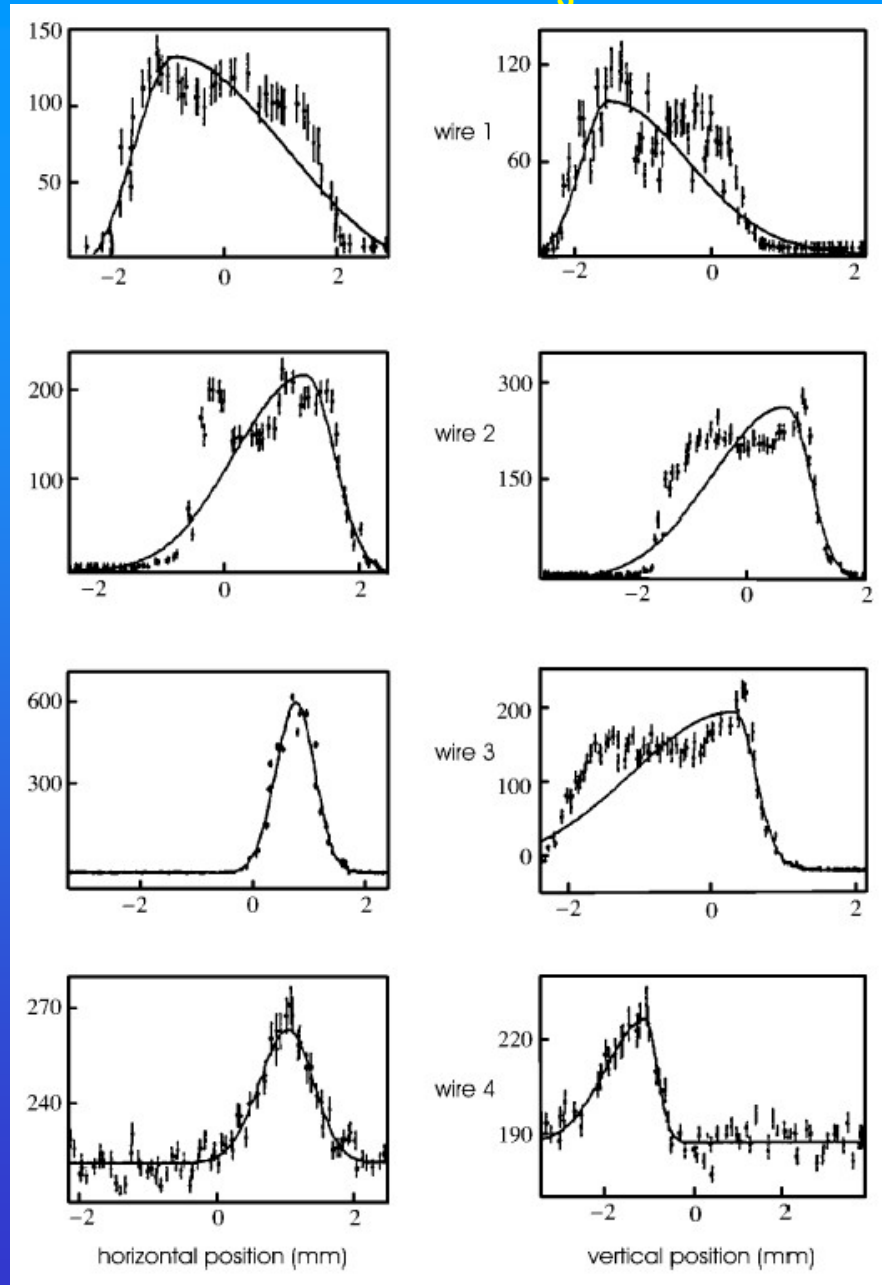


note: coordinate axes are so normalized (design phase ellipse is a circle):

lines show phase space coverage of wires:

$$\left(\frac{x}{\sqrt{\beta_x}}, \frac{\alpha_x x + \beta_x x'}{\sqrt{\beta_x}} \right)$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{ref point}} = R^{-1} \begin{pmatrix} \sigma_{x,w} \\ x'_w \end{pmatrix}$$



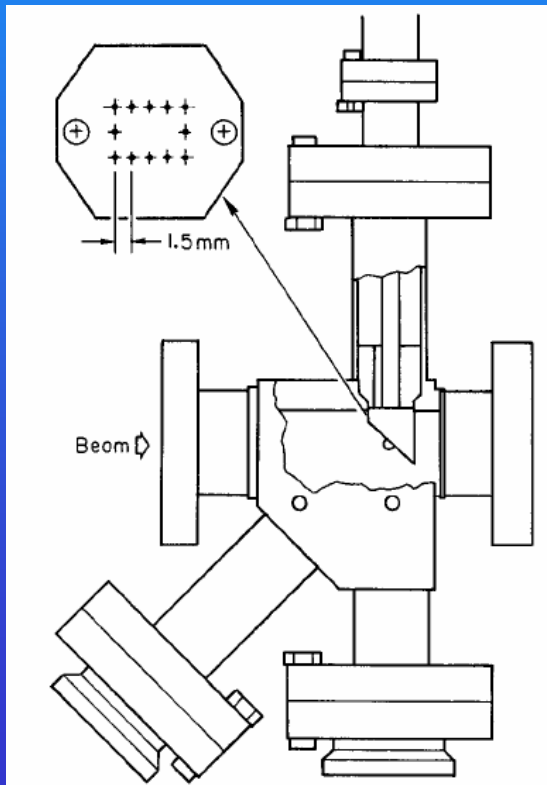
With methods I & II, the beam sizes may be measured using e.g. screens or wires

Transverse Beam Emittance - Screens

principle: intercepting screen (eg. $\text{Al}_2\text{O}_3\text{Cr}$ possibly with phosphorescent coating) inserted into beam path (usually 45°); image viewed by camera \rightarrow direct observation of x - y ($\eta = 0$) or y - δ ($\eta \neq 0$) distribution

R. Jung et al, "Single pass optical profile monitoring" (DIPAC 2003)

fluorescence - light emitted ($t \sim 10$ ns) as excited atoms decay to ground state
phosphorescence - light continues to emit ($\sim \mu\text{s}$) after exciting mechanism has ceased (i.e. oscilloscope "afterglow")
luminescence - combination of both processes



emittance measurement

image is digitized, projected, fitted with Gaussian calibration: often grid lines directly etched onto screen or calibration holes drilled, either with known spacing

issues

spacial resolution (20 - $30 \mu\text{m}$) given by phosphor grain size and phosphor transparency
temporal resolution - given by decay time
radiation hardness of screen and camera
dynamic range (saturation of screen)

(courtesy P. Tenenbaum, 2003)

Transverse Beam Emittance - Wire Scanners (1)

principle: precision stage with precision encoder propels shaft with wire support wires (e.g. C, Be, or W) scanned across beam (or beam across wire) interaction of beam with wire detected, for example with PMT

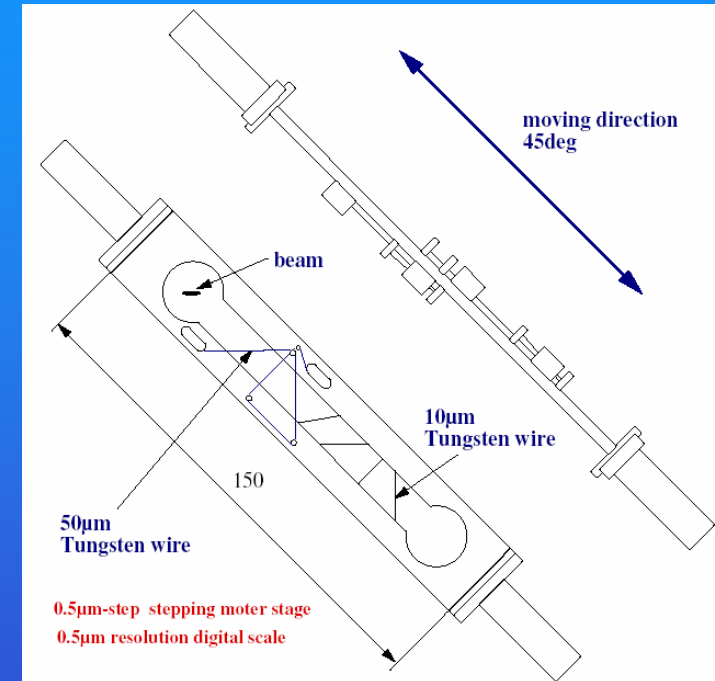
wire mount used at the ATF (at KEK) with thin W wires and 5 μm precision stepper-motors and encoders (courtesy H. Hayano, 2003)

wire velocity: depends on desired interpoint spacing and on the bunch repetition frequency

detection of beam with wire:

1. change in voltage on wire induced by secondary emission
2. hard Bremsstrahlung - forward directed γ 's which are separated from beam via an applied magnetic field and converted to e^+/e^- in the vacuum chamber wall and detected with a Cerenkov counter or PMT (after conversion to γ 's in front end of detector)
3. via detection at 90° (δ -rays)
4. using PMTs to detect scattering and electromagnetic showers
5. via change in tension of wire for beam-tail measurements)

emittance measurement: as for screens (quad scan or 3-monitor method)

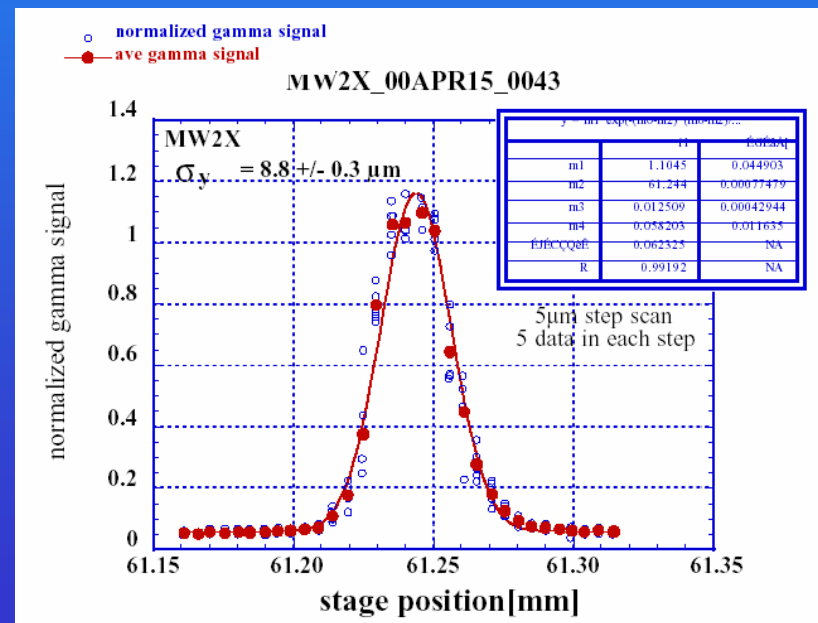


Transverse Beam Emittance - Wire Scanners (2)

issues:

- different beam bunch for each data point
- no information on x-y coupling with 1 wire (need 3 wires at common location)
- dynamic range: saturation of detectors (PMTs)
- single-pulse beam heating
- wire thickness (adds in quadrature with beam size)
- higher-order modes

(left) wire scanner chamber installed in the ATF (KEK) extraction line and (right) example wire scan (courtesy H. Hayano, 2003)



Transverse Beam Emittance - Laser Wire Scanners (1)

principle: laser wire provides a non-invasive and non-destructable target
wire scanned across beam (or beam across wire)

constituents: laser, optical transport line, interaction region and optics, detectors

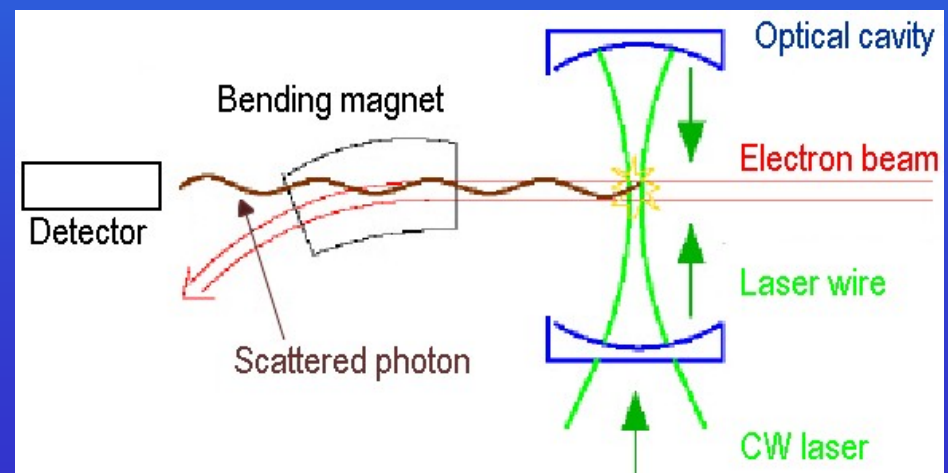
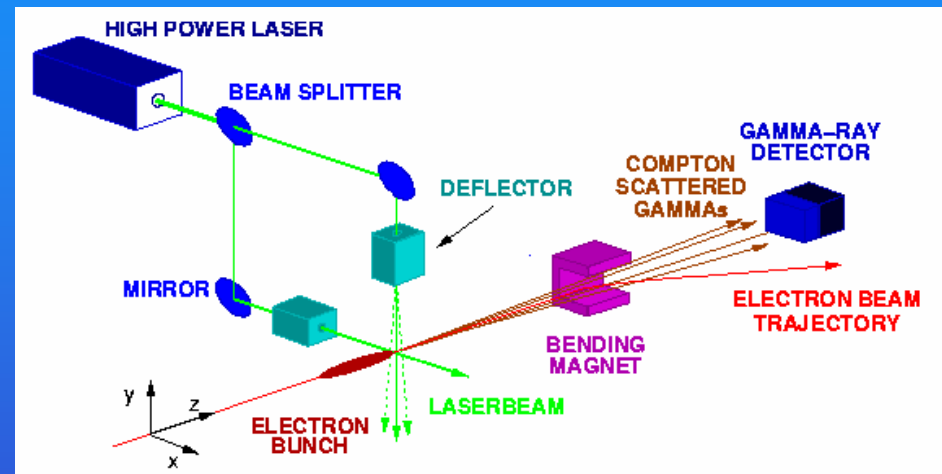
beam size measurements: forward scattered Compton γ 's or
lower-energy electrons after deflection by a magnetic field

schematic of the laser wire system planned for use at PETRA and for the third generation synchrotron light source PETRA 3 (courtesy S. Schreiber, 2003)

high power pulsed laser

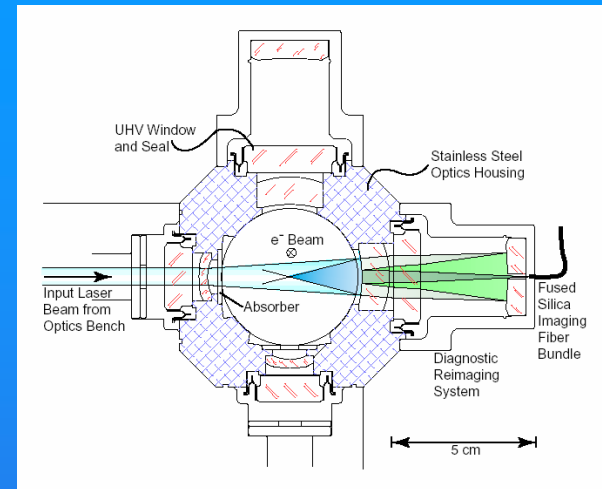
overview of the laser wire system at the ATF (courtesy H. Sakai, 2003)

optical cavity pumped by CW laser (mirror reflectivity $\sim 99+\%$)



Transverse Beam Emittance - Laser Wire Scanners (2)

laser-electron interaction point
from the pioneering experiment
at the SLC final focus (courtesy
M. Ross, 2003)



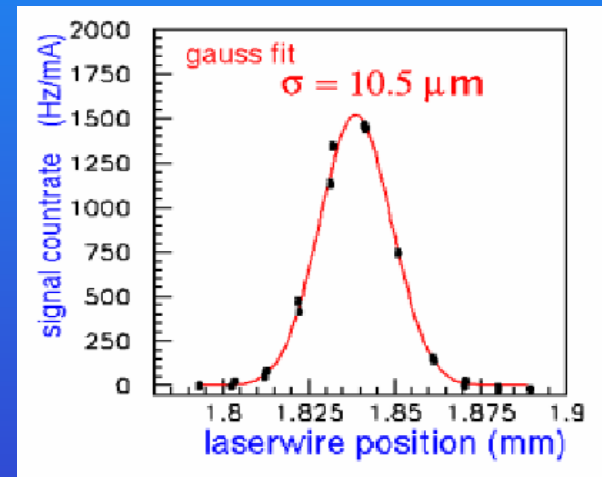
beam size measured at the
laser wire experiment of the
ATF (courtesy H. Sakai, 2003)

$$\sigma_y = \sqrt{\sigma_{\text{obs}}^2 - \left(\frac{w_0}{2}\right)^2}$$

$$\beta_y \epsilon_y = (\sigma_y)^2 - \left(\eta_y \frac{\sigma_p}{p}\right)^2$$

(as with normal
wires, the wire
size must be taken
into account)

(here w_0 is the 2σ wire thickness)



issues: photon density
waist of laser < beam size (in practice, waist size $\sim \lambda$)
background and background subtraction
synchronization (for pulsed lasers)

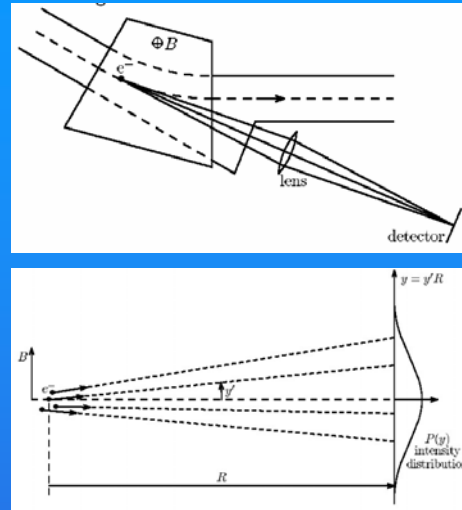
Transverse Beam Emittance - Synchrotron Radiation (1)

principle: charged particles, when accelerated, emit synchrotron radiation

measurements:

imaging \rightarrow beam cross section

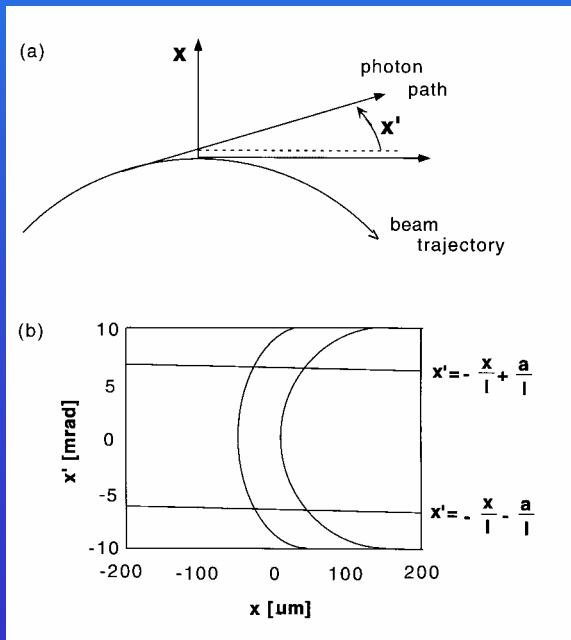
direct observation \rightarrow angular spread



A. Hofmann,
CERN accelerator
school, 2003

depth of field effect in direct imaging (ref. A. Sabersky)

phase space coordinates of the photon beam



photon beam phase space at distance l from source (for a 100 μm beam at emission point)

a = half-width of and
 l = distance to defining aperture

measured beam parameters correspond to a projection of this phase space onto the horizontal axes

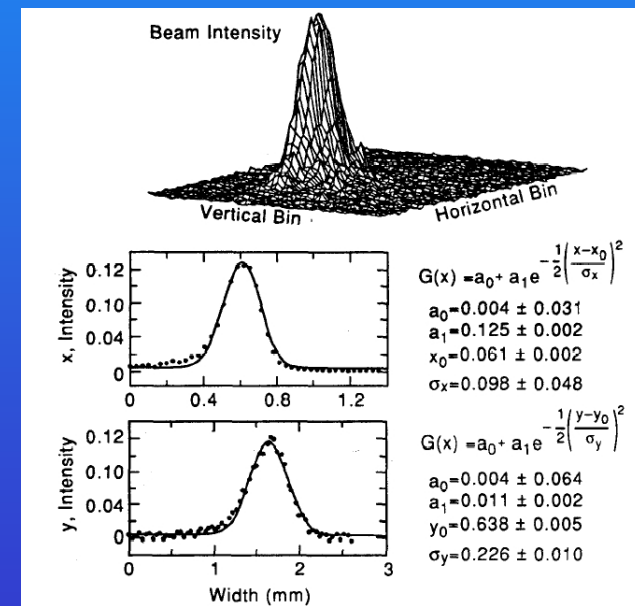
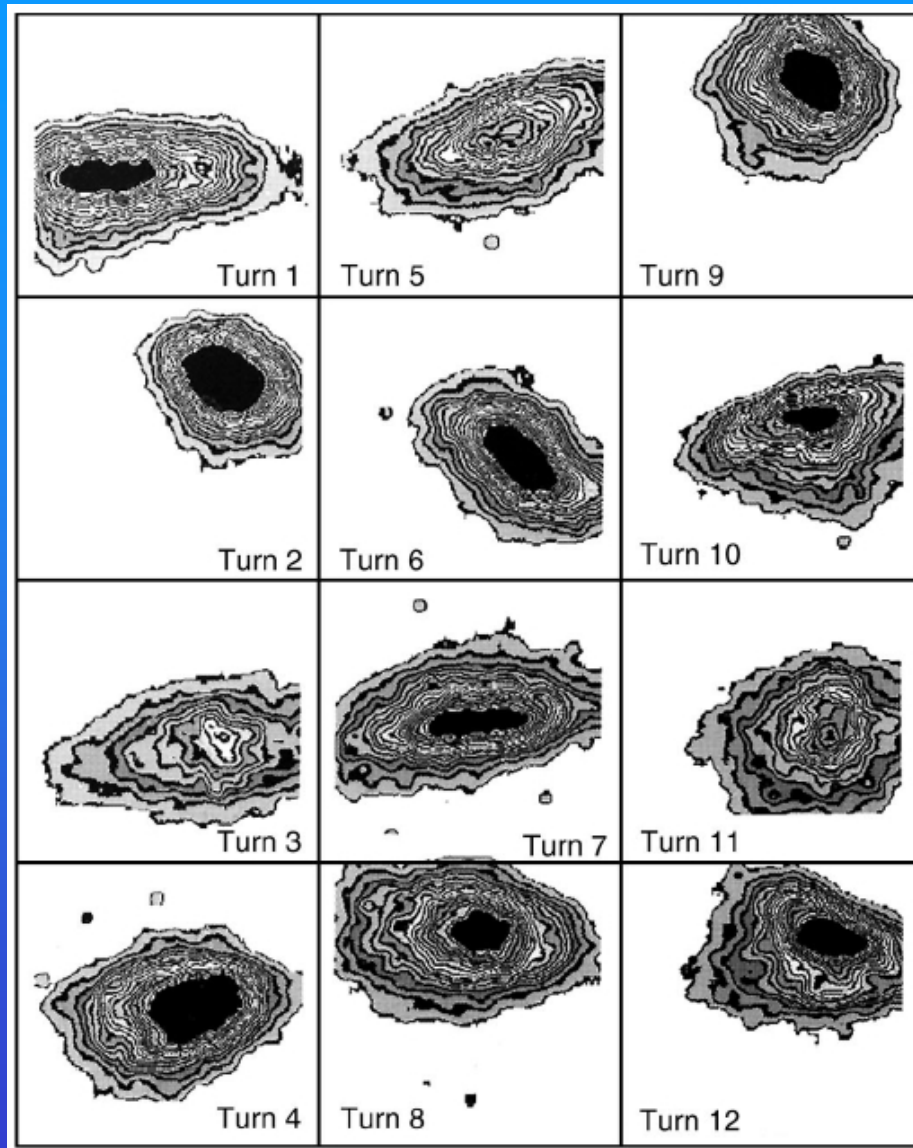
$$\bar{x} = \frac{\int I(x)x dx}{\int I(x) dx}$$

$$\sigma_{r,x}^2 = \frac{\int I(x)(x - \bar{x})^2 dx}{\int I(x) dx}$$

Transverse Beam Emittance - Synchrotron Radiation (2)

(left) raw data showing turn-by-turn images at injection (average over 8 pulses)

(bottom) digitized image in 3-D and projections together with Gaussian fits (bottom)

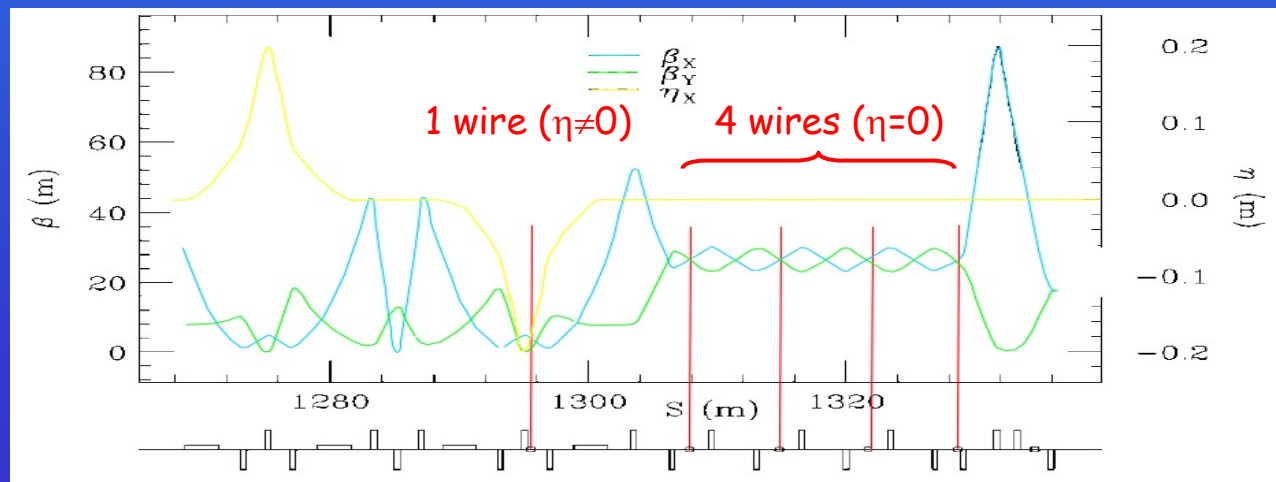


originally, these studies aimed at measuring the transverse damping times, but were then extended to measure emittance mismatch and emittance of the injected beams ...

A closing thought concerning emittance measurements: "diagnostic sections"
the layout of diagnostics for emittance measurements should
be considered together with the particle beam optics

Examples

- KEK-B (Japan): dedicated weak bending magnets (to minimize heat loading)
- TTF-I (DESY): 4 wire scanners/plane placed in symmetric fashion as geometry allowed between permanent magnet undulators; phase advance between scanners energy-dependent (quadrupoles focus less strongly higher energy beams); result: desired SASE wavelength dictated beam energy for which each pair of wires was separated by $180^\circ \rightarrow$ only 2 unique measurements
 \rightarrow no measurement of emittance at certain beam energies
- LCLS (SLAC): design optics at 15 GeV with equal beam sizes at all scanners ; optimum wire scanner spacing with 45° separation between wires



Summary

We reviewed multiple, equivalent methods for describing the transport of beam parameters between 2 points

Two methods for measuring the transverse beam emittance were presented:
the quadrupole scan - optics are varied, single measurement location
the fixed optics method with at least three independent beam size measurements

Methods for measuring the beam size were reviewed including

screens

(conventional) wire scanners

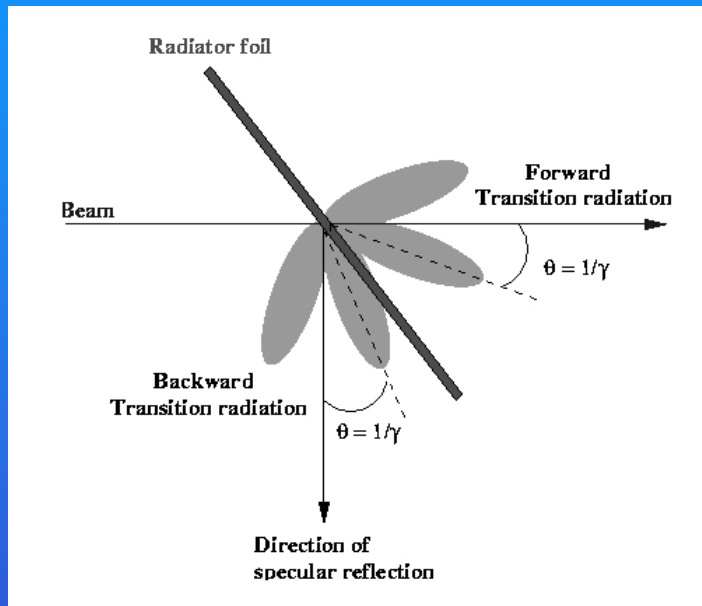
laser wire scanners

(direct imaging of) synchrotron radiation

(With a finite number of measurement devices), optimum measurement conditions may be strongly influenced by the design of the particle beam optics

Transverse Beam Emittance - Transition Radiation (1)

principle: when a charged particle crosses between two materials of different dielectric constant (e.g. between vacuum and a conductor), transition radiation is generated, temporal resolution is ~ 1 ps \rightarrow useful for high bunch repetition frequencies



forward and backward transition radiation with foil at 45° allowing for simple vacuum chamber geometry (courtesy K. Honkavaara, 2003)

foil: Al, Be, Si, Si + Al coating, for example

emittance measurement (as for screens):

digitized and fitted image

calibration: also with etched lines of known spacing

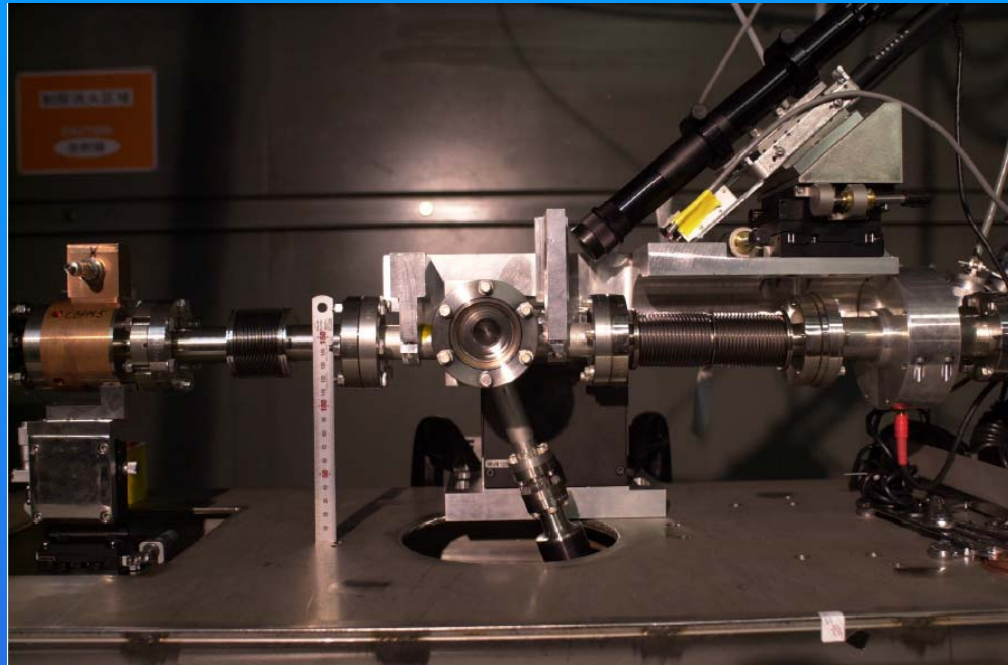
issues:

spacial resolution ($< 5 \mu\text{m}$) as introduced by the optics

damage to radiator with small spot sizes

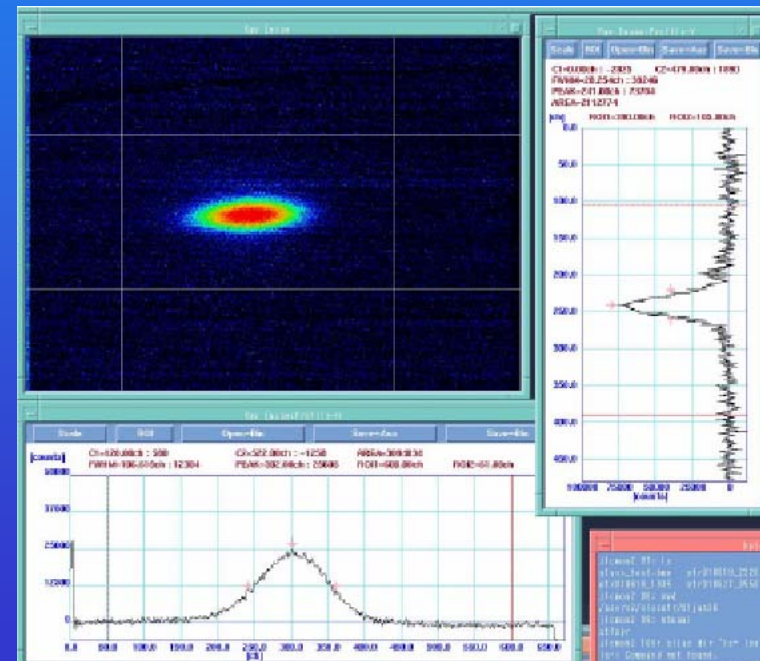
geometrical depth of field effect when imaging backward TR

Transverse Beam Emittance - Transition Radiation (2)

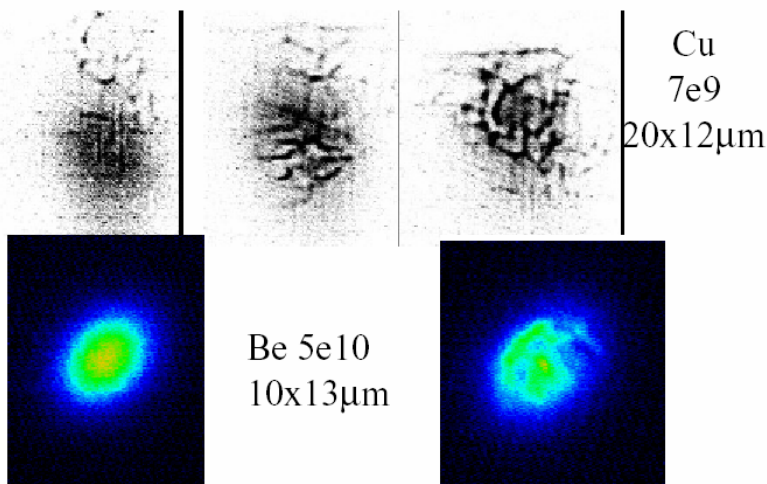


the SLAC-built OTR as installed in the extraction line of the ATF

beam spot as measured with the OTR at the ATF at KEK



successive images illustrating damage:

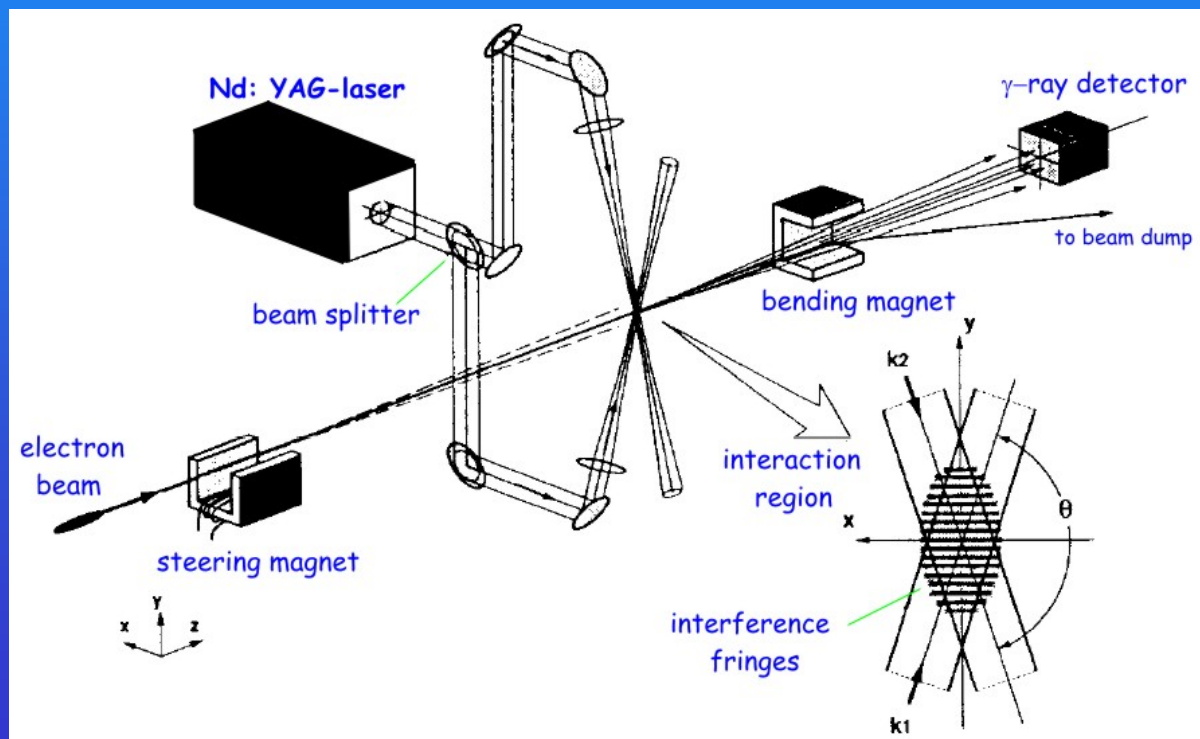


(all figures courtesy M. Ross, 2003)

Transverse Beam Emittance - (single-pass) interferometric monitor (T. Shintake)

principle: laser provides a non-invasive and non-destructable target by splitting and recombining the laser light a standing wave (sw) is made the intercepting beam produces Compton-scattered photons of high density in the sw-peaks and of low density in the sw-valleys

constituents: laser and laser beam splitter, optical transport line, particle beam steering magnet, interaction region and optics, detectors



beam size measurements: scattered Compton γ 's as a function of beam position

by scanning the beam position along the standing wave, the modulation depth of the observed photon spectrum gives information about the beam size

number of scattered photons $\longrightarrow \langle N_\gamma \rangle = A + B \cos(2k_L \Delta x + \psi)$

(arbitrary phase)

laser wave number ($= 2\pi/\lambda_L$)

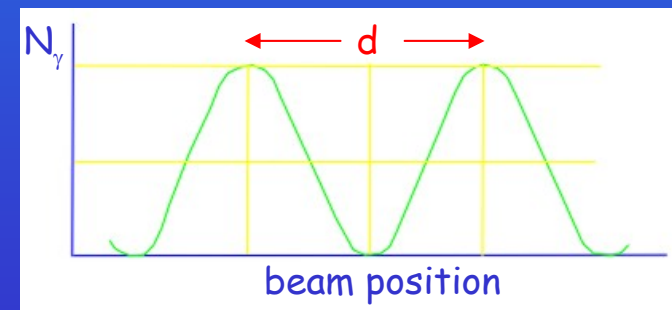
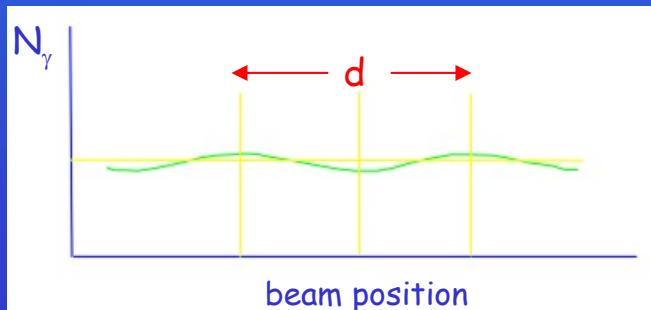
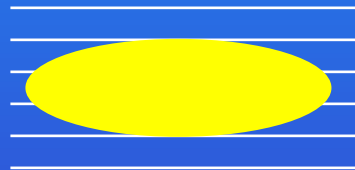
beam size large compared to fringe spacing, d : $B/A \rightarrow 0$

beam size small compared to fringe spacing, d : $B/A \rightarrow 1$

horizontal beam size:



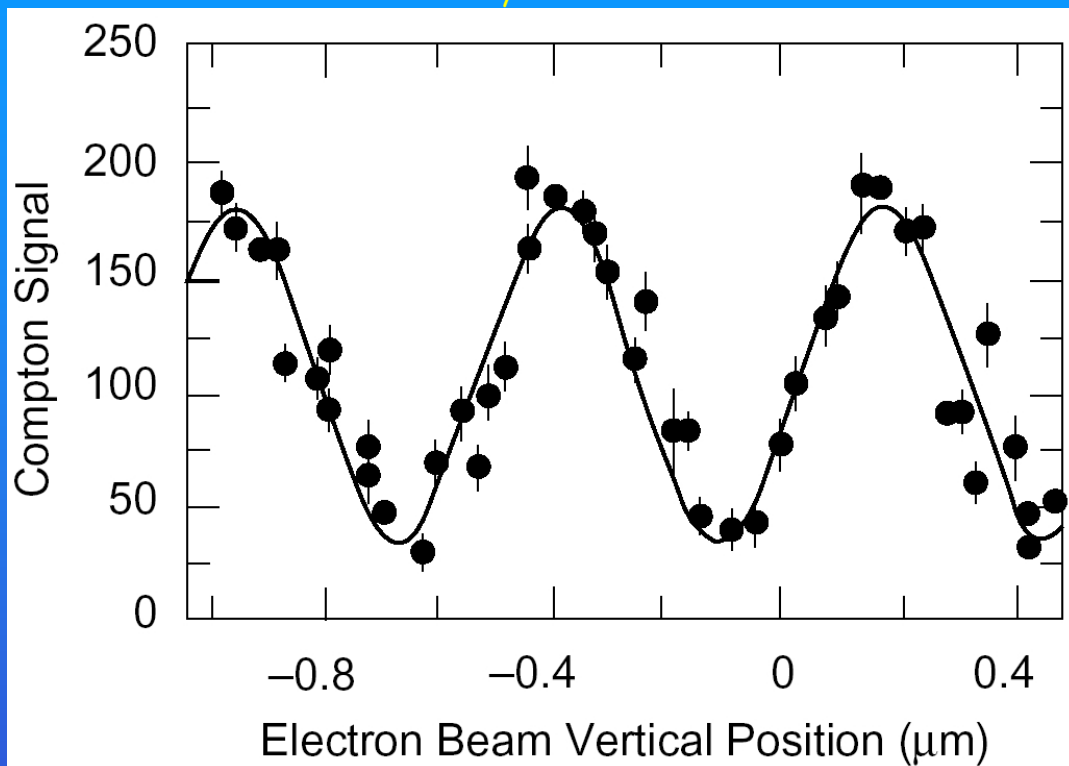
vertical beam size:



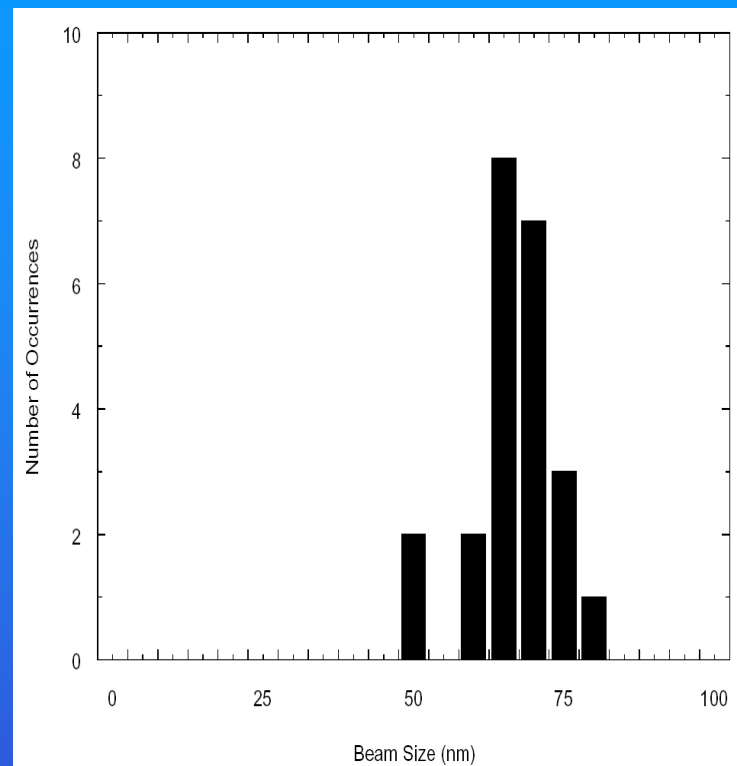
in practice, the optimum fringe spacing (for best resolution) depends on the beam size itself

Measurements of small beam sizes at the FFTB (courtesy T. Shintake, 2004)

$\sigma_y = 77$ nm



distribution of measurements



issues for the single-pass interferometric monitor:

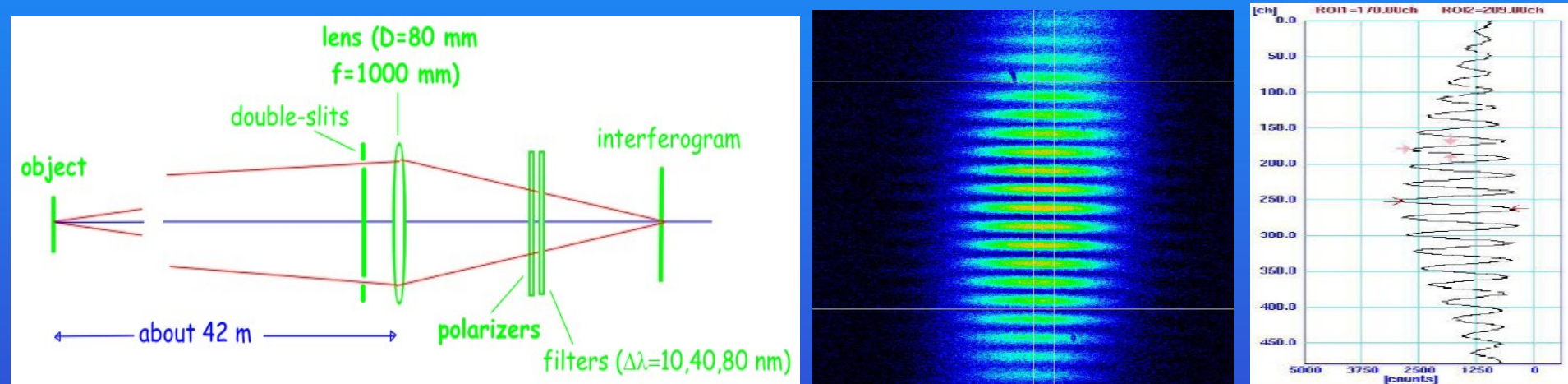
- laser power imbalance (mirror "asymmetries") and laser alignment
- electron beam crossing angle (electron beam not parallel to the plane of the fringes)
- longitudinal extent of the interference pattern (compare "hour-glass" effect)
- spatial and temporal coherence of the laser (alignment distortions)
- laser jitter

Transverse Beam Emittance -

(multiple-pass) interferometric monitor (T. Mitsuhashi)

principle: detection of interference pattern generated by synchrotron radiation after passage through "double-slits" (Michelson's stellar interferometer)

constituents: bending dipole and exit chamber for synchrotron light, photon beam optics, detector (i.e. a CCD)



beam size measurement: modulation depth of interference pattern

issues: thermal mirror distortions (KEK-B uses dedicated weak bending magnets)
precision control of slit width
depth of field effects (leads to intensity imbalance in the 2 slits)

Summary

We reviewed multiple, equivalent methods for describing the transport of beam parameters between 2 points

Two methods for measuring the transverse beam emittance were presented:
the quadrupole scan - optics are varied, single measurement location
the fixed optics method with at least three independent beam size measurements

Methods for measuring the beam size were reviewed including

- screens

- transition radiation (skipped, but in notes)

- (conventional) wire scanners

- laser wire scanners

- (direct imaging of) synchrotron radiation

- laser interference methods:

 - single-pass interferometric monitor

 - multiple-pass interferometric monitor

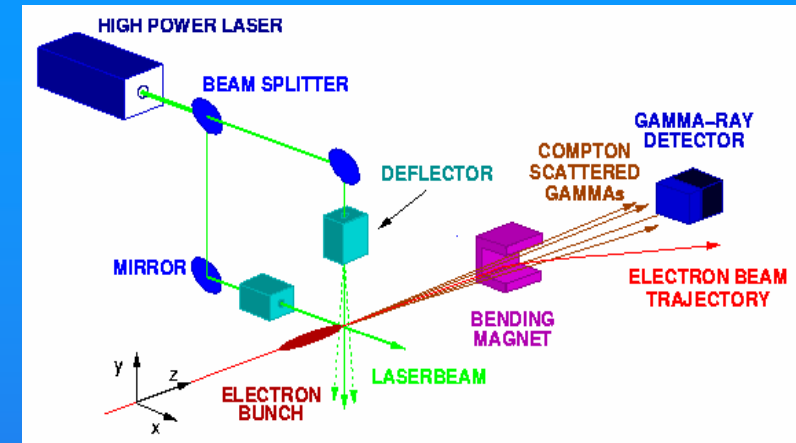
(With a finite number of measurement devices), optimum measurement conditions may be strongly influenced by the design of the particle beam optics

Transverse Beam Emittance - Laser Wire Scanners

principle: laser wire provides a non-invasive and non-destructible target; wire scanned across beam (or beam across wire)

constituents: laser (pulsed or CW with optical cavity), optical transport line, interaction region and optics, detectors

beam size measurements: forward scattered Compton γ 's or lower-energy electrons after deflection by a magnetic field



schematic of the laser wire at PETRA (courtesy S. Schreiber, 2003)

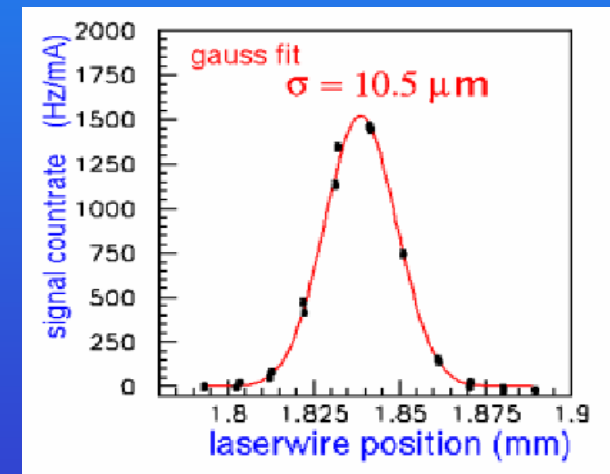
beam size measured at the laser wire experiment of the ATF (courtesy H. Sakai, 2003)

$$\sigma_y = \sqrt{\sigma_{\text{obs}}^2 - \left(\frac{w_0}{2}\right)^2}$$

$$\beta_y \epsilon_y = (\sigma_y)^2 - \left(\eta_y \frac{\sigma_p}{p}\right)^2$$

(as with normal wires, the wire size must be taken into account)

(here w_0 is the 2σ wire thickness)



issues: photon density
waist of laser < beam size (in practice, waist size $\sim \lambda$)
background and background subtraction
synchronization (for pulsed lasers)