Luminosity and Beam-Beam Effects in the Large Hadron Collider (LHC)

Tatiana Pieloni
Laboratory of Particle Accelerator Physics
Ecole Politechnique Federal de Lausanne
Circular Accelerators: acceleration occurs at every turn!

E. Lawrence 1930
Circular Accelerators: acceleration occurs at every turn!

Two Beams of

\[ E_1, \vec{p}_1, E_2, \vec{p}_2, m_1 = m_2 = m \]

\[ E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2} \]

Beam 2 is a Target \( \rightarrow \)

\[ \vec{p}_2 = 0 \]

\[ E_{cm} = \sqrt{2m^2 + 2E_1m} \]

7 TeV proton beam against fix target \( \rightarrow \) 115 GeV
Colliders: higher energy

Anello di Accumulazione AdA
B. Touschek 1960
Colliders: higher energy

Two Beams of $E_1, \vec{p}_1, E_2, \vec{p}_2, m_1 = m_2 = m$

$$E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$$

Beam 2 is a counter rotating beam

$\vec{p}_1 = \vec{p}_2$

Anello di Accumulazione AdA
B. Touschek 1960

$$E_{cm} = E_1 + E_2$$

7 TeV proton beam colliding $\rightarrow$ 14 TeV
The Large Hadron Collider

- 27 Km length
- Protons or heavy ions
- Maximum 14 TeV center of mass energy
- 4 Interaction Regions for Experiments
Circular colliders: Luminosity

Collider Luminosity $L$ is the proportionality factor between the cross section $\sigma_{\text{event}}$ and the number of events per second $\frac{dR}{dt}$.

$$\frac{dR}{dt} = L \times \sigma_{\text{event}}$$

units: $cm^{-2} s^{-1}$

Luminosity is a machine parameter
- Independent of the physical reaction
- Reliable procedure to compute and measure
Luminosity calculation

The overlap integral of two bunches crossing each other head-on is proportional to the luminosity and it is given by:

\[ \mathcal{L} \propto K N_1 N_2 \int \int \int \int_{-\infty}^{+\infty} \rho_1(x, y, s, -s_0) \rho_2(x, y, s, +s_0) \, dx \, dy \, ds \, ds_0 \]

\[ s_0 = c \cdot t \]

\[ K = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}/c^2 \]

Time variable

Kinematic Factor
Luminosity formula

\[ \mathcal{L} \propto KN_1N_2 \int \int \int \int_{-\infty}^{+\infty} \rho_1(x, y, s, -s_0)\rho_2(x, y, s, +s_0) \, dx \, dy \, ds \, ds_0 \]

Uncorrelated densities in all planes
→ Factorize the distribution density as:
\[ \rho_1(x, y, s, -s_0) = \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s - s_0) \]

For head-on collisions where
→ "Kinematic Factor" \( K = 2 \)

To have the luminosity per second
→ Needs to multiply by revolution frequency \( f \)
In the presence of many bunches \( n_b \)

\[ \mathcal{L} = 2 \cdot N_1N_2 \cdot f \cdot n_b \cdot \int \int \int \int_{-\infty}^{+\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s - s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s + s_0) \, dx \, dy \, ds \, ds_0 \]
Closed solution for Gaussian distributions

Simplest case assumptions:

- Gaussian distributions
- No dispersion at the collision point
- Head-on collision

\[ \rho_{i,z}(z) = \frac{1}{\sigma_z \sqrt{2\pi}} \exp\left(\frac{z^2}{2\sigma_z^2}\right) \]

\[ \sigma_{x,y} = \sqrt{\epsilon \cdot \beta_{x,y}^*} \]

\[ K = 2 \]

\[
\mathcal{L} = \frac{2N_1N_2f_n b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int \int \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s_x^2}{\sigma_s^2}} e^{-\frac{s_y^2}{\sigma_s^2}} \, dx \, dy \, ds \, ds_0
\]

Equal Transverse beams “Round” beams

\[
\sigma_{1x} = \sigma_{2x} \\
\sigma_{1y} = \sigma_{2y}
\]

\[
\mathcal{L} = \frac{N_1N_2f_n b}{4\pi \sigma_x \sigma_y}
\]

Un-Equal Transverse beams “Flat” beams or optics

\[
\sigma_{1x} \neq \sigma_{2x} \\
\sigma_{1y} \neq \sigma_{2y}
\]

\[
\mathcal{L} = \frac{N_1N_2f_n b}{2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}
\]
The LHC design parameters

**LHC Design**
- \( N_1 = N_2 = 1.15 \times 10^{11} \) protons per bunch
- \( \sigma_x = \sigma_y = 16.6 \, \mu m \)
- \( \beta^* = 55 \, \text{cm} \)
- \( \Rightarrow L = 10^{34} \, \text{cm}^{-2} \, \text{s}^{-1} \)

**LHC Record**
- \( N_1 = N_2 = 1.15 \times 10^{11} \) protons per bunch
- \( \sigma_x = \sigma_y = 9.5 \, \mu m \)
- \( \beta^* = 30 \, \text{cm} \)
- \( \Rightarrow L = 2 \times 10^{34} \, \text{cm}^{-2} \, \text{s}^{-1} \)

**High Luminosity Upgrade of LHC**
- \( N_1 = N_2 = 2.2 \times 10^{11} \) protons per bunch
- \( \sigma_x = \sigma_y = 7.0 \, \mu m \)
- \( \beta^* = 64 \rightarrow 15 \, \text{cm} \)
- \( \Rightarrow L = (10-20) \times 10^{34} \, \text{cm}^{-2} \, \text{s}^{-1} \)
The LHC design parameters

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- $\rightarrow L = (10-20) \times 10^{34} \, cm^{-2}s^{-1}$
Different types of collisions

- They occur when two beams get closer and collide

- Two types
  - High energy collisions between two particles (wanted)
  - Distortions of beam by electromagnetic forces (unwanted)

- Unfortunately: usually both go together...
- 0.001% (or less) of particles collide
- 99.999% (or more) of particles are distorted
Proton Beams $\rightarrow$ Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

Force on itself (space charge) and opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted

**Focusing quadrupole**

**Opposite Beam**

A beam acts on particles like an electromagnetic lens, but...
Proton Beams $\rightarrow$ Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

Force on itself: **space charge**

Effects go with $1/\gamma^2$ factor for high energy colliders, this contribution is negligible (i.e., force scales LHC $1/\gamma^2 = 1.8 \times 10^{-8}$)

A beam acts on particles like an electromagnetic lens, but...
Proton Beams → Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

Electromagnetic force from opposing beam (*beam-beam effects*)

Single particle motion and whole bunch motion **distorted**

<table>
<thead>
<tr>
<th>Focusing quadrupole</th>
<th>Opposite Beam</th>
</tr>
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<tbody>
<tr>
<td><img src="image1.png" alt="Focusing quadrupole diagram" /></td>
<td><img src="image2.png" alt="Opposite Beam diagram" /></td>
</tr>
</tbody>
</table>

A beam acts on particles like an electromagnetic lens, but...
Beam-beam Force derivation

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

\[ \Delta U = -\frac{1}{\varepsilon_0} \rho(x, y, z) \]

Derive potential from Poisson equation for charges with distribution \( \rho \)

Solution of Poisson equation

\[ U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi\varepsilon_0} \int \int \int \frac{\rho(x_0, y_0, z_0) dx_0 dy_0 dz_0}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} \]

Then compute the fields

\[ \vec{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z) \]

From Lorentz force one calculates the force acting on test particle with charge \( q \)

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]

Making some assumptions we can simplify the problem and derive analytical formula for the force...
Beam-Beam Force for Round Gaussian distributions

Gaussian distribution for charges
Round beams:
Very relativistic, Force has only radial component:

\[
F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right]
\]

\[
\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) \, dt
\]

\[
\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right]
\]

\[
\sigma_x = \sigma_y = \sigma \\
\beta \approx 1 \\
r^2 = x^2 + y^2
\]

Beam-beam Force

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

Only radial component in relativistic case

How does this force looks like?
Beam-beam Force

\[ F_r(r) = \pm \frac{n\epsilon^2(1 + \beta^2_{rel})}{2\pi\epsilon_0} \frac{1}{r} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right] \]
Beam-beam Force

\[ F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]
Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

Strongest non-linearity in a collider YOU CANNOT AVOID!

Strong non-linear electromagnetic distortion $\rightarrow$ impact on beam quality (particle losses and emittance blow-up)

Physics fill lasts for many hours 10h – 24h
Crossing angle operation

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

Num. of maximum bunches \( n_b = 2808 \)

Multi Bunch operations brings unwanted interactions left and right of the 4 Experiments

A finite crossing angle has to be applied to avoid multiple collision points.
Crossing angle operation and beam-beam interactions

Two type of interactions:
Other beam passing in the center force
→ **HEAD-ON** beam-beam interaction
→ LHC has 4 corresponding to the 4 experiments ATLAS, CMS, Alice, LHCb

Other beam passing at an offset $r$
→ **LONG-RANGE** beam-beam interaction
→ LHC has up to 120 LR interactions

Beam-beam force

![Graph showing beam-beam force vs distance from beam center](image)
Multiple bunch Complications

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

Num. of bunches : \( n_b = 2808 \)

Due to the train structure of the beams \( \rightarrow \) different bunches will experience a different number of interactions!
Long-Range separations

\[ \beta(s) = \beta^* + \frac{s^2}{\beta^*} \]

Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments
Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

\[
\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot S
\]

\[
S = \frac{1}{\sqrt{1 + (\frac{\sigma_x}{\sigma_s} \tan \frac{\phi}{2})^2}} \cdot \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2})^2}}
\]

\[
\sigma_s \gg \sigma_x, \sigma_y
\]

Always valid for LHC and HL-LHC

\[
\sigma_x = 17-7 \ \mu m, \ \sigma_s = 7.5 \ \text{cm}
\]

\[
S \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x} \frac{\phi}{2})^2}}
\]

LHC design: \( \phi = 285 \ \mu rad, \ \sigma_x = 17 \ \mu m, \ \sigma_s = 7.5 \ \text{cm}, \ S=0.84 \)

LHC 2018: \( \phi = 320 \ \mu rad, \ \sigma_x = 9.3 \ \mu m, \ \sigma_s = 7.5 \ \text{cm}, \ S=0.61 \)
Luminosity Geometric reduction factor

Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot S$$

S is the geometric reduction factor

$$S = \frac{1}{\sqrt{1 + \left(\frac{\sigma_x}{\sigma_s} \tan \frac{\phi}{2}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2}\right)^2}}$$

$$\sigma_s \gg \sigma_x, y$$

Always valid for LHC and HL-LHC

$$\sigma_x = 17-7 \, \mu m, \sigma_s = 7.5 \, cm$$

LHC design: $$\phi = 285 \, \mu rad, \sigma_x = 17 \, \mu m, \sigma_s = 7.5 \, cm, S=0.84$$

LHC 2018: $$\phi = 320 \, \mu rad, \sigma_x = 9.3 \, \mu m, \sigma_s = 7.5 \, cm, S=0.61$$
LHC operates at finite crossing angle

HL-LHC will have bunches of $2.2 \times 10^{11}$ protons per bunch

$\phi = 590 \, \mu \text{rad}$, $\sigma_x = 9.3 \, \mu \text{m}$, $\sigma_s = 7.5 \, \text{cm}$, $S=0.26 \rightarrow 73\%$ of luminosity lost!

$$S \approx \frac{1}{\sqrt{1 + \left( \frac{\sigma_s \phi}{\sigma_x} \frac{1}{2} \right)^2}}$$

Crab Cavities used to tilt the bunches longitudinally and compensate for the crossing angle at the collision point!

Testing of crab cavities on-going in SPS!
Beam-Beam Force: single particle head-on collision

For small amplitudes: linear force
For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!

\[ F = -k \cdot r \]

\[ F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]
Beam-Beam transverse kick

Gaussian distribution for charges

\[ \sigma_x = \sigma_y = \sigma \]

Round beams:

Very relativistic, Force has only radial component:

\[ \beta \approx 1 \quad r^2 = x^2 + y^2 \]

\[ F_r(r, s, t) = \frac{Ne^2(1 + \beta^2)}{\sqrt{(2\pi)^3 \epsilon_0 \sigma_s}} \cdot \frac{1}{r} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right] \cdot \left[ \exp\left(-\frac{(s + vt)^2}{2\sigma_s^2}\right) \right] \]

Radial deflection on single particle at \( r \) from the center of opposite beams

\[ \Delta r' = \frac{1}{mc\beta \gamma} \int F_r(r, s, t) \, dt \]

\[ \Delta r' = -\frac{N_pr_0}{r} \cdot \frac{r}{r^2} \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

Only radial component in relativistic case
Can we quantify the beam-beam strength?

Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force

**Beam-beam force**

For small amplitudes: linear force

\[ F \propto -\xi \cdot r \]

The slope of the force gives you the beam-beam parameter \( \xi \)
Beam-Beam Parameter

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) \, dt$$

For small amplitudes: linear force

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \left(1 - \frac{r^2}{2\sigma^2} + \ldots\right)\right]$$

$$\Delta r'|_{r \to 0} = \frac{N r_0 r}{\gamma \sigma^2} = +f \cdot r$$

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2}$$
Beam-Beam parameter:

For round beams:

\[ \xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} \]

For non-round beams:

\[ \xi_{x,y} = \frac{N r_0 \beta^*_x, y}{2\pi \gamma \sigma_{x,y} (\sigma_x + \sigma_y)} \]

Examples:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LHC TDR</th>
<th>LHC 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity ( N_{p,e}/\text{bunch} )</td>
<td>1.15 (10^{11})</td>
<td>1.8 (10^{11})</td>
</tr>
<tr>
<td>Energy GeV</td>
<td>7000</td>
<td>4000</td>
</tr>
<tr>
<td>Beam size H</td>
<td>16.6 (\mu\text{m})</td>
<td>16.6 (\mu\text{m})</td>
</tr>
<tr>
<td>Beam size V</td>
<td>16.6 (\mu\text{m})</td>
<td>16.6 (\mu\text{m})</td>
</tr>
<tr>
<td>(\beta_{x,y}^*) (\text{m})</td>
<td>0.55-0.55</td>
<td>0.55-0.55</td>
</tr>
<tr>
<td>Crossing angle (\mu\text{rad})</td>
<td>290</td>
<td>285</td>
</tr>
<tr>
<td>(\xi_{\text{bbb}})</td>
<td>\textbf{0.0037}</td>
<td>\textbf{0.007}</td>
</tr>
</tbody>
</table>
Beam-Beam parameter:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi \gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

Examples:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LEP (e⁺e⁻)</th>
<th>LHC(pp)</th>
<th>LHC 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity N_{p,e}/bunch</td>
<td>4 \times 10^{11}</td>
<td>1.15 \times 10^{11}</td>
<td>1.7 \times 10^{11}</td>
</tr>
<tr>
<td>Energy GeV</td>
<td>100</td>
<td>7000</td>
<td>4000</td>
</tr>
<tr>
<td>Beam size H</td>
<td>160-200 μm</td>
<td>16.6 μm</td>
<td>18 μm</td>
</tr>
<tr>
<td>Beam size V</td>
<td>2-4 μm</td>
<td>16.6 μm</td>
<td>18 μm</td>
</tr>
<tr>
<td>(\beta_{x,y} ) m</td>
<td>1.25-0.05</td>
<td>0.55-0.55</td>
<td>0.6-0.6</td>
</tr>
<tr>
<td>Crossing angle μrad</td>
<td>0</td>
<td>285</td>
<td>290</td>
</tr>
<tr>
<td>(\xi_{bb} )</td>
<td>0.07</td>
<td>0.0037</td>
<td>0.009</td>
</tr>
</tbody>
</table>
Linear Tune shift due to head-on collision

For small amplitude particles beam-beam can be approximated as linear force as a quadrupole

\[ F \propto -\xi \cdot r \]

Focal length is given by the beam-beam parameter:

\[
\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}
\]

Beam-beam matrix:

\[
\begin{pmatrix}
1 & 0 \\
-\frac{\xi \cdot 4\pi}{\beta^*} & 1
\end{pmatrix}
\]

Equivalent to tune shift
Perturbed one turn matrix

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

\[ F \propto -\xi \cdot r \]

Focal length:

\[ \frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*} \]

Beam-beam matrix:

\[
\begin{pmatrix}
1 & 0 \\
-\frac{\xi \cdot 4\pi}{\beta^*} & 1 \\
\end{pmatrix}
\]

Perturbed one turn matrix with perturbed tune \( \Delta Q \) and beta function at the IP \( \beta^* \):

\[
\begin{pmatrix}
\cos(2\pi(Q + \Delta Q)) & \beta^* \sin(2\pi(Q + \Delta Q)) \\
-\frac{1}{\beta^*} \sin(2\pi(Q + \Delta Q)) & \cos(2\pi(Q + \Delta Q)) \\
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & 0 \\
-\frac{1}{2f} & 1 \\
\end{pmatrix} \cdot \begin{pmatrix}
\cos(2\pi Q) & \beta^*_0 \sin(2\pi Q) \\
-\frac{1}{\beta^*_0} \sin(2\pi Q) & \cos(2\pi Q) \\
\end{pmatrix} \cdot \begin{pmatrix}
1 & 0 \\
-\frac{1}{2f} & 1 \\
\end{pmatrix}
\]
Tune shift and dynamic beta

Solving the one turn matrix one can derive the tune shift $\Delta Q$ and the perturbed beta function at the IP $\beta^*$:

Tune is changed

$$cos(2\pi (Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi \xi}{\beta^*} sin(2\pi Q)$$

...how does the tune changes?
Tune shift due to beam-beam interactions

Tune shift as a function of tune

Larger $\xi$

Strongest variation with $Q$

Effects of multiple Interaction Points does not add linearly (phase advance between IP..)
Linear head-on Tune shift

Tune shift in 2 dimensional case equally charged beams and tunes far from integer and half

Zero amplitude particle will fill an extra defocusing term

\[ \xi_{bb} = 0.02 \]

\[ \Delta Q \approx \xi_{bb} \]
A beam is a collection of particles

Beam-beam force

Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude (detuning with amplitude or tune spread)
A beam will experience all the force range

Beam-beam force

Second beam passing in the center
HEAD-ON beam-beam interaction

Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude.

\[ \Delta Q \propto \frac{\delta F}{\delta r} \]

\[ \Delta Q_{quad} = const \]

\[ \Delta Q_{bb} \approx const \]

For small amplitude test particle linear tune shift

\[ \lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} = \xi \]
Beam with many particles this results in a tune spread

\[ \Delta Q \propto \frac{\delta F}{\delta r} \]

\[ \Delta Q_{quad} = \text{const} \]

\[ \Delta Q(x) = \frac{N r_0 \beta}{4 \pi \gamma \sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -\left(\frac{x}{2}\right)^2 I_0(\frac{x}{2})^2 - 1) \]

Head-on detuning with amplitude

1-D plot of detuning with amplitude for opposite and equally charged beams

\[ \Delta Q \propto \frac{\delta F}{\delta r} \]

Maximum tune shift for small amplitude particles
Zero tune shift for very large amplitude particles

And in the other plane? **THE SAME DERIVATION**
Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude

2-D mapping of the detuning with amplitude of particles

FOOTPRINT
Long Range detuning with amplitude

1-D plot of detuning with amplitude for opposite and equally charged beams

Maximum tune shift for large amplitude particles
Smaller tune shift detuning for zero amplitude particles and opposite sign
2-D Long Range detuning with amplitude

Long range tune shift scaling for distances

\[ d > 6\sigma \]

\[ \Delta Q_{lr} \propto -\frac{N}{d^2} \]
Beam-beam tune shift and tune spread

Head-on and Long range interactions detuning with amplitude

Footprints depend on:

- number of interactions (124 per turn)
- Type (Head-on and long-range)
- Separation
- Plane of interaction

Very complicated depending on collision scheme

Pushing luminosity increases this area while we need to keep it small to avoid resonances and preserve the stability of particles

Strongest non-linearity in a collider
Higher Luminosity $\rightarrow$ increases this area
We need to keep it small to avoid resonances and preserve the long term stability of particles

The footprint from beam-beam sits in the tune diagram
LHC Footprints and multiple experiments

...operationally it is even more complicated!
...different intensities, emittances...
Dynamical Aperture and Particle Losses

Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle

Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, $\beta^*$, intensity) is the result of careful study of different effects!
Dynamical Aperture and Particle Losses

Beam-beam linear dependency with Intensity

\[ F_{bb} \propto N_p \]

Our goal: keep dynamical aperture above 6 \( \sigma \) \( \rightarrow \) all particles up to 6 \( \sigma \) amplitude not lost over long tracking time (\( 10^6 \) turns in simulation) equivalent to 1 minute of collider

Example collider collision time: 24 hours
Round optics 15 cm, 590μrad: intensity scan

\[ \beta = 15\text{cm} \quad \text{Xangle} = 590\mu\text{m} \]

![Graph showing beam charge vs. minimum DA (σ) with green dotted line indicating 6D, HLLHCV1.0 and black dashed line showing 6D + crab, HLLHCV1.0].

![Graph showing Qx vs. Qy with color scale representing Diffusion Factor from -2.0 to -6.0].
Round optics 15 cm, 590μrad: intensity scan
AT high intensity the beam-beam force gets too strong and makes particles unstable and eventually are lost.
Round 15cm, 2.2E11, 690μrad

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2 \gamma}{\epsilon_n}} \]

Smaller beam-beam separation at parasitic long-range encounters stronger non-linearities \( \rightarrow \) smaller dynamical aperture
Round 15cm, 2.2E11, 650μrad
Round 15cm, 2.2E11, 590μrad
Round 15cm, 2.2E11, 540μrad
Round 15cm, 2.2E11, 490μrad
Round 15cm, 2.2E11, 440μrad
Round 15cm, 2.2E11, 390μrad

Crossing angle changes the separation and the strength of BB-LR that strongly affect the tails. 0σ particle are almost not affected.

At small separation particles gets unstable and eventually lost
How does it look like in the LHC?

Particle losses follow number of Long range interactions

Relative intensity decay 2012 experiment

$\alpha/2$

$N$ of LRs

$\begin{array}{c|c|c|c|c|c|c} N_{of\,LRs} & 16 & 18 & 20 & 22 & 24 & 26 \\ \hline Rel.\,Intensity & 145 & 117 & 96 & 87 & 79 & 72 \\ \hline 0.96 & 65 & 58 & 51 \\ \hline \end{array}$

$d_{sep} = 6 \sigma$

Small crossing angle = small separation

If separation of long range too small particles become unstable and are lost proportionally to the number of long range encounters

Beam-Beam separation at first LR

$d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}}$

Particle losses follow number of Long range interactions
Do we see the particle losses?

Particle losses follow number of Long range interactions
Machine protection implication and beam lifetimes gets worse...

Best performance of collider always a trade off between beam-beam and luminosity

Regular Physics Fill of 2012 RUN LHC

Beam-Beam separation at first LR

$$d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}}$$

Small crossing angle = small separation

Luminosity decays following the long range numbers... higher number of long range interactions larger losses
**Long-range Beam-Beam effects: orbit**

Long Range Beam-beam interactions lead to several effects...

**Long range angular kick** \[ \Delta x'(x + d, y, r) = -\frac{2N r_0 (x + d)}{\gamma r^2} [1 - \exp \left(-\frac{r^2}{2\sigma^2}\right) \]

For well separated beams \[ d \gg \sigma \]

The force has several components at first order we have an amplitude independent contribution: **ORBIT KICK**

\[ \Delta x' = \frac{\text{const}}{d} \left[ 1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \ldots \right] \]

In simple case (1 interaction) one can compute it analytically.
Orbit effect as a function of separation

Angular Deflections: \[ \theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma) \]

Closed Orbit effect: \[ Orb_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2\tan(\pi \cdot Q_{x,y})} \]
Orbit effect as a function of separation

Angular Deflections:
\[ \theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma) \]

Closed Orbit effect:
\[ \text{Orb}_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2\tan(\pi \cdot Q_{x,y})} \]

Orbit can be corrected but we should remember PACMAN effects.
LHC orbit effects

Many long range interactions could become important effect! Holes in bunch structure leads to PACMAN effects this cannot be corrected!

Self consistent evaluation

\[ L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}} \]

1-2% Luminosity loss due to beam-beam orbit effects
...not covered here...

- Beam-Beam compensation schemes
- Landau damping and beam-beam
- Beam-Beam coherent effects
- Asymmetric beams effects
- Noise on colliding beams
- Van der Meer scans
- Leveling luminosity
- ...
- ...
Thank you!

Questions?
References:


...much more on the LHC Beam-beam webpage: http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/
From the potential of charge beam to the Beam-beam Force

We need to calculate the field $E$ and $B$ of opposing beam

In rest frame only electrostatic field: $\vec{E} \neq 0$, $\vec{B} = 0$

We can derive the electrostatic field

In the lab frame the electric and magnetic fields can be obtained:

$E_\parallel = E'_\parallel$, $E_\perp = \gamma \cdot E'_\perp$ with: $\vec{B} = \vec{\beta} \times \vec{E}/c$

Lorentz force gives: $\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$

Ultra-relativistic case $F_r = qE_\perp (1 + \beta^2)$

Beam-Beam Effect is mainly a TRANSVERSE EFFECT
Beam-beam potential and force

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

We need to calculate the field E and B of opposing beam

In rest frame only electrostatic field: \( \vec{E} \neq 0, \vec{B} = 0 \)

\[
\Delta U = -\frac{1}{\epsilon_0} \rho(x, y, z)
\]

Scalar Potential can be derived from Poisson equation which relates the potential to the charge density \( \rho \)

\[
\vec{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z)
\]

Then compute the Electric Field from Gauss Law

Then back to the Lab frame we can compute the force

Lorentz force gives:

\[
\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})
\]
Beam-beam potential

In the case of Gaussian Beam density distribution we can factorize the density distribution

\[ \rho(x_0, y_0, z_0) = \rho(x_0) \cdot \rho(y_0) \cdot \rho(z_0) \]

\[ \rho(x_0, y_0, z_0) = \frac{Ne}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}} e^{-\left(\frac{x_0^2}{2\sigma_x^2} - \frac{y_0^2}{2\sigma_y^2} - \frac{z_0^2}{2\sigma_z^2}\right)} \]

N is the number of particles in bunch

The poison equation can be formally solved using the Green’s function \( G(x, y, z, x_0, y_0, z_0) \) method [25]

Solution of Poisson equation

\[ U(x, y, z) = \frac{1}{\varepsilon_0} \int G(x, y, z, x_0, y_0, z_0) \cdot \rho(x_0, y_0, z_0) dx_0 dy_0 dz_0 \]

The potential get’s the form:

\[ U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi\varepsilon_0} \frac{Ne}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}} \int \int \int \frac{e^{-\left(\frac{x_0^2}{2\sigma_x^2} - \frac{y_0^2}{2\sigma_y^2} - \frac{z_0^2}{2\sigma_z^2}\right)}}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} dx_0 dy_0 dz_0 \]

This is difficult to solve but following [29] we can solve the diffusion equation.
Crossing angle effect

\[ L = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot S \]

S is the geometric reduction factor
For small crossing angle

\[ S \approx \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \frac{\phi}{2}\right)^2}} \]

Examples: LHC (7 TeV): \( \phi = 285 \) \( \mu \)rad, \( \sigma_x = 17 \) \( \mu \)m, \( \sigma_s = 7.5 \) cm, \( S=0.84 \)
HL-LHC (7 TeV) \( \phi=590 \) \( \mu \)rad, \( \sigma_x = 7 \) \( \mu \)m, \( \sigma_s = 7.5 \) cm, \( S=0.3 \)

70% loss of luminosity if not compensated
From the diffusion equation:

\[ \Delta V - A^2 \cdot \frac{\delta V}{\delta t} = -\frac{1}{\epsilon_0} \rho(x, y, z) \quad \text{(for } t \geq 0) \]

We obtain the potential \( U \) by going to the limit of \( A \to 0 \)

\[ U = \lim_{A \to 0} V \]

Solving the diffusion equation instead of Poisson gives a Green’s function of the form:

\[ G(x, y, z, t, x_0, y_0, z_0) = \frac{A^3}{(2\sqrt{\pi t})^3} \cdot e^{-A^2/4t \cdot ((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)} \]

We can then compute the potential

\[ U(x, y, z, \sigma_x, \sigma_y, \sigma_z) \]

\[ \frac{Ne}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3 \epsilon_0} \int_0^t \int \int \int e^{-\left( -\frac{x_0^2}{2\sigma_x^2} - \frac{y_0^2}{2\sigma_y^2} - \frac{z_0^2}{2\sigma_z^2} \right)} A^3 \cdot e^{-A^2/4\tau ((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)} \frac{dx_0 dy_0 dz_0}{(2\sqrt{\pi \tau})^3} \]
S. Kheifets proposal

From Poisson Equation:

\[ U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi\epsilon_0} \frac{Ne}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3} \int \int \int e^{\left(-\frac{x_0^2}{2\sigma_x^2} - \frac{y_0^2}{2\sigma_y^2} - \frac{z_0^2}{2\sigma_z^2}\right)} \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} dx_0 dy_0 dz_0 \]

From Diffusion equation:

\[ U(x, y, z, \sigma_x, \sigma_y, \sigma_z) \]

\[ = \frac{Ne}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3 \epsilon_0} \int_0^t dt \int \int \int e^{\left(-\frac{x_0^2}{2\sigma_x^2} - \frac{y_0^2}{2\sigma_y^2} - \frac{z_0^2}{2\sigma_z^2}\right)} A^3 \cdot e^{-A^2/4\tau((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)} (2\sqrt{\pi \tau})^3 dx_0 dy_0 dz_0 \]

This allows to avoid the denominator in the integral and to collect the exponential which can be integrated
The potential of charge beam: 2D case

Changing the independent variable $\tau$ to $q = 4\tau/A^2$ and using the three integrations:

$$\int_{-\infty}^{\infty} e^{-(au^2+2bu+c)} du = \sqrt{\frac{\pi}{a}} e^{\left(\frac{b^2-ac}{a}\right)} \quad (for: u = x_0, y_0, z_0)$$

Our potential assumes the form of:

$$U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi\varepsilon_0} \frac{Ne}{\sqrt{\pi}} \int_0^{\infty} \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q} - \frac{z^2}{2\sigma_z^2+q}\right)}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)(2\sigma_z^2+q)}} dq$$

Since we are interested in the transverse fields, in a two-dimensional case

$$\rho(x,y) = \rho(x) \cdot \rho(y)$$

$$\rho_u(u) = \frac{1}{\sigma_u\sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \text{ where } u = x, y$$
The two dimensional potential is then given by:

\[
U(x, y, \sigma_x, \sigma_y) = \frac{ne}{4\pi\varepsilon_0} \int_0^\infty \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q}\right)}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}} dq
\]

- \(n\) is the line density of particles in the beam
- \(e\) is the elementary charge
- \(\varepsilon\) is the permittivity of free space

From the potential we can derive the field

\[
\vec{E} = -\nabla U(x, y, \sigma_x, \sigma_y)
\]
Radial Force

In cylindrical coordinates

\[ r^2 = x^2 + y^2 \]

\[ E_r = -\frac{ne}{4\pi\varepsilon_0} \cdot \frac{\delta}{\delta r} \int_0^\infty \frac{\exp\left(\frac{-r^2}{(2\sigma^2 + q)}\right)}{(2\sigma^2 + q)} dq \]

\[ B_\Phi = -\frac{ne\beta c\mu_0}{4\pi} \cdot \frac{\delta}{\delta r} \int_0^\infty \frac{\exp\left(\frac{-r^2}{(2\sigma^2 + q)}\right)}{(2\sigma^2 + q)} dq \]

From Lorentz Force

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]

Force has a radial component
Beam-Beam Force: round beams

For the case of $q=-e$ opposite charges

In cylindrical Coordinates

$$F_r(r) = \frac{n e^2 (1 + \beta^2)}{2 \pi \varepsilon_0} \cdot \frac{1}{r} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2 \sigma^2}\right) \right]$$

In Cartesian Coordinates:

$$F_x(r) = \frac{n e^2 (1 + \beta^2)}{2 \pi \varepsilon_0} \cdot \frac{x}{r^2} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2 \sigma^2}\right) \right]$$

$$F_y(r) = \frac{n e^2 (1 + \beta^2)}{2 \pi \varepsilon_0} \cdot \frac{y}{r^2} \cdot \left[ 1 - \exp\left(-\frac{r^2}{2 \sigma^2}\right) \right]$$

$$r^2 = x^2 + y^2$$
If we normalize the separations in units of the beam transverse rms size:

$$F_r(r) = \pm \frac{n e^2 (1 + \beta^2_{rel})}{2\pi \epsilon_0} \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$
Why do we care?

- Tune shift has opposite sign in plane of separation
- Break the symmetry between the planes, much more resonances are excited
- Mostly affect particles at large amplitude
- Cause effects on closed orbit, tune shift, chromaticity...
- PACMAN effects complicates the picture
Dynamic beta effect and beating

- The beam-beam collision at the experiment changes also the optics of the machine
- This leads to changes in the phase $\Delta \mu$ and to an “optical error” $\Delta \beta^*$
- Source of force at the position $s$, and the effect at position $s_0$ in perturbation theory is given by:

$$\Delta \beta(s_0) = -\frac{\beta(s_0)}{2 \sin(2\pi Q)} \int_{s_1}^{s_1+C} \beta(s) \Delta k(s) \cos [2(\mu(s) - \mu(s_0)) - 2\pi Q] \, ds$$

If our case if optics changes → beam-beam force changes → optics changes → beam-beam force changes ...

Self-consistent calculation is required to evaluate the effect
Dynamic Beta effect

In a simple case with one beam-beam interaction and seen as a perturbation.
And taking the effect at the source of the error \((s=s_0)\)

\[
\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi (Q + \Delta Q))} = \frac{1}{\sqrt{1 + 4\pi \xi \cot(2\pi Q) - 4\pi^2 \xi^2}}
\]

Beam-beam interaction leads to optical distortion at interaction point itself

Dynamic beta

Beam-beam interaction leads to optical distortion at all other interaction points

Dynamic beating

Expression above not valid during scan or several interaction points \(\rightarrow\) needs optics code for calculation
Dynamic Beta effect single Interaction point

\[
\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi (Q + \Delta Q))} = \frac{1}{\sqrt{1 + 4\pi \xi \cot(2\pi Q) - 4\pi^2 \xi^2}}
\]

Sensitive to:
- Beam-beam parameter: \( \xi \)
- Tune: \( Q \)
- Configuration (IPS) and optics (phase advance)

LHC case has 1-2 %
HL-LHC 3-6 %
...or more
Dynamic beta-beating due to beam-beam effects

Maximum beta change as a function of unperturbed tune

\[
\max \left( \frac{\Delta \beta}{\beta} \right) = \frac{2\pi \xi}{\sin(2\pi Q_0)}
\]

\[\xi_{bb} = 0.02\]

Maximum beating as a function of tune
Dynamic beta-beating due to beam-beam effects

From optics codes beating along the accelerator
How will cleaning efficiency and machine protection deal with such beating?