Linacs
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CERN BE/ABP

- Basics of Accelerator Science and Technology at CERN
outline

• Introduction and focus of the lecture
  o WHAT is a LINAC, WHEN was it invented and HOW does it work

• A selection of the CERN LINACs
  o Linac2,3,4 : hadron linacs injecting into a synchrotron
  o (Rex) – will not talk about this
  o (Clic/ctf3) - will not talk about this

• LINAC building blocks
  o Acceleration : Radio Frequency Cavities
  o Focusing : Quadrupoles
What is a linac

- LINear ACcelerator: single pass device that increases the energy of a charged particle by means of a (radio frequency) electric field and it is equipped with magnetic elements (quadrupoles, solenoids, bending magnets) to keep the charged particle on a given trajectory.

- Motion equation of a charged particle in an electromagnetic field

\[
\frac{d\vec{p}}{dt} = q \cdot (\vec{E} + \vec{v} \times \vec{B})
\]

- \( \vec{p} = \text{momentum} = \gamma m_0 \vec{v} \)
- \( q, m_0 = \text{charge, mass} \)
- \( \vec{E}, \vec{B} = \text{electric, magnetic field} \)
- \( t = \text{time} \)
- \( \vec{x} = \text{position vector} \)
- \( \vec{v} = \frac{d\vec{x}}{dt} = \text{velocity} \)
What is a linac-cont’ed

\[ \frac{d}{dt} \left( \gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \left( \vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right) \]

- **Type of particle**: charge couples with the field, mass slows the acceleration
- **Relativistic or not**
- **Type of focusing**
- **Type of RF structure**
Type of particles

- **electron**, mass 0.511 MeV, quickly relativistic, easier to accelerate
- **proton**, mass 938.28 MeV, q/m=1
- **lead ions**, mass 195.16 GeV, q/m=1/8.32 (in the linac)
- **Xenon ions**, mass 122 GeV, q/m=1/5.8

\[ \gamma = \sqrt{\frac{1}{1 - \beta^2}} \]
Velocity of the particles

Synchronicity with the accelerating electric field (accelerator is single purpose)

1) Geometry of the accelerator has to be adapted step-by-step
2) The phase of the radio frequency field has to be optimised independently

Space charge effects (more severe the lower the velocity)

1) The beam cannot be compressed in a small volume
2) The beam quality degradation is more severe
When ? A short history

• Acceleration by time varying electromagnetic field overcomes the limitation of static acceleration

• First experiment towards an RF linac: Wideroe linac 1928 on a proposal by Ising dated 1925. A bunch of potassium ions were accelerated to 50 keV in a system of drift tubes in an evacuated glass cylinder. The available generator provided 25 keV at 1 MHz.

• First realization of a linac: 1931 by Sloan and Lawrence at Berkeley laboratory

• From experiment to a practical accelerator: Wideroe to Alvarez
  • to proceed to higher energies it was necessary to increase by order of magnitude the frequency and to enclose the drift tubes in a RF cavity (resonator)

  • this concept was proposed and realized by Luis Alvarez at University of California in 1955: A 200 MHz 12 m long Drift Tube Linac accelerated protons from 4 to 32 MeV.

  • the realization of the first linac was made possible by the availability of high-frequency power generators developed for radar application during World War
Why not an electrostatic field?

750 kV Cockcroft-Walton

750 keV Radio Frequency accelerator (2m long, 0.5 m across)
How? principle of an RF linac

1) RF power source: generator of electromagnetic wave of a specified frequency. It feeds a

2) Cavity: space enclosed in a metallic boundary which resonates with the frequency of the wave and tailors the field pattern to the

3) Beam: flux of particles that we push through the cavity when the field is maximized as to increase its

4) Energy.
### LINAC2 - 50 MeV protons

**LINAC2 machine layout - 200MHz**

<table>
<thead>
<tr>
<th>Drift Tube Linac</th>
<th>Pre-injector</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 MeV (beta=0.3)</td>
<td>0.750 MeV (beta=0.04)</td>
</tr>
<tr>
<td>34 m</td>
<td>5 m</td>
</tr>
<tr>
<td>3 Tanks</td>
<td>Source</td>
</tr>
<tr>
<td>3 rf power source:5 MW</td>
<td>2 solenoids</td>
</tr>
<tr>
<td>130 quadrupoles</td>
<td>Radio Frequency Quadrupole</td>
</tr>
<tr>
<td>2 steerers</td>
<td>4 EM Quadrupoles</td>
</tr>
<tr>
<td></td>
<td>2 Cavities</td>
</tr>
</tbody>
</table>
LINAC3- heavy ions

LINAC3 machine layout- 100 and 200MHz

<table>
<thead>
<tr>
<th>IH –LINAC</th>
<th>Pre-injector</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2 MeV/u (beta=0.094)</td>
<td>0.0025 MeV/u (beta=0.0023)</td>
</tr>
<tr>
<td>7 m</td>
<td>10 m</td>
</tr>
<tr>
<td>3 Tanks</td>
<td>ECRIS Source with multiple charges</td>
</tr>
<tr>
<td>3 rf power source:5 MW</td>
<td>2 solenoids</td>
</tr>
<tr>
<td>12 quadrupoles</td>
<td>Radio Frequency Quadrupole</td>
</tr>
<tr>
<td>2 steerers</td>
<td>8 EMQuadrupoles</td>
</tr>
<tr>
<td></td>
<td>2 Cavities</td>
</tr>
<tr>
<td></td>
<td>2 bendings</td>
</tr>
</tbody>
</table>

Stripper: Pb^{29+} → Pb^{54+}
# LINAC4 – 160 MeV

<table>
<thead>
<tr>
<th>Π-mode</th>
<th>CC-DTL</th>
<th>Drift Tube L</th>
<th>Pre-injector</th>
</tr>
</thead>
<tbody>
<tr>
<td>160 MeV (beta=0.52)</td>
<td>100 MeV (beta=0.42)</td>
<td>50 MeV (beta=0.3)</td>
<td>3 MeV (beta=0.08)</td>
</tr>
<tr>
<td>23 m</td>
<td>25 m</td>
<td>19 m</td>
<td>9 m</td>
</tr>
<tr>
<td>12 Modules</td>
<td>7 Modules</td>
<td>3 Tanks</td>
<td>Source(s)</td>
</tr>
<tr>
<td>8 Klystrons: 12MW</td>
<td>7 Klystrons : 7 MW</td>
<td>3 Klystrons: 5 MW</td>
<td>2 solenoids</td>
</tr>
<tr>
<td>12 Quadrupoles</td>
<td>21 Quadrupoles</td>
<td>115 quadrupoles</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>12 steerer</td>
<td>7 steerers</td>
<td>2 steerers</td>
<td>Quadrupole</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11 EMQ</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3 Cavities</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 Chopper units</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In-line dump</td>
</tr>
</tbody>
</table>

- $2\times$ solenoids
- Radio Frequency Quadrupole
- 11 EMQ
- 3 Cavities
- 2 Chopper units
- In-line dump
What is a linac-cont’ed

\[ \frac{d}{dt} (\gamma \frac{d\vec{x}}{dt}) = \frac{q}{m_0} \left( \vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right) \]

q/m = 1/1 to q/m = 1/8

type of RF structure

type of focusing

From 1 to 1.17
## Types of RF structures

<table>
<thead>
<tr>
<th>Type of structure</th>
<th>Used at CERN in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio Frequency Quadrupole</td>
<td>LINAC2, LINAC3, LINAC4, LINAC5, REX-ISOLDE</td>
</tr>
<tr>
<td>Interdigital-H structure</td>
<td>LINAC3 , REX-ISOLDE</td>
</tr>
<tr>
<td>Drift Tube Linac</td>
<td>LINAC2, LINAC4, LINAC5</td>
</tr>
<tr>
<td>CellCoupled DTL</td>
<td>LINAC4</td>
</tr>
<tr>
<td>PIMS</td>
<td>LINAC4</td>
</tr>
</tbody>
</table>
wave equation -recap

- Maxwell equation for $E$ and $B$ field:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = 0$$

- In free space the electromagnetic fields are of the transverse electromagnetic, TEM, type: the electric and magnetic field vectors are $\perp$ to each other and to the direction of propagation.

- In a bounded medium (cavity) the solution of the equation must satisfy the boundary conditions:

$$\vec{E}_{\parallel} = \vec{0} \quad \vec{B}_{\perp} = \vec{0}$$

- In a cavity we have Tranverse Electric (TE modes) or Transverse magnetic (TM modes)
$\text{TE}_{nml}$

- **n**: azimuthal, 
- **m**: radial, 
- **l**: longitudinal component

**Empty cavity; mode $\text{TE}_{11}$**

- Dipole mode used in the IH structures

**Empty cavity; mode $\text{TE}_{21}$**

- Quadrupole mode used in Radio Frequency Quadrupole
TM*$_{nml}$

n: azimuthal,  
m: radial  
l longitudinal component

TM010 mode, most commonly used accelerating mode
Radio Frequency Quadrupoles

TE or TM?
transverse field in an RFQ

alternating gradient focussing structure with period length $\beta\lambda$
(in half RF period the particles have travelled a length $\beta\lambda/2$)

Beam goes into the paper in between the 4 vanetips
transverse field in an RFQ

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acceleration in RFQ

longitudinal modulation on the electrodes creates a longitudinal component in the TE mode.

$\alpha = \text{minimum distance from axis}$

$m_\alpha = \text{maximum distance from axis}$

$m = \text{modulation factor}$
Modulation and Rhol

Looking in from the RF port: these are adjacent

\[
\frac{\beta \lambda}{2} \left(1 - \frac{\Delta \varphi}{2\pi}\right)
\]

longitudinal radius of curvature

modulation X aperture

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important parameters of the RFQ

\[
B = \left( \frac{q}{m_0} \right) \left( \frac{V}{a} \right) \left( \frac{1}{f^2} \right) \frac{1}{a} \left( \frac{I_o(ka) + I_o(mka)}{m^2 I_o(ka) + I_o(mka)} \right)
\]

Accelerating efficiency: fraction of the field deviated in the longitudinal direction (=0 for un-modulated electrodes)

\[
E_0T = \frac{m^2 - 1}{m^2 I_o(ka) + I_o(mka)} \cdot V \frac{2}{\beta \cdot \lambda} \frac{\pi}{4}
\]

Transverse field distortion due to modulation (=1 for un-modulated electrodes)

Cell length

Transit time factor
\[
\left( \frac{I_0(ka) + I_0(mka)}{m^2 I_0(ka) + I_0(mka)} \right) + \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)} \cdot I_0(ka) = 1
\]

\[a=\text{bore radius}, \ \beta, \gamma=\text{relativistic parameters}, \ c=\text{speed of light}, \ f=\text{rf frequency},
\]
\[I_0, 1=\text{zero, first order Bessel function}, \ k=\text{wave number}, \ \lambda=\text{wavelength},
\]
\[m=\text{electrode modulation}, \ m_0=\text{rest q=charge}, \ r=\text{average transverse beam dimension}, \ r_0=\text{average bore}, \ V=\text{vane voltage}
\]
RFQ

- The resonating mode of the cavity is a focusing mode (TE mode)
- Alternating the voltage on the electrodes produces an alternating focusing channel
- A longitudinal modulation of the electrodes produces a field in the direction of propagation of the beam which bunches and accelerates the beam
- Both the focusing as well as the bunching and acceleration are performed by the RF field
- Not very efficient accelerator
- The RFQ is the only linear accelerator that can accept a low energy CONTINOUS beam of particles
Interdigital H structure
Interdigital H structure

the mode is the TE110
Interdigital H structure

- stem on alternating side of the drift tube force a longitudinal field between the drift tubes
- focalisation is provided by quadrupole triplets places OUTSIDE the drift tubes or OUTSIDE the tank
The resonating mode of the cavity is a dipole mode.
The cavity is equipped with thin drift tubes.
Alternating the stems on each side of the drift tubes produces a field in the direction of propagation of the beam which accelerates the beam.
Focusing is provided by quadrupole triplets located inside the tank in a dedicated section.

Very efficient in the low beta region ($\beta \approx 0.02$ to $0.08$) and low frequency (up to 200MHz).
not for high intensity beam due to long focusing period.
ideal for low beta ion acceleration.
Drift Tube Linac
DTL : electric field

Mode is TM010
Synchronous particle

• it’s the (possibly fictitious) particle that we use to calculate and determine the phase along the accelerator. It is the particle whose velocity is used to determine the synchronicity with the electric field. Design for that particle and provide longitudinal focusing so that the other stick with it!
Perfect synchronicity

- The length of the accelerating gap is either
  \[ L = \frac{\beta \lambda}{2} \] or \[ L = \beta \lambda \]

- Each cavity is adapted to the speed of the particle

- Best possible longitudinal beam dynamics

- Full control of the longitudinal phase space
Perfect synchronicity

- The absolute phase $\phi_i$ and the velocity $\beta_{i-1}$ of this particle being known at the entrance of cavity $i$, its RF phase $\phi_i$ is calculated to get the wanted synchronous phase $\phi_{si}$, $\phi_i = \phi_i - \phi_{si}$.
- The new velocity $\beta_i$ of the particle can be calculated from,
  1. if the phase difference between cavities $i$ and $i+1$ is given, the distance $D_i$ between them is adjusted to get the wanted synchronous phase $\phi_{si+1}$ in cavity $i+1$.
  2. if the distance $D_i$ between cavities $i$ and $i+1$ is set, the RF phase $\phi_i$ of cavity $i+1$ is calculated to get the wanted synchronous phase $\phi_{si+1}$ in it.

<table>
<thead>
<tr>
<th>RF phase</th>
<th>$\phi_{i-1}$</th>
<th>$\phi_i$</th>
<th>$\phi_{i+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle velocity</td>
<td></td>
<td>$\beta_{si-1}$</td>
<td>$\beta_{si}$</td>
</tr>
<tr>
<td>Distances</td>
<td></td>
<td>$D_{i-1}$</td>
<td>$D_i$</td>
</tr>
<tr>
<td>Synchronous phase</td>
<td>$\phi_{si-1}$</td>
<td>$\phi_{si}$</td>
<td>$\phi_{si+1}$</td>
</tr>
<tr>
<td>Cavity number</td>
<td>$i-1$</td>
<td>$i$</td>
<td>$i+1$</td>
</tr>
</tbody>
</table>

Synchronism condition:

$$\phi_{si+1} - \phi_{si} = \omega \cdot \frac{D_i}{\beta_{si} c} + \phi_{i+1} - \phi_i + 2\pi n$$
Synchronous structures
Beta vs W

\[ \text{beta} = \frac{\text{velocity}}{c} \]

kinetic energy (MeV)

beta_protons
Almost perfect synchronicity

- for simplifying construction and therefore keeping down the cost, cavities are not individually tailored to the evolution of the beam velocity but they are constructed in blocks of identical cavities (tanks). Several tanks are fed by the same RF source.

- This simplification implies a “phase slippage” i.e. a motion of the centre of the beam. The phase slippage is proportional to the number of cavities in a tank and it should be carefully controlled for successful acceleration.
phase slippage

\[ L_{\text{cavity}} = \beta_g \lambda / 2 \]

particle enters the cavity with \( \beta_s < \beta_g \). It is accelerated.

the particle has not left the cavity when the field has changed sign: it is also a bit decelerated.

the particle arrives at the second cavity with a “delay”

........and so on and so on

we have to optimize the initial phase for minimum phase slippage.

for a given velocity there is a maximum number of cavity we can accept in a tank.
Adapting the structure to the velocity of the particle

• Case 1: the geometry of the cavity/structure is continuously changing to adapt to the change of velocity of the “synchronous particle”

• Case 2: the geometry of the cavity/structure is adapted in step to the velocity of the particle. Loss of perfect synchronicity, phase slippage.

• Case 3: the particle velocity is beta=1 and there is no problem of adapting the structure to the speed.
Cell Coupled-DTL

Single Accelerating
CCDTL tank
1 Power coupler / klystron
Module
Side Coupled Linac

Chain of cells, coupled via slots and off-axis coupling cells. Invented at Los Alamos in the 60's. Operates in the $\pi/2$ mode (stability).

CERN SCL design:
Each klystron feeds 5 tanks of 11 accelerating cells each, connected by 3-cell bridge couplers. Quadrupoles are placed between tanks.
PI-mode structure
Beam centre phase in LINAC4

Synchronous phase (deg) vs gap number

DTL  CCDTL  PIMS

Technology at CERN
How to choose
cavity geometry and related parameters definition

1-Maximum field/average field
2-Shunt impedance
3-Quality factor
4-Filling time
5-Transit time factor
6-Effective shunt impedance
Average electric field (E₀ measured in V/m) is the space average of the electric field along the direction of propagation of the beam in a given moment in time when F(t) is maximum.

\[ E(x, y, z, t) = E(x, y, z) \cdot e^{-j\omega t} \]

\[ E_0 = \frac{1}{L} \int_{0}^{L} E_z(x = 0, y = 0, z) \, dz \]

- physically it gives a measure how much field is available for acceleration
- it depends on the cavity shape, on the resonating mode and on the frequency
Kilpatrick sparking criterion

\[ f = 1.64 E^2 \exp(-8.5/E) \]

GUIDELINE nowadays: peak surface field up to 2* kilpatrick field

Quality factor for normal conducting cavity is \( E_{\text{peak}}/E_o T \)
The higher the frequency

<table>
<thead>
<tr>
<th>Year</th>
<th>Linac Type</th>
<th>Frequency</th>
<th>Energy/m</th>
<th>Weight/m</th>
<th>External Diameter</th>
<th>Beam Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>LINAC2 RFQ2</td>
<td>200 MHz</td>
<td>0.5 MeV/m</td>
<td>900 kg/m</td>
<td>~45 cm</td>
<td>200 mA</td>
</tr>
<tr>
<td>2007</td>
<td>LINAC4 RFQ</td>
<td>352 MHz</td>
<td>1 MeV/m</td>
<td>400 kg/m</td>
<td>29 cm</td>
<td>80 A</td>
</tr>
<tr>
<td>2014</td>
<td>HF RFQ</td>
<td>750 MHz</td>
<td>2.5 MeV/m</td>
<td>100 kg/m</td>
<td>13 cm</td>
<td>0.1 mA</td>
</tr>
</tbody>
</table>
Transit time factor

- **transit time factor** \( T \) (dimensionless) is defined as the maximum energy gain per charge of a particle traversing a cavity over the average voltage of the cavity.

- Write the field as

  \[
  E_z(x, y, z, t) = E_z(x, y, z)e^{-i\omega t}
  \]

- The energy gain of a particle entering the cavity on axis at phase \( \phi \) is

  \[
  \Delta W = \int_0^L qE_z(o, o, z)e^{-i(\omega t + \phi)}
  \]
cavity parameters-5

• assume constant velocity through the cavity (APPROXIMATION!!) we can relate position and time via

\[ z = v \cdot t = \beta ct \]

• we can write the energy gain as

\[ \Delta W = qE_0 LT \cos(\phi) \]

• and define transit time factor as

\[ T = \left| \frac{\int_0^L E_z(z) e^{-j\left(\frac{\omega z}{\beta c}\right)} dz}{\int_0^L E_z(z) dz} \right| \]

T depends on the particle velocity and on the gap length. IT DOESN”T depend on the field.
Transit time factor

$t_{\text{ff}}$ for 100 keV protons, 200 MHz., parabolic distribution

if we don’t get the length right we can end up decelerating!!!
Effective shunt impedance (Z measured in Ω/m) is defined as the ratio of the average effective electric field squared (E₀T) to the power (P) per unit length (L) dissipated on the wall surface.

- it is independent of the field level and cavity length, it depends on the cavity mode and geometry and on the velocity of the particle to be accelerated.

\[ Z_{TT} = \left( \frac{E_0 T}{E_0 T} \right)^2 \cdot \frac{L}{P} \]

measure if the structure is optimized and adapted to the velocity of the particle to be accelerated.

Measure of how much energy a charged particle can gain for 1 W of power when travelling over 1 m of structure.
### overview

<table>
<thead>
<tr>
<th></th>
<th>Ideal range of beta</th>
<th>frequency</th>
<th>Effective gradient</th>
<th>Protons / Ions</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFQ</td>
<td>Low!!! - 0.05</td>
<td>40-400 MHz</td>
<td>1 MV/m (350MHz)</td>
<td>Ions / protons</td>
</tr>
<tr>
<td>IH</td>
<td>0.02 to 0.08</td>
<td>40-200 MHz</td>
<td>4.5 MV/m (200MHz)</td>
<td>Ions and also protons</td>
</tr>
<tr>
<td>DTL</td>
<td>0.04-0.5</td>
<td>100-400 MHz</td>
<td>3.5 MV/m (350MHz)</td>
<td>Ions / protons</td>
</tr>
<tr>
<td>SCL</td>
<td>Ideal Beta=1</td>
<td>800 - 3000 MHz</td>
<td>20 MV/m (3000MHz)</td>
<td>protons</td>
</tr>
<tr>
<td></td>
<td>But as low as beta 0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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What is a linac-cont’ed

\[ \frac{d}{dt} \left( \gamma \frac{d\vec{x}}{dt} \right) = \frac{q}{m_0} \cdot \left( \vec{E} + \frac{d\vec{x}}{dt} \times \vec{B} \right) \]

Relativistic or not

Type of particle:
charge couples with the field, mass slows the acceleration

Type of focusing

Type of RF structure
Magnetic quadrupoles

ElectroMQ

Permanent MQ
Focusing force

$B = $magnetic field / $F = $force

Positively charged particles going into the screen
Magnetic quadrupole

Magnetic field
\[
\begin{align*}
B_x &= G \cdot y \\
B_y &= G \cdot x
\end{align*}
\]

Magnetic force
\[
\begin{align*}
F_x &= -q \cdot v \cdot G \cdot x \\
F_y &= q \cdot v \cdot G \cdot y
\end{align*}
\]

Focusing in one plan, defocusing in the other

sequence of focusing and defocusing quadrupoles
The beam is matched, after every period, the twiss parameters are identical.
Building blocks-recap
# Choices / Questions

| Frequency? | Frequency and size  
Frequency and acceptance  
Frequency and maximum accelerating field  
Frequency and duty cycle |
|---|---|
| RFQ output energy? | Into which structure do we inject  
How long is the RFQ (compared to wavelength) |
| Continue with TE mode or switch to TM mode? | Start thinking about transverse focusing  
Think about the final energy  
Think about the energy at the transition (NB threshold for copper activation is around few MeV) |
| At what energy we start standardising the RF structures | Quasi-synchronous condition |
| PMQ or EMQ? | Cheap and easy or maximum flexibility |